

# Position-dependent Power Spectrum

*~Attacking an old, but unsolved, problem with a new method~*

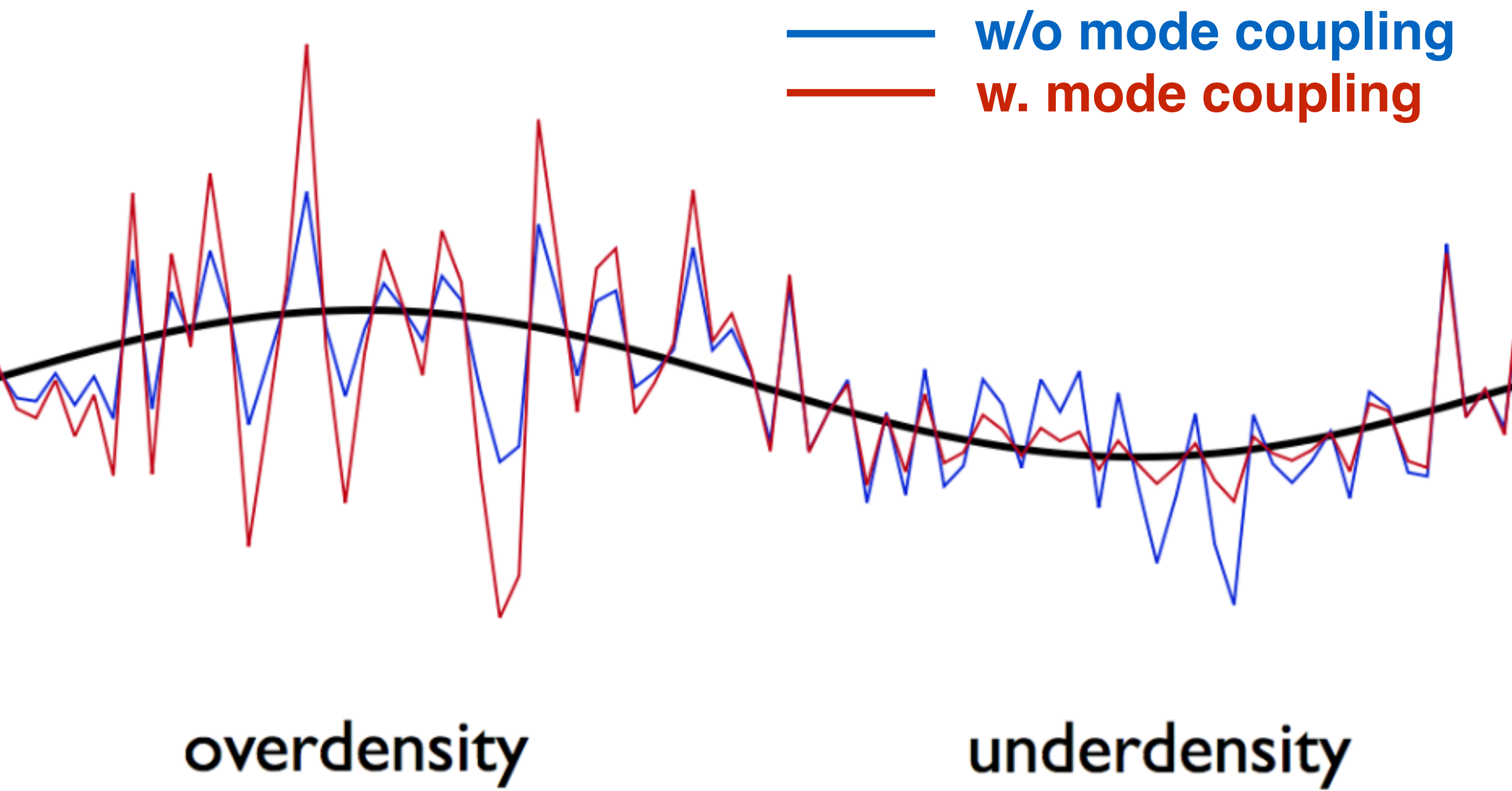
Eiichiro Komatsu (Max Planck Institute for Astrophysics)

*Understanding Cosmological Observations*

Benasque, August 1, 2017

# Motivation

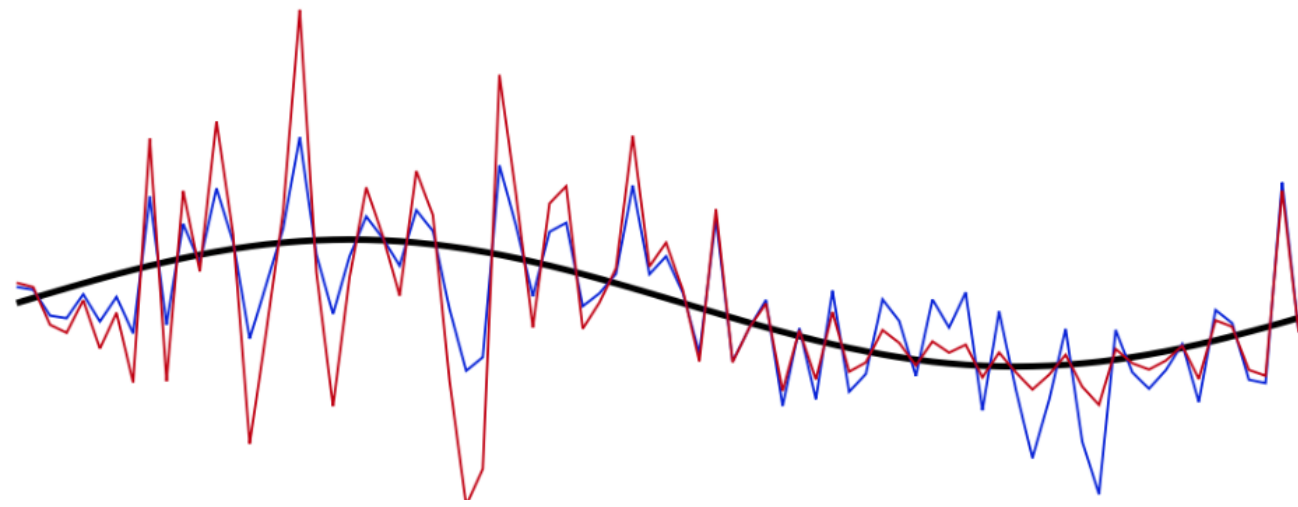
- To gain a better insight into “**mode coupling**”
  - An interaction between short-wavelength modes and long-wavelength modes
  - Specifically, how do short wavelength modes **respond** to a long wavelength mode?



# Understanding Cosmological Observations

- My talk would fit the scope of this workshop because:
  - The goal is to understand the three-point correlation function - a beast that many people are scared of...
  - So, we wanted to create something that is nicer and easier to deal with than the three-point function, yet to retain some of its power for cosmology. Hope you will like it!

# Two Approaches



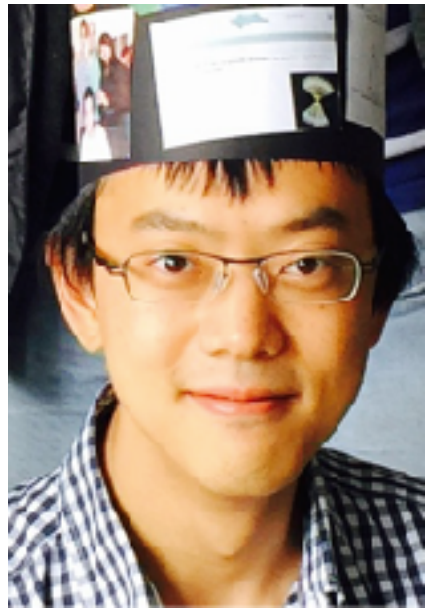
- **Global**

- “Bird’s view”: see both long- and short-wavelength modes, and compute coupling between the two directly

- **Local**

- “Ant’s view”: Absorb a long-wavelength mode into a new background solution that a local observer sees, and compute short wavelength modes in the new background.

# This presentation is based on



- **Chiang** et al. “*Position-dependent power spectrum of the large-scale structure: a novel method to measure the squeezed-limit bispectrum*”, JCAP 05, 048 (2014)
- **Chiang** et al. “*Position-dependent correlation function from the SDSS-III BOSS DR10 CMASS Sample*”, JCAP 09, 028 (2015)



- **Wagner** et al. “*Separate universe simulations*”, MNRAS, 448, L11 (2015)
- **Wagner** et al. “*The angle-averaged squeezed limit of nonlinear matter N-point functions*”, JCAP 08, 042 (2015)

# Preparation I:

## Comoving Coordinates

- Space expands. Thus, a physical length scale increases over time
- Since the Universe is homogeneous and isotropic on large scales, the stretching of space is given by a time-dependent function,  **$a(\mathbf{t})$** , which is called the “scale factor”
- Then, the physical length,  $r(t)$ , can be written as
  - $r(t) = a(t) \mathbf{x}$
  - $\mathbf{x}$  is independent of time, and called the “**comoving coordinates**”

# Preparation II: Comoving Waveumbers

- Then, the physical length,  $r(t)$ , can be written as
  - $r(t) = a(t) \mathbf{x}$
  - $\mathbf{x}$  is independent of time, and called the “**comoving coordinates**”
- When we do the Fourier analysis, **the wavenumber,  $k$ , is defined with respect to  $\mathbf{x}$** . This “comoving wavenumber” is related to the physical wavenumber by  $k_{\text{physical}}(t) = k_{\text{comoving}}/a(t)$



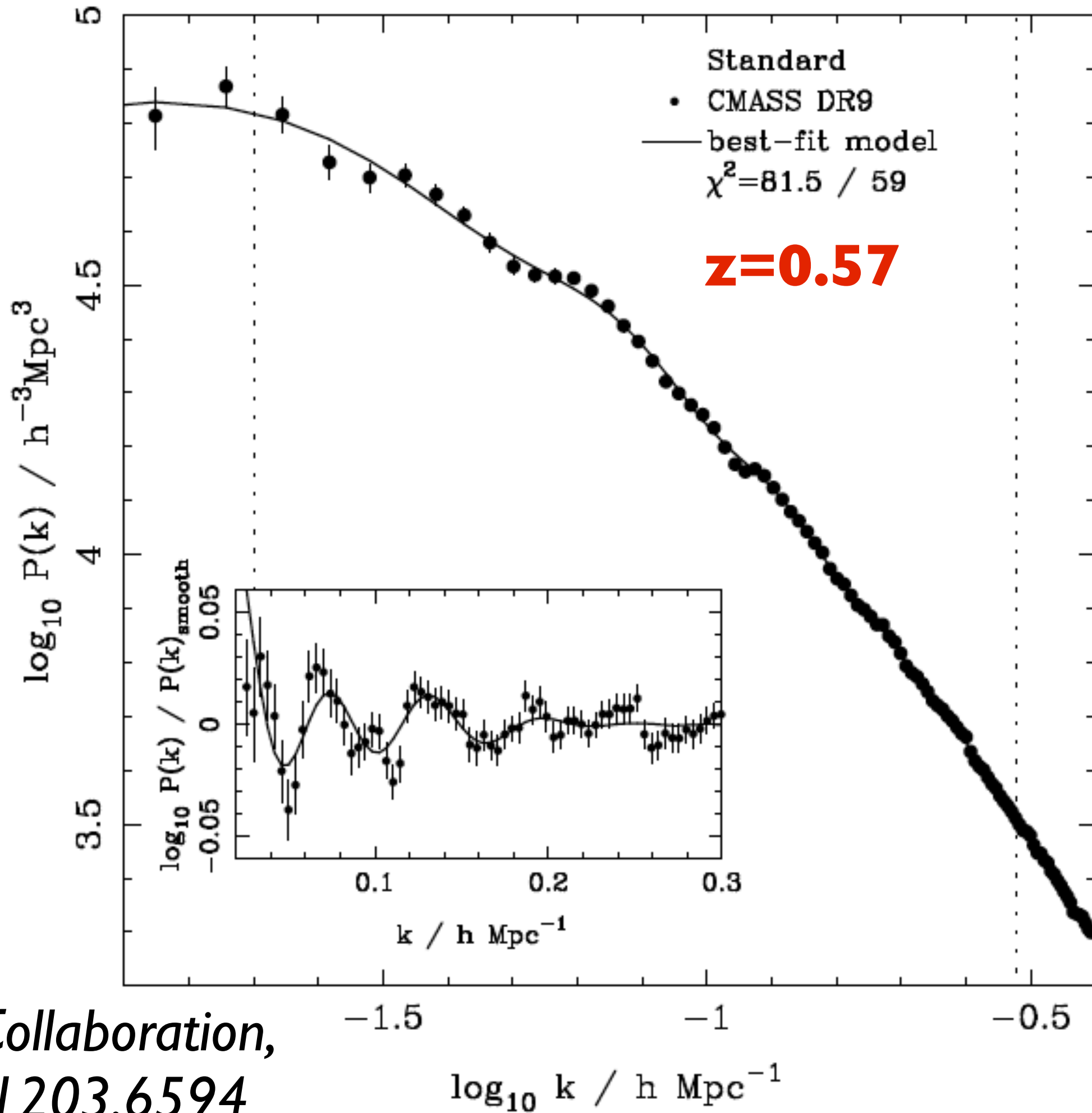
# Preparation III: Power Spectrum

500 Mpc/h



- **Take these density fluctuations, and compute the density contrast:**
  - $\delta(\mathbf{x}) = [ \rho(\mathbf{x}) - \rho_{\text{mean}} ] / \rho_{\text{mean}}$
- **Fourier-transform this, square the amplitudes, and take averages. The power spectrum is thus:**
  - $P(\mathbf{k}) = \langle |\delta_{\mathbf{k}}|^2 \rangle$





# A simple question within the context of cosmology

- How do the cosmic structures evolve in an overdense region?



