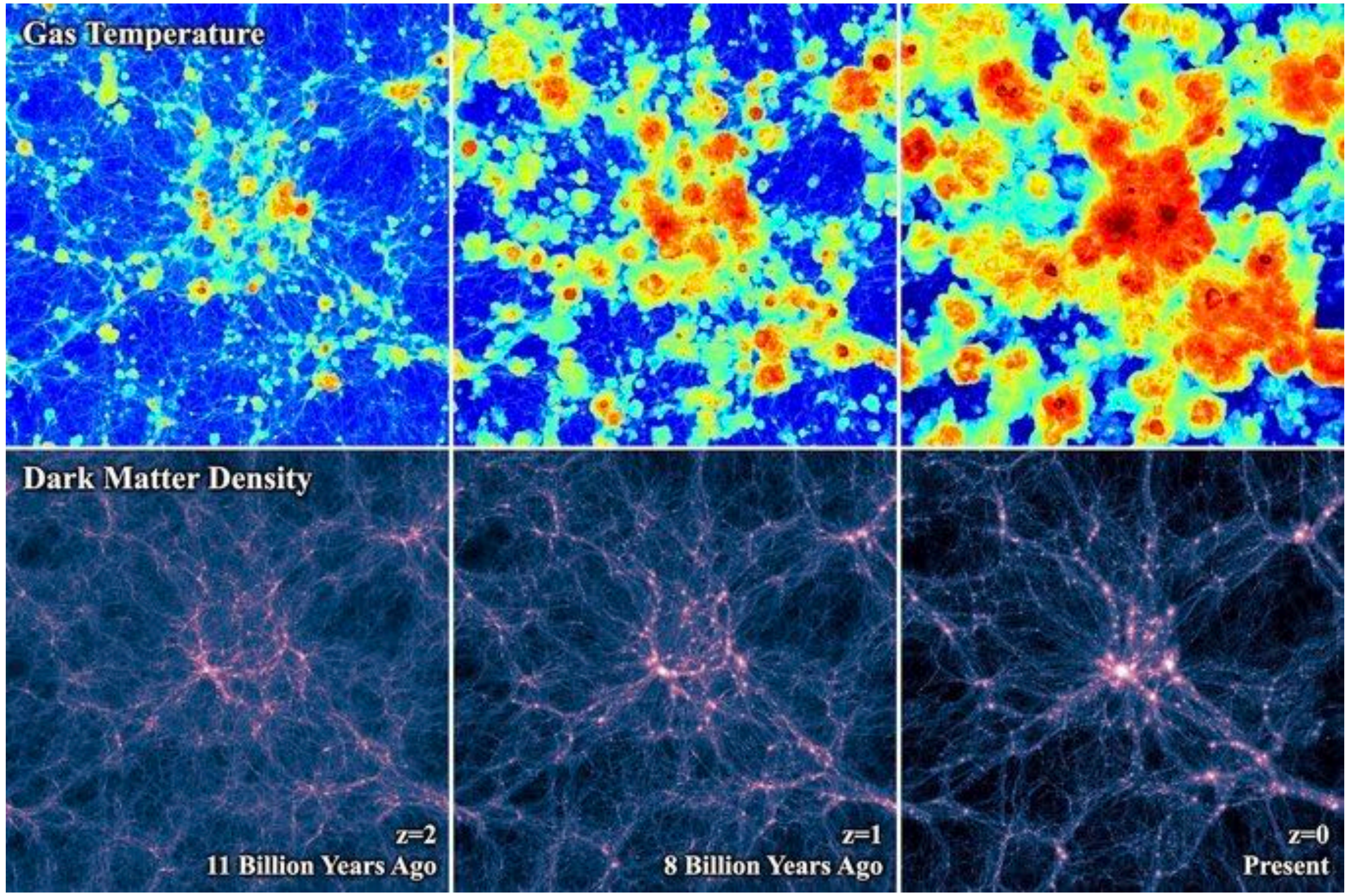
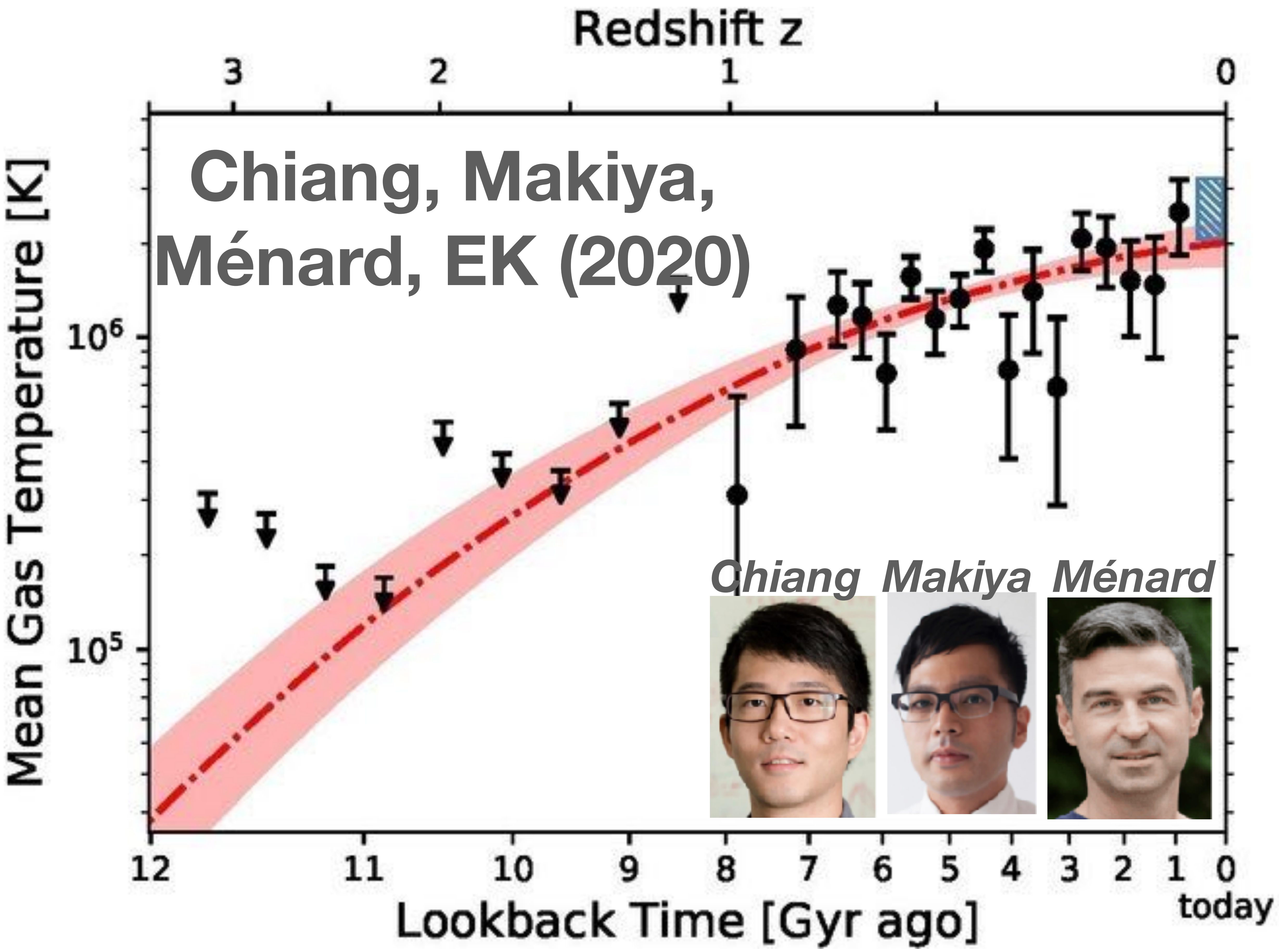
The Temperature of Hot Gas in the Universe

MAX-PLANCK-INSTITUT
FÜR ASTROPHYSIK

Credit: Dylan Nelson, Illustris Collaboration

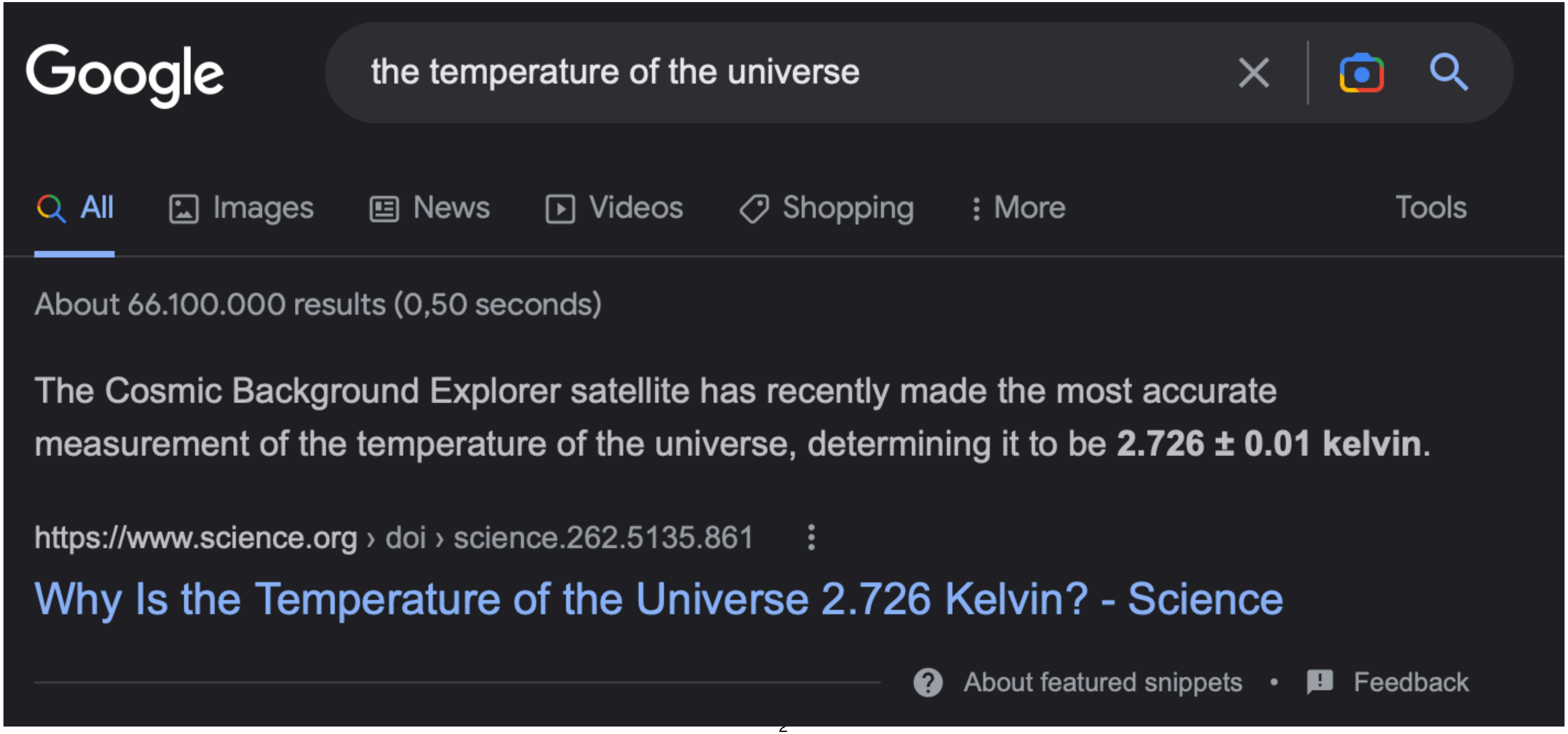




Eiichiro Komatsu, *Astrophysics Seminar, ARI@LJMU*, April 10, 2024

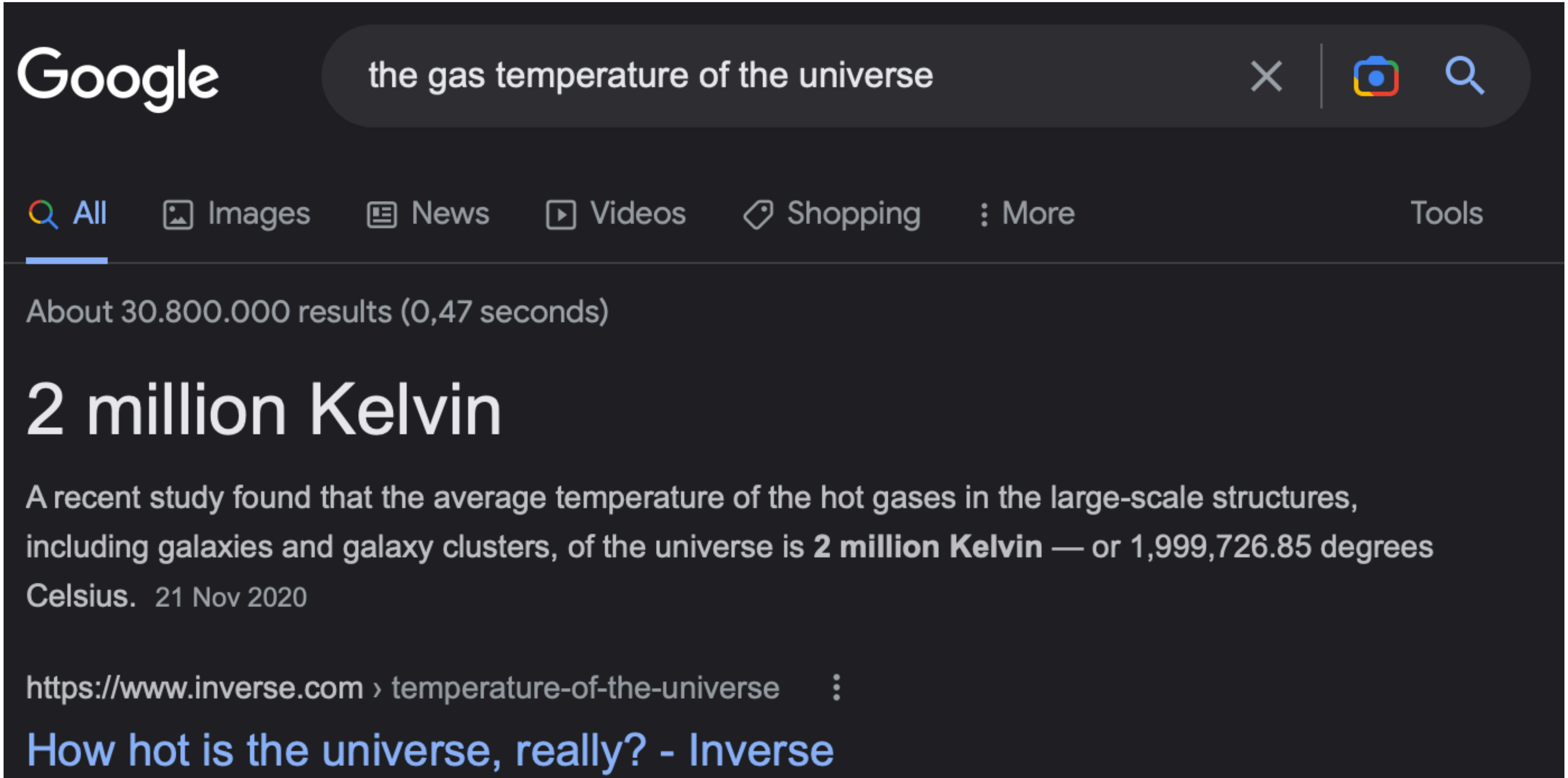
If you ask Google…

“The temperature of the Universe”



If you ask Google…

“The gas temperature of the Universe”



If you ask AI…

what is the temperature of the universe?

Welcome to the new Bing

Your AI-powered copilot for the web

The temperature of the universe is a complex topic. According to a recent study, the average temperature of the hot gases in the large-scale structures, including galaxies and galaxy clusters, of the universe is **2 million Kelvin** or **1,999,726.85 degrees Celsius** [1]. However, it's important to note that space itself doesn't have a temperature at all as temperature is a measurement of the speed at which particles are moving and heat is how much energy the particles of an object have. In a truly empty region of space, there would be no particles and radiation, meaning there's also no temperature [2].

The temperature of space is generally **2.73 Kelvin** or **-270.42 Celsius** or **-454.75 Fahrenheit** [3]. This is actually the temperature of Cosmic Microwave Background Radiation, which is spread throughout the universe [3].

It's worth noting that the mean temperature of gas across the universe has increased more than 10 times over the last 10 billion years and reached about **2 million degrees Kelvin** today [4].

Temperature of what?

The composition of the Universe

- Ordinary stuff that we are familiar with:
  - **Gas of the ordinary matter** (mostly hydrogen and helium) This talk
  - Photons (cosmic microwave background): 2.7 Kelvin
  - Neutrinos (cosmic neutrino background): 1.9 Kelvin [indirect measurement]
- Extraordinary stuff that we know exists:
  - *Dark matter*: ??
  - *Dark energy*: ??

Outline

Three questions to answer during this talk:

1. How hot is the large-scale structure of the Universe today? How was it before?
   - *Chiang, Makiya, Ménard, EK, ApJ, 902, 56 (2020)*
2. Where did the thermal energy come from?
   - *Chiang, Makiya, EK, Ménard, ApJ, 910, 32 (2021)*
3. What is our result good for?
   - Is it just beautiful physics, or is it also useful for something?
   - *Young, EK, Dolag, PRD, 104, 083538 (2021)*
   - *Chen, Jamieson, EK, et al., PRD, 109, 063513 (2024)*

The cosmic energy inventory
Fukugita & Peebles (2004)

- We know the mean total mass density of the Universe: $\mathbf{\Omega_m \sim 0.3}$.
- We also know the mean baryonic mass density of the Universe: $\mathbf{\Omega_B \sim 0.05}$.
- We also have estimates for many other energy densities in the Universe:

THE COSMIC ENERGY INVENTORY

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Received 2004 June 3; accepted 2004 August 9

ABSTRACT

We present an inventory of the cosmic mean densities of energy associated with all the known states of matter and radiation at the present epoch. The observational and theoretical bases for the inventory have become rich enough to allow estimates with observational support for the densities of energy in some 40 forms. The result is a global portrait of the effects of the physical processes of cosmic evolution.

Fukugita & Peebles (2004)

TABLE 1

THE COSMIC ENERGY INVENTORY

Category	Parameter		Components[a]	Totals[a]
1	Dark sector:			0.954 ± 0.003
1.1	Dark energy		0.72 ± 0.03	
1.2	Dark matter		0.23 ± 0.03	
1.3	Primeval gravitational waves		$\lesssim 10^{-10}$	
2	Primeval thermal remnants:			0.0010 ± 0.0005
2.1	Electromagnetic radiation		$10^{-4.3 \pm 0.0}$	
2.2	Neutrinos		$10^{-2.9 \pm 0.1}$	
2.3	Prestellar nuclear binding energy		$-10^{-4.1 \pm 0.0}$	
3	Baryon rest mass:			0.045 ± 0.003
3.1	Warm intergalactic plasma		0.040 ± 0.003	
3.1a	Virialized regions of galaxies	0.024 ± 0.005		
3.1b	Intergalactic	0.016 ± 0.005		
3.2	Intracluster plasma		0.0018 ± 0.0007	
3.3	Main-sequence stars: spheroids and bulges		0.0015 ± 0.0004	
3.4	Main-sequence stars: disks and irregulars		0.00055 ± 0.00014	
3.5	White dwarfs		0.00036 ± 0.00008	
3.6	Neutron stars		0.00005 ± 0.00002	
3.7	Black holes		0.00007 ± 0.00002	
3.8	Substellar objects		0.00014 ± 0.00007	
3.9	H I + He I		0.00062 ± 0.00010	
3.10	Molecular gas		0.00016 ± 0.00006	
3.11	Planets		10^{-6}	
3.12	Condensed matter		$10^{-5.6 \pm 0.3}$	
3.13	Sequestered in massive black holes		$10^{-5.4}(1 + \epsilon_n)$	
4	Primeval gravitational binding energy:			$-10^{-6.1 \pm 0.1}$
4.1	Virialized halos of galaxies		$-10^{-7.2}$	
4.2	Clusters		$-10^{-6.9}$	
4.3	Large-scale structure		$-10^{-6.2}$	
5	Binding energy from dissipative gravitational settling:			$-10^{-4.9}$
5.1	Baryon-dominated parts of galaxies		$-10^{-8.8 \pm 0.3}$	
5.2	Main-sequence stars and substellar objects		$-10^{-8.1}$	

Fukugita & Peebles (2004)

Category	Parameter	Components[a]	Totals[a]
5.3	White dwarfs	$-10^{-7.4}$	
5.4	Neutron stars	$-10^{-5.2}$	
5.5	Stellar mass black holes	$-10^{-4.2}\epsilon_s$	
5.6	Galactic nuclei: early type	$-10^{-5.6}\epsilon_n$	
5.7	Galactic nuclei: late type	$-10^{-5.8}\epsilon_n$	
6	Poststellar nuclear binding energy:		$-10^{-5.2}$
6.1	Main-sequence stars and substellar objects	$-10^{-5.8}$	
6.2	Diffuse material in galaxies	$-10^{-6.5}$	
6.3	White dwarfs	$-10^{-5.6}$	
6.4	Clusters	$-10^{-6.5}$	
6.5	Intergalactic	$-10^{-6.2\pm0.5}$	
7	Poststellar radiation:		$10^{-5.7\pm0.1}$
7.1	Resolved radio-microwave	$10^{-10.3\pm0.3}$	
7.2	FIR	$10^{-6.1}$	
7.3	Optical	$10^{-5.8\pm0.2}$	
7.4	X-ray–γ-ray	$10^{-7.9\pm0.2}$	
7.5	Gravitational radiation: stellar mass binaries	$10^{-9\pm1}$	
7.6	Gravitational radiation: massive black holes	$10^{-7.5\pm0.5}$	
8	Stellar neutrinos:		$10^{-5.5}$
8.1	Nuclear burning	$10^{-6.8}$	
8.2	White dwarf formation	$10^{-7.7}$	
8.3	Core collapse	$10^{-5.5}$	
9	Cosmic rays and magnetic fields		$10^{-8.3^{+0.6}_{-0.3}}$
10	Kinetic energy in the IGM		$10^{-8.0\pm0.3}$

