Cosmology

TUM WS 2019/2020

Lecture 5

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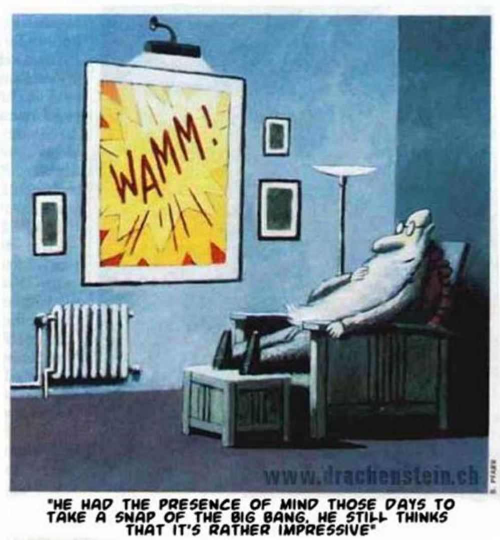
(http://www.eso.org/~bleibund/Cosmology)

Big Bang/Early Universe

Big Bang/Early Universe



Where we are …

- Smooth, uniform universe
  - no local density fluctuations
- Contents are described as perfect fluids
  - no perturbations, stresses, shears, or directional differences
- Local (linear) expansion H_0 determined
- Questions:
  - early evolution? how was matter created? elemental evolution? growth of structure?

Big Bang Theory

- Evidence for a Big Bang
  - Expansion
    - reversing the current expansion leads to a denser (and hotter) past
  - Cosmic Microwave Background
    - hot past
  - Nucleosynthesis
    - formation of the first elements

Note: If the Universe is indeed homogenous and isotropic, and if GR is the correct theory of gravitation, a 'singularity' at t=0 (a(0) = 0) is unavoidable!

Georgy Gamov (1904-1968)

- If the universe is expanding, then there has been a big bang
- Therefore, the early universe must have been very dense and hot
- Optimum environment to breed the elements by nuclear fusion (Alpher, Bethe & Gamow, 1948)
  - success: predicted that helium abundance is 25%
  - failure: could not reproduce elements more massive than lithium and beryllium ($\Rightarrow$ formed in stars)

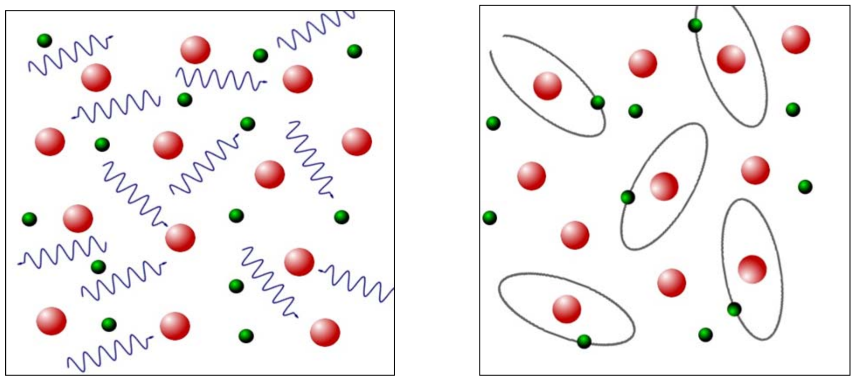
Other signs of an evolving universe

- Various parameters change over time
  - appearance of galaxies
- Change of the elemental abundances as a function of redshift
- Build up of structure
  - clusters of galaxies
  - “cosmic web”

Thermal history of the universe

- Reversing expansion
  - increasing density
  - hotter universe
- In the early universe consider most particles in thermal equilibrium
  - black body radiation
- Adiabatic expansion
  - $T=T_0(1+z) = 2.73K\ (1+z)$