# Simple Statistics

500 Mpc/h



- Divide the survey volume into many sub-volumes  $V_L$ , and compare locally-measured power spectra with the corresponding local over-densities



# Simple Statistics

500 Mpc/h



$V_L$

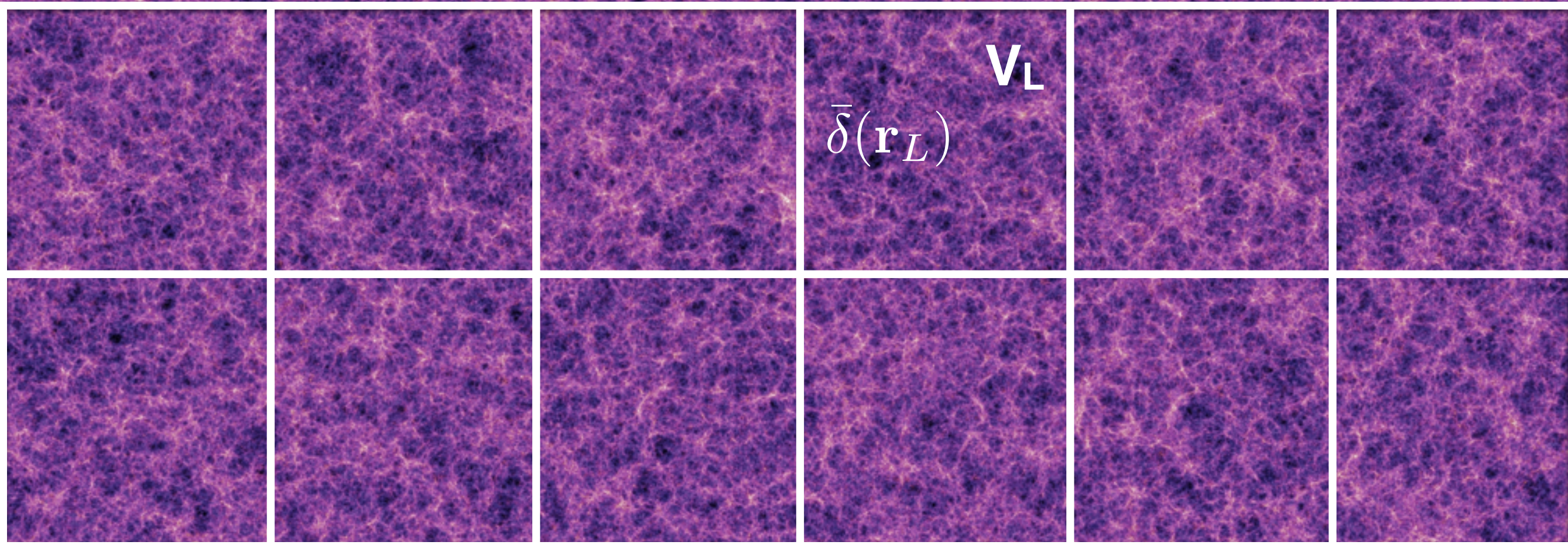
- Divide the survey volume into many sub-volumes  $V_L$ , and compare locally-measured power spectra with the corresponding local over-densities



# Simple Statistics

$$\bar{\delta}(\mathbf{r}_L) = \frac{1}{V_L} \int_{V_L} d^3r \delta(\mathbf{r})$$

500 Mpc/h



- Divide the survey volume into many sub-volumes  $V_L$ , and compare locally-measured power spectra with the corresponding local over-densities



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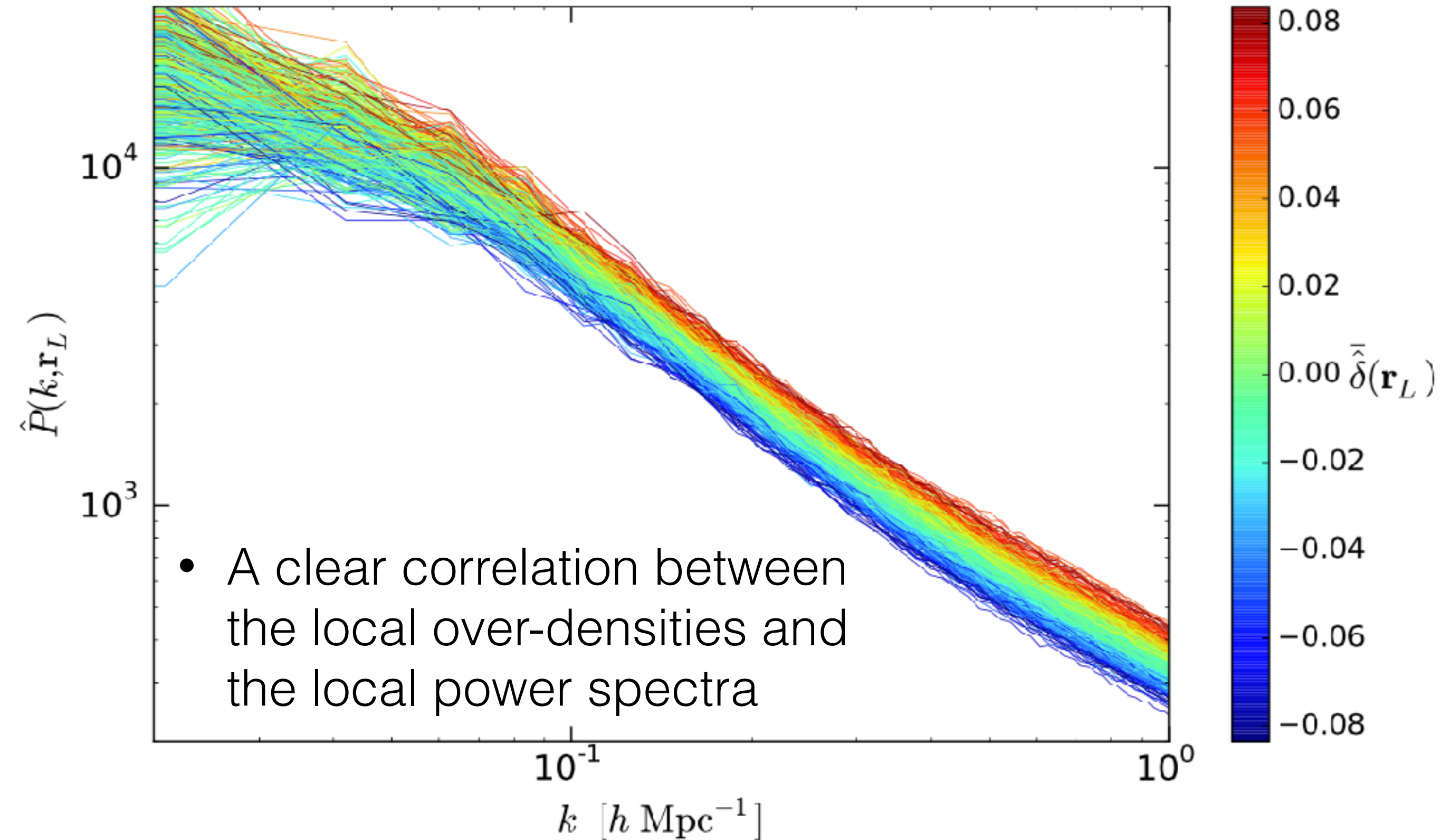
$$\hat{P}(\mathbf{k}, \mathbf{r}_L) \equiv \frac{1}{V_L} |\delta(\mathbf{k}, \mathbf{r}_L)|^2$$

$V_L$   
 $\bar{\delta}(\mathbf{r}_L)$   
 $\hat{P}(\mathbf{k}, \mathbf{r}_L)$

- Divide the survey volume into many sub-volumes  $V_L$ , and compare locally-measured power spectra with the corresponding local over-densities



# Position-dependent $\hat{P}(k)$





# Integrated Bispectrum, $iB(k)$

- Correlating the local over-densities and power spectra, we obtain the “integrated bispectrum”:

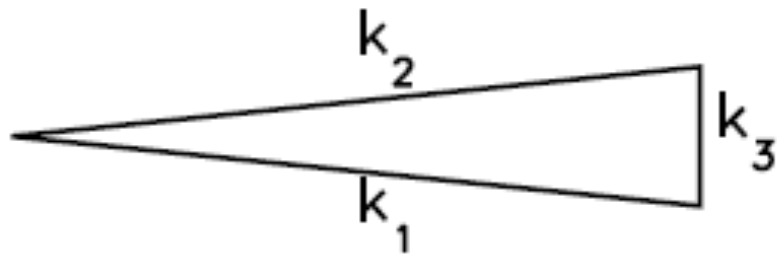
$$i\hat{B}_L(k) = \frac{1}{N_{\text{cut}}^3} \sum_{i=1}^{N_{\text{cut}}^3} \hat{P}(k, \mathbf{r}_{L,i}) \hat{\delta}(\mathbf{r}_{L,i})$$

- This is a (particular configuration of) **three-point function**. The three-point function in Fourier space is called the “**bispectrum**”, and is defined as

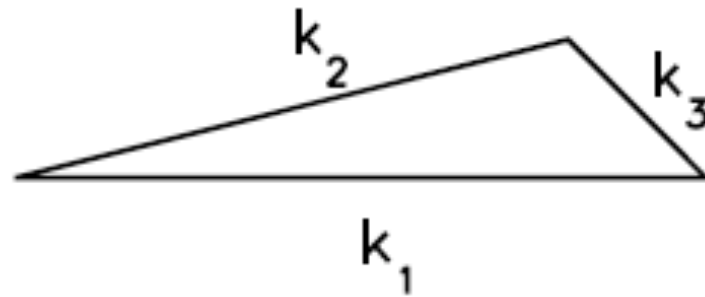
$$\langle \delta(\mathbf{q}_1) \delta(\mathbf{q}_2) \delta(\mathbf{q}_3) \rangle = B(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) (2\pi)^3 \delta_D(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3)$$

# Shapes of the Bispectrum

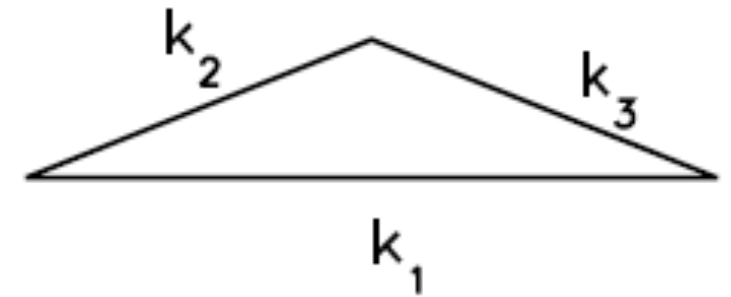
(a) squeezed triangle  
( $k_1 \approx k_2 \gg k_3$ )



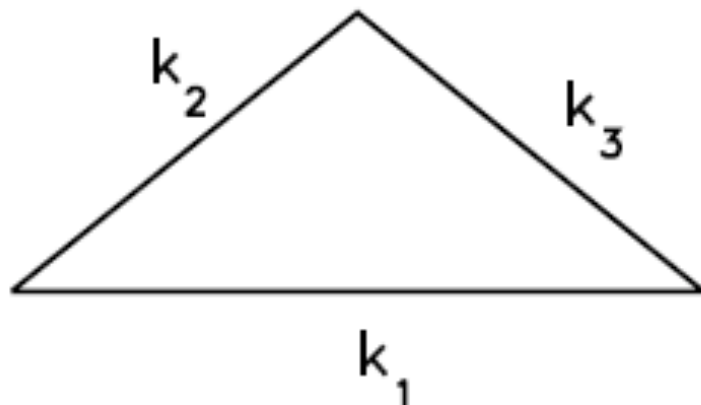
(b) elongated triangle  
( $k_1 = k_2 + k_3$ )



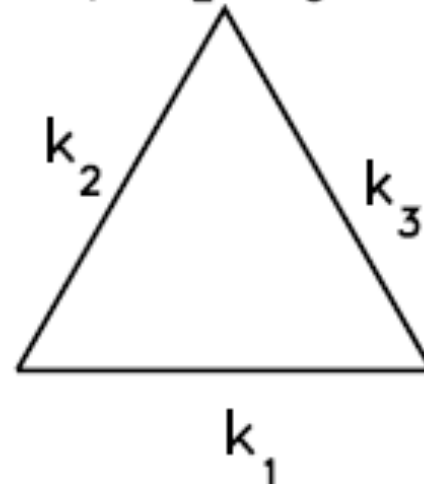
(c) folded triangle  
( $k_1 = 2k_2 = 2k_3$ )



(d) isosceles triangle  
( $k_1 > k_2 = k_3$ )

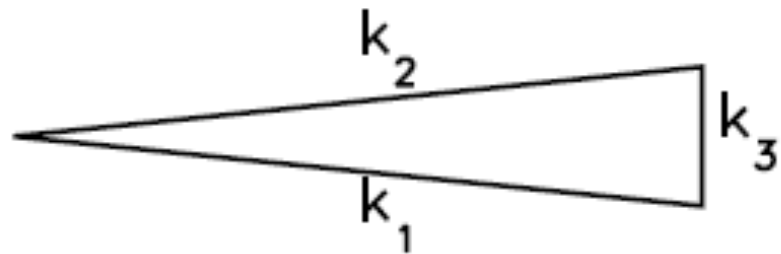


(e) equilateral triangle  
( $k_1 = k_2 = k_3$ )



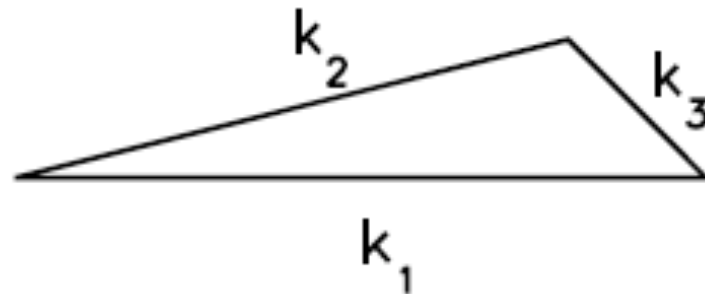
# Shapes of the Bispectrum

(a) squeezed triangle  
( $k_1 \simeq k_2 \gg k_3$ )

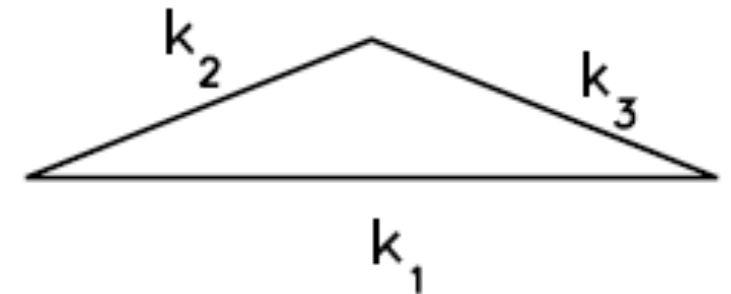


**This Talk**

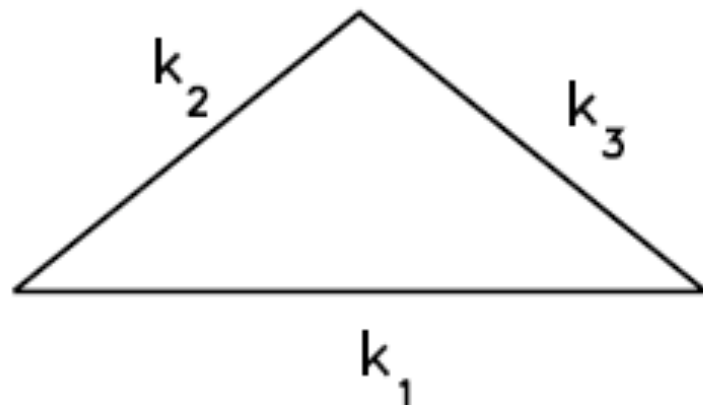
(b) elongated triangle  
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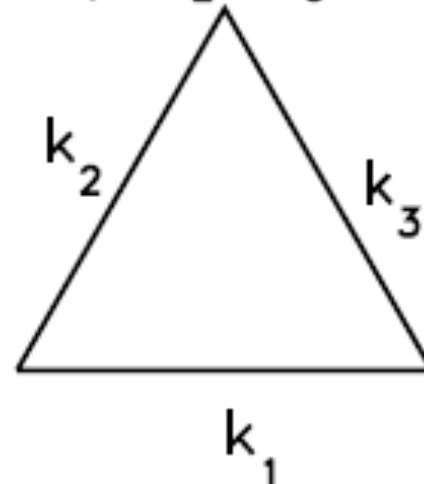
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(e) equilateral triangle  
( $k_1 = k_2 = k_3$ )



