Position-dependent Power Spectrum

~Attacking an old, but unsolved, problem with a new method~

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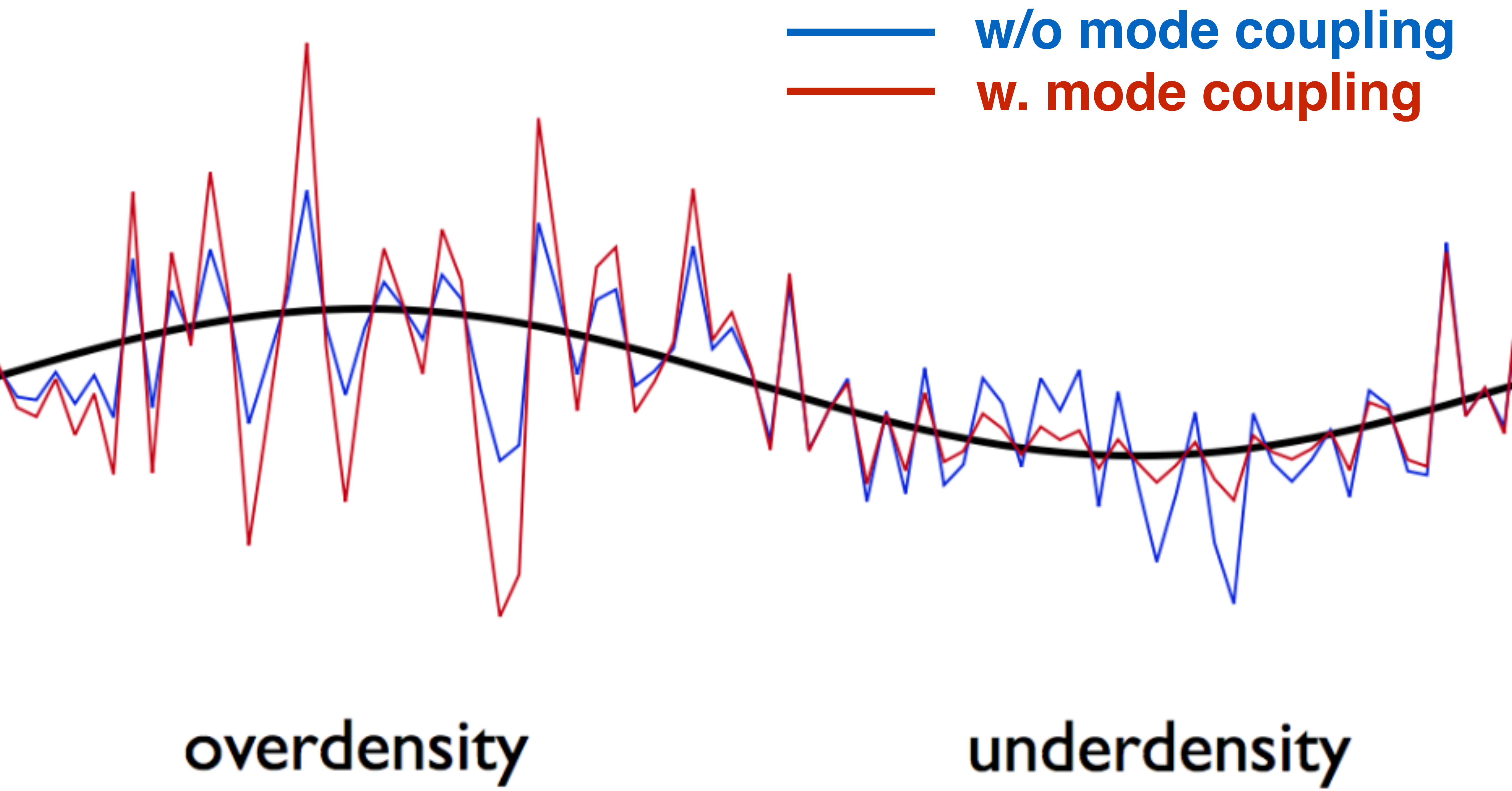
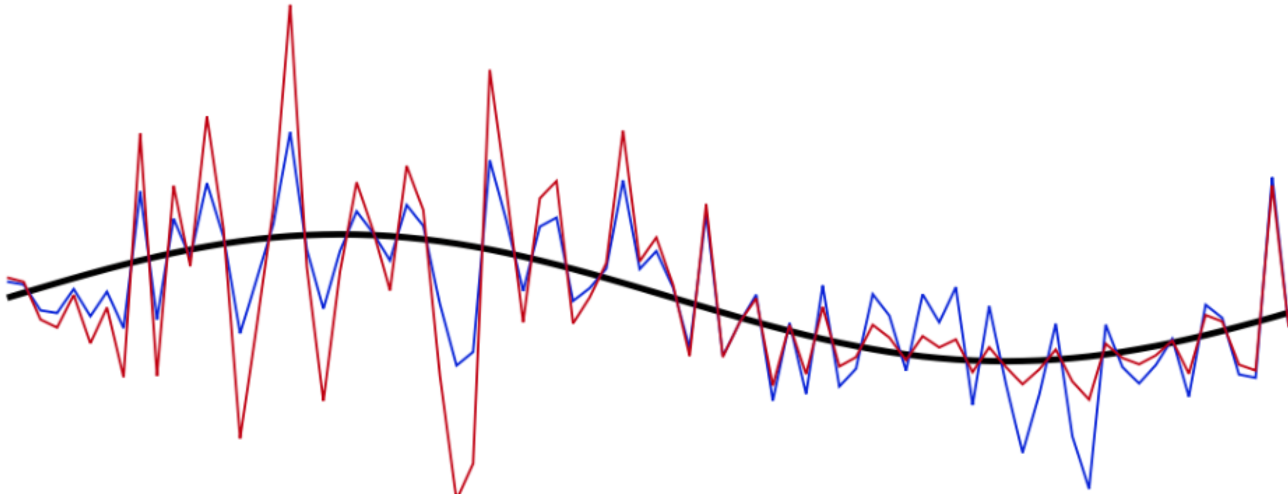
Motivation

- To gain a better insight into “**mode coupling**”
    - An interaction between short-wavelength modes and long-wavelength modes
    - Specifically, how do short wavelength modes **respond** to a long wavelength mode?


w/o mode coupling
w. mode coupling
overdensity
underdensity

Two Approaches



- **Global**
  - "Bird's view": see both long- and short-wavelength modes, and compute coupling between the two directly
- **Local**
  - "Ant's view": Absorb a long-wavelength mode into a new background solution that a local observer sees, and compute short wavelength modes in the new background.

This presentation is based on

- **Chiang** et al. "*Position-dependent power spectrum of the large-scale structure: a novel method to measure the squeezed-limit bispectrum*", JCAP 05, 048 (2014)

- **Chiang** et al. "*Position-dependent correlation function from the SDSS-III BOSS DR10 CMASS Sample*", JCAP 09, 028 (2015)

- **Wagner** et al. "*Separate universe simulations*", MNRAS, 448, L11 (2015)

- **Wagner** et al. "*The angle-averaged squeezed limit of nonlinear matter N-point functions*", JCAP 08, 042 (2015)

Preparation I: Comoving Coordinates

- Space expands. Thus, a physical length scale increases over time

- Since the Universe is homogeneous and isotropic on large scales, the stretching of space is given by a time-dependent function, **a(t)**, which is called the "scale factor"

- Then, the physical length, r(t), can be written as

    - $r(t) = a(t)\,\mathbf{x}$

    - $\mathbf{x}$ is independent of time, and called the "**comoving coordinates**"

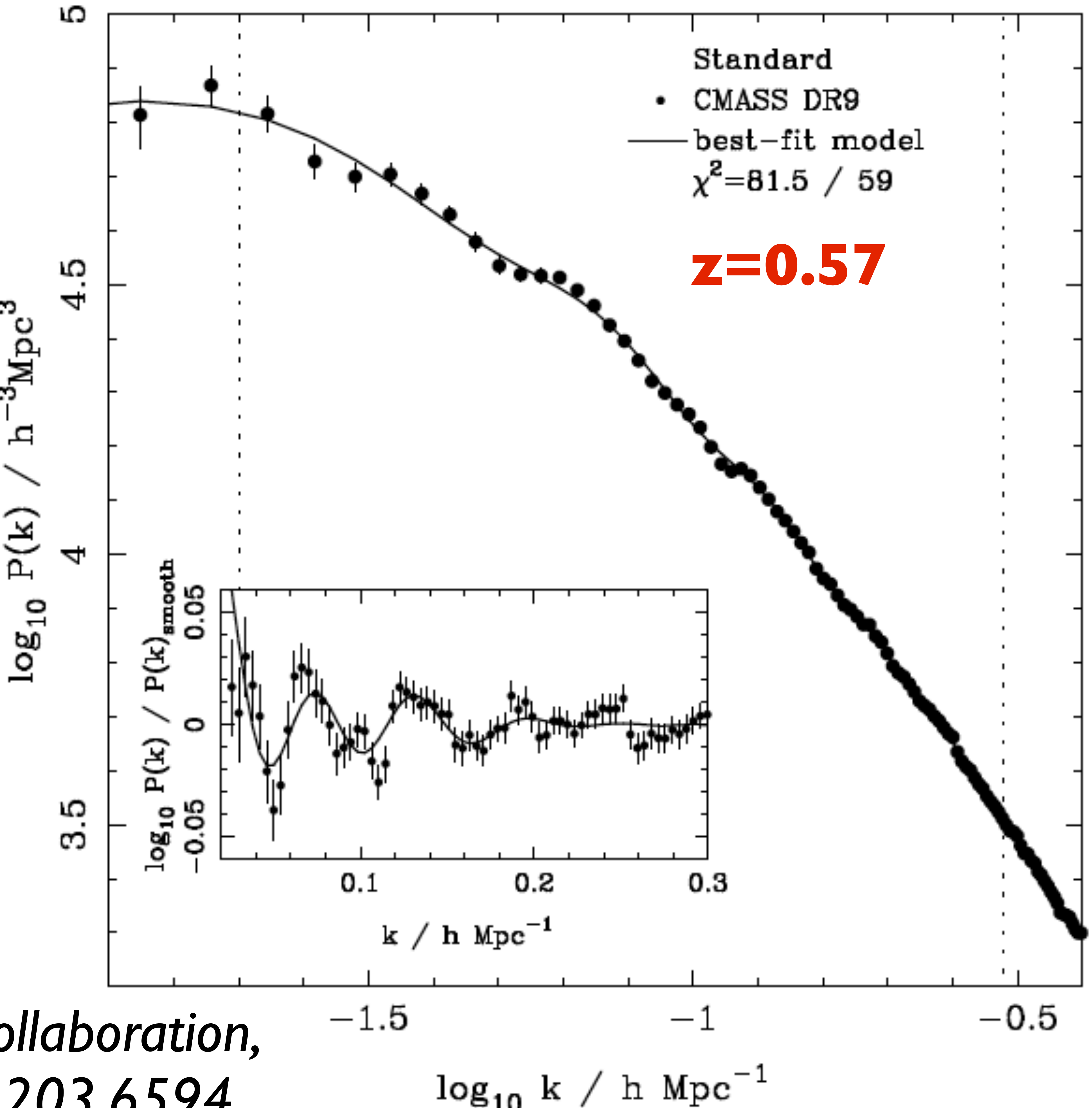
Preparation II: Comoving Waveumbers

- Then, the physical length, $r(t)$, can be written as
    - $r(t) = a(t)\,\mathbf{x}$
    - $\mathbf{x}$ is independent of time, and called the "**comoving coordinates**"
- When we do the Fourier analysis, **the wavenumber, k, is defined with respect to x**. This "comoving wavenumber" is related to the physical wavenumber by $k_{physical}(t) = k_{comoving}/a(t)$

Preparation III: Power Spectrum

500 Mpc/h

- **Take these density fluctuations, and compute the density contrast:**
  - $\delta(x) = [\ \rho(x) - \rho_{mean}\ ] / \rho_{mean}$
- **Fourier-transform this, square the amplitudes, and take averages. The power spectrum is thus:**
  - $P(k) = \langle |\delta_k|^2 \rangle$

