Position-dependent Power Spectrum

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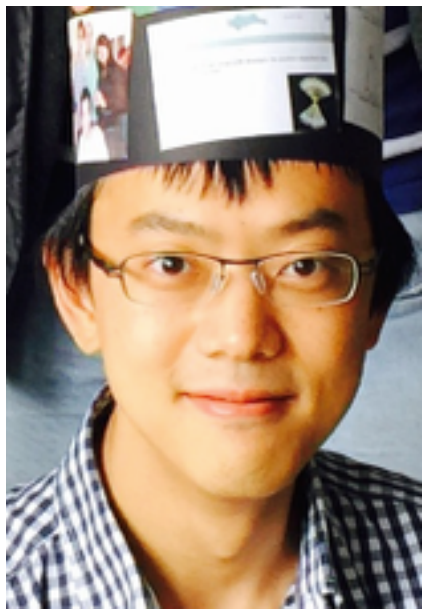
This talk is based on



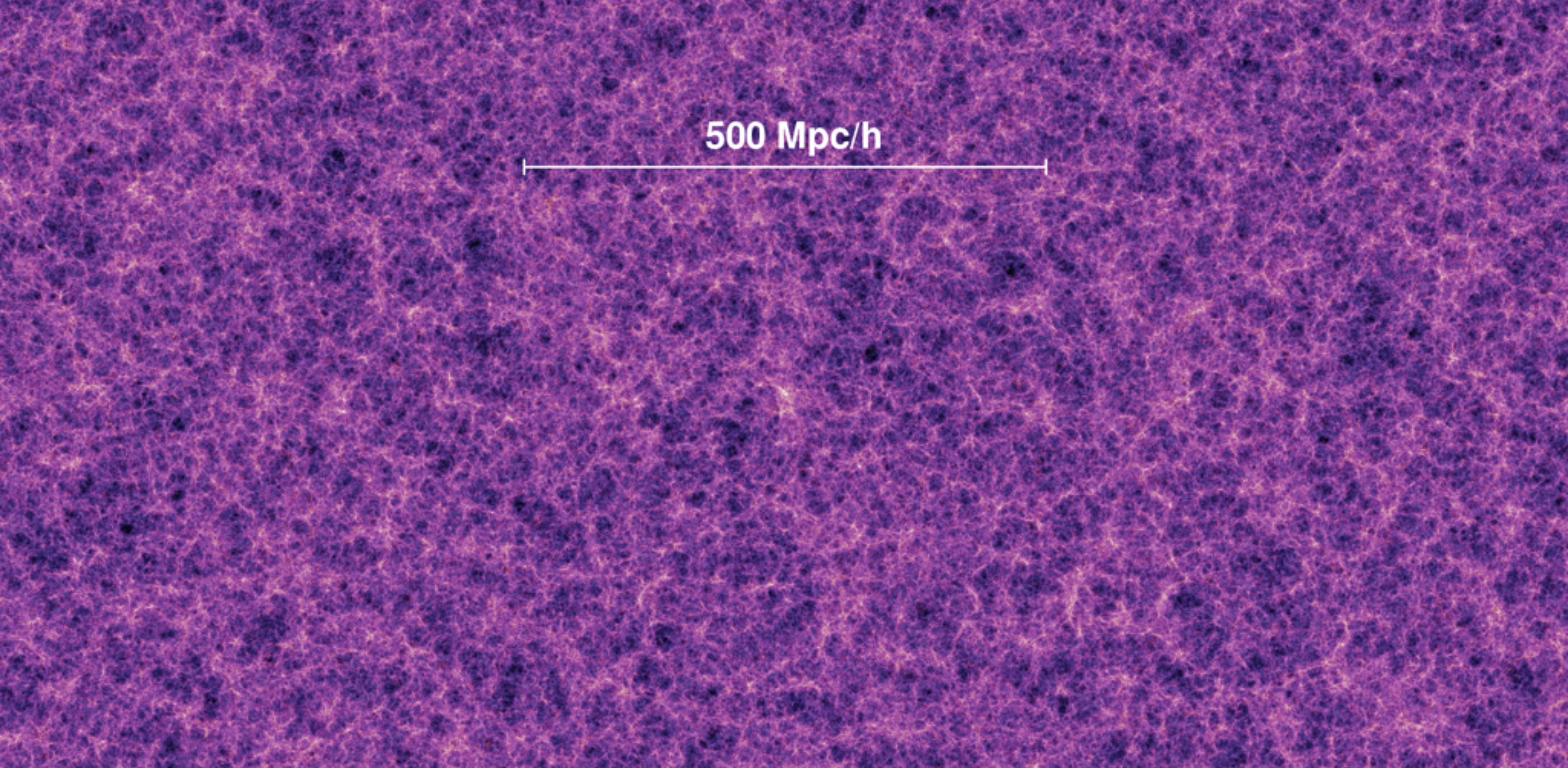
- **Chiang** et al. "*Position-dependent power spectrum of the large-scale structure: a novel method to measure the squeezed-limit bispectrum*", JCAP 05, 048 (2014)
- **Chiang** et al. "*Position-dependent correlation function from the SDSS-III BOSS DR10 CMASS Sample*", JCAP 09, 028 (2015)

- **Wagner** et al. "*Separate universe simulations*", MNRAS, 448, L11 (2015)
- **Wagner** et al. "*The angle-averaged squeezed limit of nonlinear matter N-point functions*", JCAP 08, 042 (2015)

A Simple Question

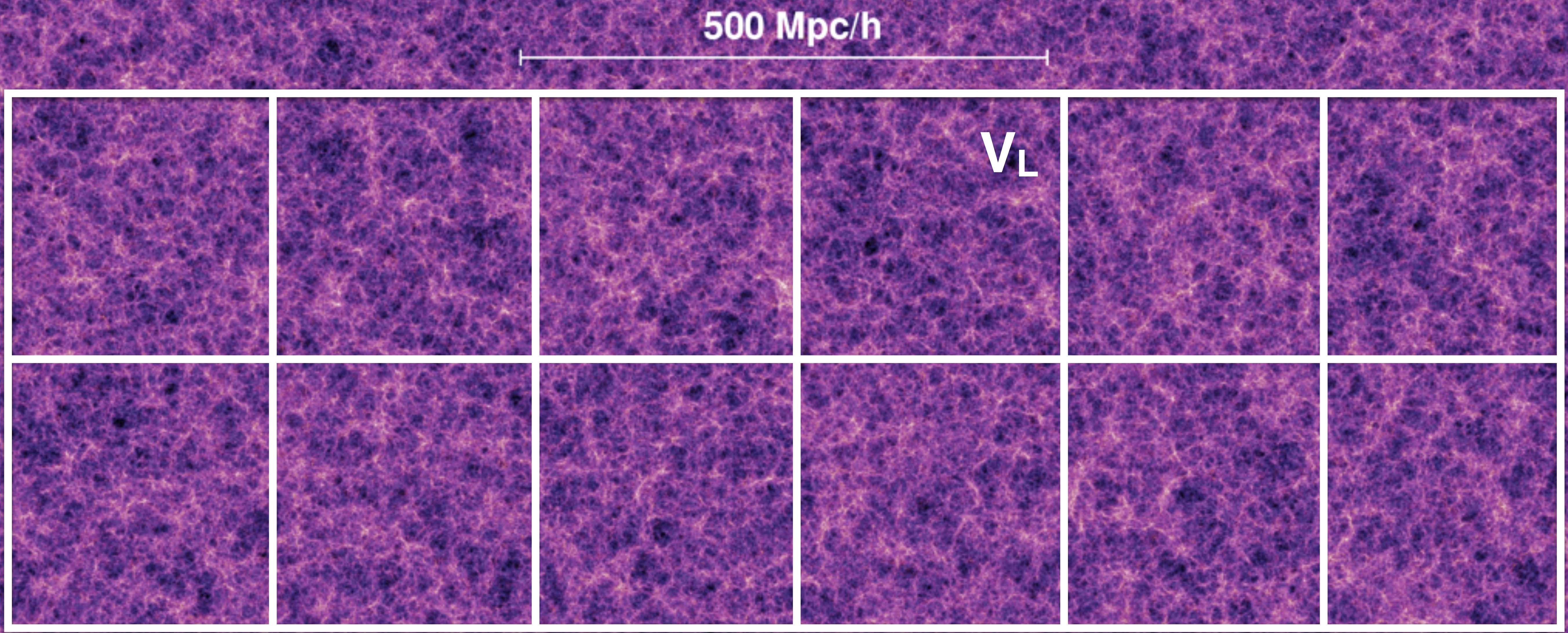
- How do the cosmic structures evolve in an over-dense region?

Simple Statistics



- Divide the survey volume into many sub-volumes V_L, and compare locally-measured power spectra with the corresponding local over-densities

Simple Statistics



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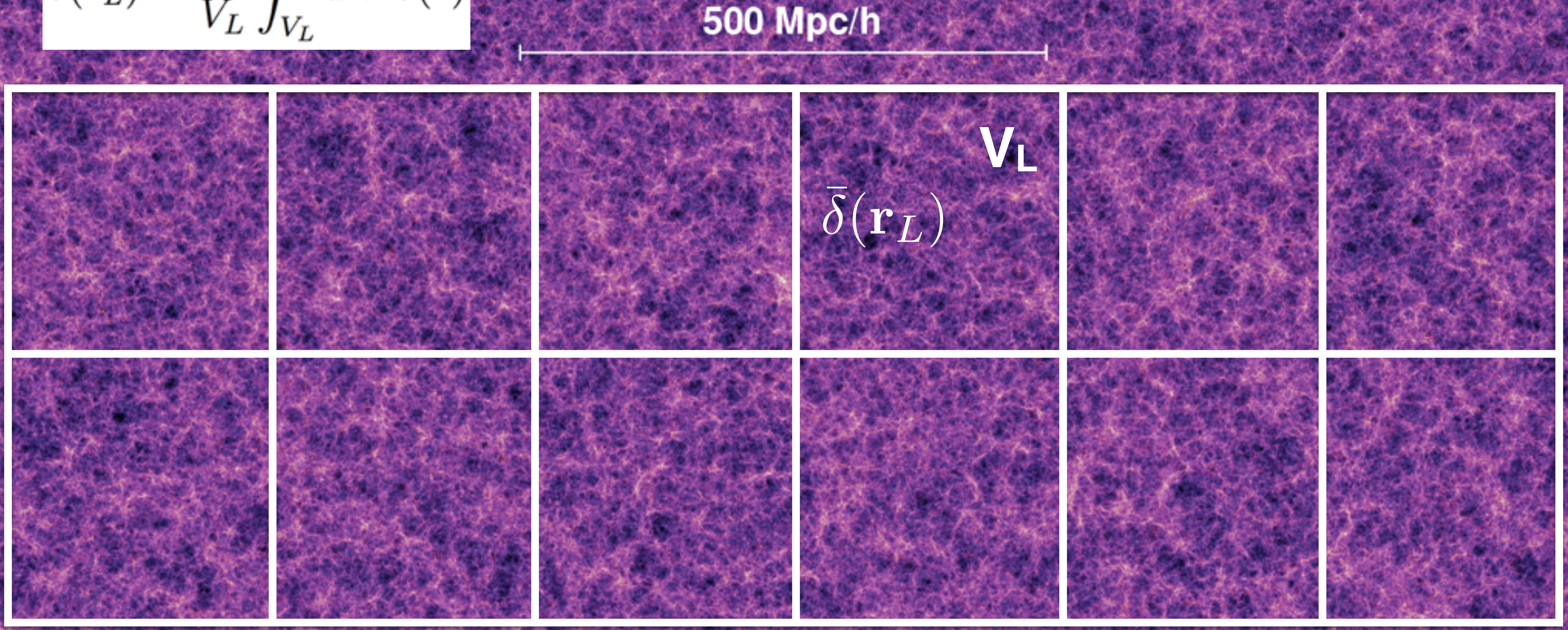
Simple Statistics

$$\bar{\delta}(\mathbf{r}_L) = \frac{1}{V_L}\int_{V_L} d^3r\ \delta(\mathbf{r})$$