# Integrated Bispectrum, $iB(k)$

- Correlating the local over-densities and power spectra, we obtain the “integrated bispectrum”:

$$i\hat{B}_L(k) = \frac{1}{N_{\text{cut}}^3} \sum_{i=1}^{N_{\text{cut}}^3} \hat{P}(k, \mathbf{r}_{L,i}) \hat{\delta}(\mathbf{r}_{L,i})$$

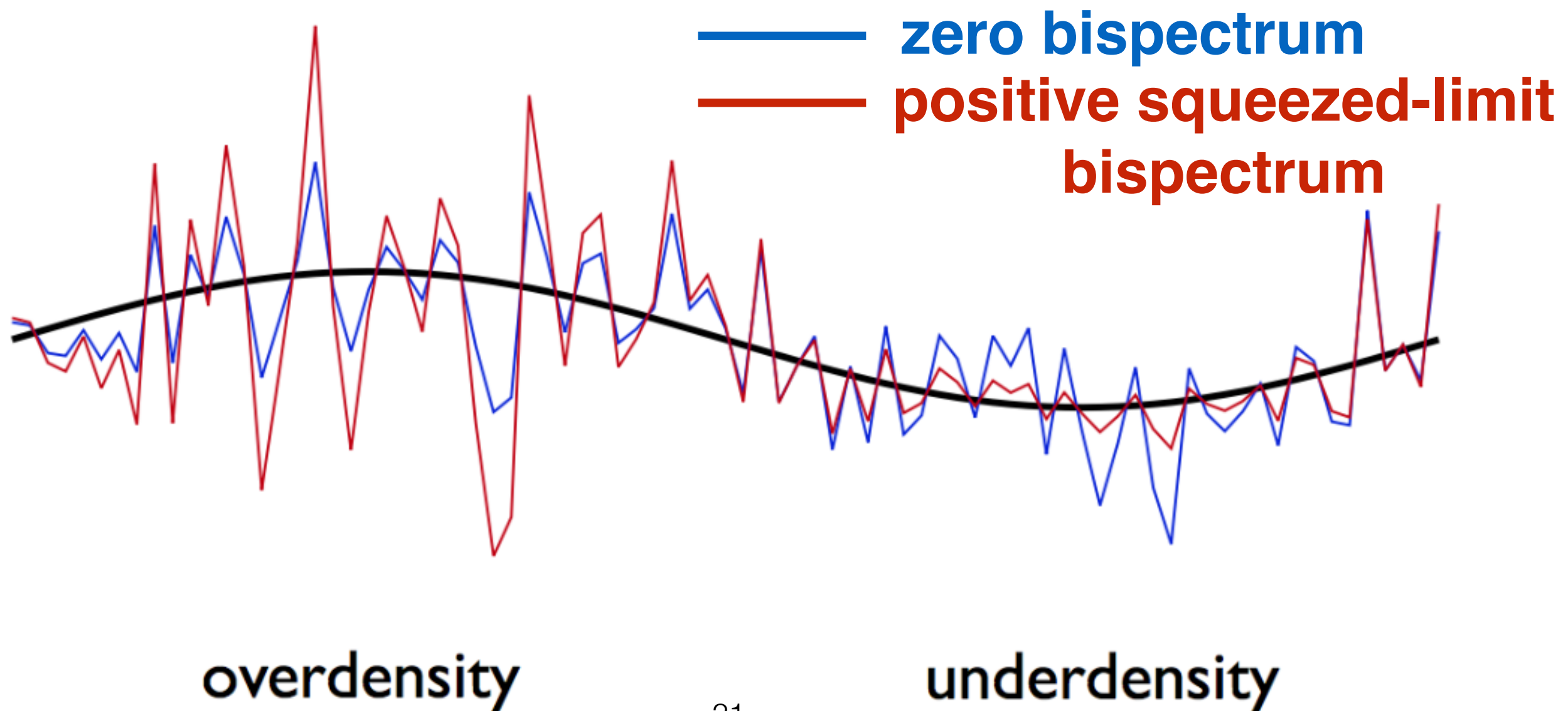
- The expectation value of this quantity is an integral of the bispectrum that picks up the contributions **mostly from the squeezed limit**:

$$\begin{aligned}
 iB_L(k) &= \langle \hat{P}(k, \mathbf{r}_L) \bar{\delta}(\mathbf{r}_L) \rangle \\
 &= \frac{1}{V_L^2} \int \frac{d^2 \hat{k}}{4\pi} \int \frac{d^3 q_1}{(2\pi)^3} \int \frac{d^3 q_3}{(2\pi)^3} B(\mathbf{k} - \mathbf{q}_1, -\mathbf{k} + \mathbf{q}_1 + \mathbf{q}_3, -\mathbf{q}_3) \\
 &\quad \times W_L(\mathbf{q}_1) W_L(-\mathbf{q}_1 - \mathbf{q}_3) W_L(\mathbf{q}_3)
 \end{aligned}$$

*“taking the squeezed limit and then angular averaging”*

# Power Spectrum Response

- The integrated bispectrum measures how the local power spectrum responds to its environment, i.e., a long-wavelength density fluctuation



# Response Function

- So, let us Taylor-expand the local power spectrum in terms of the long-wavelength density fluctuation:

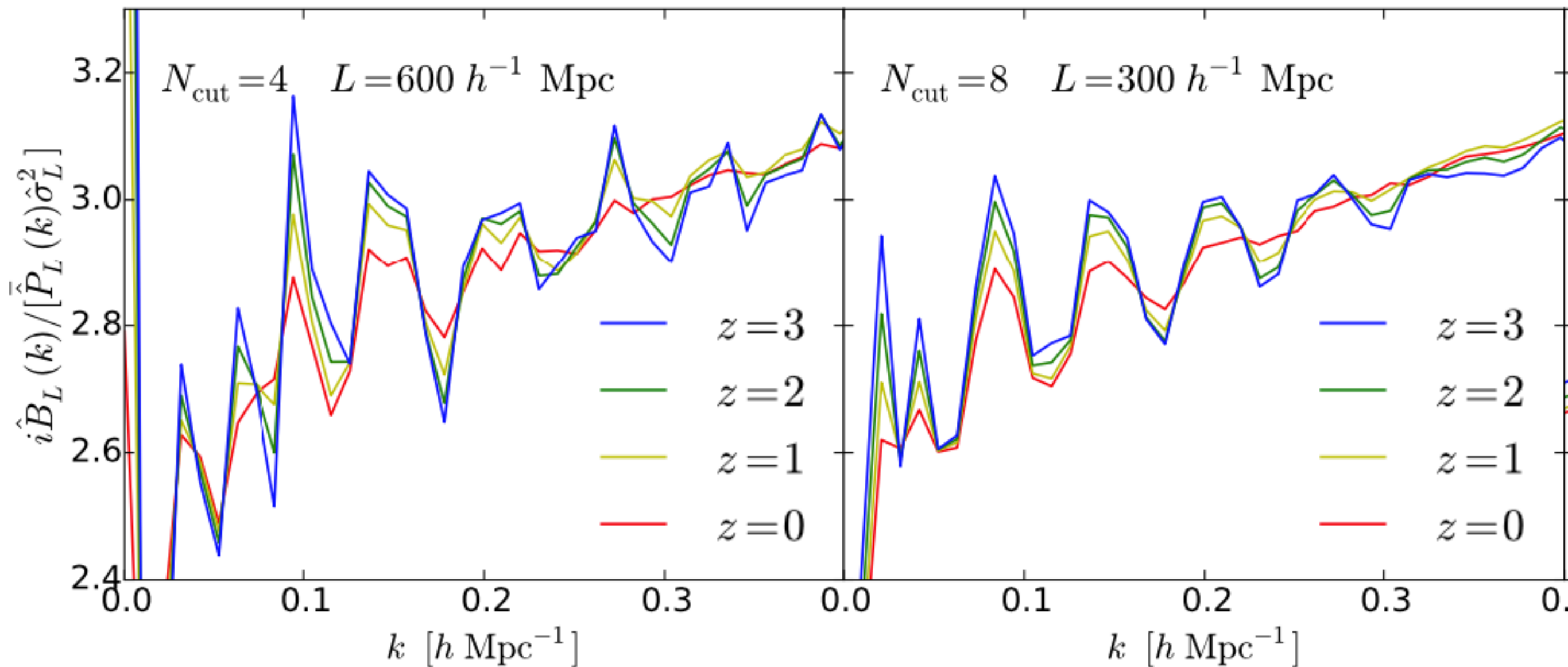
$$\hat{P}(k, \mathbf{r}_L) = P(k) \Big|_{\bar{\delta}=0} + \frac{dP(k)}{d\bar{\delta}} \Big|_{\bar{\delta}=0} \bar{\delta} + \dots$$

- The integrated bispectrum is then give as

$$iB_L(k) = \sigma_L^2 \left[ \frac{d \ln P(k)}{d\bar{\delta}} \Big|_{\bar{\delta}=0} \right] P(k)$$

**response function**

# Response Function: N-body Results



- Almost a constant, but a weak scale dependence, and clear oscillating features. How do we understand this?



# Non-linearity generates a bispectrum

- If the initial conditions were Gaussian, linear perturbations remain Gaussian
- However, **non-linear** gravitational evolution makes density fluctuations at late times non-Gaussian, generating a non-vanishing bispectrum

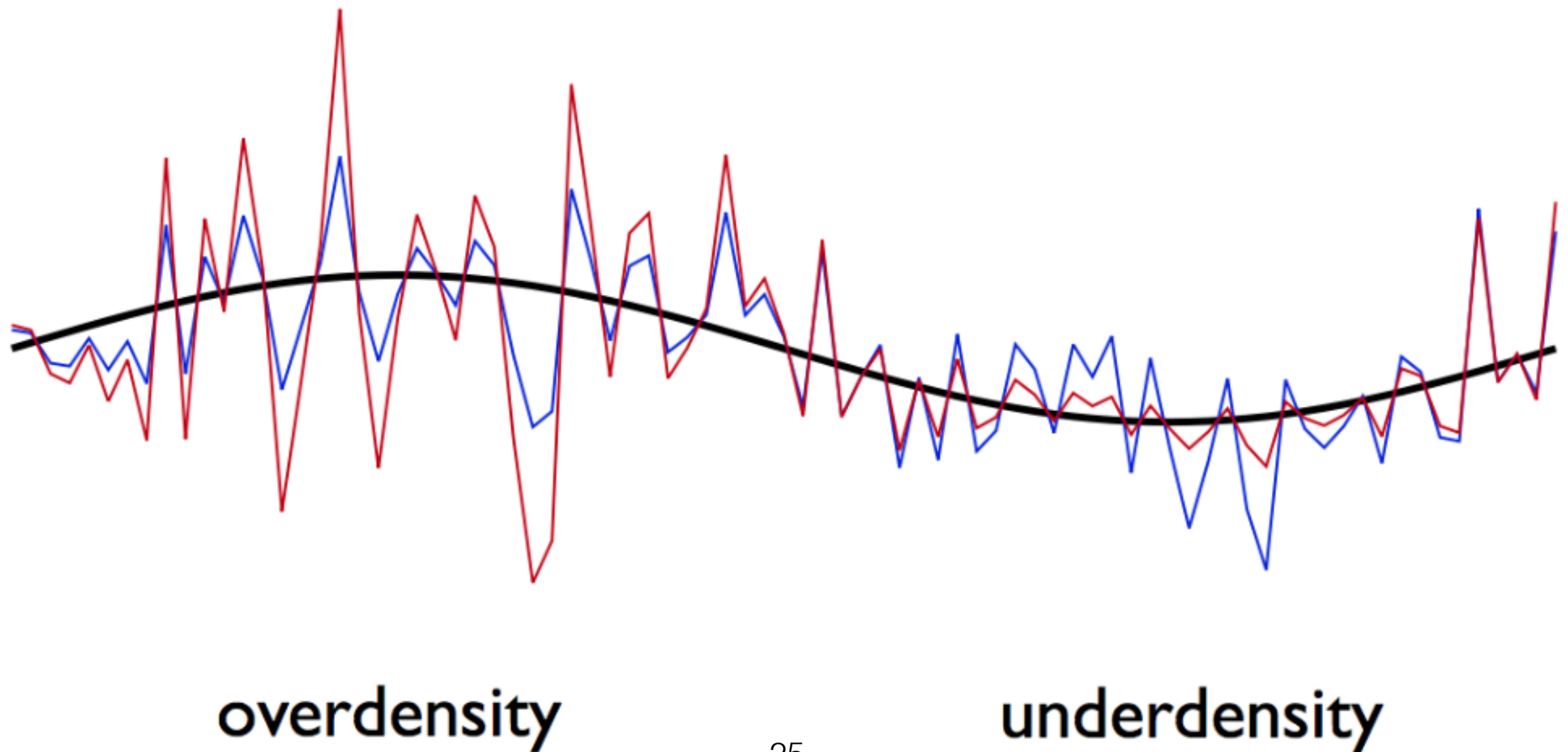
$$\delta' + \nabla \cdot [(1 + \delta)\mathbf{v}] = 0 ,$$

$$\mathbf{v}' + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\mathcal{H}\mathbf{v} - \nabla\phi , \quad H=a'/a$$

$$\nabla^2\phi = 4\pi G a^2 \bar{\rho} \delta ,$$



# 1. Global, “Bird’s View”



# Illustrative Example: SPT

- Second-order perturbation gives the lowest-order bispectrum as

$$B_{\text{SPT}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2[P_l(k_1)P_l(k_2)F_2(\mathbf{k}_1, \mathbf{k}_2) + 2 \text{ cyclic}]$$

“l” stands for “linear”

$$F_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{5}{7} + \frac{1}{2} \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1 k_2} \left( \frac{k_1}{k_2} + \frac{k_2}{k_1} \right) + \frac{2}{7} \left( \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1 k_2} \right)^2$$

