Cosmology

TUM WS 2015/2016

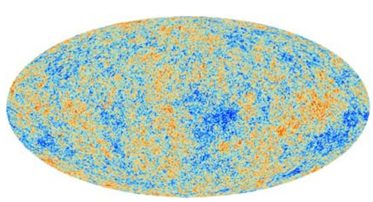
Lecture 11

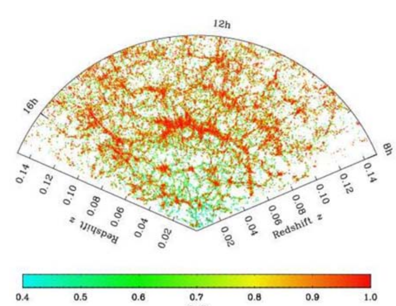
Wolfgang Hillebrandt and Bruno Leibundgut

(http://www.eso.org/~bleibund/Cosmology)

Formation of large scale structure

?





Planck

SDSS

Implementation

- Background metric $g_{\alpha\beta}$ (FRW) and sources $T_{\alpha\beta}$
- Perturbations: $\delta g_{\alpha\beta}$; $\delta T_{\alpha\beta}$
- Linearize Einstein equations
- Lin. differential eq. of 2nd order

$\mathcal{L}(g_{\alpha\beta})\delta g_{\alpha\beta} = \delta T_{\alpha\beta}$

Example: ‘Dust cosmos’

$p = 0$; $\rho = \rho_o + \delta\rho$

$a(t) = a_o(t) + \delta a$ (‘o’: unperturbed solution)

Friedmann equation:

$$\left(\frac{\dot{a}_o(t)}{a_o(t)}\right)^2 = \frac{8\pi G}{3}\rho_o - \frac{k_0}{a_0^2};\quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_o + \delta\rho) - \frac{k}{a^2}$$

Continuity equation: $\frac{\delta\rho}{\rho_0} \equiv \delta = -3\,\frac{\delta a}{a_0}$

$$\Rightarrow\quad \ddot{\delta} + 2\,\frac{\dot{a_0}}{a_0}\,\delta = 4\pi G\rho_o\delta$$

- For $k_o \approx k = 0$: $a_o \propto t^{3/2}$; $\rho_o \propto a_0^{-3}$ (flat Universe)

$\Rightarrow$ $\delta \propto t^{2/3} \propto a_o$

For radiation dominated cosmos (scales > horizon):

$\delta \propto t \propto a_o^2$

More general approach

- Expand solutions in an appropriate orthonormal basis (e.g., plane waves in case of a flat Universe) $\Rightarrow$ Fourier transform
  $\hat{L}(\vec{k})\ \delta g(\vec{k}) = \delta T(\vec{k})$, $\vec{k}$: wave vector

Solutions describe the evolution of independent modes (consequence of linearization).

However, equations are no longer covariant.

$\Rightarrow$ Growth rates of modes depend on coordinates!

Ways out

- For $\lambda << d_H$: GR effects small $\Rightarrow$ use standard comoving coordinates + Newtonian gravity
- But: For every mode there exists a time when $\lambda > d_H \Rightarrow$ Newtonian theory incorrect
- In GR: Choice of coordinates such that an interpretation of quantities is “physical”, e.g., $\delta T^0{}_0$ is the matter (energy) density
- Or: form scalars $\Rightarrow$ independent of coordinates but difficult to interpret

Primordial potential fluctuations

- Models of inflation:
  scalar field ϕ (‘inflaton’), potential $V(\phi)$
- Perturbations in the energy density:
  $\delta\rho_\Phi = \frac{\partial V}{\partial \Phi}\delta\Phi$
- Power spectrum:
  $< \delta_k \delta_{k'} >= \frac{2\pi}{k^3}\delta(k-k')P(k),$
- Many models of inflation yield:
  $P_{scalar} \propto k^{n_s-1},\ n_s \cong 1$

(see, e.g., http://pdg.lbl.gov/2016/reviews/rpp2016-rev-inflation.pdf)

Qualitative evolution of perturbations

- ‘Dark energy’: cosmological constant
- ‘Dark matter’: WIMPS
- Assumption: Decouple at $T = T_{weak} \approx 1\text{MeV}$
- Baryons, photons: decouple at $T = T_{rec} \approx 0.26\text{eV}$

$\Rightarrow$ Different evolution; treat each component seperately

Qualitative evolution of perturbations

- Each component (x) has its equation of state

$$p_x = W_x \cdot \rho_x c^2;\ W_x = \text{const.} = \begin{cases} 0 \text{ (dust)} \\ 1/3 \text{ (radiation)} \\ -1\ (\Lambda) \end{cases}$$

- Then: $\delta p_x = W_x \cdot \delta(\rho_x c^2)$
- Define: ‘velocity dispersion’ $v_x^2 \equiv \frac{\dot{p}_x}{\dot{\rho}_x}$ (= c_s^2)
- $\Rightarrow$ $v_x^2 = W_x c^2$