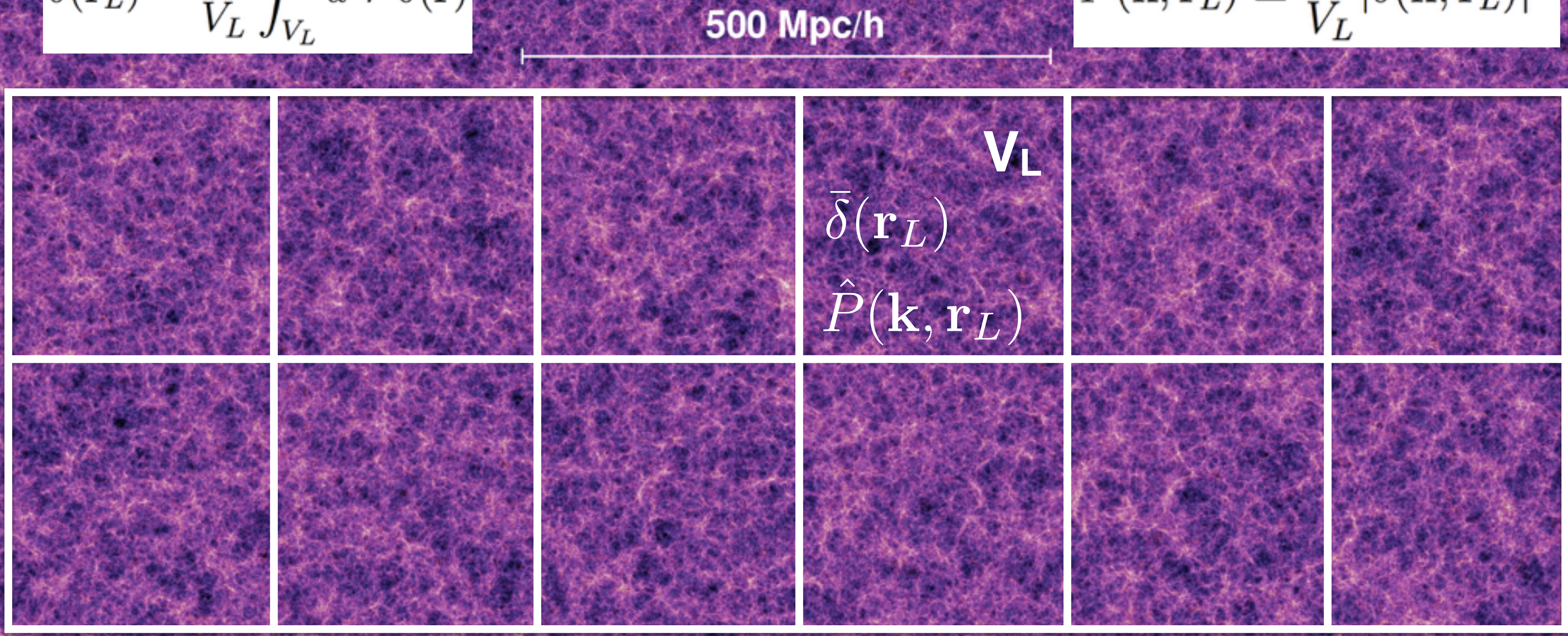


- Divide the survey volume into many sub-volumes V_L, and compare locally-measured power spectra with the corresponding local over-densities

Simple Statistics

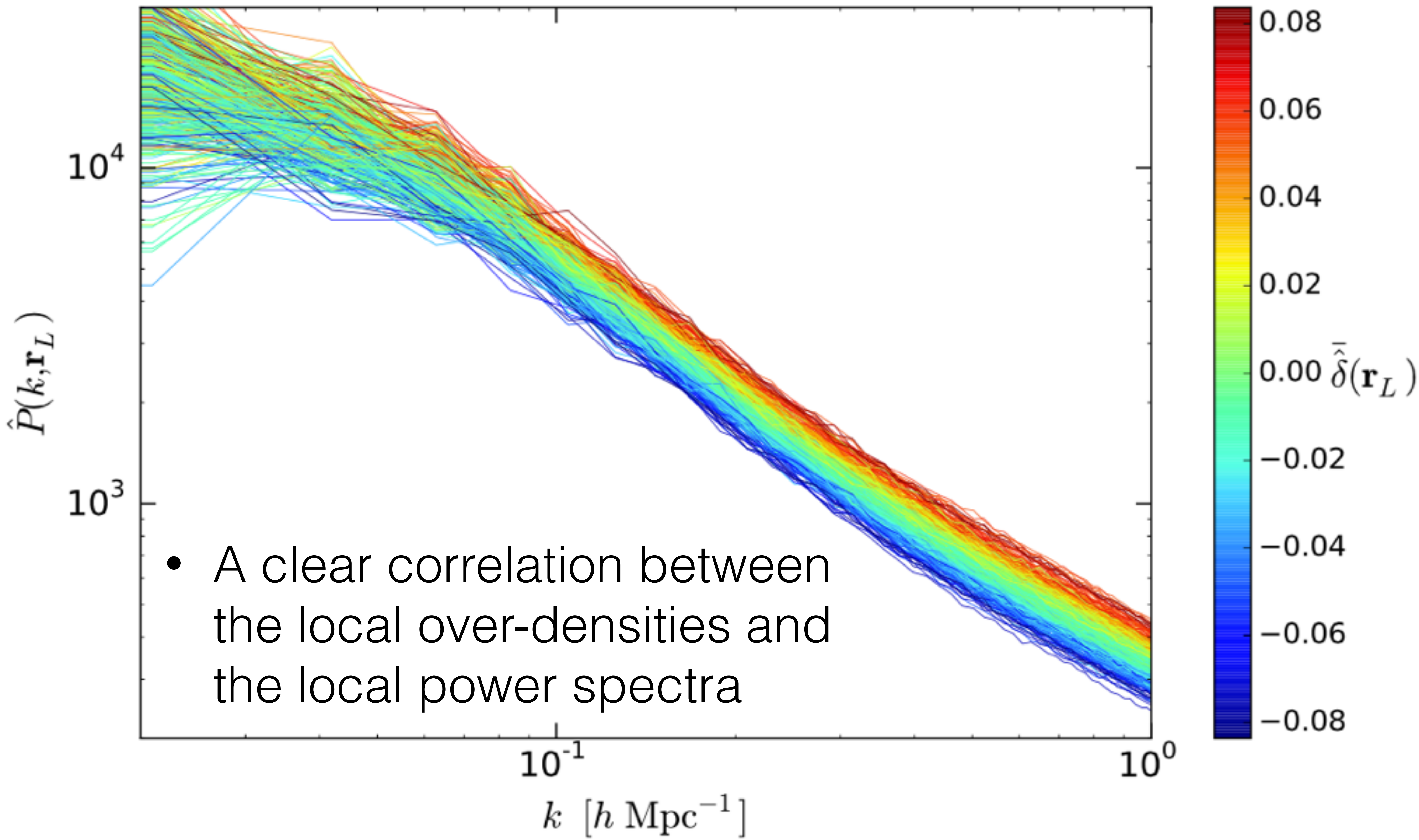
$$\bar{\delta}(\mathbf{r}_L) = \frac{1}{V_L}\int_{V_L} d^3r\ \delta(\mathbf{r})$$

$$\hat{P}(\mathbf{k}, \mathbf{r}_L) \equiv \frac{1}{V_L}|\delta(\mathbf{k}, \mathbf{r}_L)|^2$$



- **Divide the survey volume into many sub-volumes V_L, and compare locally-measured power spectra with the corresponding local over-densities**

Position-dependent $\hat{P}(k)$



- A clear correlation between the local over-densities and the local power spectra

Integrated Bispectrum, *iB*(*k*)

- Correlating the local over-densities and power spectra, we obtain the "integrated bispectrum":

$$i\hat{B}_L(k) = \frac{1}{N_{\rm cut}^3} \sum_{i=1}^{N_{\rm cut}^3} \hat{P}(k, \mathbf{r}_{L,i}) \hat{\bar{\delta}}(\mathbf{r}_{L,i})$$

- This is a (particular configuration of) **three-point function**! The three-point function in Fourier space is the bispectrum, and is defined as

$$\langle \delta(\mathbf{q}_1)\delta(\mathbf{q}_2)\delta(\mathbf{q}_3)\rangle = B(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)(2\pi)^3 \delta_D(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3)$$

Integrated Bispectrum, *iB*(*k*)

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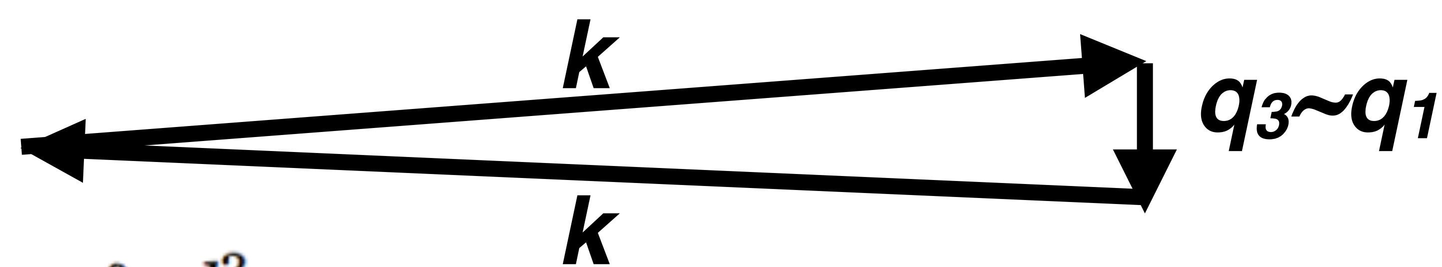
- The expectation value of this quantity is an integral of the bispectrum that picks up the contributions **mostly from the squeezed limit**:

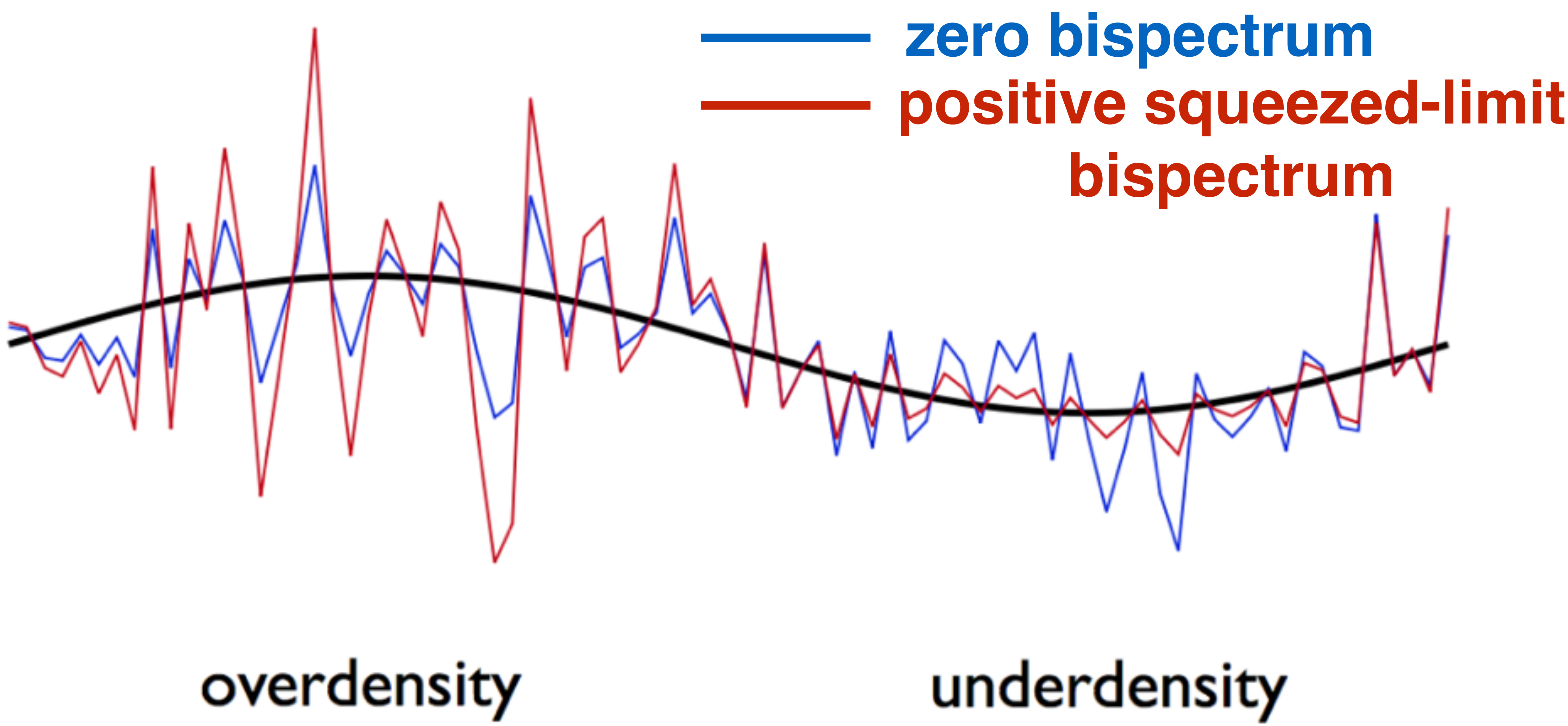


$$\begin{aligned} iB_L(k) &= \langle \hat{P}(k, \mathbf{r}_L) \bar{\delta}(\mathbf{r}_L) \rangle \\ &= \frac{1}{V_L^2} \int \frac{d^2\hat{k}}{4\pi} \int \frac{d^3 q_1}{(2\pi)^3} \int \frac{d^3 q_3}{(2\pi)^3} \, B(\mathbf{k} - \mathbf{q}_1, -\mathbf{k} + \mathbf{q}_1 + \mathbf{q}_3, -\mathbf{q}_3) \\ &\quad \times W_L(\mathbf{q}_1) W_L(-\mathbf{q}_1 - \mathbf{q}_3) W_L(\mathbf{q}_3) \end{aligned}$$

"taking the squeezed limit and then angular averaging"

Power Spectrum Response

- The integrated bispectrum measures how the local power spectrum responds to its environment, i.e., a long-wavelength density fluctuation



Response Function

- So, let us Taylor-expand the local power spectrum in terms of the long-wavelength density fluctuation:

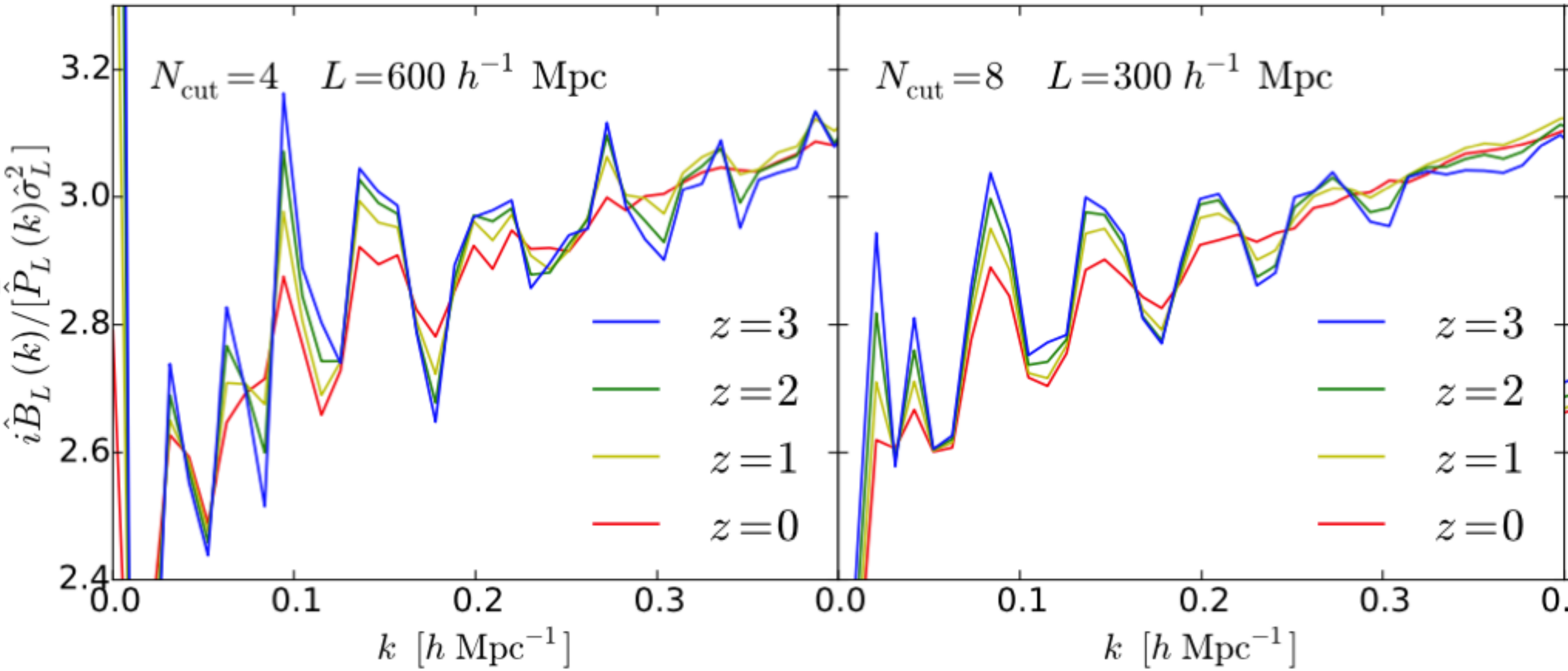
$$\hat{P}(k, \mathbf{r}_L) = P(k)|_{\bar{\delta}=0} + \left.\frac{dP(k)}{d\bar{\delta}}\right|_{\bar{\delta}=0} \bar{\delta} + \ldots$$

- The integrated bispectrum is then give as

$$iB_L(k) = \sigma_L^2 \boxed{\left.\frac{d\ln P(k)}{d\bar{\delta}}\right|_{\bar{\delta}=0}} P(k)$$

response function

Response Function: N-body Results



- Almost a constant, but a weak scale dependence, and clear BAO features. How do we understand this?

Non-linearity generates bispectrum

- If the initial conditions were Gaussian, linear perturbations remain Gaussian

- However, **non-linear** gravitational evolution makes density fluctuations at late times non-Gaussian, generating non-vanishing bispectrum

$$\delta' + \nabla \cdot [(1+\underline{\delta)\mathbf{v}}] = 0 \ ,$$

$$\mathbf{v}' + \underline{(\mathbf{v} \cdot \nabla)\,\mathbf{v}} = -\mathcal{H}\mathbf{v} - \nabla\phi \ ,$$

$$\nabla^2\phi = 4\pi G a^2 \bar{\rho}\delta \ ,$$

Illustrative Example: SPT

- Second-order perturbation gives the lowest-order ("tree-level") bispectrum as

$$B_{\rm SPT}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=2[P_l(k_1)P_l(k_2)F_2(\mathbf{k}_1,\mathbf{k}_2)+2\,\mathrm{cyclic}]$$

"l" stands for "linear"

$$F_2(\mathbf{k}_1,\mathbf{k}_2)=\frac{5}{7}+\frac{1}{2}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)+\frac{2}{7}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\right)^2$$

- Then

$$iB_L(k)=\frac{1}{V_L^2}\int\frac{d^2\hat{k}}{4\pi}\int\frac{d^3q_1}{(2\pi)^3}\int\frac{d^3q_3}{(2\pi)^3}\,B(\mathbf{k}-\mathbf{q}_1,-\mathbf{k}+\mathbf{q}_1+\mathbf{q}_3,-\mathbf{q}_3)$$
$$\times W_L(\mathbf{q}_1)W_L(-\mathbf{q}_1-\mathbf{q}_3)W_L(\mathbf{q}_3)$$

Illustrative Example: SPT

- Standard Eulerian perturbation theory gives the lowest-order ("tree-level") bispectrum as

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- Then

$$iB_{L,\rm SPT}(k)\overset{kL\to\infty}{=}\left[\frac{68}{21}-\frac{1}{3}\frac{d\ln k^3P_l(k)}{d\ln k}\right]P_l(k)\sigma_L^2$$

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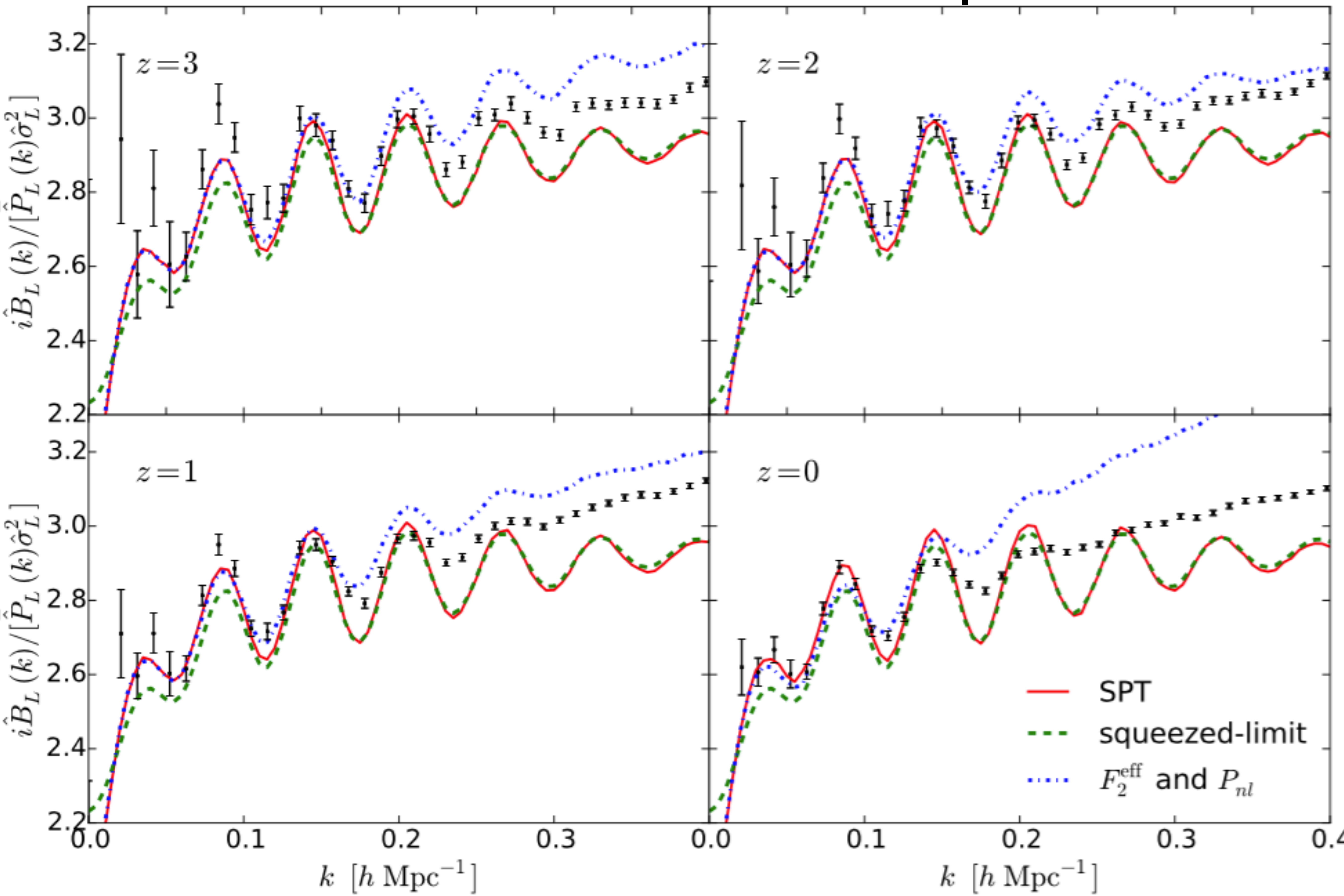
$$F_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) + \frac{2}{7}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\right)^2$$

