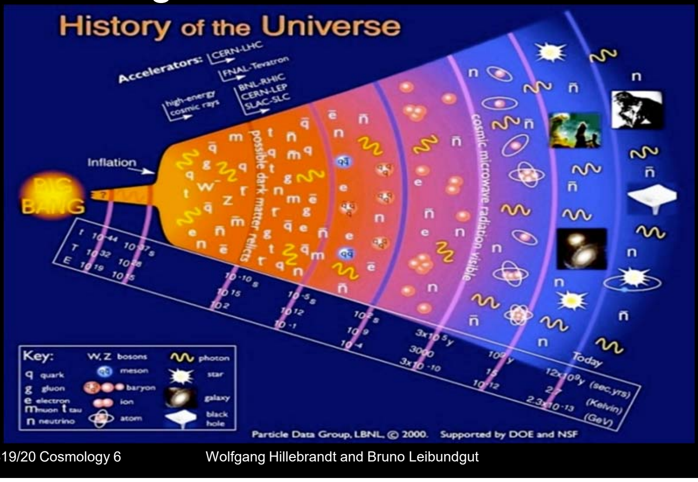
Cosmology

TUM WS 2019/2020
Lecture 6
Wolfgang Hillebrandt and Bruno Leibundgut
(http://www.eso.org/~bleibund/Cosmology)

Thermal history of the universe

From the early, hot universe to the cold accelerating universe



Energy and Temperature

Connected through

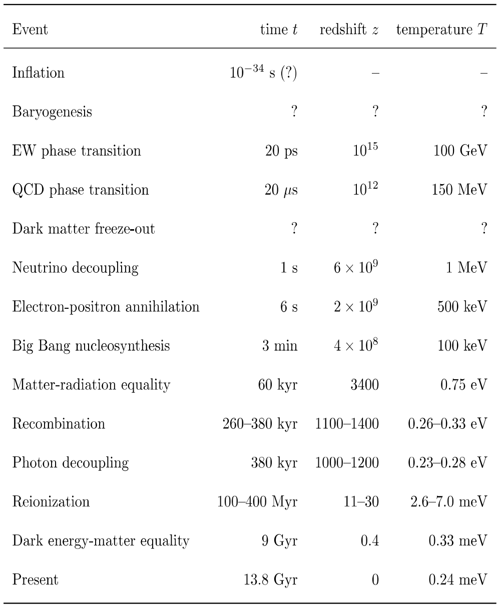
$$E = k_B T = mc^2 = h\nu$$

(k_B=8.6173 10^{-5} eV K^{-1})

(c will be dropped, i.e. set to 1 in the following)

Energy (eV)	Temperature (K)
1.00E+12	1.1605E+16
1.00E+11	1.1605E+15
1.00E+10	1.1605E+14
1.00E+09	1.1605E+13
1.00E+08	1.1605E+12
1.00E+07	1.1605E+11
1.00E+06	1.1605E+10
1.00E+05	1.1605E+09
1.00E+04	1.1605E+08
1.00E+03	1.1605E+07
1.00E+02	1.1605E+06
1.00E+01	1.1605E+05
1.00E+00	1.1605E+04
1.00E-01	1.1605E+03
1.00E-02	1.1605E+02
1.00E-03	1.1605E+01
1.00E-04	1.1605E+00
1.00E-05	1.1605E-01
1.00E-06	1.1605E-02
1.00E-07	1.1605E-03
1.00E-08	1.1605E-04
1.00E-09	1.1605E-05
1.00E-10	1.1605E-06
1.00E-11	1.1605E-07
1.00E-12	1.1605E-08

Thermal History Overview



Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

Table 3.1: *Key events in the thermal history of the universe.*

(From Daniel Baumann; http://www.damtp.cam.ac.uk/user/db275/Cosmology/)

