The lecture slides are available at
https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Lecture 8: Understanding the Power Spectrum of the Temperature Anisotropy

and

Introduction to CMB Polarisation

Part I: Cosmological Parameter Dependence of the Temperature Power Spectrum

Before starting: Let's recap the Big Picture

In words!

- *How does the power spectrum constrain the baryon density?*
  - Via the speed of sound, the increased inertia of a photon-baryon fluid, and Silk damping.

$$\frac{\delta\rho_\gamma}{4\bar{\rho}_\gamma} + \Phi = \frac{\zeta}{5}\left\{3R\mathcal{T}(q) - (1+R)^{-1/4}\mathcal{S}(q)\cos[qr_s + \theta(q)]\right\}$$

  - They all depend on R (the baryon-photon energy density **ratio**), which is proportional to $\Omega_B h^2$. **Note h^2! It is not just Ω_B.**

$$R = \frac{3\Omega_B}{4\Omega_\gamma}\frac{a}{a_0} = 0.6120\left(\frac{\Omega_B h^2}{0.022}\right)\frac{1091}{1+z}$$

$$\Omega_\gamma \equiv \frac{8\pi G\rho_{\gamma 0}}{3H_0^2} = 2.471\times 10^{-5}\ h^{-2}$$

Before starting: Let's recap the Big Picture

In words!

- *How does the power spectrum constrain the total matter density?*
  - Via the boost of the amplitude of sound waves and the ISW due to a decaying gravitational after the horizon re-entry.

$$\frac{\Delta T(\hat{n})}{T_0} = \frac{\delta T(t_L, \hat{n} r_L)}{\bar{T}(t_L)} + \Phi(t_L, \hat{n} r_L) - \Phi(t_0, 0) + \int_{t_L}^{t_0} dt \; (\dot{\Phi} + \dot{\Psi})(t, \hat{n} r)$$

$$\frac{\delta \rho_\gamma}{4 \bar{\rho}_\gamma} + \Phi = \frac{\zeta}{5} \left\{ 3 R \mathcal{T}(q) - (1+R)^{-1/4} \mathcal{S}(q) \cos[q r_s + \theta(q)] \right\}$$

  - They all depend on q_{EQ} (the wavenumber of the matter-radiation equality), which is proportional to $\Omega_M h^2$. **Note h^2! It is not just Ω_M.**

$$q_{EQ} = a_{EQ} H_{EQ} \sim 0.01 \, (\Omega_M h^2 / 0.14) \text{ Mpc}^{-1}$$

Before starting: Things you haven't learned yet

- *How does the power spectrum constrain the Hubble constant?*
  - Via the (comoving) distance to the last scattering surface, r_L. Particularly interesting now because of the "Hubble constant tension".
- *How does the power spectrum constrain the dark energy?*
  - The same: via r_L.
- *How does the power spectrum constrain the epoch of reionization?*
  - The temperature power spectrum cannot constrain this.

How does r_L depend on H_0 and dark energy?

$$a_0 r_L = c a_0 \int_{t_L}^{t_0} \frac{dt}{a(t)} = c a_0 \int_{a_L}^{a_0} \frac{da}{a\dot{a}} = c a_0 \int_{a_L}^{a_0} \frac{da}{a^2 H(a)}$$

Friedmann's equation

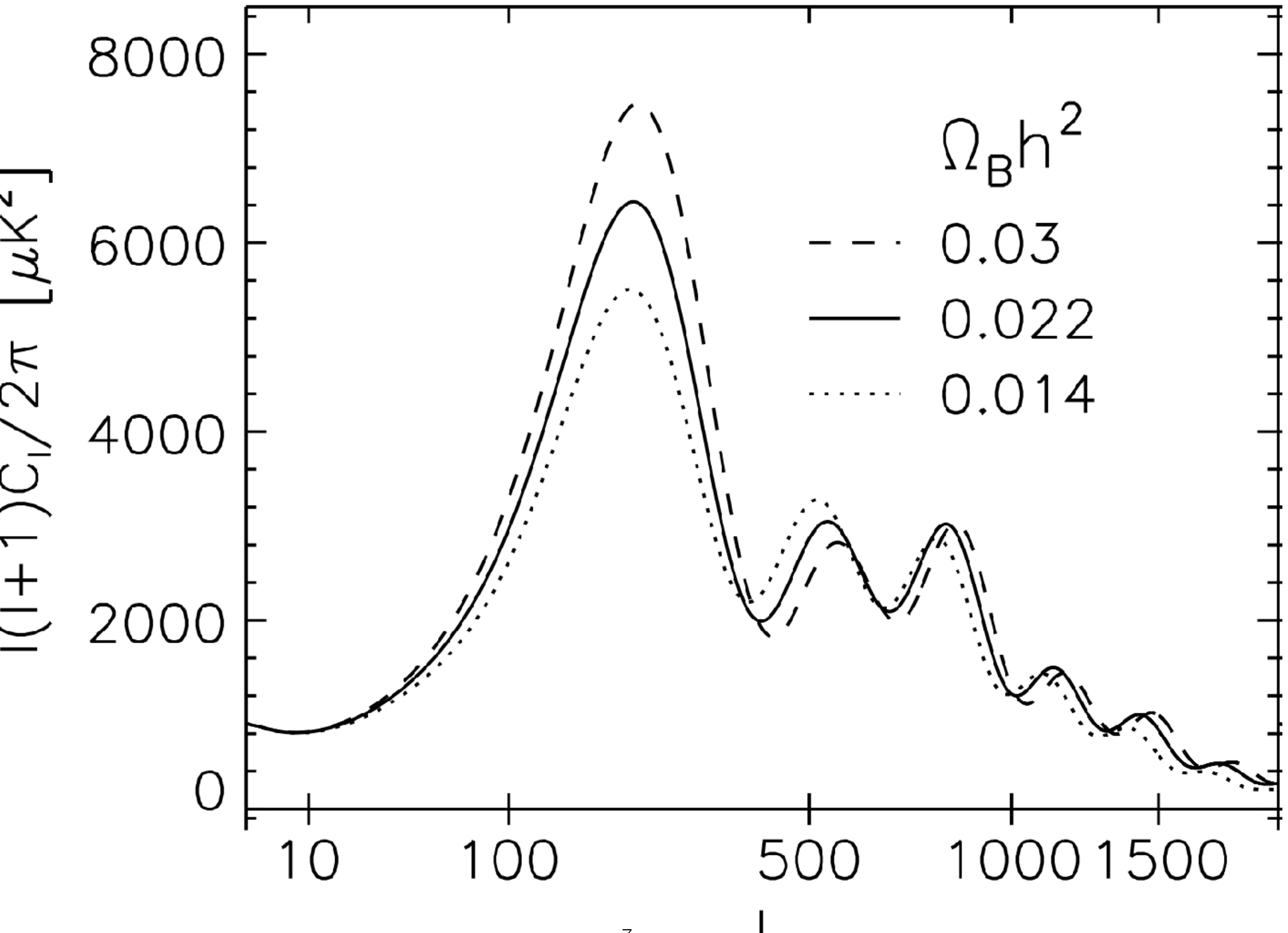
$$H^2(a) = H_0^2 \left(\Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda\right) \rightarrow H_0^2 \left(\Omega_M a^{-3} + 1 - \Omega_M\right)$$

$\Omega_M + \Omega_k + \Omega_\Lambda = 1$

Flat Universe
$\Omega_k=0$

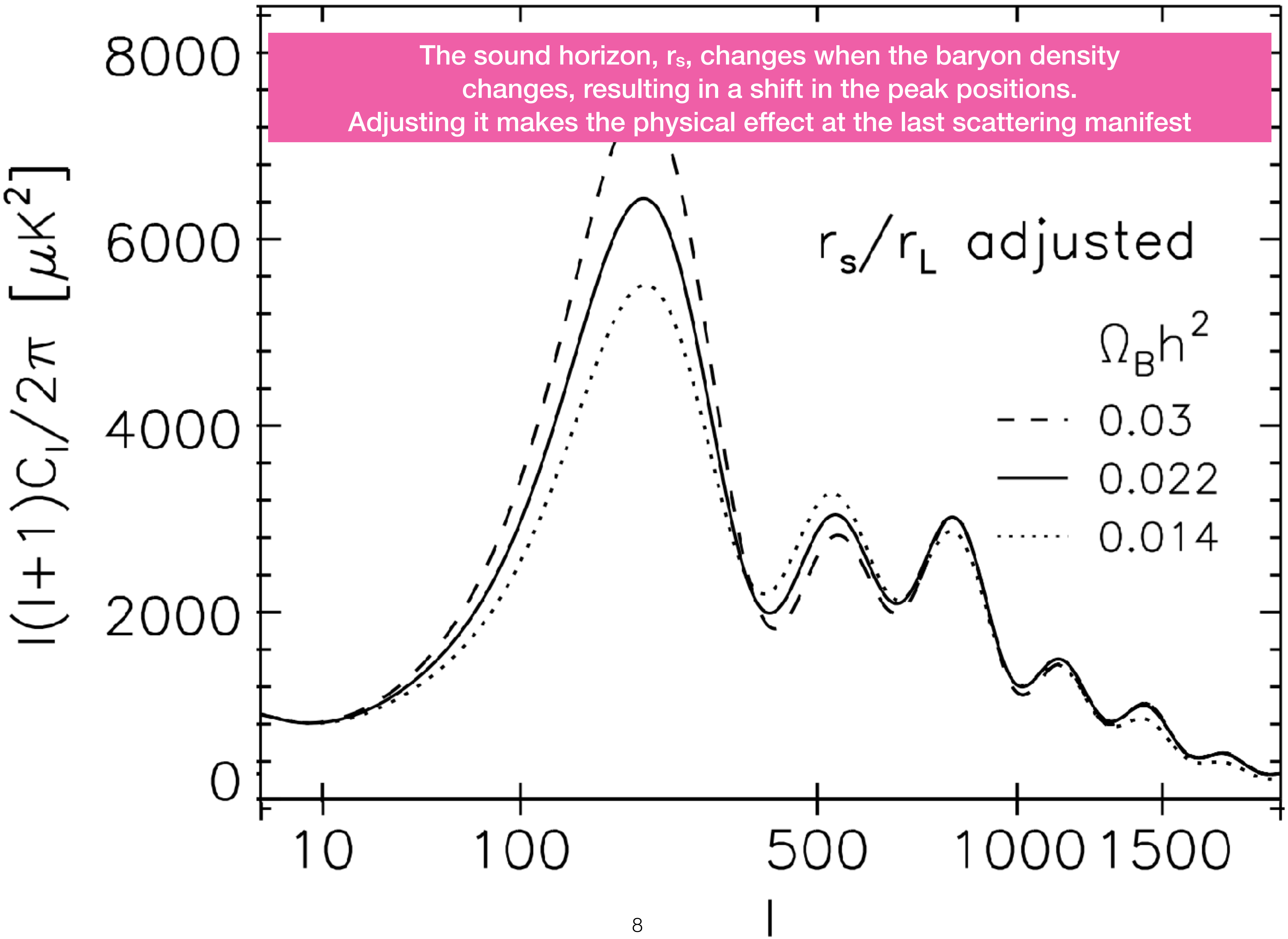
$$\propto \left(\Omega_M h^2 a^{-3} + h^2 - \Omega_M h^2\right)$$