- Then

Response, dlnP(k)/dδ

$$iB_{L,\rm SPT}(k) \overset{kL\to\infty}{=} \left[\frac{68}{21} - \frac{1}{3}\frac{d\ln k^3P_l(k)}{d\ln k}\right]P_l(k)\sigma_L^2$$

Tree-level SPT comparison



Lemaitre (1933); Peebles (1980)

Separate Universe Approach

- The meaning of the position-dependent power spectrum becomes more transparent within the context of the “separate universe approach”
- Each sub-volume with un over-density (or under-density) behaves as if it were a separate universe with different cosmological parameters
- In particular, if the global metric is a flat FLRW, then **each sub-volume can be regarded as a different FLRW with non-zero curvature**

Mapping between two cosmologies

- The goal here is to compute the power spectrum in the presence of a long-wavelength perturbation δ. We write this as P(k,a|δ)

- We try to achieve this by computing the power spectrum in a **modified cosmology** with non-zero curvature. Let us put the tildes for quantities evaluated in a modified cosmology

$$\tilde{P}(\tilde{k}, \tilde{a}) \to P(k, a|\bar{\delta})$$

Separate Universe Approach: The Rules

- We evaluate the power spectrum in both cosmologies at the **same physical time** and **same physical spatial coordinates**
  - Thus, the evolution of the scale factor is different:

$$\tilde{a}(t) = a(t)\left[1 - \frac{1}{3}\bar{\delta}(t)\right]$$

*tilde: separate universe cosmology

Separate Universe Approach: The Rules

- We evaluate the power spectrum in both cosmologies at the **same physical time** and **same physical spatial coordinates**

  - Thus, comoving coordinates are different too:

$$\tilde{\mathbf{x}} = \frac{a(t)}{\tilde{a}(t)}\mathbf{x} = \left[1 + \frac{1}{3}\bar{\delta}(t)\right]\mathbf{x}$$

*tilde: separate universe cosmology

Effect 1: Dilation

- Change in the comoving coordinates gives **dln(k³P)/dlnk**

$$\tilde{P}(k,t) \to \left[1 - \frac{1}{3}\bar{\delta}(t)\right]^3 P\left(k\left[1 - \frac{1}{3}\bar{\delta}(t)\right], t\right)$$

$$= \left[1 - \bar{\delta}(t)\right] P(k,t) \left[1 - \frac{1}{3}\frac{d\ln P(k,t)}{d\ln k}\bar{\delta}(t)\right]$$

$$= P(k,t)\left[1 - \frac{1}{3}\frac{d\ln k^3 P(k,t)}{d\ln k}\bar{\delta}(t)\right] .$$

Effect 2: Reference Density

- Change in the denominator of the definition of δ:

$$\tilde{P}(\tilde{k},t) \to \left[1+\bar{\delta}(t)\right]^2 \tilde{P}(\tilde{k},t) = \left[1+2\bar{\delta}(t)\right] \tilde{P}(\tilde{k},t)$$

- Putting both together, we find **a generic formula**, valid to linear order in the long-wavelength δ:

$$P(k,a|\bar{\delta}) = \left[1+2\bar{\delta}(t)\right] \tilde{P}\left(k,\tilde{a}\right) \left[1 - \frac{1}{3}\frac{d\ln k^3 P(k,t)}{d\ln k}\bar{\delta}(t)\right]$$

$$= \tilde{P}\left(k, a\left[1-\frac{1}{3}\bar{\delta}(a)\right]\right) \left[1+\left(2-\frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k}\right)\bar{\delta}(a)\right]$$

Example: Linear P(k)

- Let's use the formula to compute the response of the linear power spectrum, $P_l(k)$, to the long-wavelength δ. Since $P_l \sim D^2$ [D: linear growth],

$$\tilde{P}_l\left(k, a\left[1-\frac{1}{3}\bar{\delta}(a)\right]\right) = \left(\frac{\tilde{D}\left(a\left[1-\frac{1}{3}\bar{\delta}(a)\right]\right)}{D(a)}\right)^2 P_l(k,a)$$

- Spherical collapse model gives

$$\tilde{D}\left(a\left[1-\frac{1}{3}\bar{\delta}(a)\right]\right) = D(a)\left[1+\frac{13}{21}\bar{\delta}(a)\right]$$

Response of $P_l(k)$

- Then we obtain:

$$\frac{d\ln P_l(k,a)}{d\bar{\delta}(a)} = \frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P_l(k,a)}{d\ln k}$$

- Remember the response computed from the tree-level SPT **bispectrum**:

$$iB_{L,\mathrm{SPT}}(k) \overset{kL\to\infty}{=} \left[\frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P_l(k)}{d\ln k}\right] P_l(k)\sigma_L^2$$

- **So, the tree-level SPT bispectrum gives the response of the linear P(k). Neat!!**

Response of $P_{\text{1-loop}}(k)$

called "1 loop"

- So, let's do the same using **third-order** perturbation theory! $P(k,a) = P_l(k,a) + P_{22}(k,a) + 2P_{13}(k,a)$

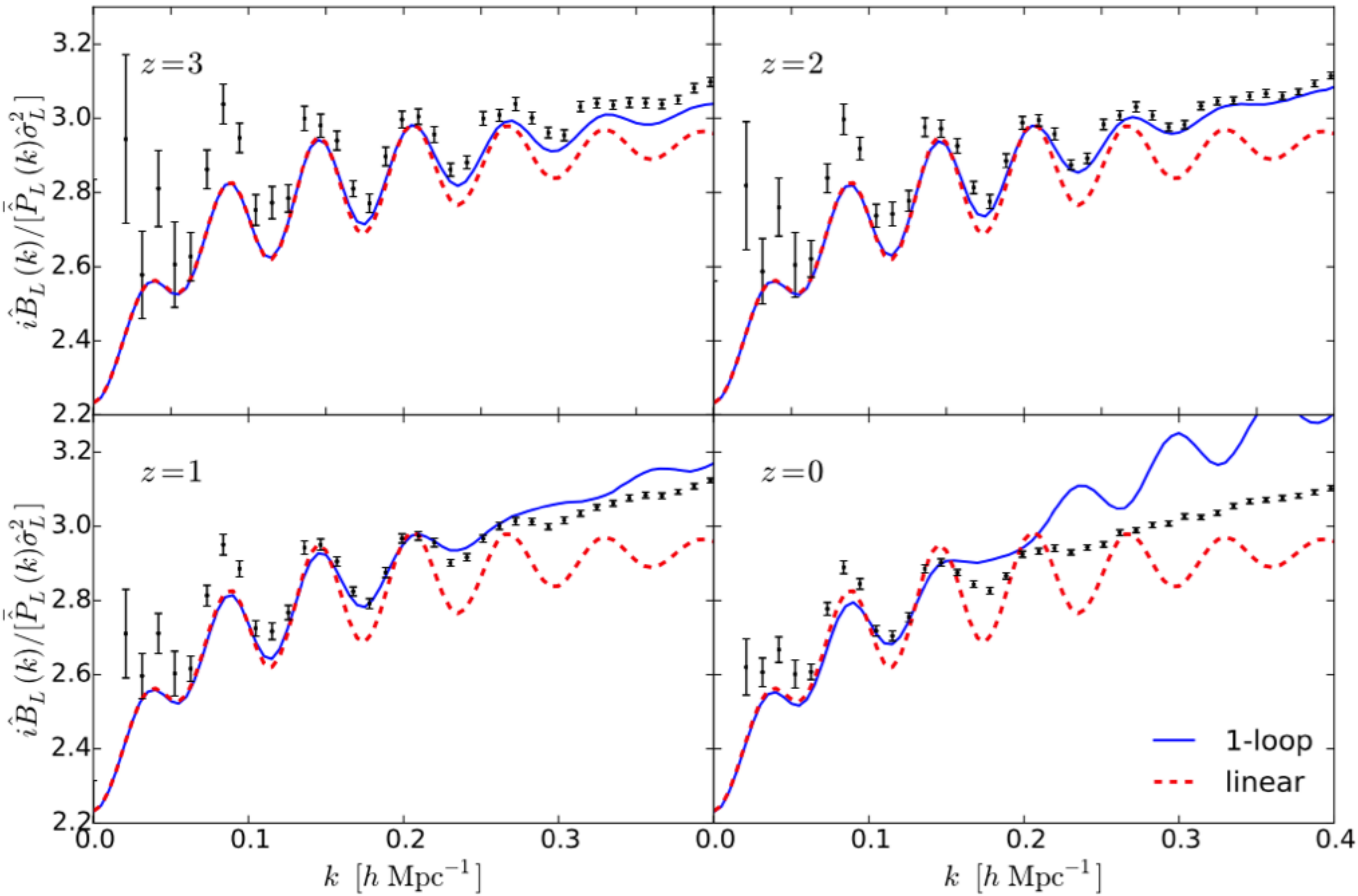
$$P_{22}(k,a) = 2\int \frac{d^3q}{(2\pi)^3}\, P_l(q,a) P_l(|\mathbf{k}-\mathbf{q}|,a)\,[F_2(\mathbf{q},\mathbf{k}-\mathbf{q})]^2$$

$$2P_{13}(k,a) = \frac{2\pi k^2}{252} P_l(k,a) \int_0^\infty \frac{dq}{(2\pi)^3}\, P_l(q,a) \times \left[100\frac{q^2}{k^2} - 158 + 12\frac{k^2}{q^2} - 42\frac{q^4}{k^4} + \frac{3}{k^5 q^3}(q^2-k^2)^3(2k^2+7q^2)\ln\left(\frac{k+q}{|k-q|}\right)\right]$$

- Then we obtain:

$$\frac{d\ln P(k,a)}{d\bar{\delta}(a)} = \frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k} + \frac{26}{21}\frac{P_{22}(k,a) + 2P_{13}(k,a)}{P(k,a)}$$

1-loop does a decent job



This is a powerful formula

$$P(k, a|\bar{\delta}) = \tilde{P}\left(k, a\left[1 - \frac{1}{3}\bar{\delta}(a)\right]\right)\left[1 + \left(2 - \frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k}\right)\bar{\delta}(a)\right]$$

- The separate universe description is powerful, as it provides physically intuitive, transparent, and straightforward way to compute the effect of a long-wavelength perturbation on the small-scale structure growth
  - The small-scale structure can be arbitrarily non-linear!