- But we did not know the mean thermal energy density of the Universe, Ω_{th}
    - **Let's measure this!**

Our definition of the thermal energy density

nk_BT rather than $(3/2)nk_BT$

- We define the thermal energy from **k_BT**, rather than the kinetic energy, $(3/2)k_BT$.
    - If you do not like this definition, keep this factor of 3/2 in your mind.
- Then the mean (comoving) thermal energy density is equal to the mean thermal pressure in the comoving volume:

Density-weighted mean temperature, <ρT>/<ρ>

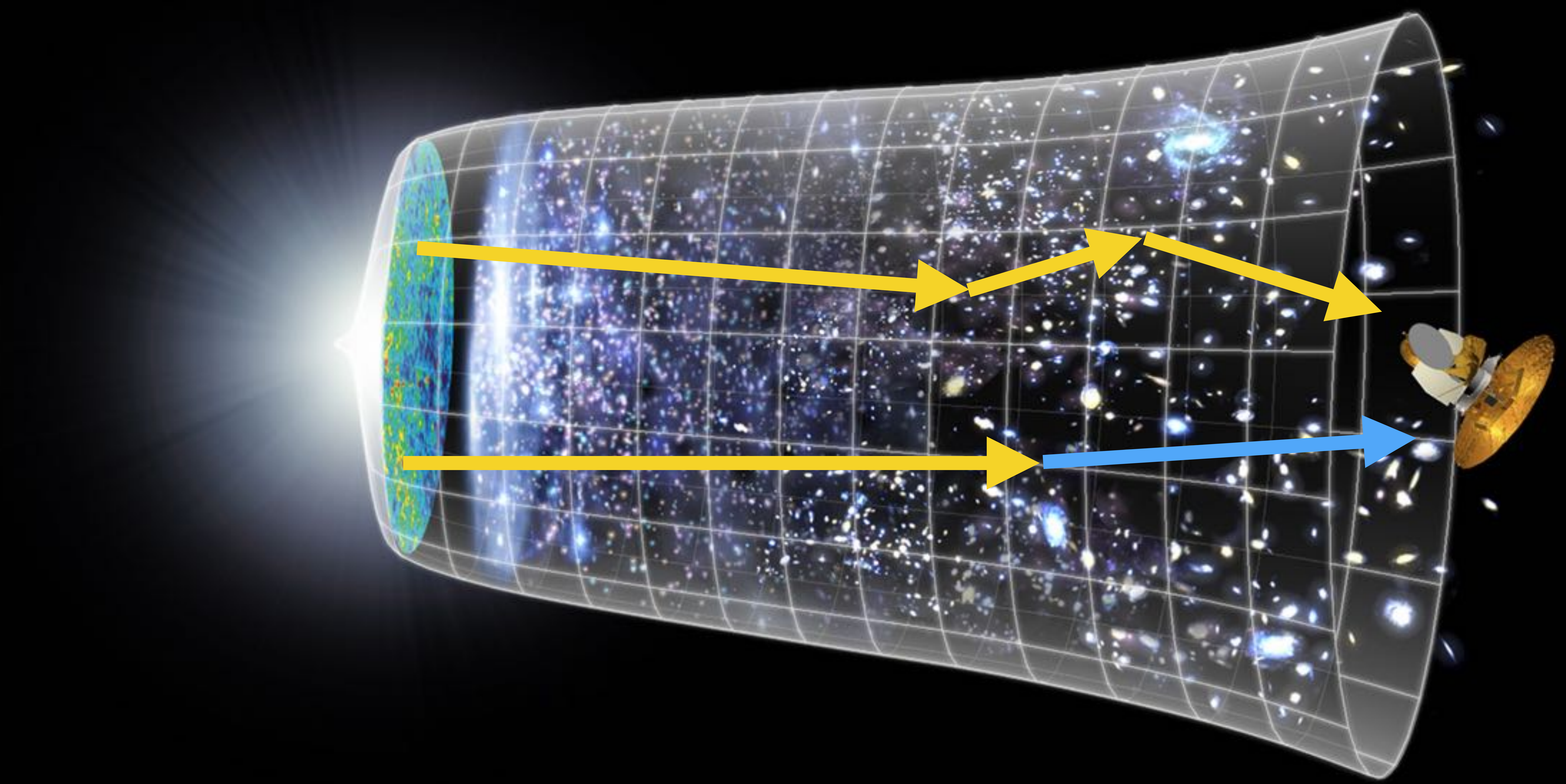
$$\Omega_{\rm th}(z) \equiv \frac{\rho_{\rm th}(z)}{\rho_{\rm crit}} = \frac{\langle P_{\rm th}(z)\rangle}{\rho_{\rm crit}(1+z)^3}, \quad = 1.78\times 10^{-8}\,\frac{k_{\rm B}\boxed{\bar{T}_\rho(z)}}{0.2\ {\rm keV}}\,\frac{\Omega_{\rm b}}{0.049}$$

where $\rho_{\rm crit} = 1.054\times 10^4\ h^2\ {\rm eV\ cm^{-3}}$ is the present-day critical energy density.

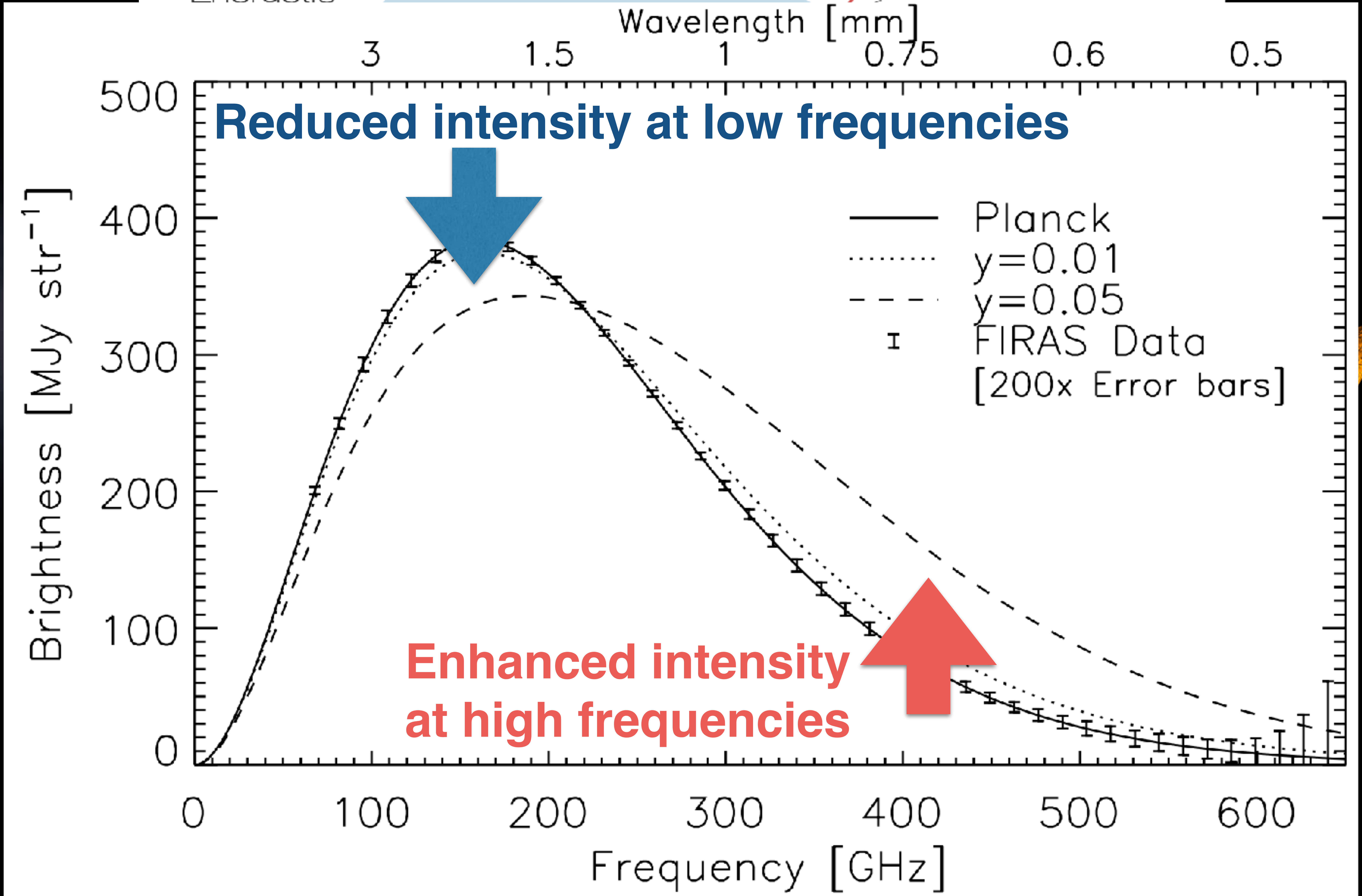
Order-of-magnitude estimate

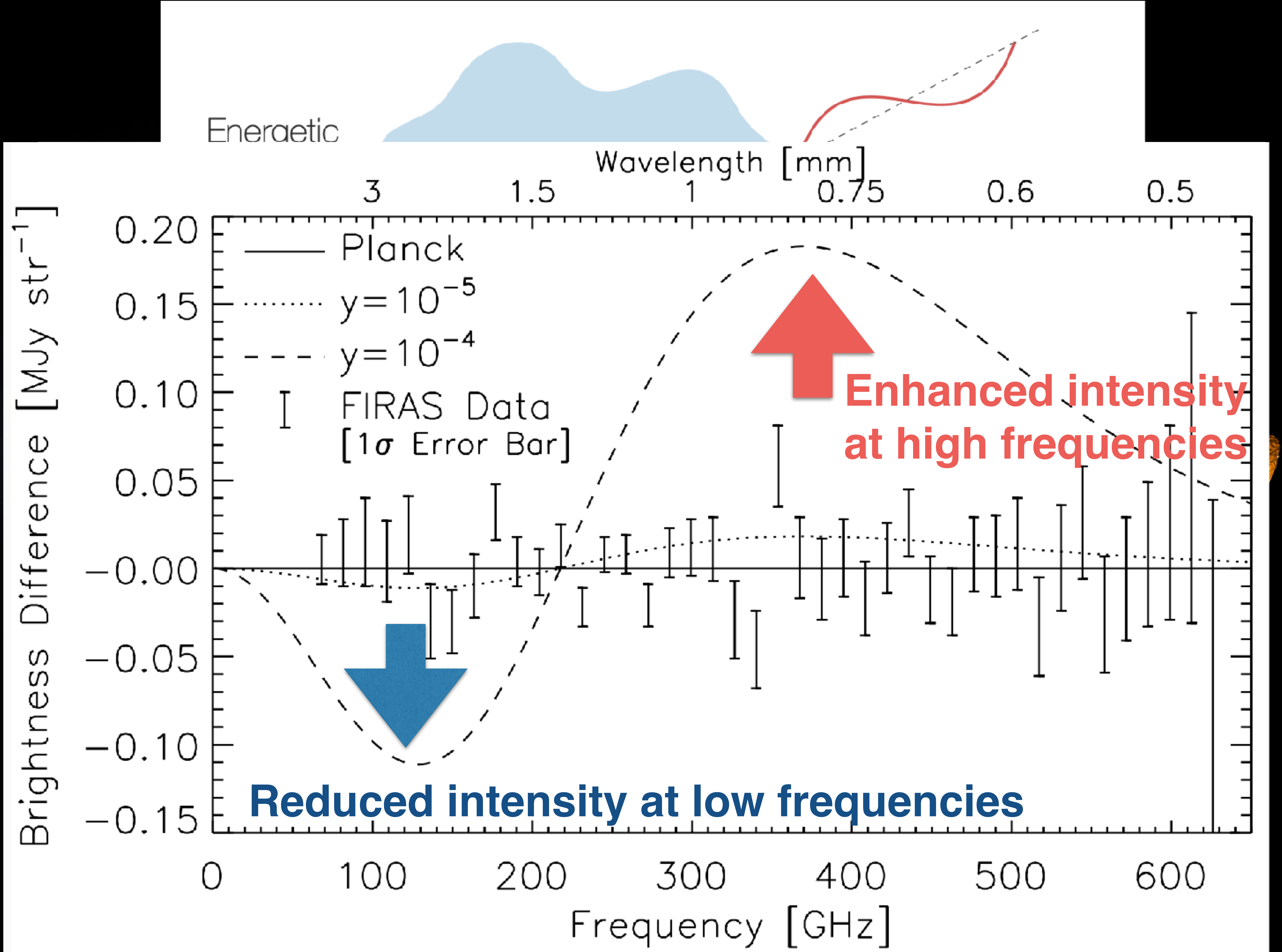
There is more than one way to do this. Here is one example.

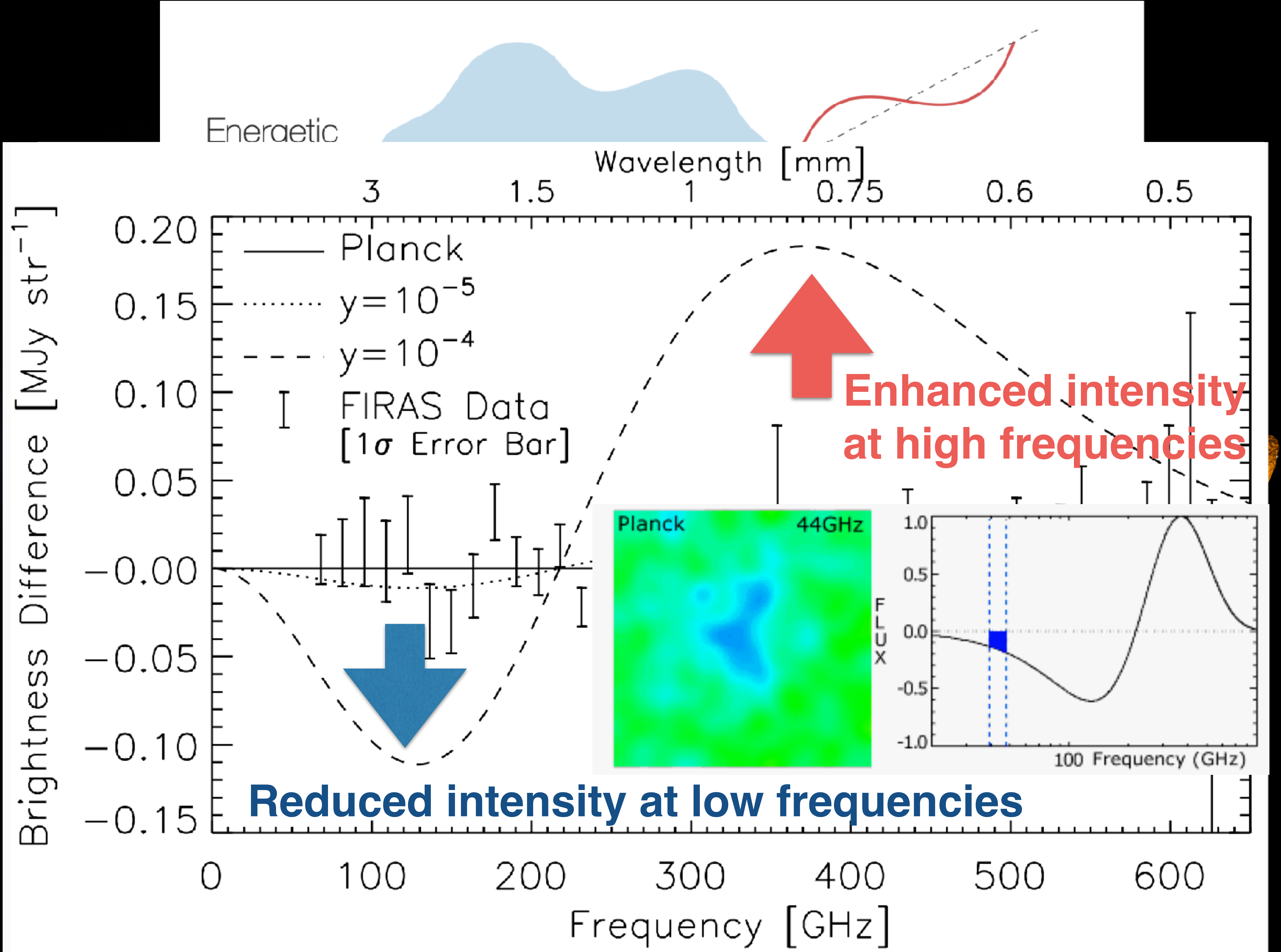
- $P_{th} = \rho_{gas}\sigma^2$, where σ^2 is some typical 1D velocity dispersion in the large-scale structure.
- $\Omega_{th} = \Omega_{gas}\sigma^2 \sim$ **$2\text{x}10^{-8}$** $(\Omega_{gas}/0.05)(\sigma/200\ \text{km/s})^2$ $$\Omega_{\rm th}(z) = 1.78 \times 10^{-8}\,\frac{k_{\rm B}\bar{T}_\rho(z)}{0.2\ {\rm keV}}\,\frac{\Omega_{\rm b}}{0.049}$$
- *Spoiler: our measurement gives* ***Ω_{th} = $(1.7\pm0.1)\text{x}10^{-8}$*** *at z=0. Not bad, but this isn't actually the right way to do it in detail.*
- OK, let's go. We use the thermal **Sunyaev-Zeldovich effect** to do this measurement.



Mroczkowski et al. (2019)


Wavelength [mm]
3
1.5
1
0.75
0.6
0.5
500
400
300
200
100
0
Brightness [MJy str^{-1}]
Reduced intensity at low frequencies
Planck
y=0.01
y=0.05
FIRAS Data
[200x Error bars]
Enhanced intensity
at high frequencies
0
100
200
300
400
500
600
Frequency [GHz]

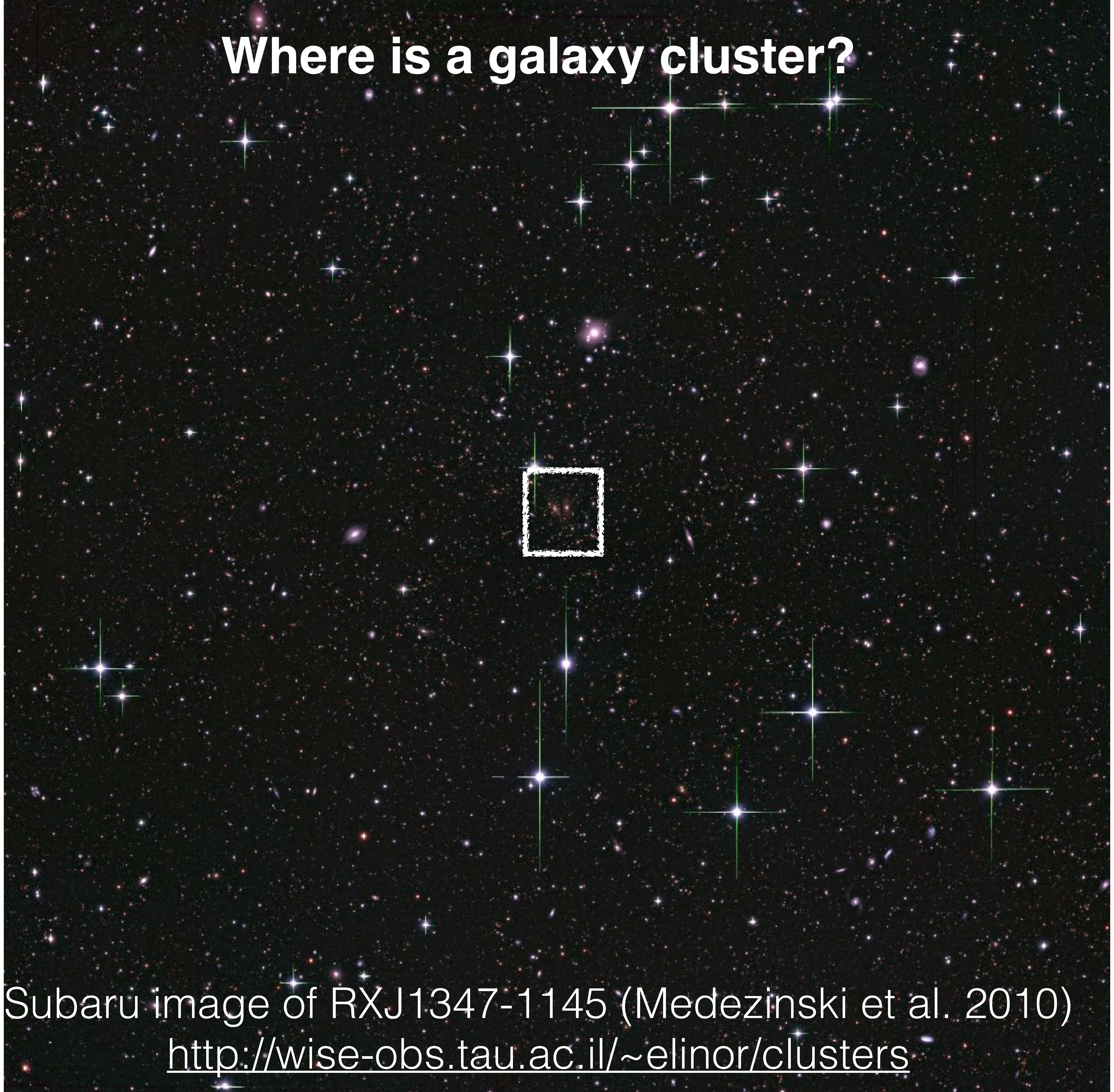

Energetic
Wavelength [mm]
3
1.5
1
0.75
0.6
0.5
Brightness Difference [MJy str^{-1}]
0.20
0.15
0.10
0.05
−0.00
−0.05
−0.10
−0.15
Planck
$y=10^{-5}$
$y=10^{-4}$
FIRAS Data
[1σ Error Bar]
Enhanced intensity
at high frequencies
Reduced intensity at low frequencies
0
100
200
300
400
500
600
Frequency [GHz]


Wavelength [mm]
3
1.5
1
0.75
0.6
0.5
Brightness Difference [MJy str^{-1}]
0.20
0.15
0.10
0.05
−0.00
−0.05
−0.10
−0.15
Planck
y=10^{-5}
y=10^{-4}
FIRAS Data
[1σ Error Bar]
Enhanced intensity at high frequencies
Reduced intensity at low frequencies
Planck
44GHz
FLUX
1.0
0.5
0.0
-0.5
-1.0
100 Frequency (GHz)
0
100
200
300
400
500
600
Frequency [GHz]

Where is a galaxy cluster?