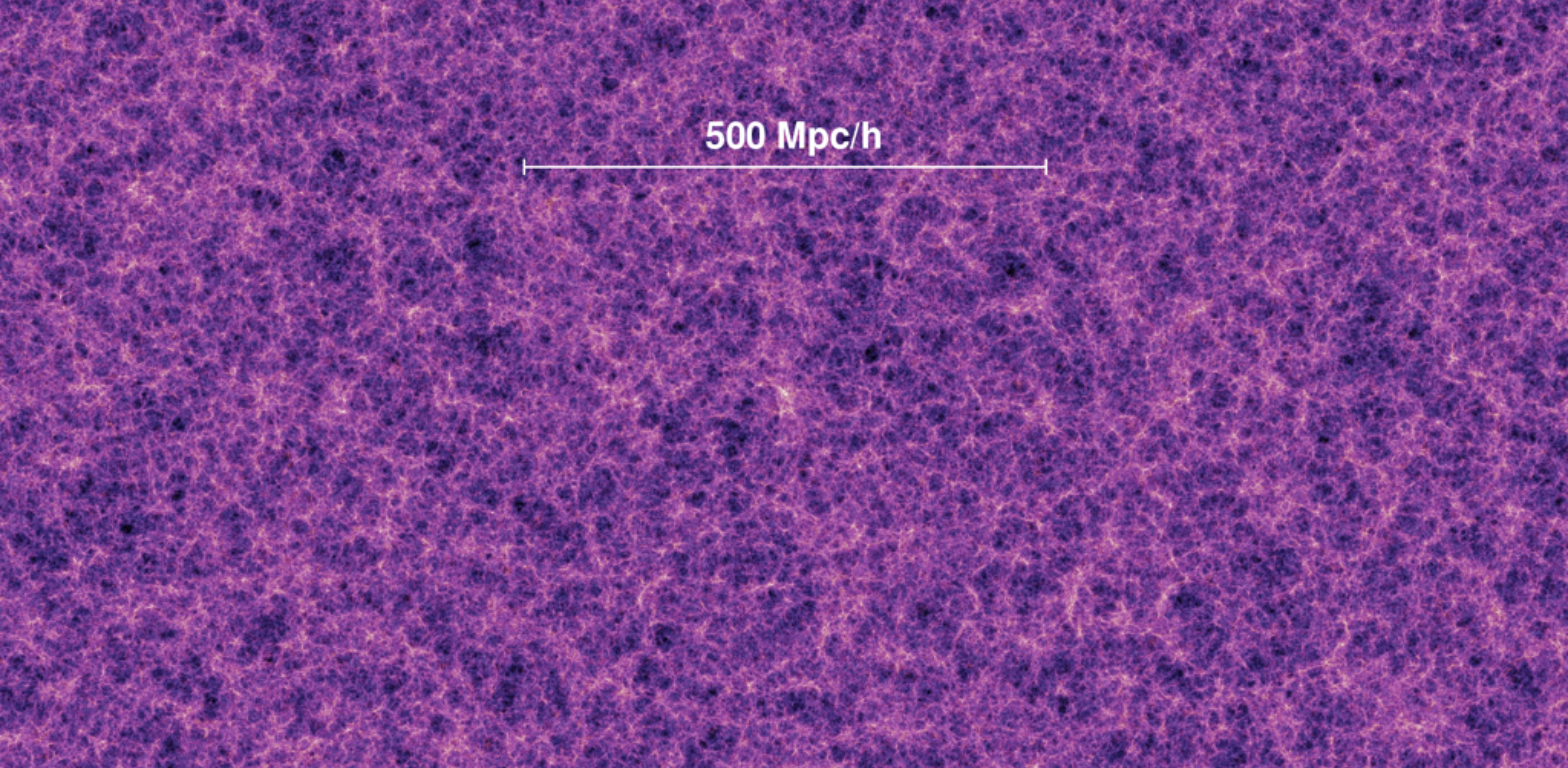


BOSS Collaboration, arXiv:1203.6594

A simple question within the context of cosmology

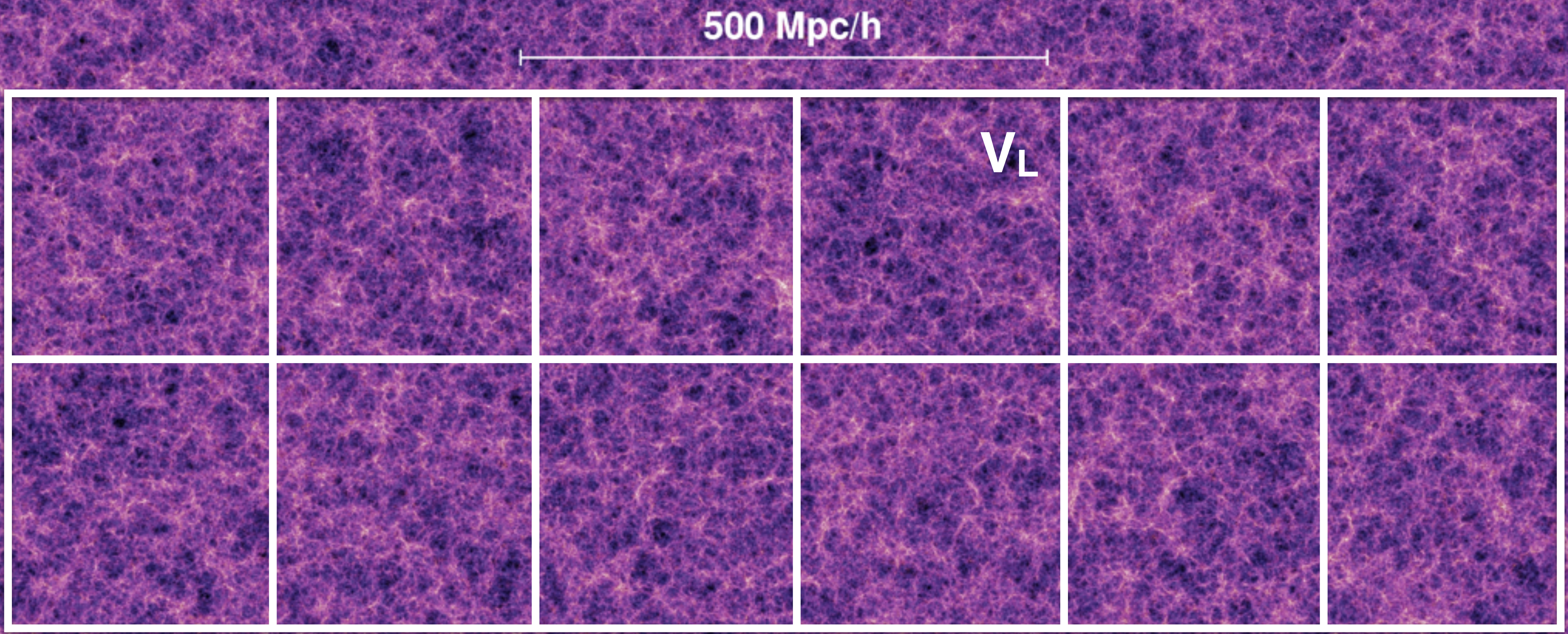
- How do the cosmic structures evolve in an over-dense region?

Simple Statistics



- Divide the survey volume into many sub-volumes V_L, and compare locally-measured power spectra with the corresponding local over-densities

Simple Statistics



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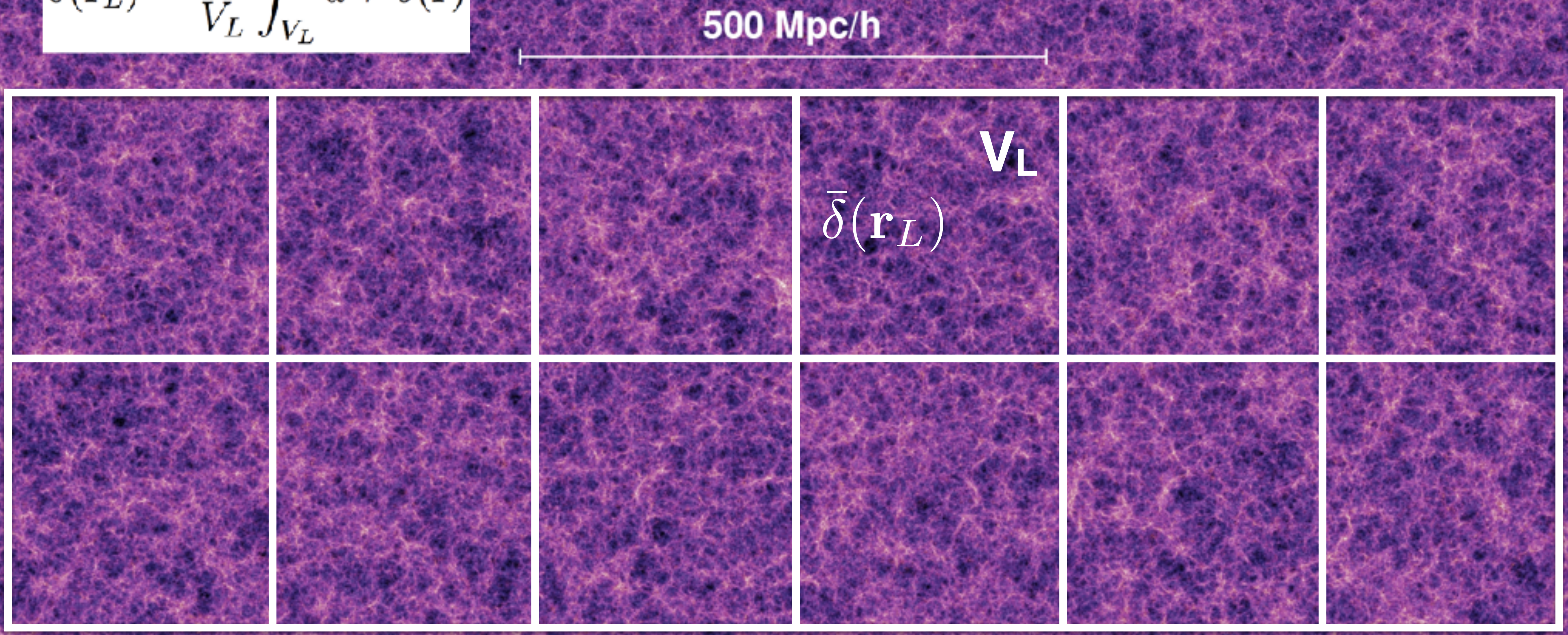
Simple Statistics

$$\bar{\delta}(\mathbf{r}_L) = \frac{1}{V_L} \int_{V_L} d^3r \; \delta(\mathbf{r})$$



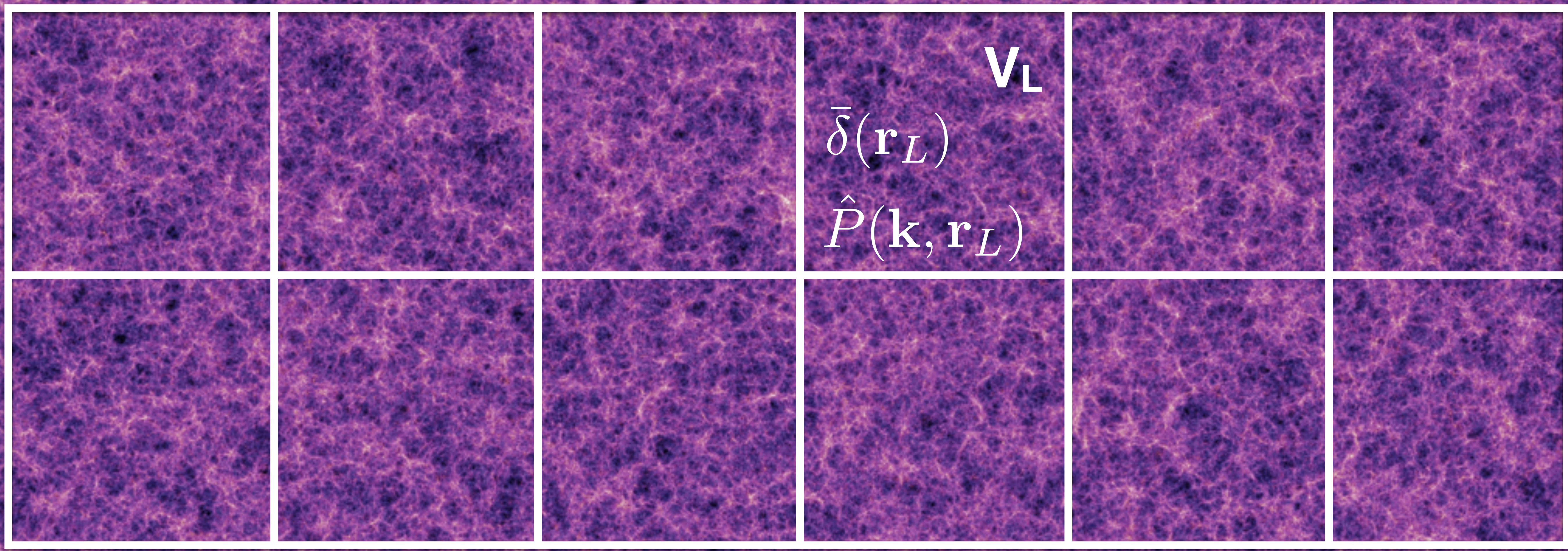
- Divide the survey volume into many sub-volumes V_L, and compare locally-measured power spectra with the corresponding local over-densities

Simple Statistics

$$\bar{\delta}(\mathbf{r}_L) = \frac{1}{V_L}\int_{V_L} d^3r\ \delta(\mathbf{r})$$

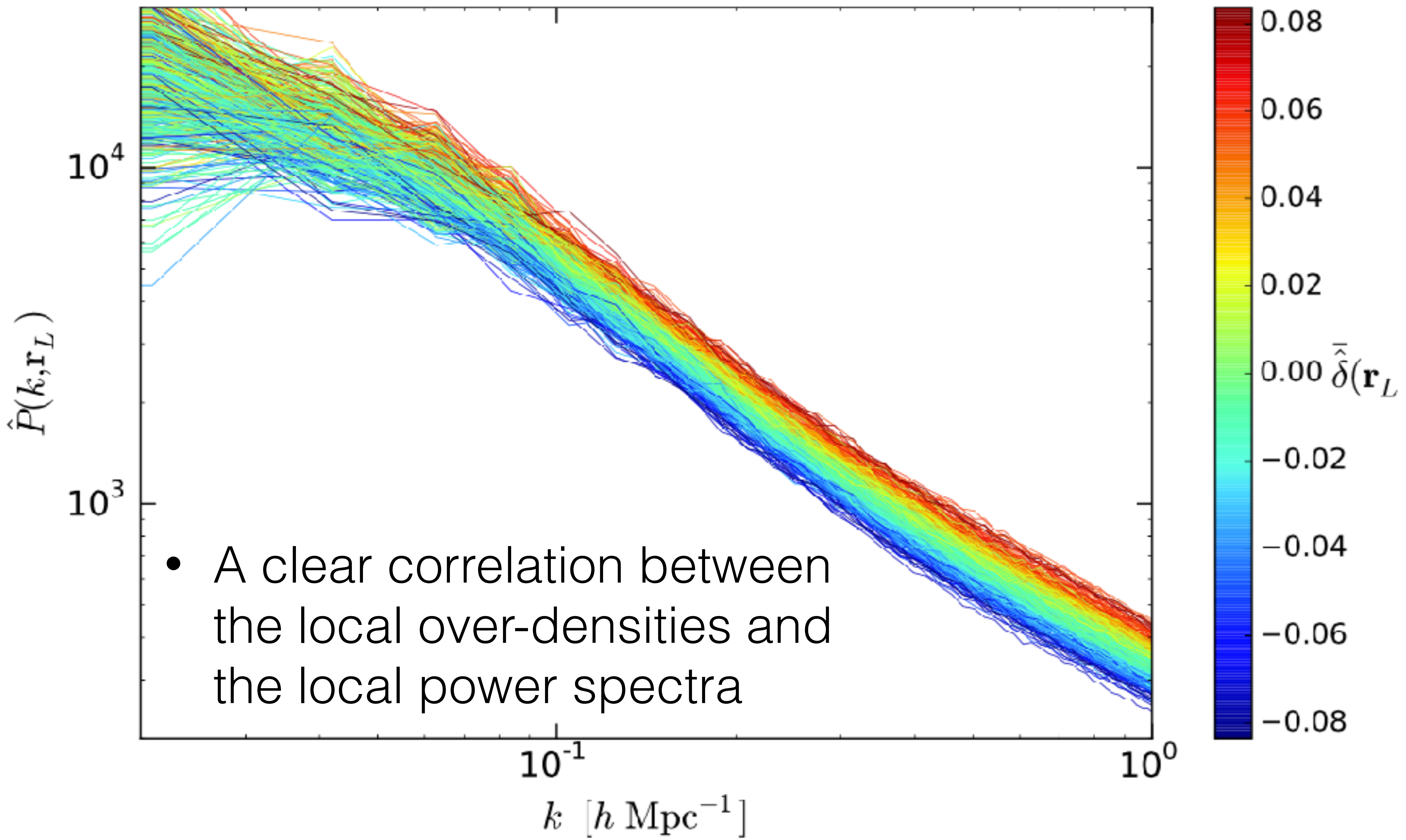
$$\hat{P}(\mathbf{k}, \mathbf{r}_L) \equiv \frac{1}{V_L}|\delta(\mathbf{k}, \mathbf{r}_L)|^2$$

500 Mpc/h



- Divide the survey volume into many sub-volumes V_L, and compare locally-measured power spectra with the corresponding local over-densities

Position-dependent $\hat{P}$(k)



- A clear correlation between the local over-densities and the local power spectra

Integrated Bispectrum, *iB*(*k*)