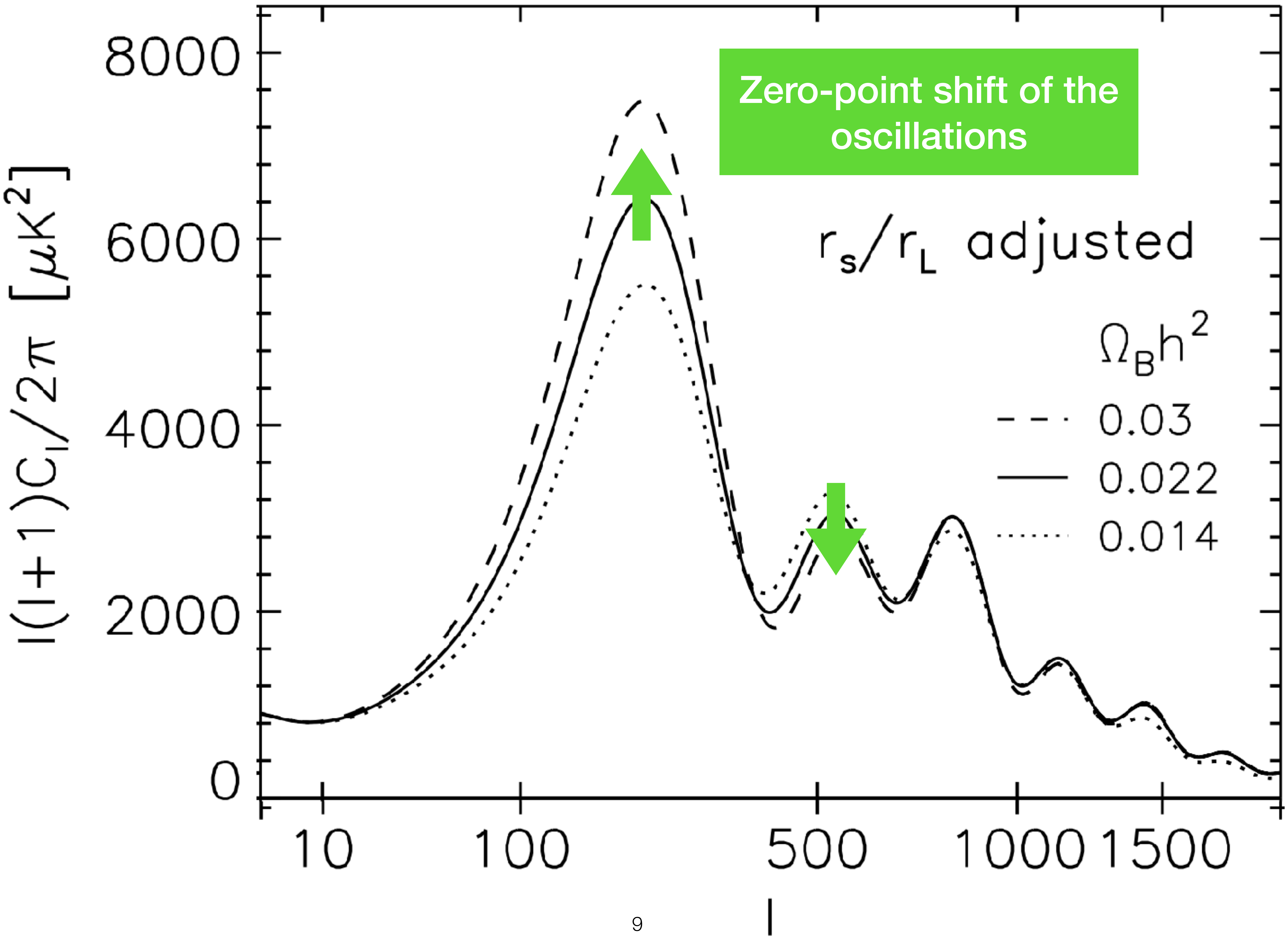
- We must know $\Omega_M h^2$ in advance. The CMB peak heights tell us $\Omega_M h^2$, which then enables us to determine H_0 **if we assume a flat Universe**. Otherwise, we cannot really determine H_0 or Ω_Λ! (Unless we use gravitational lensing of the CMB.)

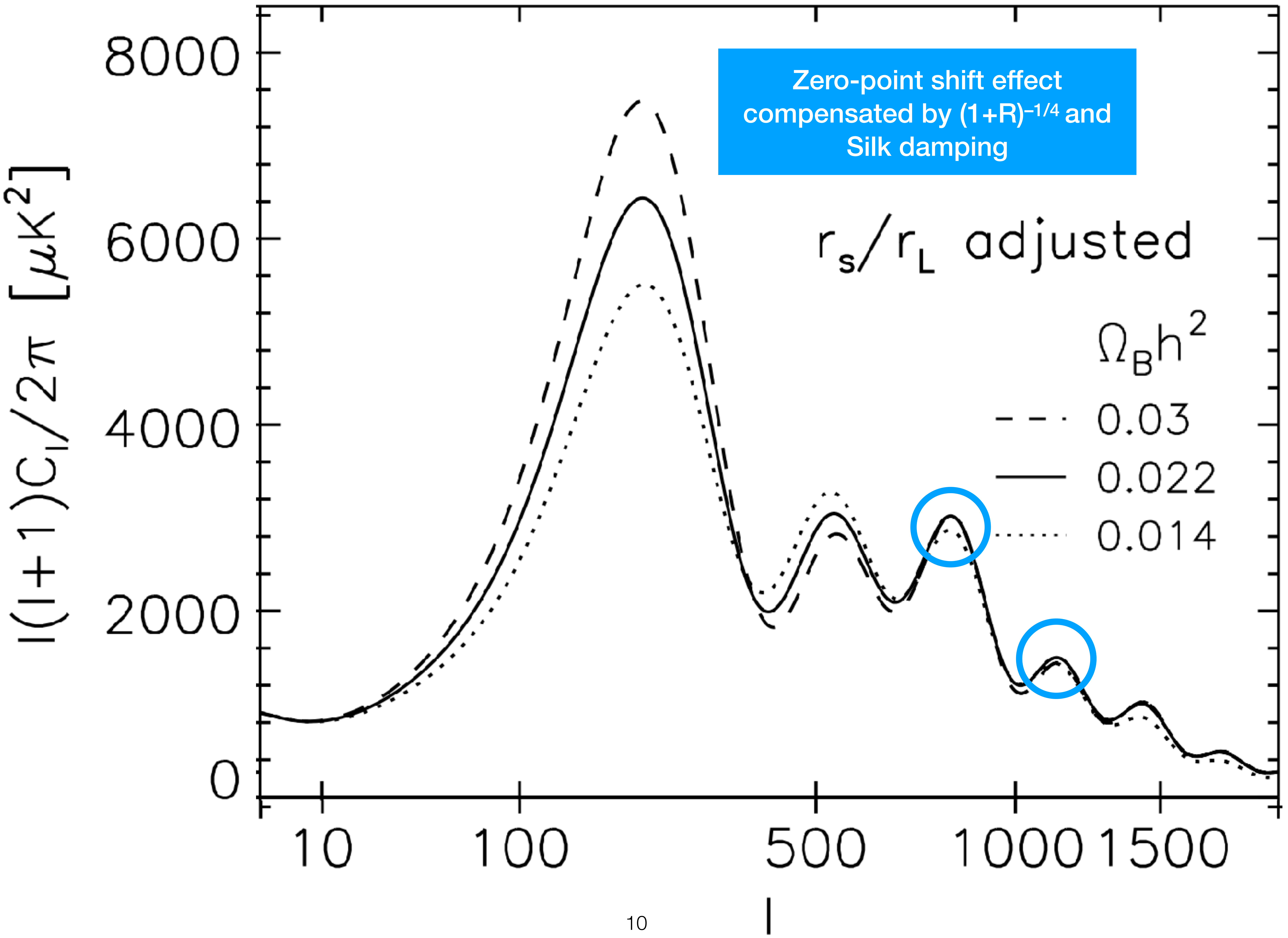


8000
6000
4000
2000
0
l(l+1)C_l/2π [μK^2]
10
100
500
1000
1500
l
Ω_B h^2
0.03
0.022
0.014

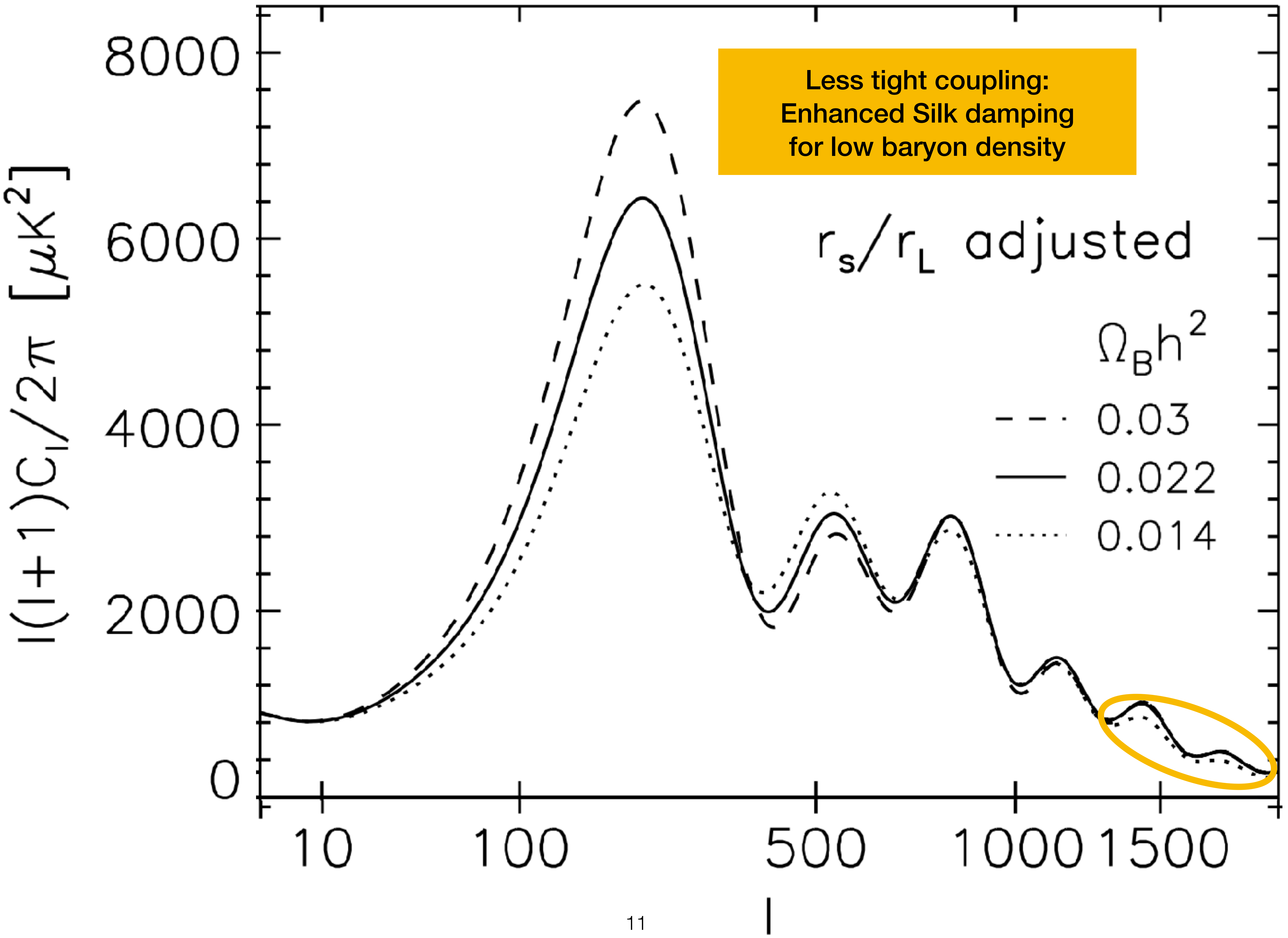
The sound horizon, r_s, changes when the baryon density changes, resulting in a shift in the peak positions.
Adjusting it makes the physical effect at the last scattering manifest



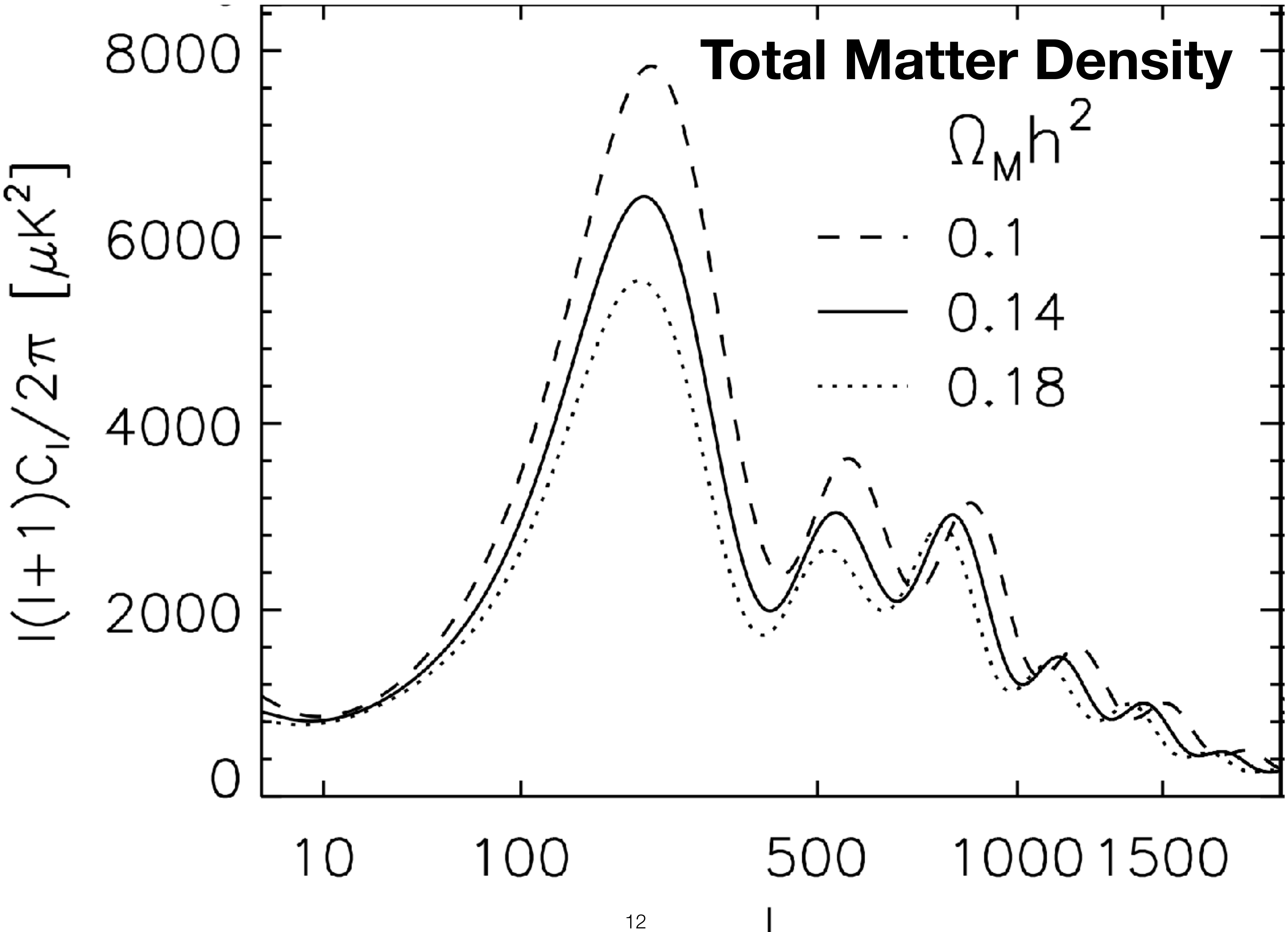

Zero-point shift of the oscillations
r_s/r_L adjusted
$\Omega_B h^2$
0.03
0.022
0.014
8000
6000
4000
2000
0
$l(l+1)C_l/2\pi$ [μK^2]
10
100
500
1000
1500
l

