Cosmology

TUM WS 2019/2020

Lecture 10

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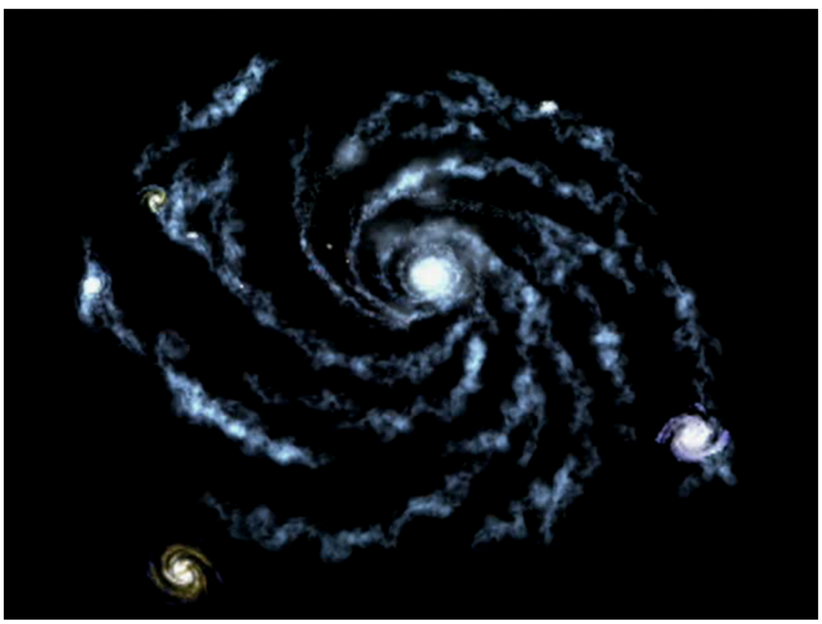
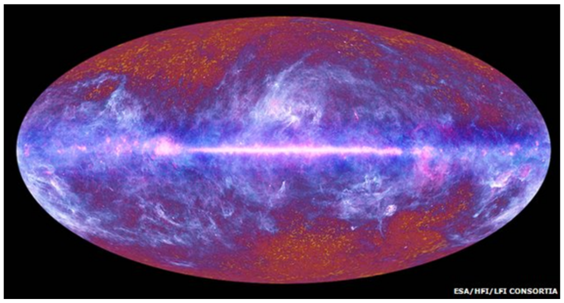
(http://www.eso.org/~bleibund/Cosmology)

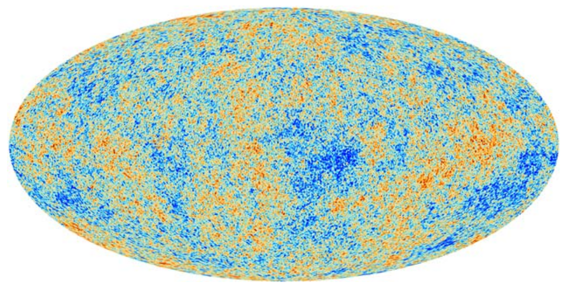




Planck Surveyor

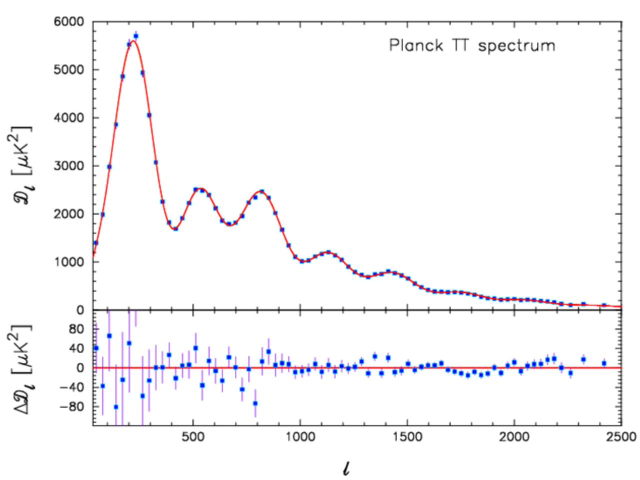
Data

After foreground subtraction



CMB cleaned from foregrounds

- After removing the foreground radiation and adding the small scales from other experiments (ACT, SPT)

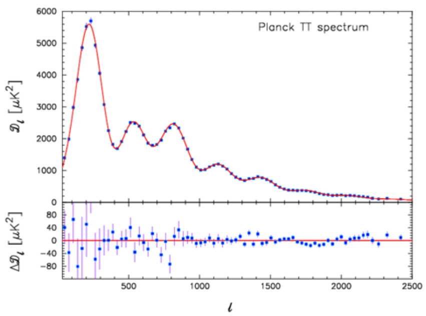


Red line: best fit cosmological model (‘ΛCDM’)

residuals

The final step

- How do we get from the power spectrum to cosmological parameters?