Qualitative evolution of perturbations

- Assume: spherical perturbation, radius $\lambda > d_H$ (Hubble radius), matter density ρ_1 (pressure does not effect growth rate)
- embedded in Friedmann cosmos k=0, ρ_0
- $\rho_1 = \rho_0 + \delta\rho$, $\delta\rho$ 'small'

$$\longrightarrow \left(\frac{\dot{a}_o(t)}{a_o(t)}\right)^2 = \frac{8\pi G}{3}\rho_o; \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_o + \delta\rho) - \frac{c^2}{a^2}$$

Qualitative evolution of perturbations

- In general: For $\delta\rho_x > 0$ perturbations grow for $\lambda > d_H$

$$\left(\frac{\delta\rho_x}{\rho_x}\right) \propto \begin{cases} a^2, t < t_{eq}^x \\ a, t > t_{eq}^x \end{cases} \quad \begin{matrix} \rho \propto a^{-4}\ (radiation) \\ \rho \propto a^{-3}\ (rel. matter) \end{matrix}$$

- What happens for $\lambda < d_H$?

Qualitative evolution of perturbations

- Pressure can stop growth if

$$\frac{t_p(pressure\ readjustment)}{t_{grav}(gravitational\ collapse)} < 1$$

$$t_p \approx \frac{\lambda}{v} < t_{grav} \approx \frac{1}{\sqrt{G\rho}}$$

('sound travel') < ('free fall')

$\Rightarrow$ Modes with $\lambda < \lambda_J \propto \frac{v}{\sqrt{G\rho}}$ do not grow!

<u>'Jeans length'</u>: $\lambda_J \equiv \sqrt{\pi}\,\frac{v}{\sqrt{G\rho}}$

Qualitative evolution of perturbations

- <u>Note:</u> Also collisionless matter ('dark matter') has a Jeans length because of its velocity dispersion. v^2 acts like a pressure!

Qualitative evolution of perturbations

- For a universe consisting of several components: v is of the perturbed and ρ the dominant gravitating component (governs $\dot{a}$(t)).
- Example: dark matter during radiation dominated phase

  t_{grav}(DM) < t_p $\Rightarrow$ $\lambda > \lambda_J \Rightarrow$ collapse
- But: $t_{exp} \propto (G\rho_{rad})^{1/2} < t_{grav}$(DM)

$\Rightarrow$ Expansion faster then the collapse of DM clumps

Qualitative evolution of perturbations

- In radiation dominated phase:

  Only modes with $\lambda > d_H$ are unstable ($\propto a^2$)
- In matter dominated phase:

  No effects of pressure for modes with $\lambda >> \lambda_J$

  $\Rightarrow$ all modes with $\lambda >> \lambda_J$ (i.e., $\lambda > d_H$ and $d_H > \lambda >> \lambda_J$) grow ($\propto$ a) and modes with $\lambda \geq \lambda_J$ grow more slowly

Perturbations in DM component

- <u>Denote</u>

  a_{nr}: scale factor at which DM becomes non-relat.

  a_{eq}: transition radiation dominated (RD) to matter dominated (MD)

  a_e: $\lambda = d_H$
- Modes with $\lambda \propto a$ drop below d_H during the RD phase (most relevant modes): $a_{nr} < a_e < a_{eq}$
- For DM: $a < a_{nr}$: $v \approx c$; $a > a_{nr}$: $v \propto a^{-1}$

Perturbations in DM component

$$\rho_{dom}^{-1/2} = \begin{Bmatrix} \rho_{rad}^{-1/2} \propto a^2; a < a_{eq} \\ \rho_{DM}^{-1/2} \propto a^{\frac{3}{2}}; a > a_{eq} \end{Bmatrix}$$

$$\Rightarrow \quad \lambda_J \propto \frac{v}{\rho_{dom}^{1/2}} \propto \begin{Bmatrix} a^2;\ a < a_{nr} \\ a;\ a_{nr} < a < a_{eq} \\ a^{1/2};\ a_{eq} < a \end{Bmatrix}$$

Perturbations in baryonic component

$$\lambda_J \propto \begin{cases} a^2;\ a < a_{eq} \\ a^{3/2};\ a_{eq} < a < a_{B\gamma} \\ a;\ a_{B\gamma} < a < a_{dec} \end{cases} \quad (with\ a_{B\gamma}\colon \rho_\gamma = \rho_B)$$

and $\lambda_j \propto a^{1/2};\ a_{dec} < a$

Define: 'Jeans mass' of component x

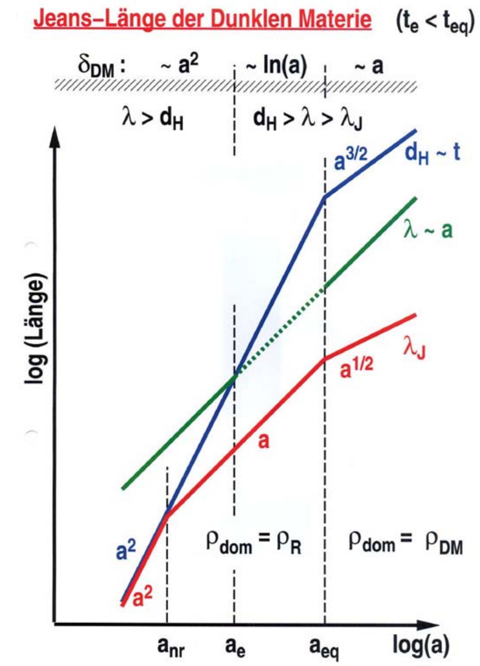
$$M_J^{(x)} \equiv \frac{4\pi}{3}\rho_x\left(\frac{\lambda_J^{(x)}}{2}\right)^3$$