This is a powerful formula

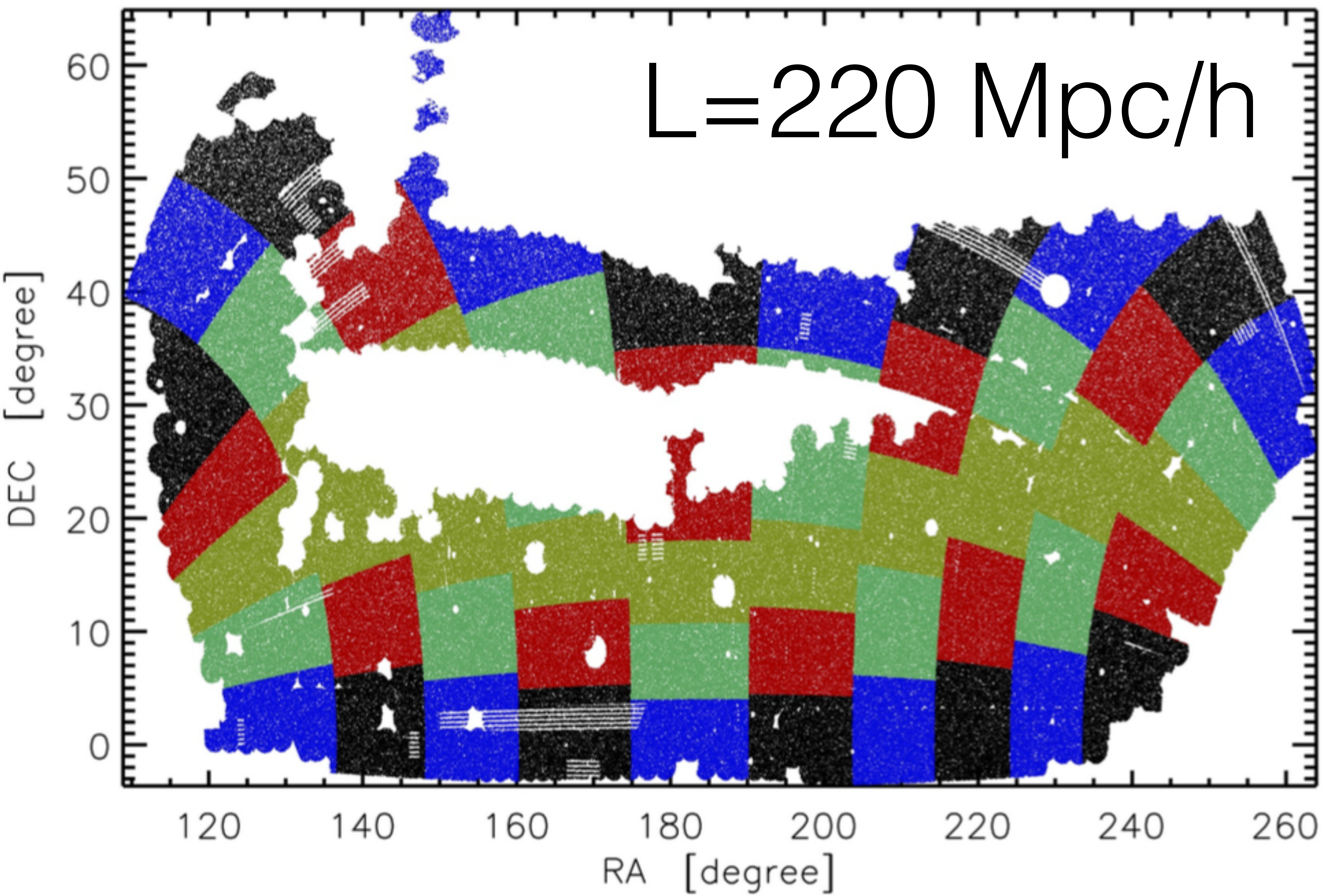
$$P(k,a|\bar{\delta}) = \tilde{P}\left(k, a\left[1-\frac{1}{3}\bar{\delta}(a)\right]\right)\left[1+\left(2-\frac{1}{3}\frac{d\ln k^3P(k,a)}{d\ln k}\right)\bar{\delta}(a)\right]$$

- How can we compute \tilde{P}(k,a) in practice?
  - Small N-body simulations with a modified cosmology (“Separate Universe Simulation”)
  - Perturbation theory
    - We can compute the bispectrum with n-th order PT by the power spectrum in (n–1)-th order PT!

SDSS-III/BOSS DR11

- OK, now, let's look at the real data (BOSS DR11) to see if we can detect the expected influence of environments on the small-scale structure growth

- Bottom line: **we have detected the integrated bispectrum at 7.4σ**. Not bad for the first detection!



L=220 Mpc/h
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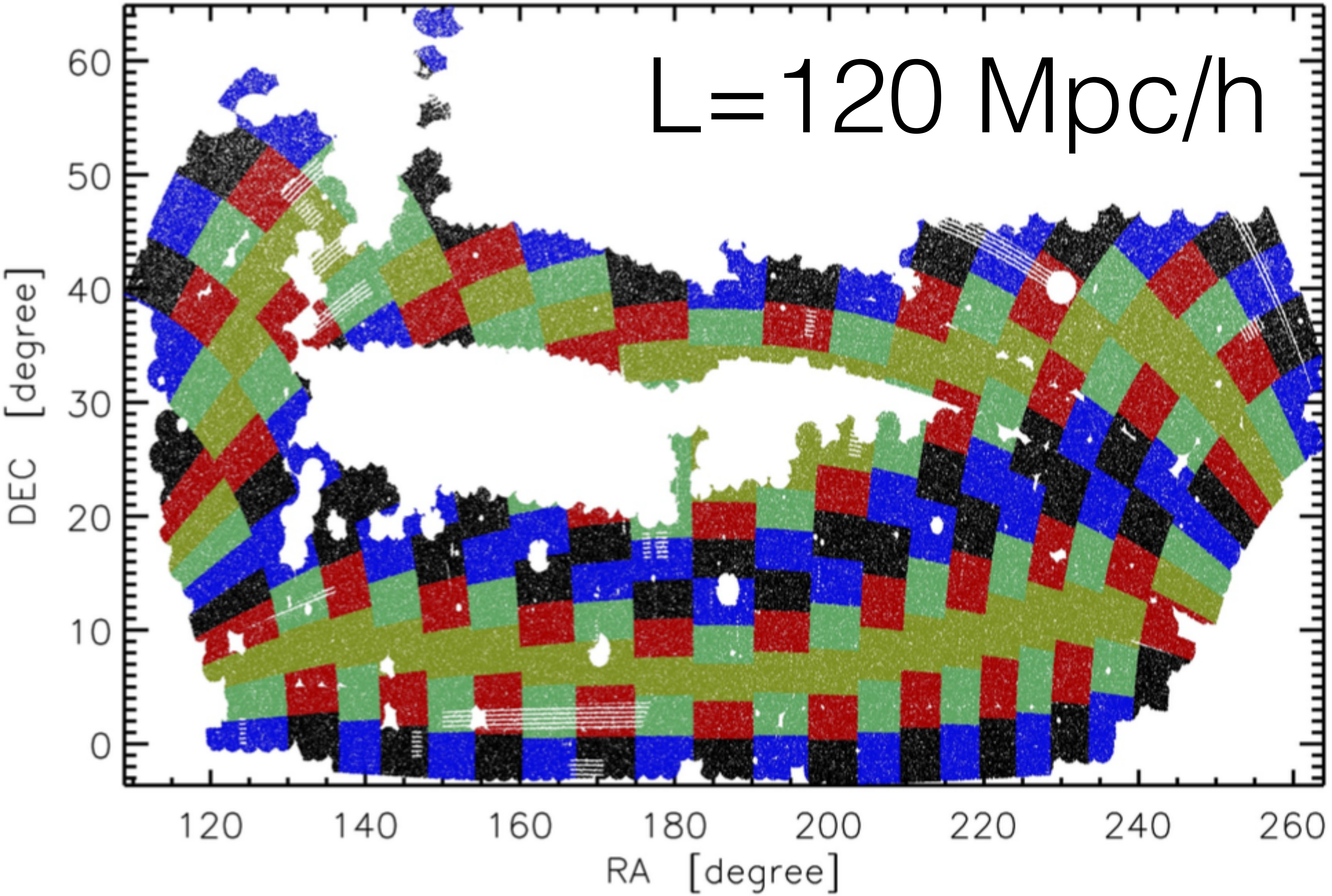

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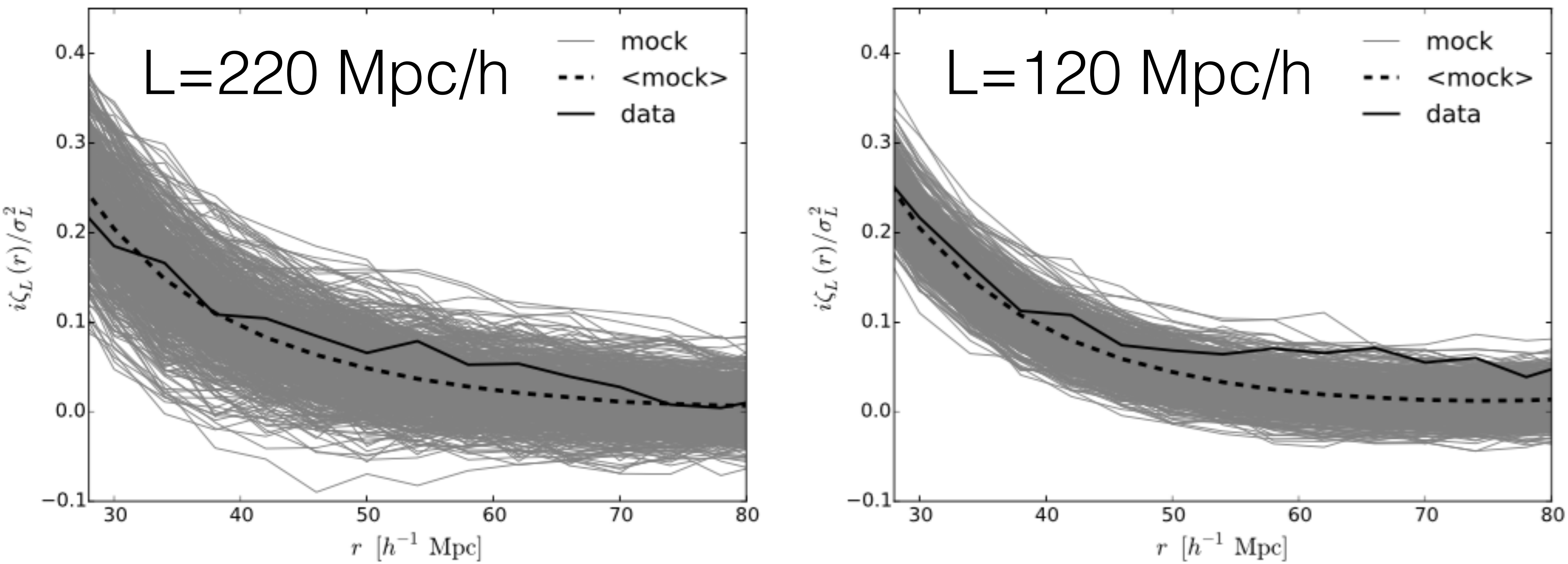
260

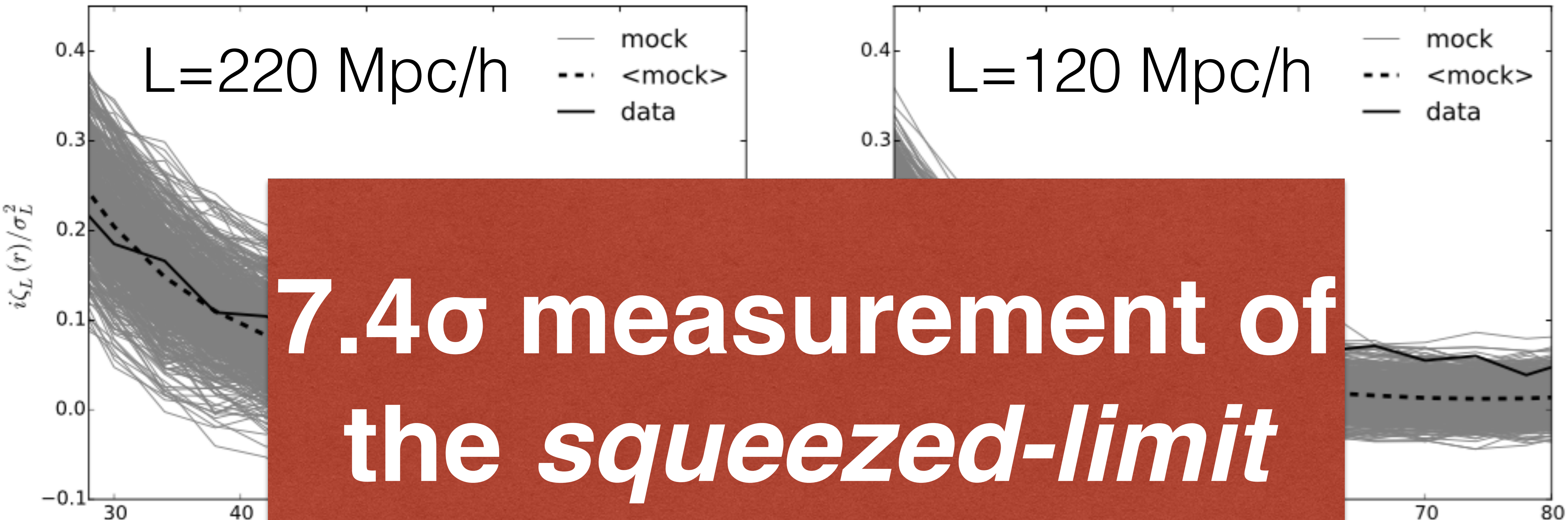
Results: χ^2/DOF = 46.4/38



- Because of complex geometry of DR11 footprint, we use the local correlation function, instead of the power spectrum. Power spectrum will be presented using DR12 in the future

- Integrated three-point function, iζ(r), is just Fourier transform of iB(k): $$i\zeta_L(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\, iB_L(\mathbf{k}) e^{i\mathbf{r}\cdot\mathbf{k}}$$

Results: χ^2/DOF = 46.4/38



7.4σ measurement of the *squeezed-limit bispectrum*!!