- Then

$$iB_L(k) = \frac{1}{V_L^2} \int \frac{d^2 \hat{k}}{4\pi} \int \frac{d^3 q_1}{(2\pi)^3} \int \frac{d^3 q_3}{(2\pi)^3} B(\mathbf{k} - \mathbf{q}_1, -\mathbf{k} + \mathbf{q}_1 + \mathbf{q}_3, -\mathbf{q}_3) \\ \times W_L(\mathbf{q}_1) W_L(-\mathbf{q}_1 - \mathbf{q}_3) W_L(\mathbf{q}_3)$$

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- Then

$$iB_{L,\text{SPT}}(k) \stackrel{kL \rightarrow \infty}{=} \left[ \frac{68}{21} - \frac{1}{3} \frac{d \ln k^3 P_l(k)}{d \ln k} \right] P_l(k) \sigma_L^2$$

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- Then

**Response,  $d \ln P(k)/d\delta$**

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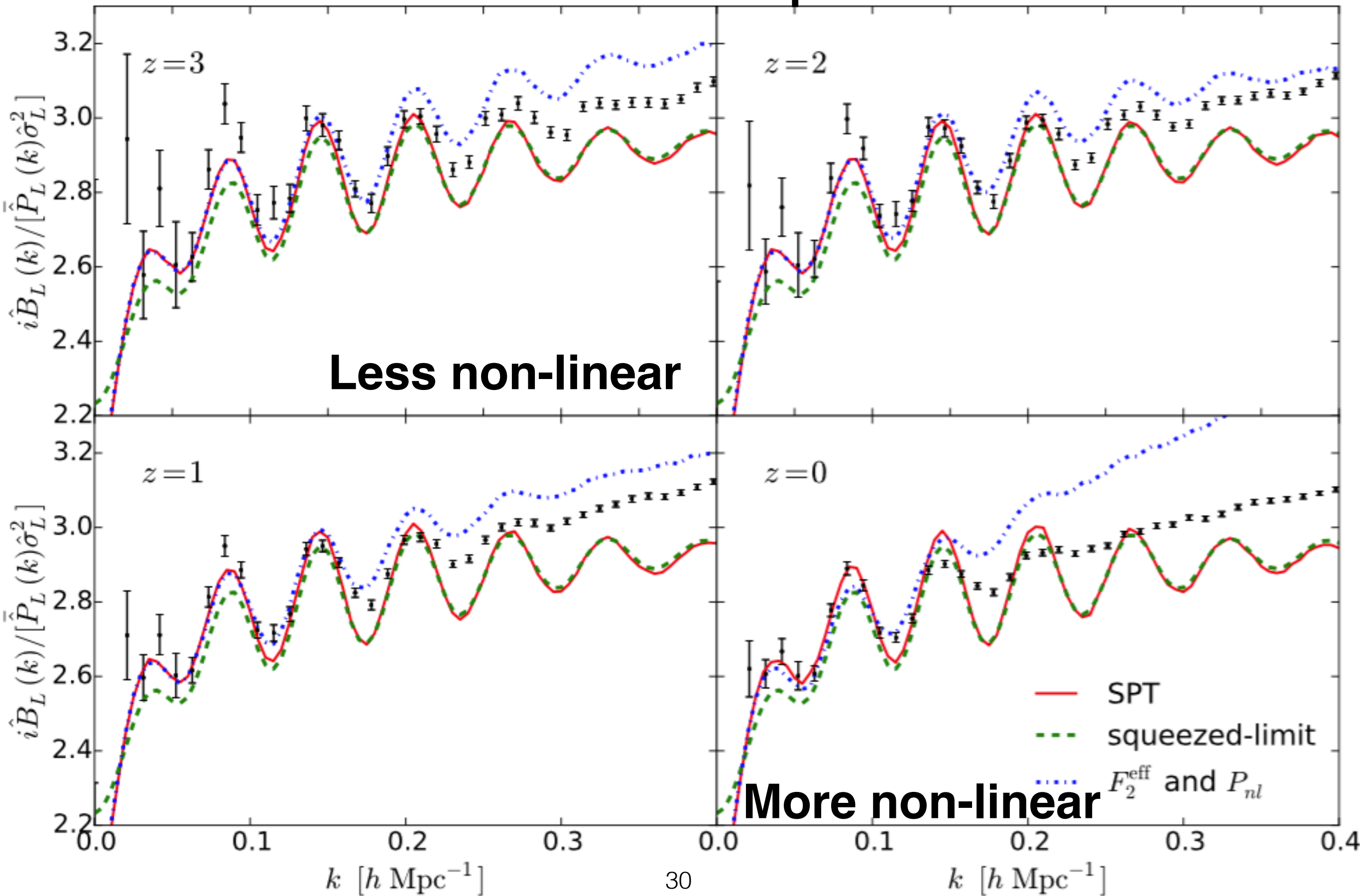
- Then

**Oscillation in  $P(k)$  is enhanced**

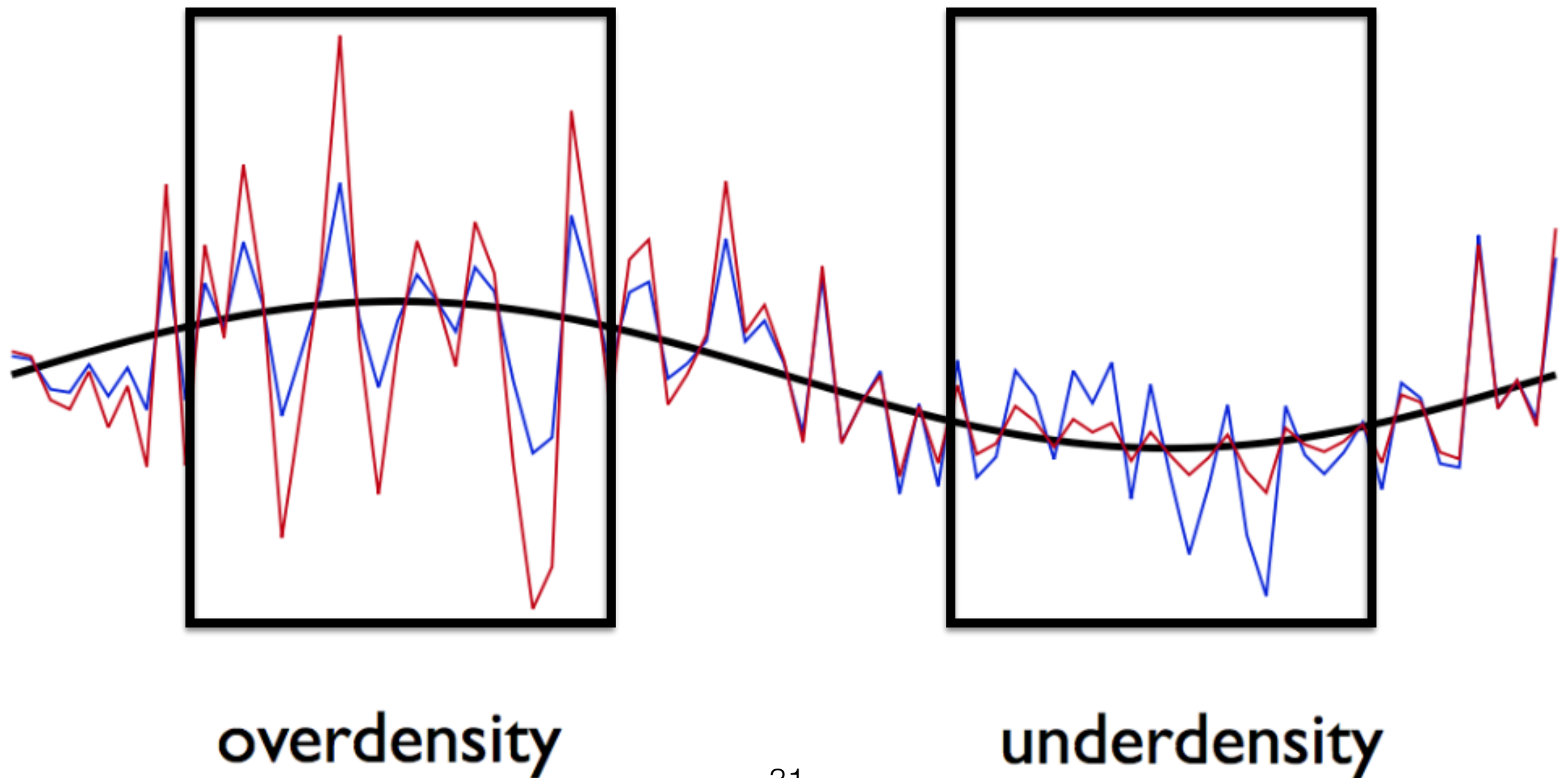
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# Lowest-order prediction



# 1. Local, “Ant’s View”



# Separate Universe Approach

- The meaning of the position-dependent power spectrum becomes more transparent within the context of the “separate universe approach”
  - Each sub-volume with an over-density (or under-density) behaves as if it were **a separate universe with different cosmological parameters**
- In particular, if the global metric is a flat universe, then **each sub-volume can be regarded as a different universe with non-zero curvature**



# Mapping between two cosmologies

- The goal here is to compute the power spectrum in the presence of a long-wavelength perturbation  $\delta$ . We write this as  $P(k, a | \delta)$
- We try to achieve this by computing the power spectrum in a **modified cosmology** with non-zero curvature. Let us put the tildes for quantities evaluated in a modified cosmology

$$\tilde{P}(\tilde{k}, \tilde{a}) \longrightarrow P(k, a | \bar{\delta})$$

# Separate Universe Approach: The Rules

- We evaluate the power spectrum in both cosmologies at the **same physical time** and **same physical spatial coordinates**
- Thus, the evolution of the scale factor is different:

$$\tilde{a}(t) = a(t) \left[ 1 - \frac{1}{3} \bar{\delta}(t) \right]$$

\*tilde: separate universe cosmology

# Separate Universe Approach: The Rules

- We evaluate the power spectrum in both cosmologies at the **same physical time** and **same physical spatial coordinates**
- Thus, comoving coordinates are different too:

$$\tilde{\mathbf{x}} = \frac{a(t)}{\tilde{a}(t)} \mathbf{x} = \left[ 1 + \frac{1}{3} \bar{\delta}(t) \right] \mathbf{x}$$

\*tilde: separate universe cosmology

# Effect 1: Dilation

- Change in the comoving coordinates gives  **$d \ln(k^3 P) / d \ln k$**

$$\begin{aligned} \tilde{P}(k, t) &\rightarrow \left[ 1 - \frac{1}{3} \bar{\delta}(t) \right]^3 P \left( k \left[ 1 - \frac{1}{3} \bar{\delta}(t) \right], t \right) \\ &= [1 - \bar{\delta}(t)] P(k, t) \left[ 1 - \frac{1}{3} \frac{d \ln P(k, t)}{d \ln k} \bar{\delta}(t) \right] \\ &= P(k, t) \left[ 1 - \frac{1}{3} \frac{d \ln k^3 P(k, t)}{d \ln k} \bar{\delta}(t) \right]. \end{aligned}$$

# Effect 2: Reference Density

- Change in the denominator of the definition of  $\delta$ :

$$\tilde{P}(\tilde{k}, t) \rightarrow [1 + \bar{\delta}(t)]^2 \tilde{P}(\tilde{k}, t) = [1 + 2\bar{\delta}(t)] \tilde{P}(\tilde{k}, t)$$

- Putting both together, we find **a generic formula**, valid to linear order in the long-wavelength  $\delta$ :

$$P(k, a|\bar{\delta}) = [1 + 2\bar{\delta}(t)] \tilde{P}(k, \tilde{a}) \left[ 1 - \frac{1}{3} \frac{d \ln k^3 P(k, t)}{d \ln k} \bar{\delta}(t) \right]$$

$$= \tilde{P} \left( k, a \left[ 1 - \frac{1}{3} \bar{\delta}(a) \right] \right) \left[ 1 + \left( 2 - \frac{1}{3} \frac{d \ln k^3 P(k, a)}{d \ln k} \right) \bar{\delta}(a) \right]$$

# Example: Linear $P(k)$

- Let's use the formula to compute the response of the linear power spectrum,  $P_l(k)$ , to the long-wavelength  $\delta$ . Since  $P_l \sim D^2$  [D: linear growth],

$$\tilde{P}_l \left( k, a \left[ 1 - \frac{1}{3} \bar{\delta}(a) \right] \right) = \left( \frac{\tilde{D} \left( a \left[ 1 - \frac{1}{3} \bar{\delta}(a) \right] \right)}{D(a)} \right)^2 P_l(k, a)$$

- Spherical collapse model gives

$$\tilde{D} \left( a \left[ 1 - \frac{1}{3} \bar{\delta}(a) \right] \right) = D(a) \left[ 1 + \frac{13}{21} \bar{\delta}(a) \right]$$

# Response of $P_l(k)$

- Then we obtain:

$$\frac{d \ln P_l(k, a)}{d \bar{\delta}(a)} = \frac{68}{21} - \frac{1}{3} \frac{d \ln k^3 P_l(k, a)}{d \ln k}$$

- Remember the response computed from the leading-order SPT **bispectrum**:

$$iB_{L,\text{SPT}}(k) \stackrel{kL \rightarrow \infty}{=} \left[ \frac{68}{21} - \frac{1}{3} \frac{d \ln k^3 P_l(k)}{d \ln k} \right] P_l(k) \sigma_L^2$$

- **So, the leading-order SPT bispectrum gives the response of the linear  $P(k)$ . Neat!!**

# Response of $P_{3\text{rd-order}}(k)$

- So, let's do the same using **third-order** perturbation theory!  $P(k, a) = P_l(k, a) + P_{22}(k, a) + 2P_{13}(k, a)$

$$P_{22}(k, a) = 2 \int \frac{d^3q}{(2\pi)^3} P_l(q, a) P_l(|\mathbf{k} - \mathbf{q}|, a) [F_2(\mathbf{q}, \mathbf{k} - \mathbf{q})]^2$$