DM, $a_{eq} < a$: $M_J = 3.2 \times 10^{14} M_\odot (\Omega h^2)^{-2} (\frac{a}{a_{eq}})^{-3/2}$

- Note:
  Perturbations in DM grow for $a > a_{eq}$
  Baryonic perturbations grow for $a > a_{dec}$
- Since $a_{dec} > a_{eq} \Rightarrow$ only perturbation in DM grow for $a_{eq} < a < a_{dec}$
- When the baryons decouple, they 'feel' already the potential wells of the DM particles

$\Rightarrow \delta_B$ grows fast for a short time after a_{dec} until $\delta_B \approx \delta_{DM}$

Jeans length and mass: dark matter



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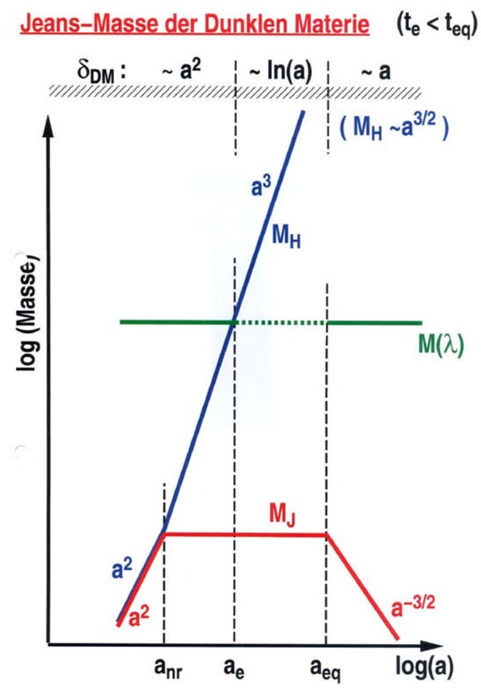
Jeans length and mass: dark matter



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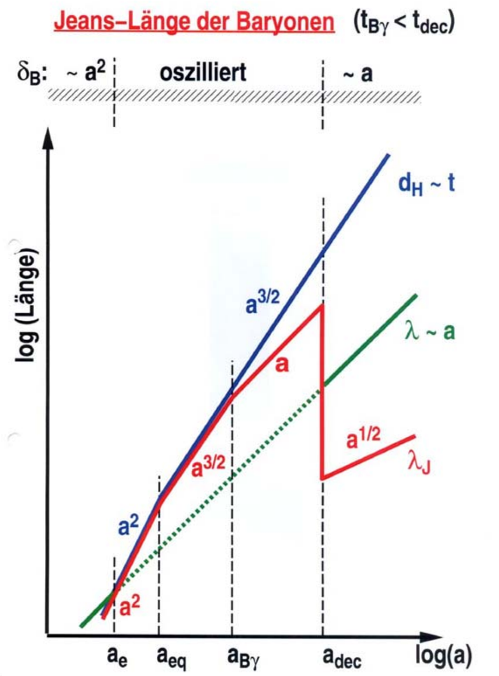
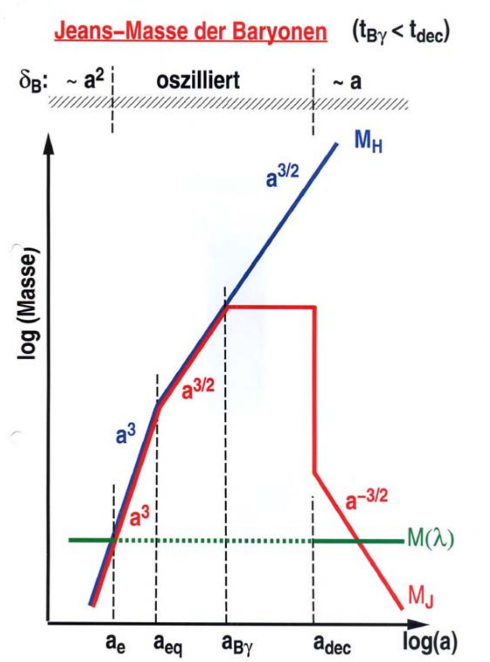
Jeans length and mass: baryonic matter



Jeans length and mass: baryonic matter



Summary

- DM perturbations grow first ($a > a_{eq}$)
- Baryons gather in potential wells of the DM ($a > a_{dec}$)
- Small-scale perturbations in the DM grow first (and strongest): merge to form larger structures ("hierarchical clustering")

Numerical solutions

1. <u>Collisionless matter; (Newtonian) N-body simulations</u>

   <u>Goal:</u> Non-linear evolution of self-gravitating matter without other forces in the expanding Universe.