Relativistic gas

- For a highly relativistic gas ($m<<k_BT$) the density and entropy become

$$\rho(T)=\frac{1}{2}N\sigma_B T^4 \qquad s(T)=\frac{2}{3}N\sigma_B T^3$$

with *N* the number of particles and anti-particles and all spin states (and a faction 7/8 for fermions)

$$t=\sqrt{\frac{3}{16\pi G N\sigma_B}}\frac{1}{T^2}+const.$$

Thermal history

Start with a very hot state:
$T=10^{11}K\approx 1MeV$

$$m_\mu >> k_B T >> m_e$$

- – photons, electrons and positrons, all neutrinos in thermal equilibrium
  - latter through the neutral current interaction

$$e^+ + e^- \Leftrightarrow \nu + \bar{\nu}$$

$$N = 2+\frac{7}{8}(6+4)=\frac{43}{4}$$

- Adding all states
- and time

$$t=\sqrt{\frac{3c^2}{172\pi G\sigma_B}}T^{-2}+const.$$

Neutrino decoupling

The critical parameter is the ratio between the reaction rate (Γ) and the expansion rate (H). Once the reaction rate is slower than the expansion the equilibrium is lost.

- For the neutrinos, the weak interaction with electrons is critical

$$\Gamma_\nu = n_e \sigma_{wk} \approx \left(\frac{k_B T}{\hbar}\right)^3 (\hbar G_{wk} k_B T)^2 = \frac{G_{wk}^2 (k_B T)^5}{\hbar}$$

here we have electron density n_e and the weak interaction coupling constant G_{wk}

Neutrino decoupling

- Expansion rate is given by the Friedmann equation

$$H = \sqrt{\frac{G(k_B T)^4}{\hbar^3}}$$

which leads to

$$\frac{\Gamma_\nu}{H} = G_{wk}^2 \sqrt{\frac{\hbar}{G}} \left(k_B T\right)^3 \approx \left(\frac{T}{10^{10} K}\right)^3$$

  - At this temperature the reaction rate falls below the expansion rate and the neutrinos are no longer in thermal equilibrium with the electrons (and photons)

Neutrino decoupling

- At this point the neutrinos start free streaming and an adiabatic expansion

$$T_\nu \propto a^{-1} = 1+z$$

- Note that 10^{10}K corresponds to roughly 1 MeV, i.e. twice the e^- mass
- This means that e^- and e^+ can no longer be assumed massless

Electrons (and positrons)

- The entropy density for the photons and electrons and positrons is

$$s(T) = \frac{4\sigma_B T^3}{3} + \frac{4}{T}\int_0^\infty \frac{4\pi p^2 dp}{(2\pi\hbar)^3}\left(\sqrt{p^2+m^2} + \frac{p^2}{3\sqrt{p^2+m^2}}\right)\left(e^{\frac{\sqrt{p^2+m^2}}{k_B T}} + 1\right)^{-1}$$

  - first term is for the photons and the second term for the electrons and positrons
- Since $s(T)a^3 = const.$ and $T_\nu \propto a^{-1}$ this means $T_\nu \propto T f^{1/3}\left(m_e / k_B T\right)$
  - temperatures for neutrinos and electrons
  - *$f(m_e/k_BT)$* is a function taking up the terms in the equation above $s(T) = \frac{4\sigma_B T^3}{3} f(m_e / k_B T)$

Electrons and Positrons

- For large temperatures $m_e << k_B T$ the neutrinos were still in equilibrium with the electrons and both temperatures were the same so that f(0) reduces to counting the number of particles $f(0)=1+2\frac{7}{8}=\frac{11}{4}$ hence $T_\nu = \left(\frac{4}{11}\right)^{1/3} T f^{1/3}(m_e / k_B T)$
- For late times when the temperature is low $f(\infty)=1$ and $\frac{T}{T_\nu} \to \left(\frac{11}{4}\right)^{1/3} = 1.401$