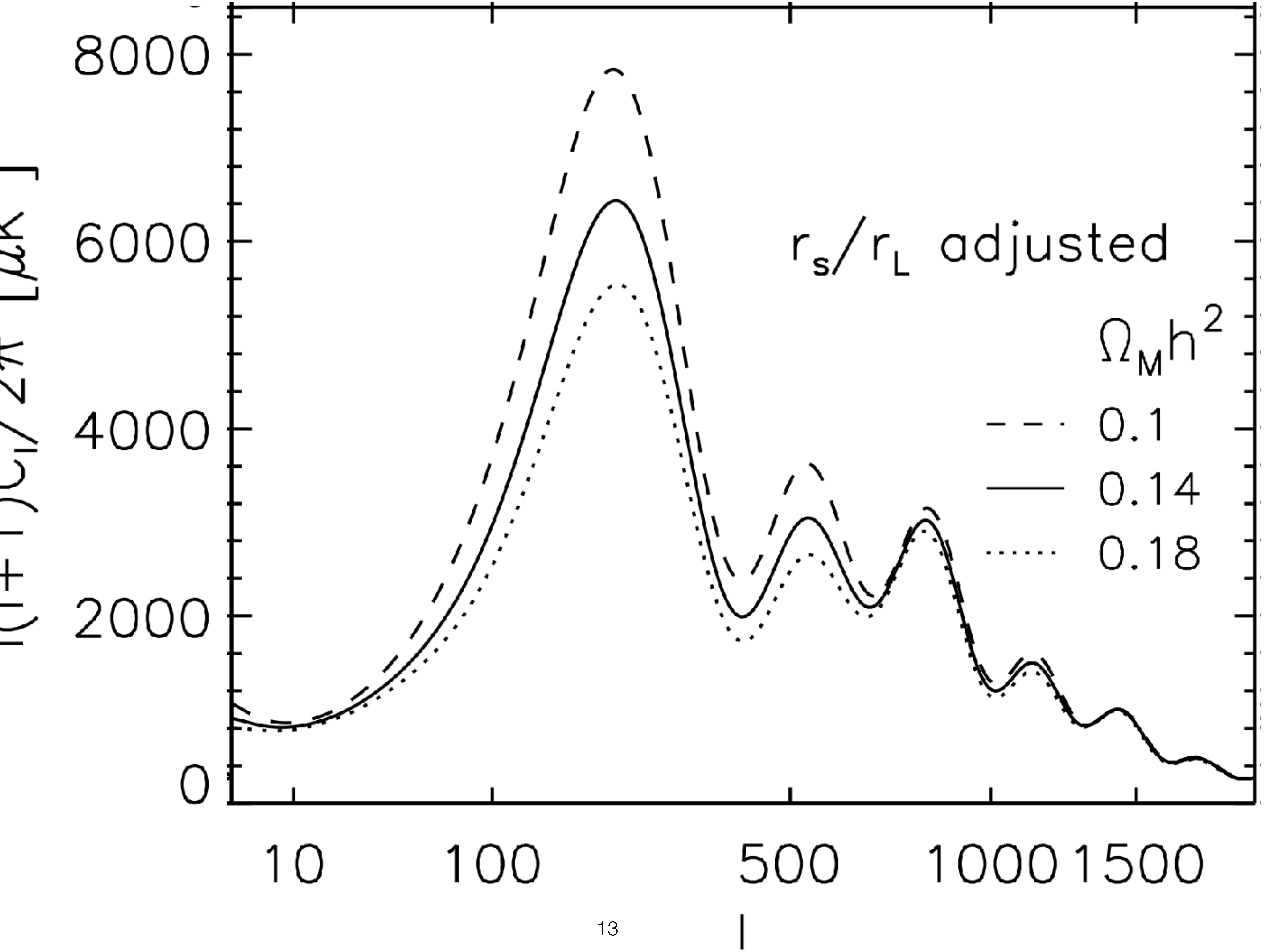


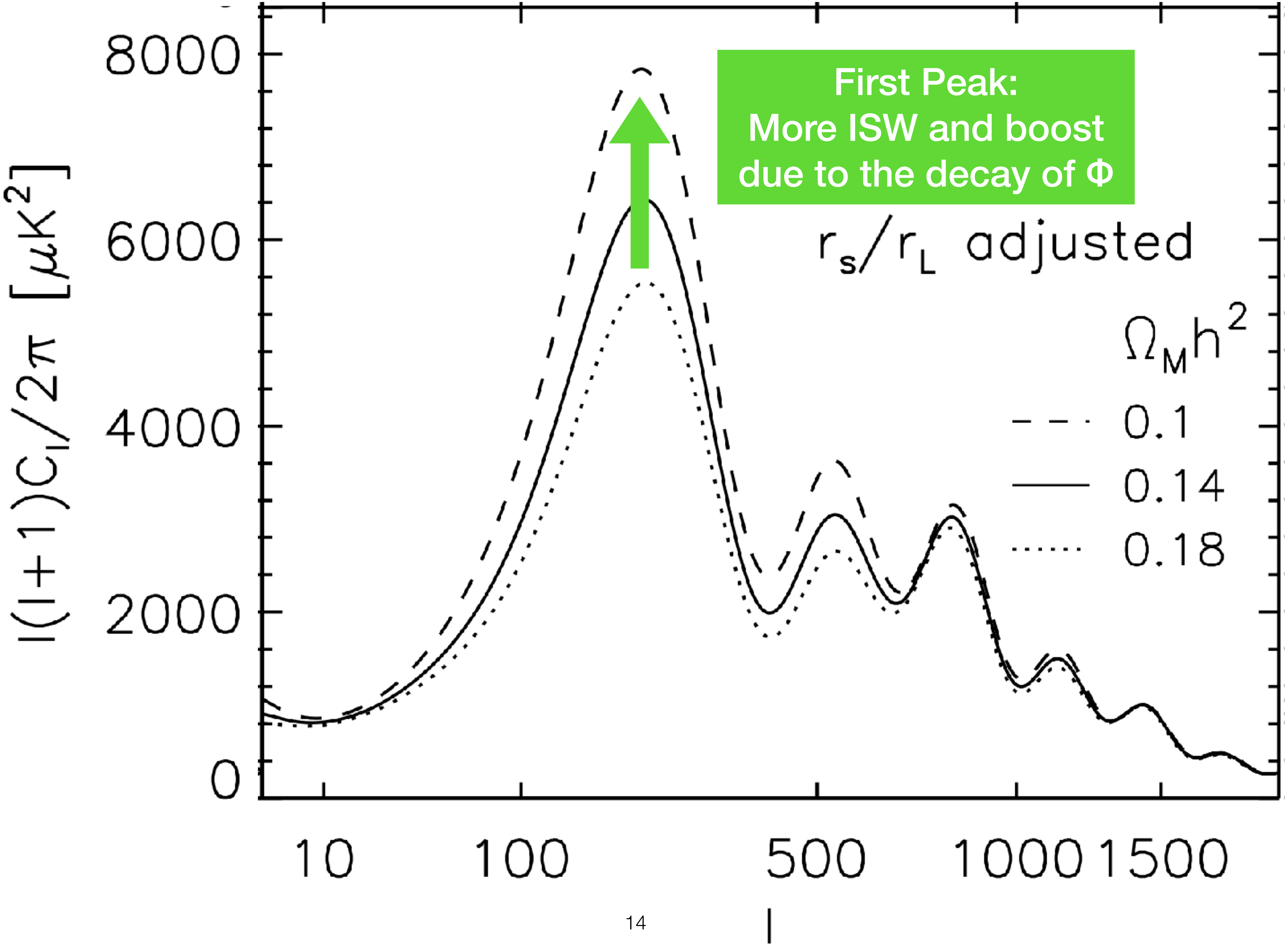
Zero-point shift effect compensated by (1+R)^−1/4 and Silk damping
r_s/r_L adjusted
Ω_B h^2
0.03
0.022
0.014
8000
6000
4000
2000
0
10
100
500
1000
1500
l
l(l+1)C_l/2π [μK^2]

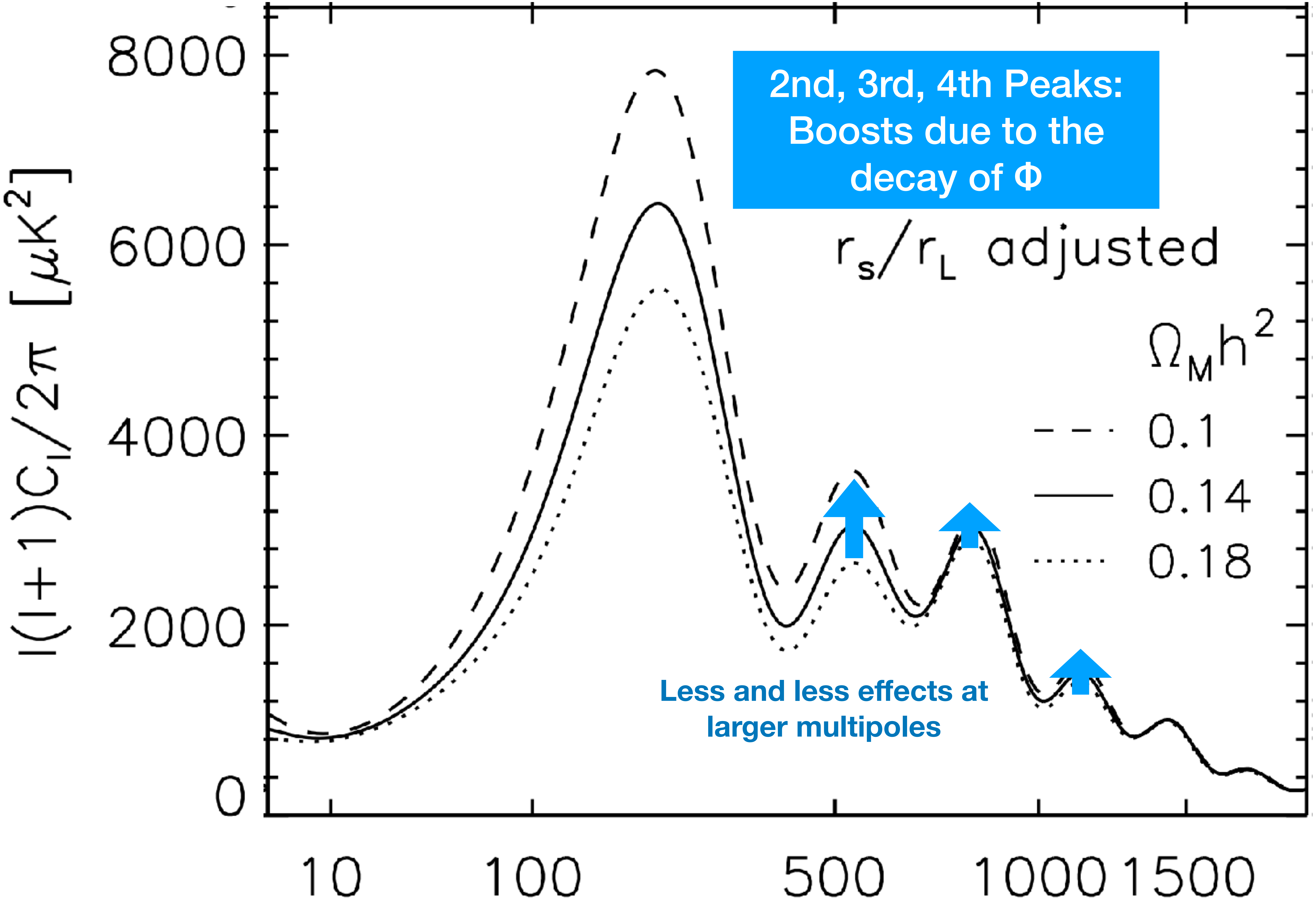


Less tight coupling:
Enhanced Silk damping
for low baryon density
r_s/r_L adjusted
$\Omega_B h^2$
0.03
0.022
0.014
8000
6000
4000
2000
0
$l(l+1)C_l/2\pi$ [μK^2]
10
100
500
1000
1500
l