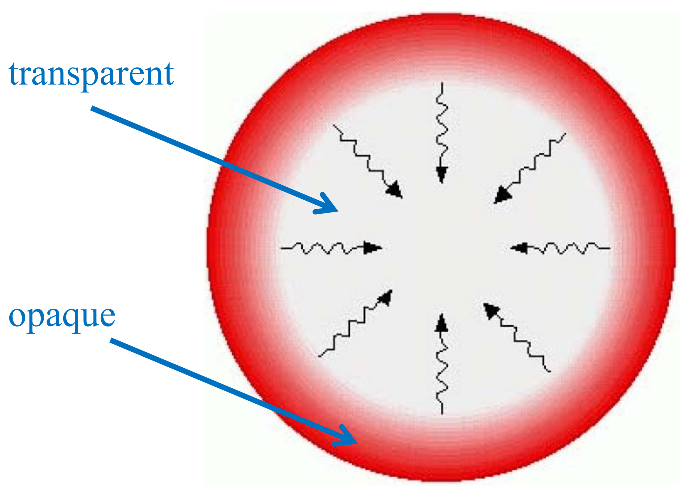
Cosmic Microwave Background



Before recombination: *The Universe is opaque*
After recombination: *The Universe is transparent*

Transition ~ 300 000 years after the Big Bang

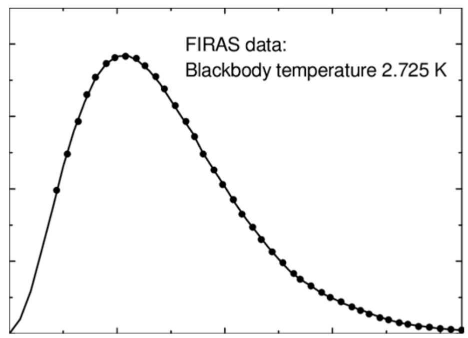
Cosmic Microwave Background



Cosmic Microwave Background

- Perfect blackbody radiation is the signature of the early equilibrium
- Observed by the balloon and satellite experiments

$$B_\nu = \frac{2h\nu^3}{c^2}\left(e^{h\nu/k_BT} - 1\right)^{-1}$$



Cobe ,Wright et al. (1994)

Invariance of black body

- Consider the number density of photons in the black body radiation $$n_T(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{e^{\frac{h\nu}{k_BT}} - 1}$$
- What happens to the black body radiation when the photons decouple?
- Consider the change in frequency $\nu(t) = \nu\frac{a(t)}{a(t_L)}$ the new number density then becomes

$$n(\nu,t)d\nu = \left(\frac{a(t_L)}{a(t)}\right)^3 n_{T(t_L)}\left(\nu\frac{a(t)}{a(t_L)}\right)d\left(\nu\frac{a(t)}{a(t_L)}\right)$$

(the cubic factor is the increase in volume)

Invariance of black body

- This leads to

$$n(\nu,t)d\nu = \frac{8\pi\nu^2 d\nu}{e^{\frac{h\nu}{k_B T(t)}} - 1} = n_{T(t)}(\nu)d\nu$$

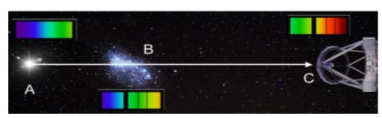
with

$$T(t) = T(T_L)\frac{a(t_L)}{a(t)}$$

- The photon number density maintains the black body distribution and the temperature simply decrease with (1+z)

Cosmic Microwave Background

- Observed also through interstellar absorption lines of CN and CH
- Rotational transitions in the ground state of these molecules in equilibrium with CMB photons
- Measure T_L



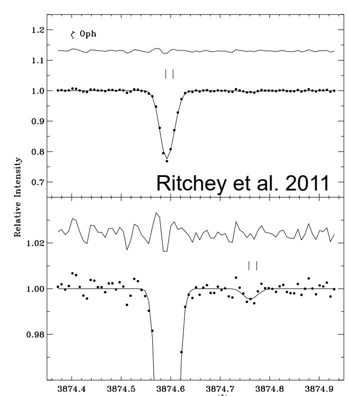


Figure 9. Same as Figure 7 except for the ^{12}CN and ^{13}CN *R*(0) lines toward ζ Oph.