- Bec … se the loca … m. Power spectrum will be presented using DR12 in the future

- Integrated three-point function, iζ(r), is just Fourier transform of iB(k):

$$i\zeta_L(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\, iB_L(\mathbf{k}) e^{i\mathbf{r}\cdot\mathbf{k}}$$

Nice, but what is this good for?

- Primordial non-Gaussianity (“local-type f_{NL}”)
  - The constraint from BOSS is work in progress, but the Fisher matrix analysis suggests that the integrated bispectrum is a **nearly optimal estimator for the local-type f_{NL}**
  - We no longer need to measure the full bispectrum, if we are just interested in f_{NL}^{local}!

Nice, but what is this good for?

- We can also learn about galaxy bias
  - Local bias model:
    - $\delta_g(x)=b_1\delta_m(x)+(\mathbf{b_2}/2)[\delta_m(x)]^2+\ldots$
- **The bispectrum can give us b_2 at the leading (tree-level) order**, unlike for the power spectrum that has b_2 at the next-to-leading order

Result on b_2

- We use the simplest, tree-level SPT bispectrum in redshift space with the local bias model to interpret our measurements
  - [We also use information from BOSS's 2-point correlation function on $f\sigma_8$ and BOSS's weak lensing data on σ_8]
- We find: $\mathbf{b_2 = 0.41 \pm 0.41}$

More on b_2

- Using slightly more advanced models, we find:

	baseline	eff kernel	tidal bias	both*
b_2	0.41 ± 0.41	0.51 ± 0.41	0.48 ± 0.41	0.60 ± 0.41

*The last value is in agreement with b_2 found by the Barcelona group (Gil-Marín et al. 2014) that used the full bispectrum analysis and the same model

Separate Universe Simulation

- How do we compute the response function beyond perturbation theory?
  - Do we have to run many big-volume simulations and divide them into sub-volumes? No.
- Fully non-linear computation of the response function is possible with **separate universe simulations**
  - E.g., we run two small-volume simulations with separate-universe cosmologies of over- and under-dense regions with the same initial random number seeds, and compute the derivative dlnP/dδ by, e.g.,

$$\frac{d\ln P(k)}{d\bar{\delta}} = \frac{\ln P(k|+\bar{\delta}) - \ln P(k|-\bar{\delta})}{2\bar{\delta}}$$

Separate Universe Cosmology

$$\rho(t)\,[1+\delta_\rho(t)] = \tilde{\rho}(t)$$

$$\frac{\Omega_m h^2}{a^3(t)}\,[1+\delta_\rho(t)] = \frac{\tilde{\Omega}_m \tilde{h}^2}{\tilde{a}^3(t)}$$

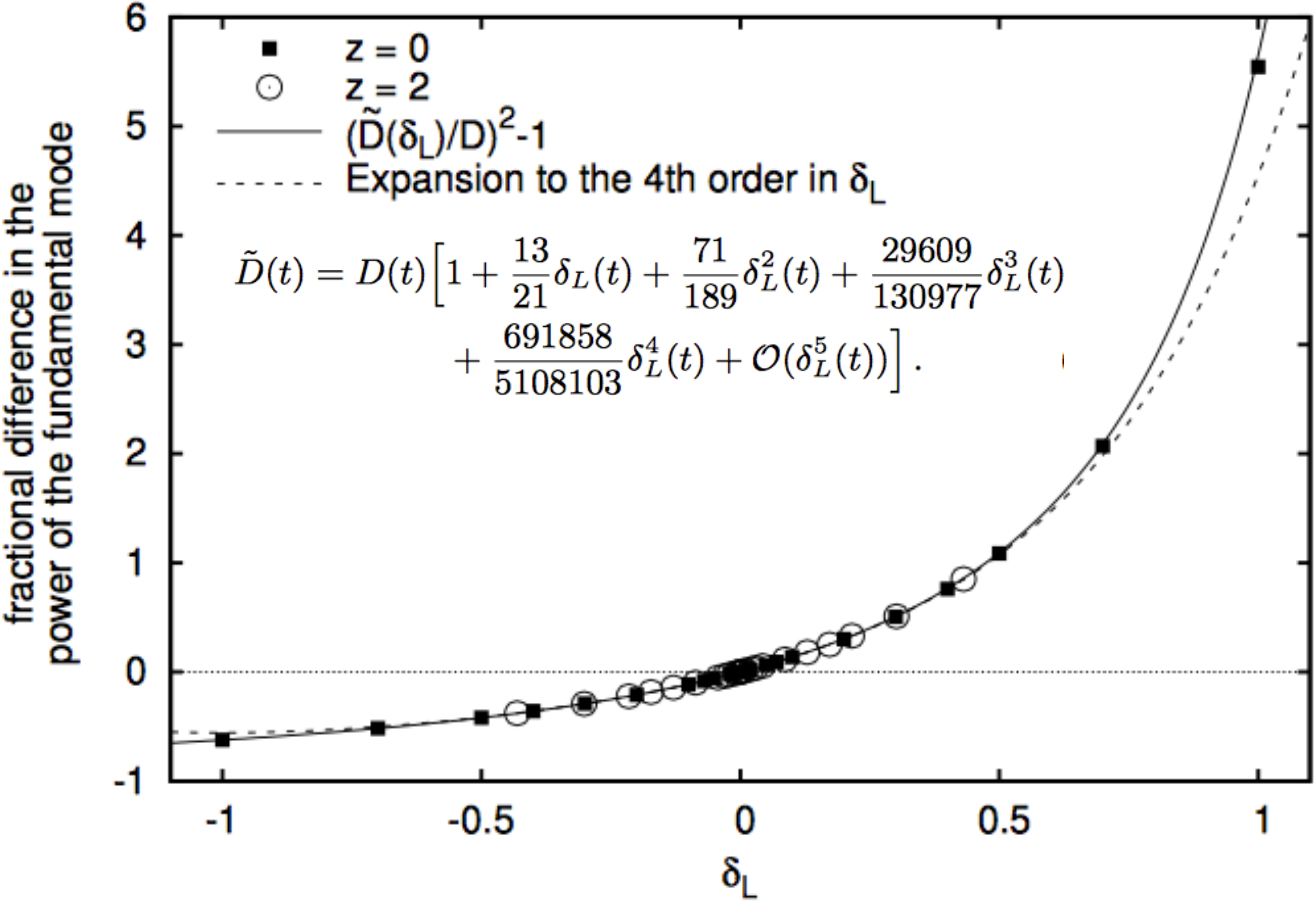
$$\frac{\tilde{K}}{H_0^2} = \frac{5}{3}\frac{\Omega_m}{a(t_i)}\delta_\rho(t_i)$$

$$\delta_H = \left(1 - \frac{\tilde{K}}{H_0^2}\right)^{1/2} - 1$$

$$\tilde{H}_0 = H_0[1+\delta_H]$$

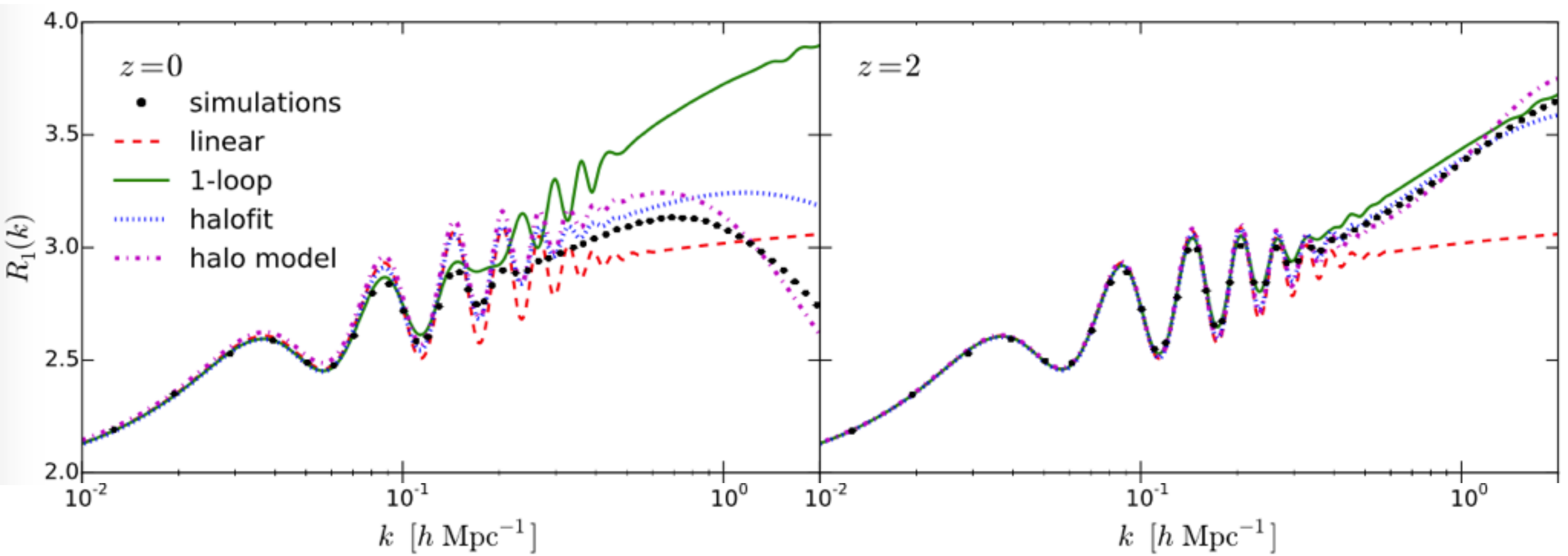
$$\tilde{\Omega}_m = \Omega_m[1+\delta_H]^{-2}$$