→ Ω, Ω_M, Ω_B, Ω_Λ, H_0,

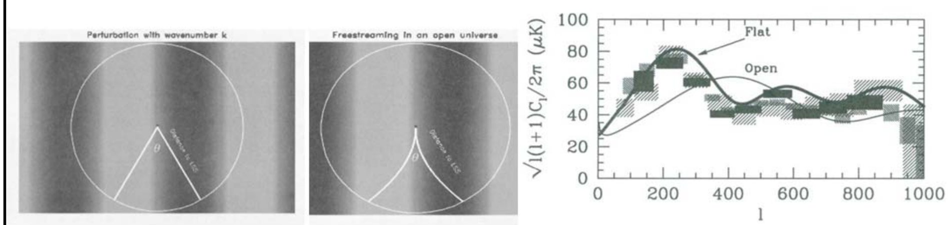
Cosmological effects in the CMB

- Influence of
  - Curvature

    $\Omega_K = 1 - \Omega_M - \Omega_\Lambda$ (ignore radiation density)
  - Normalisation
  - Primordial tilt, spectral index n_S
    - describes the scale dependence of the fluctuations as they are created
  - ratio of tensor to scalar modes *r*
  - optical depth to recombination τ
    - mostly due to reionization

Curvature

- The geodesics of a light ray will not be straight and the mean angle of the fluctuations does not reflect the true perturbation scale
- peak at about l≈180 (1 degree) indicates flat geometry

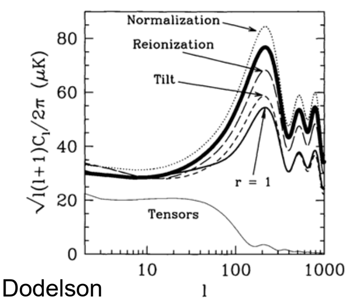


Cosmological effects on the CMB (cont.)

- Influence of
  - Baryon density $\Omega_B h^2$
  - (Dark) Matter density $\Omega_M h^2$
  - Cosmological Constant energy density Ω_Λ
- Other parameters
  - neutrino mass
  - equation of state of dark energy ***w***
  - tensor tilt n_T

Normalisation, Tilt, Reionization and Tensor modes

- Normalisation shifts curves up and down
- Tilt (spectral index) n_S decreases the power on small scales
- Reionization at late times changes the optical depth (due to free e^-)
  - restores isotropy and large l are suppressed
- Tensor modes from gravitational waves
  - decrease rapidly once they enter the horizon
  - only contribute to small l

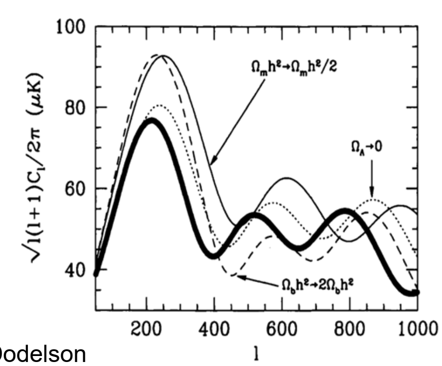


Dodelson

Figure 8.18. Changes in the anisotropy spectrum as C_{10}, τ, r, and n vary. The base model (thick curve) is a flat universe with no reionization or tensors, $n = 1$, $\Omega_m h^2 = 0.16$, $\Omega_b h^2 = 0.021$, and $\Omega_\Lambda = 0.7$. The thin curves vary one parameter each. *Reionization* corresponds to letting the optical depth back to the last scattering surface equal 0.2 instead of zero; *tilt* has a primordial spectum with $n = 0.8$; $r = 1$ has an equal contribution of scalars and tensors to the quadrupole; and *normalization* has C_{10} 10% higher than the base model. The curve labeled *tensors* is the contribution to the anisotropy from tensors only. Only the $r = 1$ curve includes this contribution; all others assume no anisotropy from tensors.

Baryon and Matter density

- Mass components shift the peaks
  - lower mass increases the distances of the peaks
- Baryon density affects the strength of the odd and even peaks
- Matter density sets the relative time between recombination and equality
  - smaller mass moves time of equality closer to recombination increasing importance of the radiation density

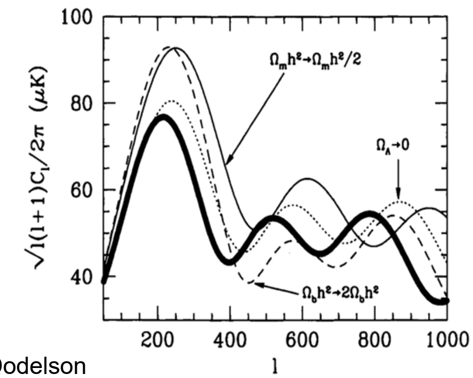


Dodelson

Figure 8.19. Changes in the anisotropy spectrum as baryon density, matter density, and cosmological constant vary. Same base model as Figure 8.18.