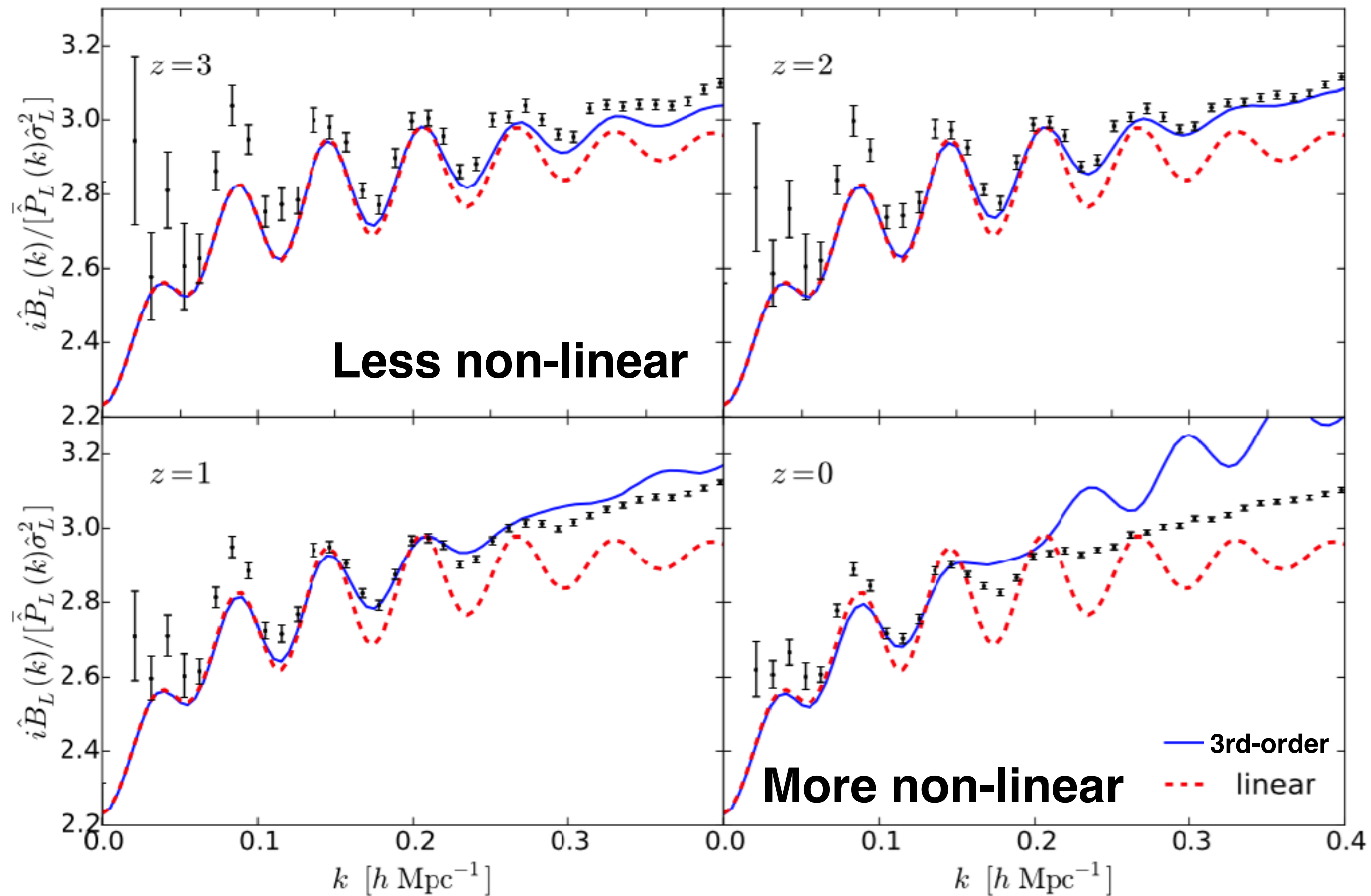
$$2P_{13}(k, a) = \frac{2\pi k^2}{252} P_l(k, a) \int_0^\infty \frac{dq}{(2\pi)^3} P_l(q, a) \times \left[ 100 \frac{q^2}{k^2} - 158 + 12 \frac{k^2}{q^2} - 42 \frac{q^4}{k^4} + \frac{3}{k^5 q^3} (q^2 - k^2)^3 (2k^2 + 7q^2) \ln \left( \frac{k+q}{|k-q|} \right) \right]$$

- Then we obtain:

$$\frac{d \ln P(k, a)}{d \bar{\delta}(a)} = \frac{68}{21} - \frac{1}{3} \frac{d \ln k^3 P(k, a)}{d \ln k} + \frac{26}{21} \frac{P_{22}(k, a) + 2P_{13}(k, a)}{P(k, a)}$$



# 3rd-order does a decent job



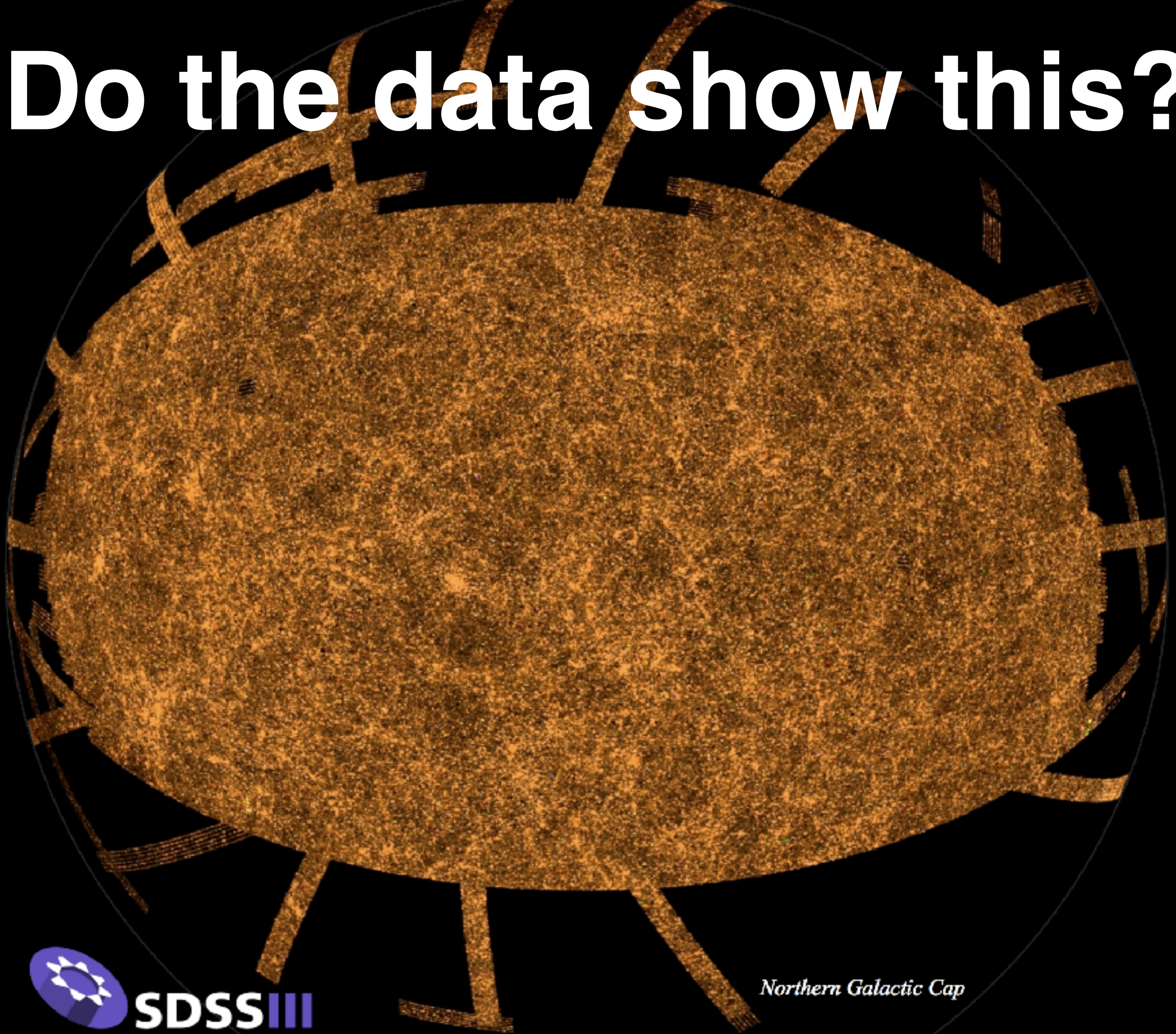
# This is a powerful formula

$$P(k, a|\bar{\delta}) = \tilde{P}\left(k, a\left[1 - \frac{1}{3}\bar{\delta}(a)\right]\right) \left[1 + \left(2 - \frac{1}{3}\frac{d \ln k^3 P(k, a)}{d \ln k}\right)\bar{\delta}(a)\right]$$

- The separate universe description is powerful, as it provides a physically intuitive, transparent, and straightforward way to compute the effect of a long-wavelength perturbation on the small-scale structure growth
- The small-scale structure can be arbitrarily non-linear!



# Do the data show this?

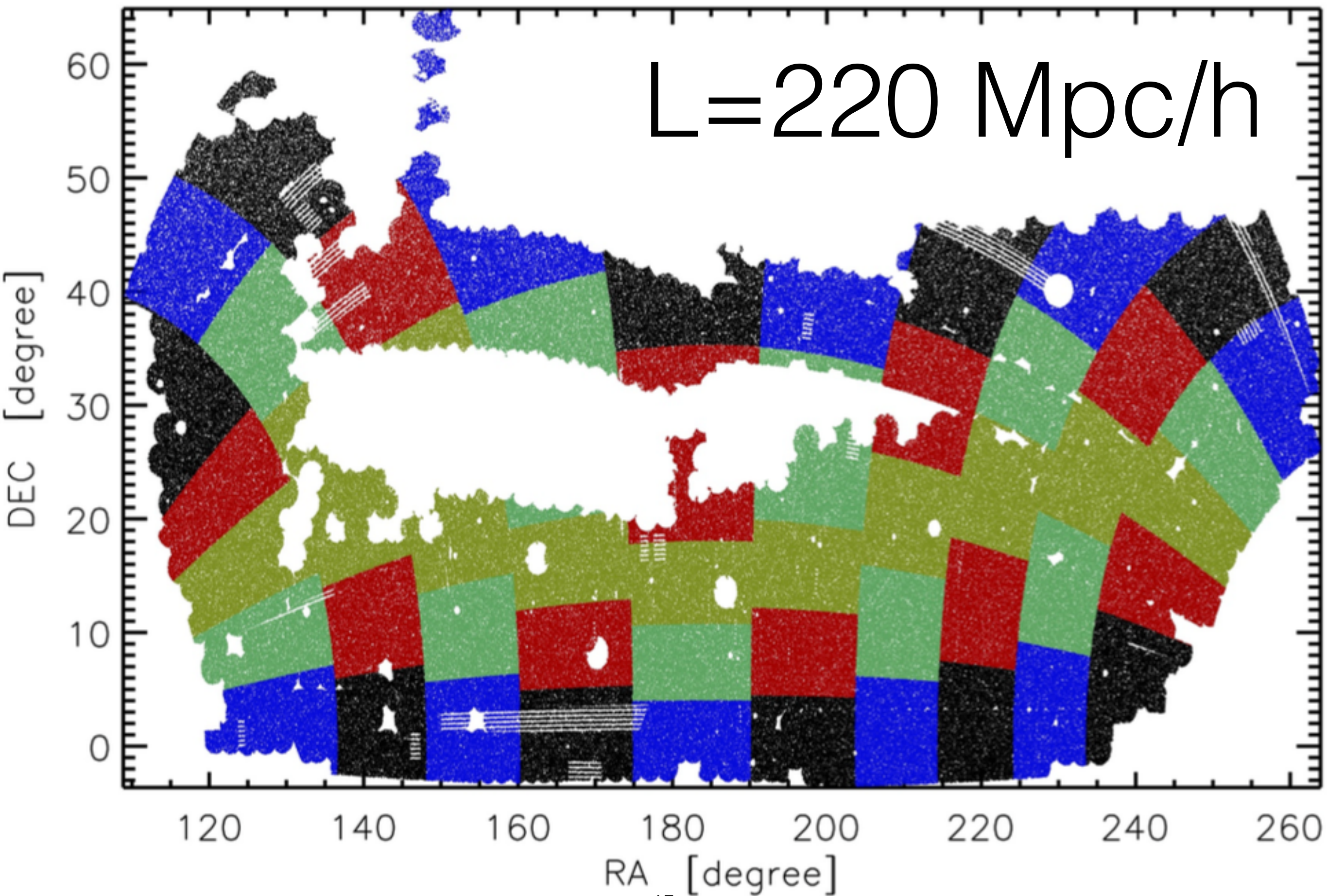




# SDSS-III/BOSS DR11

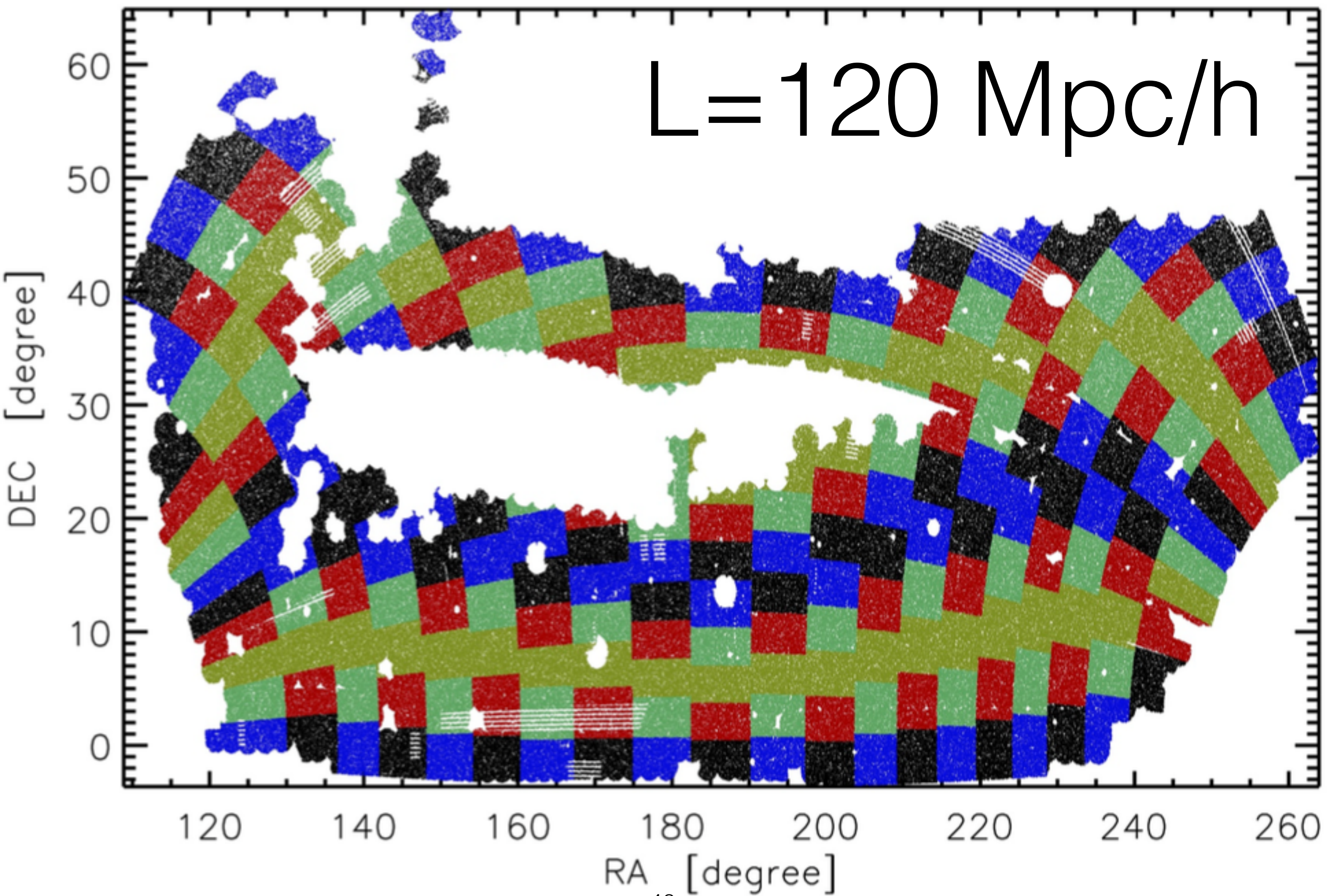
- OK, now, let's look at the real data (BOSS DR10) to see if we can detect the expected influence of environments on the small-scale structure growth
- Bottom line: **we have detected the integrated bispectrum at  $7.4\sigma$** . Not bad for the first detection!

