University of Victoria

CaNPAN

Mixing by internal gravity waves and its possible roles in the origin of the Li-rich red-clump stars and formation of the ^{13}C pocket in AGB stars

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In collaboration with

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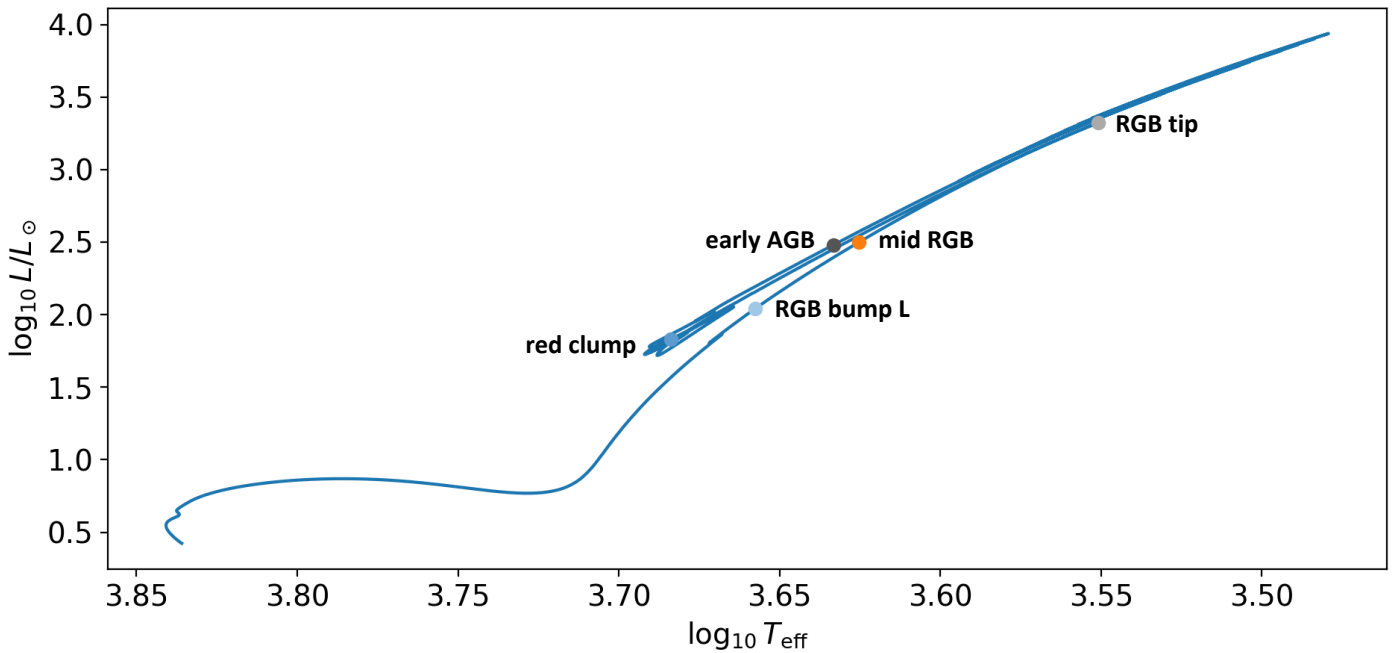
NUGRID

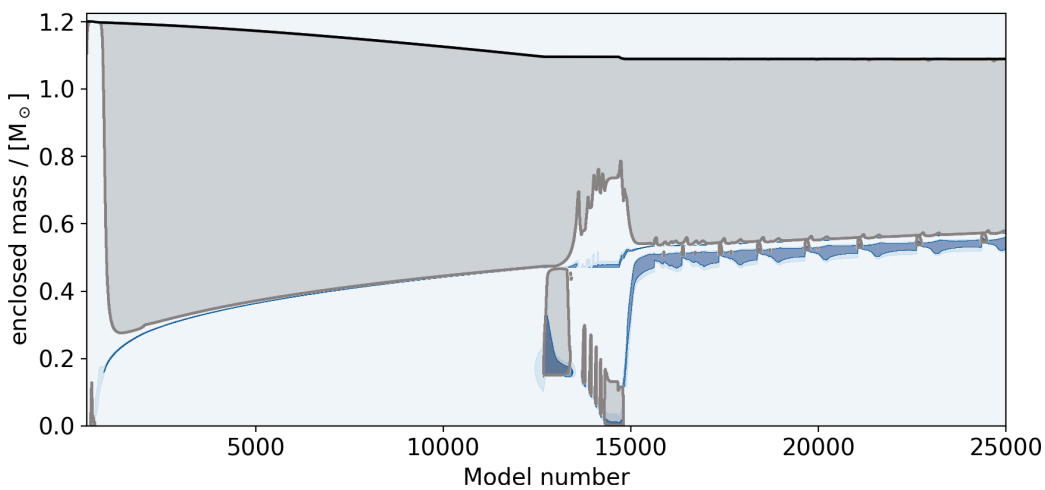
MESA

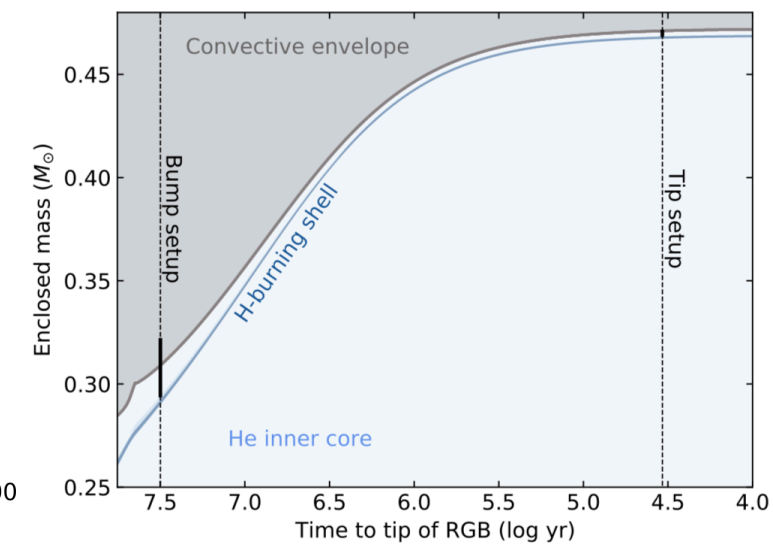
PPMstar

JINA–CEE

The evolution of a star with [Fe/H]=-0.3 and M=1.2$M_\odot$ (MESA rev. 7624)



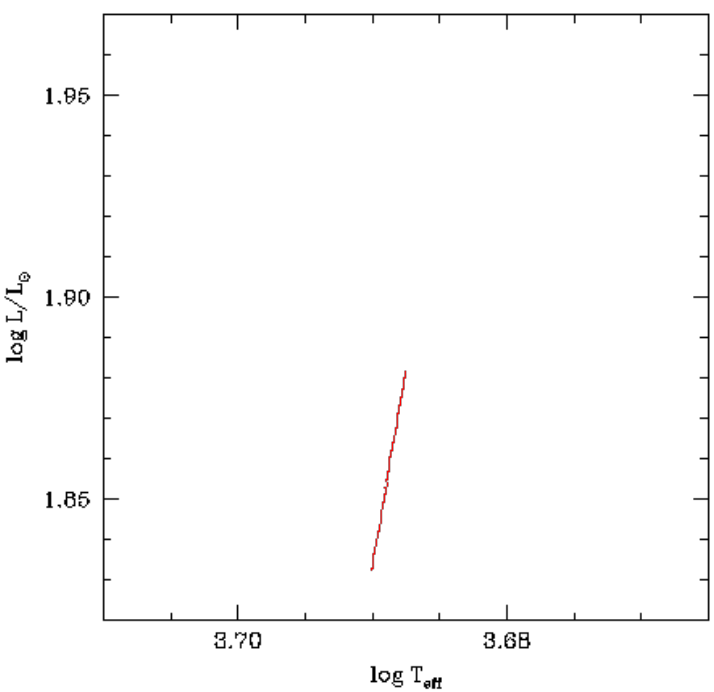


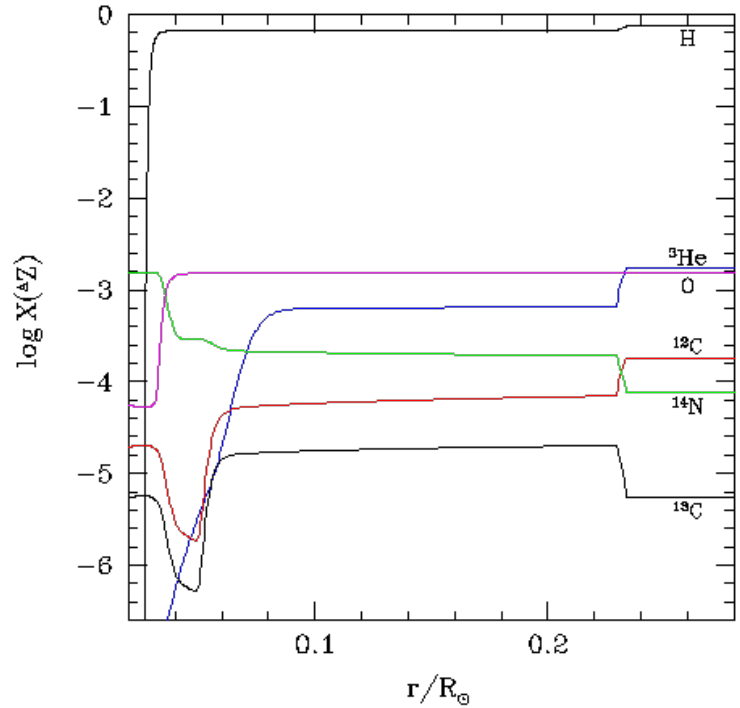


What happens at the bump luminosity?