Total Matter Density
$\Omega_M h^2$
0.1
0.14
0.18
8000
6000
4000
2000
0
$l(l+1)C_l/2\pi$ [μK^2]
10
100
500
1000
1500
l


8000
6000
4000
2000
0
l(l+1)C_l/2π [μK^2]
r_s/r_L adjusted
Ω_M h^2
0.1
0.14
0.18
10
100
500
1000
1500
l


First Peak:
More ISW and boost
due to the decay of Φ
r_s/r_L adjusted
Ω_M h^2
0.1
0.14
0.18
l(l+1)C_l/2π [μK^2]
8000
6000
4000
2000
0
10
100
500
1000
1500
l


2nd, 3rd, 4th Peaks:
Boosts due to the
decay of Φ
r_s/r_L adjusted
Ω_M h^2
0.1
0.14
0.18
Less and less effects at
larger multipoles
l(l+1)C_l/2π [μK^2]
8000
6000
4000
2000
0
10
100
500
1000
1500

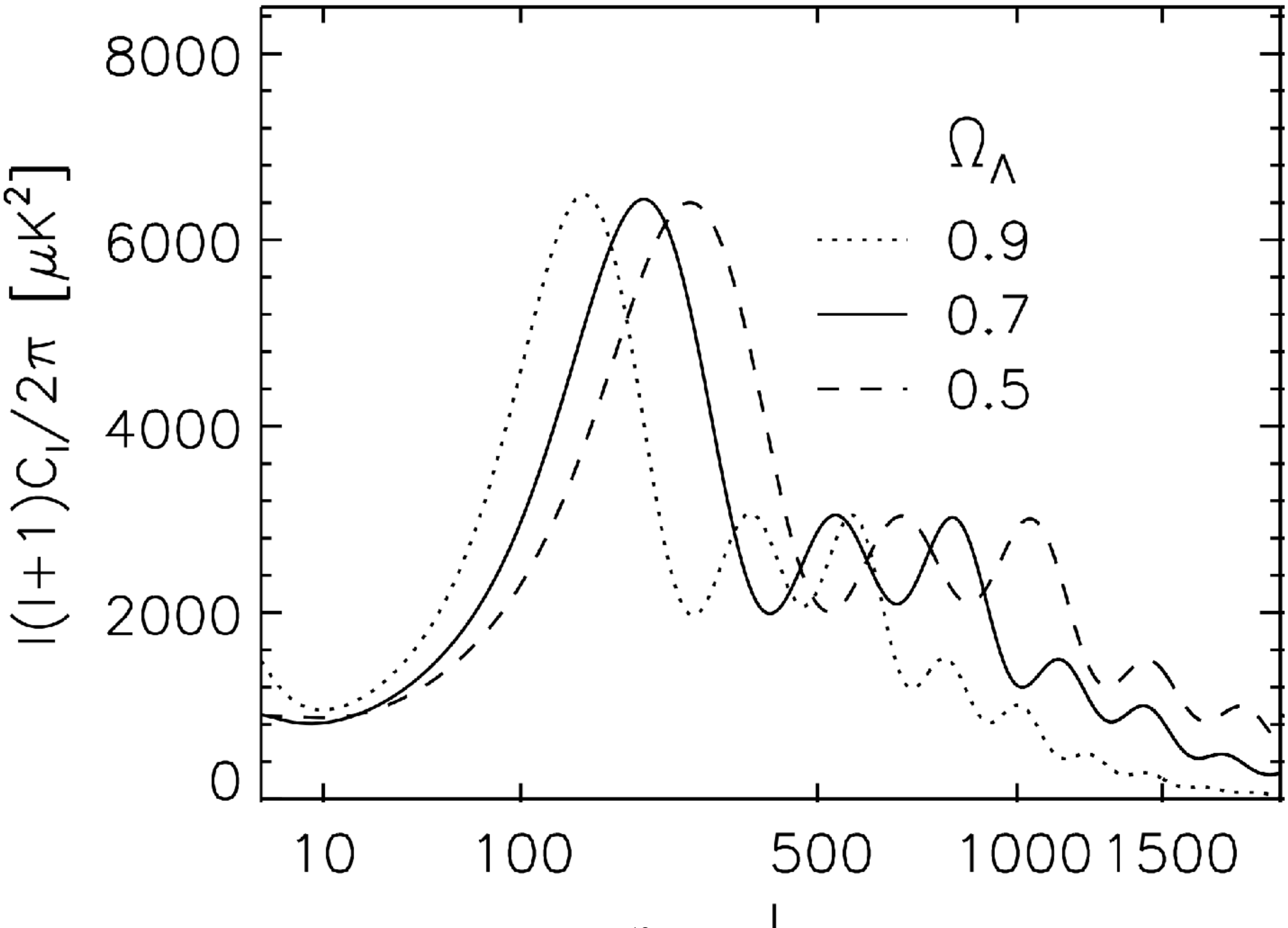
Two Other Effects

- **Spatial curvature**
  - We have been assuming a spatially-flat Universe with zero curvature (i.e., Euclidean space). What if it is curved?
- **Optical depth to Thomson scattering in a low-redshift Universe**
  - We have been assuming that the Universe is transparent to photons since the last scattering at z=1090. What if there is an extra scattering in a low-redshift Universe?

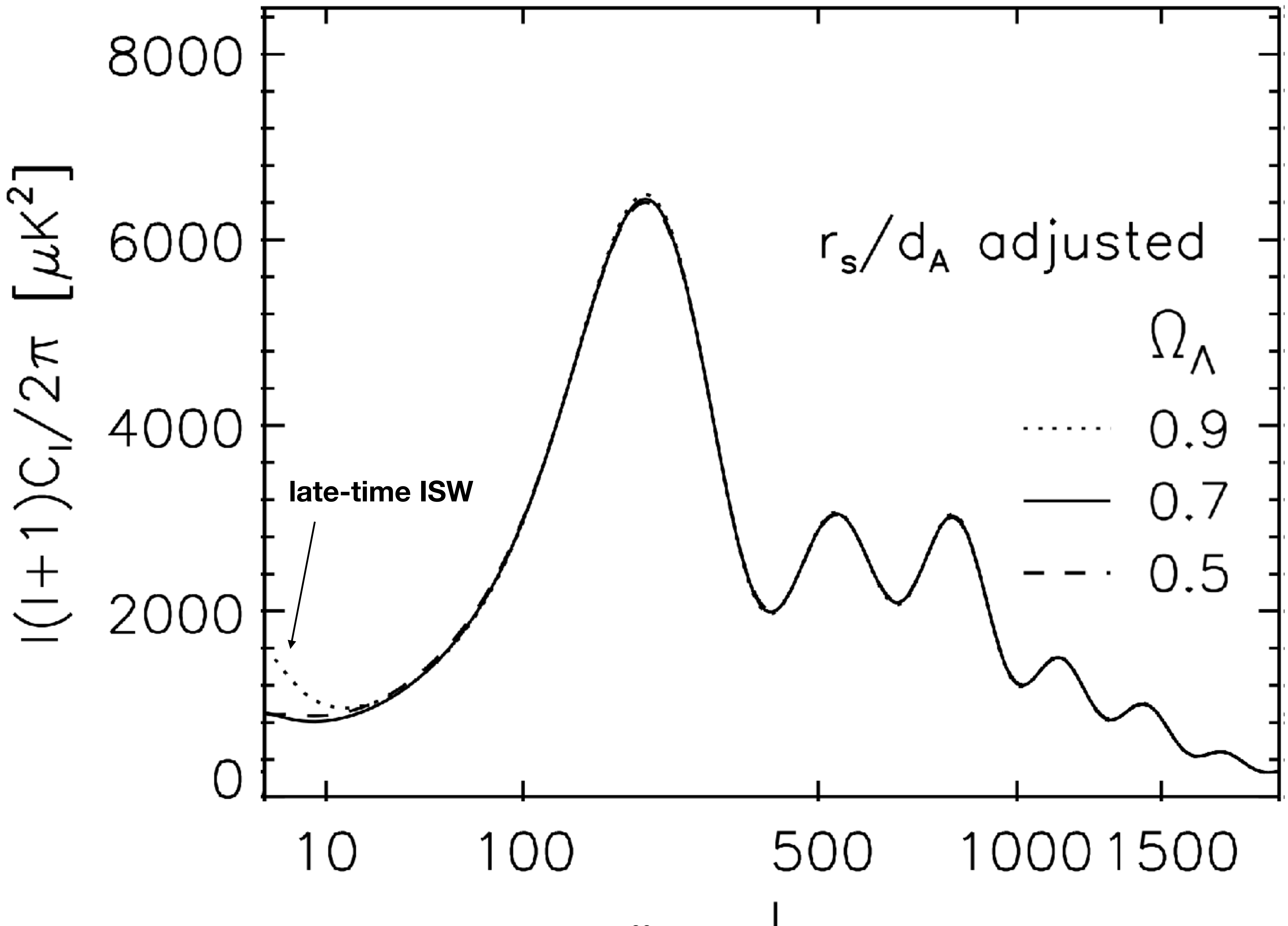
Spatial Curvature

- It changes the angular diameter distance, d_A, to the last scattering surface; namely,
  - $r_L \rightarrow d_A = R\,\mathit{sin}(r_L/R) = r_L(1 - r_L^2/6R^2) + \ldots$ for positively-curved space
  - $r_L \rightarrow d_A = R\,\mathit{sinh}(r_L/R) = r_L(1 + r_L^2/6R^2) + \ldots$ for negatively-curved space

Smaller angles (larger multipoles) for a negatively curved Universe



8000
6000
4000
2000
0
l(l+1)C_l/2π [μK^2]
10
100
500
1000
1500
l
Ω_Λ
0.9
0.7
0.5



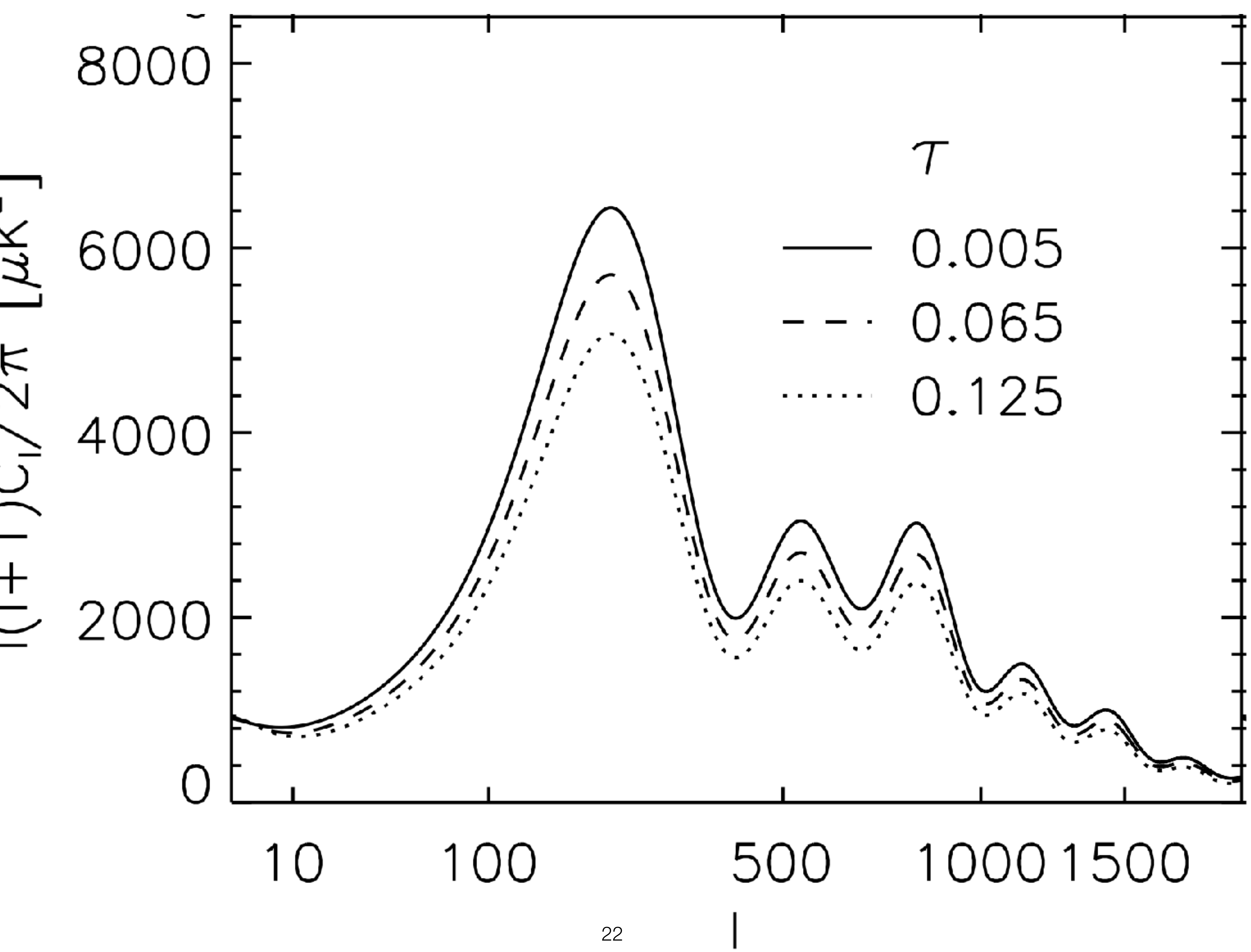
8000
6000
4000
2000
0
$l(l+1)C_l/2\pi$ [μK^2]
10
100
500
1000
1500
l
r_s/d_A adjusted
Ω_Λ
0.9
0.7
0.5
late-time ISW