$$\tilde{\Omega}_\Lambda = \Omega_\Lambda[1+\delta_H]^{-2}$$



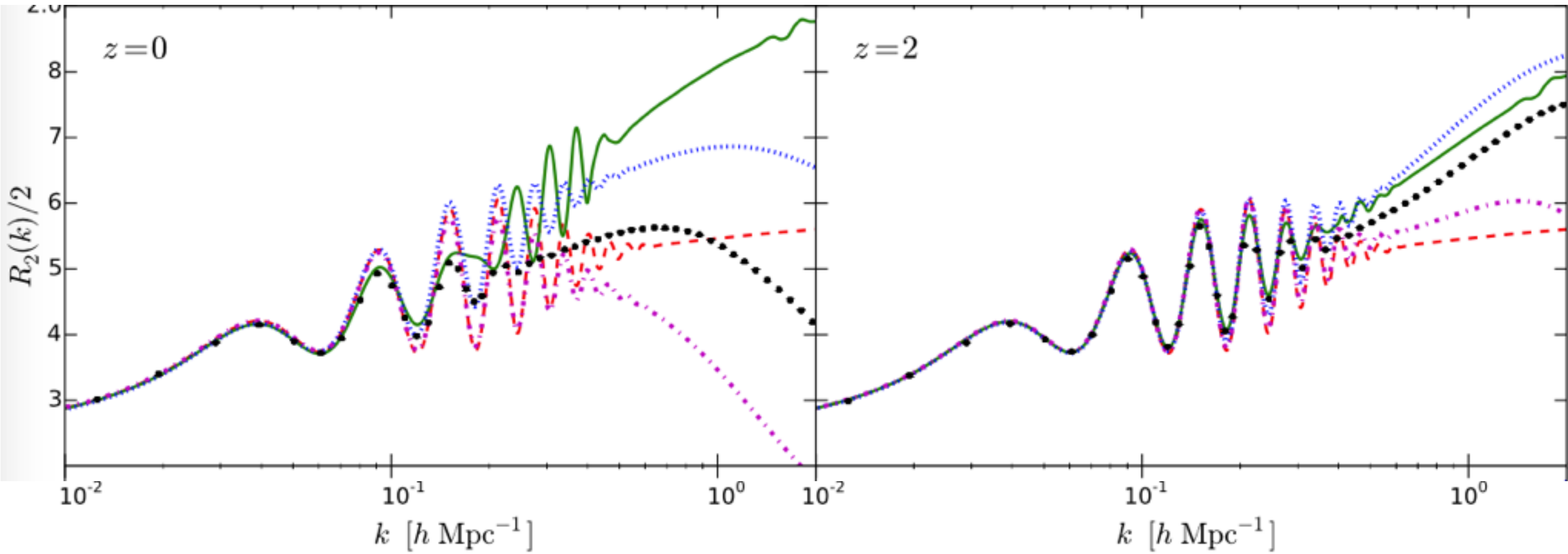
z = 0
z = 2
$(\tilde{D}(\delta_L)/D)^2-1$
Expansion to the 4th order in δ_L
$\tilde{D}(t) = D(t)\left[1 + \frac{13}{21}\delta_L(t) + \frac{71}{189}\delta_L^2(t) + \frac{29609}{130977}\delta_L^3(t) + \frac{691858}{5108103}\delta_L^4(t) + \mathcal{O}(\delta_L^5(t))\right].$
fractional differenc in the power of the fundamental mode
δ_L
6
5
4
3
2
1
0
-1
-1
-0.5
0
0.5
1

$R_1 = d\ln P / d\delta$



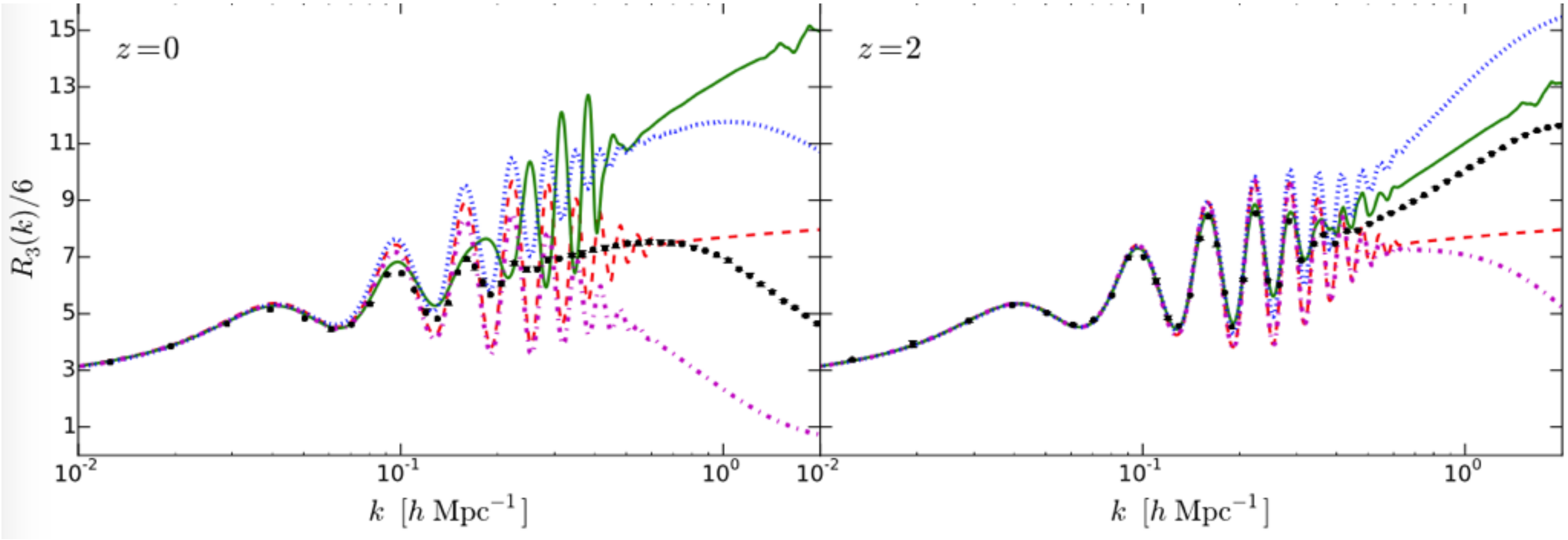
- The symbols are the data points with error bars. You cannot see the error bars!

$R_2 = d^2 \ln P / d\delta^2$



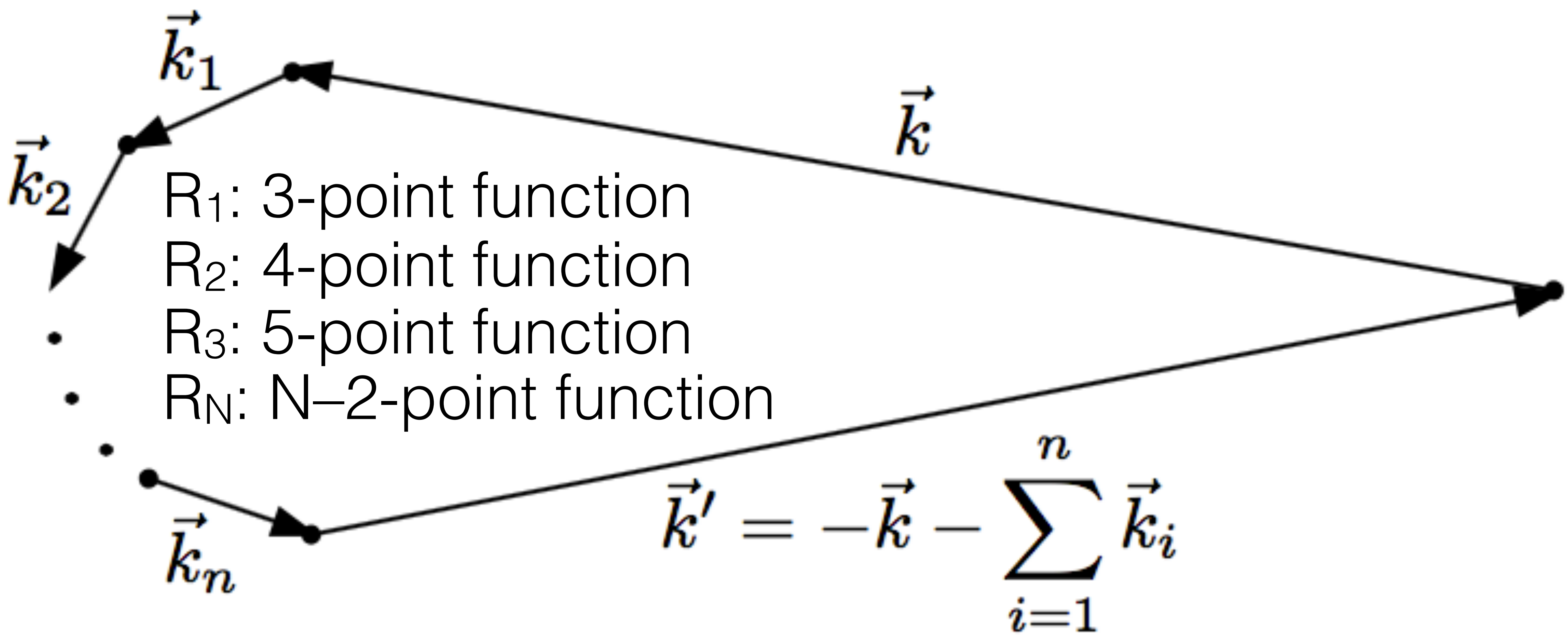
- More derivatives can be computed by using simulations run with more values of δ

$R_3 = d^3 \ln P / d\delta^3$



- But, what do $d^n \ln P / d\delta^n$ mean physically??

More derivatives: Squeezed limits of N-point functions



- Why do we want to know this? I don’t know, but it is cool and they have not been measured before!

One more cool thing

- We can use the separate universe simulations to test **validity of SPT to all orders in perturbations**

- The fundamental prediction of SPT: the non-linear power spectrum at a given time is given by the linear power spectra at the same time

  - In other words, the only time dependence arises from the linear growth factors, D(t)

One more cool thing

- We can use the separate universe simulations to test **validity of SPT to all orders in perturbations**

$$\delta' + \nabla \cdot [(1+\delta)\mathbf{v}] = 0 \ ,$$
$$\mathbf{v}' + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = -\mathcal{H}\mathbf{v} - \nabla\phi \ ,$$
$$\nabla^2\phi = 4\pi G a^2 \bar{\rho}\delta \ ,$$

SPT at all orders: Exact solution of the pressureless fluid equations

We can test validity of SPT as a description of collisions particles

Example: $P_{1\text{-loop}}(k)$

- "1-loop" SPT [3rd order]

$$P(k,a) = P_l(k,a) + P_{22}(k,a) + 2P_{13}(k,a)$$

$$P_{22}(k,a) = 2\int \frac{d^3q}{(2\pi)^3}\, P_l(q,a)P_l(|\mathbf{k}-\mathbf{q}|,a)\left[F_2(\mathbf{q},\mathbf{k}-\mathbf{q})\right]^2$$

$$2P_{13}(k,a) = \frac{2\pi k^2}{252}P_l(k,a)\int_0^\infty \frac{dq}{(2\pi)^3}\, P_l(q,a) \times \left[100\frac{q^2}{k^2} - 158 + 12\frac{k^2}{q^2} - 42\frac{q^4}{k^4} + \frac{3}{k^5q^3}(q^2-k^2)^3(2k^2+7q^2)\ln\left(\frac{k+q}{|k-q|}\right)\right.$$

- The only time-dependence is in $P_l(k,a) \sim D^2(a)$

- Is this correct?