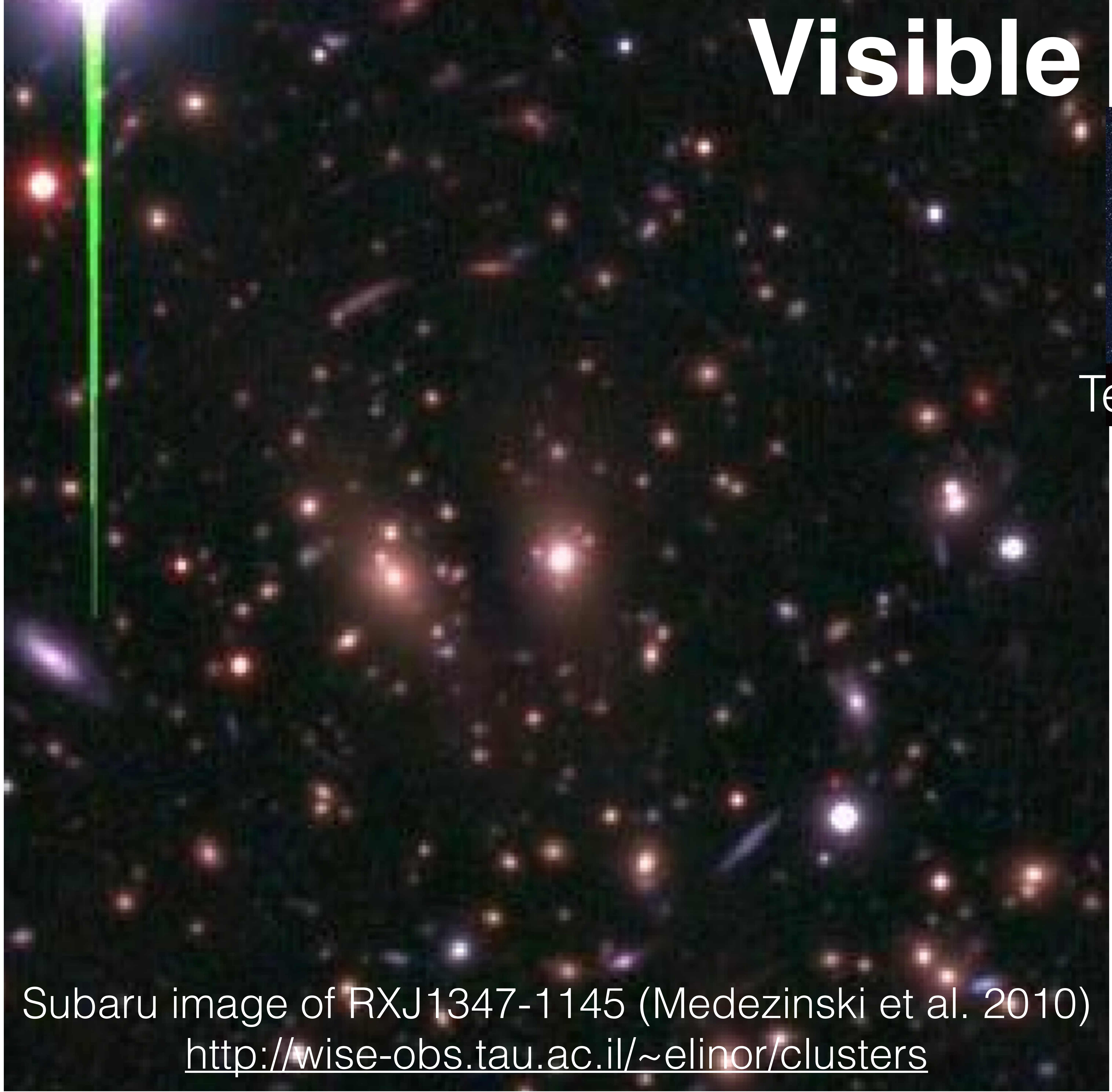
Subaru image of RXJ1347-1145 (Medezinski et al. 2010)
http://wise-obs.tau.ac.il/~elinor/clusters

Where is a galaxy cluster?



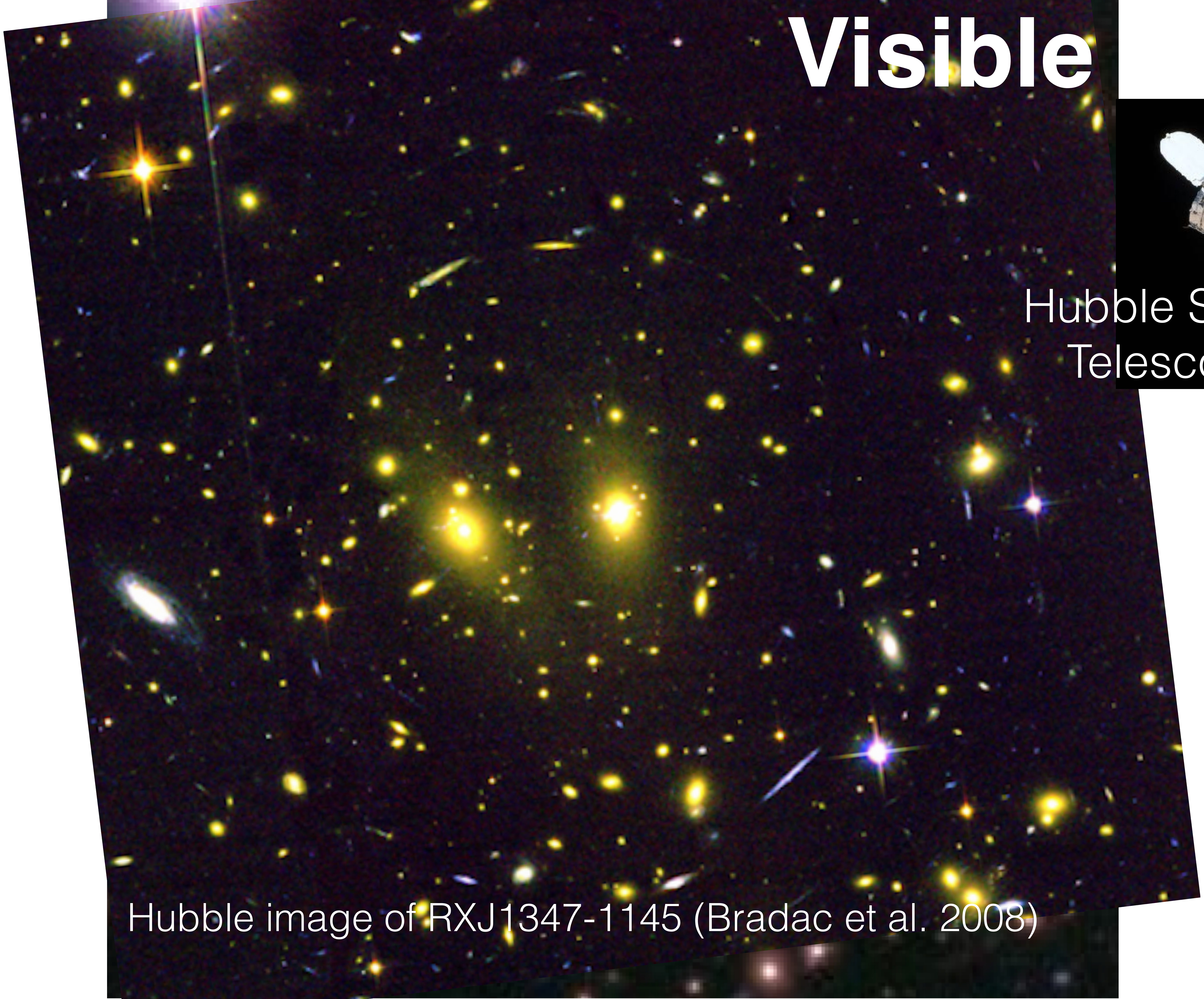
Subaru image of RXJ1347-1145 (Medezinski et al. 2010)
http://wise-obs.tau.ac.il/~elinor/clusters

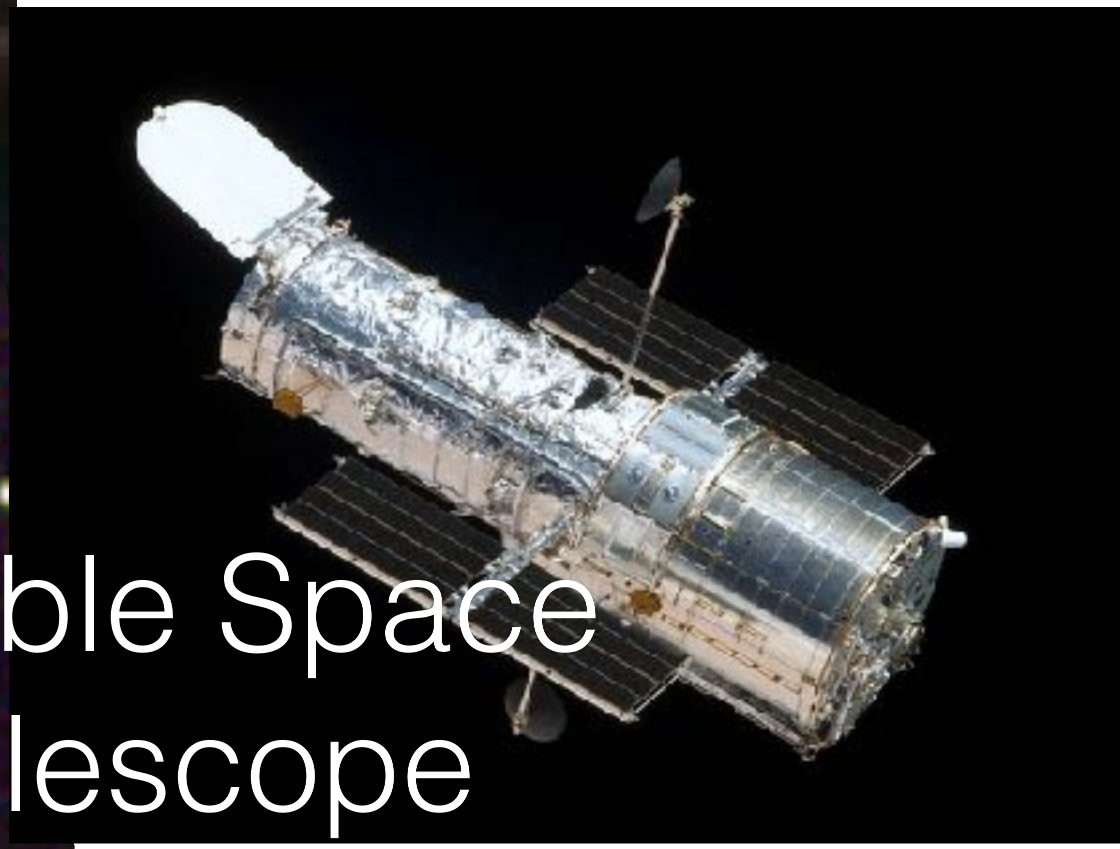
Visible



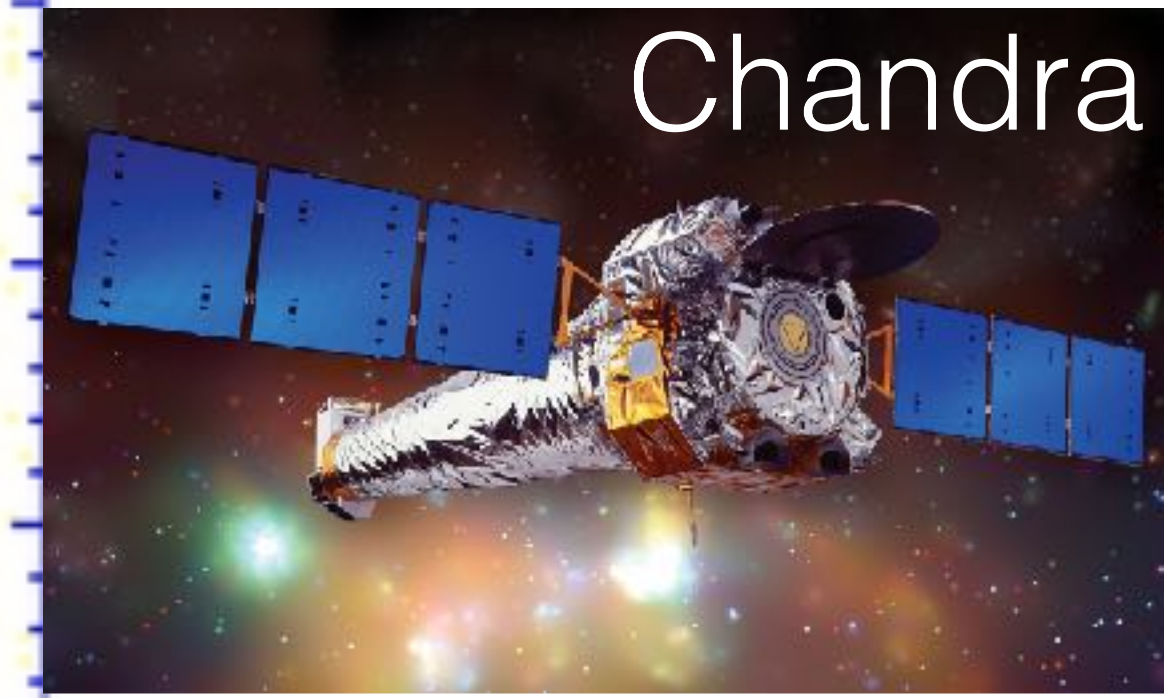
Subaru image of RXJ1347-1145 (Medezinski et al. 2010)
http://wise-obs.tau.ac.il/~elinor/clusters

Ground-based
Telescope (Subaru)


Visible
Hubble image of RXJ1347-1145 (Bradac et al. 2008)


Hubble Space
Telescope

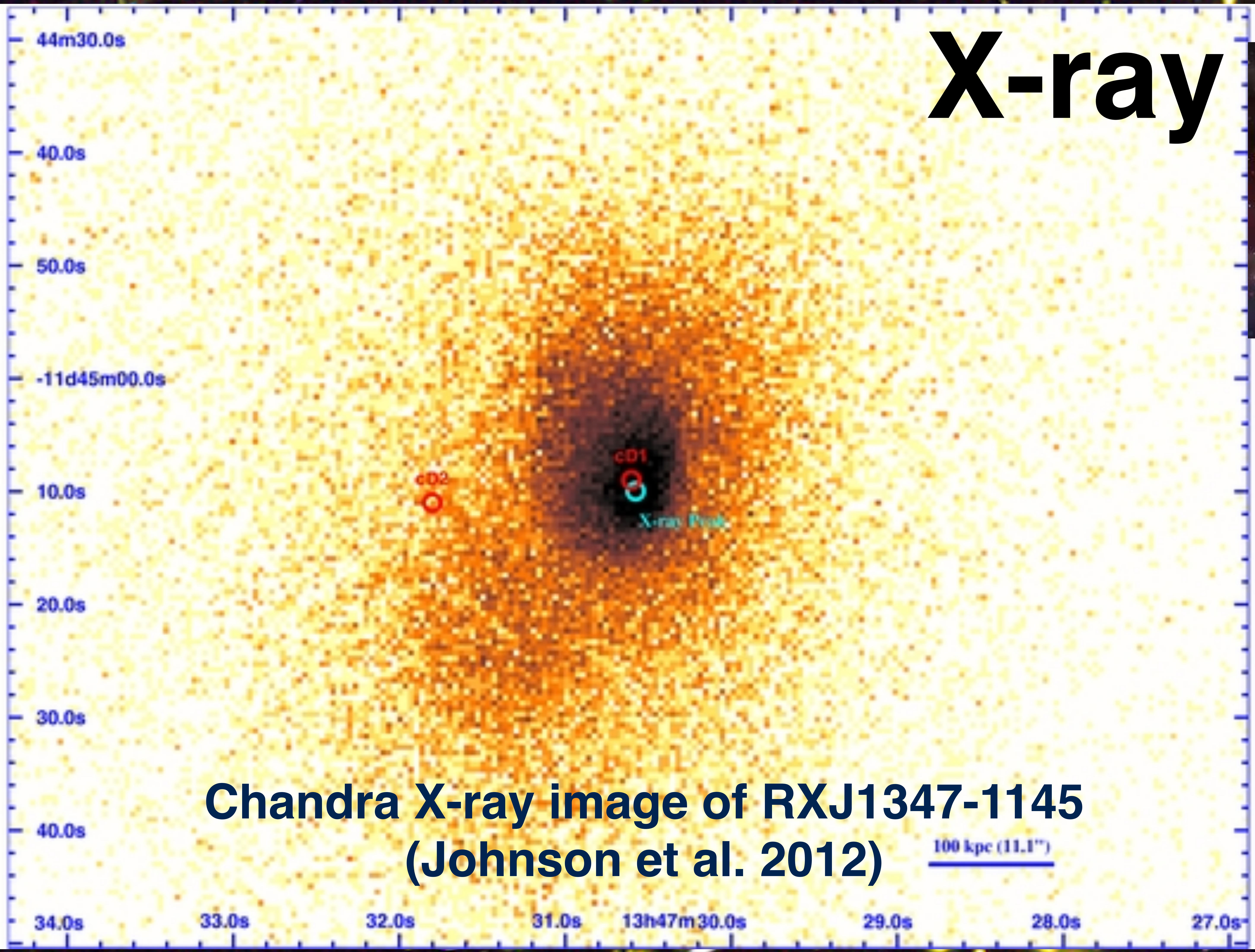
X-ray



X-ray:

- hot gas ($10^{7–8}$ K)
- spectroscopic T_X
- Intensity ~ $n_e^2 L$

$$I_X = \int dl\ n_e^2 \Lambda(T_X)$$



Chandra X-ray image of RXJ1347-1145 (Johnson et al. 2012)

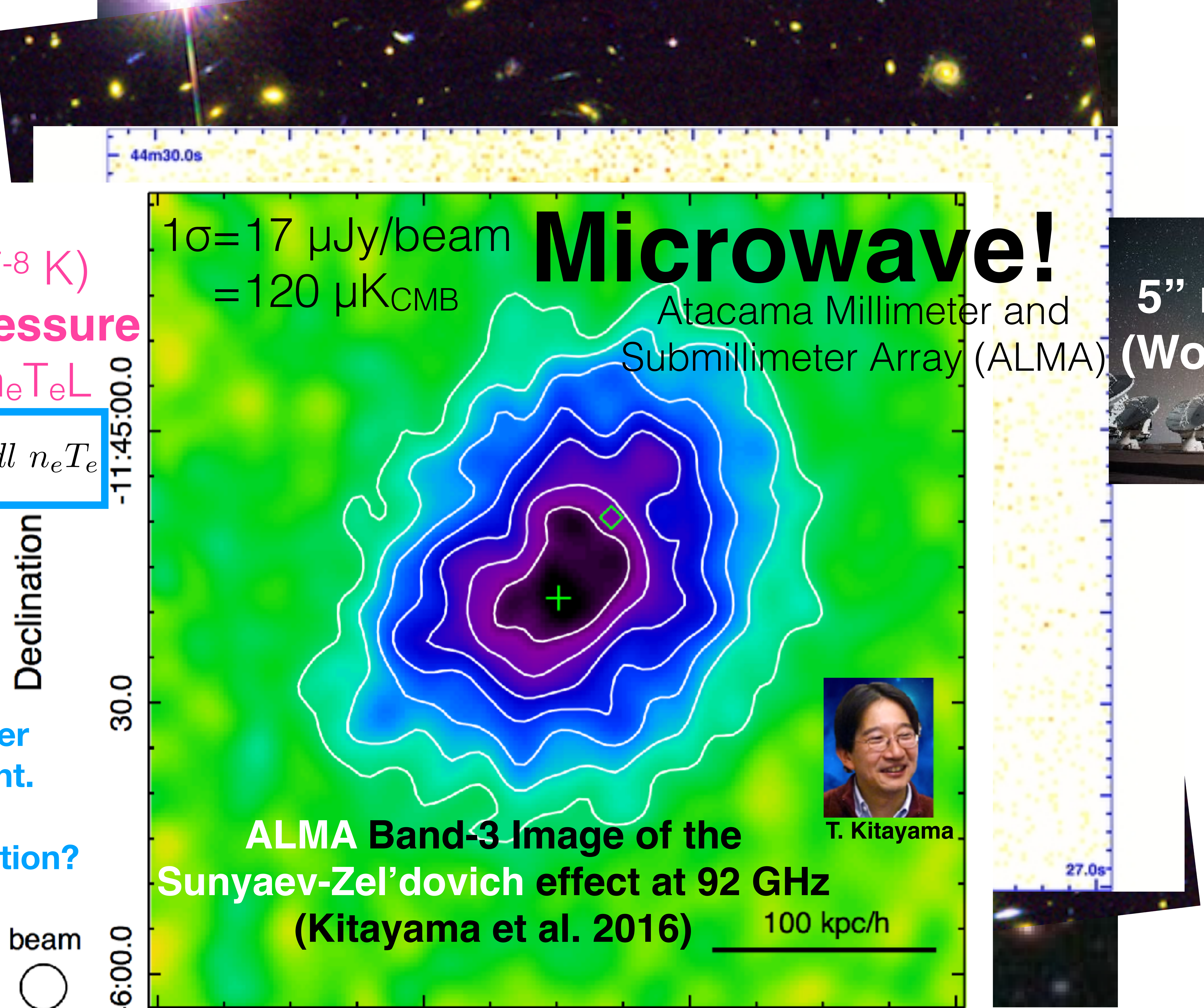
SZ

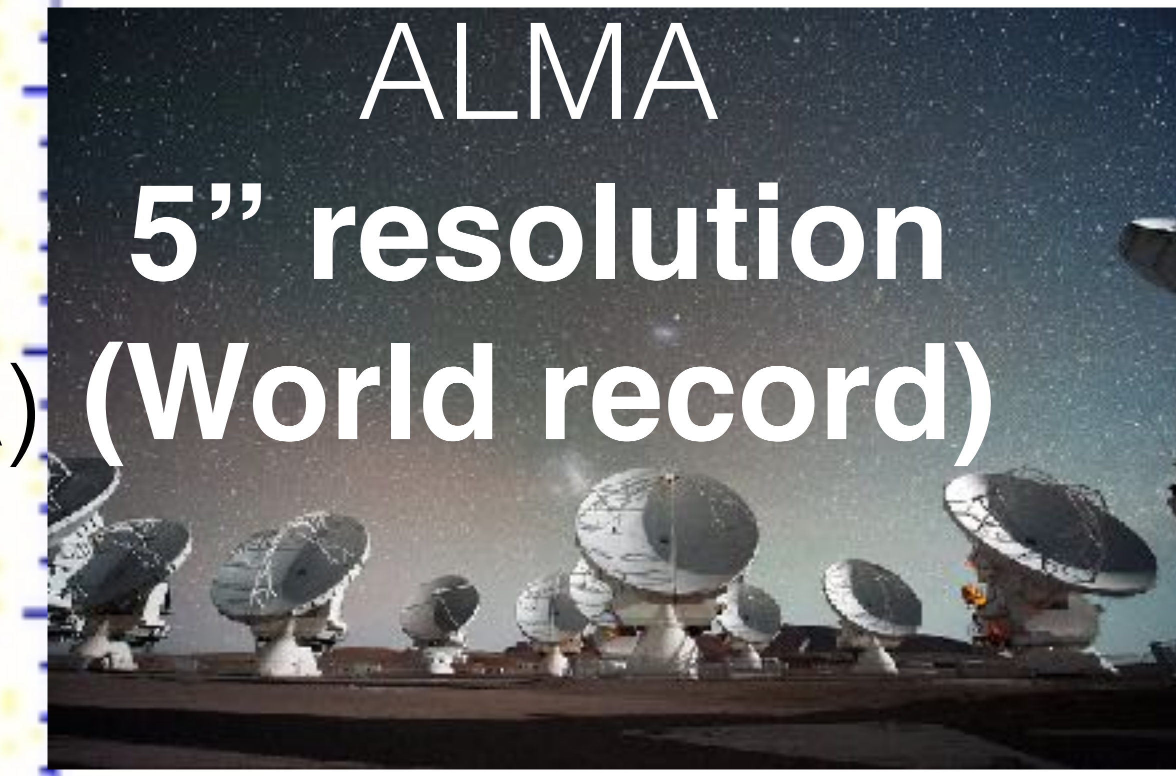
- hot gas ($10^{7\text{-}8}$ K)
- **electron pressure**
- Intensity ~ $n_e T_e L$

$$I_{SZ} = g_\nu \frac{\sigma_T k_B}{m_e c^2} \boxed{\int dl\ n_e T_e}$$

Integrated over the line of sight.