Neutrino temperature today

- In effect the photons were heated by the $e^- + e^+$ annihilation and the temperature difference today is

$$T_\nu = \frac{T_0}{1.401} = \frac{2.275K}{1.401} = 1.945K$$

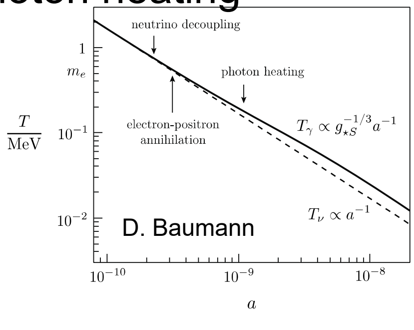
Summary of the temperature evolution

The exact values are

$T(K)$	T/T_ν	t (seconds)
10^{11}	1.000	0
$6\ 10^{10}$	1.000	0.0177
$3\ 10^{10}$	1.001	0.101
$2\ 10^{10}$	1.002	0.239
10^{10}	1.008	0.998
$6\ 10^{9}$	1.022	2.86
$3\ 10^{9}$	1.080	12.66
$2\ 10^{9}$	1.159	33.1
10^{9}	1.345	168
$3\ 10^{8}$	1.401	1980
10^{8}	1.401	$1.78\ 10^{4}$
10^{7}	1.401	$1.78\ 10^{6}$
10^{6}	1.401	$1.78\ 10^{8}$

Early temperature evolution

- The important events are neutrino decoupling, electron-positron annihilation and photon heating



D. Baumann

Big Bang Nucleosynthesis

- Relevant interactions
  - Conversions between protons and neutrons

$$n+\nu \Leftrightarrow p+e^- \qquad E_e - E_\nu = Q$$

$$n+e^+ \Leftrightarrow p+\overline{\nu} \qquad E_\nu - E_e = Q$$

$$n \Leftrightarrow p+e^- +\overline{\nu} \qquad E_\nu + E_e = Q$$

$$Q = m_n - m_p = 1.293 MeV$$

  - The energy transfer is the difference in rest mass between the neutron and the proton

Reaction rates

- The reaction rates in thermal equilibrium depend on the temperature $\frac{\lambda(p\rightarrow n)}{\lambda(n\rightarrow p)} = e^{-\frac{Q}{k_B T}}$

  and hence the fraction of neutrons and protons is

$$\frac{X_n}{X_p} = \frac{X_n}{1-X_n} = e^{-\frac{Q}{k_b T}} \qquad \text{definition of } X_n \equiv \frac{N_n}{N_n + N_p}$$

- The rates are determined by the weak interactions $\lambda(n\rightarrow p) = \lambda(p\rightarrow n) \propto (k_B T)^5$

Connect to expansion

- In the radiation dominated era the expansion age is $t = \frac{1}{2H}$
- The equilibrium stops when the reaction rate is slower than the expansion age

$$\frac{\lambda}{H} \approx 0.8\left(\frac{T}{10^{10}K}\right)^{3}$$

- Above $T = 1.1 \cdot 10^{10}K$ the reactions are still faster than the expansion

$$X_n = \left(1 + e^{\frac{Q}{k_B T}}\right)^{-1}$$

Evolution of X_n

- Dramatic decrease when the neutron decay starts

$$\tau_n = 885.$$

and the number of neutrons reduces as

$$N_n \propto e^{-\frac{t}{\tau_n}}$$

A fit to the table gives

$$X_N \rightarrow 0.1609 e^{-\frac{t}{885.7s}}$$

T(K)	***t (seconds)***	X_n
10^{12}	0.0001	0.4962
$3\ 10^{11}$	0.0011	0.4875
10^{11}	0.0099	0.4626
$3\ 10^{10}$	0.1106	0.3798
10^{10}	1.008	0.2386
$3\ 10^{9}$	12.67	0.1654
$1.3\ 10^{9}$	91.09	0.1458
$1.2\ 10^{9}$	110.2	0.1425
$1.1\ 10^{9}$	135.1	0.1385
10^{9}	168.1	0.1333
$9\ 10^{8}$	212.7	0.1268
$8\ 10^{8}$	274.3	0.1182
$7\ 10^{8}$	362.6	0.1070
$6\ 10^{8}$	496.3	0.0919
$3\ 10^{8}$	1980	0.0172
10^{8}	17780	$3.07\ 10^{-10}$

