1) Elements of Intelligence

Working hypothesis

Intelligent system = System with internal representation of aspects of the world, which uses this for its action

action = change of properties of the world

I: internal representation of something = knowledge on s

state of the world s

Perfect knowledge on s: $I_{s=s'}$, knowing that $s=s' \in S'$

imperfect " " s: $s \in A \subseteq \mathcal{P}S'$

nuanced knowledge: $P: \mathcal{P}S' \to [0,1]$

probability

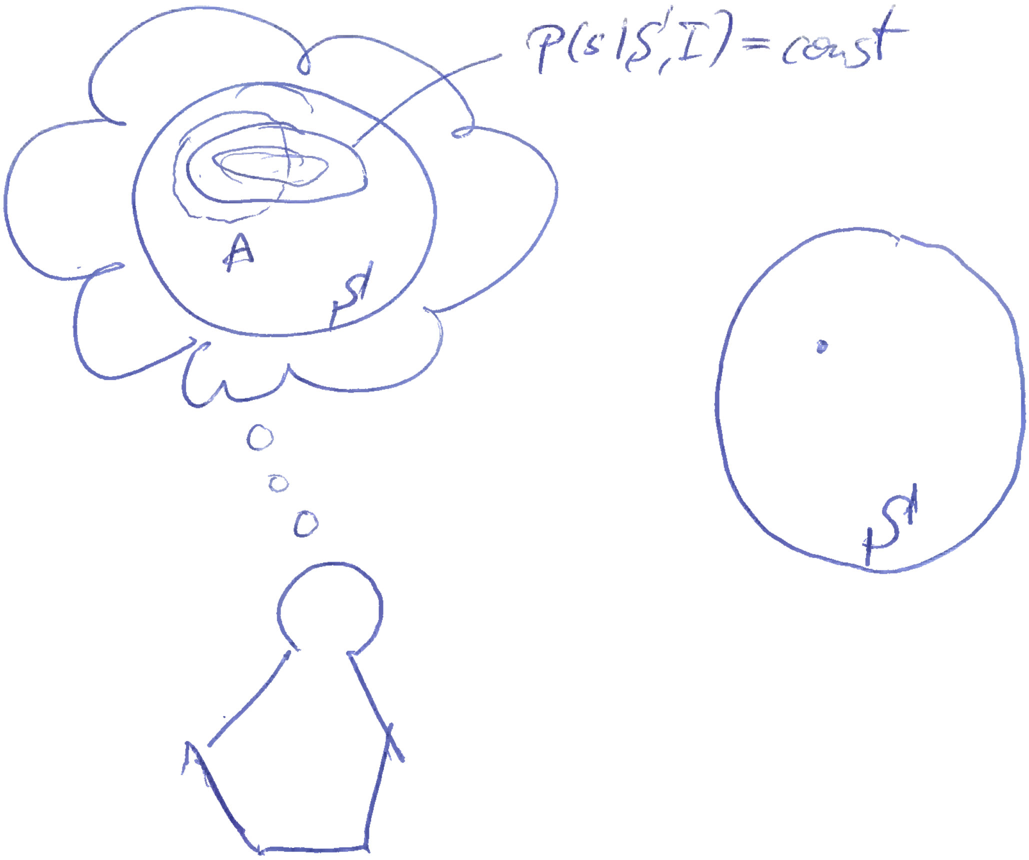
$\mathcal{P}S' = \{0,1\}^{S'}$ = power set of S', $A \in \mathcal{P}S'$ $\Rightarrow A \subseteq S'$

$P(A|B,I)$ = probability that A is true assuming B and I to be true

↑ and

requirements on probabilities (Cox '48)

- logic included as limiting case
- order of information ingestion ~~does not matter~~ is irrelevant
  (information = something that changes knowledge states)
- calibration: known frequency of an event should be its probability



agent = intelligent system
(human, animal, computer program, AI,
cell, gene regulation network, ..)

Probability

$P(A|B)$ = degree of belief in A assuming B is the case $\in [0,1]$

Frequency

$F(A|B)$ = frequency ~~of~~ of A happening (per time, volume, etc) assuming B is the case

$$= \frac{\#(A,B) \text{ happens}}{\# B \text{ happens}} \in [0,1]$$

$A,B = AB = A$ and B $= A \wedge B$

Note: The condition can only be omitted, if it is clear from the context. Many, if not most paradoxes in Probability theory arise from confusion about the condition!