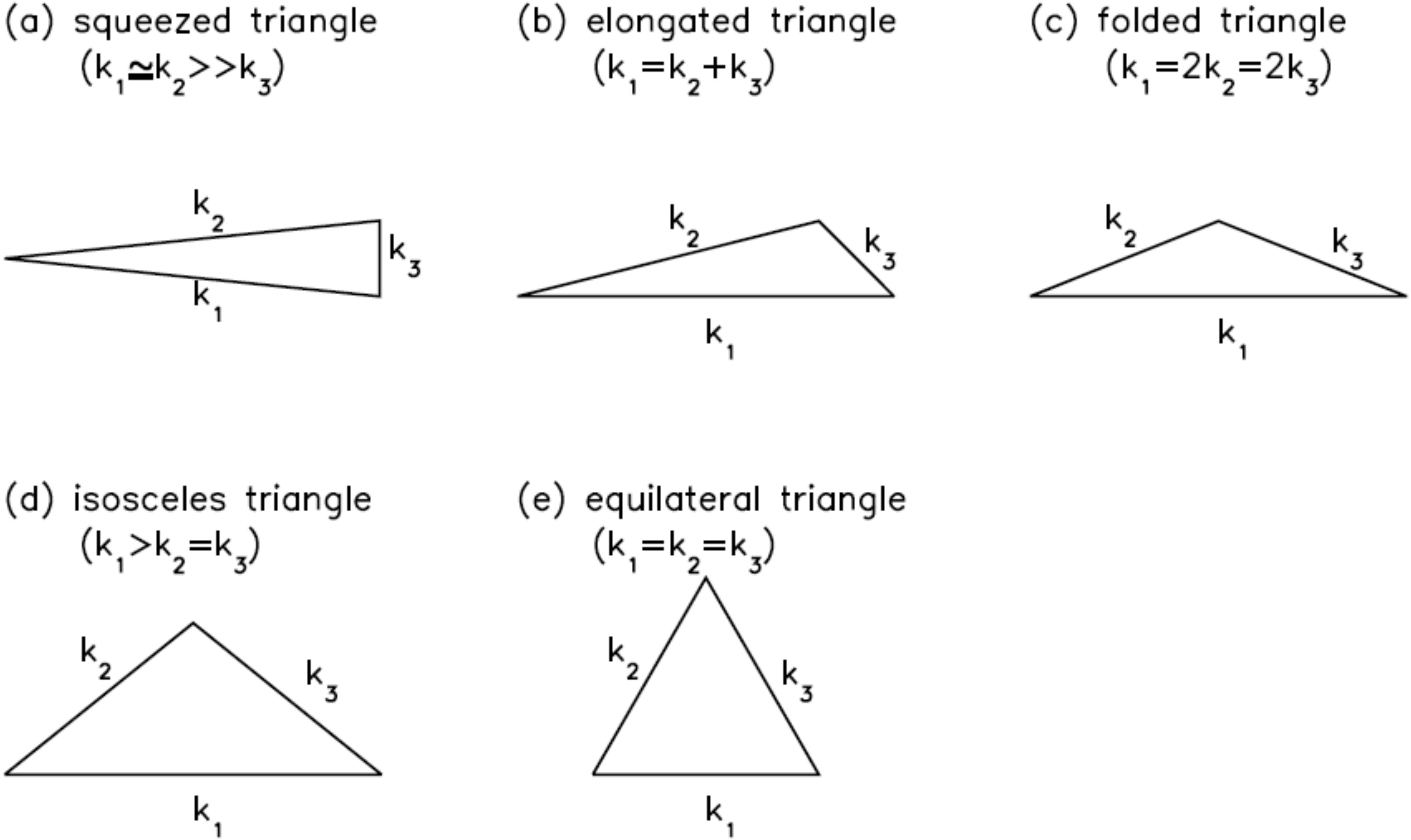
- Correlating the local over-densities and power spectra, we obtain the "integrated bispectrum":

$$i\hat{B}_L(k) = \frac{1}{N_{\rm cut}^3} \sum_{i=1}^{N_{\rm cut}^3} \hat{P}(k, \mathbf{r}_{L,i}) \hat{\bar{\delta}}(\mathbf{r}_{L,i})$$

- This is a (particular configuration of) **three-point function.** The three-point function in Fourier space is called the "**bispectrum**", and is defined as

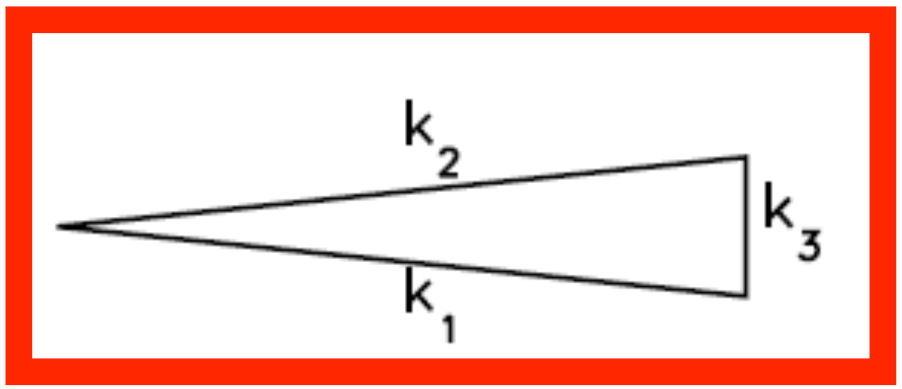
$$\langle \delta(\mathbf{q}_1)\delta(\mathbf{q}_2)\delta(\mathbf{q}_3) \rangle \; = \; B(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)(2\pi)^3 \delta_D(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3)$$

Shapes of the Bispectrum



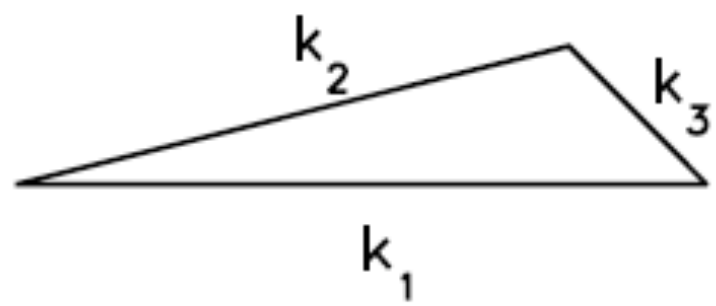
Shapes of the Bispectrum

(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)

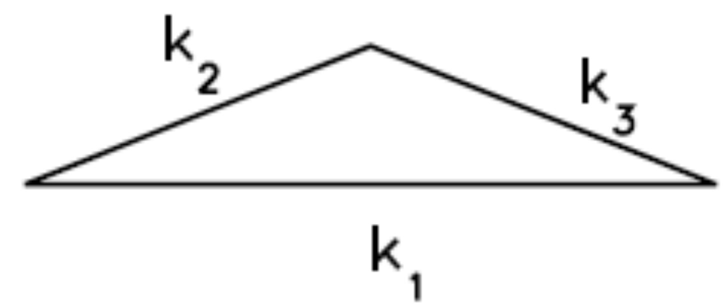


This Talk

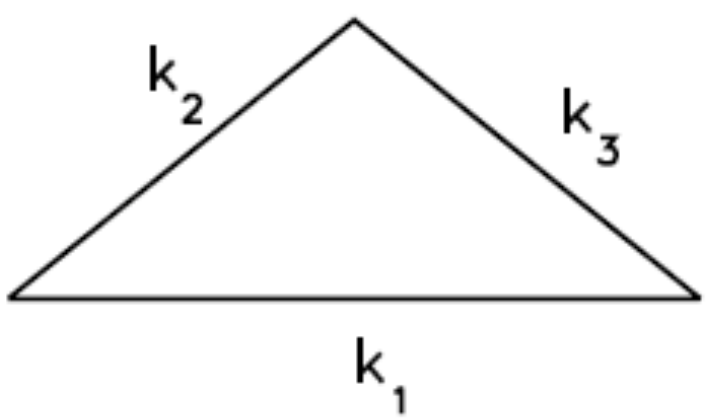
(b) elongated triangle
($k_1 = k_2 + k_3$)



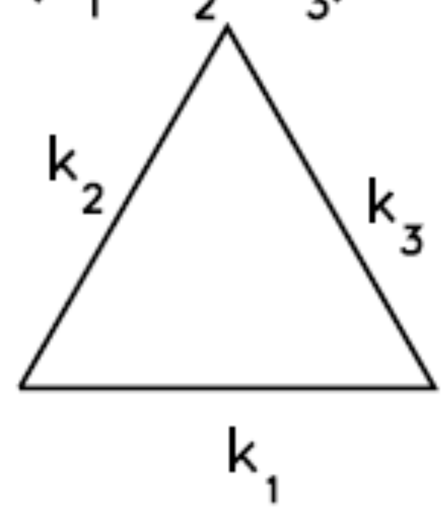
(c) folded triangle
($k_1 = 2k_2 = 2k_3$)



(d) isosceles triangle
($k_1 > k_2 = k_3$)



(e) equilateral triangle
($k_1 = k_2 = k_3$)



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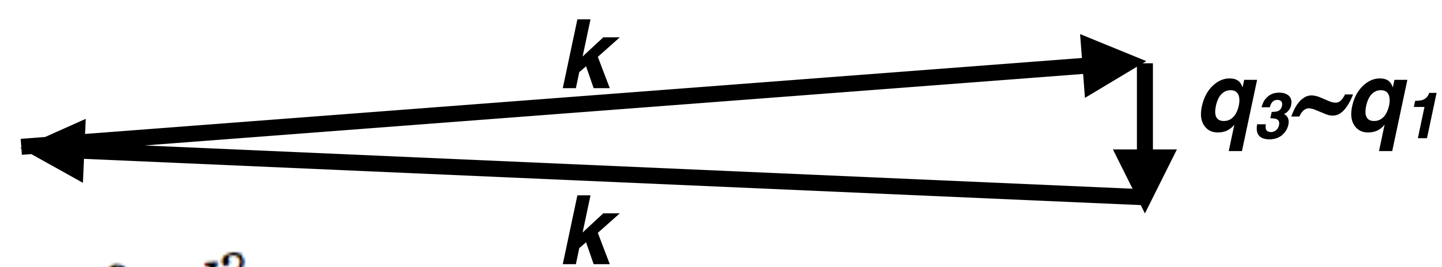
- Correlating the local over-densities and power spectra, we obtain the "integrated bispectrum":

$$i\hat{B}_L(k) = \frac{1}{N_{\rm cut}^3}\sum_{i=1}^{N_{\rm cut}^3}\hat{P}(k,\mathbf{r}_{L,i})\hat{\bar{\delta}}(\mathbf{r}_{L,i})$$

- The expectation value of this quantity is an integral of the bispectrum that picks up the contributions **mostly from the squeezed limit**:

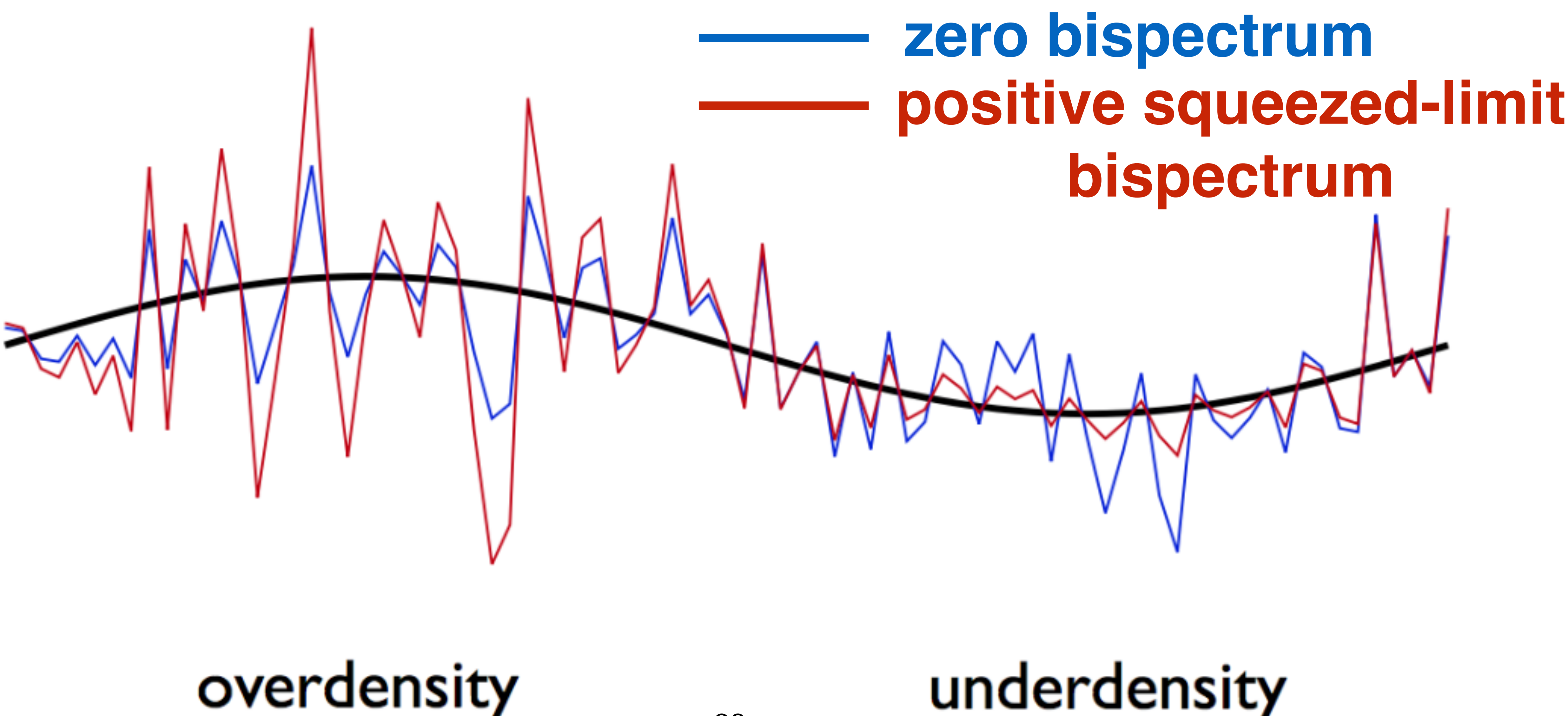


$$\begin{aligned} iB_L(k) &= \langle \hat{P}(k,\mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle \\ &= \frac{1}{V_L^2}\int\frac{d^2\hat{k}}{4\pi}\int\frac{d^3q_1}{(2\pi)^3}\int\frac{d^3q_3}{(2\pi)^3}\, B(\mathbf{k}-\mathbf{q}_1,-\mathbf{k}+\mathbf{q}_1+\mathbf{q}_3,-\mathbf{q}_3) \\ &\quad \times W_L(\mathbf{q}_1)W_L(-\mathbf{q}_1-\mathbf{q}_3)W_L(\mathbf{q}_3) \end{aligned}$$

"taking the squeezed limit and then angular averaging"

Power Spectrum Response

- The integrated bispectrum measures how the local power spectrum responds to its environment, i.e., a long-wavelength density fluctuation