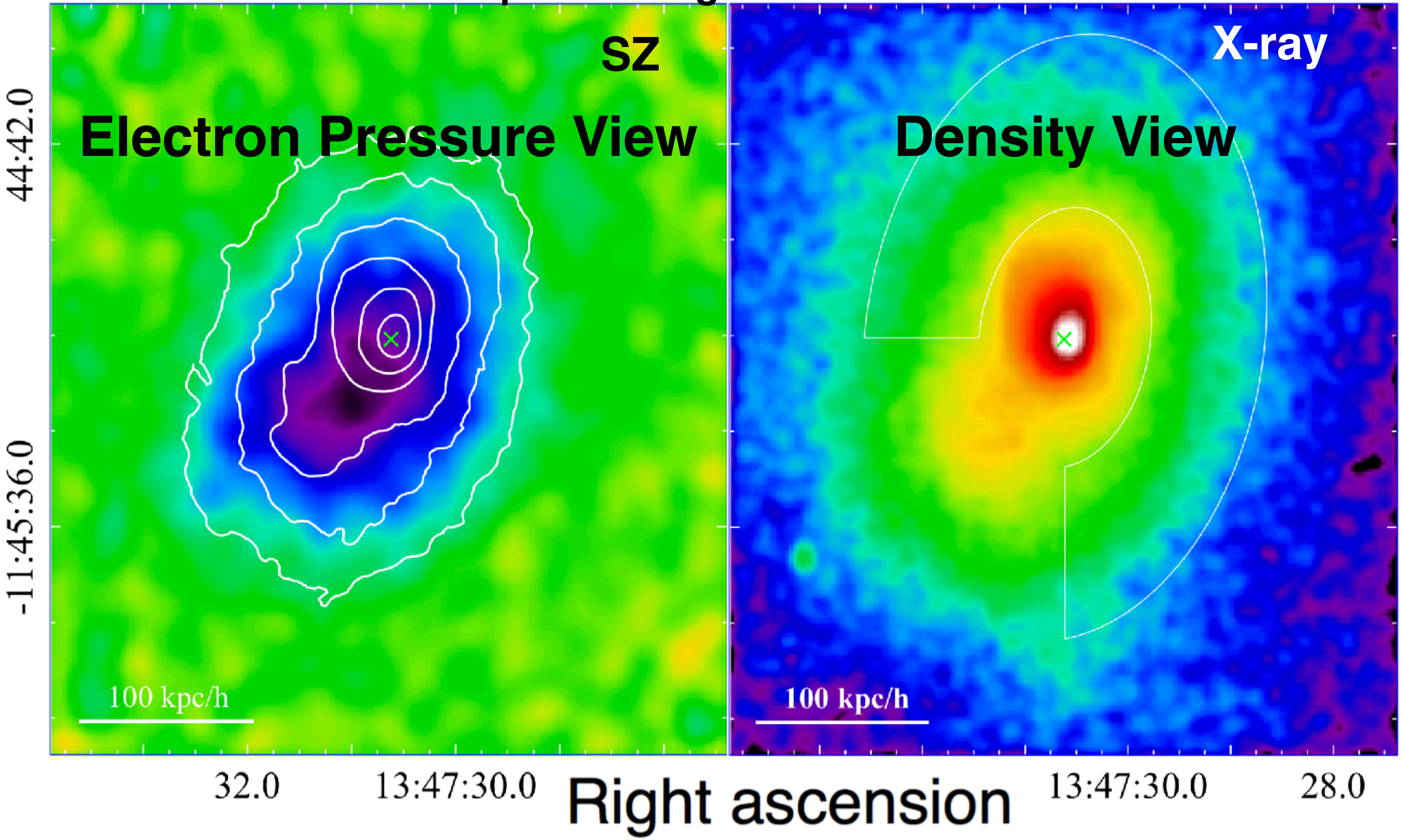
Redshift information?



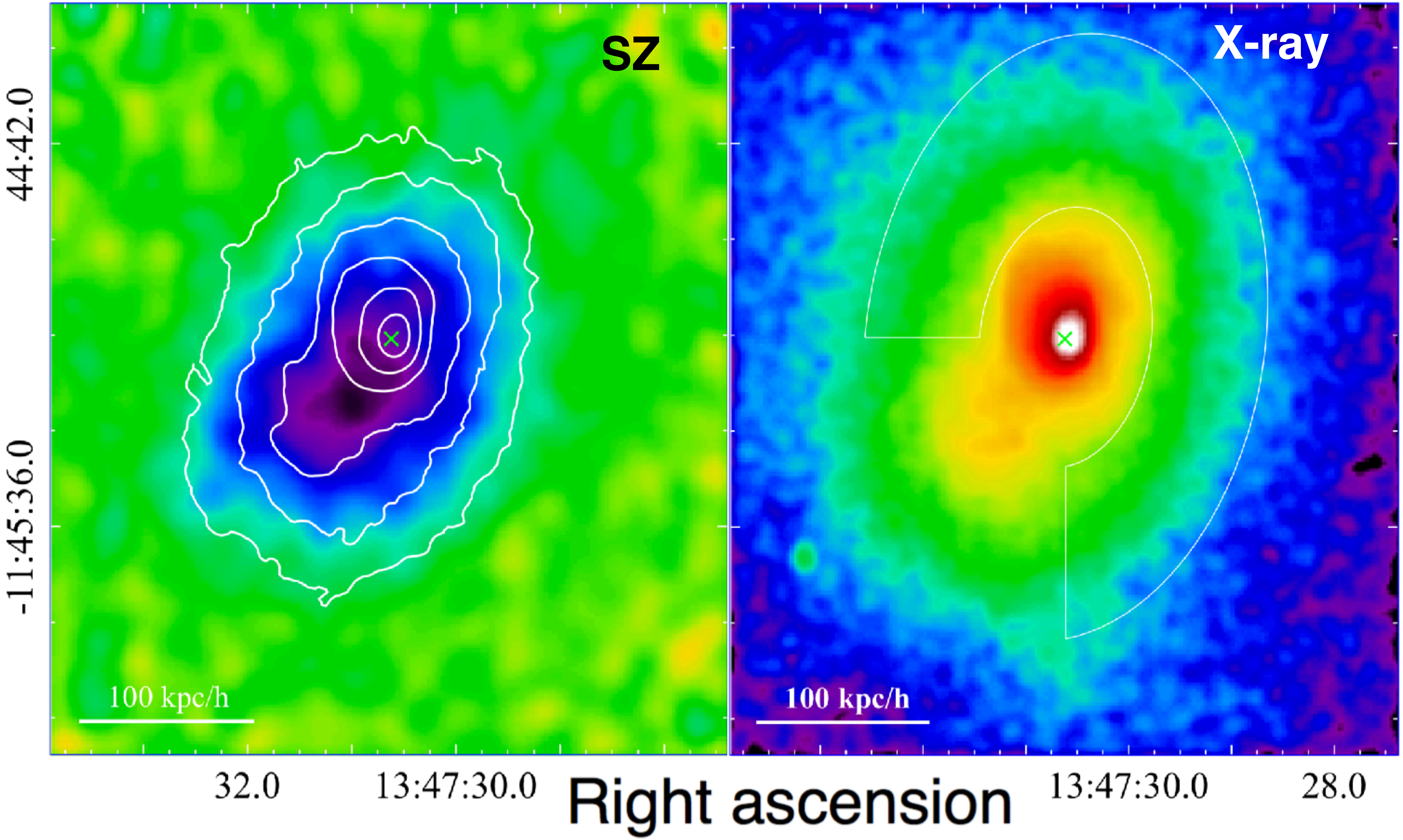


X-rays vs Microwave: Similar, but different

This is the first time to compare SZ and X-ray images at a comparable angular resolution.

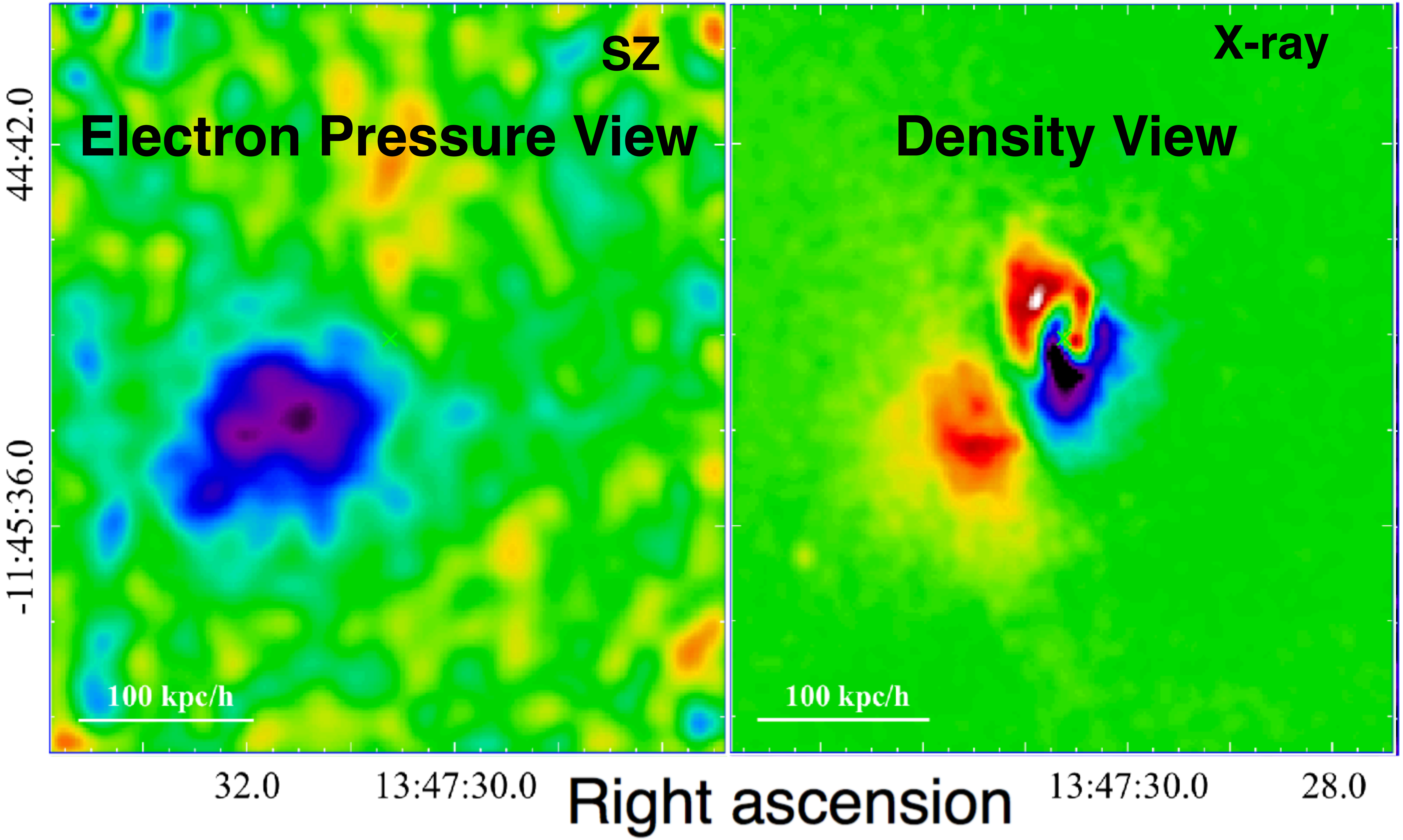


Let’s subtract a smooth component



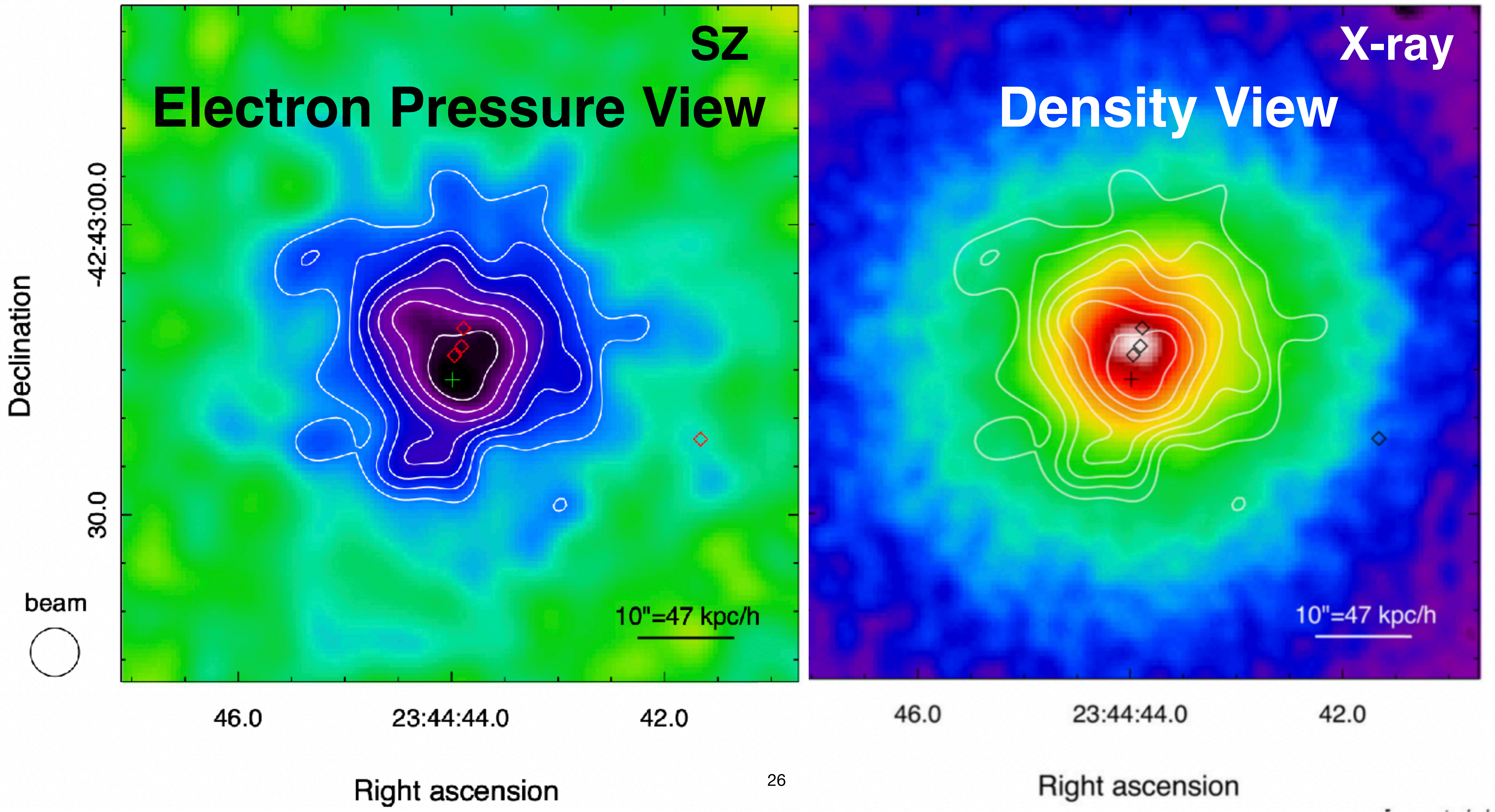
Ueda et al. (2018)

Let's subtract a smooth component

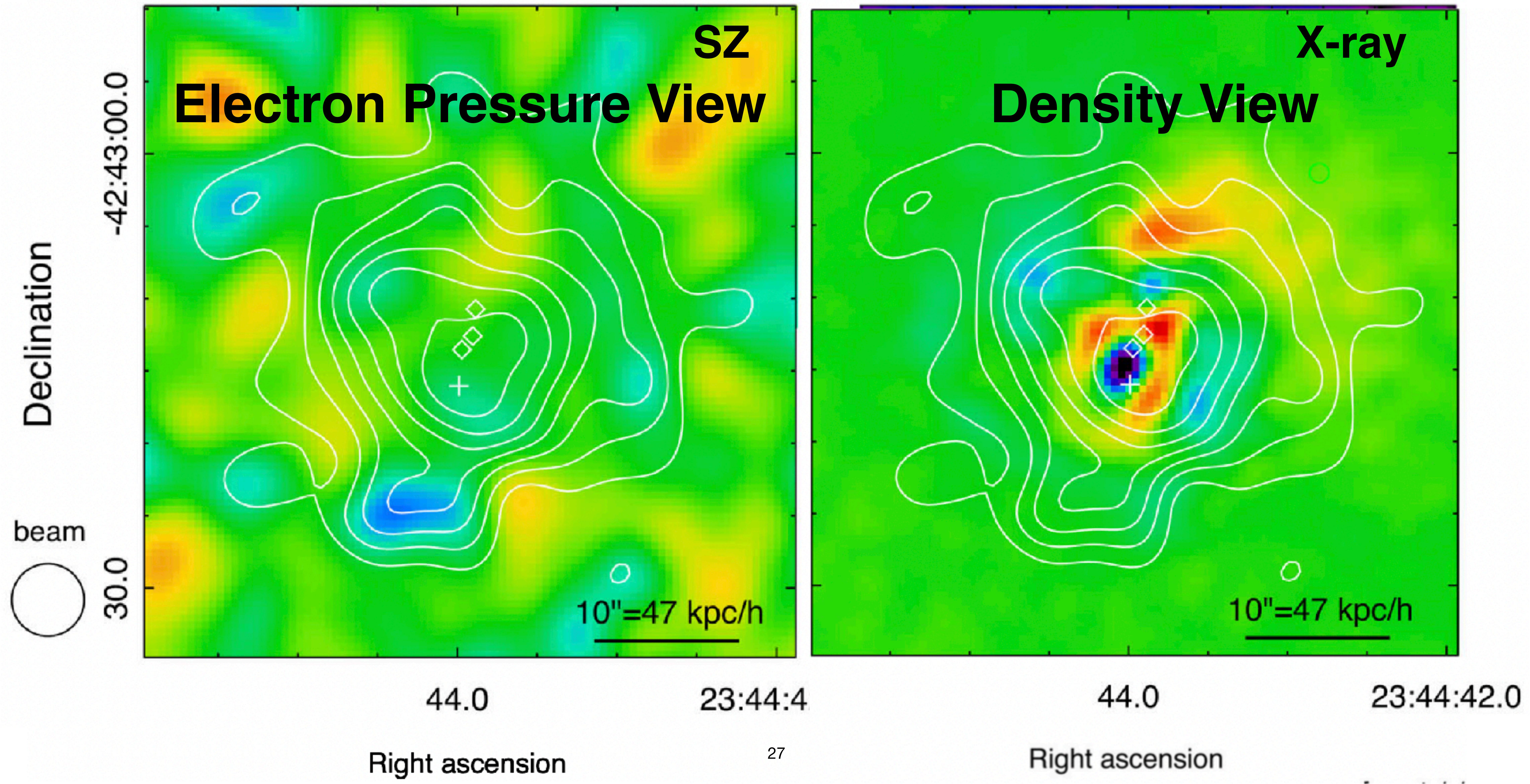


Kitayama et al. (2020)

Another example: Phoenix Cluster (z=0.597)