Cosmological Constant

- Not important at recombination
- Is a late time effect (part of Integrated Sachs-Wolfe effect)
  - affects the free streaming of the photons
  - affects largest scales (small l)



Dodelson

Figure 8.19. Changes in the anisotropy spectrum as baryon density, matter density, and cosmological constant vary. Same base model as Figure 8.18.

Effect of the different parameters on CMB spectrum

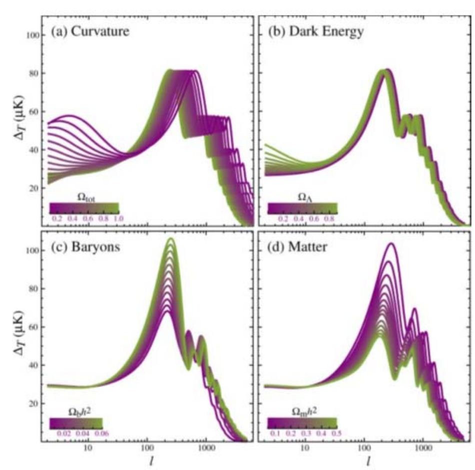
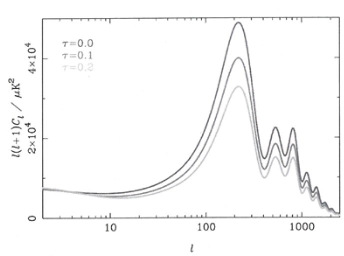


Figure 4 Sensitivity of the acoustic temperature spectrum to four fundamental cosmological parameters. (*a*) The curvature as quantified by Ω_{tot}. (*b*) The dark energy as quantified by the cosmological constant Ω_Λ ($w_\Lambda = -1$). (*c*) The physical baryon density $\Omega_b h^2$. (*d*) The physical matter density $\Omega_m h^2$. All are varied around a fiducial model of $\Omega_{tot} = 1$, $\Omega_\Lambda = 0.65$, $\Omega_b h^2 = 0.02$, $\Omega_m h^2 = 0.147$, $n = 1$, $z_{ri} = 0$, $E_i = 0$.

Hu & Dodelson 2002



Longair 2008

CMB-Leistungsspektren

Film hergestellt von
Matthias Bartelmann
mithilfe von CMBfast von
Uros Seljak und
Matias Zaldarriaga

Cosmological parameters

Planck 2013

Parameter	Definition
$\omega_b \equiv \Omega_b h^2$	Baryon density today
$\omega_c \equiv \Omega_c h^2$	Cold dark matter density today
$100\theta_{MC}$	$100\times$ approximation to r_*/D_A (CosmoMC)
τ	Thomson scattering optical depth due to reionization
Ω_K	Curvature parameter today with $\Omega_{tot} = 1 - \Omega_K$
$\sum m_\nu$	The sum of neutrino masses in eV
$m^{eff}_{\nu,\,sterile}$	Effective mass of sterile neutrino in eV
w_0	Dark energy equation of state[a], $w(a) = w_0 + (1-a)w_a$
w_a	As above (perturbations modelled using PPF)
N_{eff}	Effective number of neutrino-like relativistic degrees of freedom (see text)
Y_P	Fraction of baryonic mass in helium
A_L	Amplitude of the lensing power relative to the physical value
n_s	Scalar spectrum power-law index ($k_0 = 0.05\ \mathrm{Mpc}^{-1}$)
n_t	Tensor spectrum power-law index ($k_0 = 0.05\ \mathrm{Mpc}^{-1}$)
$dn_s/d\ln k$	Running of the spectral index
$\ln(10^{10}A_s)$	Log power of the primordial curvature perturbations ($k_0 = 0.05\ \mathrm{Mpc}^{-1}$)
$r_{0.05}$	Ratio of tensor primordial power to curvature power at $k_0 = 0.05\ \mathrm{Mpc}^{-1}$
Ω_Λ	Dark energy density divided by the critical density today
t_0	Age of the Universe today (in Gyr)
Ω_m	Matter density (inc. massive neutrinos) today divided by the critical density
σ_8	RMS matter fluctuations today in linear theory
z_{re}	Redshift at which Universe is half reionized
H_0	Current expansion rate in $\mathrm{km\,s^{-1}Mpc^{-1}}$
$r_{0.002}$	Ratio of tensor primordial power to curvature power at $k_0 = 0.002\ \mathrm{Mpc}^{-1}$
$10^9 A_s$	$10^9 \times$ dimensionless curvature power spectrum at $k_0 = 0.05\ \mathrm{Mpc}^{-1}$
$\omega_m \equiv \Omega_m h^2$	Total matter density today (inc. massive neutrinos)
z_*	Redshift for which the optical depth equals unity (see text)
$r_* = r_s(z_*)$	Comoving size of the sound horizon at $z = z_*$
$100\theta_*$	$100\times$ angular size of sound horizon at $z = z_*$ (r_*/D_A)
z_{drag}	Redshift at which baryon-drag optical depth equals unity (see text)
$r_{drag} = r_s(z_{drag})$	Comoving size of the sound horizon at $z = z_{drag}$
k_D	Characteristic damping comoving wavenumber (Mpc^{-1})
$100\theta_D$	$100\times$ angular extent of photon diffusion at last scattering (see text)
z_{eq}	Redshift of matter-radiation equality (massless neutrinos)
$100\theta_{eq}$	$100\times$ angular size of the comoving horizon at matter-radiation equality
$r_{drag}/D_V(0.57)$	BAO distance ratio at $z = 0.57$ (see Sect. 5.2)