Response Function

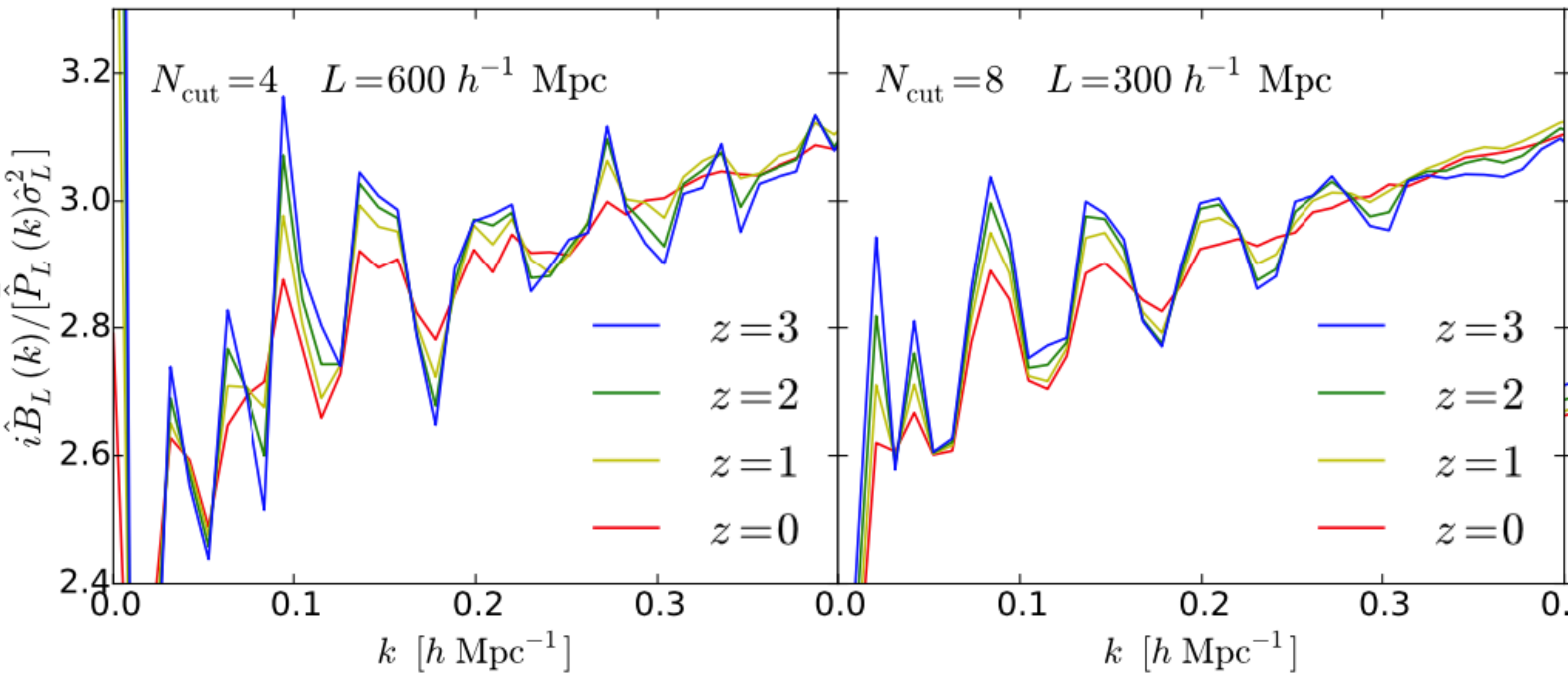
- So, let us Taylor-expand the local power spectrum in terms of the long-wavelength density fluctuation:

$$\hat{P}(k, \mathbf{r}_L) = P(k)|_{\bar{\delta}=0} + \left.\frac{dP(k)}{d\bar{\delta}}\right|_{\bar{\delta}=0} \bar{\delta} + \dots$$

- The integrated bispectrum is then give as

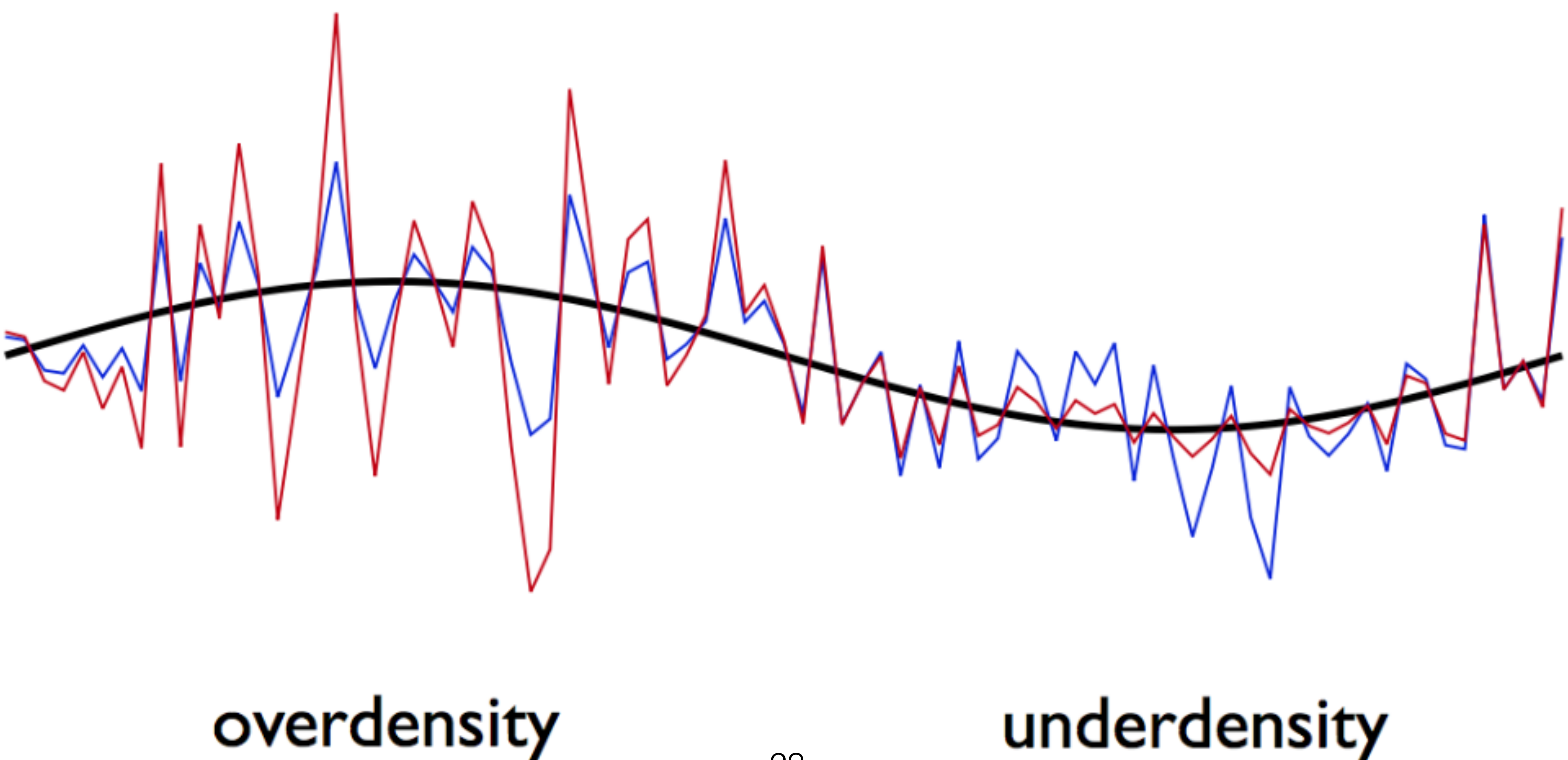
$$iB_L(k) = \sigma_L^2 \underbrace{\boxed{\left.\frac{d\ln P(k)}{d\bar{\delta}}\right|_{\bar{\delta}=0}}}_{\textbf{response function}} P(k)$$

Response Function: N-body Results



- Almost a constant, but a weak scale dependence, and clear oscillating features. How do we understand this?

1. Global, “Bird’s View”



Non-linearity generates a bispectrum

- If the initial conditions were Gaussian, linear perturbations remain Gaussian

- However, **non-linear** gravitational evolution makes density fluctuations at late times non-Gaussian, generating a non-vanishing bispectrum

$$\delta' + \nabla \cdot [(1+\underline{\delta)\mathbf{v}}] = 0 \ ,$$
$$\mathbf{v}' + \underline{(\mathbf{v} \cdot \nabla)\,\mathbf{v}} = -\mathcal{H}\mathbf{v} - \nabla\phi \ , \qquad \text{H=a'/a}$$
$$\nabla^2\phi = 4\pi G a^2 \bar{\rho}\delta \ ,$$

Fourier Transform…

$$
\begin{aligned}
& \dot{\delta}(\boldsymbol{k},\tau)+\theta(\boldsymbol{k},\tau) \\
= \; & -\int \frac{d^3k_1}{(2\pi)^3}\int d^3k_2 \delta_D(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\frac{\boldsymbol{k}\cdot\boldsymbol{k}_1}{k_1^2}\delta(\boldsymbol{k}_2,\tau)\theta(\boldsymbol{k}_1,\tau), \\
& \dot{\theta}(\boldsymbol{k},\tau)+\frac{\dot{a}}{a}\theta(\boldsymbol{k},\tau)+\frac{3\dot{a}^2}{2a^2}\Omega_{\mathrm{m}}(\tau)\delta(\boldsymbol{k},\tau) \\
= \; & -\int \frac{d^3k_1}{(2\pi)^3}\int d^3k_2 \delta_D(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\frac{k^2(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)}{2k_1^2k_2^2}\theta(\boldsymbol{k}_1,\tau)\theta(\boldsymbol{k}_2,\tau)
\end{aligned}
$$

- $\theta = \nabla \cdot \boldsymbol{v}$ is the velocity gradient

Taylor-expand in powers of linear density fields δ_1...

$$\delta(\boldsymbol{k},\tau)=\sum_{n=1}^{\infty}a^n(\tau)\int\frac{d^3q_1}{(2\pi)^3}\cdots\frac{d^3q_{n-1}}{(2\pi)^3}\int d^3q_n\delta_D(\sum_{i=1}^{n}\boldsymbol{q}_i-\boldsymbol{k})F_n(\boldsymbol{q}_1,\boldsymbol{q}_2,\cdots,\boldsymbol{q}_n)\delta_1(\boldsymbol{q}_1)\cdots\delta_1(\boldsymbol{q}_n),$$

$$\theta(\boldsymbol{k},\tau)=-\sum_{n=1}^{\infty}\dot{a}(\tau)a^{n-1}(\tau)\int\frac{d^3q_1}{(2\pi)^3}\cdots\frac{d^3q_{n-1}}{(2\pi)^3}\int d^3q_n\delta_D(\sum_{i=1}^{n}\boldsymbol{q}_i-\boldsymbol{k})G_n(\boldsymbol{q}_1,\boldsymbol{q}_2,\cdots,\boldsymbol{q}_n)\delta_1(\boldsymbol{q}_1)\cdots\delta_1(\boldsymbol{q}_n)$$

Solutions!

$$F_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2)=\frac{5}{7}+\frac{2}{7}\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2}+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{2}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)$$

$$G_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2)=\frac{3}{7}+\frac{4}{7}\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2}+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{2}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)$$

- F_n and G_n with $n \geq 3$ can be found recursively.

Illustrative Example: SPT

- Second-order perturbation gives the lowest-order bispectrum as

$$B_{\rm SPT}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2[P_l(k_1)P_l(k_2)F_2(\mathbf{k}_1, \mathbf{k}_2) + 2\,\text{cyclic}]$$

"l" stands for "linear"

$$F_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1 k_2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) + \frac{2}{7}\left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1 k_2}\right)^2$$

- Then

$$iB_L(k) = \frac{1}{V_L^2}\int \frac{d^2\hat{k}}{4\pi}\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_3}{(2\pi)^3}\; B(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_3, -\mathbf{q}_3)$$
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- Then

$$iB_{L,\rm SPT}(k) \stackrel{kL\to\infty}{=} \left[\frac{68}{21} - \frac{1}{3}\frac{d\ln k^3P_l(k)}{d\ln k}\right] P_l(k)\sigma_L^2$$

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- Then

Response, dlnP(k)/dδ

$$iB_{L,\rm SPT}(k)\overset{kL\to\infty}{=}\boxed{\left[\frac{68}{21}-\frac{1}{3}\frac{d\ln k^3P_l(k)}{d\ln k}\right]}P_l(k)\sigma_L^2$$

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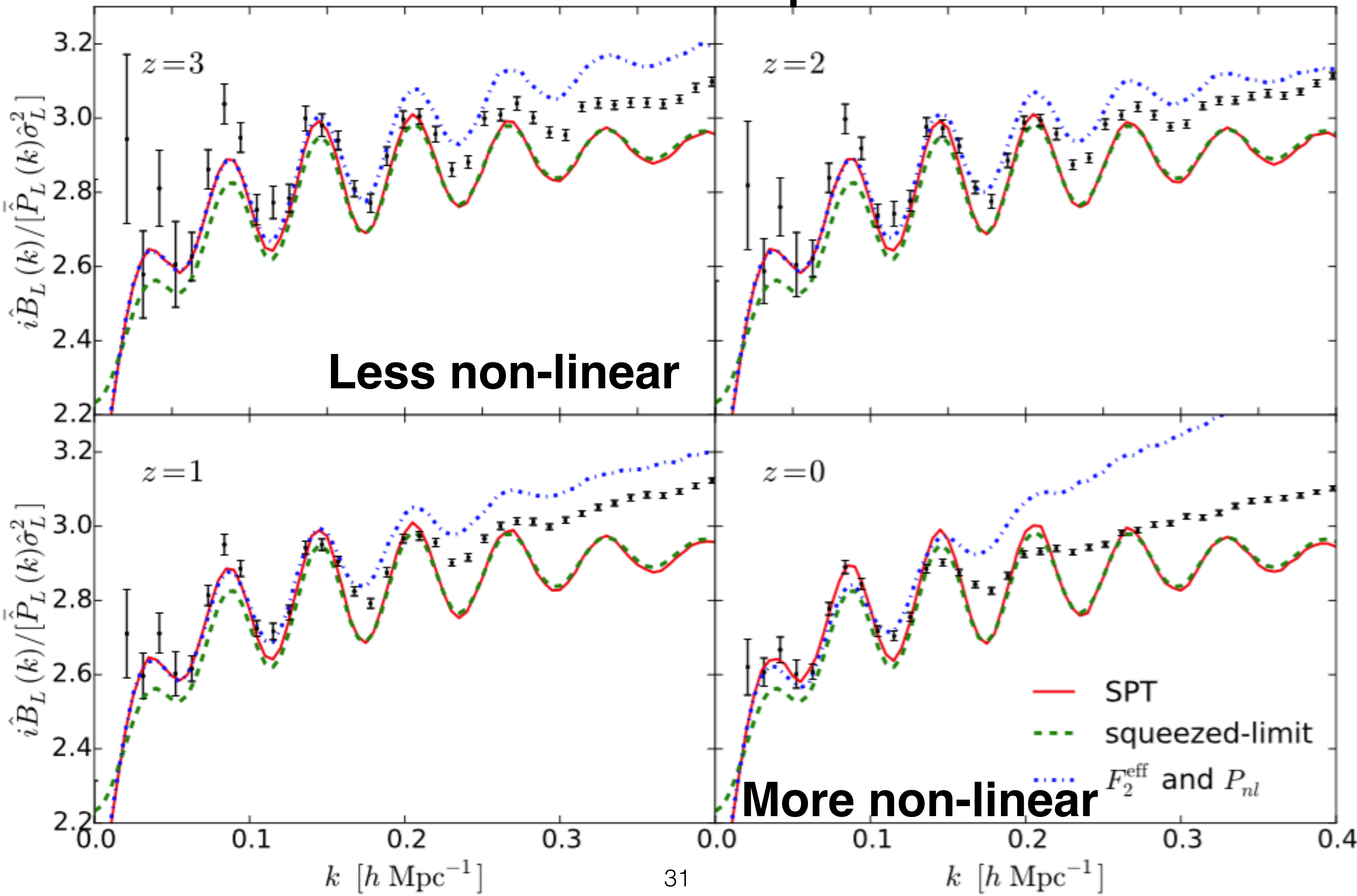
- Then