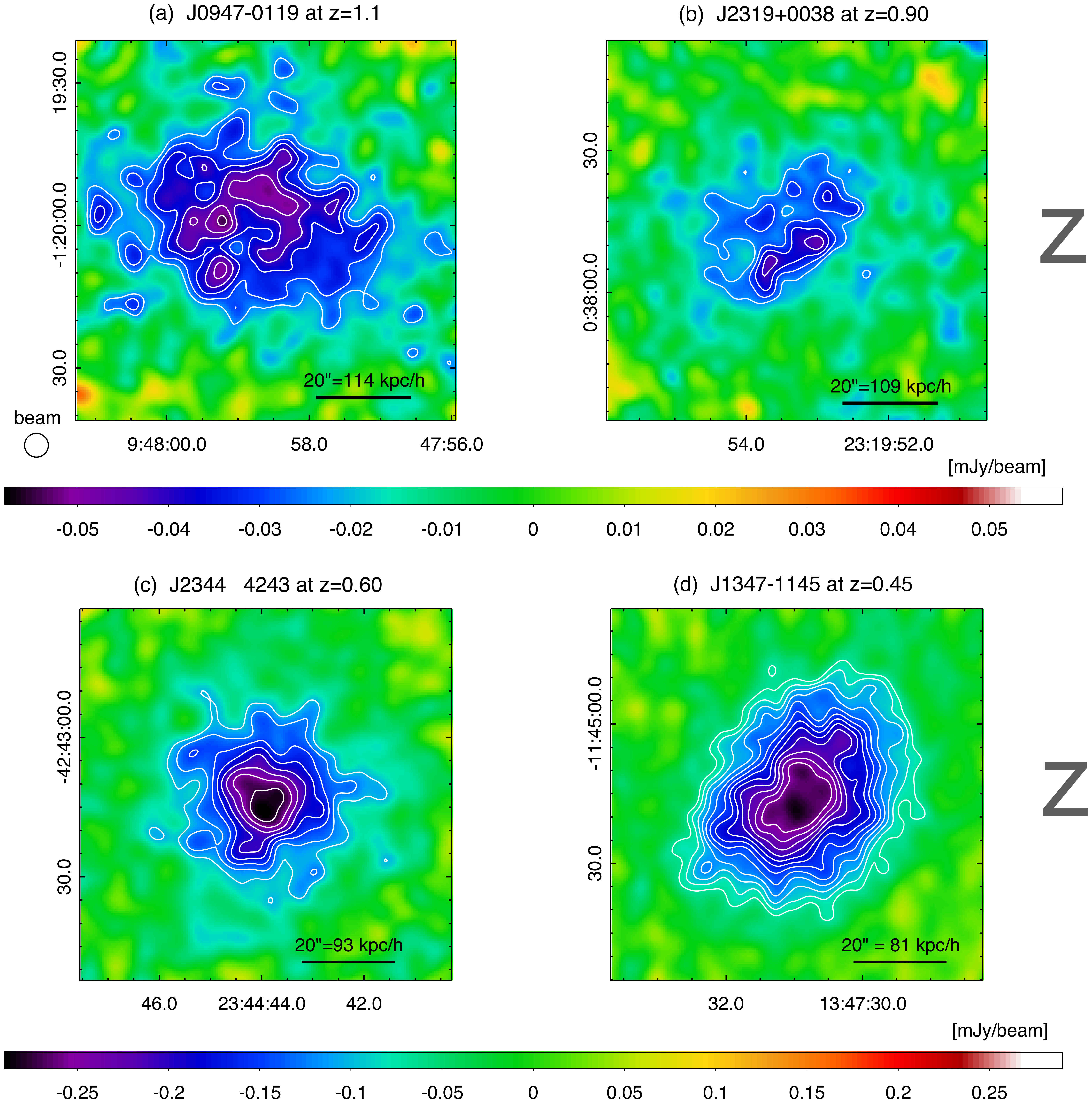
Another example: Phoenix Cluster (z=0.597)



Gallery of the highest-resolution SZ images

z=1.1

z=0.9



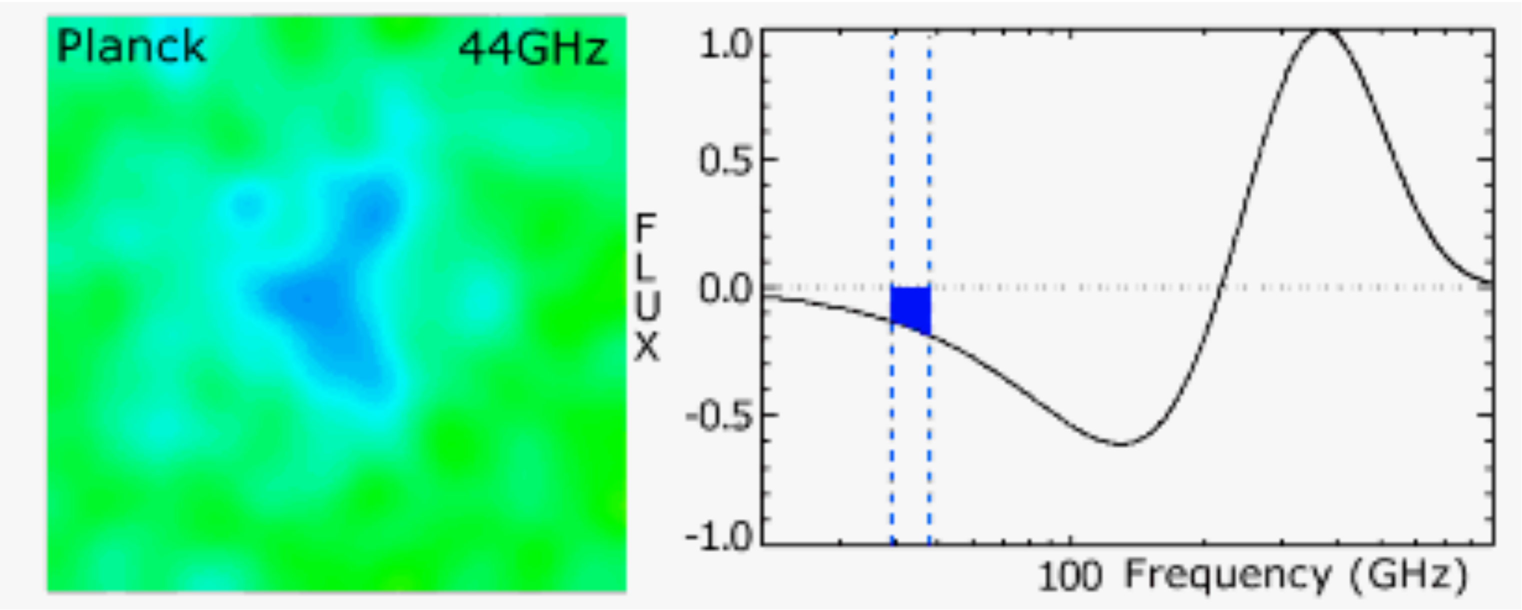
z=0.6

z=0.45

Chiang, Makiya, Ménard, EK, ApJ, 902, 56 (2020)

Q1: How hot is the large-scale structure of the Universe?

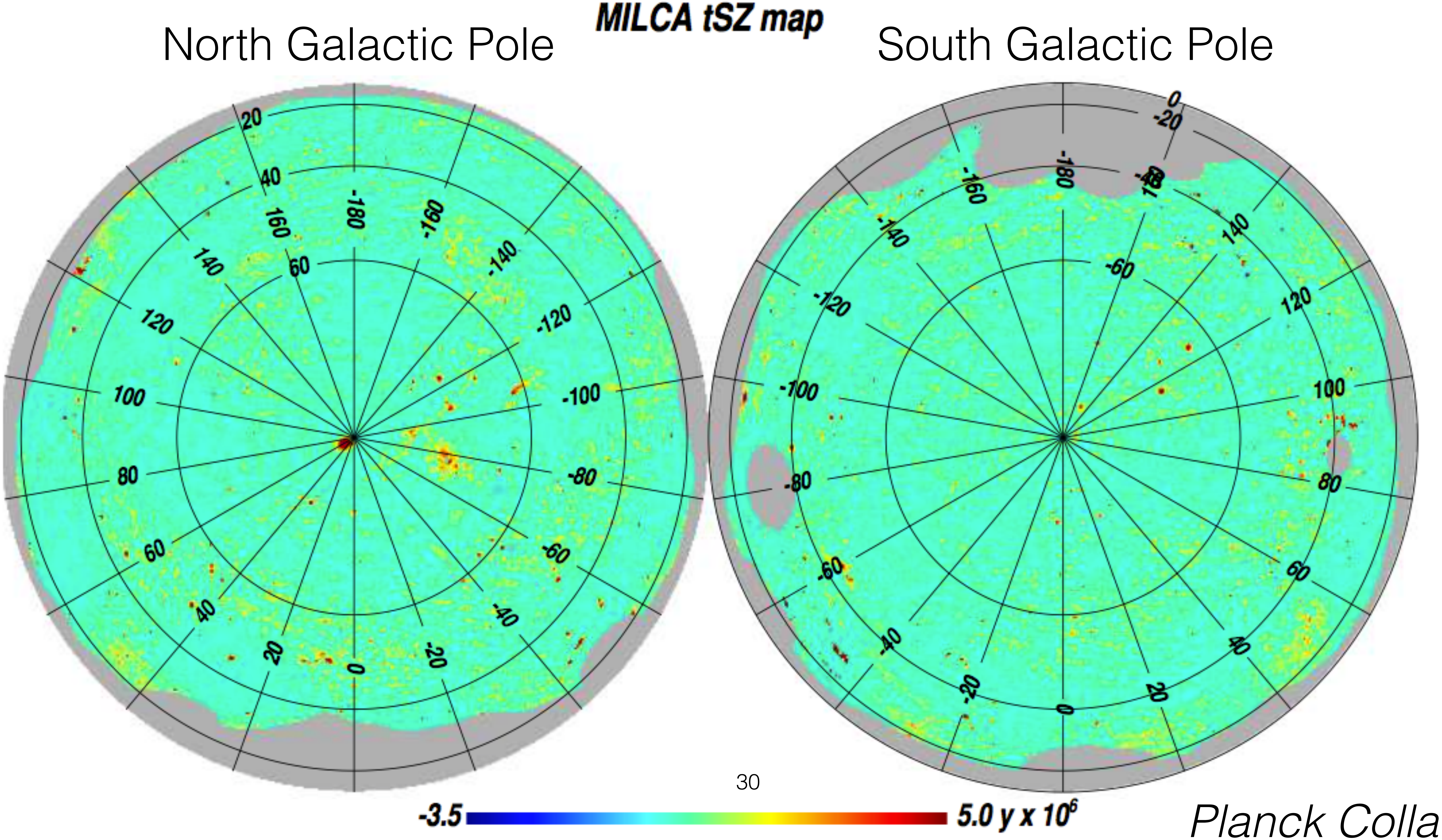
Create a full-sky SZ map using the multi-frequency data!



Full-sky Electron Pressure Map

North Galactic Pole

South Galactic Pole



Planck Collaboration

Makiya, Ando & EK (2018)

The Limitation of the SZ data

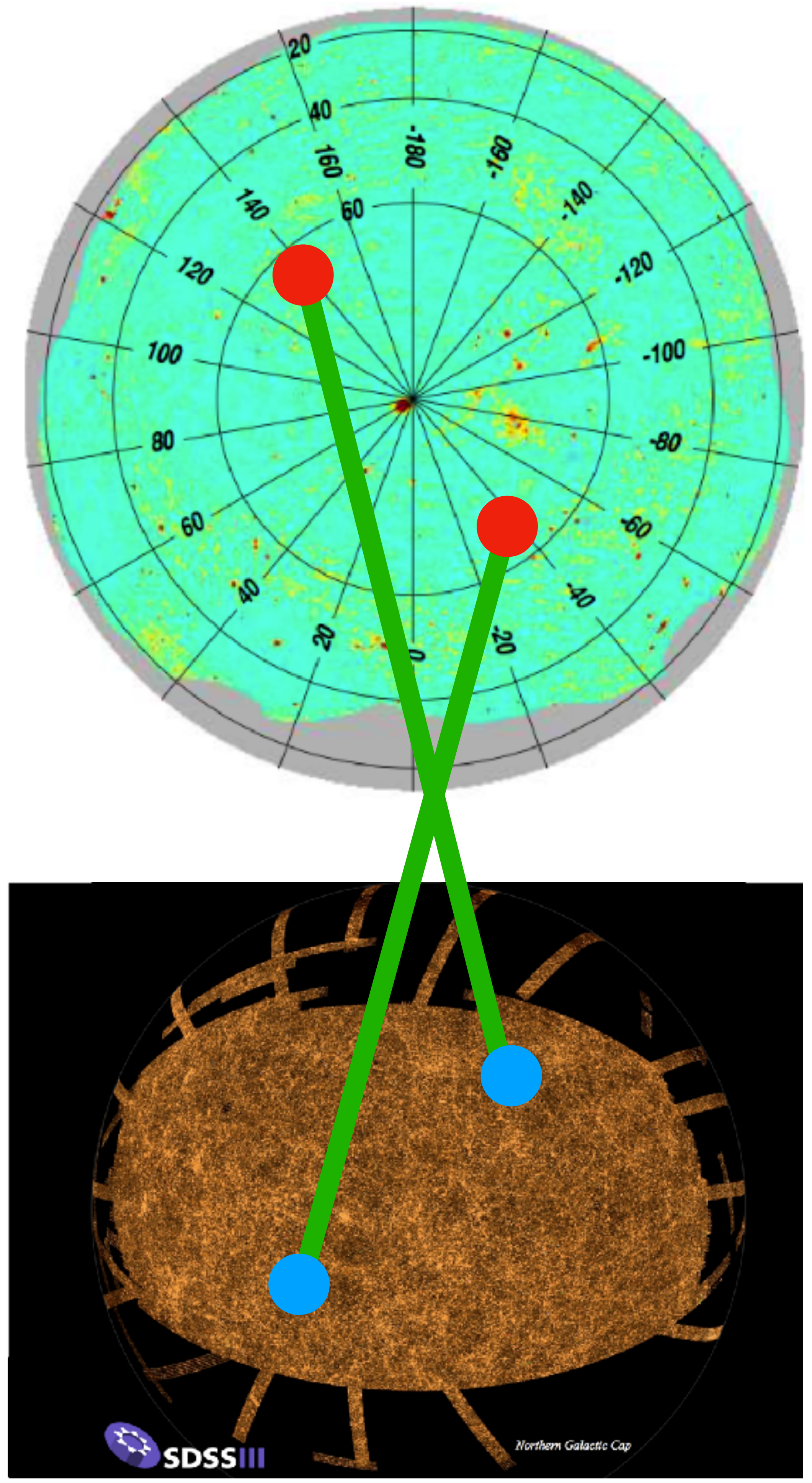
The need for “Tomography”

- This map gives us all the hot electron pressure **in projection**.
    - No redshift information.
- We can overcome this limitation by cross-correlating the SZ map with the locations of galaxies with the known redshifts => **the SZ tomography**.

See Chiang's versatile cross-correlation tool, "*Tomographer*":
http://tomographer.org

The data used

Planck and SDSS



- **For the SZ: Multi-frequency component separation**
  - The Planck High-frequency Instrument (HFI) data at 100, 143, 217, 353, 545 and 857 GHz.
  - In addition, we use the IRAS data at 3 and 5 THz for better separating the cosmic infrared background (CIB; from dusty galaxies).
- **For the galaxies and quasars: 2 million redshifts at $0<z<3$**
  - The SDSS main, SDSS-III/BOSS, and SDSS-IV/eBOSS data sets.

The basic methodology: A heuristic description

Vikram, Lids & Jain (2017)

- We focus on the clustering signal at large scales (the so-called “2-halo term” of clustering).
  - Ignore non-linear clustering inside dark matter halos, but focus only on clustering between distinct halos.
- In this limit, we can write $P_e = \langle P_e \rangle (1+b_y\delta_{matter})$ and $n_{gal} = \langle n_{gal} \rangle (1+b_{gal}\delta_{matter})$. Thus, the cross-correlation yields

What we measure from the cross-correlation

What we want in the end

$$\frac{\langle P_{\rm e} n_{\rm gal}\rangle}{\underbrace{b_{\rm gal}\langle n_{\rm gal}\rangle}_{\text{Measured from the auto galaxy correlation}}\underbrace{\langle \delta_{\rm matter}\delta_{\rm matter}\rangle}_{\text{From the }\Lambda\text{CDM model}}} = \boxed{b_y\boxed{\langle P_{\rm e}\rangle}}$$

The first key deliverable

Chiang et al. (2020)

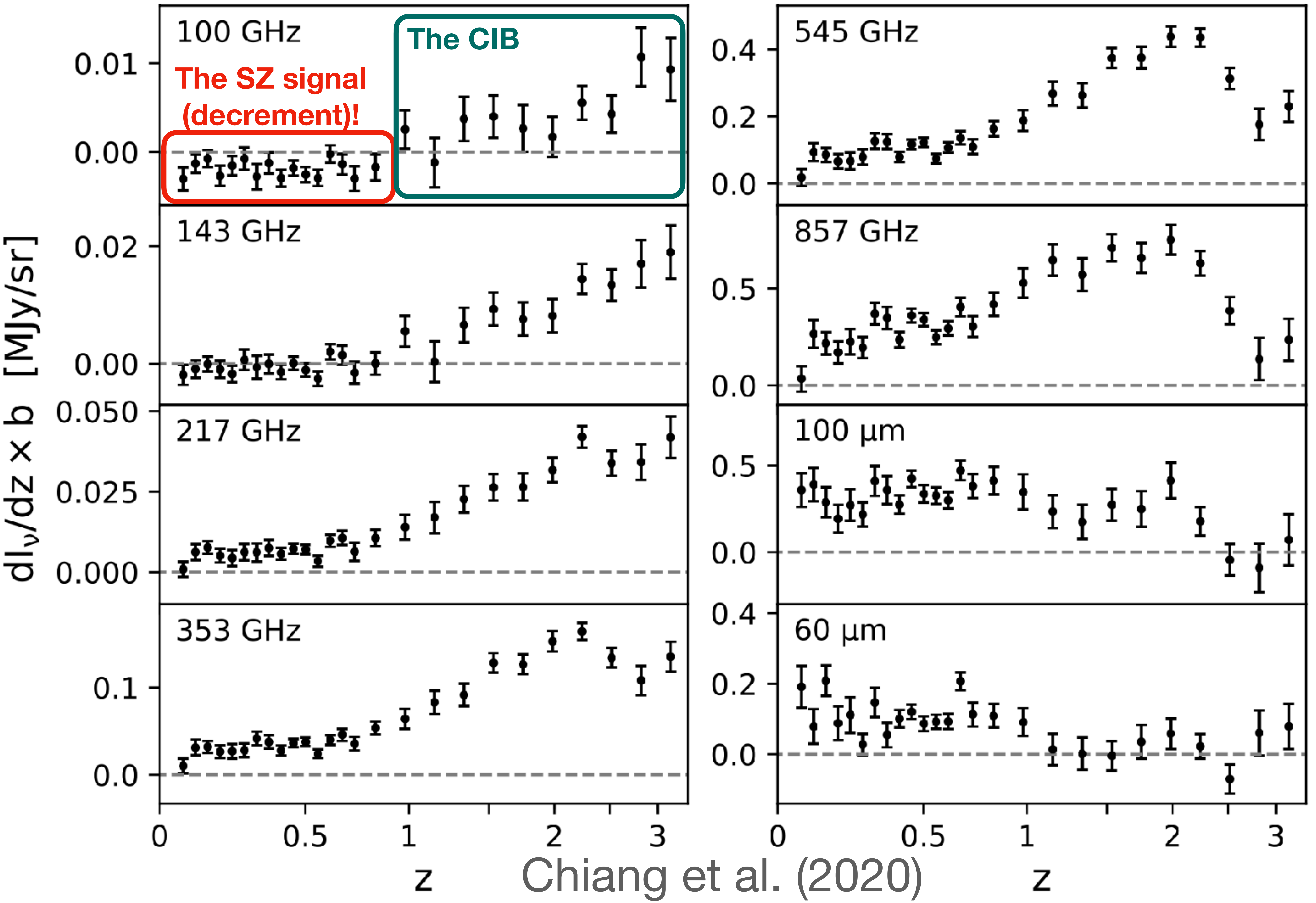
How the measurements look

To show that we are in the “linear” regime

- The data within the grey band are used for the analysis, where the ratio is a constant, justifying the extraction of the single constant amplitude in each z bin.