Left: Evolution of a low-mass star in the HRD near the bump luminosity ($M = 0.83 M_{\odot}$, $Z = 0.0015$)

Right: H-burning shell erases abundance discontinuities left behind by the CE at the FDU

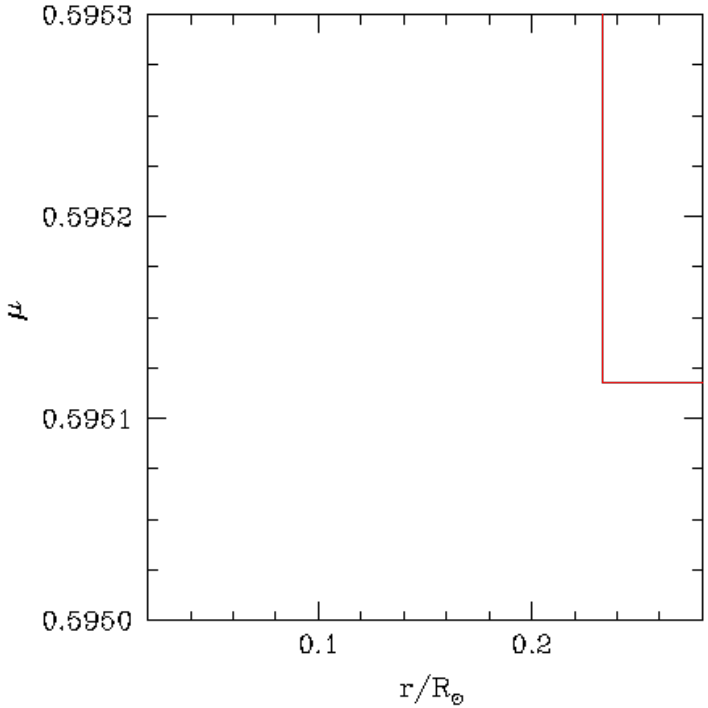


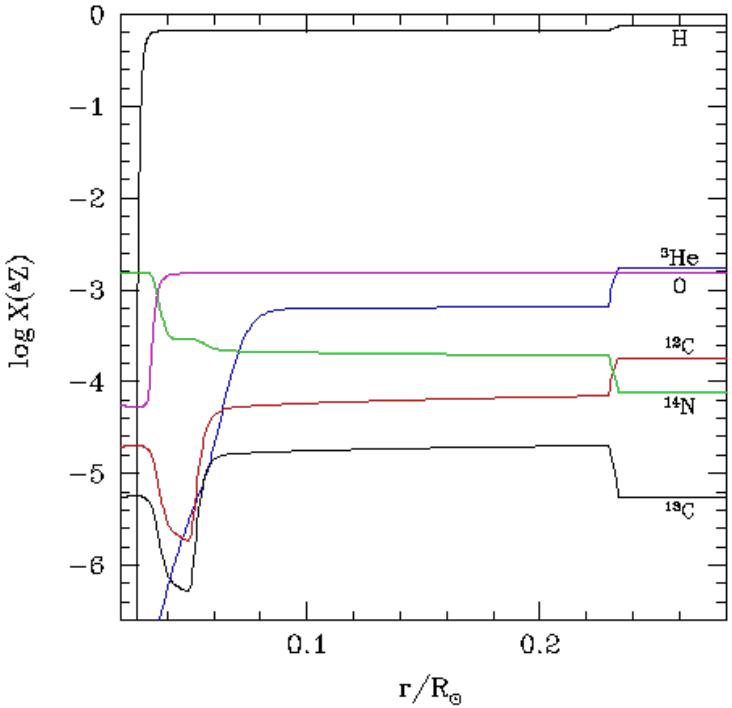


On the upper RGB, above the bump luminosity, the radiative zone between the H-burning shell and the base of the convective envelope has uniform chemical composition, which facilitates fluid buoyancy of any origin

Thermohaline mixing in upper RGB stars

The upper RGB thermohaline mixing is driven by a small difference in μ between a fluid element and its surrounding medium because the atomic diffusivity is much slower than the radiative heat diffusivity K. It develops when μ decreases with depth (usually, μ increases with depth in stars)

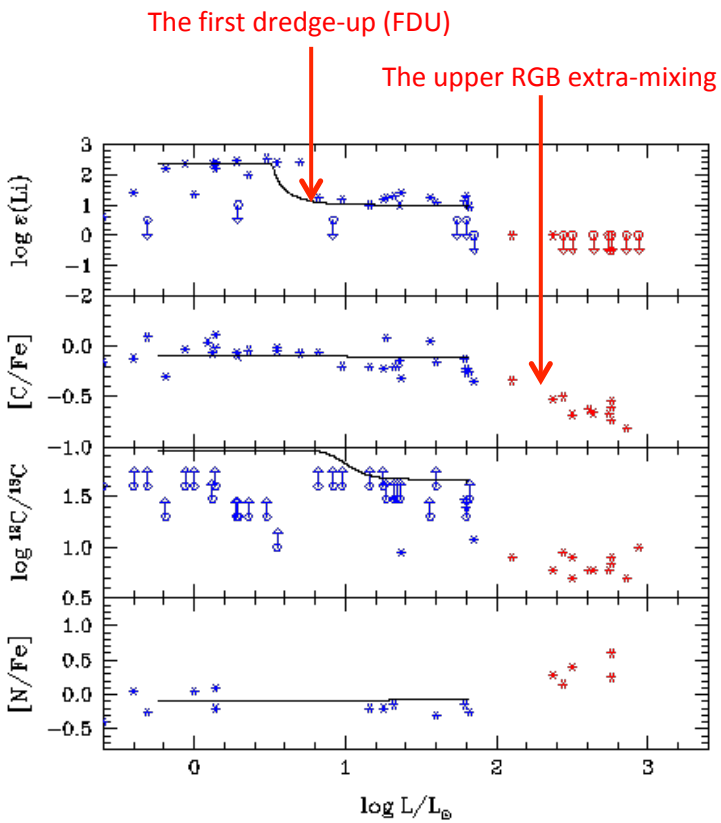


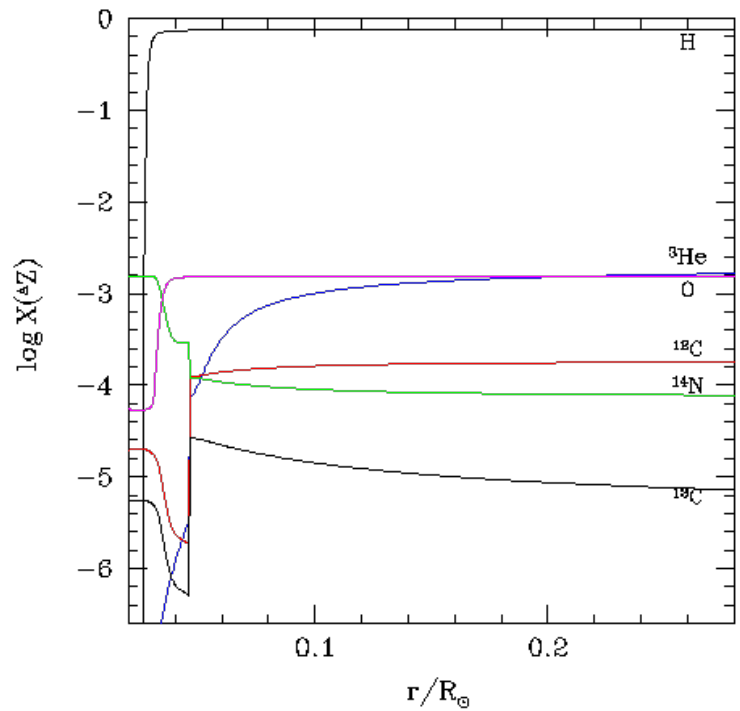


Reaction $^3He(^3He,2p)^4He$ decreases μ locally by $|\Delta\mu| < 0.0001$ (Eggleton et al. 2006)

A parametric model of extra mixing in a low-mass upper RGB star

Right: Diffusion coefficient D_{mix} = (0.02 - 0.1) × K, or D_{mix} = 4 × 10^8 cm^2 s^{-1}, with a mixing depth r_{mix} = 0.04 – 0.06 $R_\odot$ reproduce the observed abundance patterns on the upper RGB (Left)





Data from Gratton R.G. et al., 2000, A&A, 354, 169

RADIATIVE ^{13}C BURNING IN ASYMPTOTIC GIANT BRANCH STARS AND *s*-PROCESSING

O. STRANIERO,[1] R. GALLINO,[2] M. BUSSO,[3] A. CHIEFFI,[4] C. M. RAITERI,[3] M. LIMONGI,[5] AND M. SALARIS[1]

(ApJ, 440, L85, 1995)

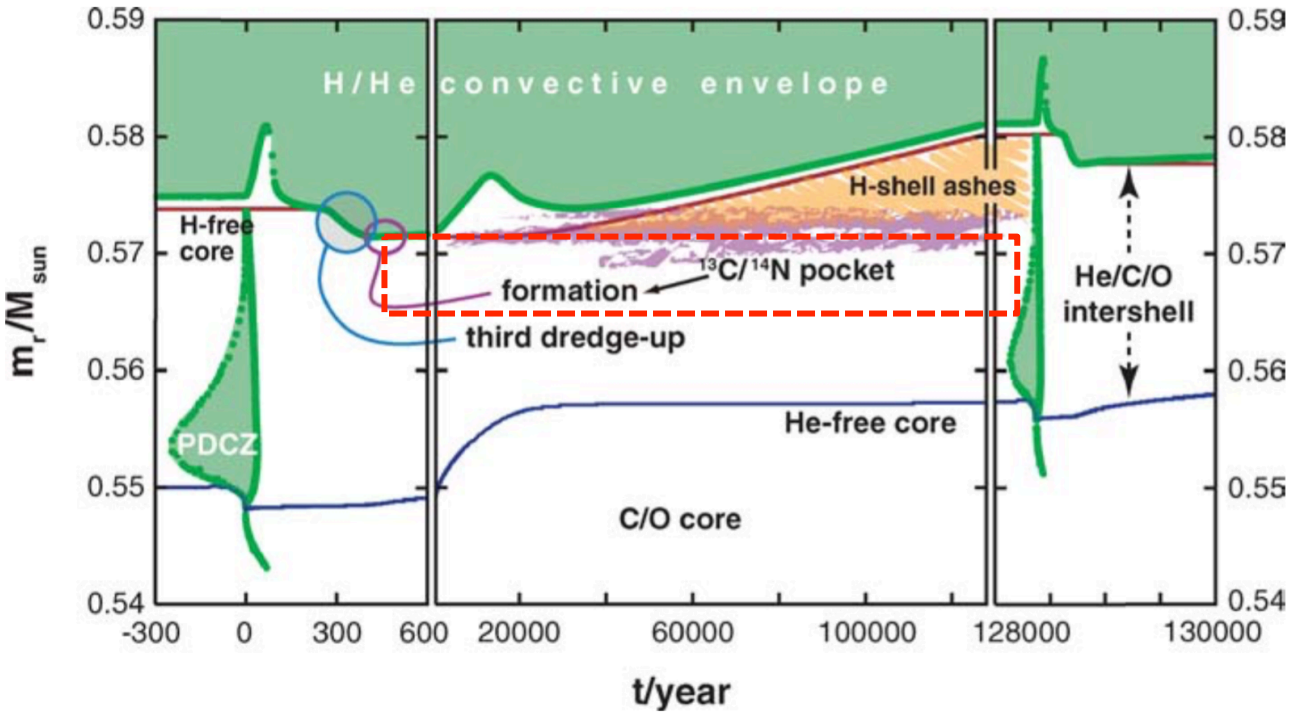


Figure 3 Thermal pulse 14, the subsequent interpulse phase and thermal pulse 15 of 2 $M_\odot$, $Z = 0.01$ sequence ET2 of Herwig & Austin (2004). The timescale is different in each panel. The *red solid line* indicates the mass coordinate of the H-free core. The *dotted green line* shows the boundaries of convection; each *dot* corresponds to one model in time. Convection zones are *light green*. The shown section of the evolution comprises 12,000 time steps. The colors indicating convection zones, layers with H-shell ashes and the region of the ^{13}C pocket match those in Figure 5. (From F. Herwig, ARA&A, 43, 435, 2005)