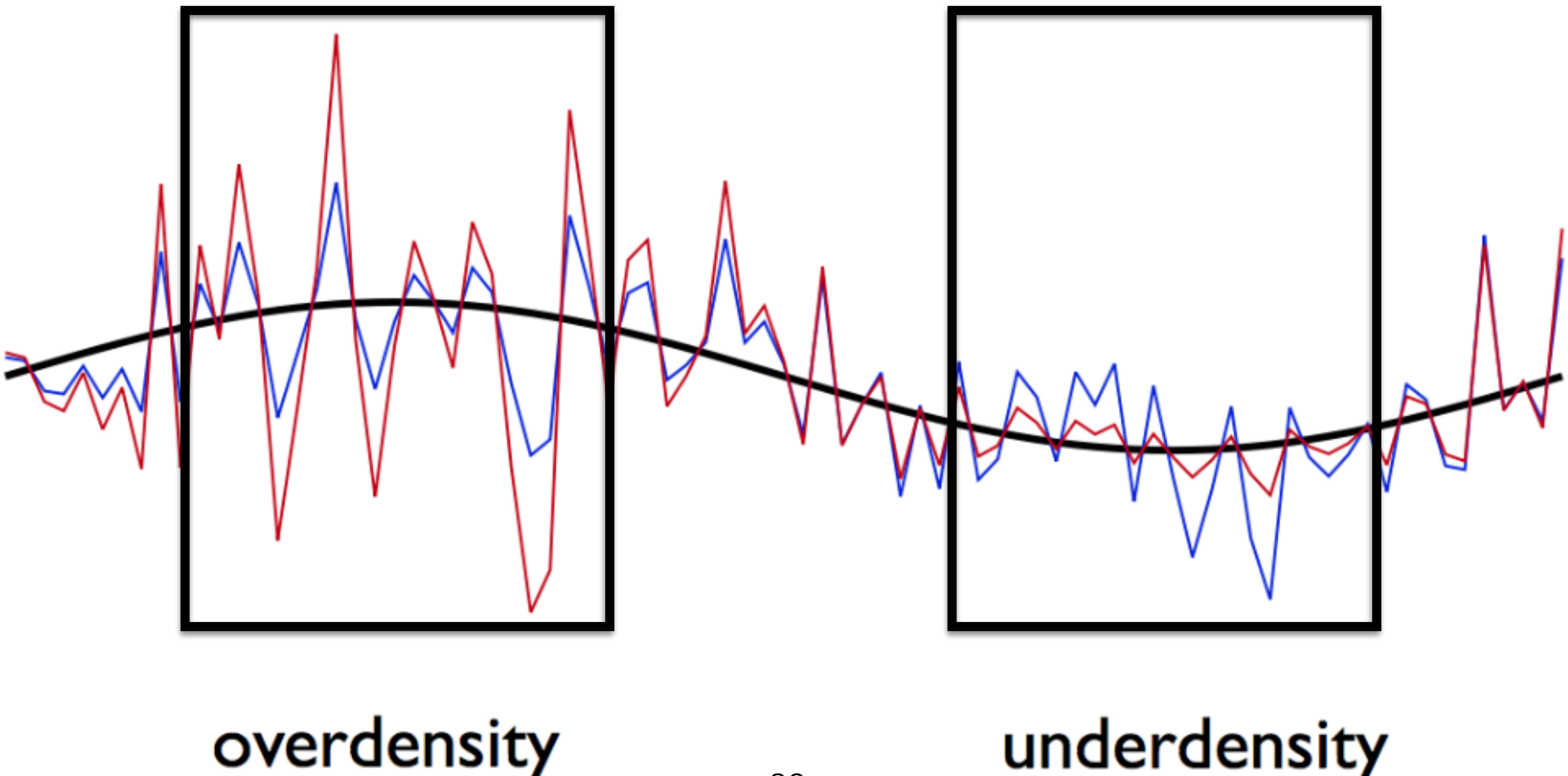
Oscillation in P(k) is enhanced

$$iB_{L,\rm SPT}(k) \overset{kL\to\infty}{=} \left[\frac{68}{21} - \frac{1}{3}\boxed{\frac{d\ln k^3 P_l(k)}{d\ln k}}\right] P_l(k)\sigma_L^2$$

Lowest-order prediction



1. Local, “Ant’s View”



Lemaitre (1933); Peebles (1980)

Separate Universe Approach

- The meaning of the position-dependent power spectrum becomes more transparent within the context of the "separate universe approach"
  - Each sub-volume with un over-density (or under-density) behaves as if it were **a separate universe with different cosmological parameters**
- In particular, if the global metric is a flat universe, then **each sub-volume can be regarded as a different universe with non-zero curvature**

Mapping between two cosmologies

- The goal here is to compute the power spectrum in the presence of a long-wavelength perturbation δ. We write this as P(k,a|δ)

- We try to achieve this by computing the power spectrum in a **modified cosmology** with non-zero curvature. Let us put the tildes for quantities evaluated in a modified cosmology

$$\tilde{P}(\tilde{k},\tilde{a}) \rightarrow P(k,a|\bar{\delta})$$

Separate Universe Approach: The Rules

- We evaluate the power spectrum in both cosmologies at the **same physical time** and **same physical spatial coordinates**
  - Thus, the evolution of the scale factor is different:

$$\tilde{a}(t) = a(t)\left[1 - \frac{1}{3}\bar{\delta}(t)\right]$$

*tilde: separate universe cosmology

Separate Universe Approach: The Rules

- We evaluate the power spectrum in both cosmologies at the **same physical time** and **same physical spatial coordinates**
  - Thus, comoving coordinates are different too:

$$\tilde{\mathbf{x}} = \frac{a(t)}{\tilde{a}(t)}\mathbf{x} = \left[1 + \frac{1}{3}\bar{\delta}(t)\right]\mathbf{x}$$

*tilde: separate universe cosmology

Effect 1: Dilation

- Change in the comoving coordinates gives **dln(k³P)/dlnk**

$$
\begin{aligned}
\tilde{P}(k,t) &\to \left[1-\frac{1}{3}\bar{\delta}(t)\right]^3 P\left(k\left[1-\frac{1}{3}\bar{\delta}(t)\right],t\right) \\
&= \left[1-\bar{\delta}(t)\right] P(k,t)\left[1-\frac{1}{3}\frac{d\ln P(k,t)}{d\ln k}\bar{\delta}(t)\right] \\
&= P(k,t)\left[1-\frac{1}{3}\frac{d\ln k^3 P(k,t)}{d\ln k}\bar{\delta}(t)\right] \ .
\end{aligned}
$$

Effect 2: Reference Density

- Change in the denominator of the definition of δ:

$$\tilde{P}(\tilde{k},t) \to \left[1+\bar{\delta}(t)\right]^2 \tilde{P}(\tilde{k},t) = \left[1+2\bar{\delta}(t)\right] \tilde{P}(\tilde{k},t)$$

- Putting both together, we find **a generic formula**, valid to linear order in the long-wavelength δ:

$$\begin{aligned} P(k,a|\bar{\delta}) &= \left[1+2\bar{\delta}(t)\right] \tilde{P}\left(k,\tilde{a}\right) \left[1 - \frac{1}{3}\frac{d\ln k^3 P(k,t)}{d\ln k}\bar{\delta}(t)\right] \\ &\boxed{= \tilde{P}\left(k, a\left[1-\frac{1}{3}\bar{\delta}(a)\right]\right)\left[1+\left(2-\frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k}\right)\bar{\delta}(a)\right]} \end{aligned}$$

Example: Linear P(k)

- Let's use the formula to compute the response of the linear power spectrum, $P_l(k)$, to the long-wavelength δ. Since $P_l \sim D^2$ [D: linear growth],

$$\tilde{P}_l\left(k, a\left[1-\frac{1}{3}\bar{\delta}(a)\right]\right) = \left(\frac{\tilde{D}\left(a\left[1-\frac{1}{3}\bar{\delta}(a)\right]\right)}{D(a)}\right)^2 P_l(k, a)$$

- Spherical collapse model gives

$$\tilde{D}\left(a\left[1-\frac{1}{3}\bar{\delta}(a)\right]\right) = D(a)\left[1+\frac{13}{21}\bar{\delta}(a)\right]$$

Response of $P_l(k)$

- Then we obtain:

$$\frac{d\ln P_l(k,a)}{d\bar{\delta}(a)} = \frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P_l(k,a)}{d\ln k}$$

- Remember the response computed from the leading-order SPT **bispectrum**:

$$iB_{L,\mathrm{SPT}}(k) \overset{kL\to\infty}{=} \left[\frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P_l(k)}{d\ln k}\right] P_l(k)\sigma_L^2$$

- **So, the leading-oder SPT bispectrum gives the response of the linear P(k). Neat!!**

Response of $P_{\text{3rd-order}}(k)$

- So, let's do the same using **third-order** perturbation theory! $P(k,a) = P_l(k,a) + P_{22}(k,a) + 2P_{13}(k,a)$

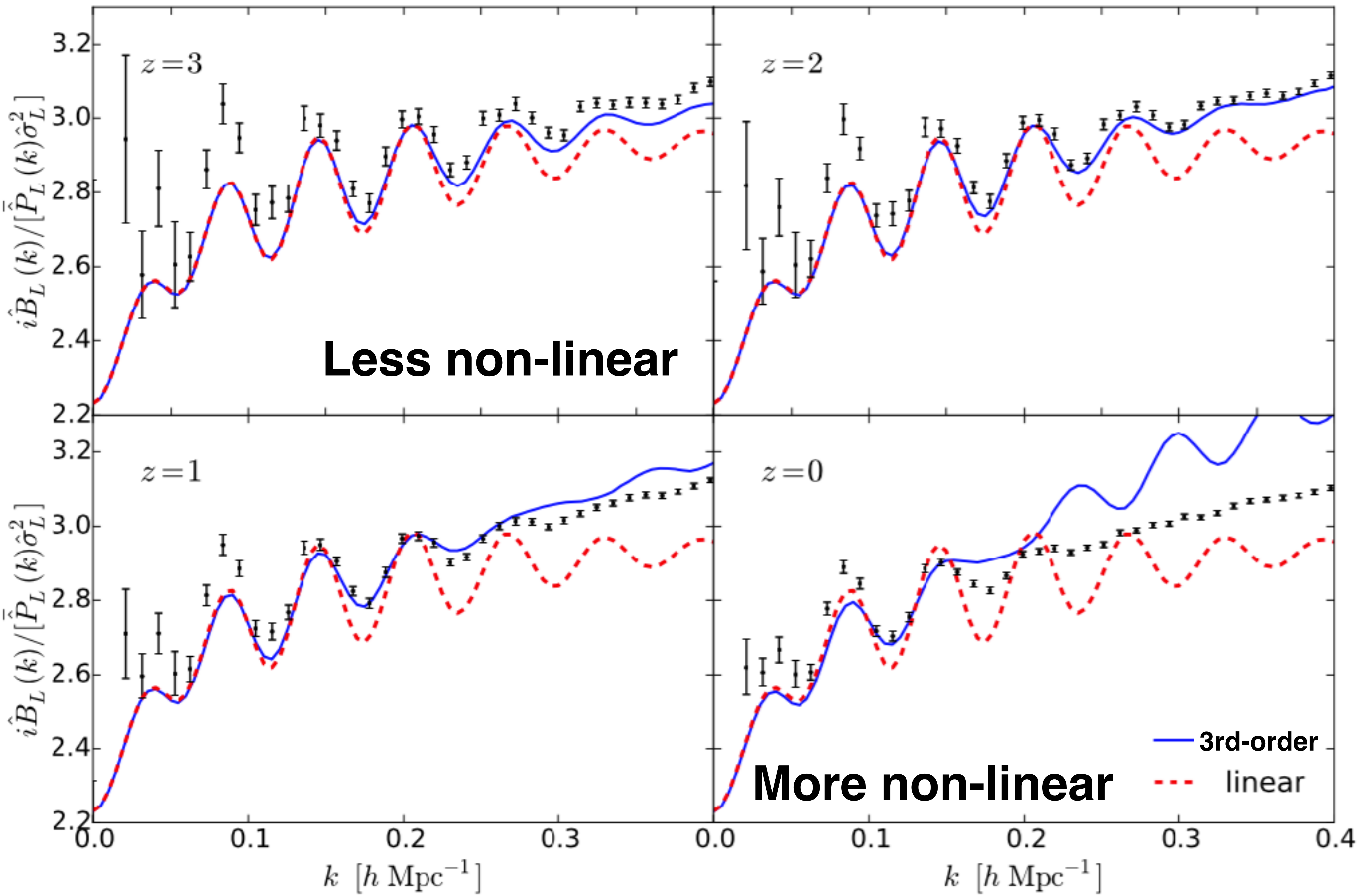
$$P_{22}(k,a) = 2\int \frac{d^3q}{(2\pi)^3}\, P_l(q,a) P_l(|\mathbf{k}-\mathbf{q}|,a)\, [F_2(\mathbf{q},\mathbf{k}-\mathbf{q})]^2$$

$$2P_{13}(k,a) = \frac{2\pi k^2}{252} P_l(k,a) \int_0^\infty \frac{dq}{(2\pi)^3}\, P_l(q,a) \times \left[100\frac{q^2}{k^2} - 158 + 12\frac{k^2}{q^2} - 42\frac{q^4}{k^4} + \frac{3}{k^5 q^3}(q^2-k^2)^3(2k^2+7q^2)\ln\left(\frac{k+q}{|k-q|}\right)\right.$$

- Then we obtain:

$$\frac{d\ln P(k,a)}{d\bar{\delta}(a)} = \frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k} + \frac{26}{21}\frac{P_{22}(k,a) + 2P_{13}(k,a)}{P(k,a)}$$

3rd-order does a decent job



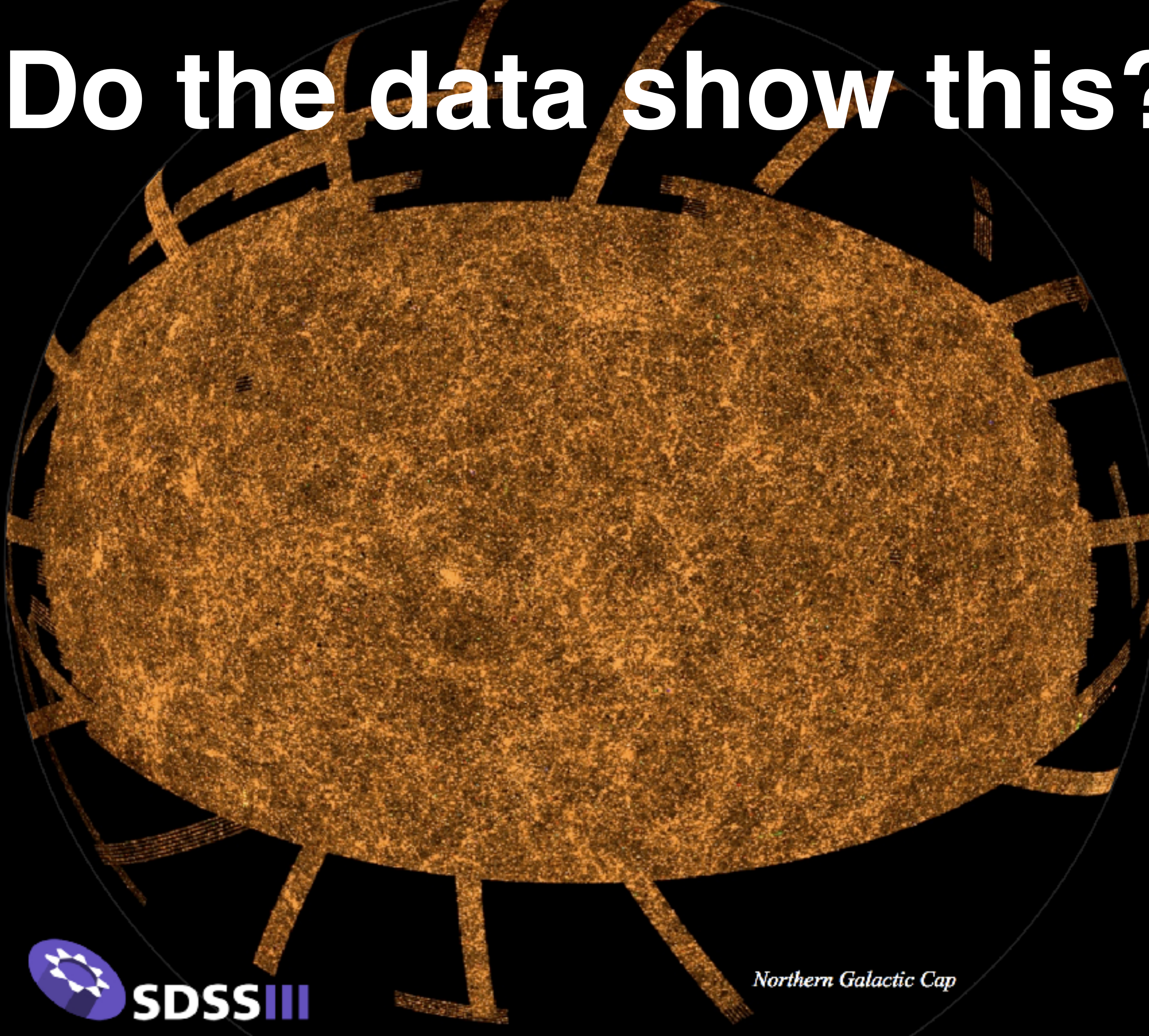
This is a powerful formula

$$P(k, a|\bar{\delta}) = \tilde{P}\left(k, a\left[1 - \frac{1}{3}\bar{\delta}(a)\right]\right)\left[1 + \left(2 - \frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k}\right)\bar{\delta}(a)\right]$$