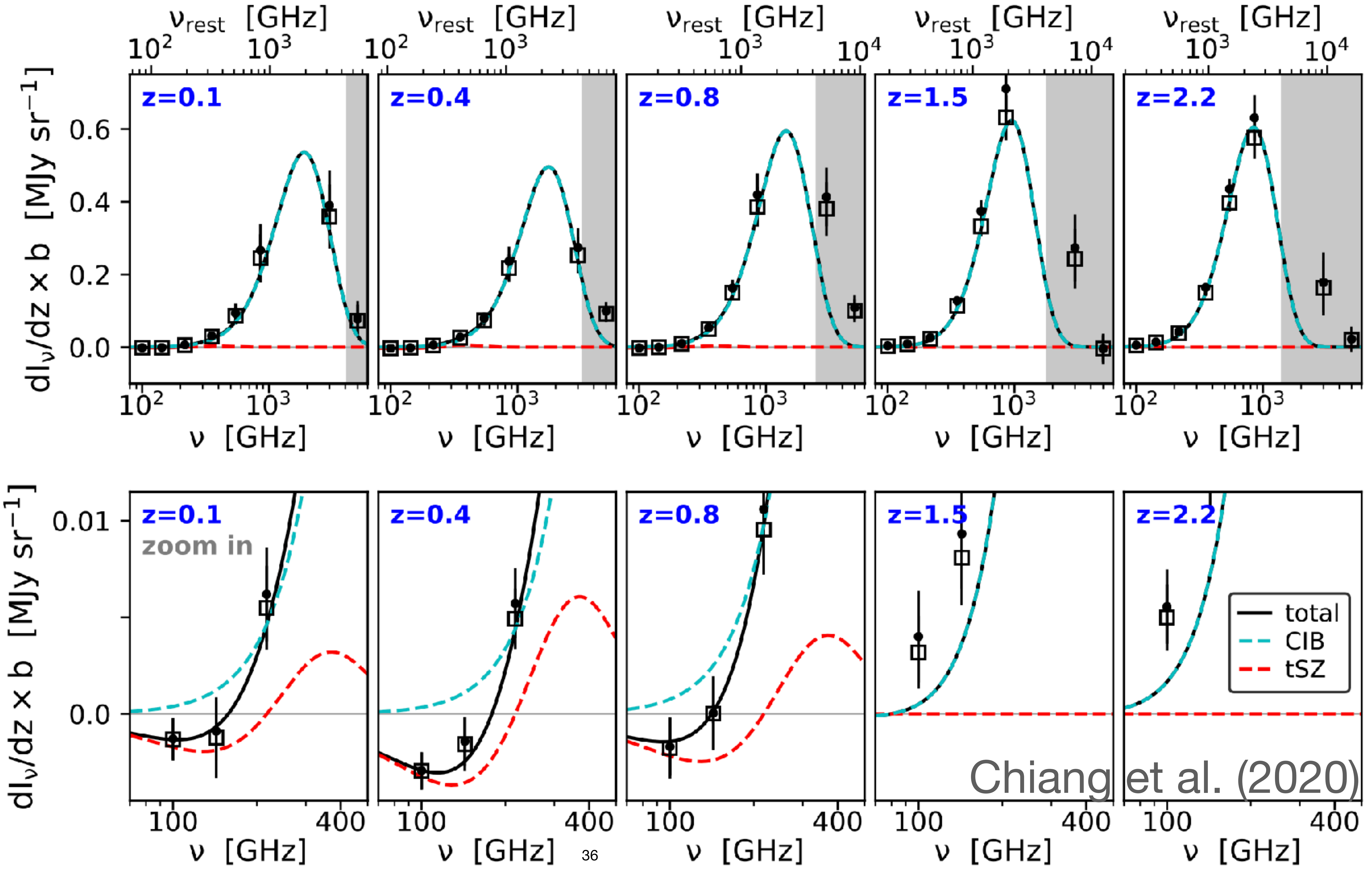
The Planck/IRAS-SDSS cross-correlations

The need for the multi-component fits (SZ+CIB).



Chiang et al. (2020)

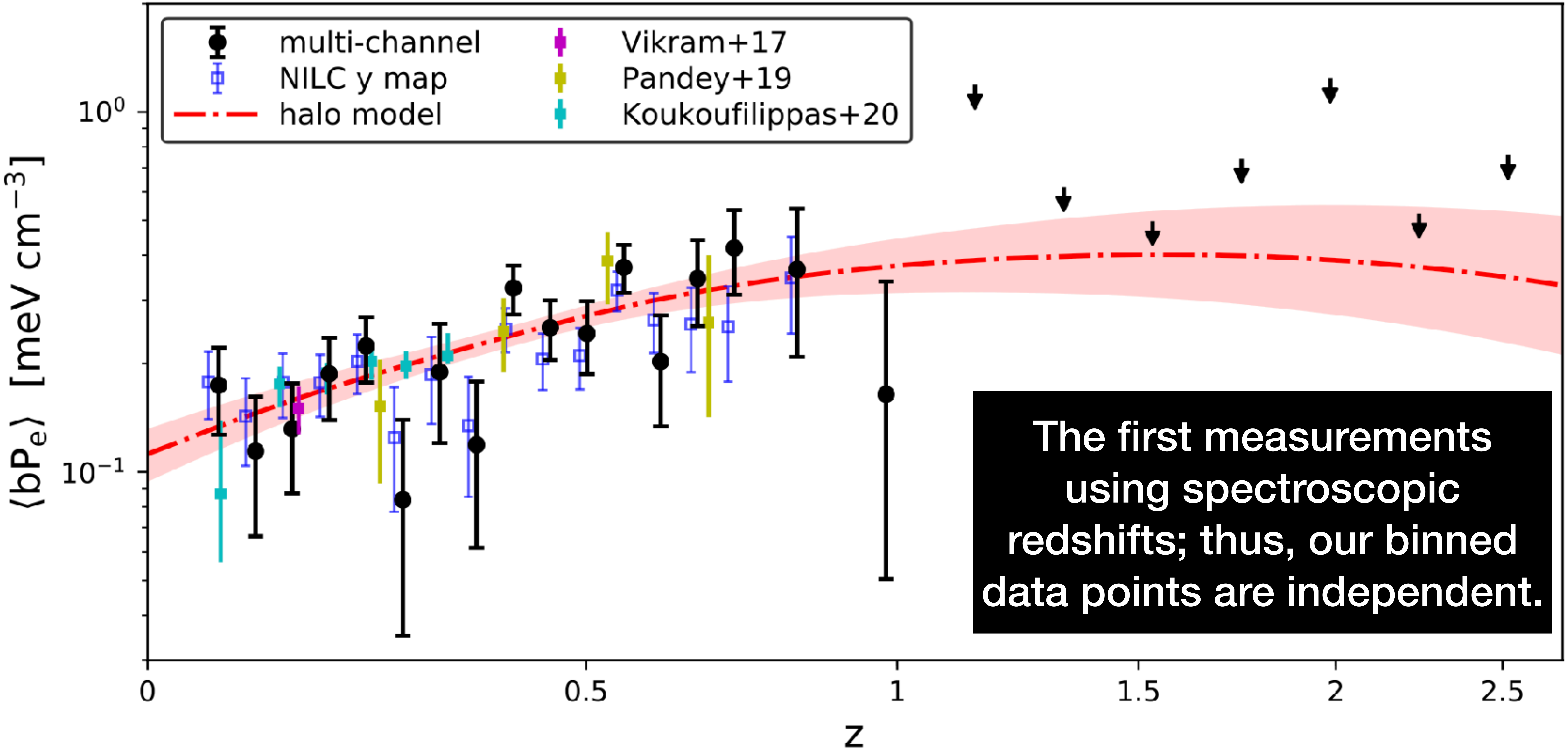
Tomography of the SED of not only SZ, but also CIB!



Chiang et al. (2020)

The first main result: Model-independent

Bias-weighted mean electron pressure of the Universe!



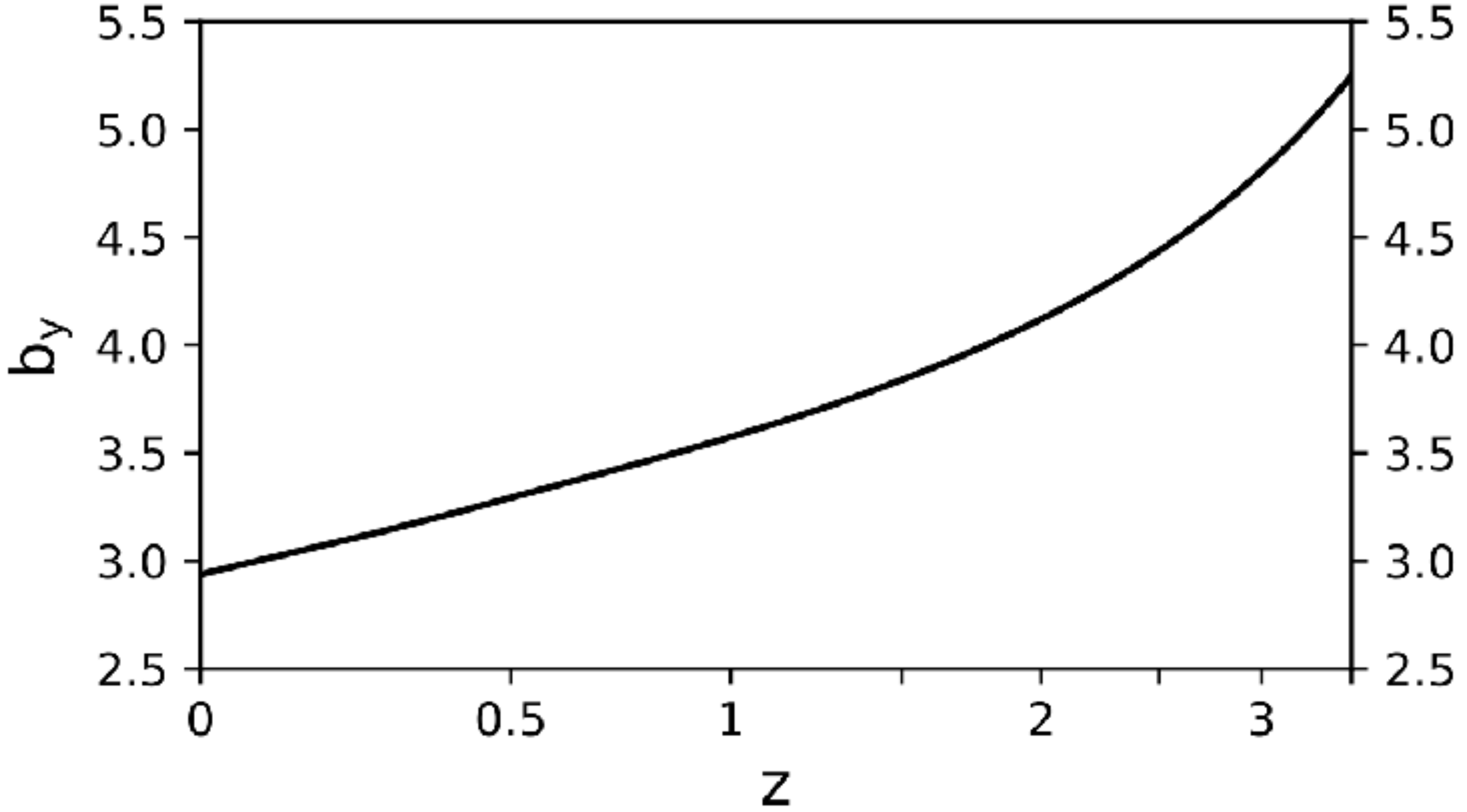
The first measurements using spectroscopic redshifts; thus, our binned data points are independent.

<bP$_e$> -> <P$_e$>

Debiasing by the physical model

- To get the mean pressure, we need to “de-bias” <bP$_e$> = b$_y$<P$_e$>. This can be done by computing and dividing by

$$b_y(z) = \frac{\langle bP_\mathrm{e}\rangle}{\langle P_\mathrm{e}\rangle} = \frac{\int dM\,\frac{dn}{dM}\,M^{5/3+\alpha_P}\,b_\mathrm{halo}(M,z)}{\int dM\,\frac{dn}{dM}\,M^{5/3+\alpha_P}}$$



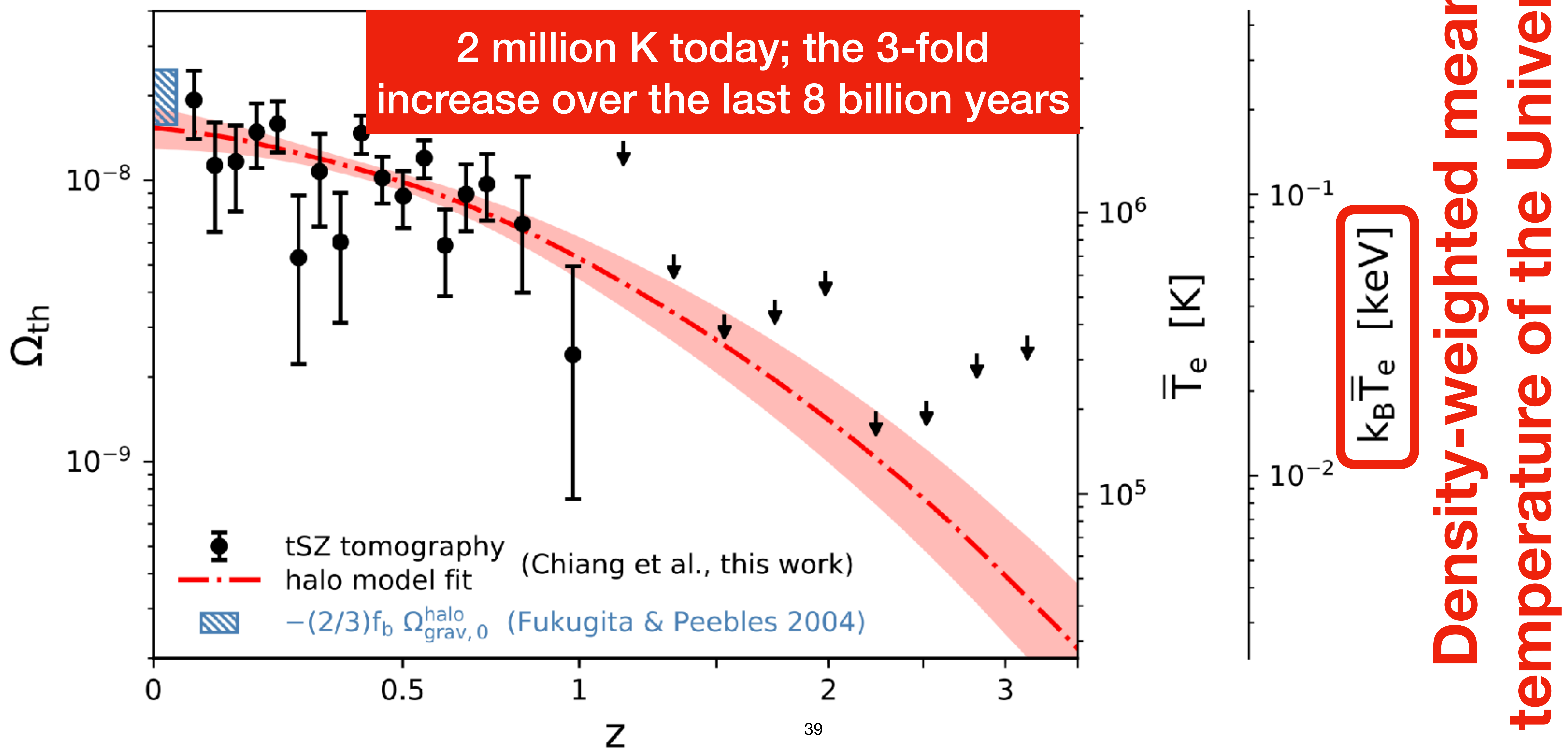
α_P = 0.12 is the empirical correction for non-self-similar scaling found by the X-ray data (Arnaud et al. 2010).

Chiang et al. (2020)

The second main result

$$\Omega_{\rm th}(z) = 1.78\times10^{-8}\ \frac{k_{\rm B}\bar{T}_\rho(z)}{0.2\ \mathrm{keV}}\,\frac{\Omega_{\rm b}}{0.049}$$

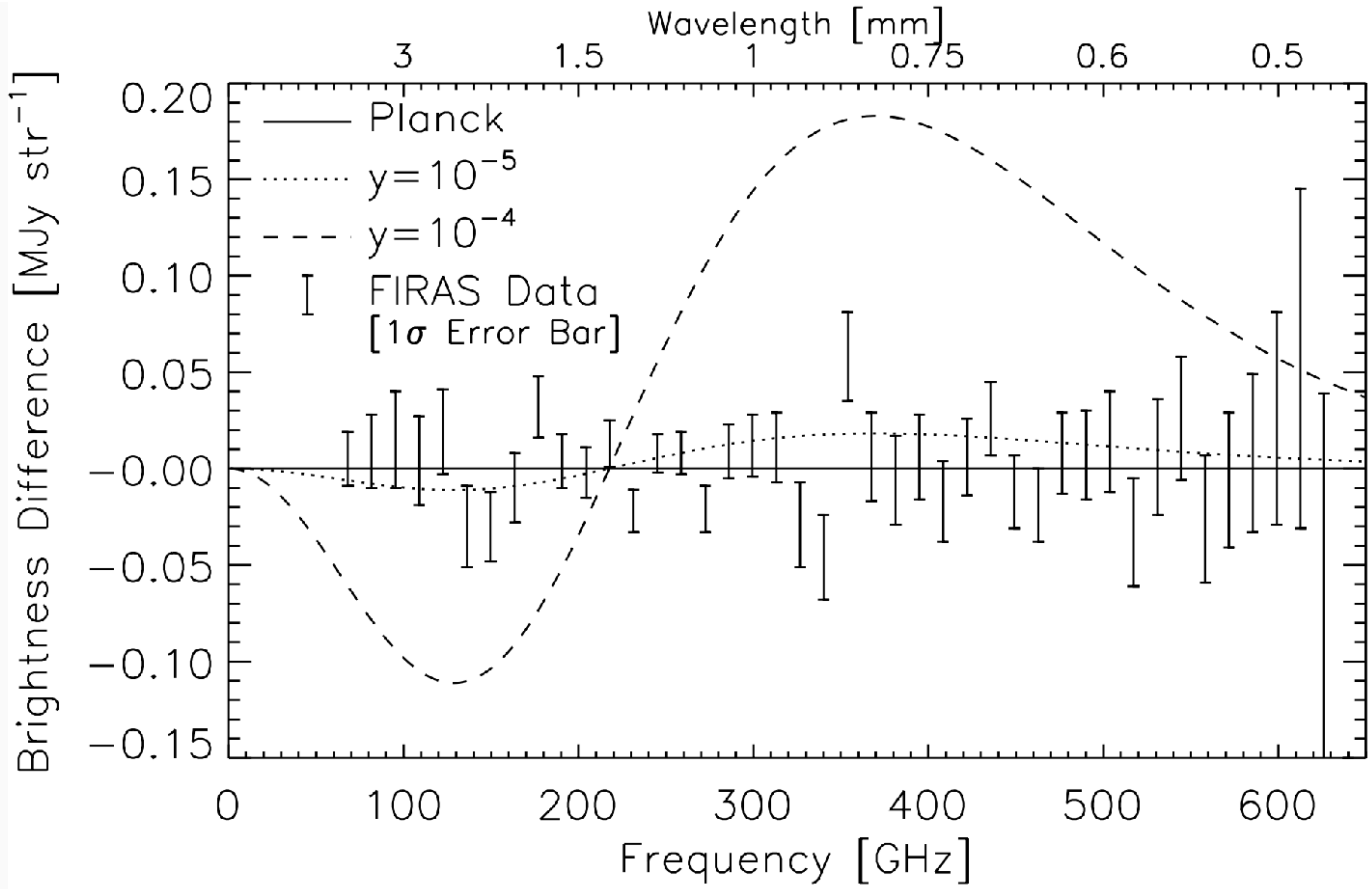
The mean thermal energy density of the Universe!



Chiang et al. (2020)

The prediction for the future space mission

The sky-averaged Compton y parameter



- Sometime in future, there will be a space mission measuring the sky-averaged (monopole) spectrum of the CMB, improving upon COBE/FIRAS by a factor of 10^{3-5}.
- Such a mission will measure the average distortion from the hot gas in the Universe.
- Our data suggest **<y>=1.2x10^{-6}**

Chiang, Makiya, EK, Ménard, ApJ, 910, 32 (2021)

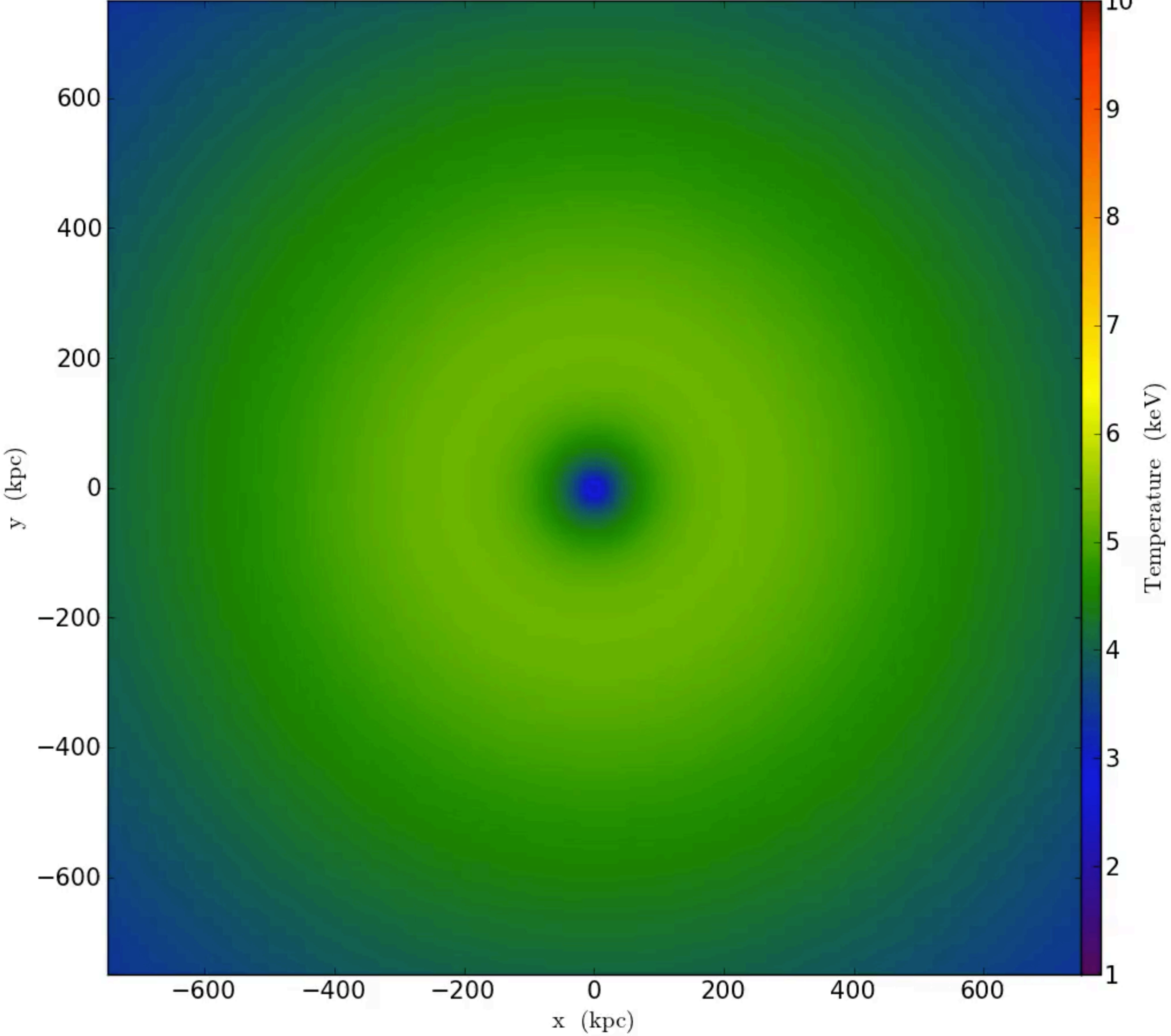
Q2: Where did the thermal energy come from?