Cosmic Microwave Background

Excitation temperature measured in the lines

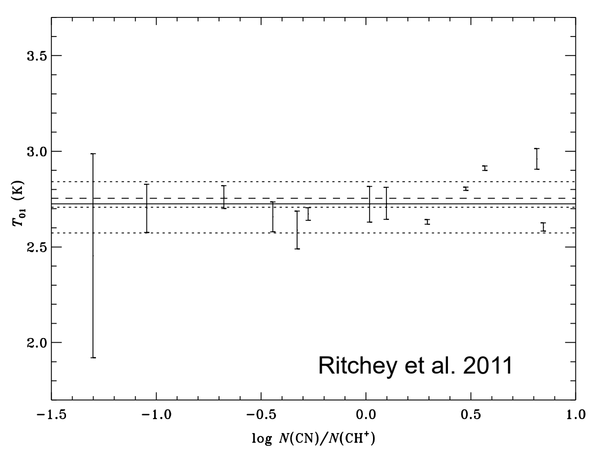
Table 9
CN Rotational Column Densities and Excitation Temperatures

Star	$N(N=0)$ (10^{12} cm^{-2})	$N(N=1)$ (10^{12}cm^{-2})	$N(N=2)$ (10^{12} cm^{-2})	T_{01} (K)	T_{12} (K)	$T_{01}(^{13}CN)$ (K)
McDonald sight lines						
ζ Per	2.31 ± 0.02	0.94 ± 0.02	0.04 ± 0.02	2.723 ± 0.031	2.931 ± 0.411	…
ρ Oph A	1.77 ± 0.01	0.68 ± 0.01	0.04 ± 0.01	2.657 ± 0.026	3.136 ± 0.323	…
ζ Oph	2.06 ± 0.02	0.82 ± 0.02	…	2.702 ± 0.042	…	…
20 Aql	2.76 ± 0.02	1.12 ± 0.02	0.06 ± 0.02	2.728 ± 0.028	3.155 ± 0.340	…
VLT/UVES sight lines						
HD 73882	27.66 ± 0.04	10.48 ± 0.02	0.31 ± 0.02	2.631 ± 0.004	2.693 ± 0.038	2.706 ± 0.200
HD 152236 (−4.8)	0.38 ± 0.01	0.12 ± 0.02	…	2.454 ± 0.178	…	…
HD 152236 (+6.0)	2.21 ± 0.02	0.81 ± 0.02	…	2.588 ± 0.033	…	…
HD 154368 (+5.2)	18.19 ± 0.03	8.40 ± 0.02	0.29 ± 0.01	2.911 ± 0.004	2.815 ± 0.033	2.781 ± 0.192
HD 161056 (−3.1)	3.34 ± 0.01	1.59 ± 0.02	0.10 ± 0.02	2.960 ± 0.018	3.333 ± 0.159	…
HD 161056 (+2.8)	3.51 ± 0.02	1.46 ± 0.02	0.10 ± 0.02	2.761 ± 0.020	3.383 ± 0.210	…
HD 169454	30.92 ± 0.05	13.30 ± 0.02	0.51 ± 0.01	2.804 ± 0.003	2.881 ± 0.017	2.599 ± 0.082
HD 170740	6.23 ± 0.02	2.44 ± 0.02	0.11 ± 0.02	2.672 ± 0.011	2.989 ± 0.123	…
HD 210121	14.33 ± 0.04	5.31 ± 0.02	0.13 ± 0.02	2.604 ± 0.007	2.589 ± 0.096	3.096 ± 0.383
Weighted mean				2.754 ± 0.002	2.847 ± 0.014	2.652 ± 0.069