- The separate universe description is powerful, as it provides physically intuitive, transparent, and straightforward way to compute the effect of a long-wavelength perturbation on the small-scale structure growth

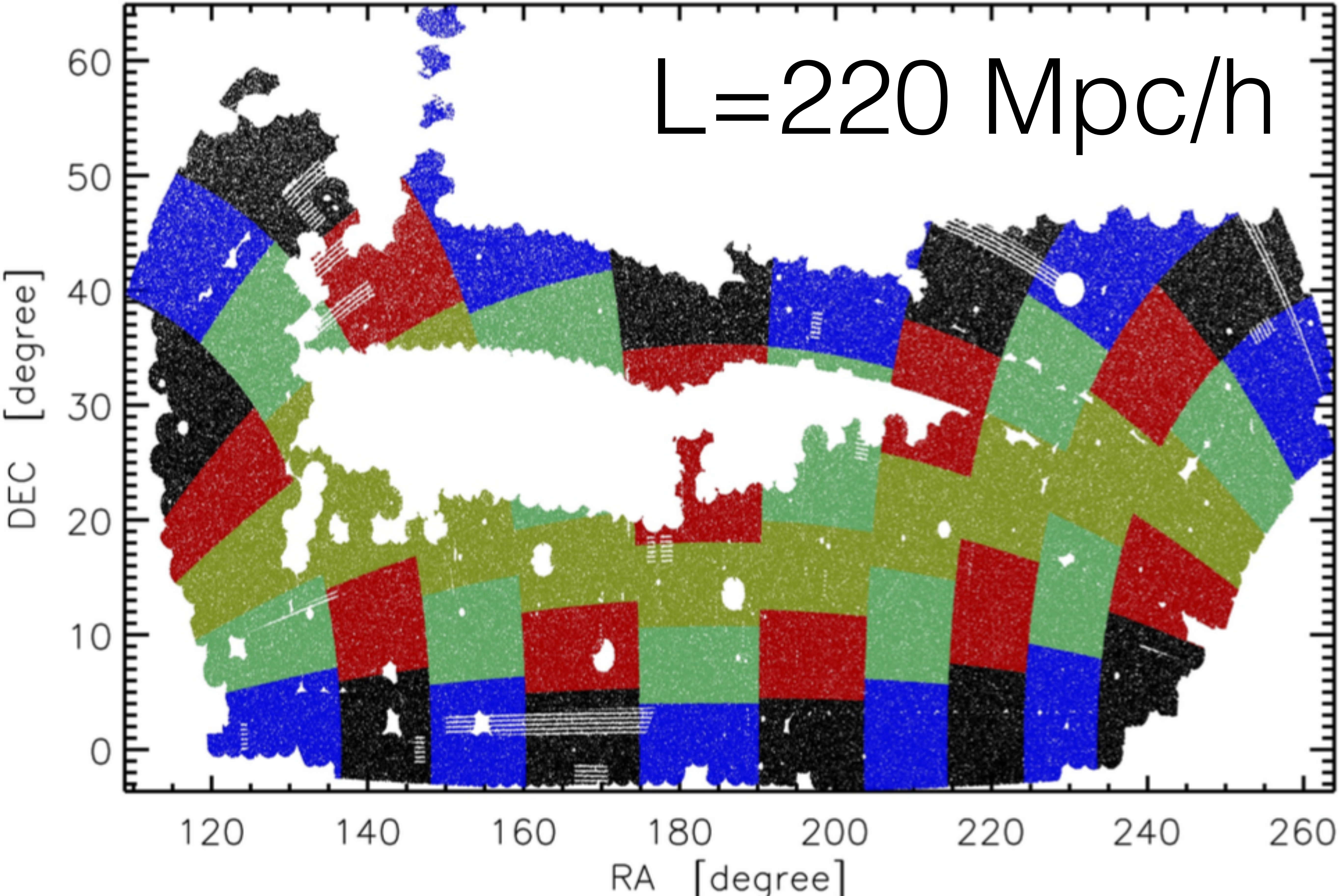
  - The small-scale structure can be arbitrarily non-linear!

Do the data show this?

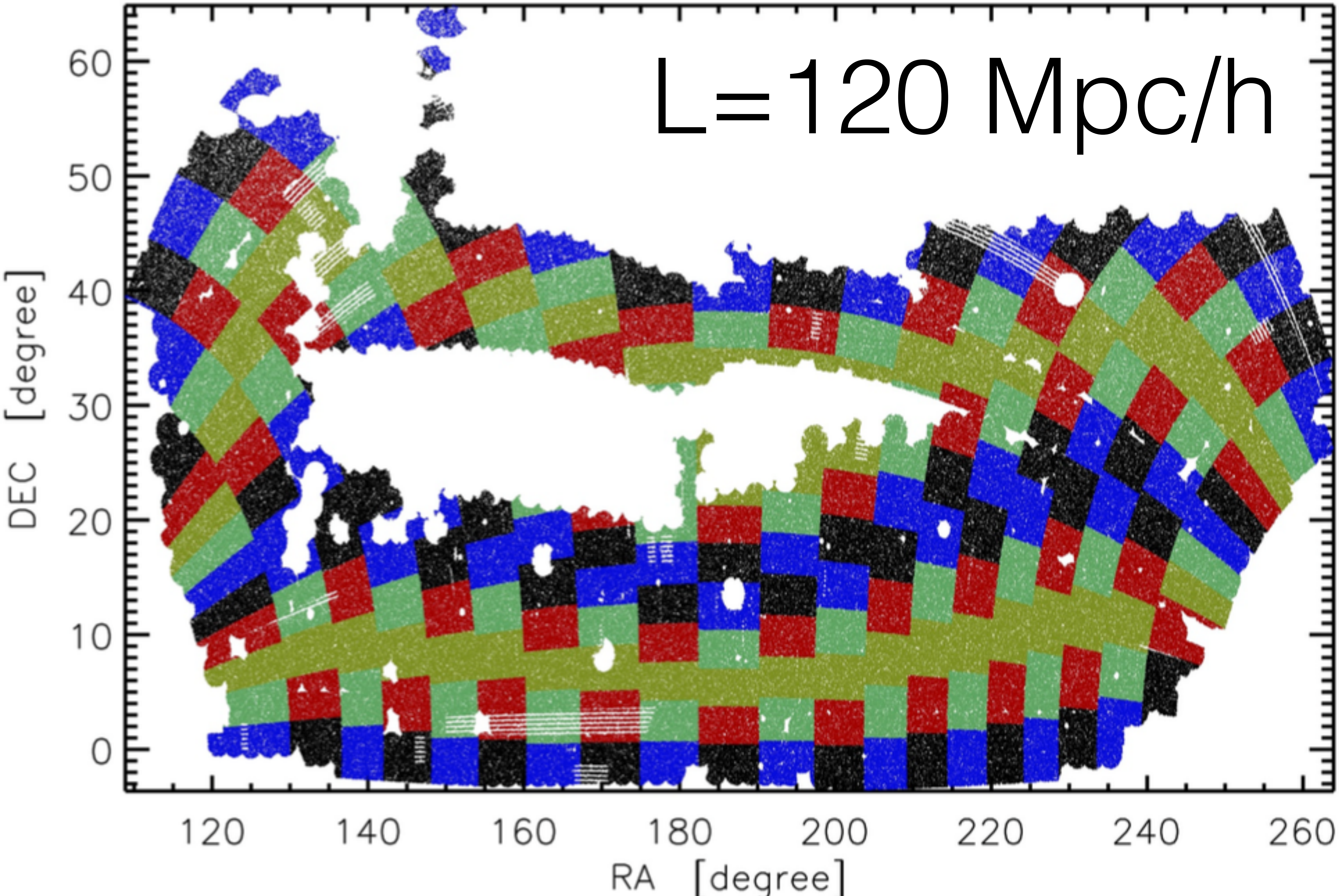


SDSS-III/BOSS DR11

- OK, now, let's look at the real data (BOSS DR10) to see if we can detect the expected influence of environments on the small-scale structure growth
- Bottom line: **we have detected the integrated bispectrum at 7.4σ**. Not bad for the first detection!

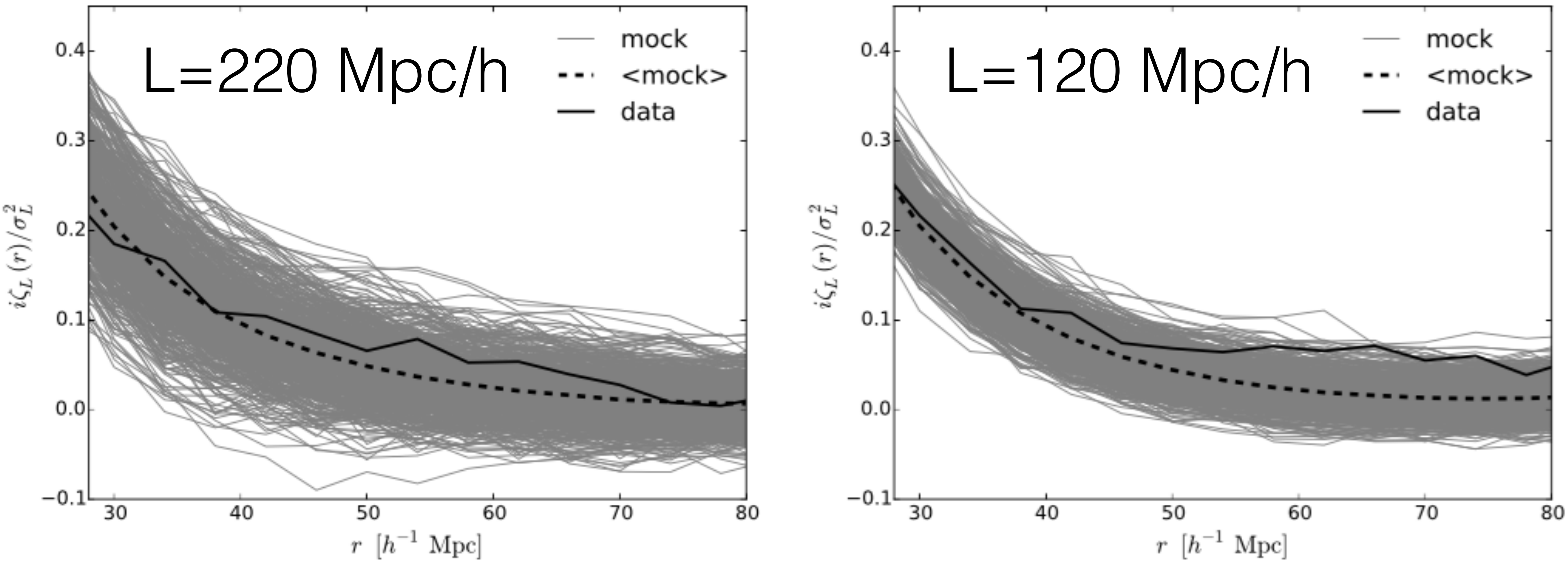


L=220 Mpc/h
60
50
40
30
20
10
0
DEC [degree]
120
140
160
180
200
220
240
260
RA [degree]



L=120 Mpc/h
DEC [degree]
RA [degree]
0
10
20
30
40
50
60
120
140
160
180
200
220
240
260

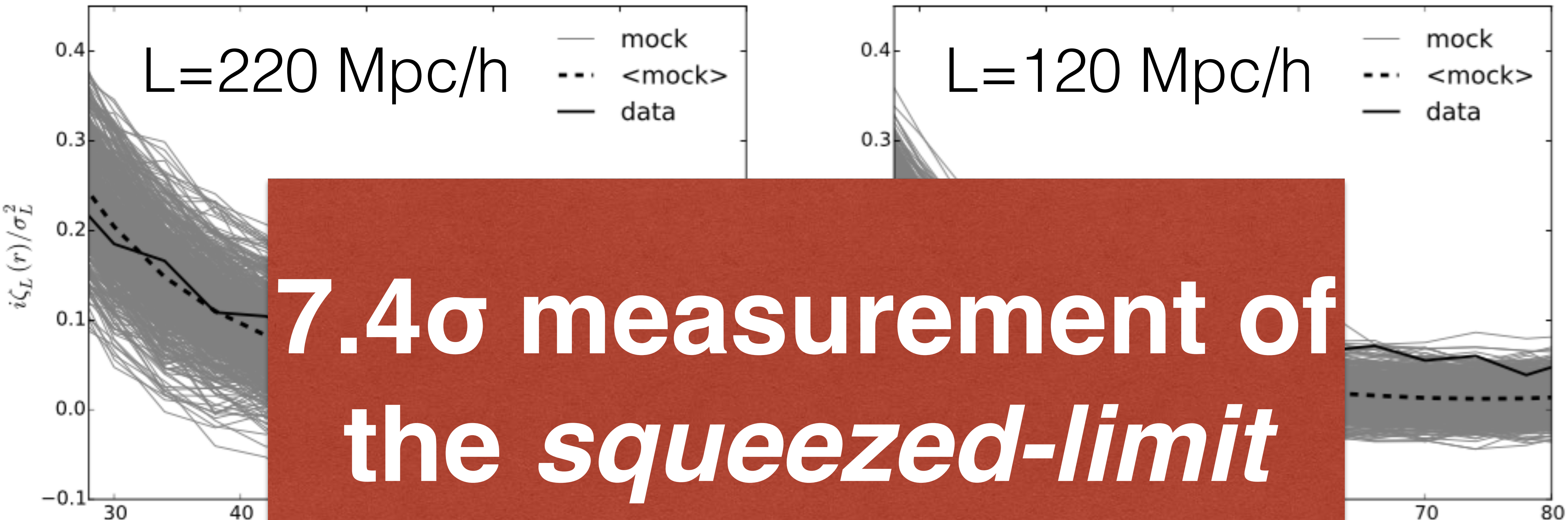
Results: χ^2/DOF = 46.4/38



- Because of complex geometry of DR10 footprint, we use the local correlation function, instead of the power spectrum

- Integrated three-point function, iζ(r), is just Fourier transform of iB(k): $$i\zeta_L(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\, iB_L(\mathbf{k}) e^{i\mathbf{r}\cdot\mathbf{k}}$$

Results: χ^2/DOF = 46.4/38



7.4σ measurement of the *squeezed-limit bispectrum*!!

- t, the power spectrum

- Integrated three-point function, iζ(r), is just Fourier transform of iB(k): $$i\zeta_L(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\, iB_L(\mathbf{k})e^{i\mathbf{r}\cdot\mathbf{k}}$$

Nice, but what is this good for?