Of course you know the answer…

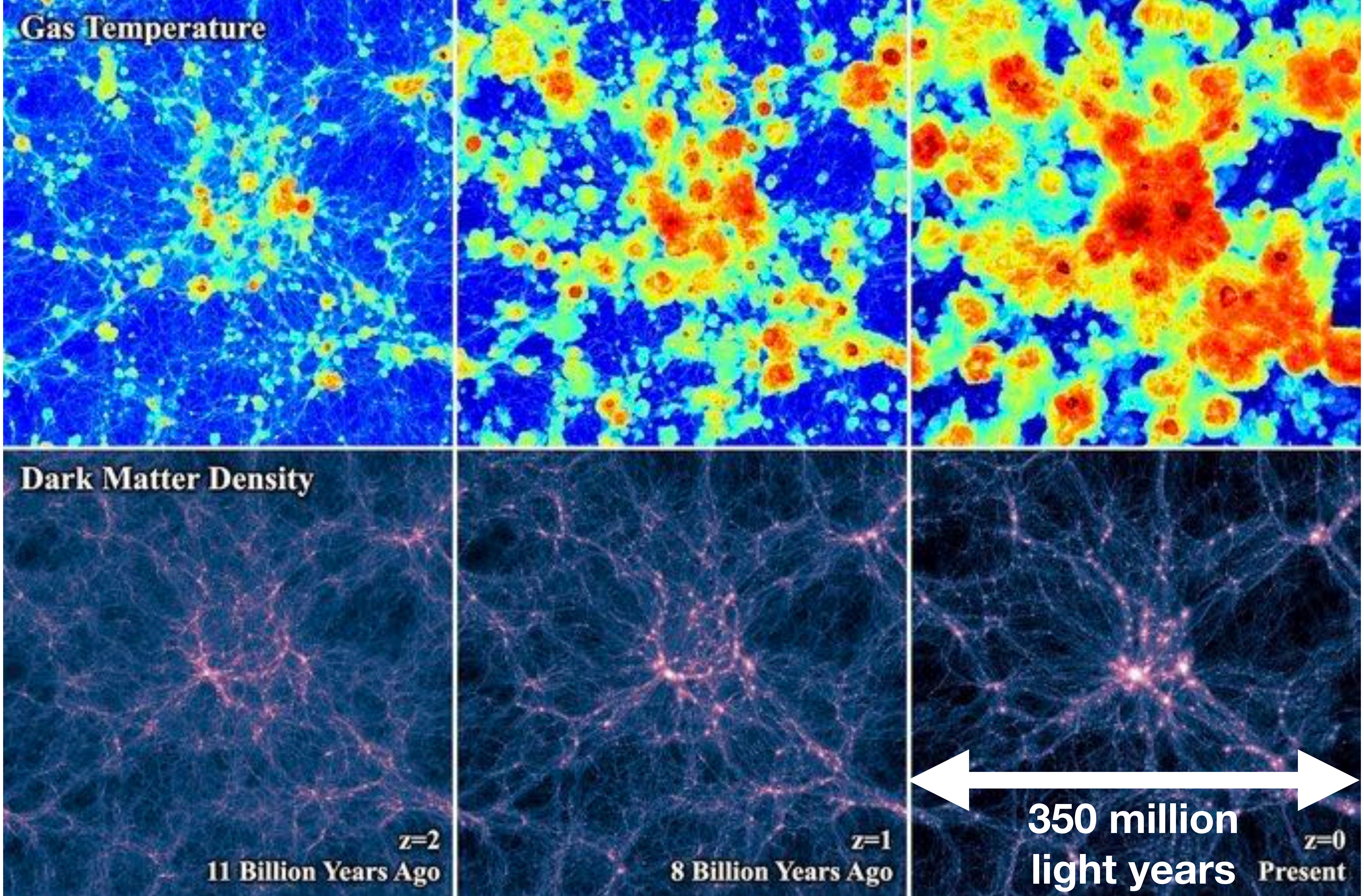
Open any textbook!

- You can find a statement like, "*As the large-scale structure forms and the matter density fluctuation collapses, the gravitational energy is converted into the thermal energy via a shock.*"
  - Yes, of course this picture is correct. However, how much do we know about this energy conversion **quantitatively**?
  - To my knowledge, no quantitative assessment of this statement has been made before.
- Our approach: We have measured Ω_{th}. We can calculate Ω_{grav} using theory of the structure formation. Let's compare the two and see if they make sense.

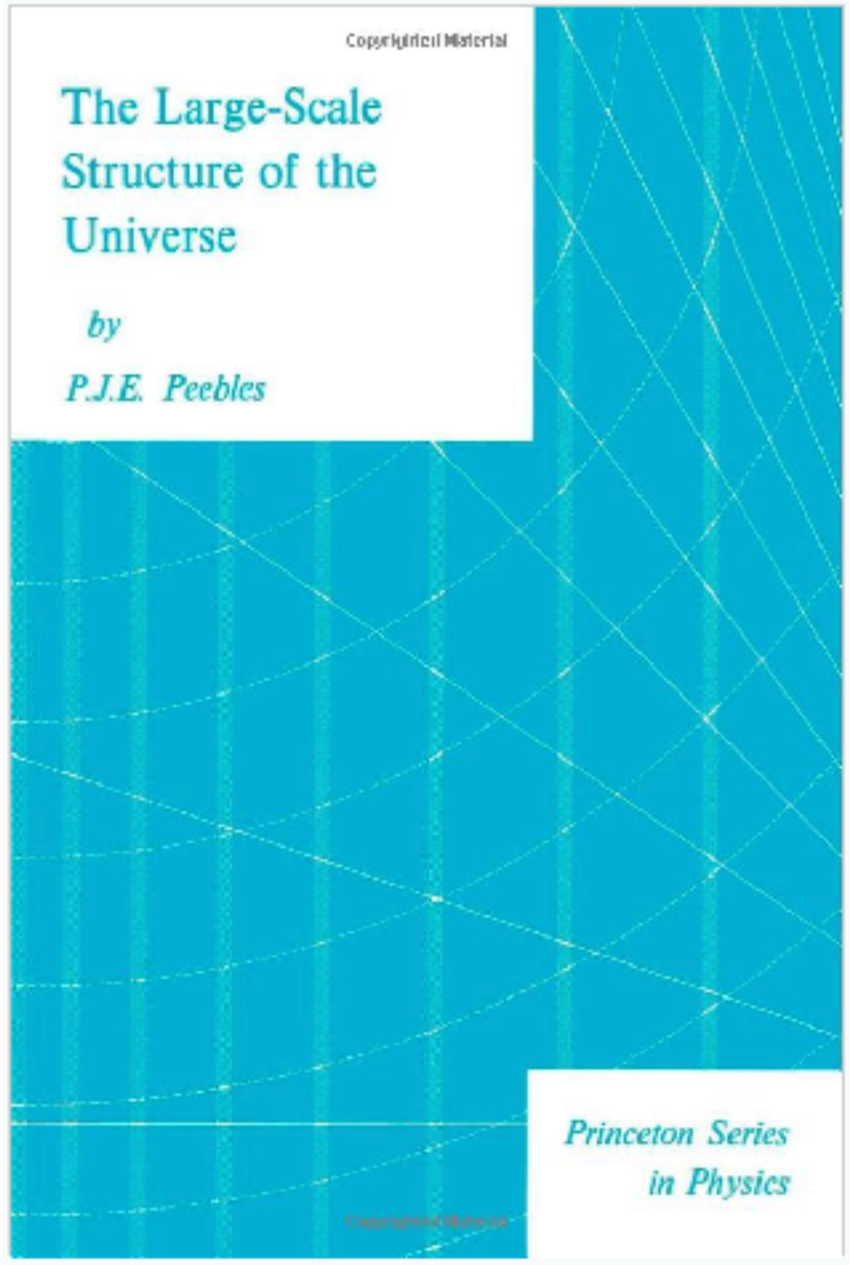
Idealized Simulation of Gas Heating by Shock Waves (Johnson et al. 2012)



More sophisticated simulation (Credit: Illustris Collaboration)



The "W": Gravitational potential energy per unit mass



Considering a system of mass M consisting of particles with mass m_i, such that $M = \sum_i m_i$,

$$MW = -\frac{1}{2}a^3\rho_{\rm m}(a)\int d^3x\ \delta(\mathbf{x},a)\phi(\mathbf{x},a)$$

$$= -\frac{1}{2}Ga^5\rho_{\rm m}^2(a)\int d^3x\int d^3x'\ \frac{\delta(\mathbf{x},a)\delta(\mathbf{x}',a)}{|\mathbf{x}-\mathbf{x}'|}$$

- The ensemble average is given by the density-potential cross power spectrum:

$$\underline{M}W = -\frac{1}{2}\rho_{\rm m0}\left(\int d^3x\right)\int \frac{d^3k}{(2\pi)^3}\boxed{P_{\phi\delta}(k,a)}$$

$$\boxed{M = \rho_{\rm m0}\int d^3x}$$

With the Poisson equation:

$$P_{\phi\delta}(k,a) = -4\pi G\frac{\rho_{\rm m0}}{a}\frac{P(k,a)}{k^2}$$

Section 9 of Peebles's Book in 1980

The "W": Gravitational potential energy per unit mass

Considering a system of mass M consisting of particles with mass m_i, such that $M = \sum_i m_i$,

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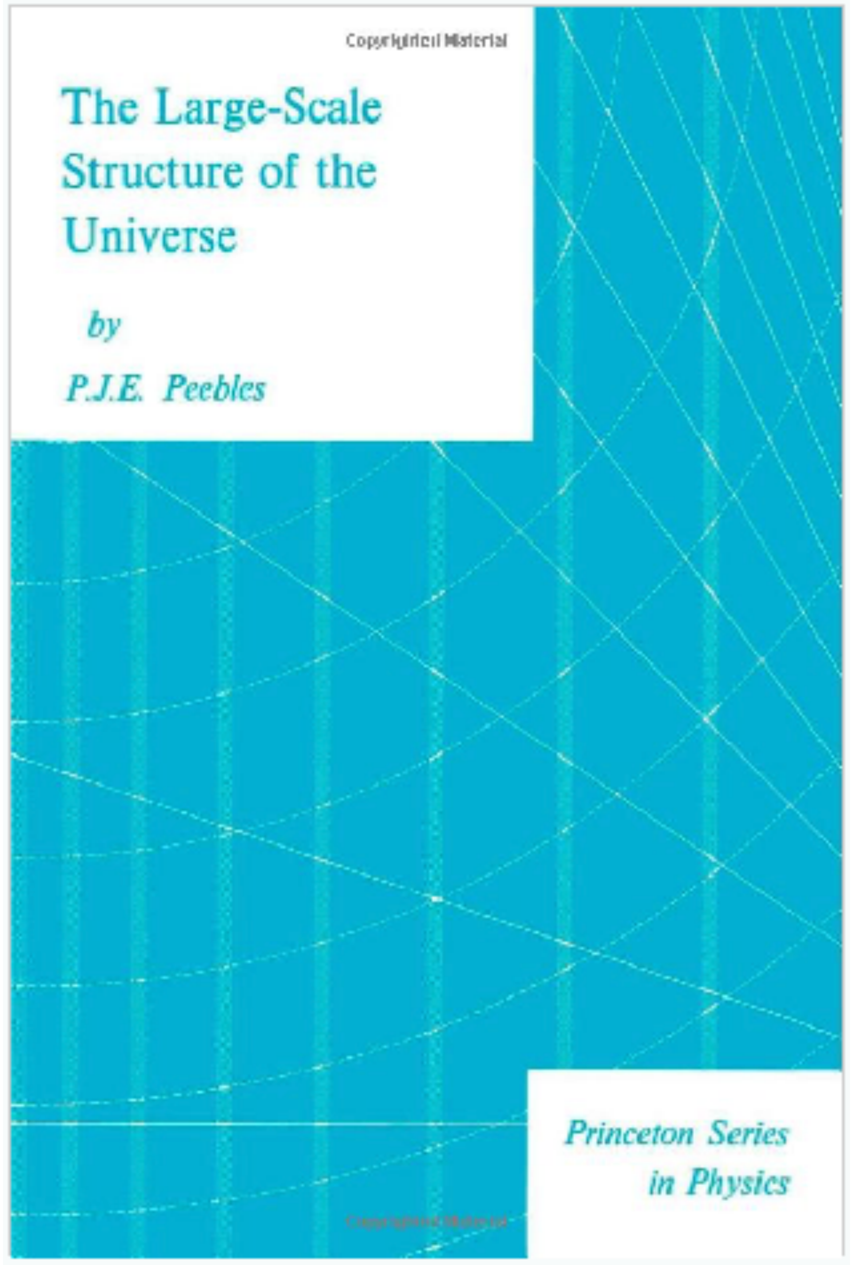
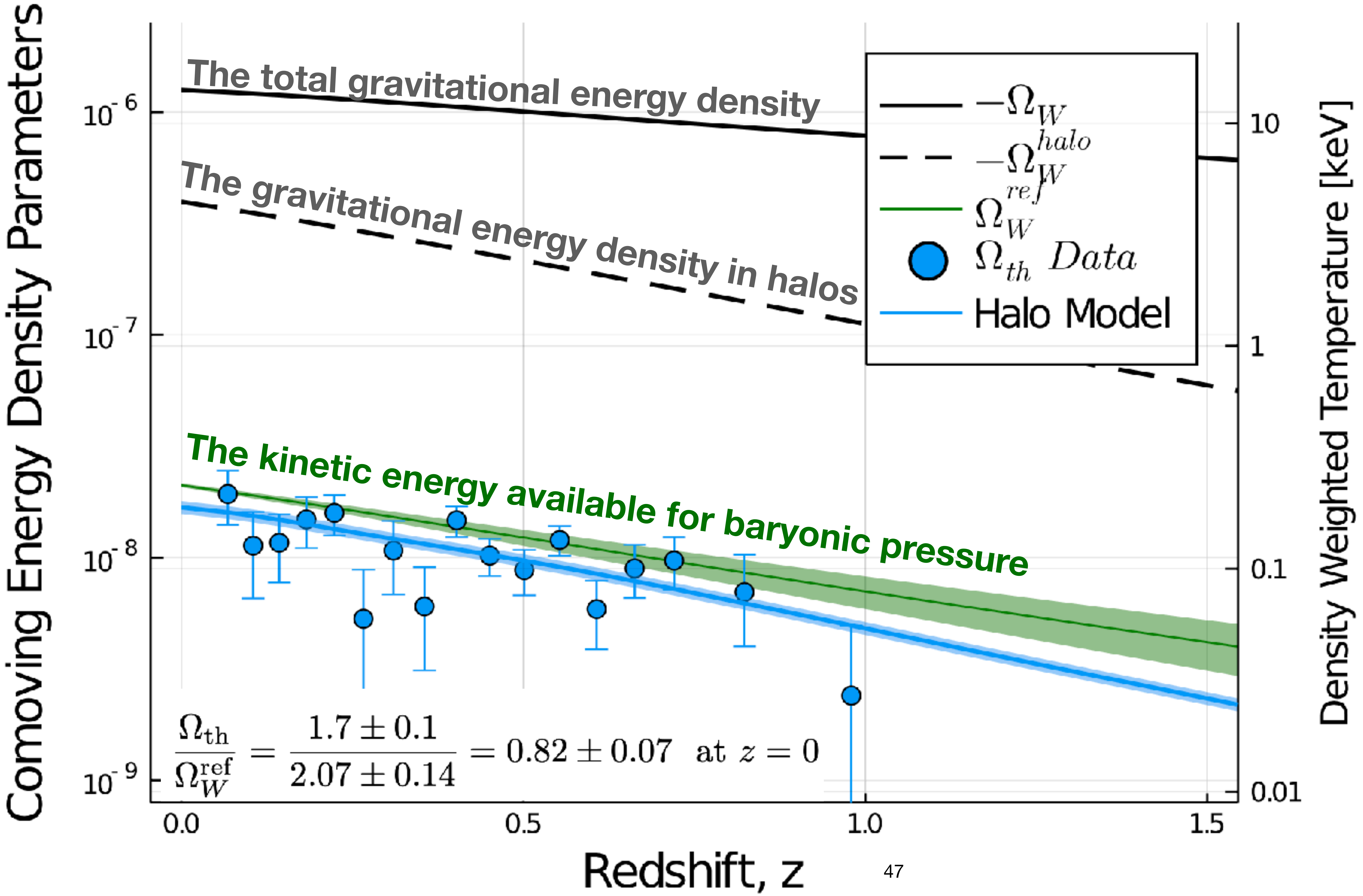


$$W = -\frac{3\Omega_{\rm m}H_0^2}{8\pi^2 a}\int_0^\infty {\rm d}k\ P(k,a)$$

This is the exact formula for W (in the Newtonian limit).

The Energy Balance in the Large-scale Structure



The energy density parameter for W:

$$\Omega_W = \frac{\Omega_{\rm m}}{c^2} W$$

The halo contribution:

$$\frac{\Omega_W^{\rm halo}}{\Omega_W^{\rm tot}} \simeq 0.3 \quad \text{at } z = 0$$

Pressure available for baryons

$$\Omega_W^{\rm ref} = -\frac{1}{3} f_{\rm b}\, \Omega_W^{\rm halo}$$

Conclusion from the second part

The energy balance does work, but where is the rest of the K.E.?

- We can now make the following statement:
  - **The measured thermal energy density accounts for ~80% of the gravitational potential energy available for kinetic energy of collapsed baryons.**
  - This is the first quantitative assessment of the textbook statement on gravitational -> thermal energy conversion in the large-scale structure formation (using the observational data).
- **What is the rest (~20%)?** => Non-thermal pressure due to the mass accretion! [Shi and EK (2014); Shi et al. (2015; 2016)]
- There is a lot more (x3) kinetic energy available in the LSS beyond collapsed baryons. **Where/how can we find it?** Kinetic SZ effect?

Young, EK, Dolag, PRD, 104, 038538 (2021)
Chen, Jamieson, EK, et al., PRD, 109, 063513 (2024)

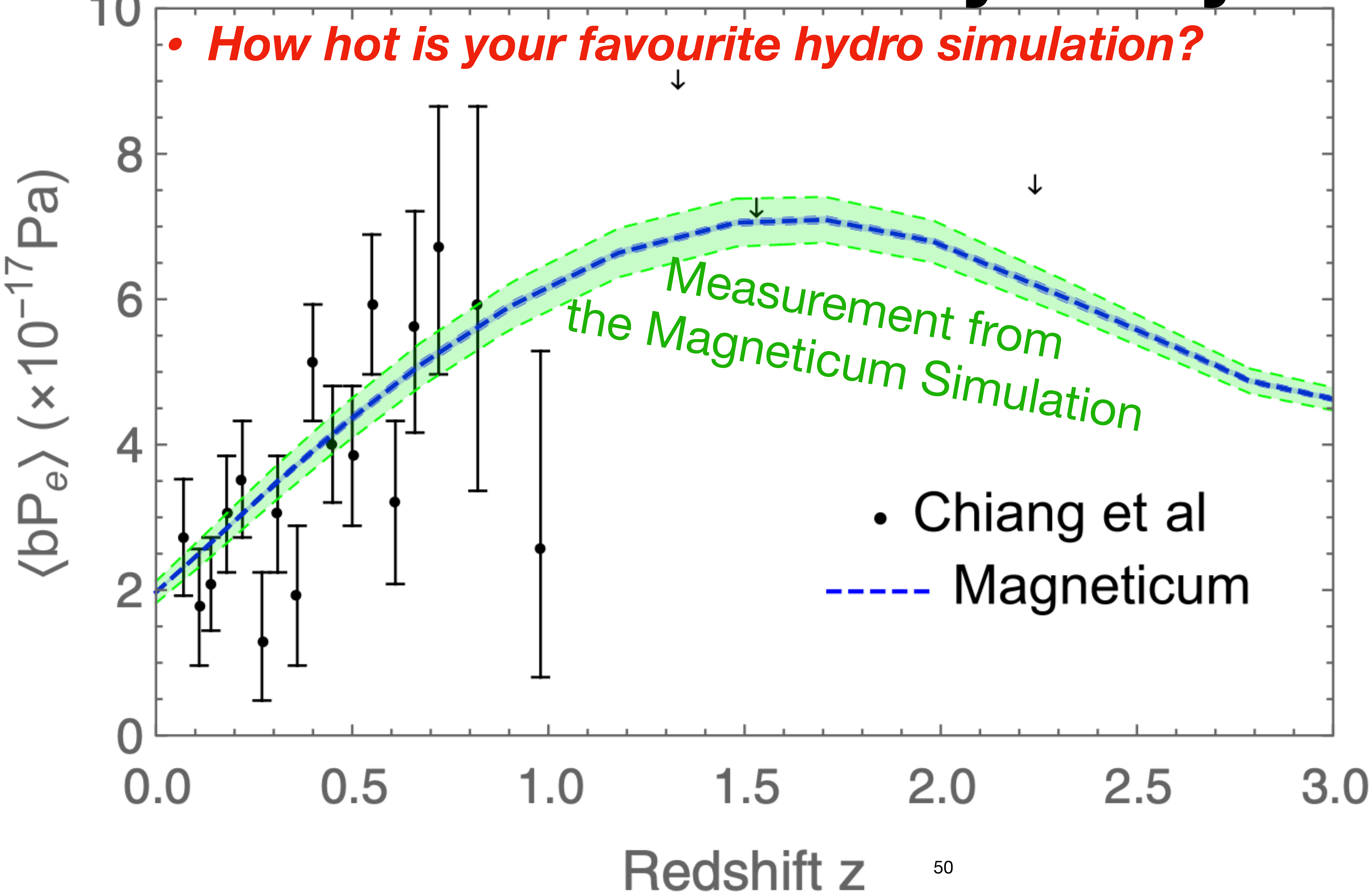
Q3: Is this good for anything?

Is this just beautiful physics, or actually useful for anyone?