Rescaled simulations vs Separate universe simulations

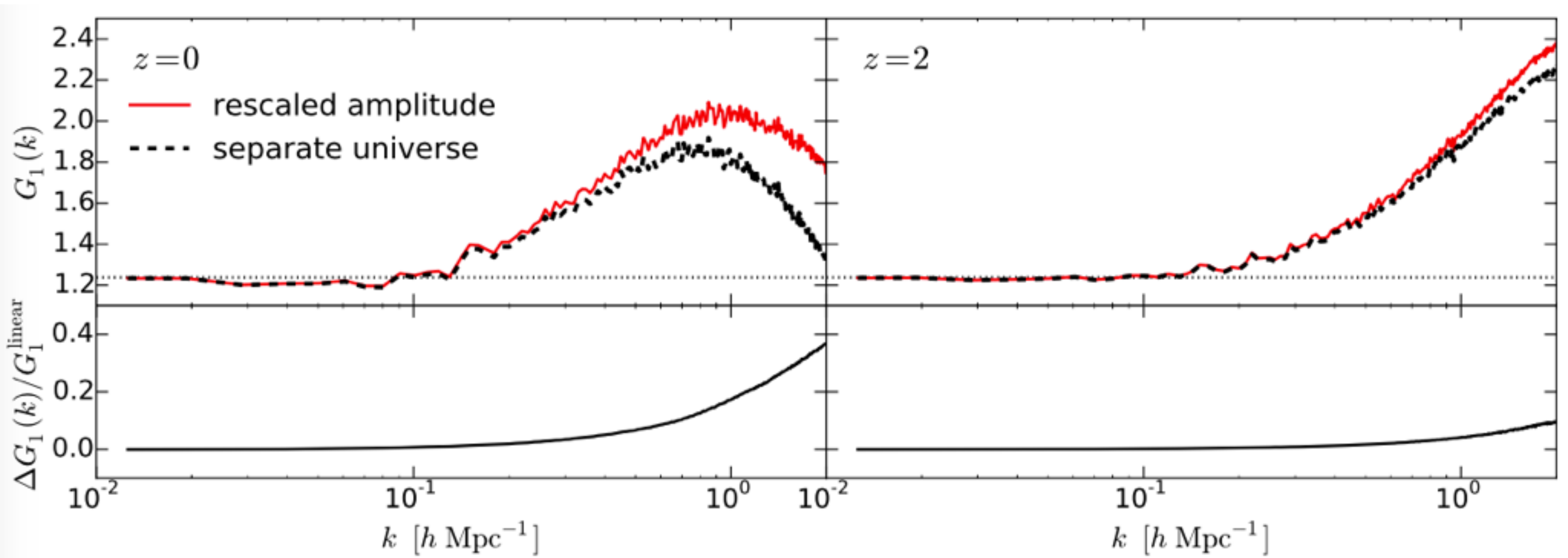
- To test this, we run two sets of simulations.

- **<u>First</u>**: we rescale the initial amplitude of the power spectrum, so that we have a given value of the linear power spectrum amplitude at some later time, t_{out}

- **<u>Second</u>**: full separate universe simulation, which changes all the cosmological parameters consistently, given a value of δ

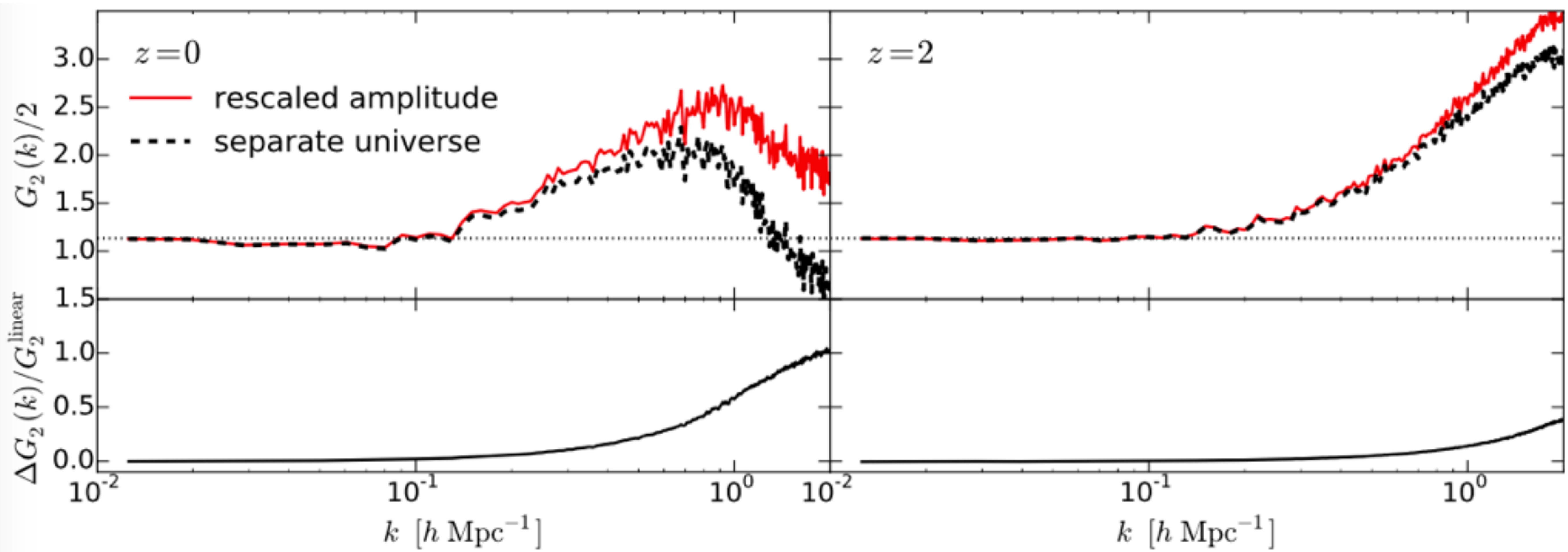
  - We choose δ so that it yields the same amplitude of the linear power spectrum as the first one at t_{out}

Results: 3-point function



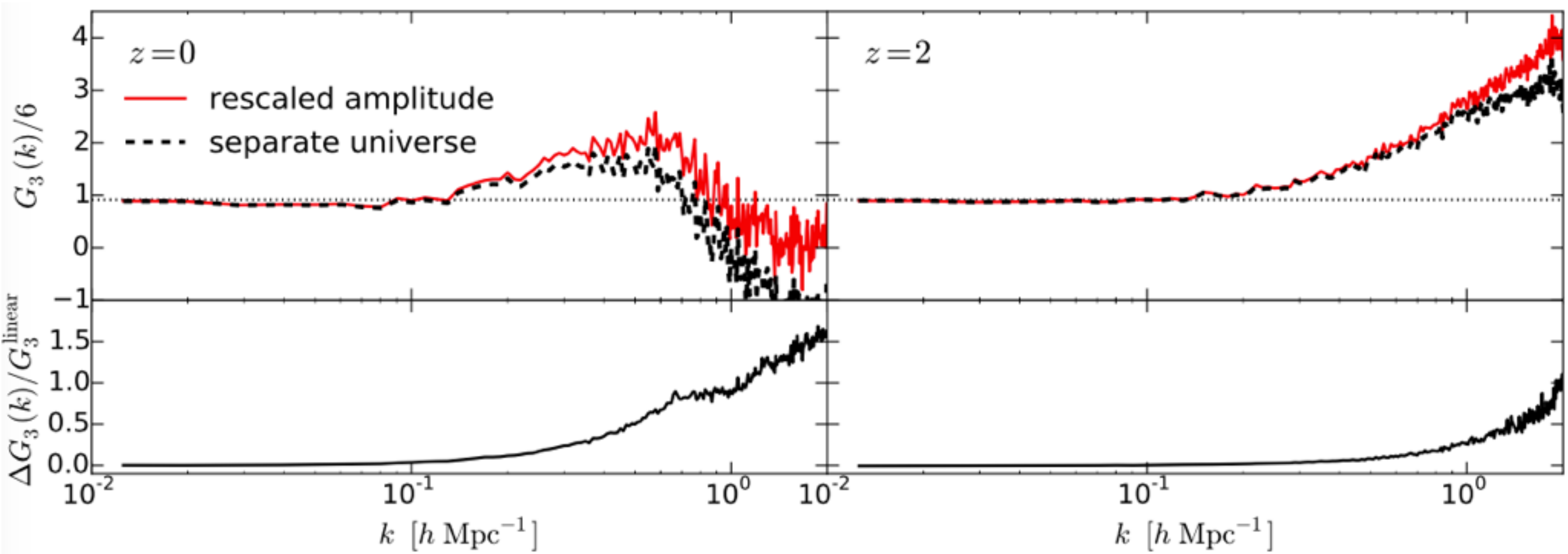
- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions

Results: 4-point function



- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions

Results: 5-point function



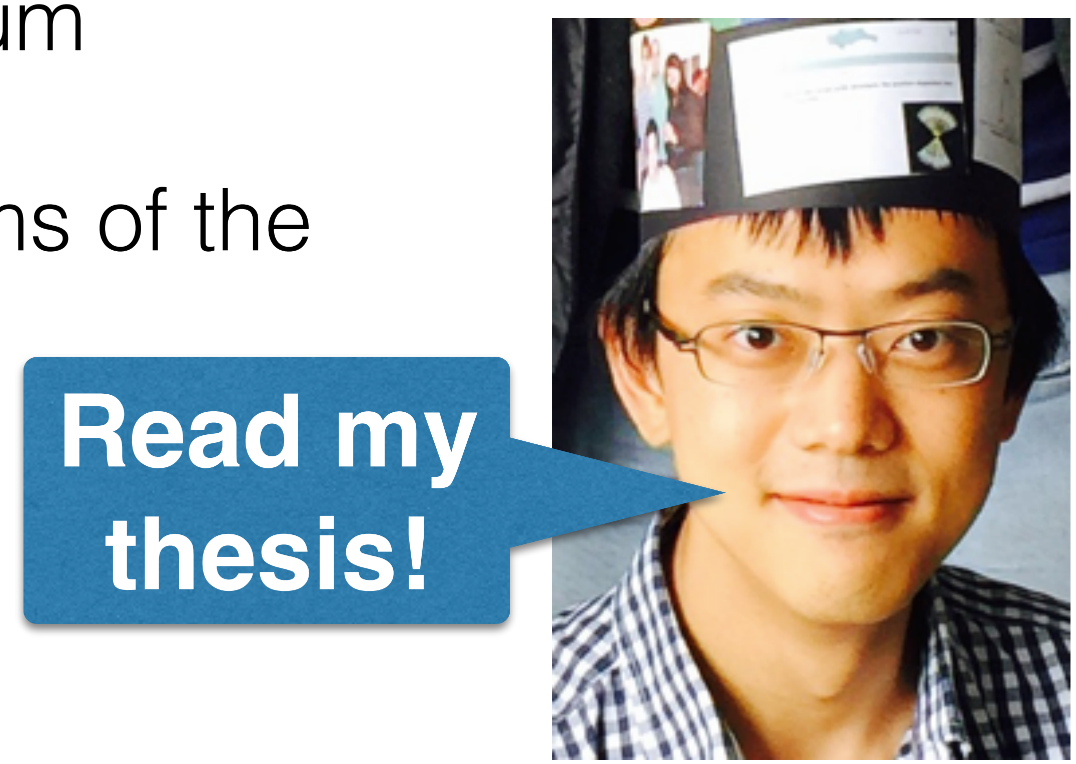
- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions

Break down of SPT at all orders

- **At z=0, SPT computed to all orders breaks down at k~0.5 Mpc/h with 10% error,** in the squeezed limit 3-point function
    - Break down occurs at lower k for the squeezed limits of the 4- and 5-point functions
    - Break down occurs at higher k at z=2
- I find this information quite useful: *it quantifies accuracy of the perfect-fluid approximation of density fields*

Summary

- ***New observable***: the position-dependent power spectrum and the integrated bispectrum
    - Straightforward interpretation in terms of the separate universe
    - Easy to measure; easy to model!
    - Useful for f_{NL}^{local} and non-linear bias
- Lots of applications: e.g., QSO density correlated with Lyman-alpha power spectrum
- All of the results and much more are summarised in Chi-Ting Chiang's PhD thesis: **arXiv:1508.03256**



r-space	b_1	b_2
baseline	1.971 ± 0.076	0.58 ± 0.31
eff kernel	1.973 ± 0.076	0.62 ± 0.31
tidal bias	1.971 ± 0.076	0.64 ± 0.31
both	1.973 ± 0.076	0.68 ± 0.31

z-space	b_1	b_2
baseline	1.931 ± 0.077	0.54 ± 0.35
eff kernel	1.933 ± 0.077	0.65 ± 0.35
tidal bias	1.932 ± 0.077	0.60 ± 0.35
both	1.933 ± 0.077	0.71 ± 0.35