   <u>Solves:</u>
   - Problem of large-scale structure
   - 1^{st} approximation to galaxy formation in a Universe dominated by dark matter and dark energy

Numerical solutions

2. Baryonic matter; hydrodynamics

Use: 'Smoothed-particle' hydrodynamics
(similar to N-body; approximates density $\rho(\vec{r})$ by distribution of N 'particles')

(*References:*
Monaghan, Comp.Phys.Rep. ***3,*** *71 (1985)*
Benz et al., Astrophys. J. ***348****, 647 (1990)*
Steinmetz & Müller, Astron.Astrophys. ***268,*** *391 (1993))*
Vogelsberger et al., arXiv:1909.07976v6 (2019)

Lookback time and redshift

- Redshift is a 'observer' parameter
  - convenient as measured directly
- Lookback time is a function of the redshift and the metric

$$t_0 - t_1 = \frac{1}{H_0}\int_0^{z_1} \frac{1}{1+z}\frac{dz}{\sqrt{(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda}}$$

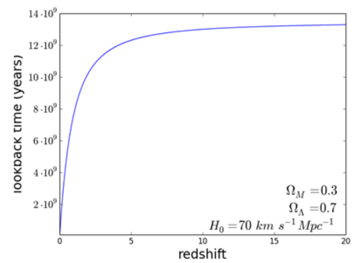
(ignored radiation density and assumed a flat universe)

Lookback time

- Three parameters
  - percentage of age of the universe
  - Hubble age tH_0
  - age in years

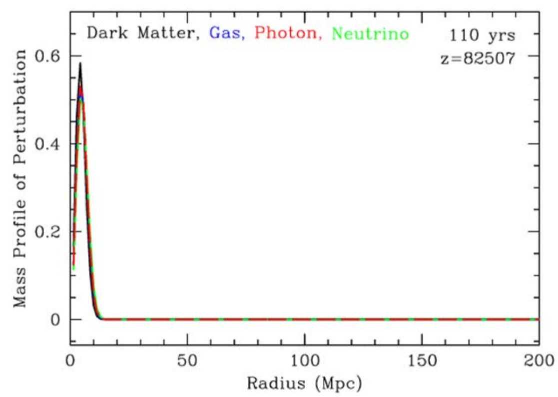
Parameters used for the calculation:
H0 = 70.0; Omega_matter=0.30; Omega_Lambda=

z	%	t(*H0)	years
0.01	0.01	0.01	1.39E+08
0.02	0.02	0.02	2.75E+08
0.03	0.03	0.029	4.10E+08
0.04	0.04	0.039	5.43E+08
0.05	0.05	0.048	6.74E+08
0.06	0.06	0.057	8.03E+08
0.07	0.07	0.067	9.30E+08
0.08	0.08	0.076	1.06E+09
0.09	0.09	0.084	1.18E+09
0.1	0.1	0.093	1.30E+09
0.11	0.11	0.102	1.42E+09
0.12	0.11	0.11	1.54E+09
0.13	0.12	0.119	1.66E+09
0.14	0.13	0.127	1.77E+09
0.15	0.14	0.135	1.89E+09
0.16	0.15	0.143	2.00E+09
0.17	0.16	0.151	2.11E+09
0.18	0.16	0.159	2.22E+09
0.19	0.17	0.167	2.33E+09
0.2	0.18	0.174	2.43E+09
0.25	0.22	0.211	2.94E+09
0.3	0.25	0.245	3.42E+09
0.35	0.29	0.277	3.86E+09
0.4	0.32	0.307	4.28E+09
0.45	0.35	0.335	4.67E+09
0.5	0.37	0.361	5.04E+09
0.55	0.4	0.386	5.39E+09
0.6	0.42	0.409	5.71E+09
0.65	0.45	0.431	6.01E+09
0.7	0.47	0.451	6.30E+09
0.75	0.49	0.471	6.57E+09
0.8	0.51	0.489	6.83E+09
0.85	0.52	0.506	7.07E+09
0.9	0.54	0.522	7.30E+09
0.95	0.56	0.538	7.51E+09
1	0.57	0.552	7.72E+09
1.1	0.6	0.579	8.09E+09
1.2	0.63	0.604	8.43E+09
1.3	0.65	0.625	8.74E+09
1.4	0.67	0.645	9.01E+09
1.5	0.69	0.663	9.27E+09
1.6	0.71	0.68	9.50E+09
1.7	0.72	0.695	9.71E+09
1.8	0.74	0.709	9.90E+09
1.9	0.75	0.721	1.01E+10
2	0.76	0.733	1.02E+10
2.5	0.81	0.78	1.09E+10
3	0.84	0.813	1.14E+10
3.5	0.87	0.837	1.17E+10
4	0.89	0.856	1.20E+10
5	0.91	0.881	1.23E+10
6	0.93	0.898	1.26E+10
7	0.94	0.91	1.27E+10
8	0.95	0.919	1.28E+10
9	0.96	0.926	1.29E+10
10	0.97	0.931	1.30E+10
11	0.97	0.935	1.31E+10
12	0.97	0.938	1.31E+10
15	0.98	0.945	1.32E+10
20	0.99	0.951	1.33E+10
1000	1	0.96	1.35E+10