Mechanisms for upper RGB extra mixing

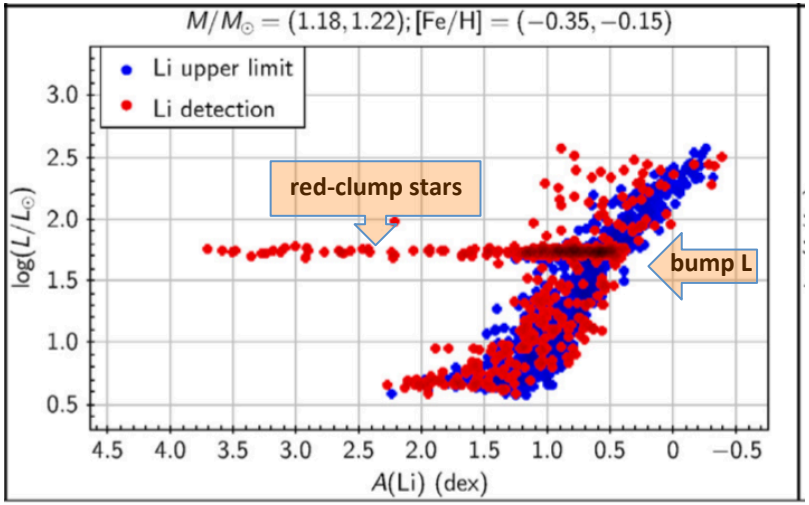
- Rotationally-induced mixing (Sweigart A.V. and Mengel J.G., 1979, ApJ, 229, 624).
- Thermohaline mixing (Charbonnel C. and Zahn J.-P., 2007, A&A, 467, L15).
- Magnetic mixing (Busso M. et al., 2007, ApJ, 671, 802).
- Internal gravity wave (IGW) shear-induced mixing? (this work)

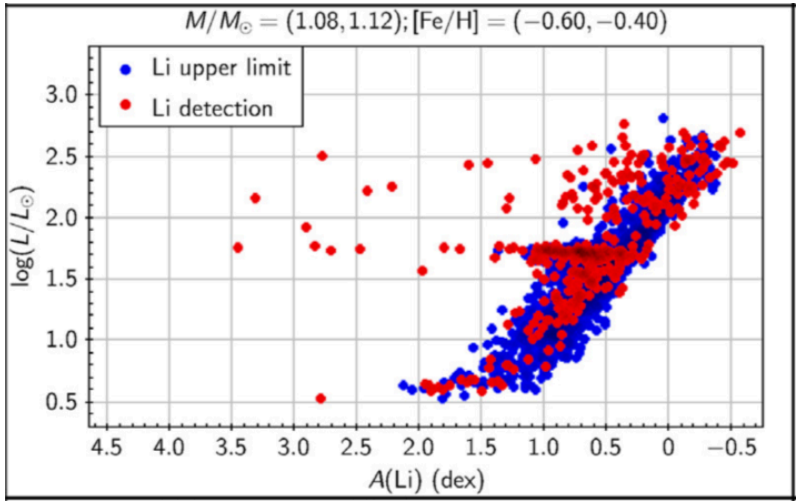
Proton ingestion mechanisms to form ^{13}C pocket in AGB stars

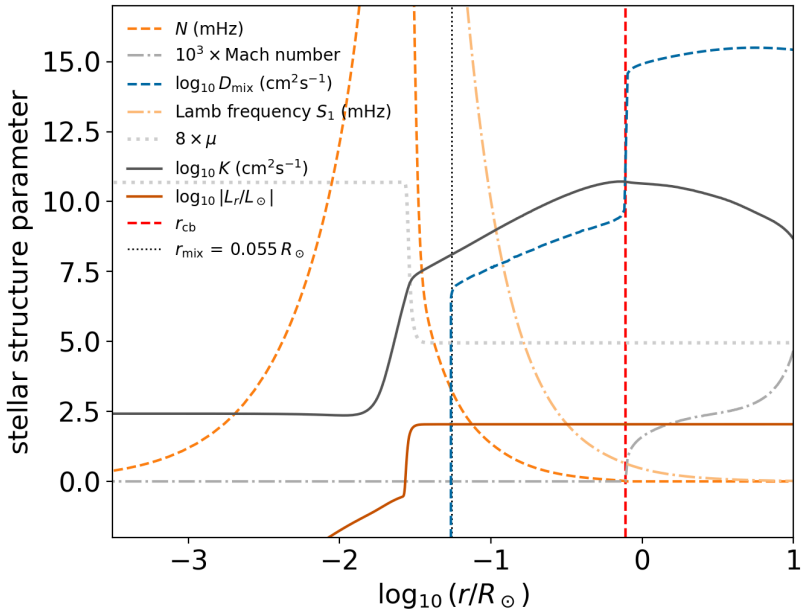
- Rotationally-induced mixing (Langer N. et al., 1999, A&A, 346, L37).
- Convective overshooting (Herwig F., 2000, A&A, 360, 952).
- Magnetic mixing (Busso M. et al., 2007, ApJ, 671, 802; Busso M. et al., 2021, ApJ, 908, 55).
- Internal gravity wave (IGW) shear-induced mixing (Denissenkov P. and Tout C., 2003, MNRAS, 340, 722).

The problems of extra mixing in low-mass upper RGB stars and Li enrichment in red-clump stars

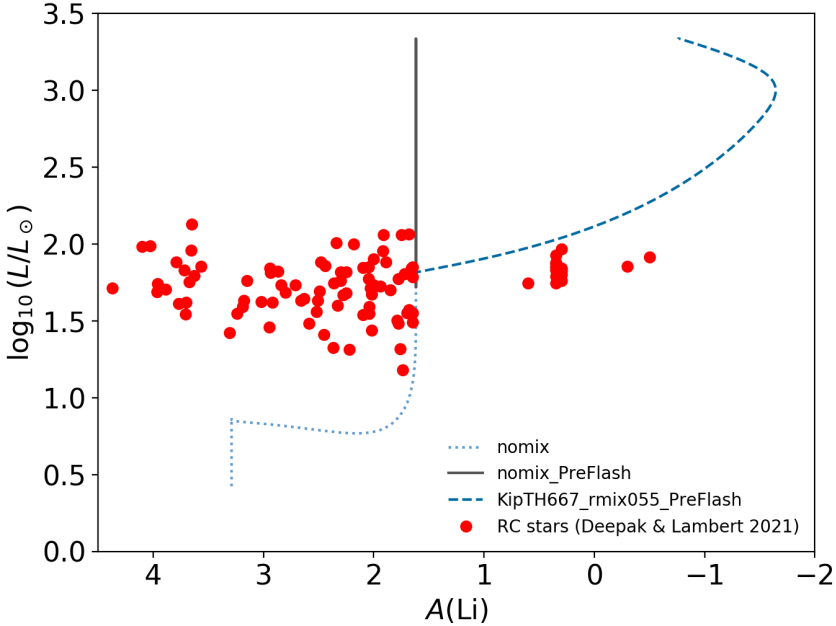
Li-rich red-clump stars (data from Deepak & Lambert D.L., 2021, MNRAS, 507, 205)





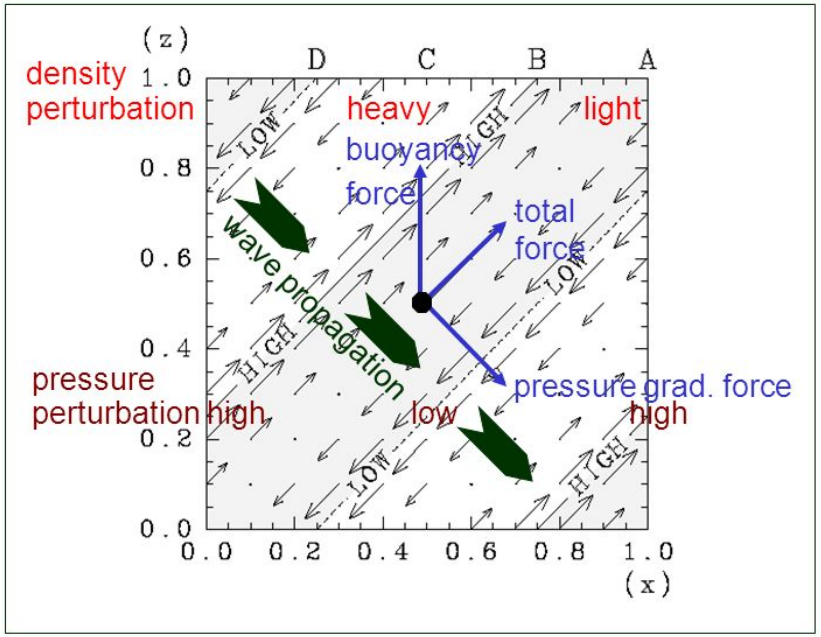


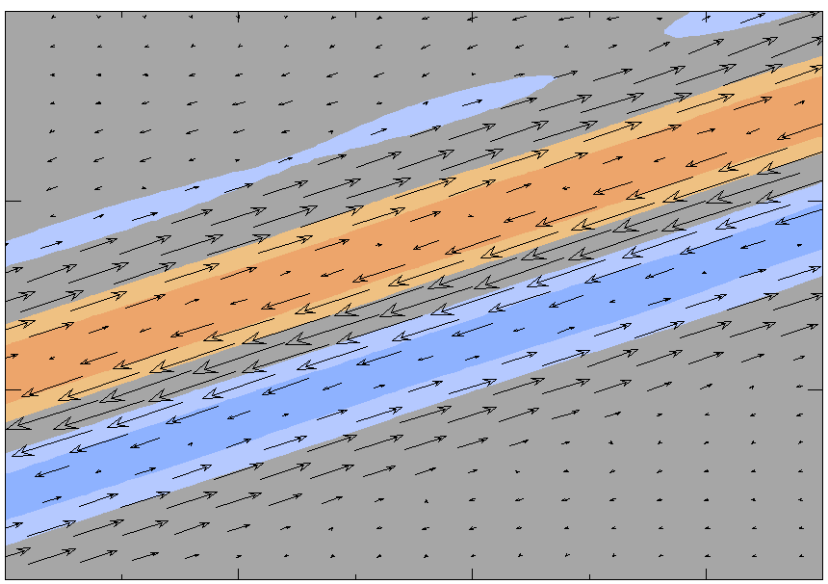
RGB bump luminosity model with [Fe/H]=-0.3, M=1.2$M_\odot$ and D_{mix} = 100×D_{th}



No-mixing and 100× thermohaine mixing on upper RGB

❖ propagation of internal gravity waves





Shear-induced mixing by IGWs

For $\omega \ll N$, IGW fluid motion is nearly horizontal with $u_\mathrm{v} \ll u_\mathrm{h}$, and the vertical velocity shear is

$$\frac{du_\mathrm{h}}{dr} \approx u_\mathrm{h} k_\mathrm{v} \approx u_\mathrm{h} k_\mathrm{h} \left(\frac{N}{\omega}\right).$$

The diffusion coefficient for shear-induced mixing by small-scale turbulence is

$$D_\mathrm{shear} \approx \eta \frac{\left(\frac{du_\mathrm{h}}{dr}\right)^2}{N^2} K,$$

where $K = \frac{4acT^3}{3\kappa\rho^2 C_P}$ is the thermal diffusivity, and $\eta \sim 0.1$ (as proposed by Zahn J.-P., A&A, 265, 115, 1992, and validated in 3D hydro sims by Prat V., Guilet J., Viallet M, Mueller E., A&A, 592, 59 2016).