Planck 2013 parameters

Table 2. Cosmological parameter values for the six-parameter base ΛCDM model. Columns 2 and 3 give results for the *Planck* temperature power spectrum data alone. Columns 4 and 5 combine the *Planck* temperature data with *Planck* lensing, and columns 6 and 7 include *WMAP* polarization at low multipoles. We give best fit parameters (i.e. the parameters that maximise the overall likelihood for each data combination) as well as 68% confidence limits for constrained parameters. The first six parameters have flat priors. The remainder are derived parameters as discussed in Sect. 2. Beam, calibration parameters, and foreground parameters (see Sect. 4) are not listed for brevity. Constraints on foreground parameters for *Planck*+WP are given later in Table 5.

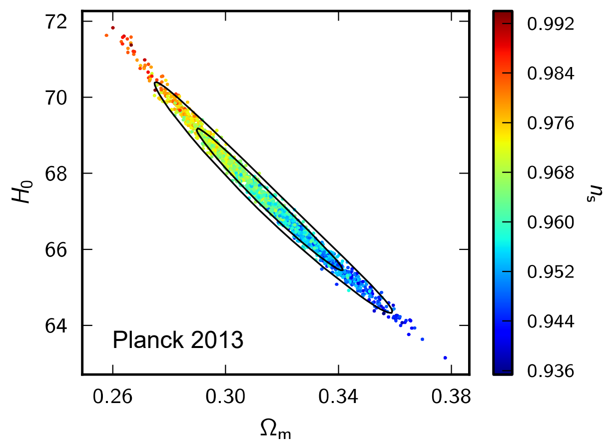
	Planck		*Planck*+lensing		*Planck*+WP	
Parameter	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033	0.022242	0.02217 ± 0.00033	0.022032	0.02205 ± 0.00028
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031	0.11805	0.1186 ± 0.0031	0.12038	0.1199 ± 0.0027
$100\theta_{MC}$	1.04122	1.04132 ± 0.00068	1.04150	1.04141 ± 0.00067	1.04119	1.04131 ± 0.00063
τ	0.0925	0.097 ± 0.038	0.0949	0.089 ± 0.032	0.0925	$0.089^{+0.012}_{-0.014}$
n_s	0.9624	0.9616 ± 0.0094	0.9675	0.9635 ± 0.0094	0.9619	0.9603 ± 0.0073
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072	3.098	3.085 ± 0.057	3.0980	$3.089^{+0.024}_{-0.027}$
Ω_Λ	0.6825	0.686 ± 0.020	0.6964	0.693 ± 0.019	0.6817	$0.685^{+0.018}_{-0.016}$
Ω_m	0.3175	0.314 ± 0.020	0.3036	0.307 ± 0.019	0.3183	$0.315^{+0.016}_{-0.018}$
σ_8	0.8344	0.834 ± 0.027	0.8285	0.823 ± 0.018	0.8347	0.829 ± 0.012
z_{re}	11.35	$11.4^{+4.0}_{-2.8}$	11.45	$10.8^{+3.1}_{-2.5}$	11.37	11.1 ± 1.1
H_0	67.11	67.4 ± 1.4	68.14	67.9 ± 1.5	67.04	67.3 ± 1.2
$10^9 A_s$	2.215	2.23 ± 0.16	2.215	$2.19^{+0.12}_{-0.14}$	2.215	$2.196^{+0.051}_{-0.060}$
$\Omega_m h^2$	0.14300	0.1423 ± 0.0029	0.14094	0.1414 ± 0.0029	0.14305	0.1426 ± 0.0025
$\Omega_m h^3$	0.09597	0.09590 ± 0.00059	0.09603	0.09593 ± 0.00058	0.09591	0.09589 ± 0.00057
Y_P	0.247710	0.24771 ± 0.00014	0.247785	0.24775 ± 0.00014	0.247695	0.24770 ± 0.00012
Age/Gyr	13.819	13.813 ± 0.058	13.784	13.796 ± 0.058	13.8242	13.817 ± 0.048
z_*	1090.43	1090.37 ± 0.65	1090.01	1090.16 ± 0.65	1090.48	1090.43 ± 0.54
r_*	144.58	144.75 ± 0.66	145.02	144.96 ± 0.66	144.58	144.71 ± 0.60
$100\theta_*$	1.04139	1.04148 ± 0.00066	1.04164	1.04156 ± 0.00066	1.04136	1.04147 ± 0.00062
z_{drag}	1059.32	1059.29 ± 0.65	1059.59	1059.43 ± 0.64	1059.25	1059.25 ± 0.58
r_{drag}	147.34	147.53 ± 0.64	147.74	147.70 ± 0.63	147.36	147.49 ± 0.59
k_D	0.14026	0.14007 ± 0.00064	0.13998	0.13996 ± 0.00062	0.14022	0.14009 ± 0.00063
$100\theta_D$	0.161332	0.16137 ± 0.00037	0.161196	0.16129 ± 0.00036	0.161375	0.16140 ± 0.00034
z_{eq}	3402	3386 ± 69	3352	3362 ± 69	3403	3391 ± 60
$100\theta_{eq}$	0.8128	0.816 ± 0.013	0.8224	0.821 ± 0.013	0.8125	0.815 ± 0.011
$r_{drag}/D_V(0.57)$	0.07130	0.0716 ± 0.0011	0.07207	0.0719 ± 0.0011	0.07126	0.07147 ± 0.00091