- **Primordial non-Gaussianity from the early Universe**
  - The constraint from BOSS is work in progress, but we find that the integrated bispectrum is a **nearly optimal estimator for the squeezed-limit bispectrum from inflation**
  - We no longer need to measure the full bispectrum, if we are just interested in the squeezed limit

Nice, but what is this good for?

- **We can also learn about galaxy bias**
  - Local bias model:
    - $\delta_g(x)=b_1\delta_m(x)+(\mathbf{b_2}/2)[\delta_m(x)]^2+\ldots$
- **The bispectrum can give us b_2 at the leading order**, unlike for the power spectrum that has b_2 at the next-to-leading order

Result on b_2

- We use the leading-order SPT bispectrum with the local bias model to interpret our measurements
  - [We also use information from BOSS's 2-point correlation function on $f\sigma_8$ and BOSS's weak lensing data on σ_8]
- We find: $\mathbf{b_2 = 0.41 \pm 0.41}$

Simulating Ant’s Views

This is a powerful formula

$$P(k,a|\bar{\delta}) = \tilde{P}\left(k, a\left[1-\frac{1}{3}\bar{\delta}(a)\right]\right)\left[1+\left(2-\frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k}\right)\bar{\delta}(a)\right]$$

- How can we compute \tilde{P}(k,a) in practice?
  - **Small N-body simulations with a modified cosmology ("Separate Universe Simulation")**
  - Perturbation theory

Separate Universe Simulation

- How do we compute the response function beyond perturbation theory?
  - Do we have to run many big-volume simulations and divide them into sub-volumes? No.
- Fully non-linear computation of the response function is possible with **separate universe simulations**
  - E.g., we run two small-volume simulations with separate-universe cosmologies of over- and under-dense regions with the same initial random number seeds, and compute the derivative dlnP/dδ by, e.g.,

$$\frac{d\ln P(k)}{d\bar{\delta}} = \frac{\ln P(k|+\bar{\delta}) - \ln P(k|-\bar{\delta})}{2\bar{\delta}}$$

Separate Universe Cosmology

$$\boxed{\rho(t)\,[1+\delta_\rho(t)] = \tilde{\rho}(t)}$$

$$\frac{\Omega_m h^2}{a^3(t)}\,[1+\delta_\rho(t)] = \frac{\tilde{\Omega}_m \tilde{h}^2}{\tilde{a}^3(t)}$$

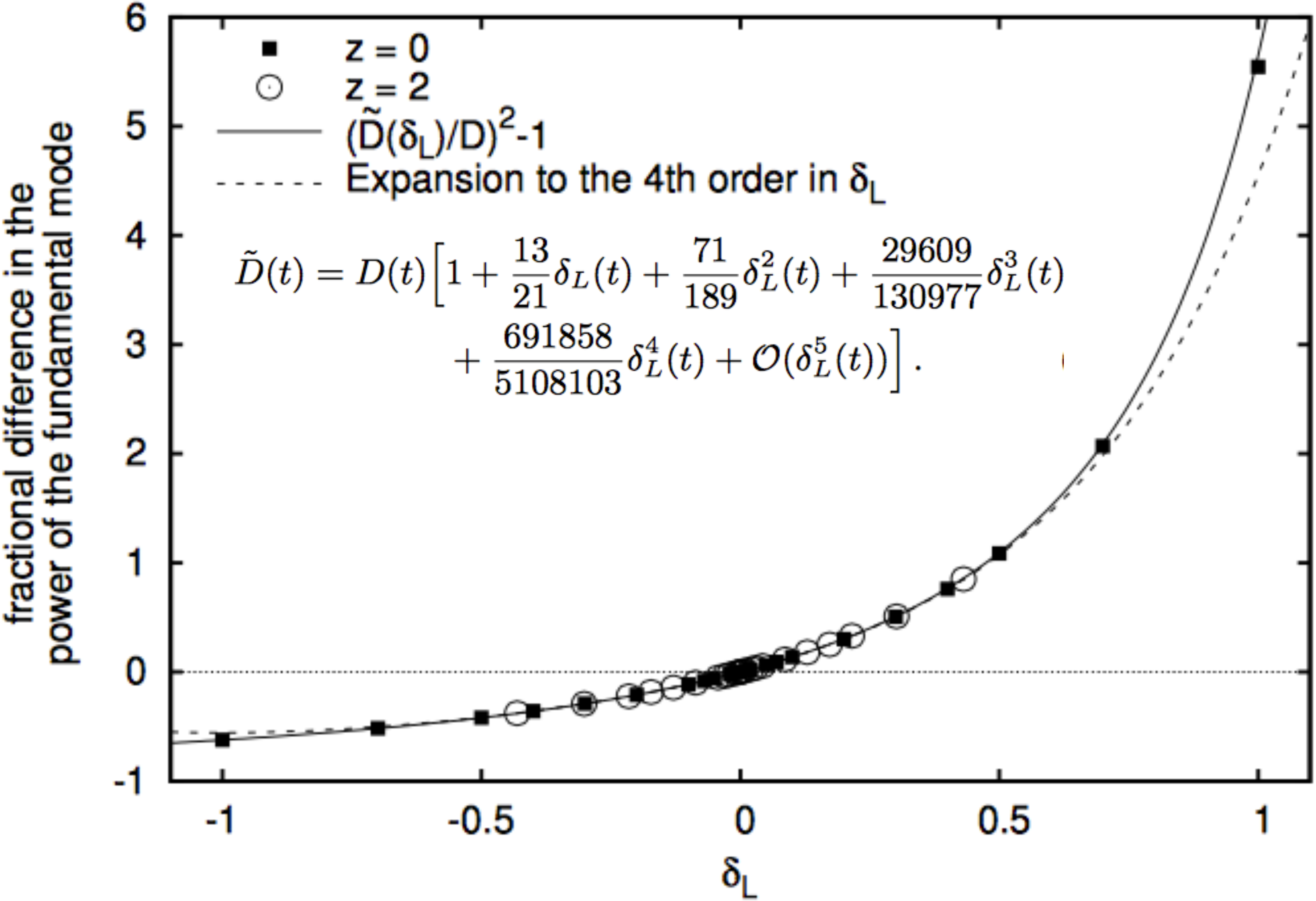
$$\frac{\tilde{K}}{H_0^2} = \frac{5}{3}\frac{\Omega_m}{a(t_i)}\delta_\rho(t_i)$$

$$\delta_H = \left(1 - \frac{\tilde{K}}{H_0^2}\right)^{1/2} - 1$$

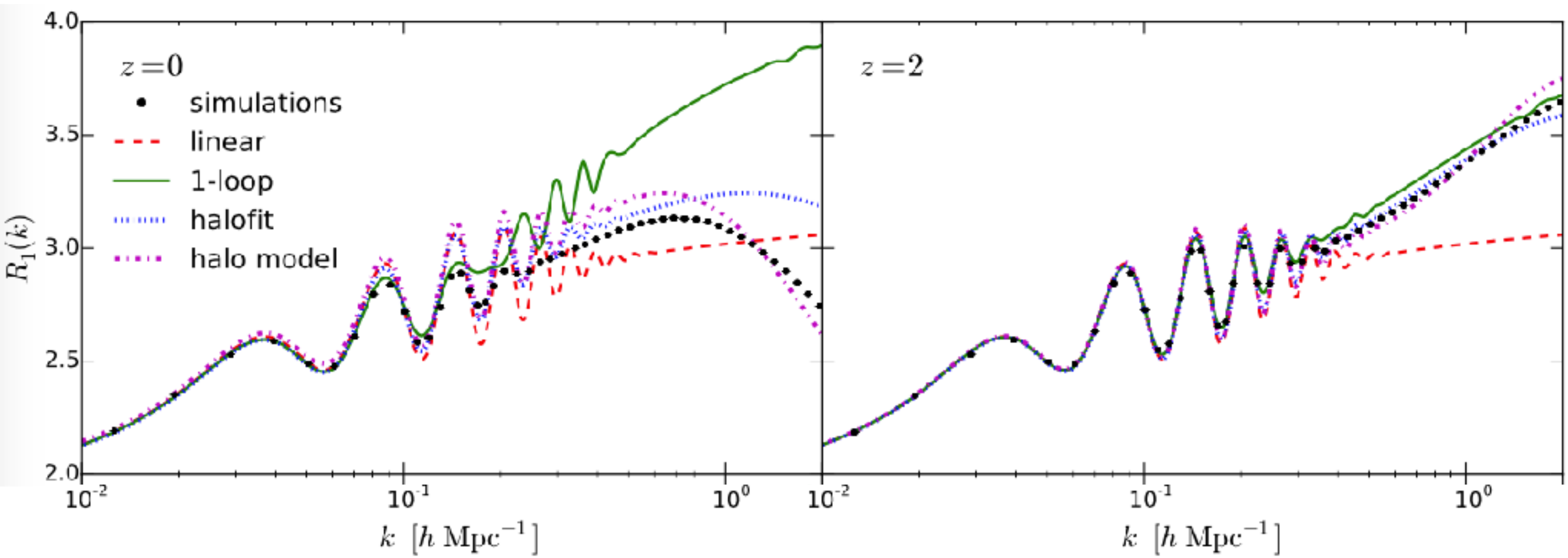
$$\tilde{H}_0 = H_0[1+\delta_H]$$

$$\tilde{\Omega}_m = \Omega_m[1+\delta_H]^{-2}$$

$$\tilde{\Omega}_\Lambda = \Omega_\Lambda[1+\delta_H]^{-2}$$

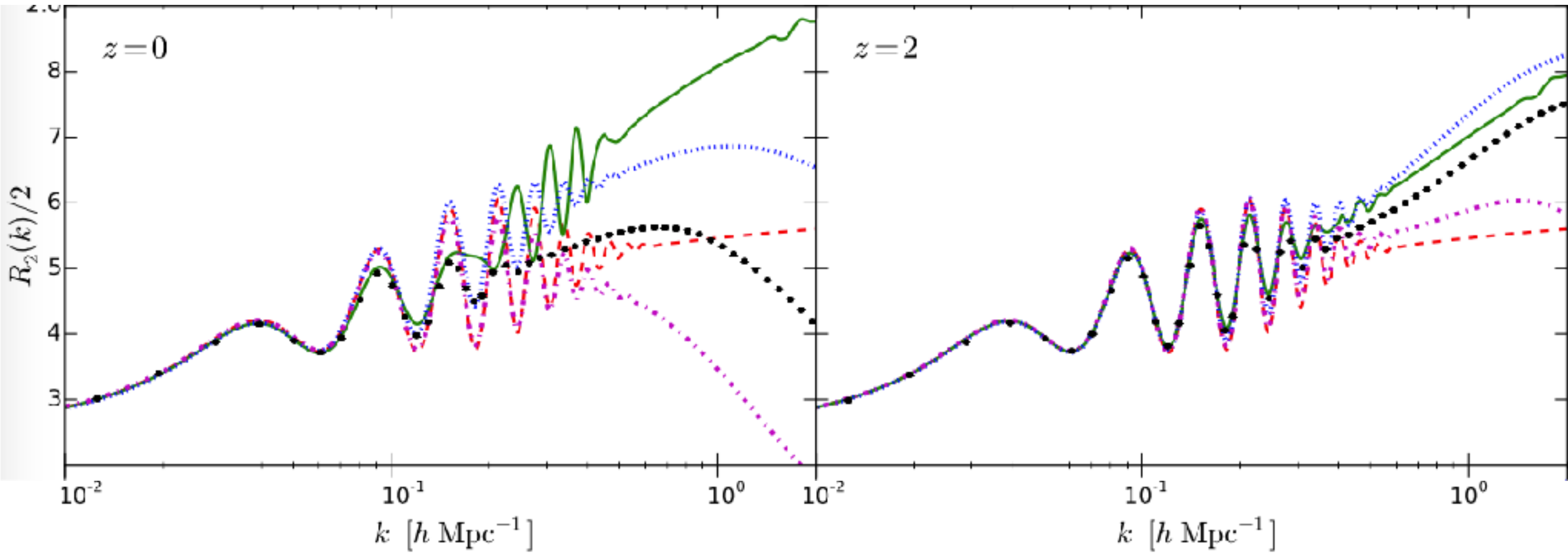

z = 0
z = 2
$(\tilde{D}(\delta_L)/D)^2-1$
Expansion to the 4th order in δ_L
$$\tilde{D}(t) = D(t)\Big[1 + \frac{13}{21}\delta_L(t) + \frac{71}{189}\delta_L^2(t) + \frac{29609}{130977}\delta_L^3(t) + \frac{691858}{5108103}\delta_L^4(t) + \mathcal{O}(\delta_L^5(t))\Big].$$
fractional differenc in the power of the fundamental mode
δ_L
6
5
4
3
2
1
0
-1
-0.5
0.5

$R_1 = d\ln P/d\delta$



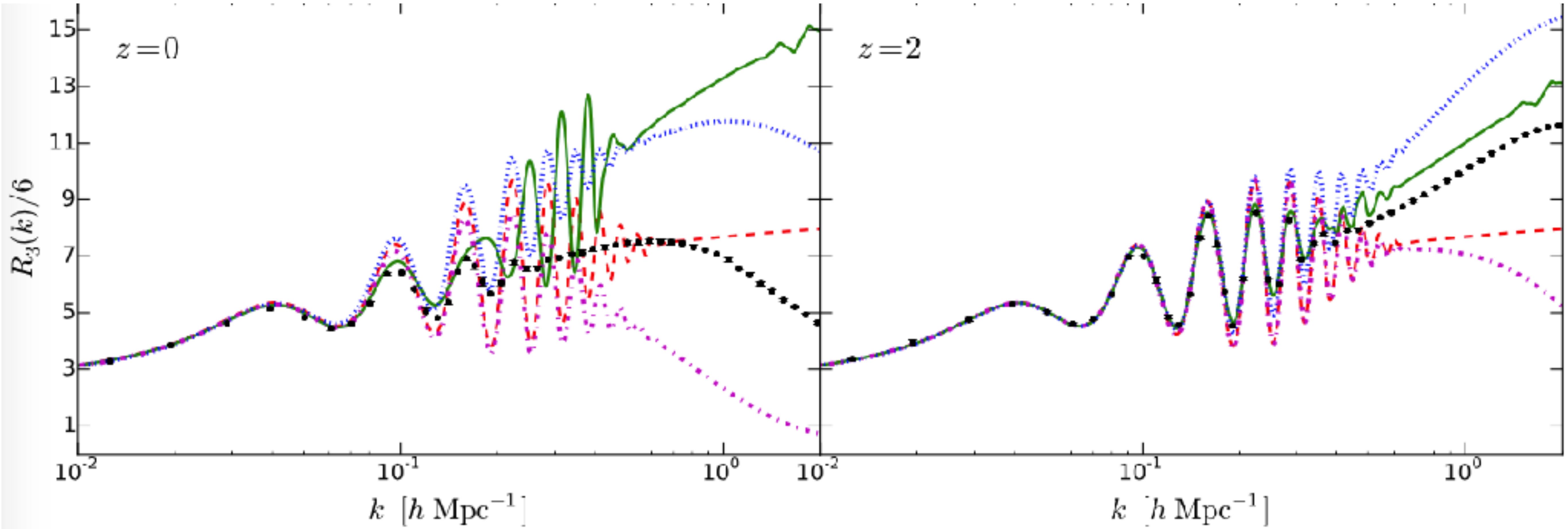
- The symbols are the data points with error bars. You cannot see the error bars!