For shear mixing by IGWs, the diffusion coefficient is then

$$D_\mathrm{IGW} \approx \eta \frac{(u_\mathrm{h} k_\mathrm{v})^2}{N^2} K.$$

The horizontal components of the vorticity of the IGW fluid motion can be estimated as

$$(\nabla \times \mathbf{u})_y = \frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} \approx u_\mathrm{h} k_\mathrm{v} - u_\mathrm{v} k_\mathrm{h} \approx u_\mathrm{h} k_\mathrm{h} \frac{N}{\omega},$$

or $(\nabla \times \mathbf{u})_y^2 \approx (u_\mathrm{h} k_\mathrm{h})^2 \left(\frac{N}{\omega}\right)^2 \approx (u_\mathrm{h} k_\mathrm{v})^2$. A similar estimate can be obtained for the x component of the vorticity.

Therefore,

$$(u_\mathrm{h} k_\mathrm{v})^2 \approx (\nabla \times \mathbf{u})^2, \text{ and } D_\mathrm{IGW} \approx \eta \frac{(\nabla \times \mathbf{u})^2}{N^2} K.$$

So, to estimate the diffusion coefficient for shear mixing by IGWs we need to know their vorticity $|(\nabla \times \mathbf{u})|$ and spectrum $u^2(\omega, k_\mathrm{h})$.

Li production in a helium-flash-induced mixing event at the RGB tip

The kinetic energy flux of IGWs is predicted to be proprtional to the convective flux $F_{\rm IGW} = \mathcal{M} F_{\rm conv}$, where $\mathcal{M} = v_{\rm conv}/c_s$ is the Mach number of turbulent convection (e.g., Rogers T.M. et al., ApJ, 772, 21, 2013). Given that also, e.g., Eq. (39) in Press W.H., ApJ, 245, 286, 1981,

$$F_{\rm IGW} = \rho u_{\rm v}^2 \frac{N}{k_{\rm h}} \sqrt{1 - \frac{\omega^2}{N^2}} = \rho u_{\rm h}^2 \frac{\omega^2}{N k_{\rm h}} (1 - \frac{\omega^2}{N^2})^{-1/2},$$

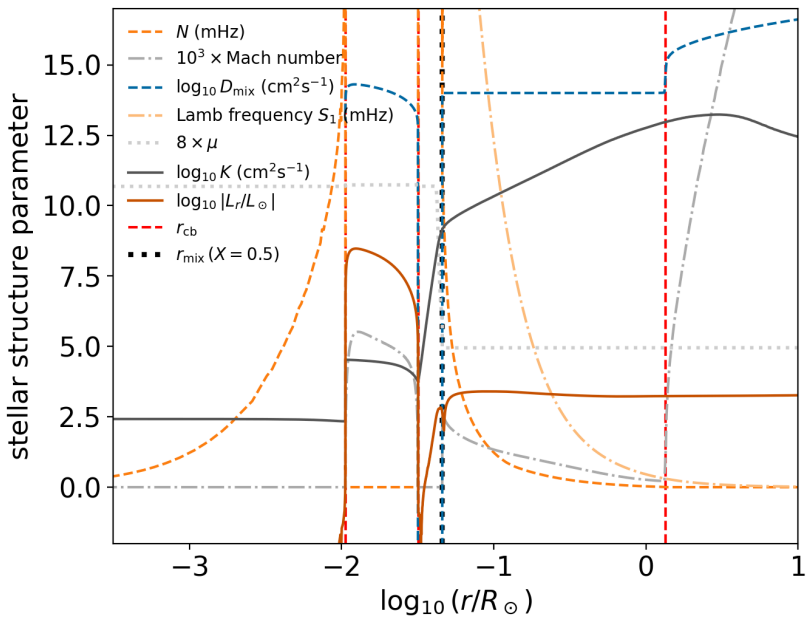
and $v_{\rm conv} \propto L^{1/3} \propto F_{\rm conv}^{1/3}$, we find that $u_{\rm v} \propto L^{2/3}$ and $u_{\rm h} \propto L^{2/3}$. For an IGW velocity field, the Richardson number is

$$\mathcal{R}i \approx \frac{N^2}{(\nabla \times \mathbf{u})^2}.$$

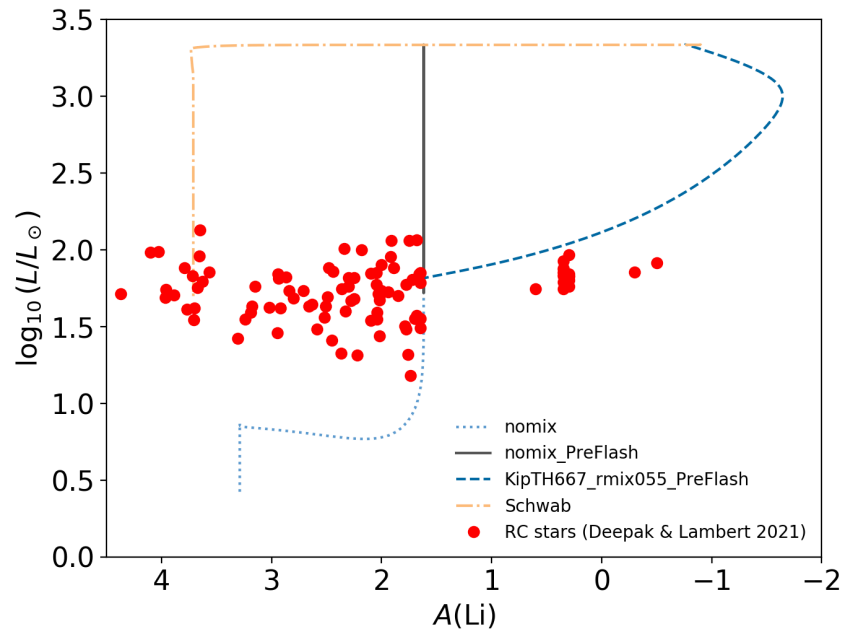
For the KH instability, $\mathcal{R}i \leq \mathcal{R}i_{\rm crit} = \frac{1}{4}$. However, even in a case of $\mathcal{R}i > \mathcal{R}i_{\rm crit}$ mixing is still possible (Zahn J.-P., A&A, 265, 115, 1992) with

$$D_{\rm IGW} \approx \eta \frac{K}{\mathcal{R}i} = \eta \frac{(\nabla \times \mathbf{u})^2}{N^2} K \propto L^{4/3} K,$$

where $\eta \approx 0.1$ (e.g., Prat V. et al., A&A, 592, 59, 2016).



RGB tip luminosity model with [Fe/H]=-0.3, M=1.2$M_\odot$ and $D_{\rm mix}$ = 10^{14} cm^2/s >> K (Schwab J., 2020, ApJ, 901, L18)



IGW mixing triggered by He-shell convection at the RGB tip.

Li production on upper RGB by extra mixing with rate increasing proportionally to $L^{4/3}K$

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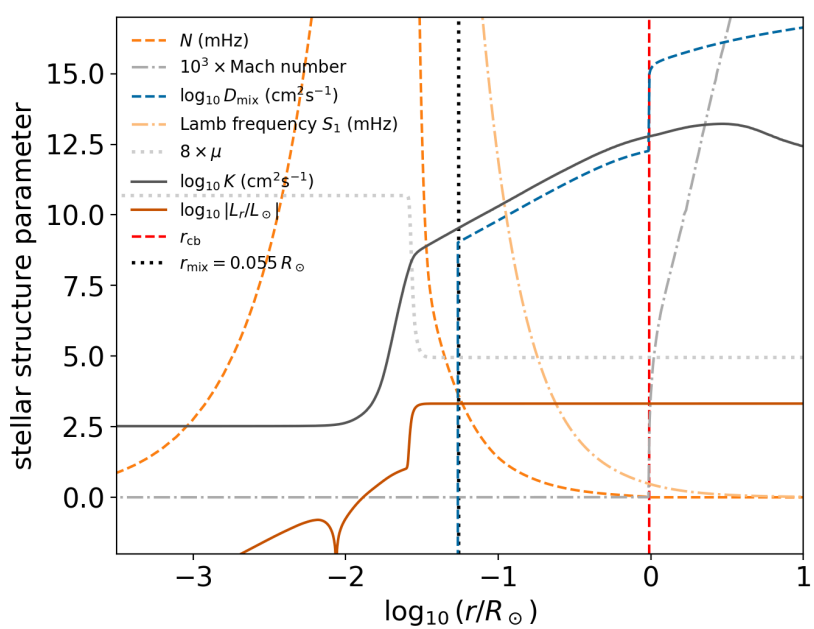
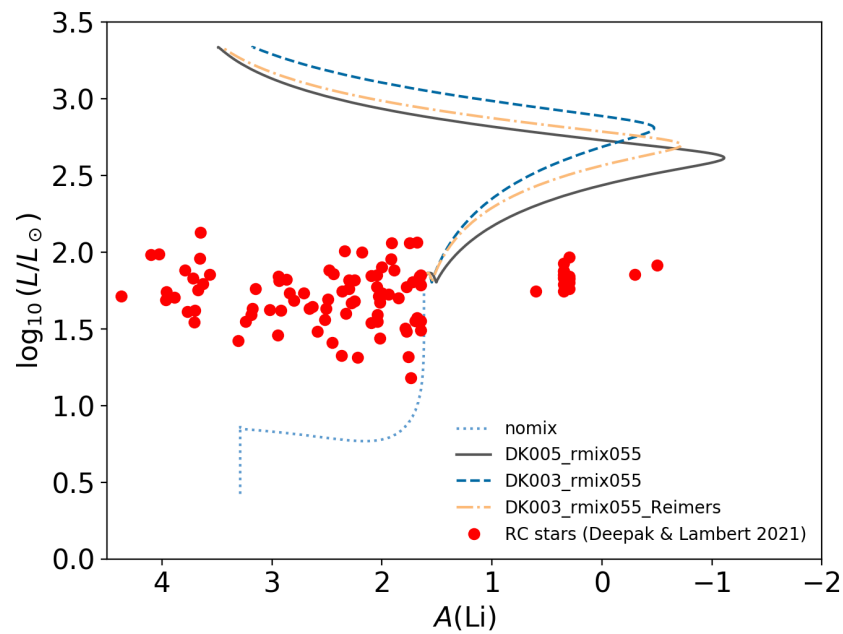