Optical Depth

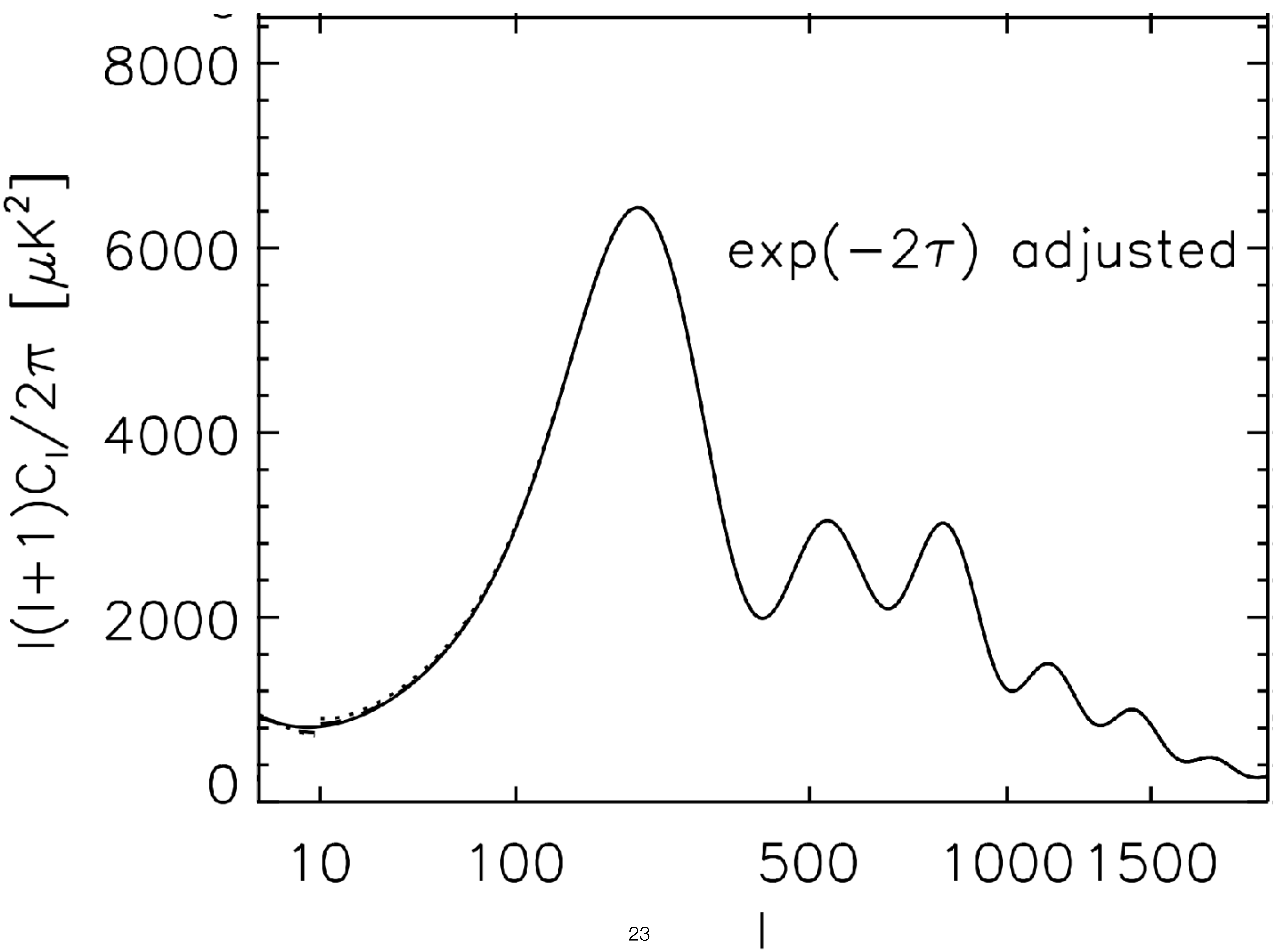
- Extra scattering by electrons in a low-redshift Universe damps temperature anisotropy

- $C_l \to C_l \exp(-2\tau)$ at $l \gtrsim 10$
  - where τ is the optical depth

$$\tau = c\sigma_T \int_{t_{\text{re-ionisation}}}^{t_0} dt\ \bar{n}_e$$



l(l+1)C_l/2π [μK²]
8000
6000
4000
2000
0
10
100
500
1000
1500
l
τ
0.005
0.065
0.125



8000
6000
4000
2000
0
l(l+1)C$_l$/2π [μK^2]
exp(−2τ) adjusted
10
100
500
1000
1500
l

Important consequence of the optical depth

- Since the power spectrum is uniformly suppressed by exp(–2τ) at l>~10, we cannot determine the amplitude of the power spectrum of the gravitational potential, $P_\Phi(q)$, independently of τ.

- Namely, what we constrain is the combination: **exp(–2τ)P_Φ(q)**

$$\propto \exp(-2\tau)A_s$$

- Breaking this degeneracy requires an independent determination of the optical depth. This requires **POLARISATION** of the CMB.

Cosmological Parameters Derived from the Power Spectrum

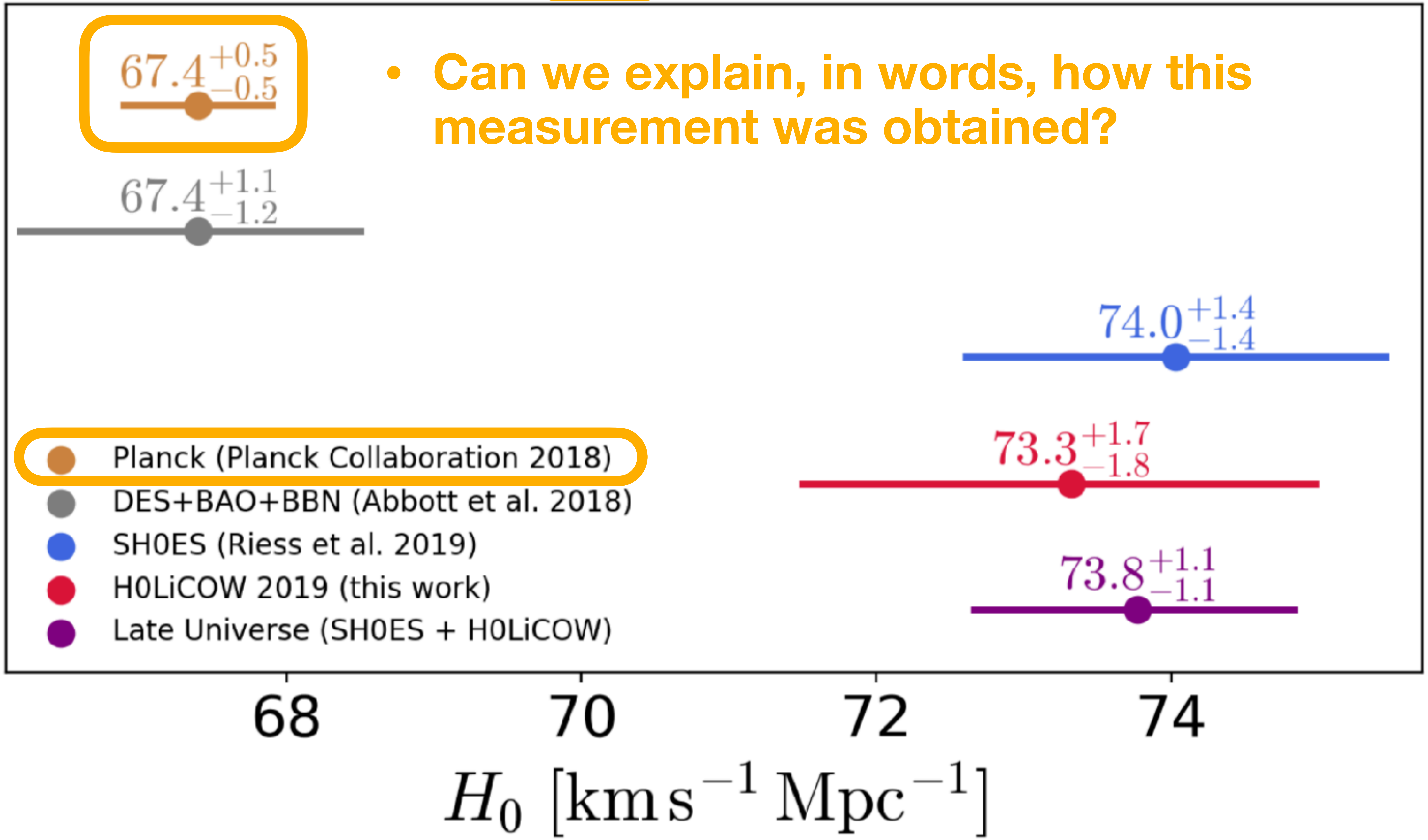
	WMAP	Planck	+CMB Lensing
$100\Omega_B h^2$	2.264 ± 0.050	2.222 ± 0.023	2.226 ± 0.023
$\Omega_D h^2$	0.1138 ± 0.0045	0.1197 ± 0.0022	0.1186 ± 0.0020
Ω_Λ	0.721 ± 0.025	0.685 ± 0.013	0.692 ± 0.012
n	0.972 ± 0.013	0.9655 ± 0.0062	0.9677 ± 0.0060
$10^9 A_s$	2.203 ± 0.067	$2.198^{+0.076}_{-0.085}$	2.139 ± 0.063
τ	0.089 ± 0.014	0.078 ± 0.019	0.066 ± 0.016
t_0 [100 Myr]	137.4 ± 1.1	138.13 ± 0.38	137.99 ± 0.38
H_0	70.0 ± 2.2	67.31 ± 0.96	67.81 ± 0.92
$\Omega_M h^2$	0.1364 ± 0.0044	0.1426 ± 0.0020	0.1415 ± 0.0019
$10^9 A_s e^{-2\tau}$	1.844 ± 0.031	1.880 ± 0.014	1.874 ± 0.013
σ_8	0.821 ± 0.023	0.829 ± 0.014	0.8149 ± 0.0093

Wong et al. (2020)

The Hubble Constant Tension

The role of the CMB flat ΛCDM

- Can we explain, in words, how this measurement was obtained?



CMB -> Distance Ratio -> (Physics) -> H_0

Physical Assumption: the sound horizon r_s

- The CMB peak positions are controlled by $\cos(qr_s)$.
- We measure q in the angular wavenumber, $l \sim qr_L$.
- Thus, the CMB power spectrum gives a direct measurement of the distance **ratio**: r_s/r_L.
  - You already know how to obtain $a_0 r_s$ = 145 Mpc (see Lecture 5).
  - Today we saw how r_L depends on cosmology (in a flat Universe):

$$a_0 r_L = c a_0 \int_{a_L}^{a_0} \frac{da}{a^2 H(a)} \propto \int_{a_L}^{a_0} \frac{da}{a^2 \sqrt{\Omega_M h^2 a^{-3} + h^2 - \Omega_M h^2}}$$

CMB -> Distance Ratio -> (Physics) -> H_0

Physical Assumption: the sound horizon r_s

- The CMB peak positions are controlled by $\cos(qr_s)$.
- We measure q in the angular wav
- Thus, the CMB power spectrum g ce **ratio**: r_s/r_L.