Nucleosynthesis

- The free neutrons are bound in atomic nuclei
  - number density of a species *i*

$$n_i = g_i e^{\frac{\mu_i}{k_B T}} \left(\frac{2\pi m_i k_B T}{h^2} \right)^{\frac{3}{2}} e^{-\frac{m_i}{k_B T}}$$

    - μ_i chemical potential, m_i mass of the particle
  - chemical potential for rapid reactions

$$\mu_i = Z_i m_p + \left(A_i - Z_i \right) m_n$$

    - Z_i charge of the nucleus, A_i atomic (mass) number

Nucleosynthesis

- The mass of the nucleus is

$$m_i = Z_i m_p + \left(A_i - Z_i \right) m_n - B_i$$

  - B_i is the binding energy of the nucleus
- Nuclei with the highest binding energy would in principle form first
- However, only 2-particle reactions and only D forms

Nucleus	**Binding Energy (MeV)**
neutron	1.29
Deuterium D (or ^{2}H)	2.22
Tritium (^{3}H)	6.92
Helium (^{3}He)	7.72
Helium (^{4}He)	28.3

Nucleosynthesis

- D forms first, but it also is dissociated by the photons
- Production depends on the (baryon) density Ω_B

$$n_N = \frac{3\Omega_B H_0^2 \left(\frac{a_0}{a}\right)^3}{8\pi G m_N}$$

Introducing $h = \frac{H}{100 km\ s^{-1}\ Mpc^{-1}}$

- Rate of D production

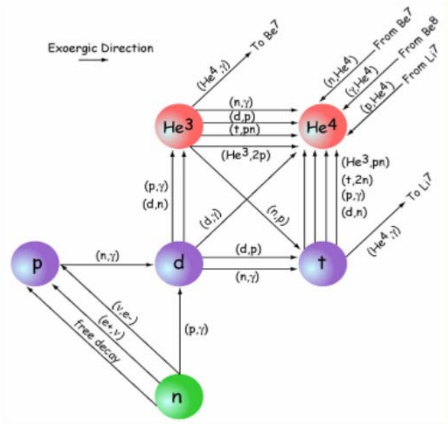
$$\lambda_D = 2.52 \cdot 10^4 s^{-1} \left(\frac{T}{10^{10} K}\right)^3 X_p \Omega_B h^2$$

Nucleosynthesis

- D starts to form at $T \approx 7 \cdot 10^8 K$
- After D production the next nuclei, D, ^{3}H, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li and ^{7}Be form quickly
- There are no stable nuclei with A=5 or A=8

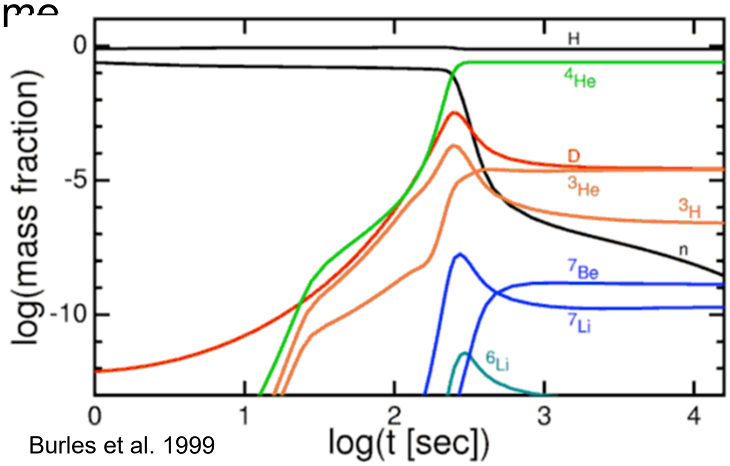
pp-reaction chain

- Formation of D, He and Li



Nucleosynthesis

Formation of elements as a function of time



Burles et al. 1999