RGB tip luminosity model with [Fe/H]=-0.3, M=1.2$M_\odot$ and $D_{\rm mix}$ = 0.003 $(L/L_{\rm bump})^{4/3}$ $K < K$



RGB mixing with rate increasing proportionally to $L^{4/3}K$

The first results of 3D hydrodynamic simulations of convection and IGWs in a 1.2$M_\odot$ RGB luminosity bump and tip model with [Fe/H]=-0.3 by S. Blouin using the PPMstar code (Woodward P., Herwig F. and Lin P.-H., 2015, ApJ, 798, 49)

A movie of the vorticity in the RGB tip model from 2880^3
Frontera TACC run at the nominal heating (1x*L*)

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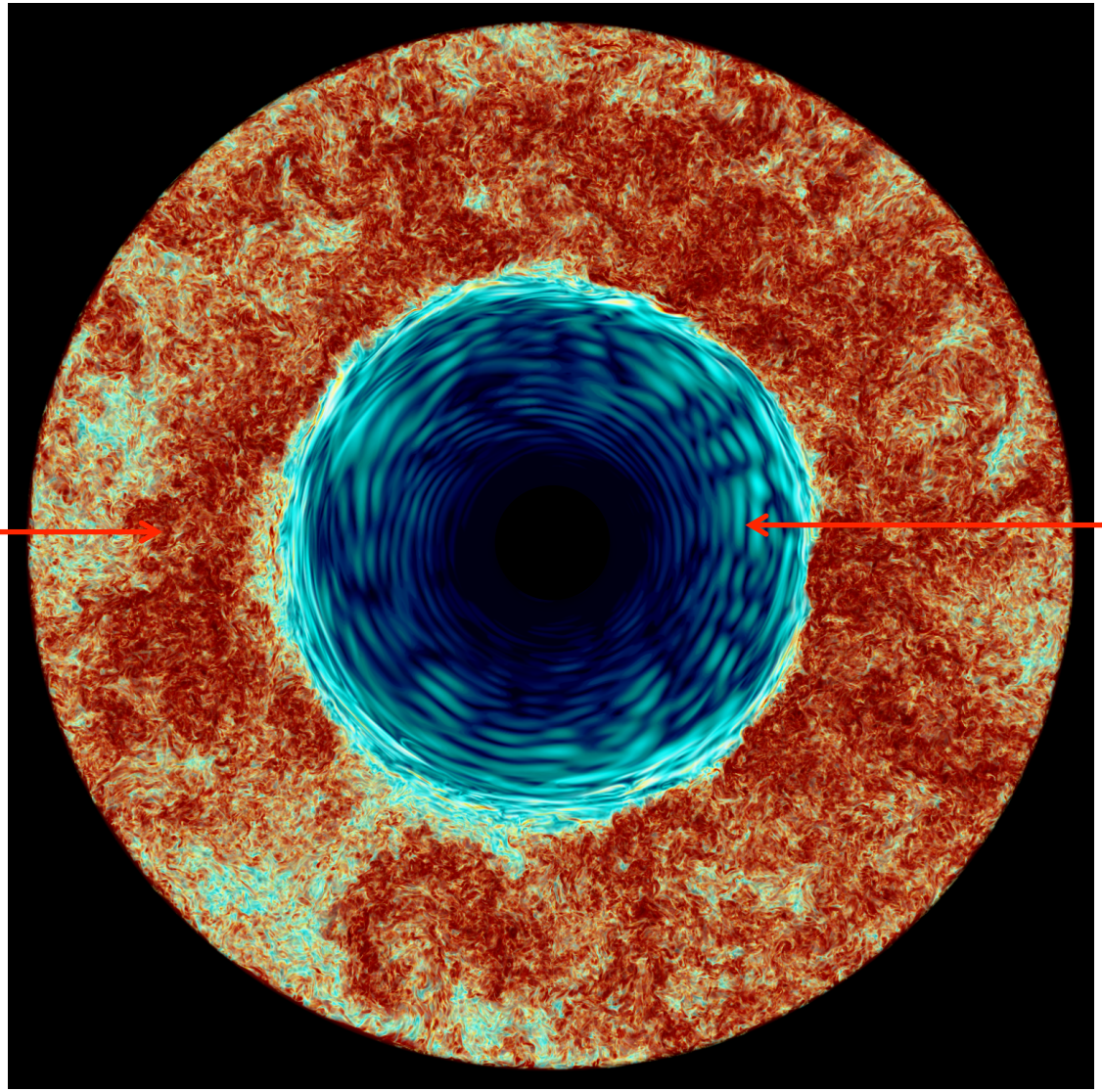
small envelope

Convective envelope



IGWs

The vorticity image from 1536^3 Frontera TACC run

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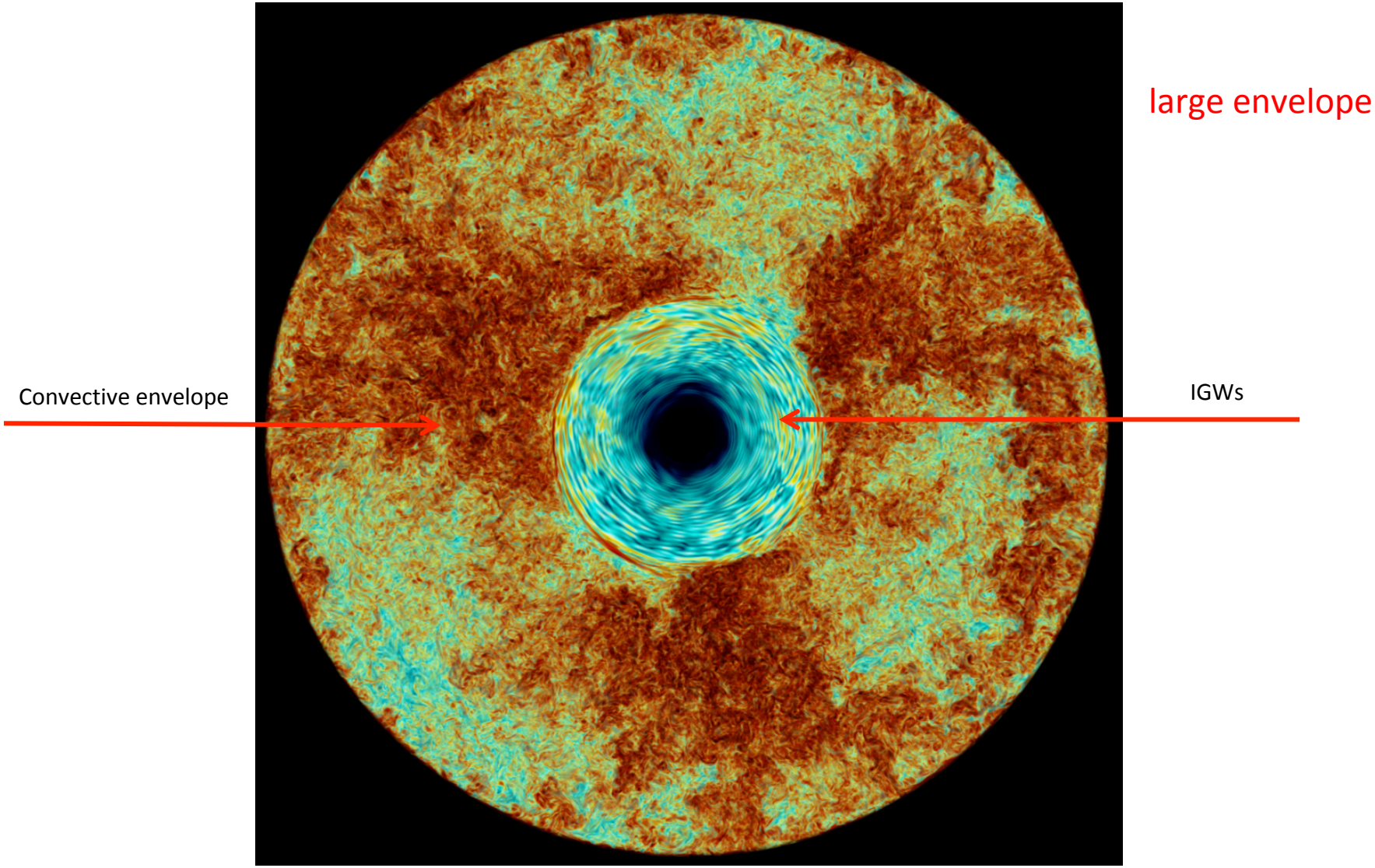