If we used an incorrect value of r_s, we would infer r_L incorrectly, hence an incorrect value of H_0!

  - You already know how to obtain **$a_0 r_s$ = 145 Mpc (see Lecture 5)**.
  - Today we saw how r_L depends on cosmology (in a flat Universe):

$$a_0 r_L = c a_0 \int_{a_L}^{a_0} \frac{da}{a^2 H(a)} \propto \int_{a_L}^{a_0} \frac{da}{a^2\sqrt{\Omega_M h^2 a^{-3} + h^2 - \Omega_M h^2}}$$

So…?

Bernal, Verde and Riess (2016); Poulin et al. (2019)

- The CMB-inferred value of H_0 is too low, by 10 percent.
  - This means that the inferred value of r_L is too high, by 10 percent.
  - This may mean that the value of r_s we calculated using the standard understanding of physics was too high by 10 percent.
  - **If we managed to reduce the calculated value of r_s by 10 percent**, we could resolve the Hubble constant tension.
- Is that possible? Not really, but one way to achieve this would be to increase H(a) by 10 percent in the radiation era. => **Early Dark Energy?**

Part II: Basics of the CMB Polarisation

Temperature (smoothed)

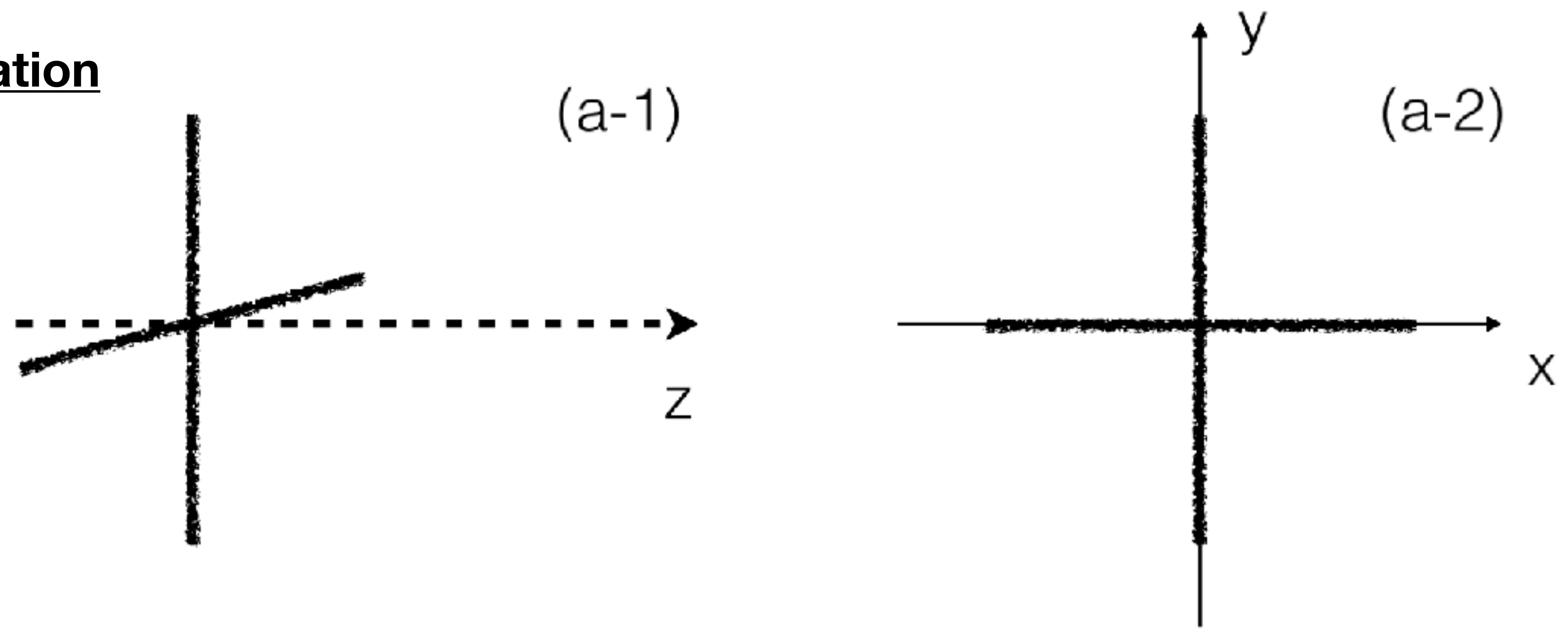
CMB is weakly polarised!

Credit: ESA

Temperature (smoothed) + Polarisation

Polarisation

<u>No polarisation</u>



<u>Polarised in x-direction</u>

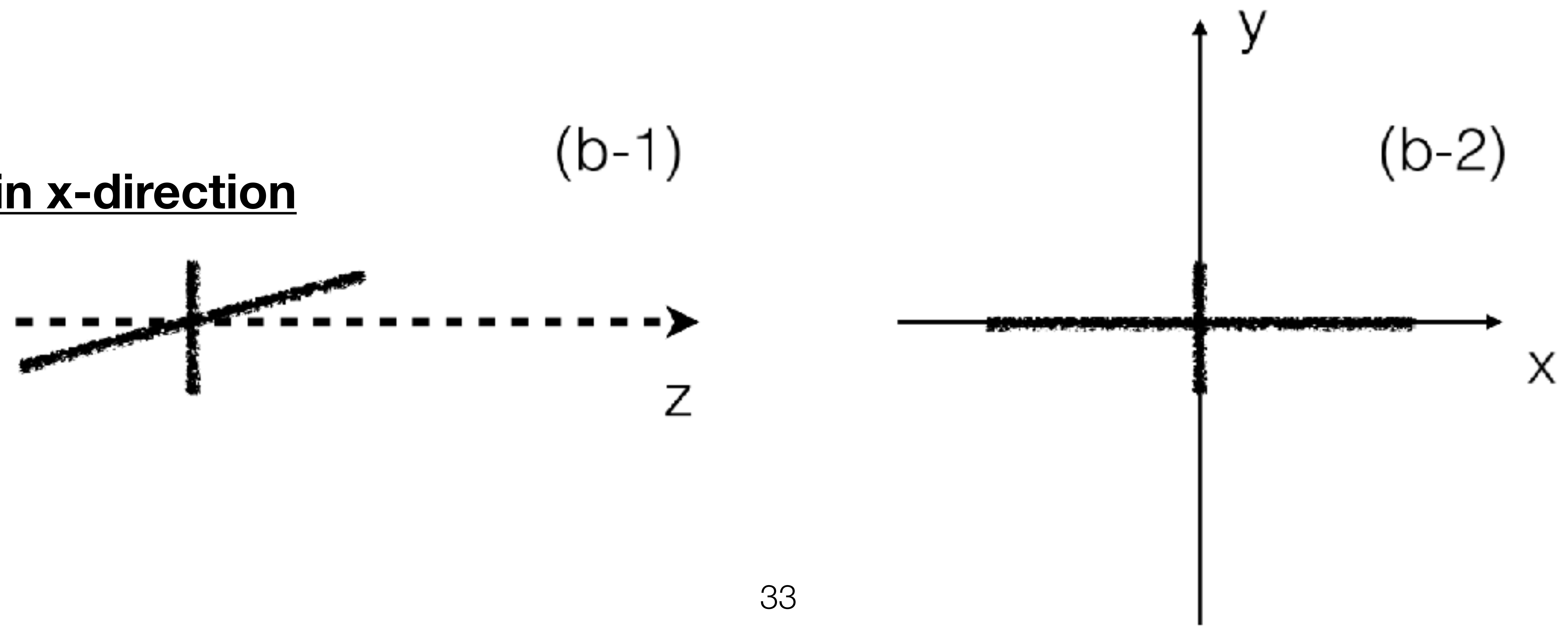


Photo Credit: TALEX


Photo Credit: TALEX
horizontally polarised

Photo Credit: TALEX

Generation of polarisation

The necessary and sufficient conditions

- To generate polarisation, we must satisfy the following two conditions
  - **Scattering**
  - **Anisotropic incident light**

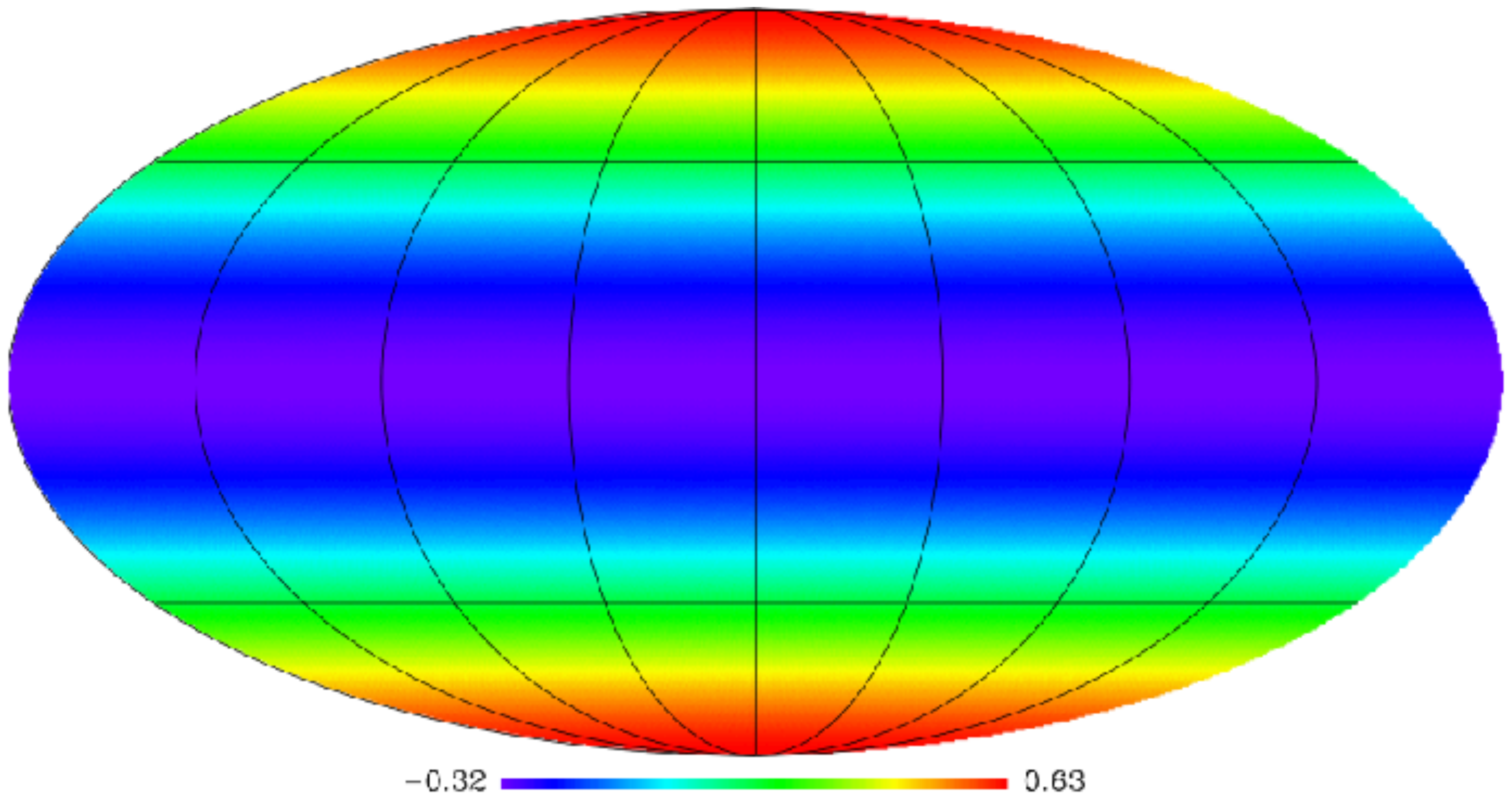
- However, the Universe does not have a preferred direction. How do we generate anisotropic incident light?