Young, EK, Dolag, PRD, 104, 038538 (2021)

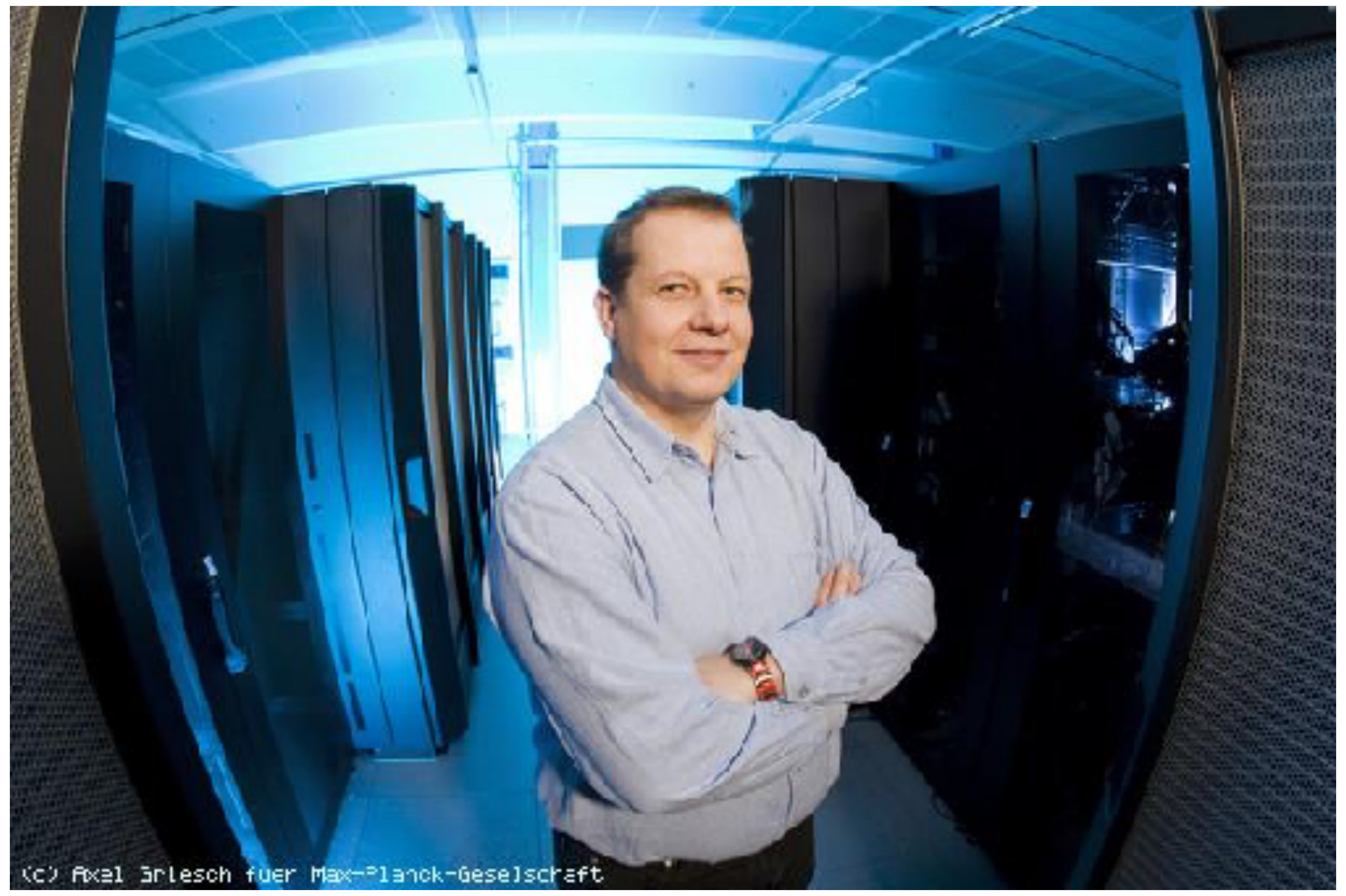
"Thermometer" test of your hydro simulation

- ***How hot is your favourite hydro simulation?***



Sam Young (Sussex)

Klaus Dolag (LMU/MPA)



Chen, Jamieson, EK, et al., PRD, 109, 063513 (2024)

"Thermometer" test of your hydro simulation

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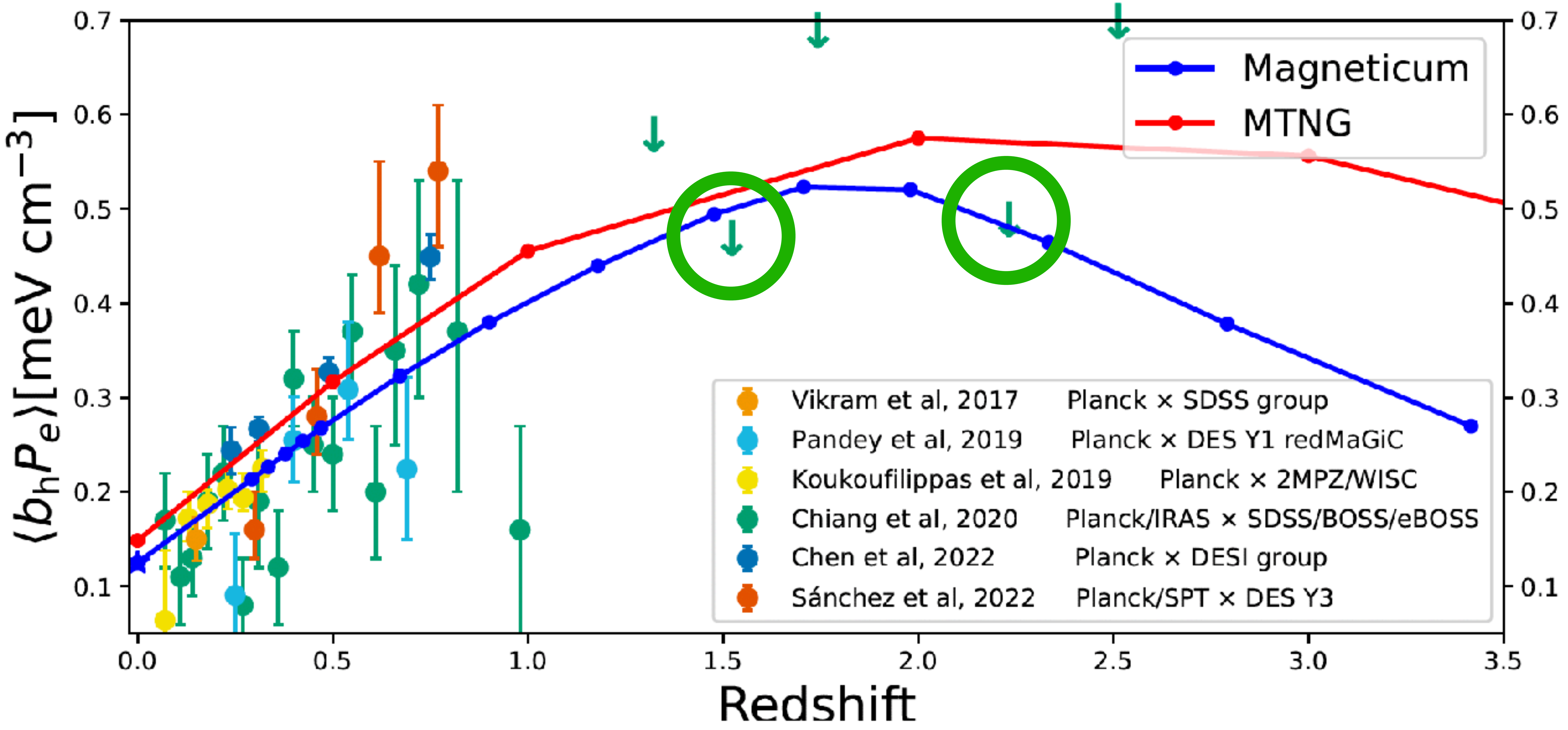

Ziyang Chen (Shanghai Jiao Tong)

Drew Jamieson (MPA)

The MillenniumTNG Simulation is in tension with some upper bounds at z>1.5?

Conclusion

The energy balance seems to work in the Universe

- We have measured the evolution of the mean thermal energy density (equivalently the density-weighted mean temperature) of the large-scale structure of the Universe out to z~1.

- **Personally**: This completes the quarter century of homework since *Refregier, EK, Spergel, Pen (2000)* [See also Cen & Ostriker (1999)].
    - We used Ue-Li Pen's moving mesh hydro code to predict the evolution of the density-weighted mean temperature. **We finally measured this.**

- Detailed comparison to the gravitational energy of the LSS shows that **the thermal energy accounts for ~80% of the kinetic energy available for thermal pressure of collapsed baryons.** The rest can be accounted for easily by non-thermal pressure (*Shi & EK 2014*).

- **The pressure statistics can be a powerful test of galaxy formation simulations!**

THE ASTROPHYSICAL JOURNAL, 514:1–6, 1999 March 20

WHERE ARE THE BARYONS?

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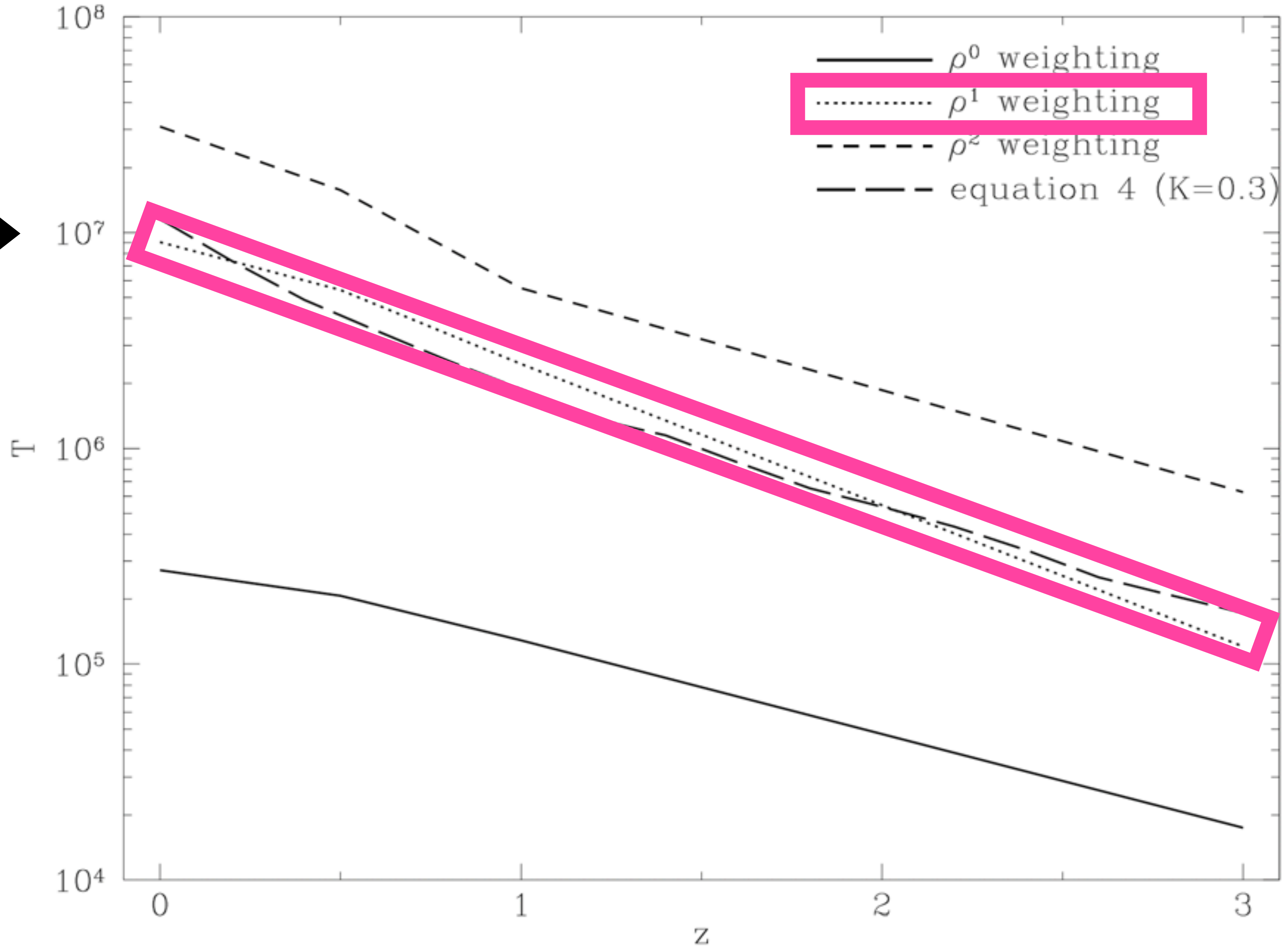
Received 1998 September 11; accepted 1998 October 29

Too hot!



PHYSICAL REVIEW D, VOLUME 61, 123001

Power spectrum of the Sunyaev-Zel'dovich effect

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(Received 10 December 1999; published 12 May 2000)

Model	$\bar{T}_\rho^a$ (keV)
SCDM	0.19 (2.2 mil. K)
ΛCDM	0.25 (2.9 mil. K)
OCDM	0.19 (2.2 mil. K)

[a] At $z=0$.

Not bad!

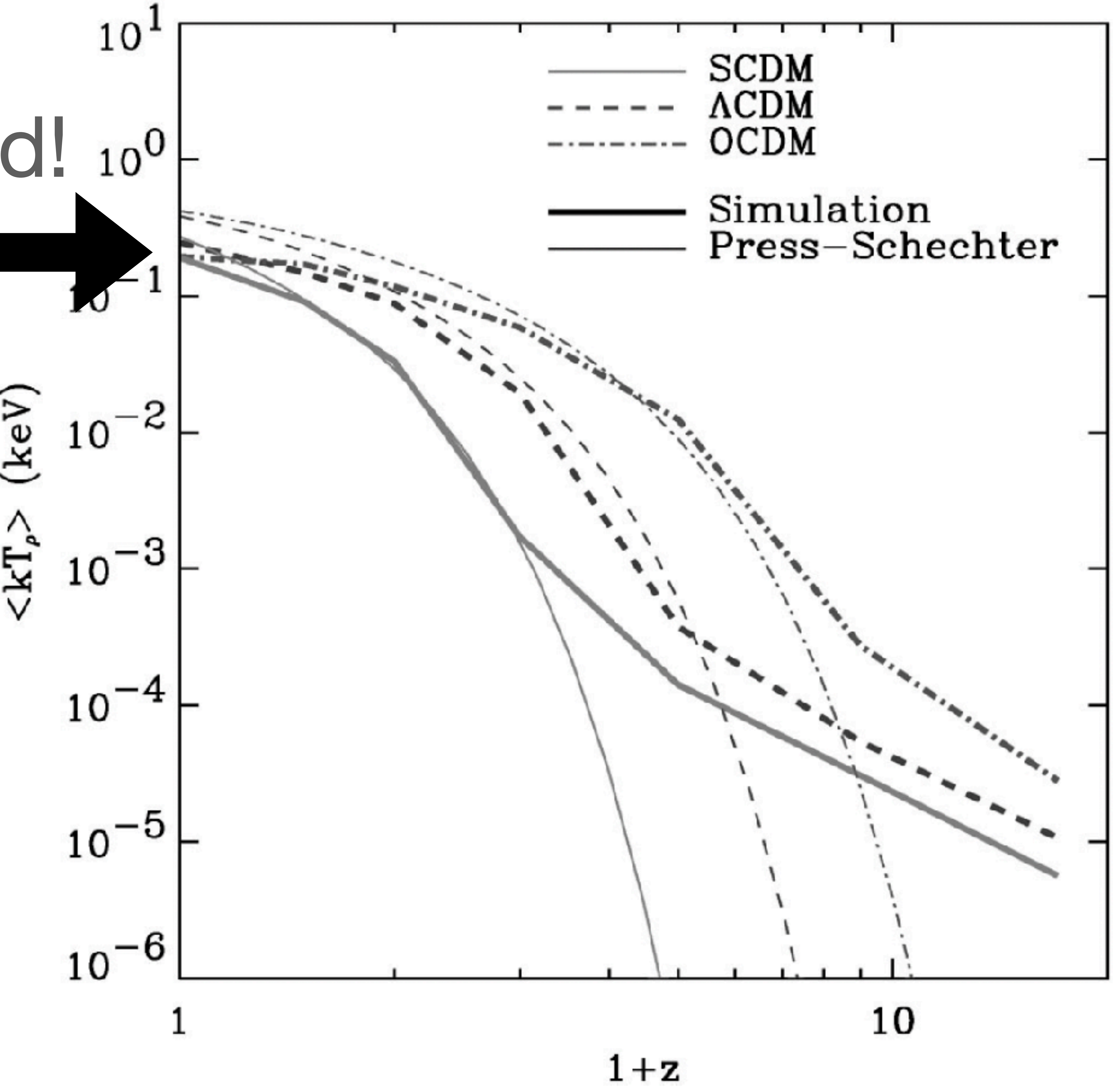


FIG. 4. Temperature history of the gas. For each model, the density weighted temperature T_ρ is shown for the simulations and for the Press-Schechter prediction.

Back up slides

W to K: the mean kinetic energy per unit mass

Layzer-Irvine equation (Layzer 1963; Irvine 1961; Dmitriev & Zeldovich 1964)

- Given the knowledge of W, we can calculate the mean kinetic energy per unit mass, K, using the Layzer-Irvine equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}(K+W)+\frac{\dot{a}}{a}(2K+W)=0$$

where K is the mean kinetic energy per unit mass, $K = \sum_i m_i v_i^2/(2\sum_i m_i)$.

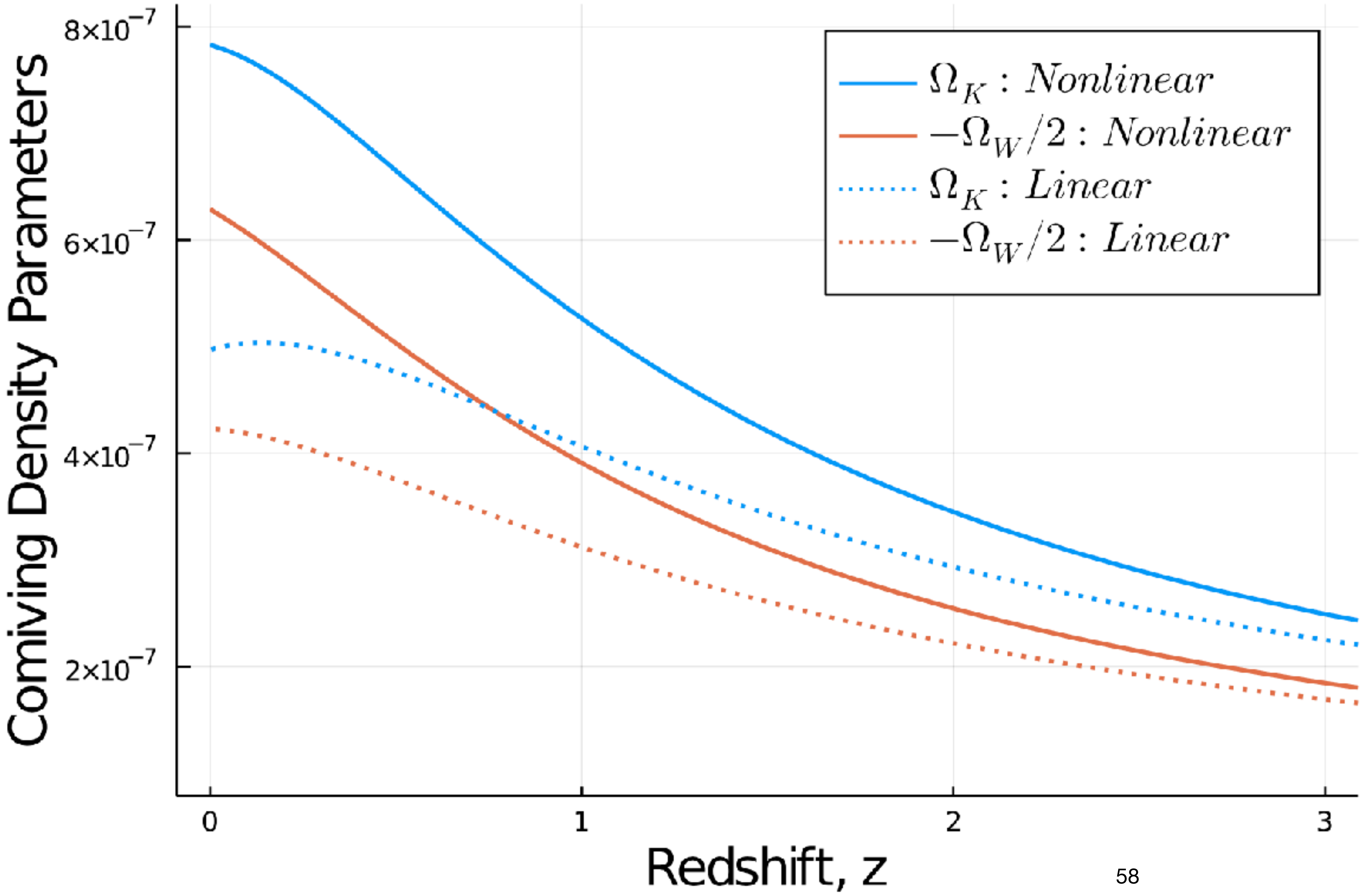
- The initial condition for K can be set using the linear theory result at sufficiently early time (Davis et al. 1997),

$$K=-\frac{2f^2}{3\Omega_\mathrm{m}(a)}W$$

where $f \equiv \mathrm{d}\ln\delta_1/\mathrm{d}\ln a$ with the linear density contrast δ_1 and $\Omega_\mathrm{m}(a) = \Omega_\mathrm{m}/[a^3E^2(a)]$ is the matter density parameter at a given a.

W to K: The Result

More kinetic energy is available than the virial theorem K = –W/2



- This result captures the kinetic energy of all structures.
- Here, we do not separate random and bulk motion of collapsed and non-collapsed structures, respectively.
- For comparison to the thermal energy, we used the virial relationship, K = –W/2.