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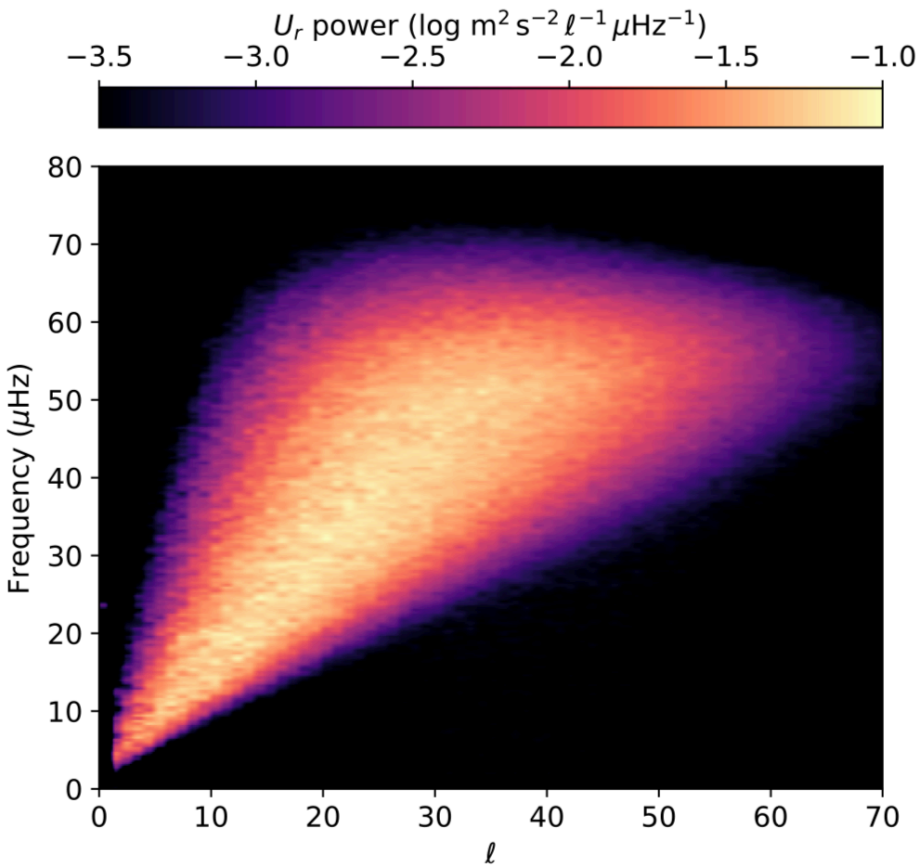
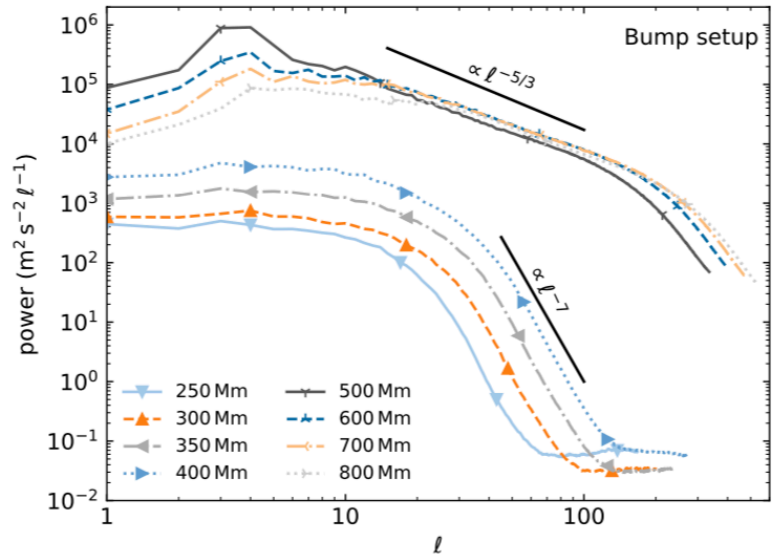
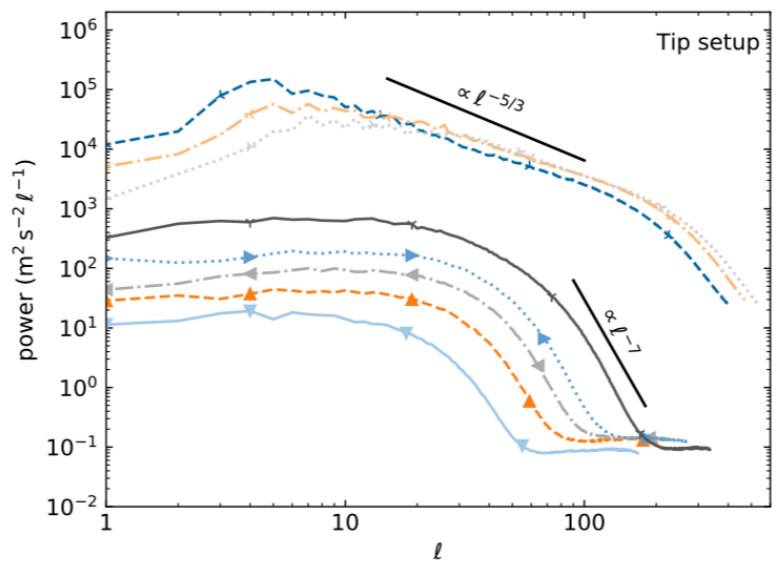
Spectra of IGWs in the radiative zone of the RGB tip model at nominal heating (1×*L*) (in the right panels, the radius increases from the bottom to the top curve; the *k*-*ω* diagram below is shown for the radius *R*=400 Mm of the blue dotted curve)







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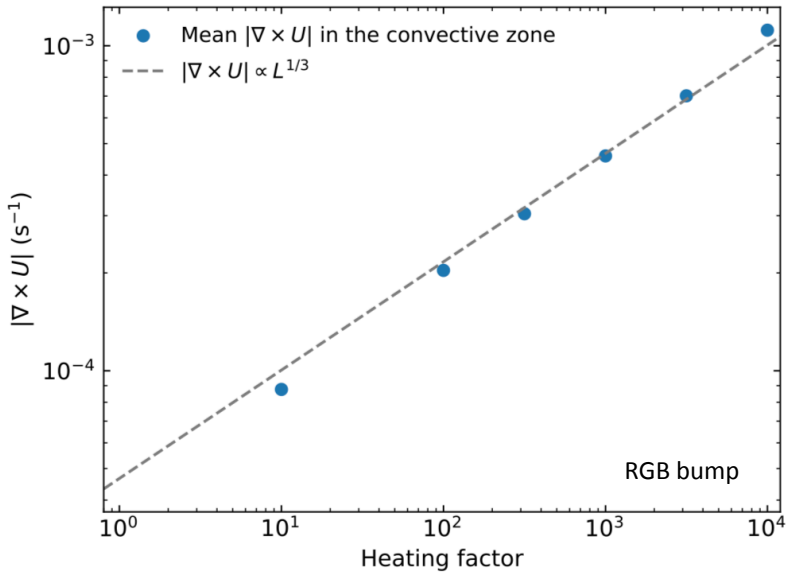