Physics of CMB Polarisation

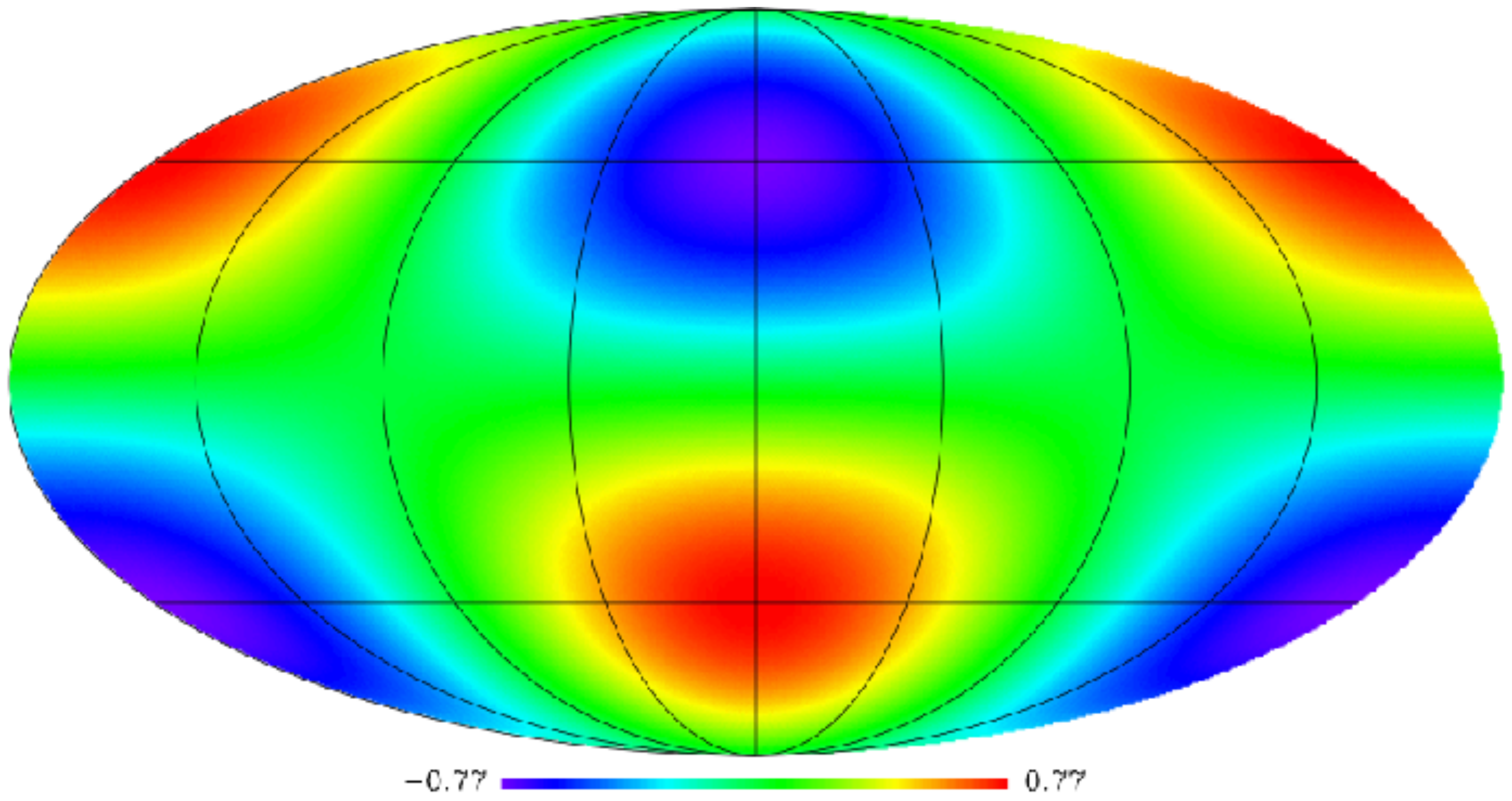
Necessary and sufficient condition: Scattering and *Local* Quadrupole Anisotropy

Credit：Wayne Hu

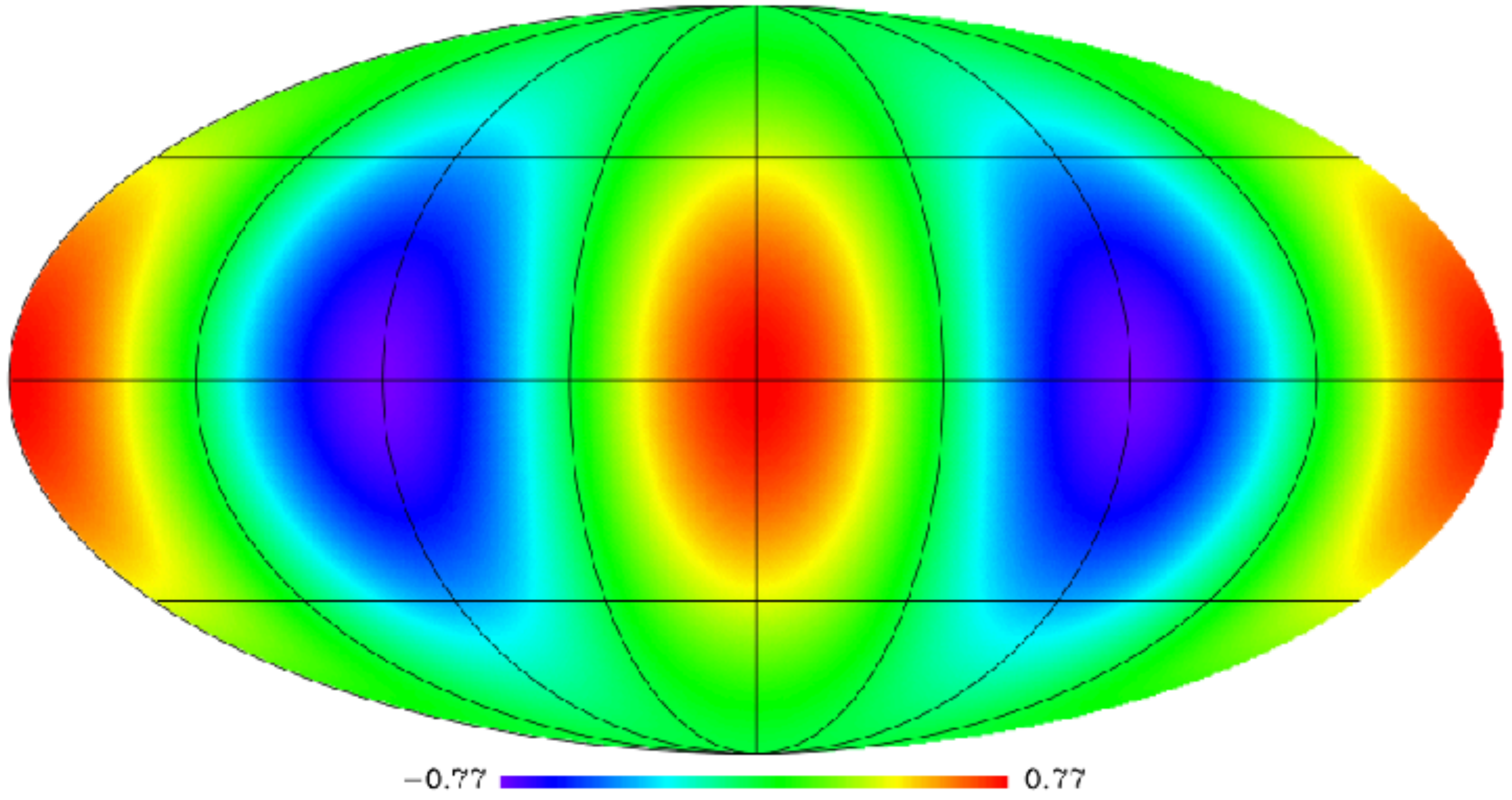
(l,m)=(2,0)



(l,m)=(2,1)



(l,m)=(2,2)



Quadrupole temperature anisotropy seen by an electron

Generation of temperature quadrupole

The punch line

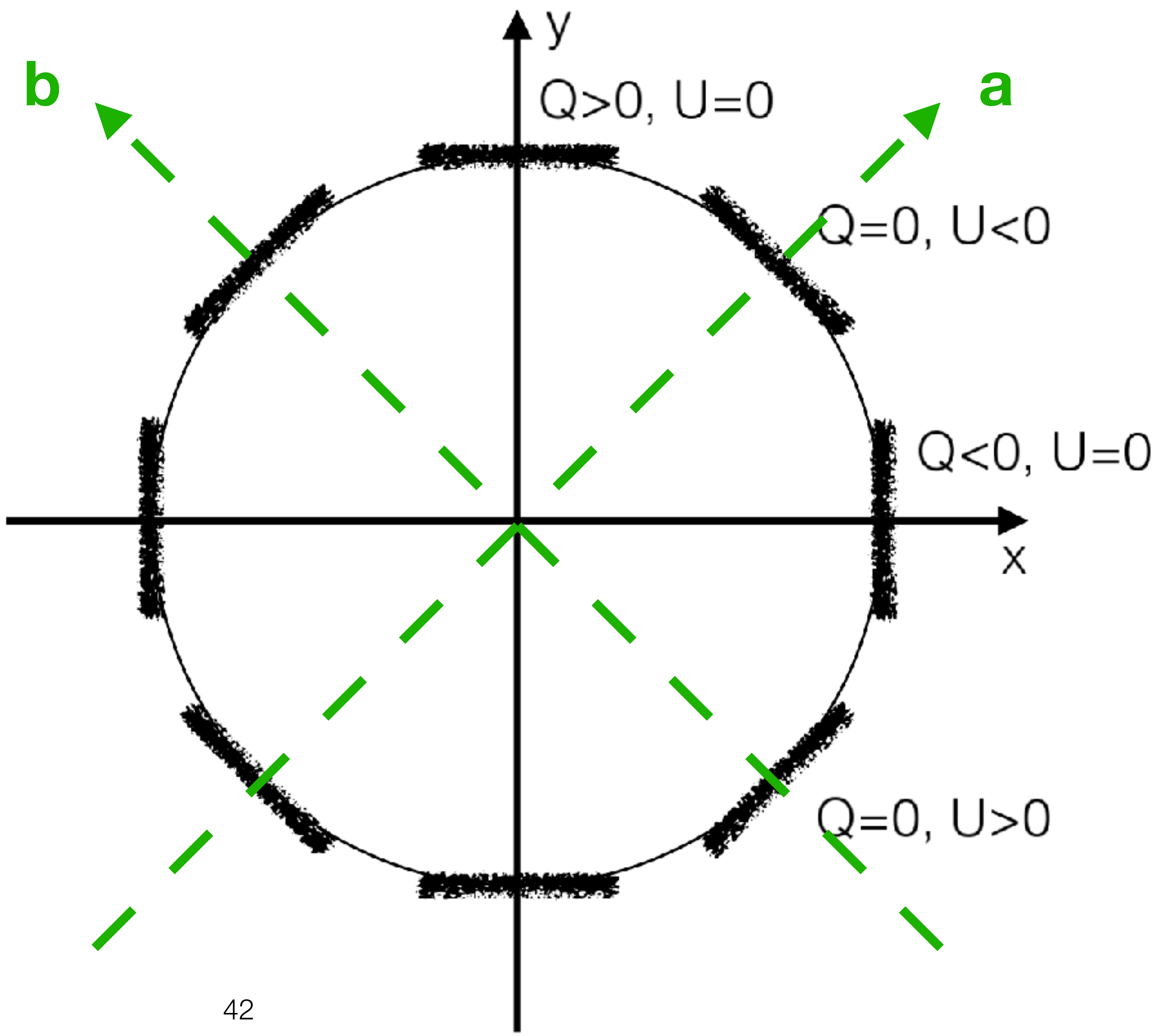
- When the Thomson scattering is efficient (i.e., tight coupling between photons and baryons via electrons), the distribution of photons from the rest frame of baryons appears isotropic.
- **Only when tight coupling weakens**, a local quadrupole temperature anisotropy in the rest frame of a photon-baryon fluid can be generated.
- In fact, "a local *temperature anisotropy in the rest frame of a photon-baryon fluid*" is equal to **viscosity.**