Calibration

$P(A|B, F(A|B)=f) = f$

If a frequency is known for AB, it is also the probability for $A|B$.

<u>product rule:</u>

$$P(AB|C) = P(A|BC)\,P(B|C)$$
$$= P(B|AC)\,P(A|C)$$

$$\Rightarrow P(A|BC) = \frac{P(B|AC)\,P(A|C)}{P(B|C)}$$

Bayes theorem

e.g.

A = some variable $s \in S$ has a specific value, $s = s^*$ $\left(= I_{s=s^*}\right)$

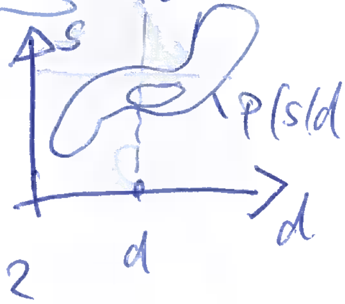
B = some data $d \in D$ has a specific value, $d = d^*$ $\left(= I_{d=d^*}\right.$

C = all the prior knowledge on s and its relation to d, basically specifying

$$P(s) :\neq P(\overbrace{s_{real} = s}^{A} | C)$$

and

$$P(s|d) := P(\underbrace{d_{real} = d}_{B} | AC) \qquad p(d|s) = \frac{P(d,s)}{P(d)}$$



$$P(s|d) = \frac{\overbrace{P(d|s)}^{\text{likelihood}}\ \overbrace{P(s)}^{\text{prior}}}{\underbrace{P(d)}_{\text{evidence}}}$$

How to calculate the evidence?

We need the sum rule!

Sum rule $P(A|C) + P(\bar{A}|C) = 1$

$\bar{A} = \neg A = \text{not } A$

marginalization

$$P(A,B|C) = P(B|AC)\,P(A|C)$$

$$P(A,\bar{B}|C) = P(\bar{B}|AC)\,P(A|C)$$

$$P(A,B|C) + P(A,\bar{B}|C) = \underbrace{\left[P(B|AC) + P(\bar{B}|AC)\right]}_{1 \text{ (sum rule)}} P(A|C)$$

$$= P(A|C)$$

extension: $\{B_i\}_{i=1}^{n}$ mutually exclusive $\left[P(B_i B_j|I) = 0 \text{ for } i \neq j\right]$

$A+B = A \vee B = A \text{ or } B$ and exhaustive $\left[P(B_1 + \dots B_n|I) = 1\right]$

in I

$$\Rightarrow P(A|I) = \sum_{i=1}^{n} P(A,B_i|I)$$

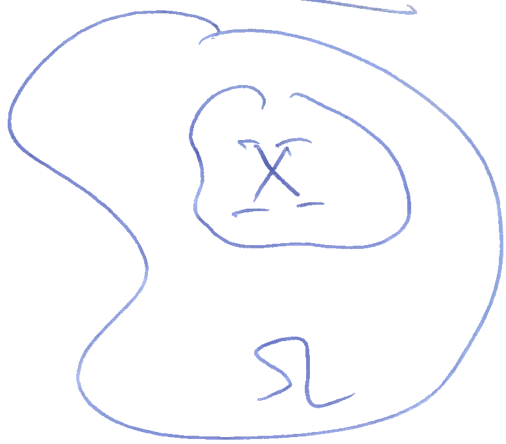
generalized sum rule:

$$P(A+B) = P(A) + P(B) - P(AB) \quad |I$$

condition for all probabilities in an equation

Probability density functions

I: $X \subset \Omega$, $x \in \Omega$



$$P(x \in X \mid I) = \int_X dx\, \mathcal{P}(x|I)$$

product rule, sum rule, marginalization all apply

Laplace's principle of insufficient reasoning

Let $\{B_i\}_{i=1}^{n}$ be mutually exclusive and exhaustive in I, and I symmetric w.r.t. to the B_i's

$$\Rightarrow P(B_i|I) = P(B_j|I) = \frac{1}{n}$$

uniform distribution

Convention

data d : known quantity

signal s : unknown quantity of interest

noise n : " " that is not of interest