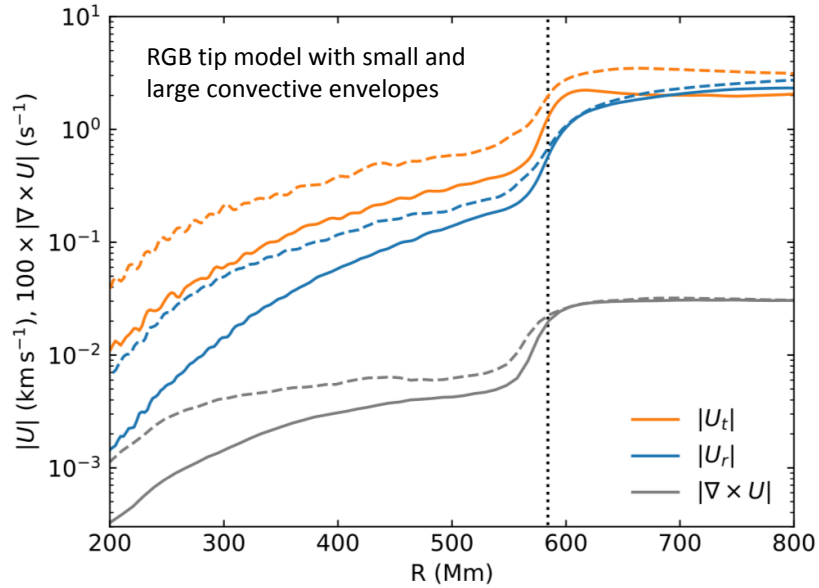

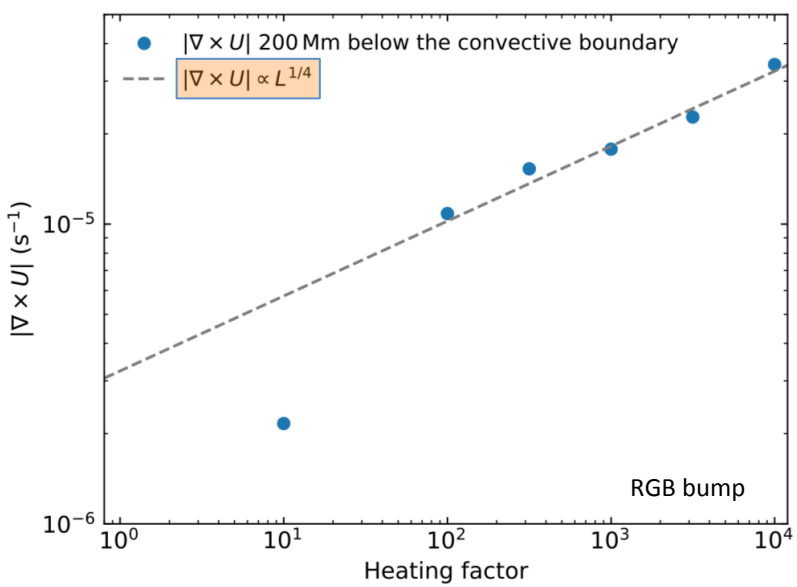

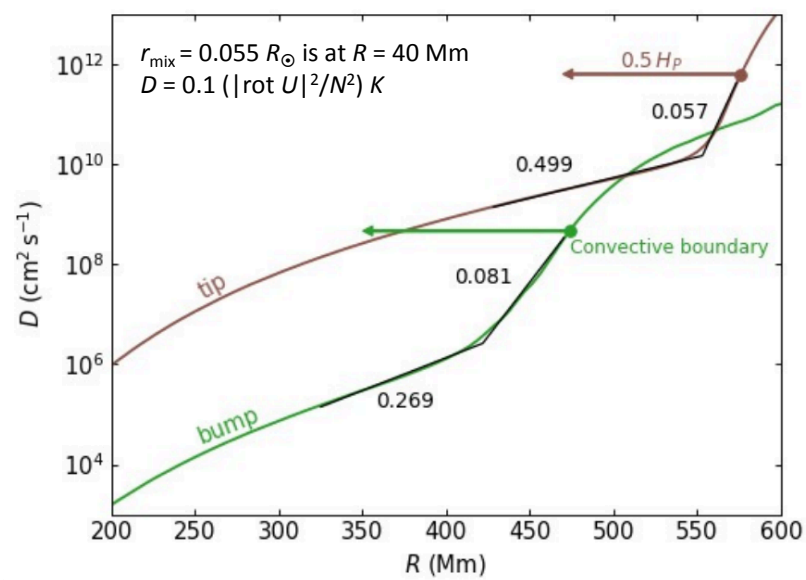

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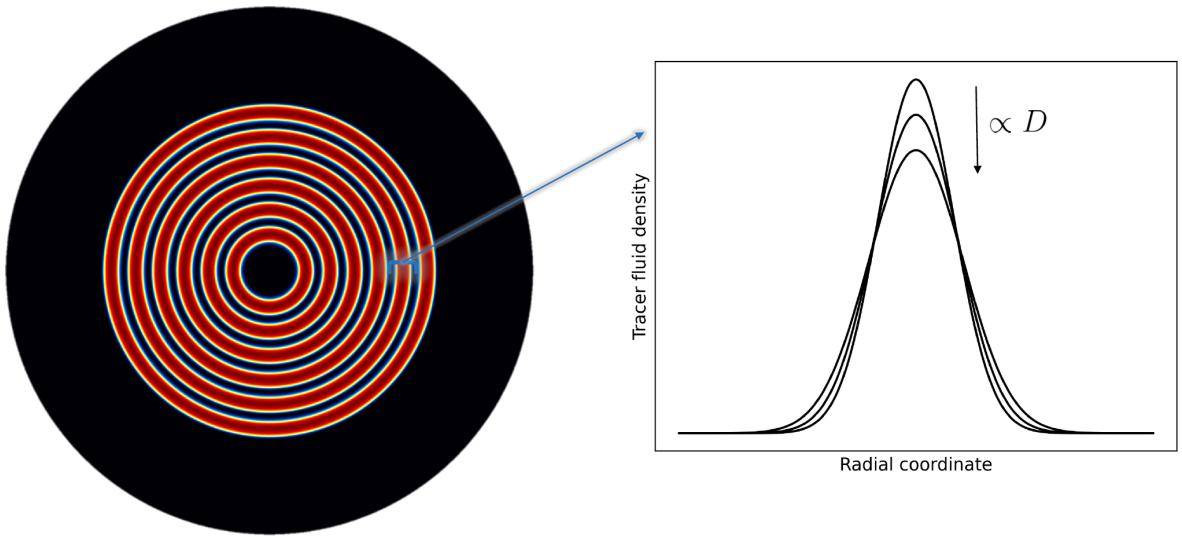
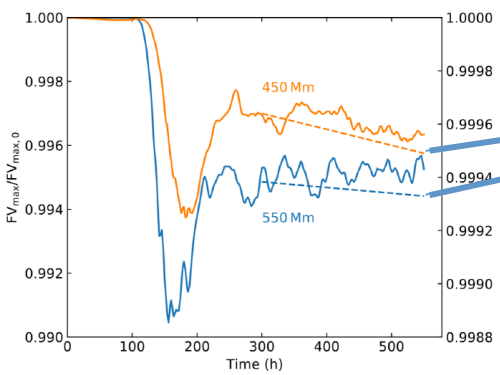
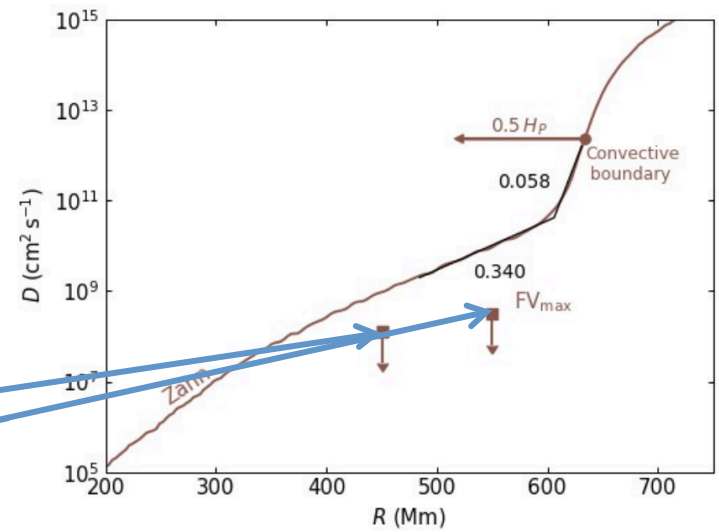


The mixing is too weak to be measured

Two conclusions:

1. Internal gravity waves are not the source of the missing RGB extra mixing
2. Beware of simple analytical expressions





Li production on upper RGB by rotational mixing with rate proportional to L^pK

In the radiative zone of an upper RGB star, angular momentum transport by rotation-induced mixing is determined by the equation

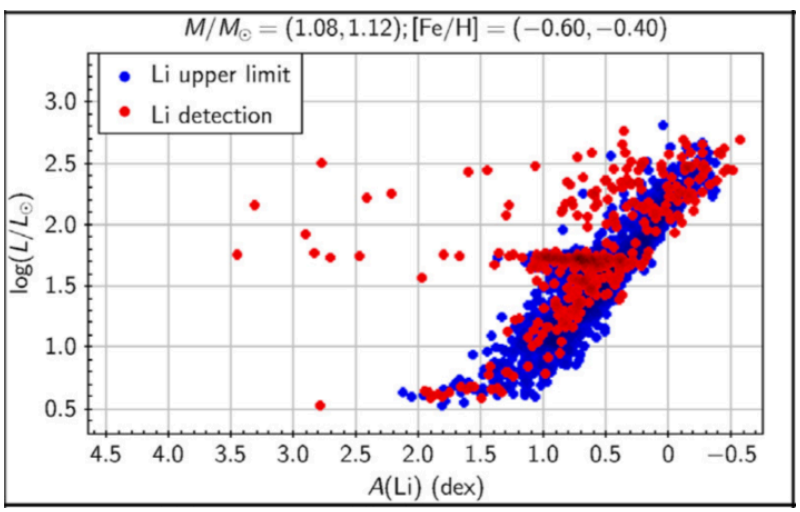
$$\frac{\partial}{\partial t}\left(\rho r^2\Omega\right) = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4\Omega\left(U-5\dot{r}\right)\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4 \nu_{\rm v}\frac{\partial\Omega}{\partial r}\right),$$

where $\dot{r} = \left(\frac{\partial r}{\partial t}\right)_{M_r}$, $\nu_{\rm v} = D_{\rm v} = \eta\frac{\left(r\frac{\partial\Omega}{\partial r}\right)^2}{N^2}$, and U is the radial component of the meridional circulation velocity.

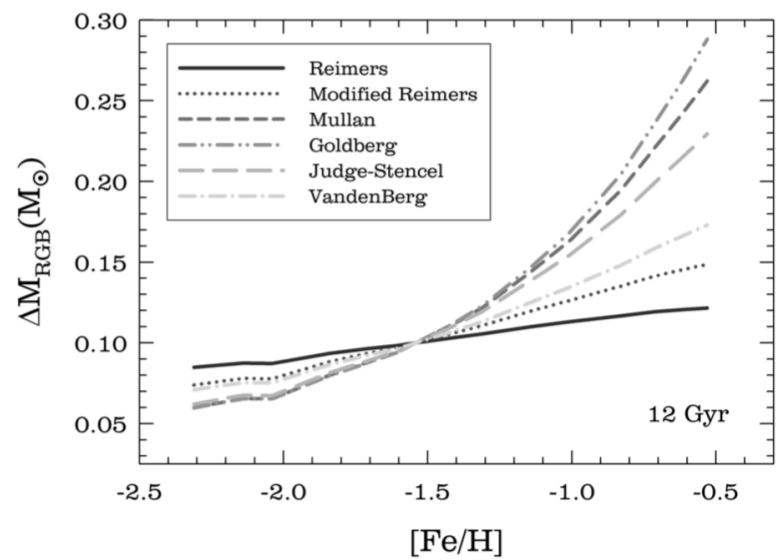
For moderate stellar wind, according to Zahn (1992),

$$D_{\rm eff} = \frac{C_{\rm h}}{50\alpha}r|U| = \frac{C_{\rm h}}{20\,\alpha}\frac{\rho_{\rm m}}{\rho}\frac{\Omega_{\rm s}R^2}{\Omega M}\left(-\frac{dM}{dt}\right),$$

where $\alpha = \frac{1}{2}\frac{\partial\ln(r^2\Omega)}{\partial r}$. For the Reimers mass-loss rate, $\frac{dM}{dt}(M_\odot{\rm yr}^{-1}) = -4\times10^{-13}\,\eta\,\frac{L}{gR} \propto L^{5/3} \propto L^{1/3}K$, while for the modified Reimers law $\frac{dM}{dt}(M_\odot{\rm yr}^{-1}) \propto \left(\frac{L}{gR}\right)^{1.4} \propto L^{7/3} \propto \boxed{L^{4/3}K}$, where L, R, and g are in solar units, and $K \propto L$.

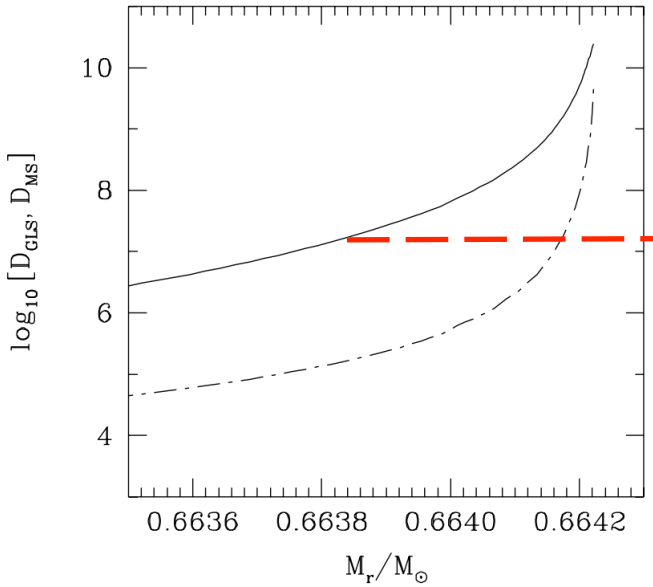


At log($L/L_\odot$)=2.5, RGB and AGB stars have very similar asteroseismology properties.