Ritchey et al. 2011

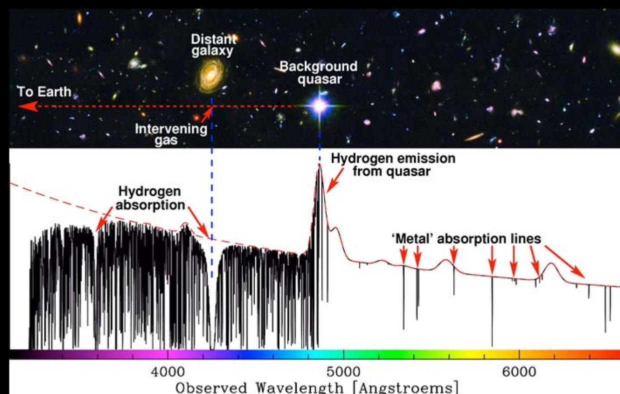
Cosmic Microwave Background

- Excitation temperature independent of line of site or density of the interstellar clouds



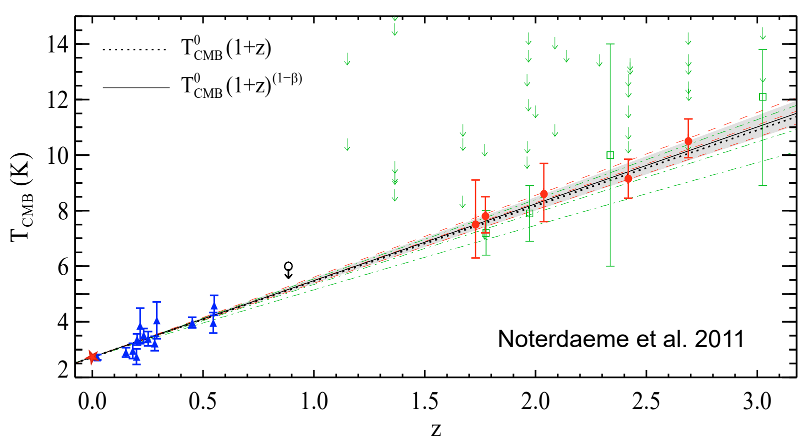
Temperature evolution

- Observe intergalactic clouds through the absorption between a bright source (typically a quasar) and us



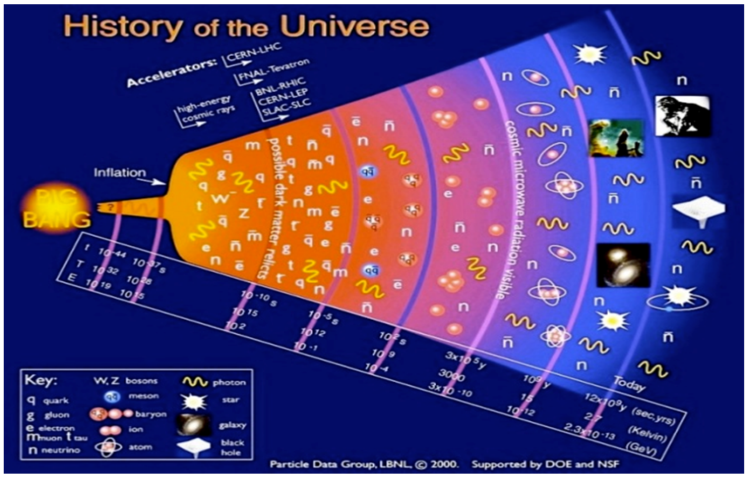
Measure temperature evolution

- Using the CO molecule in distant quasar absorption systems



Thermal history of the universe

From the early, hot universe to the cold accelerating universe



Energy and Temperature

- Connected through

$$E = k_B T = mc^2 = h\nu$$

(k_B=8.6173 10^{-5} eV K^{-1})

(c will be dropped, i.e. set to 1 in the following)