$L=220 \text{ Mpc}/h$



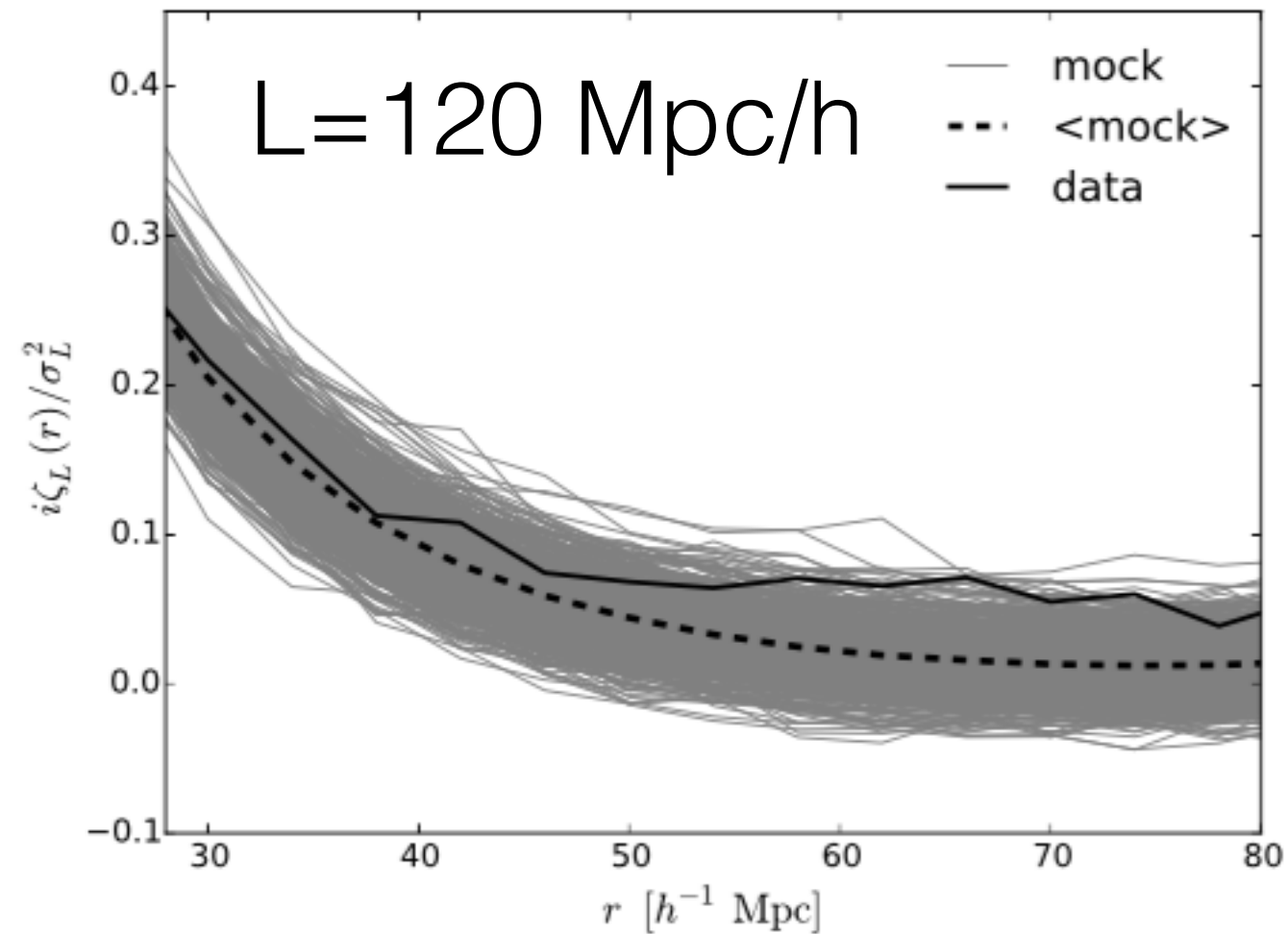
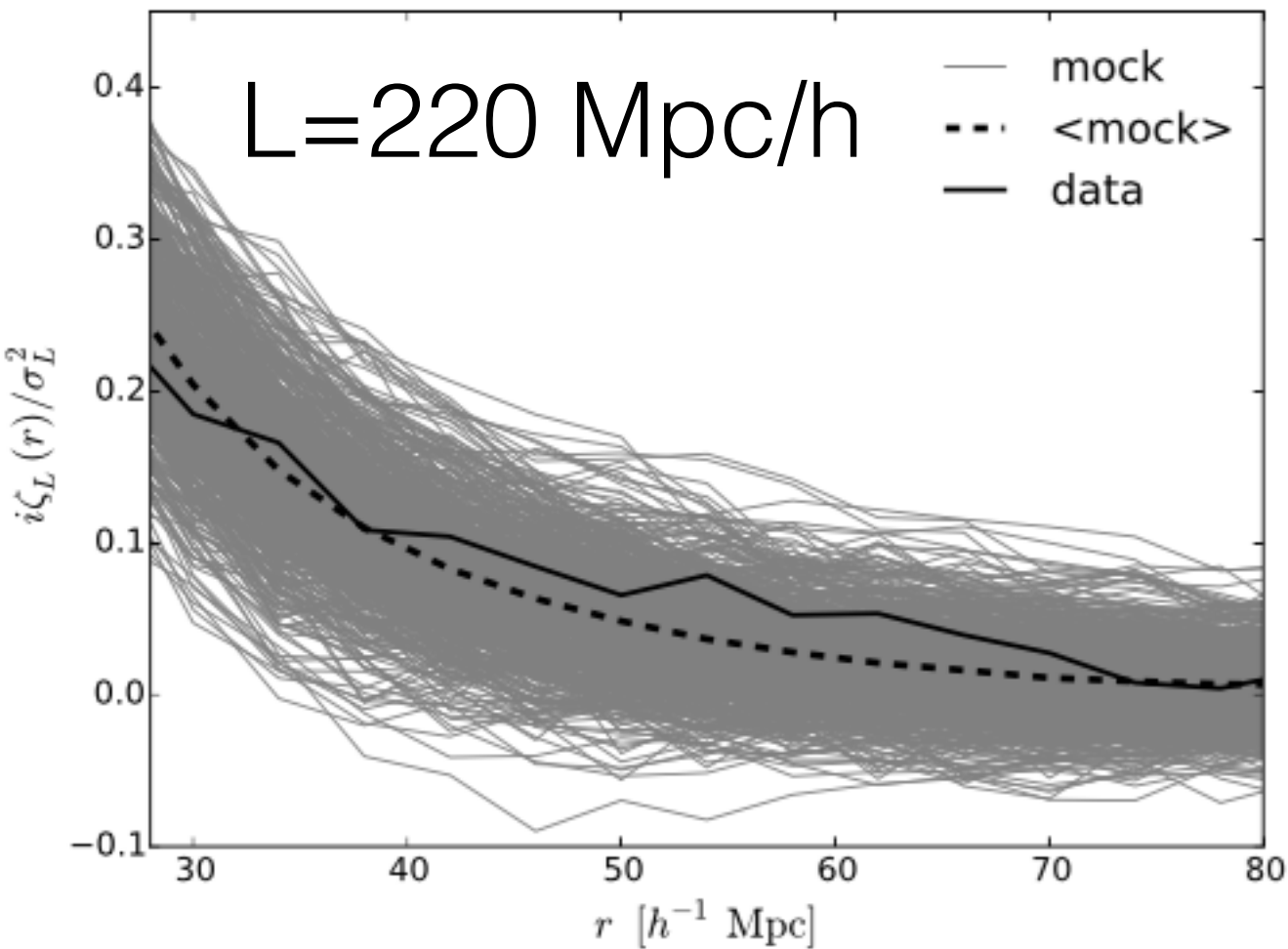


$L = 120 \text{ Mpc}/h$





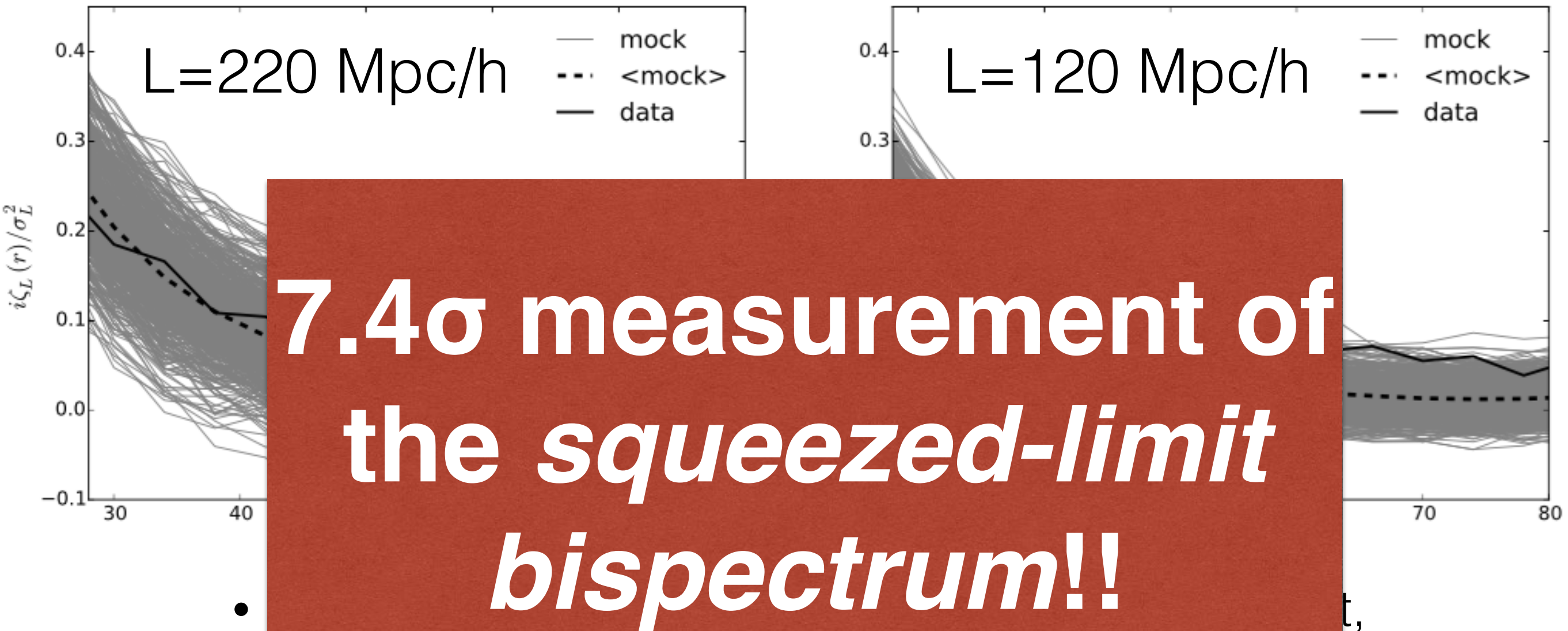
# Results: $\chi^2/\text{DOF} = 46.4/38$



- Because of complex geometry of DR10 footprint, we use the local correlation function, instead of the power spectrum
- Integrated three-point function,  $i\zeta(r)$ , is just Fourier transform of  $iB(k)$ :

$$i\zeta_L(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} iB_L(\mathbf{k}) e^{i\mathbf{r}\cdot\mathbf{k}}$$

# Results: $\chi^2/\text{DOF} = 46.4/38$



the power spectrum

- Integrated three-point function,  $i\zeta(r)$ , is just Fourier transform of  $iB(k)$ :

$$i\zeta_L(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} iB_L(\mathbf{k}) e^{i\mathbf{r}\cdot\mathbf{k}}$$



# Nice, but what is this good for?

- **Primordial non-Gaussianity from the early Universe**
  - The constraint from BOSS is work in progress, but we find that the integrated bispectrum is a **nearly optimal estimator for the squeezed-limit bispectrum from inflation**
  - We no longer need to measure the full bispectrum, if we are just interested in the squeezed limit

# Nice, but what is this good for?

- **We can also learn about galaxy bias**
  - Local bias model:
    - $\delta_g(x) = b_1 \delta_m(x) + (\mathbf{b}_2/2) [\delta_m(x)]^2 + \dots$
- **The bispectrum can give us  $b_2$  at the leading order**, unlike for the power spectrum that has  $b_2$  at the next-to-leading order

# Result on $b_2$

- We use the leading-order SPT bispectrum with the local bias model to interpret our measurements
- [We also use information from BOSS's 2-point correlation function on  $f\sigma_8$  and BOSS's weak lensing data on  $\sigma_8$ ]
- We find:  **$b_2 = 0.41 \pm 0.41$**

# Simulating Ant's Views

# This is a powerful formula

$$P(k, a|\bar{\delta}) = \tilde{P}\left(k, a \left[1 - \frac{1}{3}\bar{\delta}(a)\right]\right) \left[1 + \left(2 - \frac{1}{3} \frac{d \ln k^3 P(k, a)}{d \ln k}\right) \bar{\delta}(a)\right]$$

- How can we compute  $\tilde{P}(k, a)$  in practice?
- **Small N-body simulations with a modified cosmology (“Separate Universe Simulation”)**
- Perturbation theory

# Separate Universe Simulation

- How do we compute the response function beyond perturbation theory?
  - Do we have to run many big-volume simulations and divide them into sub-volumes? No.
- Fully non-linear computation of the response function is possible with **separate universe simulations**
- E.g., we run two small-volume simulations with separate-universe cosmologies of over- and under-dense regions with the same initial random number seeds, and compute the derivative  $d\ln P/d\delta$  by, e.g.,

$$\frac{d \ln P(k)}{d\bar{\delta}} = \frac{\ln P(k| + \bar{\delta}) - \ln P(k| - \bar{\delta})}{2\bar{\delta}}$$