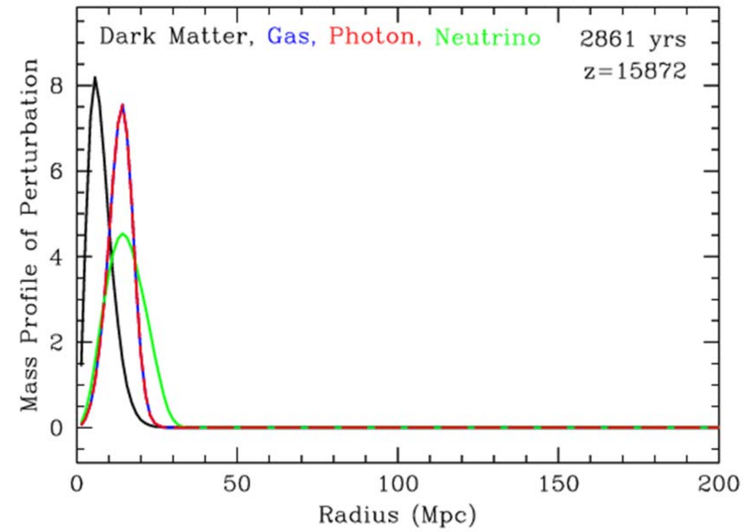
Summary of the evolution

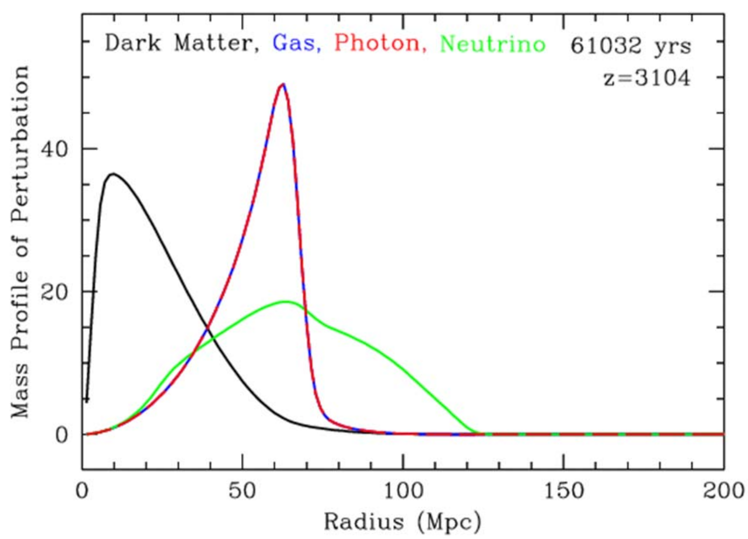
- Interplay of neutrinos, radiation, dark matter and baryons

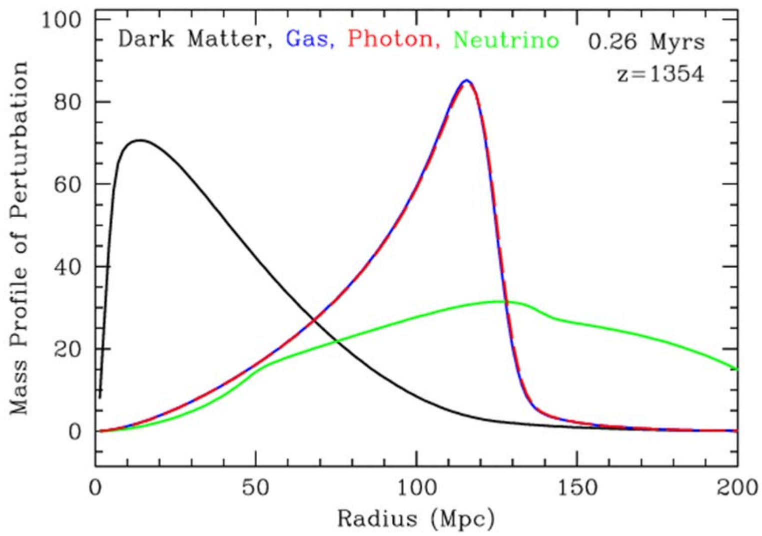


David Weinberg
Daniel Eisenstein

Snapshots of evolution of the radial mass profile vs. comoving radius of an initially pointlike overdensity located at the origin. All perturbations are fractional for that species. Units of the mass profile are arbitrary but are correctly scaled between the panels for the synchronous gauge.