Energy (eV)	Temperature (K)
1.00E+12	1.1605E+16
1.00E+11	1.1605E+15
1.00E+10	1.1605E+14
1.00E+09	1.1605E+13
1.00E+08	1.1605E+12
1.00E+07	1.1605E+11
1.00E+06	1.1605E+10
1.00E+05	1.1605E+09
1.00E+04	1.1605E+08
1.00E+03	1.1605E+07
1.00E+02	1.1605E+06
1.00E+01	1.1605E+05
1.00E+00	1.1605E+04
1.00E-01	1.1605E+03
1.00E-02	1.1605E+02
1.00E-03	1.1605E+01
1.00E-04	1.1605E+00
1.00E-05	1.1605E-01
1.00E-06	1.1605E-02
1.00E-07	1.1605E-03
1.00E-08	1.1605E-04
1.00E-09	1.1605E-05
1.00E-10	1.1605E-06
1.00E-11	1.1605E-07
1.00E-12	1.1605E-08

Overview

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^{9}	1 MeV
Electron-positron annihilation	6 s	2×10^{9}	500 keV
Big Bang nucleosynthesis	3 min	4×10^{8}	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

Table 3.1: *Key events in the thermal history of the universe.*

(From Daniel Baumann; http://www.damtp.cam.ac.uk/user/db275/Cosmology/)

Early temperature evolution

- Thermal equilibrium implies a constant entropy within a comoving volume

$$s(T)a^3 = const.$$

- An adiabatic change in the volume changes the entropy $$d(s(T)V) = \frac{d(\rho(T)V) + p(T)dV}{T}$$
- and the entropy density $$s(T) = \frac{\rho(T) + p(T)}{T}$$

Radiation in thermal equilibrium

- Radiation as an example: $p(T) = \frac{\rho(T)}{3}$
- which leads to the Stefan-Boltzmann law for radiation $$\rho = \sigma_B T^4$$
- with the radiation density constant

$$\sigma_B = \frac{8\pi^5 k_B^4}{15 h^3 c^3} = 7.57 \cdot 10^{-15} erg\ cm^{-1}\ K^{-4}$$

- entropy density is then $s(T) = \frac{4}{3}\sigma_B T^3$

Some particle physics

- Number density of particles is given by

$$n(p,T) = \frac{4\pi g p^2}{(2\pi\hbar)^3}\left(e^{\sqrt{p^2+m^2}/k_B T} \pm 1\right)^{-1} \quad \text{(note } p \text{ stands for momentum here)}$$

with g the number of spin states of particles and antiparticles.
+ is for fermions (Fermi-Dirac distribution)
− for bosons (Bose-Einstein distribution)

- Examples:
  - photons g=2, m=0
  - electrons and positrons g=4, m_e=511keV

Connecting to thermodynamics

- Density ρ and pressure P for these particles are
  $\rho(T) = \int_0^\infty n(p,T)\sqrt{p^2+m^2}dp$
  $P(T) = \int_0^\infty n(p,T)\frac{p^2}{3\sqrt{p^2+m^2}}dp$
  and the entropy density
  $$s(T) = \frac{1}{T}\int_0^\infty n(p,T)\left[\sqrt{p^2+m^2}+\frac{p^2}{3\sqrt{p^2+m^2}}\right]dp$$

For massless particles

- m=0 and then the density becomes

$$\rho(T) = g\int_0^\infty \frac{4\pi p^3 dp}{(2\pi\hbar)^3}\left(e^{\frac{p}{k_B T}} \pm 1\right)^{-1} = \begin{cases} \frac{g\sigma_B T^4}{2} & \text{bosons} \\ \frac{7g\sigma_B T^4}{16} & \text{fermions} \end{cases}$$

- Each species and spin of a massless fermion contributes to the energy density, pressure and entropy the same way as each polarisation state of a photon with the additional factor 7/8