# Separate Universe Cosmology

$$\rho(t) [1 + \delta_\rho(t)] = \tilde{\rho}(t)$$

$$\rightarrow \frac{\Omega_m h^2}{a^3(t)} [1 + \delta_\rho(t)] = \frac{\tilde{\Omega}_m \tilde{h}^2}{\tilde{a}^3(t)}$$

$$\frac{\tilde{K}}{H_0^2} = \frac{5}{3} \frac{\Omega_m}{a(t_i)} \delta_\rho(t_i)$$

$$\rightarrow \delta_H = \left(1 - \frac{\tilde{K}}{H_0^2}\right)^{1/2} - 1$$

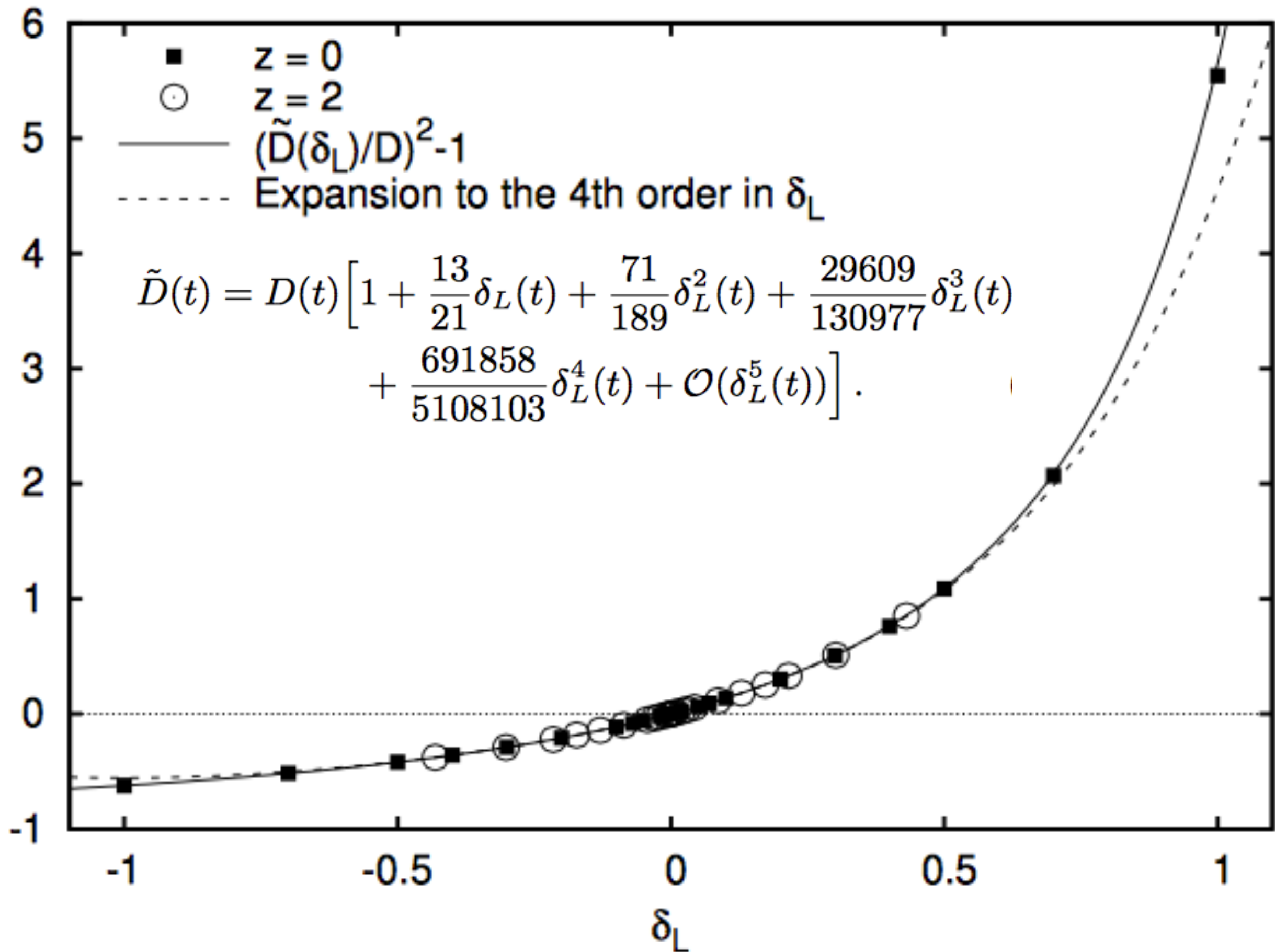
$$\tilde{H}_0 = H_0 [1 + \delta_H]$$

$$\tilde{\Omega}_m = \Omega_m [1 + \delta_H]^{-2}$$

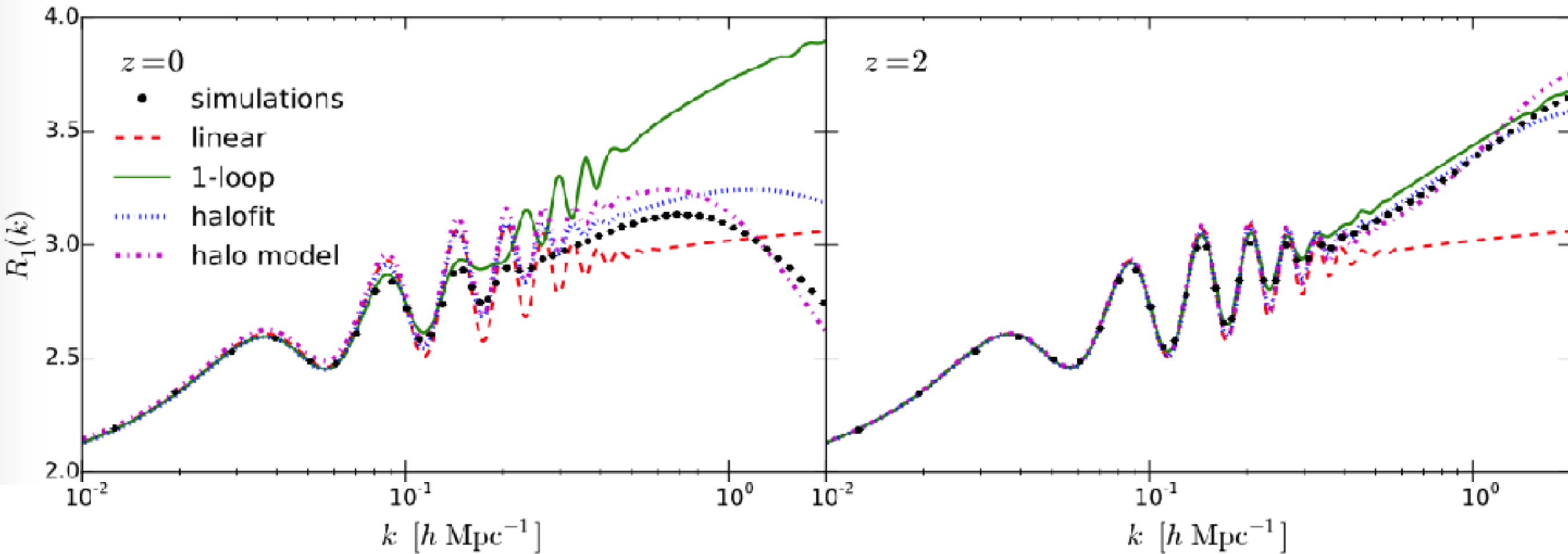
$$\tilde{\Omega}_\Lambda = \Omega_\Lambda [1 + \delta_H]^{-2}$$



fractional difference in the  
power of the fundamental mode

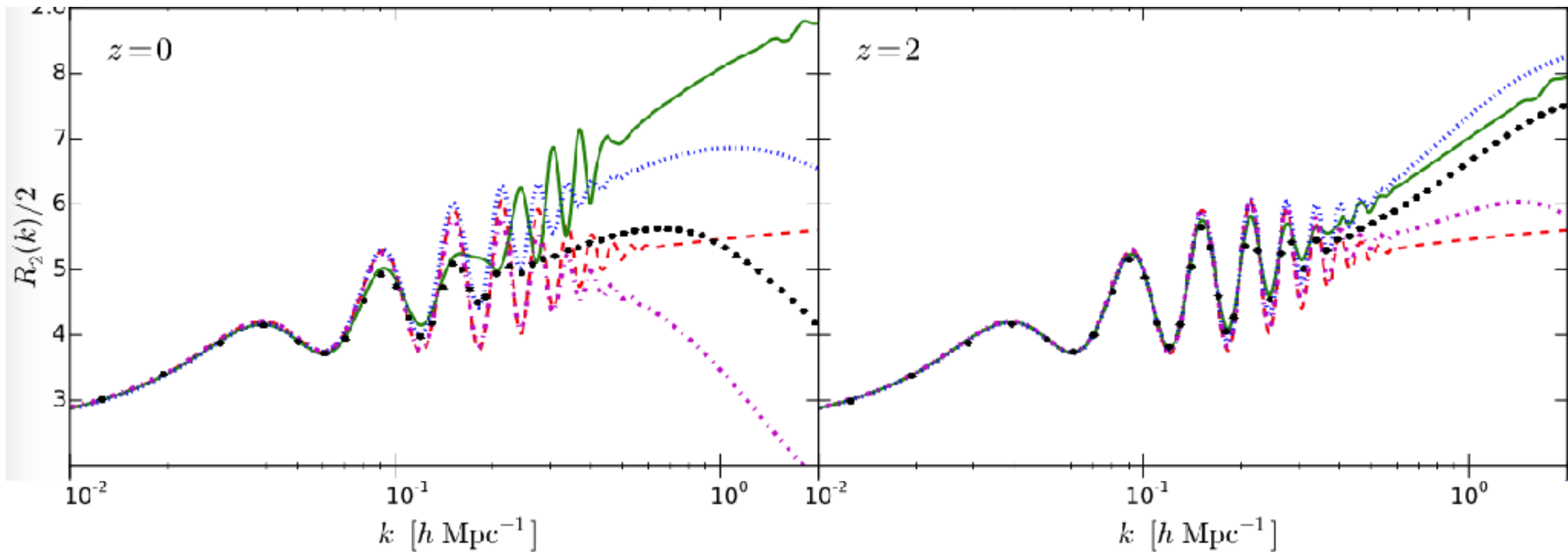


$$R_1 = d \ln P / d \delta$$



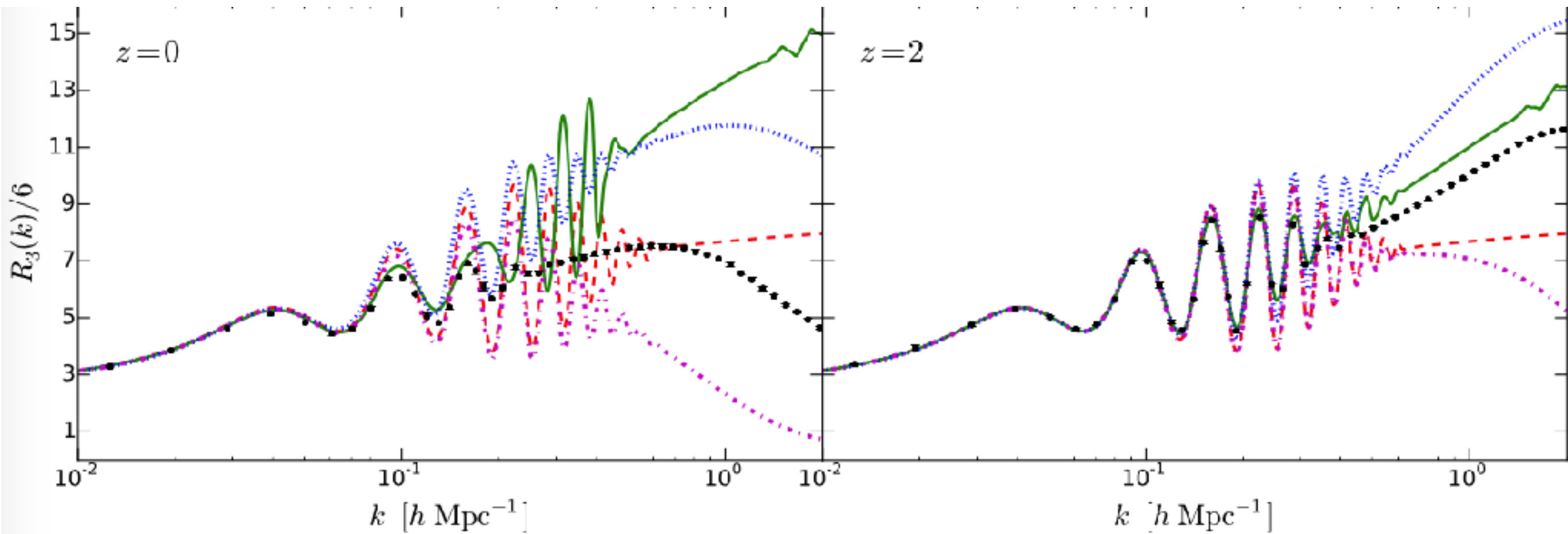
- The symbols are the data points with error bars. You cannot see the error bars!

$$R_2 = d^2 \ln P / d\delta^2$$



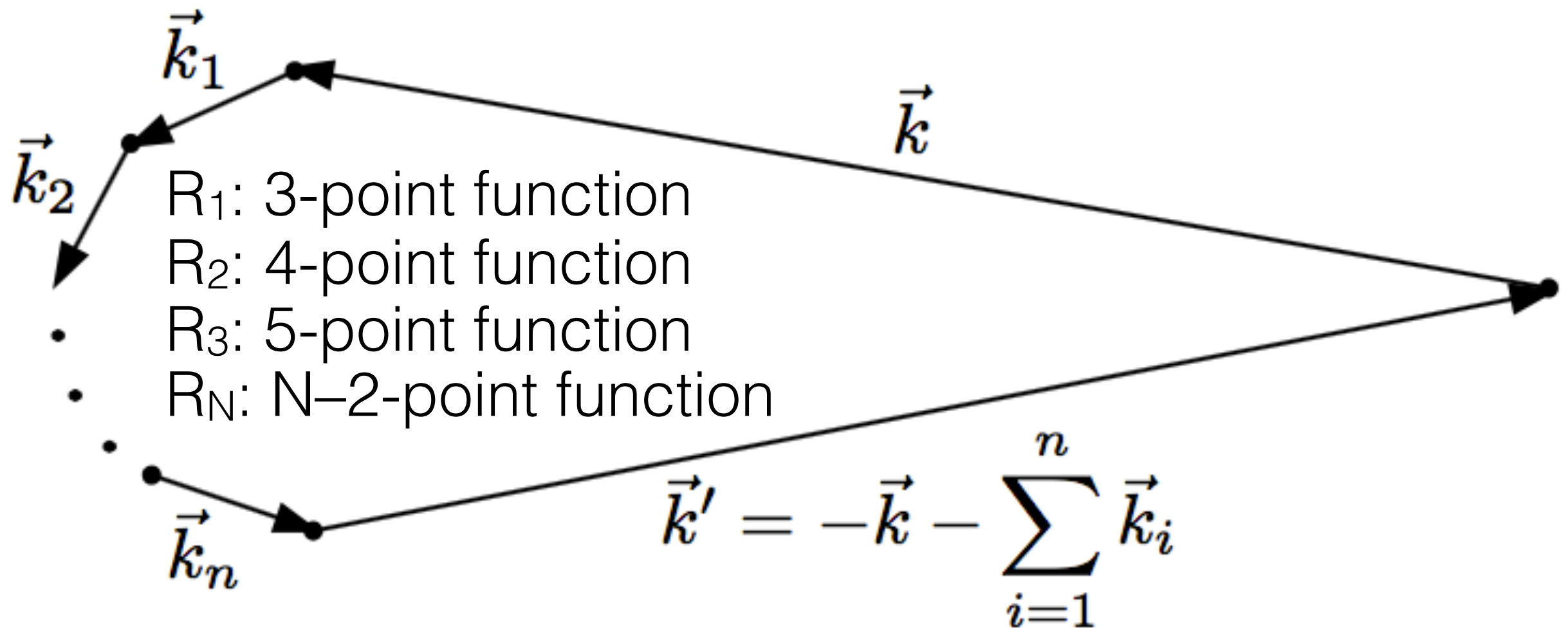
- More derivatives can be computed by using simulations run with more values of  $\delta$

$$R_3 = d^3 \ln P / d\delta^3$$



- But, what do  $d^n \ln P / d\delta^n$  mean physically??