Dark Matter, Gas, Photon, Neutrino
2861 yrs
z=15872
Mass Profile of Perturbation
Radius (Mpc)
0
2
4
6
8
0
50
100
150
200


Dark Matter, Gas, Photon, Neutrino
61032 yrs
z=3104
Mass Profile of Perturbation
Radius (Mpc)
0
20
40
0
50
100
150
200



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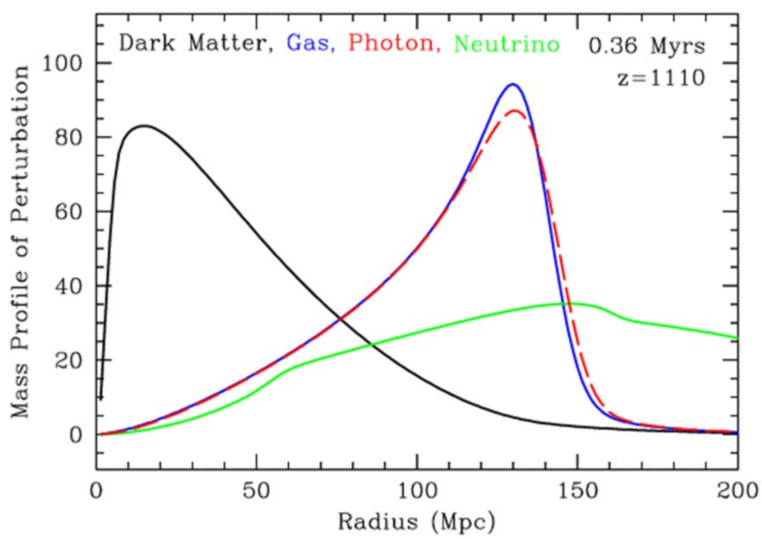

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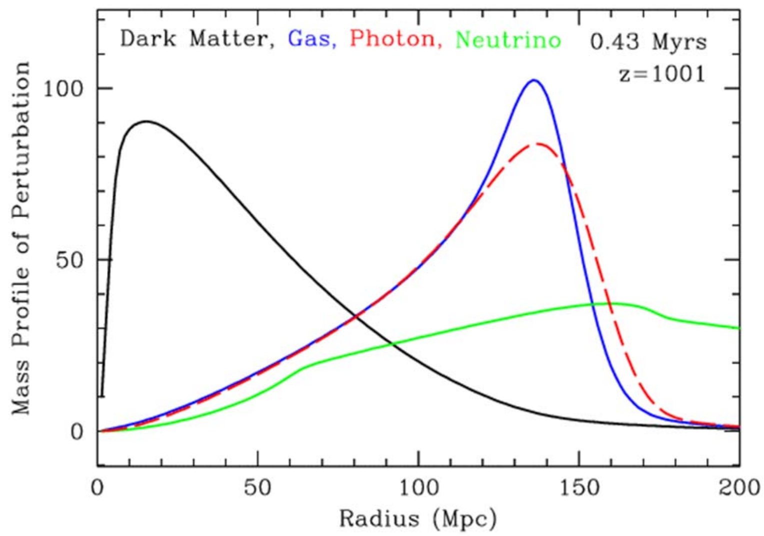
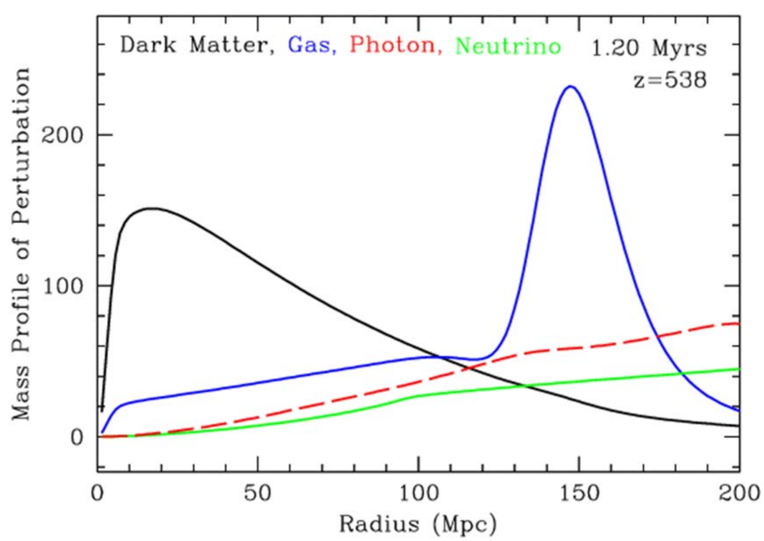
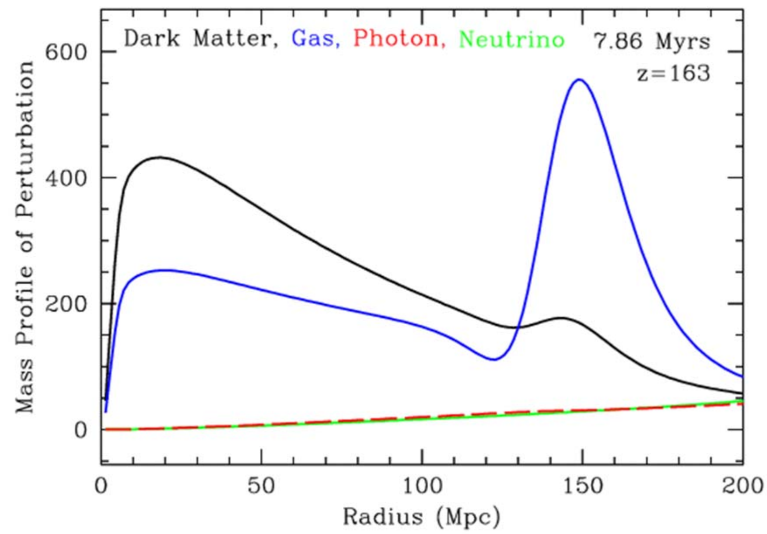