Planck 2018 parameters

Parameter	*Planck* alone	*Planck* + BAO
$\Omega_b h^2$	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8111 ± 0.0060	0.8102 ± 0.0060
$\sigma_8(\Omega_m/0.3)^{0.5}$	0.832 ± 0.013	0.825 ± 0.011
z_{re}	7.67 ± 0.73	7.82 ± 0.71
Age[Gyr]	13.797 ± 0.023	13.787 ± 0.020
r_*[Mpc]	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04110 ± 0.00031	1.04119 ± 0.00029
r_{drag}[Mpc]	147.09 ± 0.26	147.57 ± 0.22
z_{eq}	3402 ± 26	3387 ± 21
k_{eq}[Mpc^{-1}]	0.010384 ± 0.000081	0.010339 ± 0.000063
Ω_K	-0.0096 ± 0.0061	0.0007 ± 0.0019
Σm_ν [eV]	< 0.241	< 0.120
N_{eff}	$2.89^{+0.36}_{-0.38}$	$2.99^{+0.34}_{-0.33}$
$r_{0.002}$	< 0.101	< 0.106

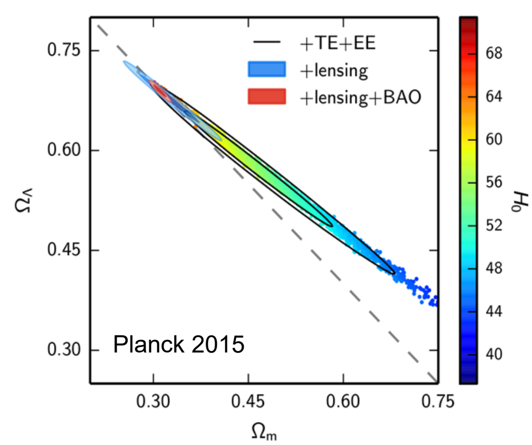
Degeneracy between H_0 and Ω_M

- Dependence on the spectral index n_S



Degeneracy with Dark Energy

- Dark Energy and curvature are degenerate in the CMB



Literature

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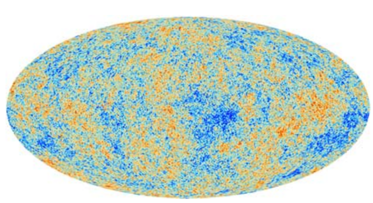
- Scott Dodelson, Modern Cosmology, Boston, Academic Press 2013 ISBN: 0122191412
- Malcolm S. Longair, Galaxy Formation, Astronomy and Astrophysics Library, Springer Verlag: ISBN 978-3-540-73477-2
- Hu & Dodelson, Annual Review of Astronomy & Astrophysics, **40**, 171 (2002)
- Joshua Friedman, Michael Turner, Dragan Huterer, Annual Reviews of Astronomy & Astrophysics **46**, 385 (2008)
- Planck publications, e.g., Ade et al., Astronomy & Astrophysics **594,** A13 (2016); Aghanim et al., arXiv:1807.06209 (2018)