# More derivatives: Squeezed limits of N-point functions



- Why do we want to know this? I don't know, but it is cool and they have not been measured before!

# Summary

- **New observable**: the position-dependent power spectrum and the integrated bispectrum
  - Straightforward interpretation in terms of the separate universe
  - Easy to measure; easy to model!
  - Useful for primordial non-Gaussianity and non-linear bias
- Lots of applications: e.g., QSO density correlated with Lyman-alpha power spectrum
- All of the results and much more are summarised in Chi-Ting Chiang's PhD thesis: **arXiv:1508.03256**

Read my  
thesis!



# More on $b_2$

- Using slightly more advanced models, we find:

|       | baseline        | eff kernel      | tidal bias      | both*           |
|-------|-----------------|-----------------|-----------------|-----------------|
| $b_2$ | $0.41 \pm 0.41$ | $0.51 \pm 0.41$ | $0.48 \pm 0.41$ | $0.60 \pm 0.41$ |

\*The last value is in agreement with  $b_2$  found by the Barcelona group (Gil-Marín et al. 2014) that used the full bispectrum analysis and the same model



| r-space    | $b_1$             | $b_2$           |
|------------|-------------------|-----------------|
| baseline   | $1.971 \pm 0.076$ | $0.58 \pm 0.31$ |
| eff kernel | $1.973 \pm 0.076$ | $0.62 \pm 0.31$ |
| tidal bias | $1.971 \pm 0.076$ | $0.64 \pm 0.31$ |
| both       | $1.973 \pm 0.076$ | $0.68 \pm 0.31$ |

| z-space    | $b_1$             | $b_2$           |
|------------|-------------------|-----------------|
| baseline   | $1.931 \pm 0.077$ | $0.54 \pm 0.35$ |
| eff kernel | $1.933 \pm 0.077$ | $0.65 \pm 0.35$ |
| tidal bias | $1.932 \pm 0.077$ | $0.60 \pm 0.35$ |
| both       | $1.933 \pm 0.077$ | $0.71 \pm 0.35$ |

# One more cool thing

- We can use the separate universe simulations to test **validity of SPT to all orders in perturbations**
- The fundamental prediction of SPT: the non-linear power spectrum at a given time is given by the linear power spectra at the same time
- In other words, the only time dependence arises from the linear growth factors,  $D(t)$

# One more cool thing

- We can use the separate universe simulations to test **validity of SPT to all orders in perturbations**

$$\delta' + \nabla \cdot [(1 + \delta)\mathbf{v}] = 0 ,$$

$$\mathbf{v}' + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\mathcal{H}\mathbf{v} - \nabla\phi ,$$

$$\nabla^2\phi = 4\pi G a^2 \bar{\rho} \delta ,$$

**SPT at all orders: Exact solution of the pressureless fluid equations**

**We can test validity of SPT as a description of collisions particles**

# Example: $P_{3\text{rd-order}}(k)$

- SPT to 3rd order

$$P(k, a) = P_l(k, a) + P_{22}(k, a) + 2P_{13}(k, a)$$

$$P_{22}(k, a) = 2 \int \frac{d^3q}{(2\pi)^3} P_l(q, a) P_l(|\mathbf{k} - \mathbf{q}|, a) [F_2(\mathbf{q}, \mathbf{k} - \mathbf{q})]^2$$

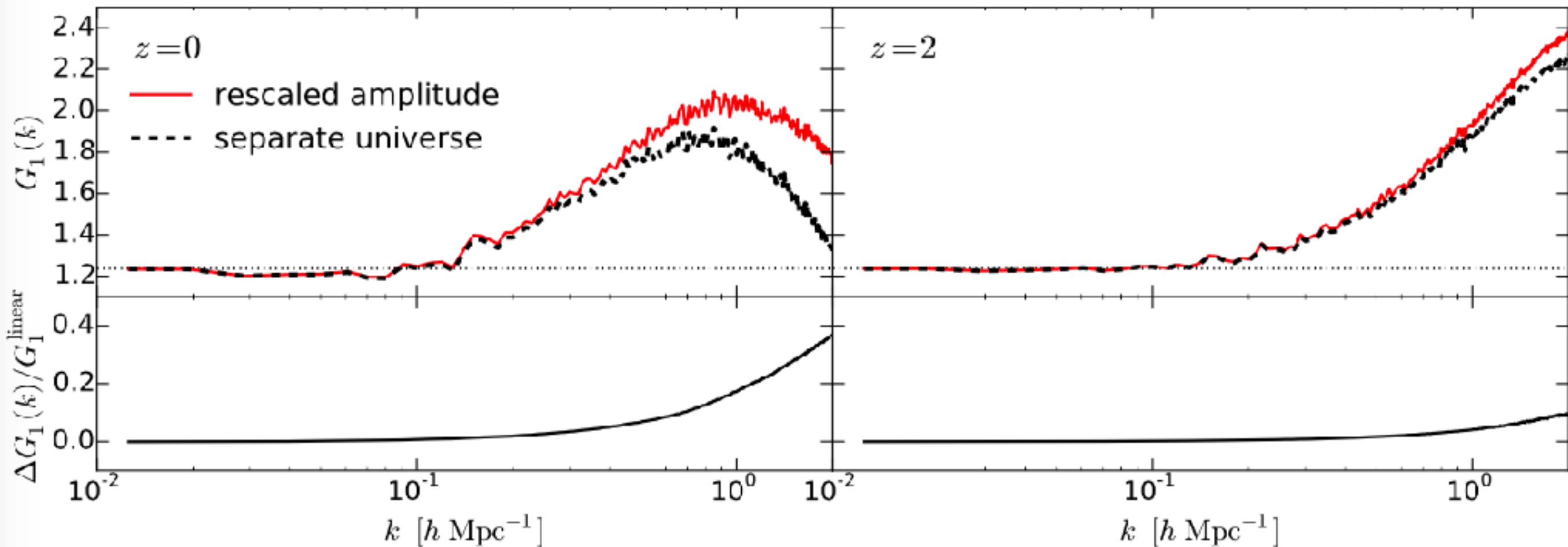
$$2P_{13}(k, a) = \frac{2\pi k^2}{252} P_l(k, a) \int_0^\infty \frac{dq}{(2\pi)^3} P_l(q, a) \\ \times \left[ 100 \frac{q^2}{k^2} - 158 + 12 \frac{k^2}{q^2} - 42 \frac{q^4}{k^4} + \frac{3}{k^5 q^3} (q^2 - k^2)^3 (2k^2 + 7q^2) \ln \left( \frac{k+q}{|k-q|} \right) \right]$$

- The only time-dependence is in  $P_l(k, a) \sim D^2(a)$
- Is this correct?

# Rescaled simulations vs Separate universe simulations

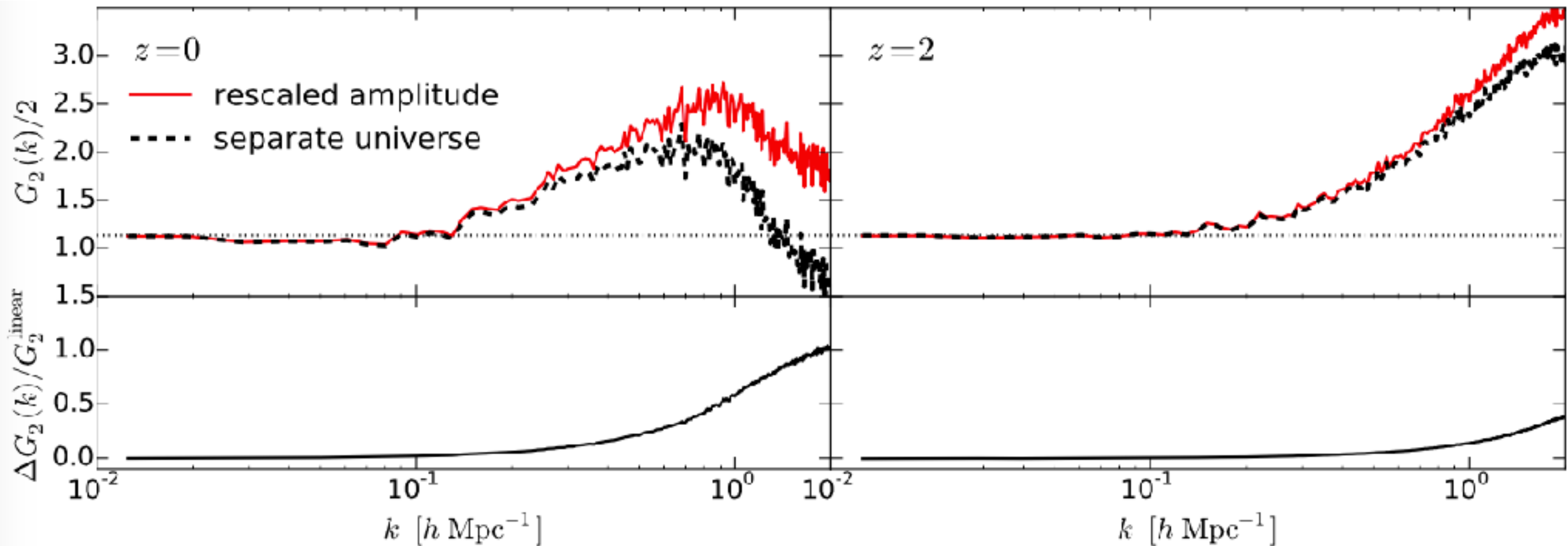
- To test this, we run two sets of simulations.
- **First**: we rescale the initial amplitude of the power spectrum, so that we have a given value of the linear power spectrum amplitude at some later time,  $t_{\text{out}}$
- **Second**: full separate universe simulation, which changes all the cosmological parameters consistently, given a value of  $\delta$ 
  - We choose  $\delta$  so that it yields the same amplitude of the linear power spectrum as the first one at  $t_{\text{out}}$

# Results: 3-point function



- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions

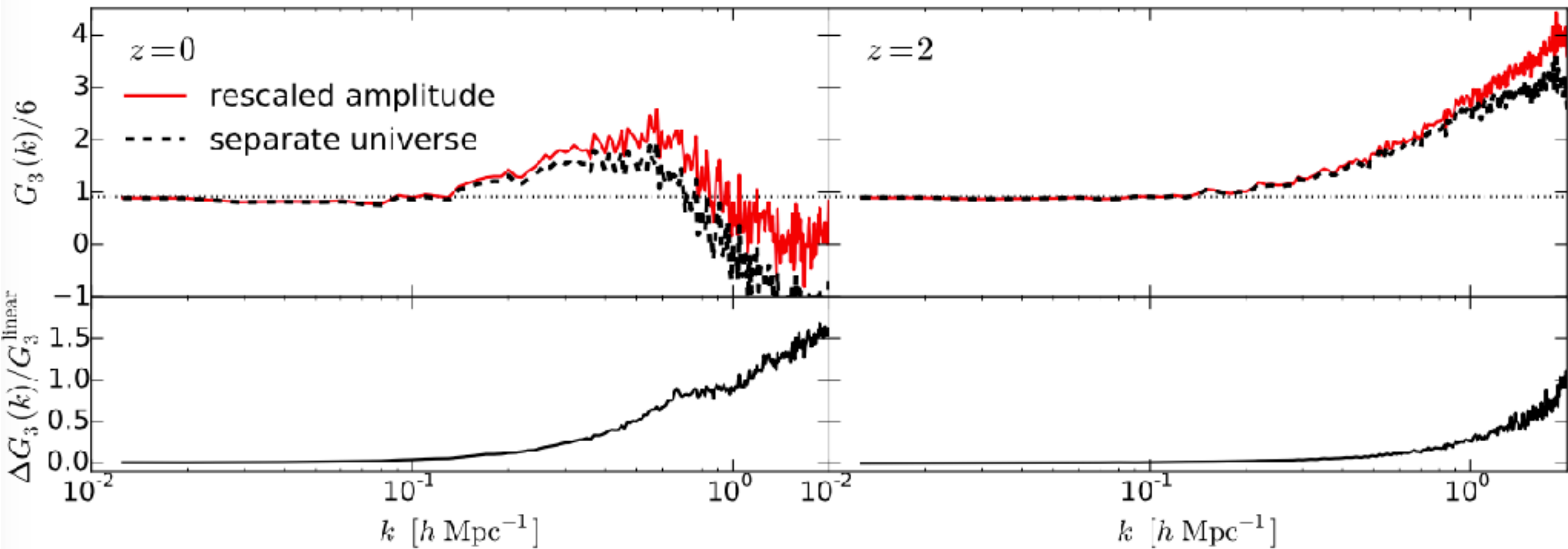
# Results: 4-point function



- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions



# Results: 5-point function



- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions



# Break down of SPT at all orders

- **At  $z=0$ , SPT computed to all orders breaks down at  $k \sim 0.5 \text{ Mpc/h}$  with 10% error**, in the squeezed limit 3-point function
  - Break down occurs at lower  $k$  for the squeezed limits of the 4- and 5-point functions
  - Break down occurs at higher  $k$  at  $z=2$
- I find this information quite useful: *it quantifies accuracy of the perfect-fluid approximation of density fields*