Dark Matter, Gas, Photon, Neutrino
0.43 Myrs
z=1001
Mass Profile of Perturbation
0
50
100
0
50
100
150
200
Radius (Mpc)


Dark Matter, Gas, Photon, Neutrino
1.20 Myrs
z=538
Mass Profile of Perturbation
0
100
200
0
50
100
150
200
Radius (Mpc)



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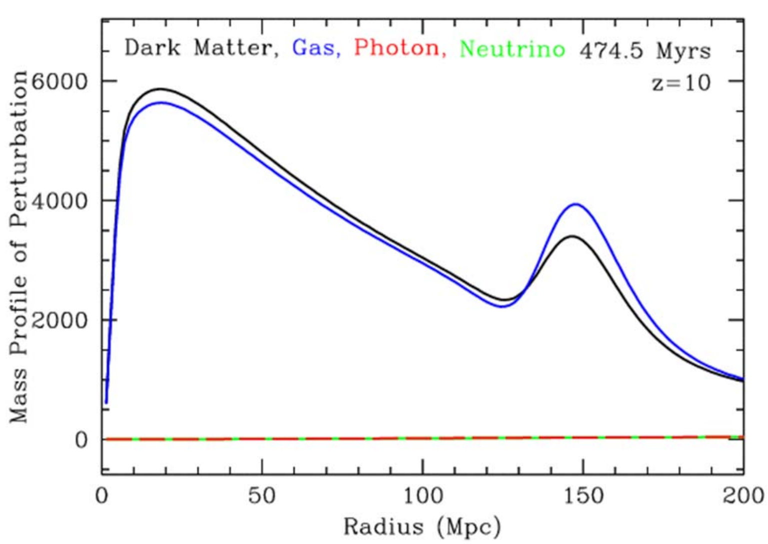

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Growth of Large Scale Structure

- The galaxy distribution reflects the matter distribution
- It growths with the amount of matter and the change of expansion rate
  - mostly on the matter density Ω_M
  - cosmological constant Ω_Λ
    - late phenomenon
    - leads to accelerated expansion
    - hence slower clustering

Growth of structure

- Depends on the contents of the universe
  - Hot Dark Matter
    - dark matter particles have relativistic velocities when they decouple
    - dampen small scales
  - Cold Dark Matter
    - move slowly (non-relativistic) at decoupling

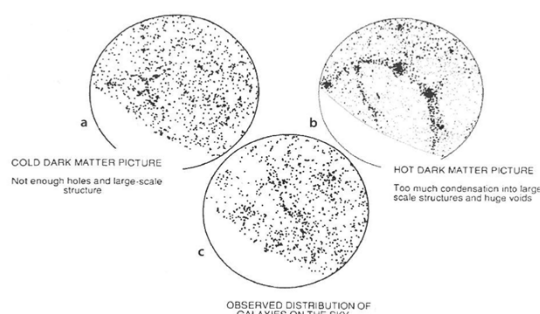
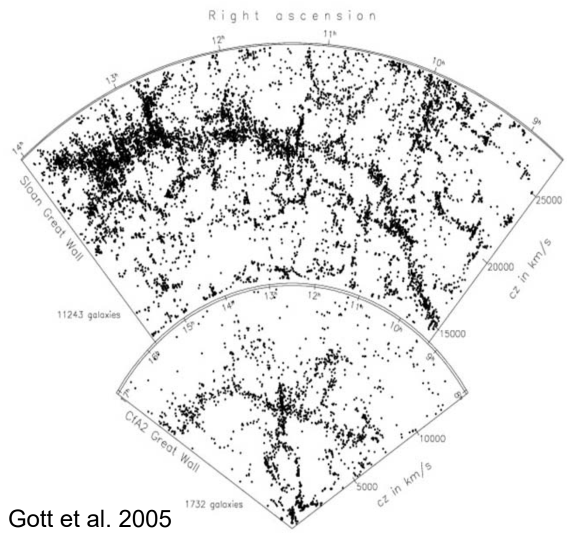