Formation of large scale structure



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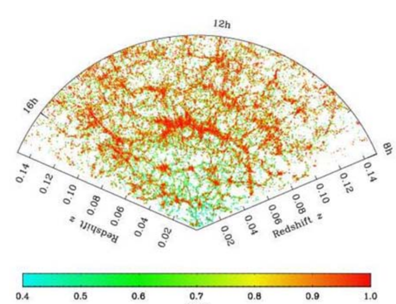
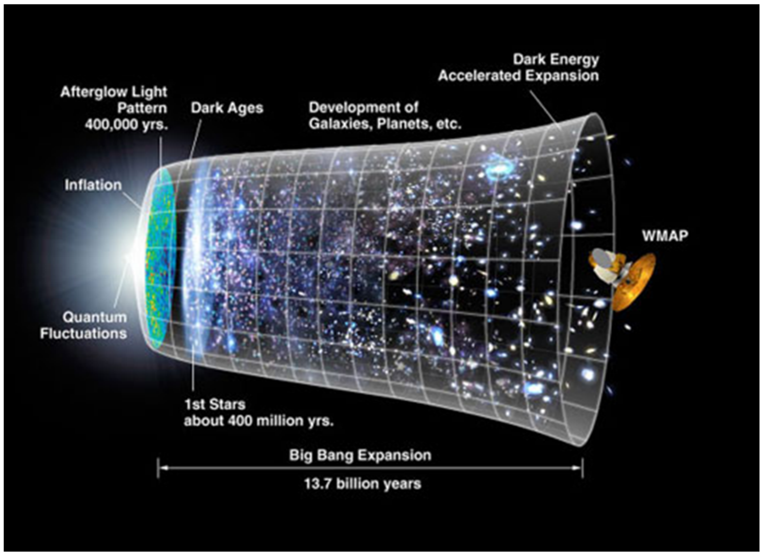


Planck SDSS

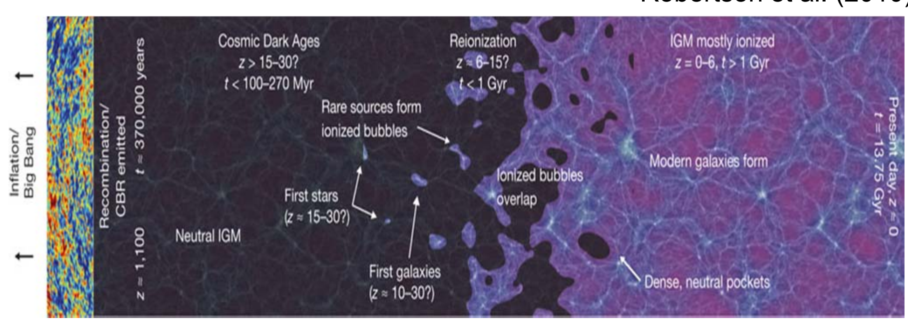
Structure formation in the Big-Bang model



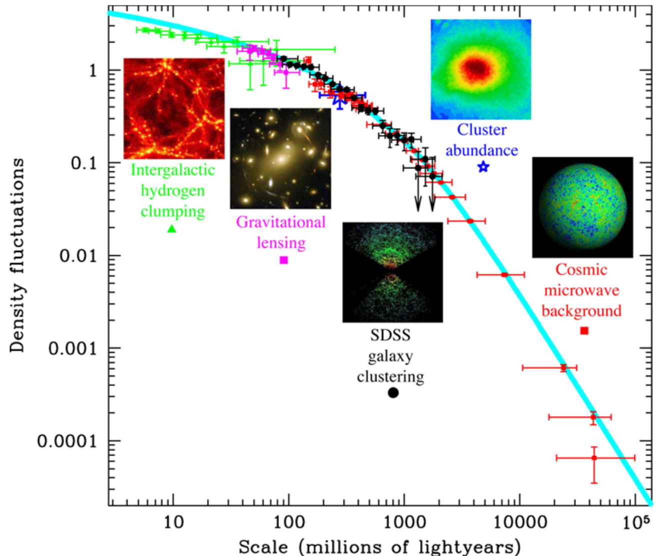
After recombination

- Only 'global' event is the reionization around z=11

Robertson et al. (2010)



Density fluctuations and relevant scales

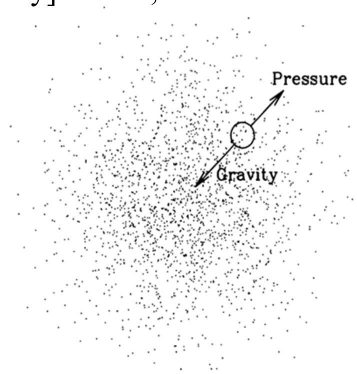


SDSS collaboration

Growth of structure

- Gravitational instability (schematically)

$$\ddot{\delta} + [\text{Pressure} - \text{Gravity}]\,\delta = 0; \quad \delta : \text{overdensity (dark matter)}$$



Scott Dodelson, *Modern Cosmology* (2013)

Concept

- Universe isotropic on large scales + GR (~72% dark energy, ~24% dark matter, ~4% baryons; + radiation)
- Small deviations from homogeneity at early times (e.g., quantum fluctuations from inflation)
- Perturbations (potential/density) grow because of gravity; linear perturbation theory as long as their amplitudes are "small"