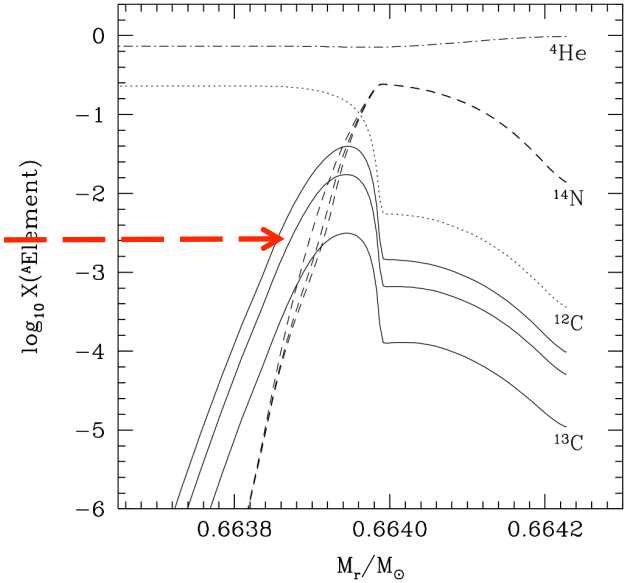


RGB mass-loss rates from Catelan M., 2009, Ap&SS, 320, 261

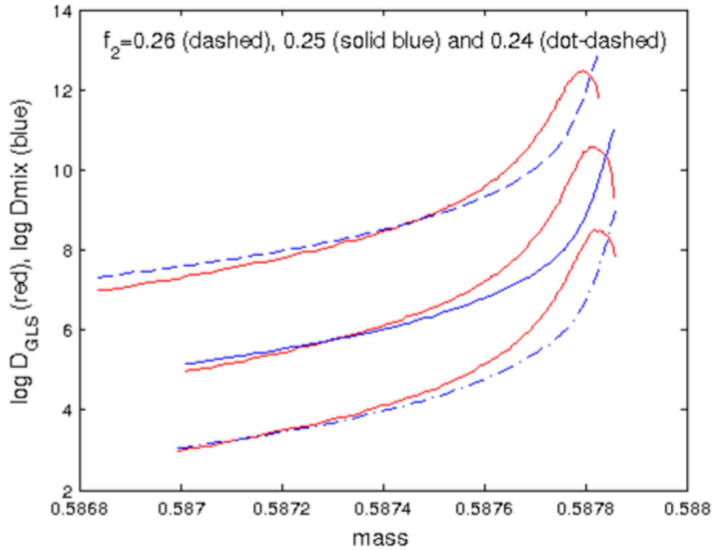
The problem of proton ingestion into C-rich radiative zone in low-mass AGB stars

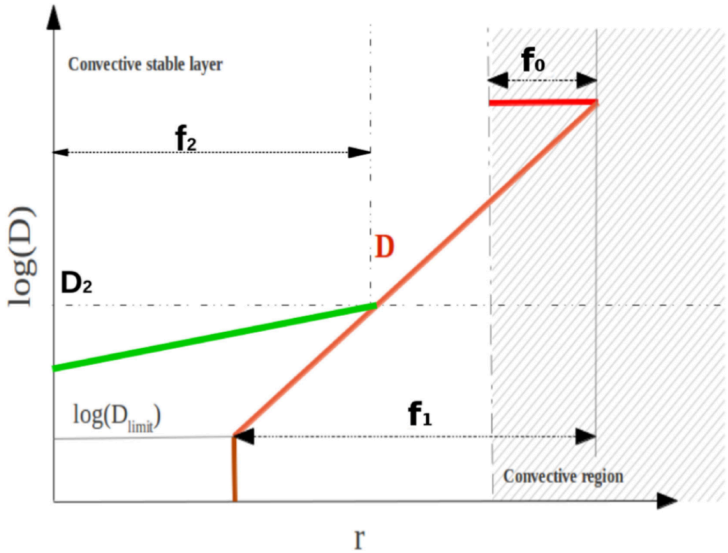


IGW diffusion coefficients below the BCE in a $3M_\odot$ AGB model with Z=0.02 after the 19th pulse obtained by PD and C. Tout (2003, MNRAS, 340, 722) using the analytical prescriptions of Garcia Lopez R.J. and Spruit H.C. (1991, ApJ, 377, 268, GLS) and Montalban J. and Schatzman E. (2000, A&A, 354, 943, MS).



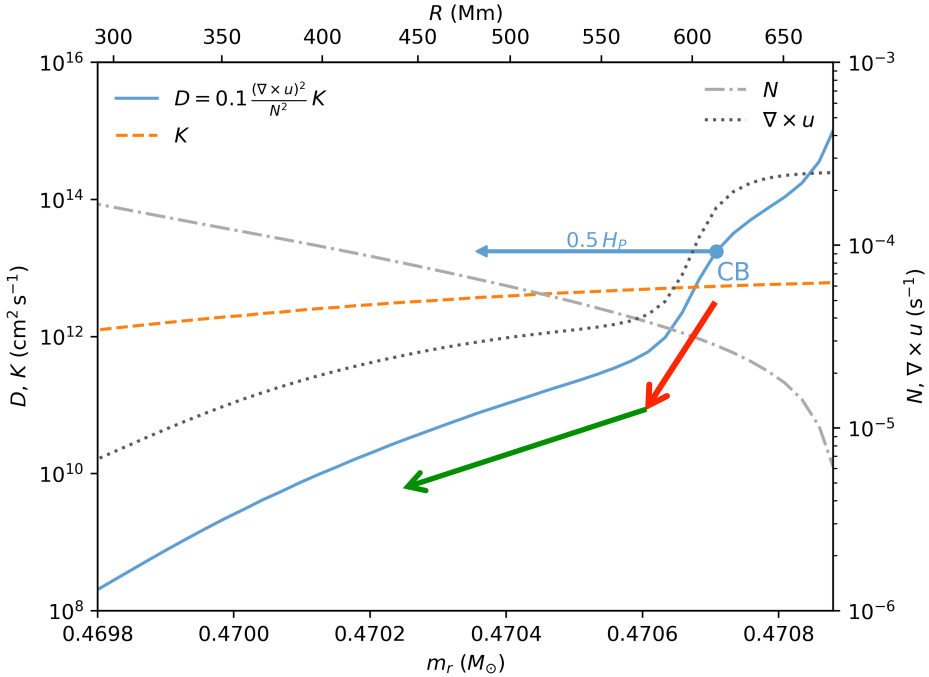
Formation of the ^{13}C pocket (solid lines) during the interpulse period after protons had been ingested into the He/C-rich radiative zone below the BCE by the GLS IGW mixing.



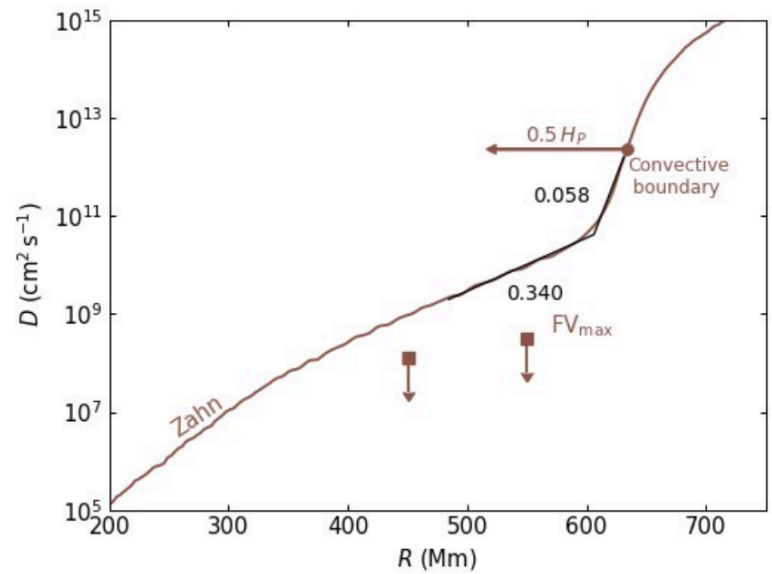


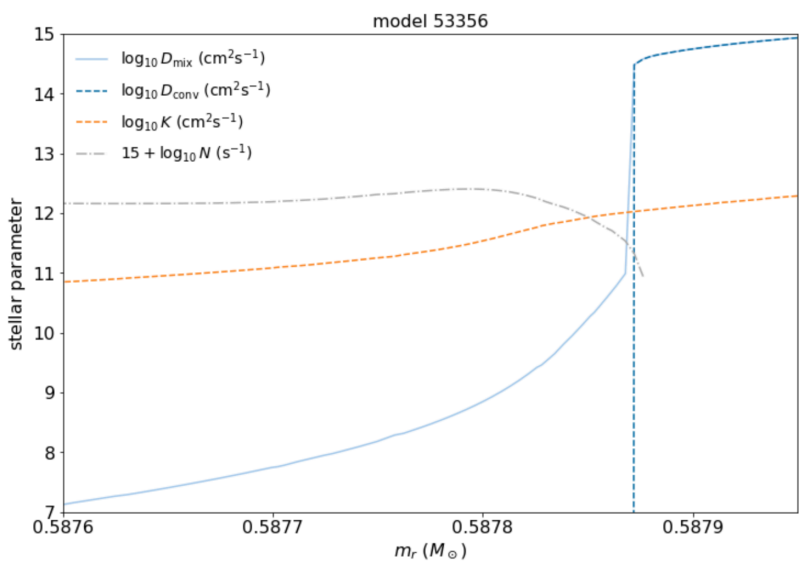
Approximation of the GLS IGW diffusion coefficient (red lines on the left) in a $2M_\odot$ AGB model with Z=0.01 after the 19th pulse by a combination of two exponentially decaying diffusion coefficients (blue lines on the left) that was proposed by Herwig F., Freytag B. and Fuchs T. (2007, ASP Conf. Ser. 378, 43) to better describe convective boundary mixing (CBM) and that is implemented in MESA (the right plot from Battino U. et al., 2016, ApJ, 827, 30).

The formation of the ^{13}C pocket by IGW mixing



Diffusion coefficient for the IGW shear-induced mixing (D), thermal diffusivity (K), buoyancy frequency (N), and vorticity profiles below the BCE in the RGB tip model from the 3D hydro sims.





Diffusion coefficients for mixing in the radiative zone (D_{mix}) and convection (D_{conv}), thermal diffusivity (K), and buoyancy frequency (N) for the AGB model. The mixing diffusion coefficient is calculated here using the MESA prescription for the two-exponent CBM diffusion profile with the parameters adjusted to approximate the GLS IGW mixing.

Conclusions

- Mixing by IGWs in the radiative zone of the low-mass upper RGB star measured in our 3D hydrodynamic simulations is far too slow to explain the observational data, and its rate scales with the luminosity differently, D_{IGW} is proportional to $L^{1/2}$, than expected ($L^{4/3}$), therefore IGW mixing is also unlikely to be efficient enough to produce Li during the He-core flash in amounts comparable to those observed in red-clump stars
- On the other hand, IGW mixing can still be sufficiently fast to form a ^{13}C pocket in a low-mass AGB star (the corresponding 3D hydrodynamic simulations are underway)
- As an alternative, rotation-driven mixing by meridional circulation and shear-induced turbulent diffusion (Zahn J.-P., 1992, A&A, 265, 115; Charbonnel C., 1995, ApJ, 453, L41; Denissenkov P.A. & Tout C., 2000, MNRAS, 316, 395) may have a rate proportional to L^pK with $p \geq 4/3$ (for the modified Reimers and other RGB mass-loss rates) and lead to the enrichment of RGB stars in Li when they approach the RGB tip.

Thank you!