Fig. 14.2a–c. Simulations of the expectations of **a** the cold dark matter model with $\Omega_0 = 0.2$ and $\Omega_\Lambda = 0$ and **b** the hot dark matter model with $\Omega_0 = 1$ and $\Omega_\Lambda = 0$ for origin of large-scale structure of the Universe (Frenk, 1986). **c** These simulations can be compared with the large-scale distribution of galaxies observed in the Harvard–Smithsonian Center for Astrophysics Survey of Galaxies (Fig. 2.7). The unbiased cold dark matter model does not produce sufficient large-scale structure in the form of voids and filaments of galaxies, whereas the unbiased hot dark matter model produces too much clustering

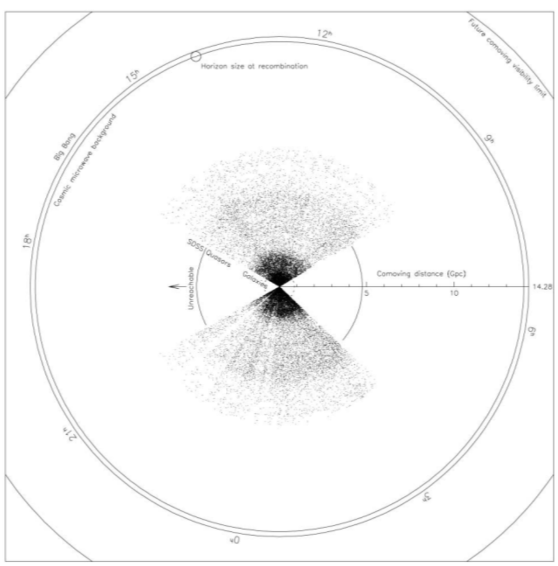
Observed matter distribution

- Redshift surveys



Gott et al. 2005

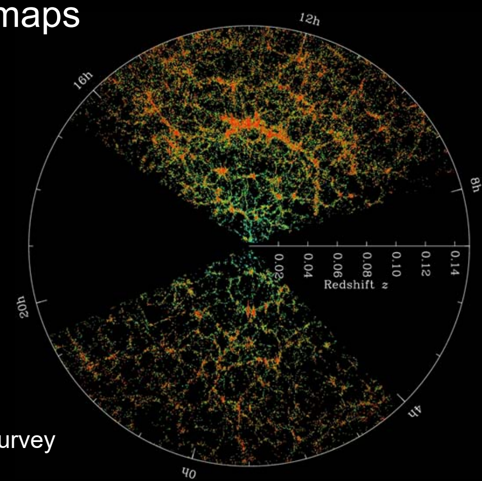
Mapping the Universe



Gott et al. 2005

Matter distribution in todays universe

- Galaxy maps

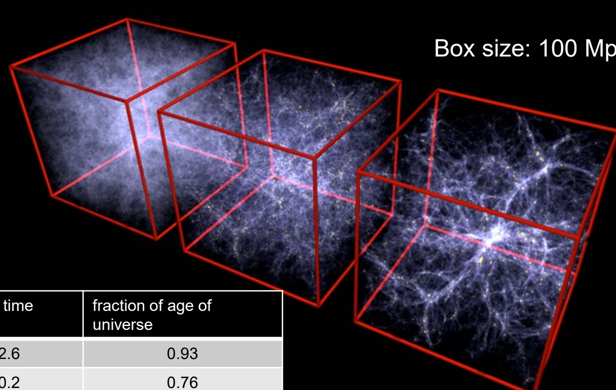


Sloan Digital Sky Survey

Example of a simulation

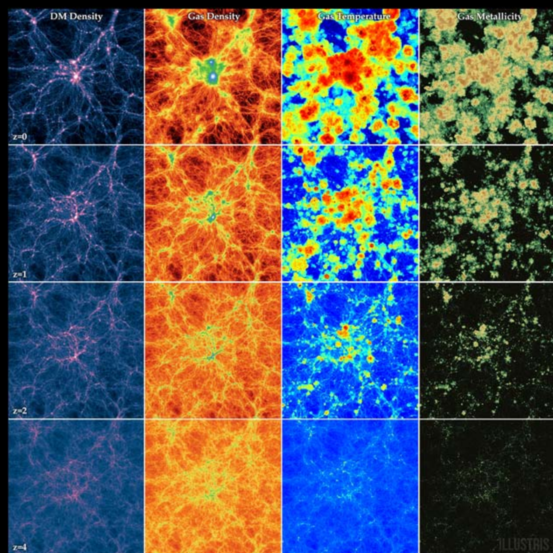
- Springel et al. (Virgo consortium)
  - snapshots at z=6, z=2, and z=0

Box size: 100 Mpc/h



redshift	lookback time (Gyr)	fraction of age of universe
6	12.6	0.93
2	10.2	0.76

Example of a simulation



box size: 15 Mpc/h

Illusstris simlation
(http://www.illustris-project.org)

To be continued ....