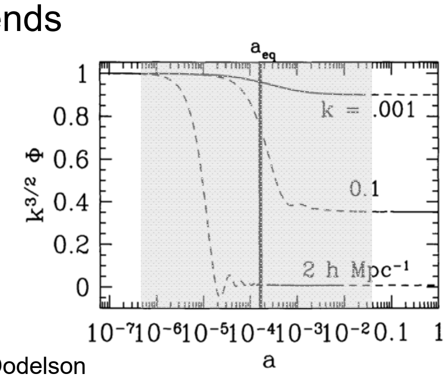
$$\Phi(\vec{k},a) = \Phi_{Prim}(\vec{k}) \times \{Transfer\ Function\ (k)\} \times \{Growth\ Function\ (a)\}$$

From original perturbation to todays fluctuation

- Transition from original potential perturbation
- In shaded region: depends on 'Transfer Function'

$$T(k) = \frac{\Phi(k, a_{late})}{\Phi_{Large-Scale}(k, a_{late})}$$

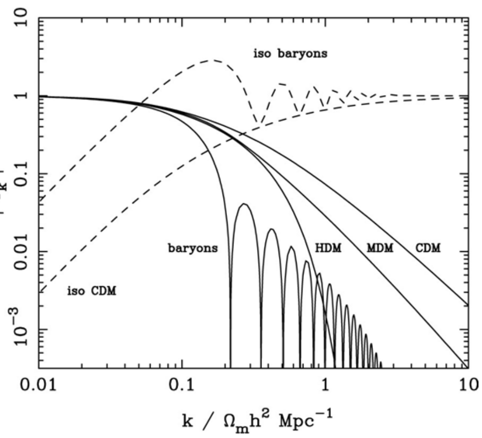
- Critical whether perturbation enters horizon before or after equality of matter and radiation



Dodelson

Figure 7.2. The linear evolution of the gravitional potential Φ. Dashed line denotes that the mode has entered the horizon. Evolution through the shaded region is described by the transfer function. The potential is unnormalized, but the relative normalization of the three modes is as it would be for scale-invariant perturbations. Here baryons have been neglected, $\Omega_m = 1$, and $h = 0.5$.

Examples of Transfer Functions



Transfer functions for various adiabatic models, in which $T_k \rightarrow 1$ at small k.

(J. Peacock, astro-ph/0309240, 2003)

Power spectrum of matter fluctuations

- Peak defined by epoch of equality
  - “standard” Cold Dark Matter (sCDM)

$$\Omega_M = 1$$

  - Cosmological Constant plus CDM : ΛCDM

$$\Omega_M = 0.3$$

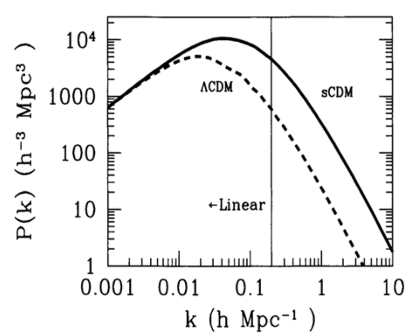


Figure 7.4. The power spectrum in two Cold Dark Matter models, with (ΛCDM) and without (sCDM) a cosmological constant. The spectra have been normalized to agree on large scales. The spectrum in the cosmological constant model turns over on larger scales because of a later a_{eq}. Scales to the left of the vertical line are still evolving linearly.

Influenced by the growth in the matter density

- Radiation affects the growth before the time of equality
- Note this diagram is the matter density (before was potential)

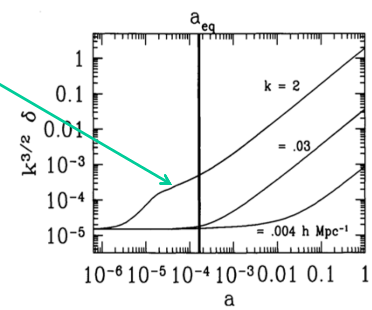
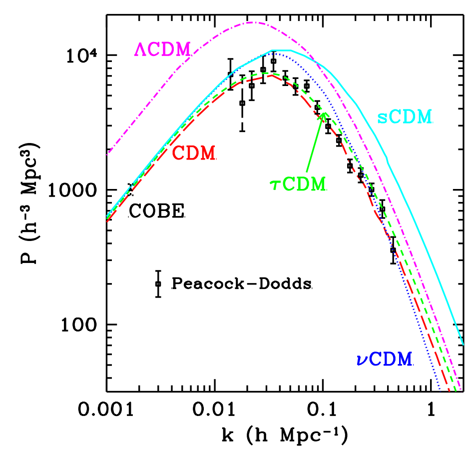


Figure 7.3. The evolution of perturbations to the dark matter in the same model as plotted in Figure 7.2. Amplitude starts to grow upon horizon entry (different times for the three different modes shown here). Well after a_{eq}, all sub-horizon modes evolve identically, scaling as the growth factor. In the case plotted, a flat, matter dominated universe, the growth factor is simply equal to a.