Expansion

- Governed by Einstein equation (no curvature, k=0)

$$\frac{\dot{a}^2}{a^2} = H^2 = \frac{8\pi G}{3}\rho(T)$$

Combining all together the time (from the time derivative in the scale factor) becomes

$$t = -\int \frac{s'(T)dT}{s(T)\sqrt{24\pi G\rho(T)}} + const.$$

Relativistic gas

- For a highly relativistic gas (*m*<<k_B*T)* the density and entropy become

$$\rho(T) = \frac{1}{2}N\sigma_B T^4 \qquad s(T) = \frac{2}{3}N\sigma_B T^3$$

with *N* the number of particles and anti-particles and all spin states (and a faction 7/8 for fermions)

$$t = \sqrt{\frac{3}{16\pi G N\sigma_B}}\frac{1}{T^2} + const.$$

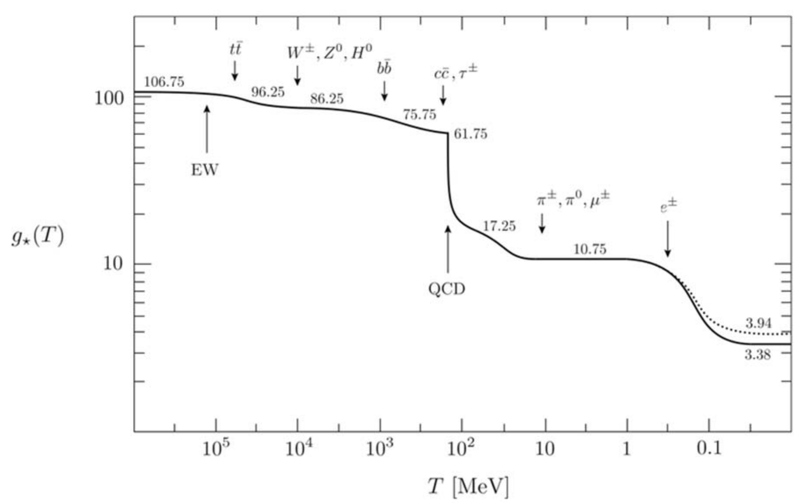
Summary: The 'equation of state' of cosmic matter, general

occupation number → number density → energy density;
grand-canonical partition sum → pressure, entropy

	relativistic		non-relativistic
	Bosons	Fermions	
number density n	$g_B \frac{\zeta(3)}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3$	$\frac{3}{4}\frac{g_F}{g_B}n_B$	$g\left(\frac{kT}{2\pi\hbar}\right)^{3/2} e^{-kT/mc^2}$
energy density u	$g_B \frac{\pi^2}{30}\frac{(kT)^4}{(\hbar c)^3}$	$\frac{7}{8}\frac{g_F}{g_B}u_B$	$\frac{3}{2}nkT$
pressure P	$g_B \frac{\pi^2}{90}\frac{(kT)^4}{(\hbar c)^3} = \frac{u_B}{3}$	$\frac{7}{8}\frac{g_F}{g_B}P_B$	nkT
entropy density s	$g_B k \frac{2\pi^2}{45}\left(\frac{kT}{\hbar c}\right)^3$	$\frac{7}{8}\frac{g_F}{g_B}s_B$	

Relativistic effective degrees of freedom g_*

$$g_* = \Sigma_B g_B \left(\frac{T_B}{T_\gamma}\right)^4 + \frac{7}{8}\Sigma_F\, g_F \left(\frac{T_F}{T_\gamma}\right)^4$$



Thermal history

Start with a very hot state: *T=10^11 K≈1MeV*

$$m_\mu >> k_B T >> m_e$$

- photons, electrons and positrons, all neutrinos in thermal equilibrium
  - latter through the neutral current interaction

$$e^+ + e^- \Leftrightarrow \nu + \overline{\nu}$$

- Adding all states $N = 2 + \frac{7}{8}(6+4) = \frac{43}{4}$
- and time

$$t = \sqrt{\frac{3c^2}{172\pi G\sigma_B}} T^{-2} + const.$$