$R_2 = d^2 \ln P / d\delta^2$



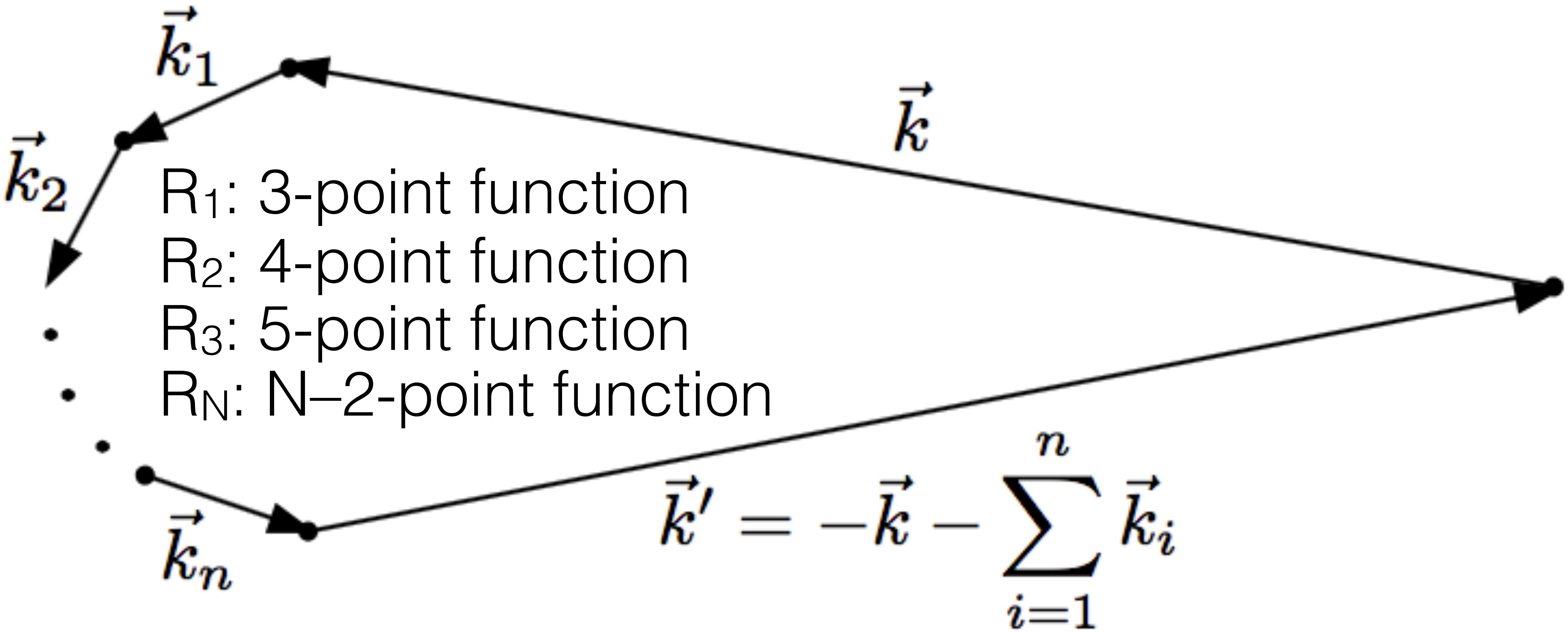
- More derivatives can be computed by using simulations run with more values of δ

$R_3 = d^3 \ln P / d\delta^3$



- But, what do $d^n \ln P / d\delta^n$ mean physically??

More derivatives: Squeezed limits of N-point functions



R_1: 3-point function
R_2: 4-point function
R_3: 5-point function
R_N: N–2-point function

- Why do we want to know this? I don't know, but it is cool and they have not been measured before!

Summary

- ***<u>New observable</u>***: the position-dependent power spectrum and the integrated bispectrum
    - Straightforward interpretation in terms of the separate universe
    - Easy to measure; easy to model!
    - Useful for primordial non-Gaussianity and non-linear bias

- Lots of applications: e.g., QSO density correlated with Lyman-alpha power spectrum
- All of the results and much more are summarised in Chi-Ting Chiang's PhD thesis: **arXiv:1508.03256**

One more cool thing

- We can use the separate universe simulations to test **validity of SPT to all orders in perturbations**

- The fundamental prediction of SPT: the non-linear power spectrum at a given time is given by the linear power spectra at the same time

    - In other words, the only time dependence arises from the linear growth factors, D(t)

One more cool thing

- We can use the separate universe simulations to test **validity of SPT to all orders in perturbations**

$$\delta' + \nabla \cdot [(1+\delta)\mathbf{v}] = 0 \ ,$$
$$\mathbf{v}' + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = -\mathcal{H}\mathbf{v} - \nabla\phi \ ,$$
$$\nabla^2\phi = 4\pi G a^2 \bar{\rho}\delta \ ,$$

SPT at all orders: Exact solution of the pressureless fluid equations

We can test validity of SPT as a description of collisions particles

Example: $P_{\text{3rd-order}}(k)$

- SPT to 3rd order

$$P(k,a) = P_l(k,a) + P_{22}(k,a) + 2P_{13}(k,a)$$

$$P_{22}(k,a) = 2\int \frac{d^3q}{(2\pi)^3}\, P_l(q,a)P_l(|\mathbf{k}-\mathbf{q}|,a)\left[F_2(\mathbf{q},\mathbf{k}-\mathbf{q})\right]^2$$

$$2P_{13}(k,a) = \frac{2\pi k^2}{252}P_l(k,a)\int_0^\infty \frac{dq}{(2\pi)^3}\, P_l(q,a) \times \left[100\frac{q^2}{k^2} - 158 + 12\frac{k^2}{q^2} - 42\frac{q^4}{k^4} + \frac{3}{k^5q^3}(q^2-k^2)^3(2k^2+7q^2)\ln\left(\frac{k+q}{|k-q|}\right)\right.$$

- The only time-dependence is in $P_l(k,a) \sim D^2(a)$

- Is this correct?

Rescaled simulations vs Separate universe simulations

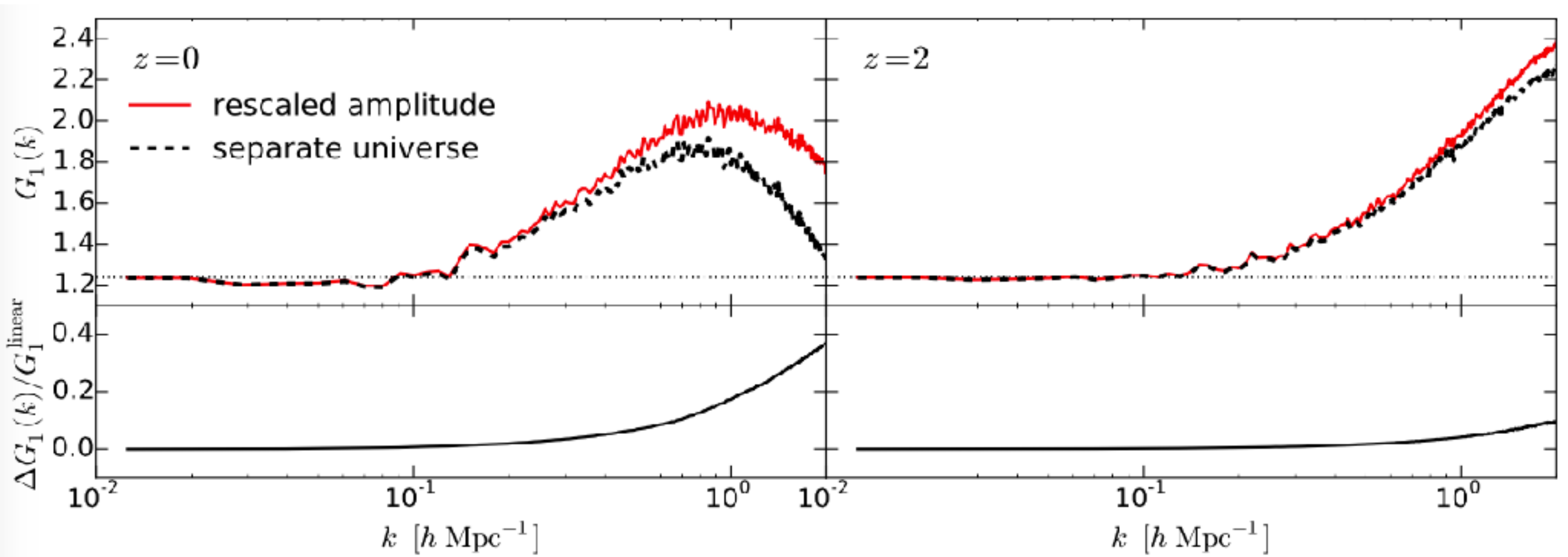
- To test this, we run two sets of simulations.

- **First**: we rescale the initial amplitude of the power spectrum, so that we have a given value of the linear power spectrum amplitude at some later time, t_{out}

- **Second**: full separate universe simulation, which changes all the cosmological parameters consistently, given a value of δ

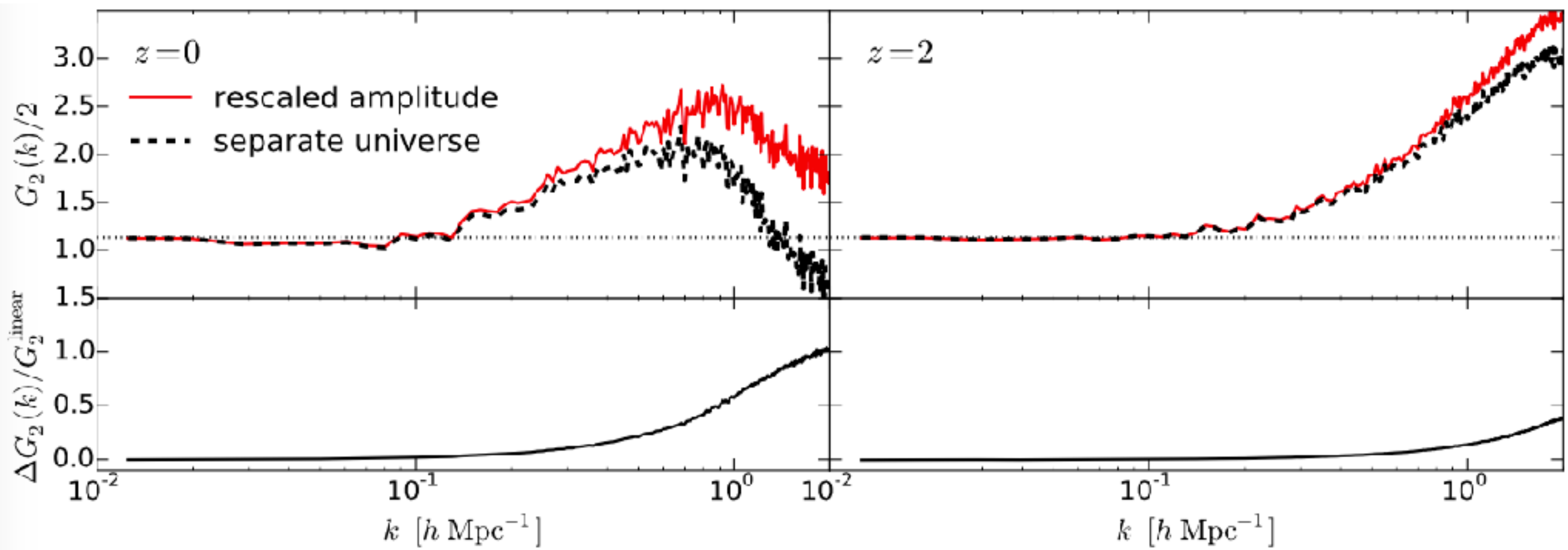
    - We choose δ so that it yields the same amplitude of the linear power spectrum as the first one at t_{out}

Results: 3-point function



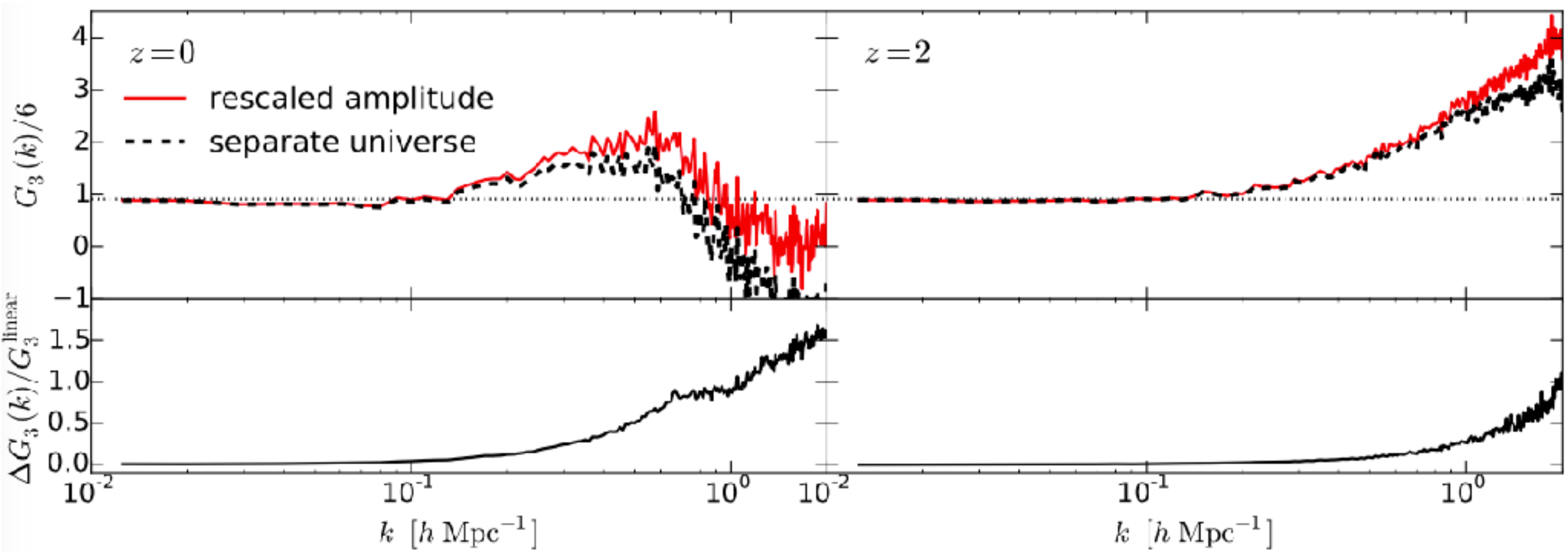
- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions

Results: 4-point function



- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions

Results: 5-point function



- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions

Break down of SPT at all orders

- **At z=0, SPT computed to all orders breaks down at k~0.5 Mpc/h with 10% error,** in the squeezed limit 3-point function
  - Break down occurs at lower k for the squeezed limits of the 4- and 5-point functions
  - Break down occurs at higher k at z=2
- I find this information quite useful: *it quantifies accuracy of the perfect-fluid approximation of density fields*

More on b_2

- Using slightly more advanced models, we find:

	baseline	eff kernel	tidal bias	both*
b_2	0.41 ± 0.41	0.51 ± 0.41	0.48 ± 0.41	0.60 ± 0.41

*The last value is in agreement with b_2 found by the Barcelona group (Gil-Marín et al. 2014) that used the full bispectrum analysis and the same model

r-space	b_1	b_2
baseline	1.971 ± 0.076	0.58 ± 0.31
eff kernel	1.973 ± 0.076	0.62 ± 0.31
tidal bias	1.971 ± 0.076	0.64 ± 0.31
both	1.973 ± 0.076	0.68 ± 0.31

z-space	b_1	b_2
baseline	1.931 ± 0.077	0.54 ± 0.35
eff kernel	1.933 ± 0.077	0.65 ± 0.35
tidal bias	1.932 ± 0.077	0.60 ± 0.35
both	1.933 ± 0.077	0.71 ± 0.35