Comparison with real data

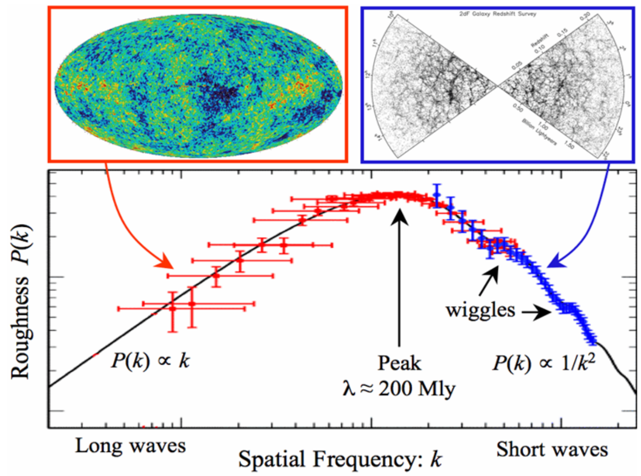
- CMB sets the normalisation at large scales and galaxy distribution the small scales



Dodelson et al. (1996)

Another example

- Modern version



Implementation

- Background metric $g_{\alpha\beta}$ (FRW) and sources $T_{\alpha\beta}$
- Perturbations: $\delta g_{\alpha\beta}$; $\delta T_{\alpha\beta}$
- Linearize Einstein equations
- Lin. differential eq. of 2nd order

$$\mathcal{L}(g_{\alpha\beta})\delta g_{\alpha\beta} = \delta T_{\alpha\beta}$$

Example: ‘Dust cosmos’

$p = 0$; $\rho = \rho_o + \delta\rho$

$a(t) = a_o(t) + \delta a$ (‘o’: unperturbed solution)

Friedmann equation:

$$\left(\frac{\dot{a}_o(t)}{a_o(t)}\right)^2 = \frac{8\pi G}{3}\rho_o - \frac{k_0}{a_0^2};\quad \left(\frac{\dot{a}}{a}\right) = \frac{8\pi G}{3}(\rho_o + \delta\rho) - \frac{k}{a^2}$$

Continuity equation: $\frac{\delta\rho}{\rho_0} \equiv \delta = -3\,\frac{\delta a}{a_0}$

$$\Rightarrow \quad \ddot{\delta} + 2\frac{\dot{a_0}}{a_0}\,\delta = 4\pi G\rho_o\delta$$

- For $k_o \approx k = 0$: $a_o \propto t^{3/2}$; $\rho_o \propto a_0^{-3}$ (flat Universe)

$\Rightarrow$ $\boxed{\delta \propto t^{2/3} \propto a_o}$

For radiation dominated cosmos:

$\boxed{\delta \propto t \propto a_o^2}$

More general approach

- Expand solutions in an appropriate orthonormal basis (e.g., plane waves in case of a flat Universe) $\Rightarrow$ Fourier transform

  $\hat{L}(\vec{k})\ \delta g(\vec{k}) = \delta T(\vec{k})$, $\vec{k}$: wave vector

Solutions describe the evolution of independent modes (consequence of linearization).

However, equations are no longer covariant.

$\Rightarrow$ Growth rates of modes depend on coordinates!

Ways out

- For $\lambda \ll d_H$: GR effects small $\Rightarrow$ use standard comoving coordinates + Newtonian gravity
- But: For every mode there exists a time when $\lambda > d_H \Rightarrow$ Newtonian theory incorrect
- In GR: Choice of coordinates such that an interpretation of quantities is "physical", e.g., $\delta T^0{}_0$ is the matter (energy) density
- Or: form scalars $\Rightarrow$ independent of coordinates but difficult to interpret

Qualitative evolution of perturbations

- ‘Dark energy’: cosmological constant
- ‘Dark matter’: WIMPS
- Assumption: Decouple at $T = T_{weak} \approx 1$MeV
- Baryons, photons: decouple at $T = T_{rec} \approx 0.26$eV

$\Rightarrow$ Different evolution; treat each component seperately

Qualitative evolution of perturbations

- Each component (x) has its equation of state

$$P_x = W_x \cdot \rho_x c^2;\ W_x = \text{const.} = \begin{cases} 0 \text{ (dust)} \\ 1/3 \text{ (radiation)} \\ -1\ (\Lambda) \end{cases}$$

- Then: $\delta p_x = W_x \cdot \delta(\rho_x c^2)$
- Define: ‘velocity dispersion’ $v_x^2 \equiv \frac{\dot{p}_x}{\dot{\rho}_x}$ (= c_s^2)
- $\Rightarrow$ $v_x^2 = W_x c^2$

Qualitative evolution of perturbations

- Solution: For $\delta\rho_x$>0 perturbations grow if
  λ>d_H (Hubble radius)

$$\left(\frac{\delta\rho_x}{\rho_x}\right) \propto \begin{Bmatrix} a^2, t < t_{eq}^x \\ a, t > t_{eq}^x \end{Bmatrix} \quad \begin{matrix} \rho \propto a^{-4}\ (radiation) \\ \rho \propto a^{-3}\ (rel. matter) \end{matrix}$$

- What happens for $\lambda < d_H$?

To be continued ....