Part III: Stokes Parameters

Stokes Parameters
[Flat Sky, Cartesian coordinates]

$$Q \propto E_x^2 - E_y^2$$

$$U \propto E_a^2 - E_b^2$$

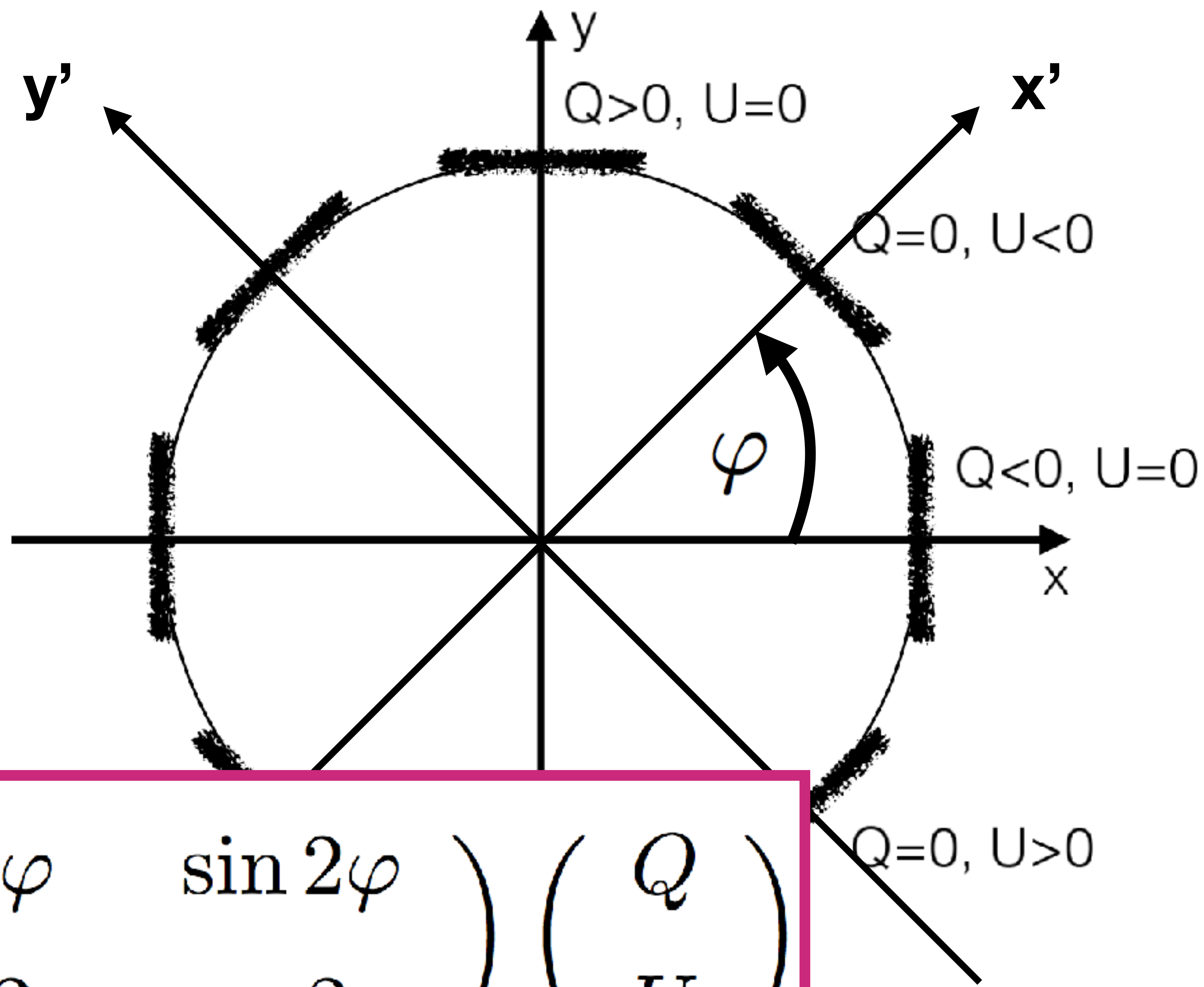


Stokes Parameters
change under coordinate rotation

Under (x,y) -> (x',y'):

$$Q \longrightarrow \tilde{Q}$$

$$U \longrightarrow \tilde{U}$$



$$\begin{pmatrix} \tilde{Q} \\ \tilde{U} \end{pmatrix} = \begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ -\sin 2\varphi & \cos 2\varphi \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}$$

Compact Expression

- Using an imaginary number, write $Q + iU$

Then, under the coordinate rotation we have

$$\tilde{Q} + i\tilde{U} = \exp(-2i\varphi)(Q + iU)$$

$$\tilde{Q} - i\tilde{U} = \exp(2i\varphi)(Q - iU)$$

Alternative Expression

- With the polarisation amplitude, P, and angle, α , defined by

$$P \equiv \sqrt{Q^2 + U^2}, \;\; U/Q \equiv \tan 2\alpha$$

We write

$$Q + iU = P \exp(2i\alpha)$$

- **Then, under coordinate rotation we have**

$$\tilde{\alpha} = \alpha - \varphi$$

and P is invariant under rotation.

Help!

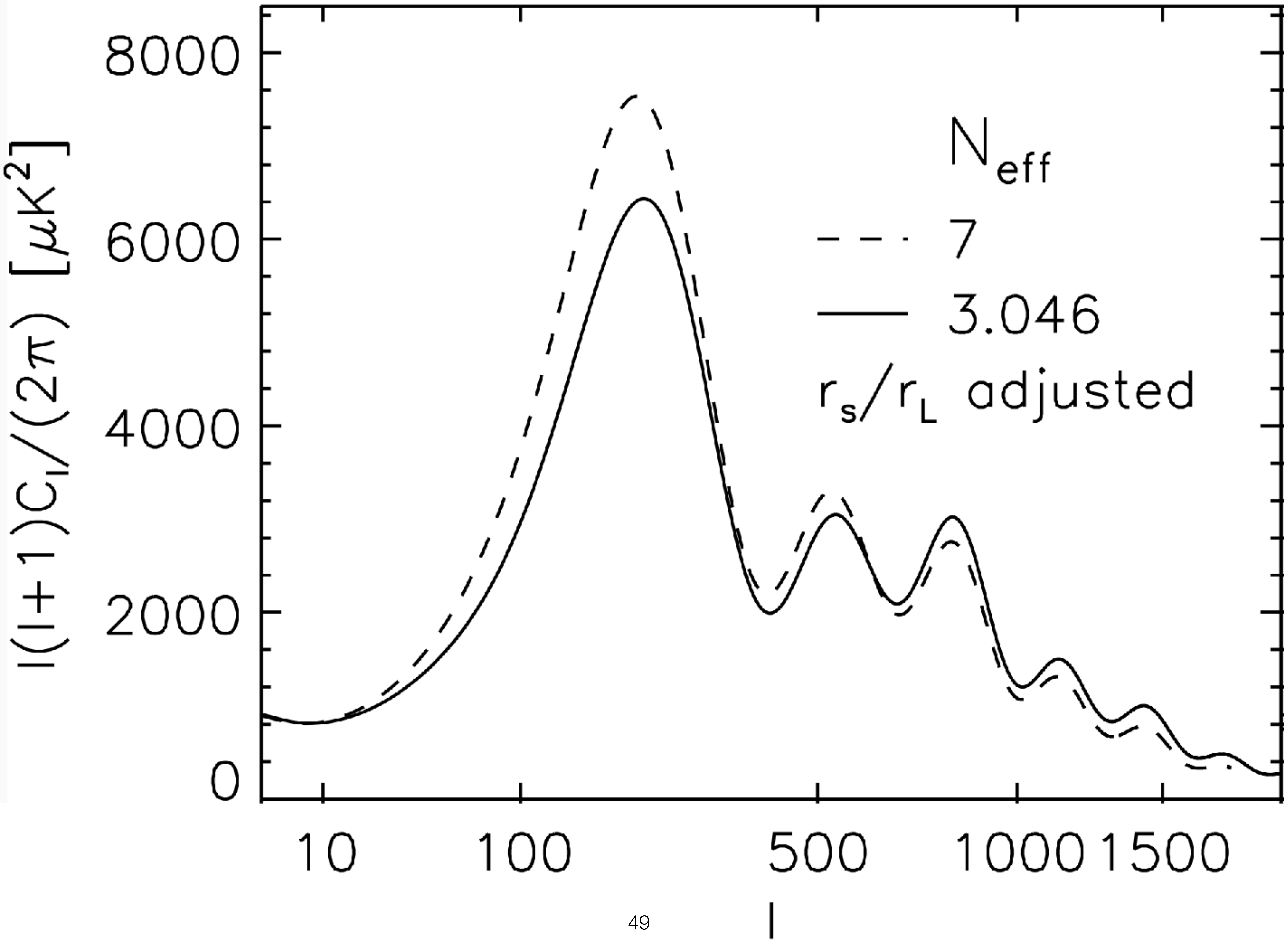
- That Q and U depend on coordinates is not very convenient…
    - Someone said, “*I measured Q!*” but then someone else may say, “*No, it’s U!*”. They flight to death, only to realise that their coordinates are 45 degrees rotated from one another…
- The best way to avoid this unfortunate fight is to define a **coordinate-independent quantity** for the distribution of polarisation patterns in the sky

To achieve this, we need to go to Fourier space

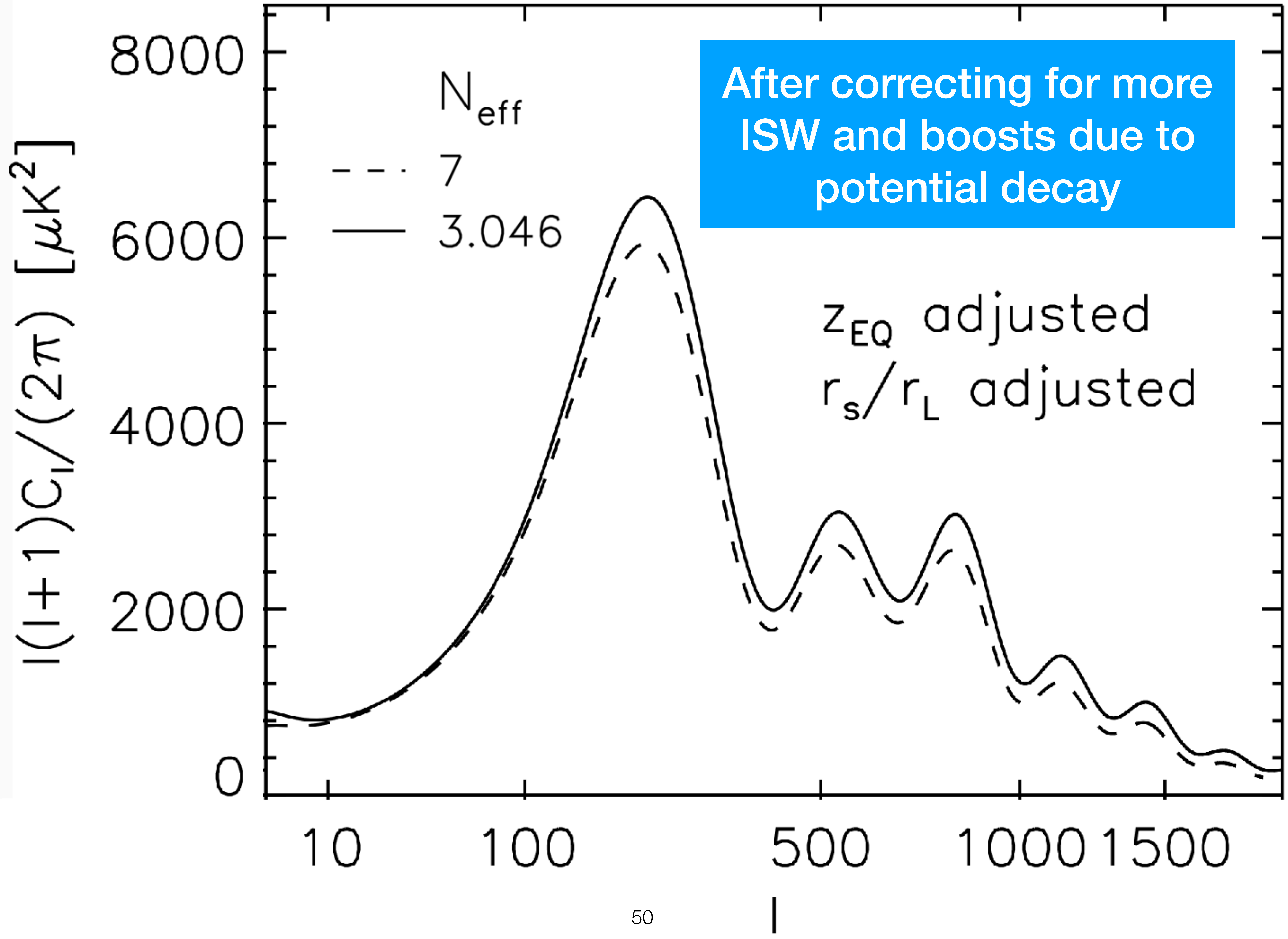
Appendix: Effects of Neutrinos on the Temperature Power Spectrum

The Effects of Relativistic Neutrinos

- To see the effects of relativistic neutrinos, we artificially increase the number of neutrino species from 3 to 7
    - Great energy density in neutrinos, i.e., greater energy density in radiation
    - **(1)** Longer radiation domination -> More ISW and boosts due to potential decay



8000
6000
4000
2000
0
l(l+1)C_l/(2π) [μK^2]
10
100
500
1000
1500
l
N_eff
7
3.046
r_s/r_L adjusted


After correcting for more ISW and boosts due to potential decay
N_{eff}
7
3.046
z_{EQ} adjusted
r_s/r_L adjusted
$l(l+1)C_l/(2\pi)$ [μK^2]
8000
6000
4000
2000
0
10
100
500
1000
1500
l

(2): Viscosity Effect on the Amplitude of Sound Waves

The solution is

$$X = -C\cos(\varphi + \theta)$$

where

$$C \equiv \sqrt{(-\zeta + \Delta A_\nu)^2 + \Delta B_\nu^2}$$

$$\approx \zeta(1 + 4R_\nu/15)^{-1}$$

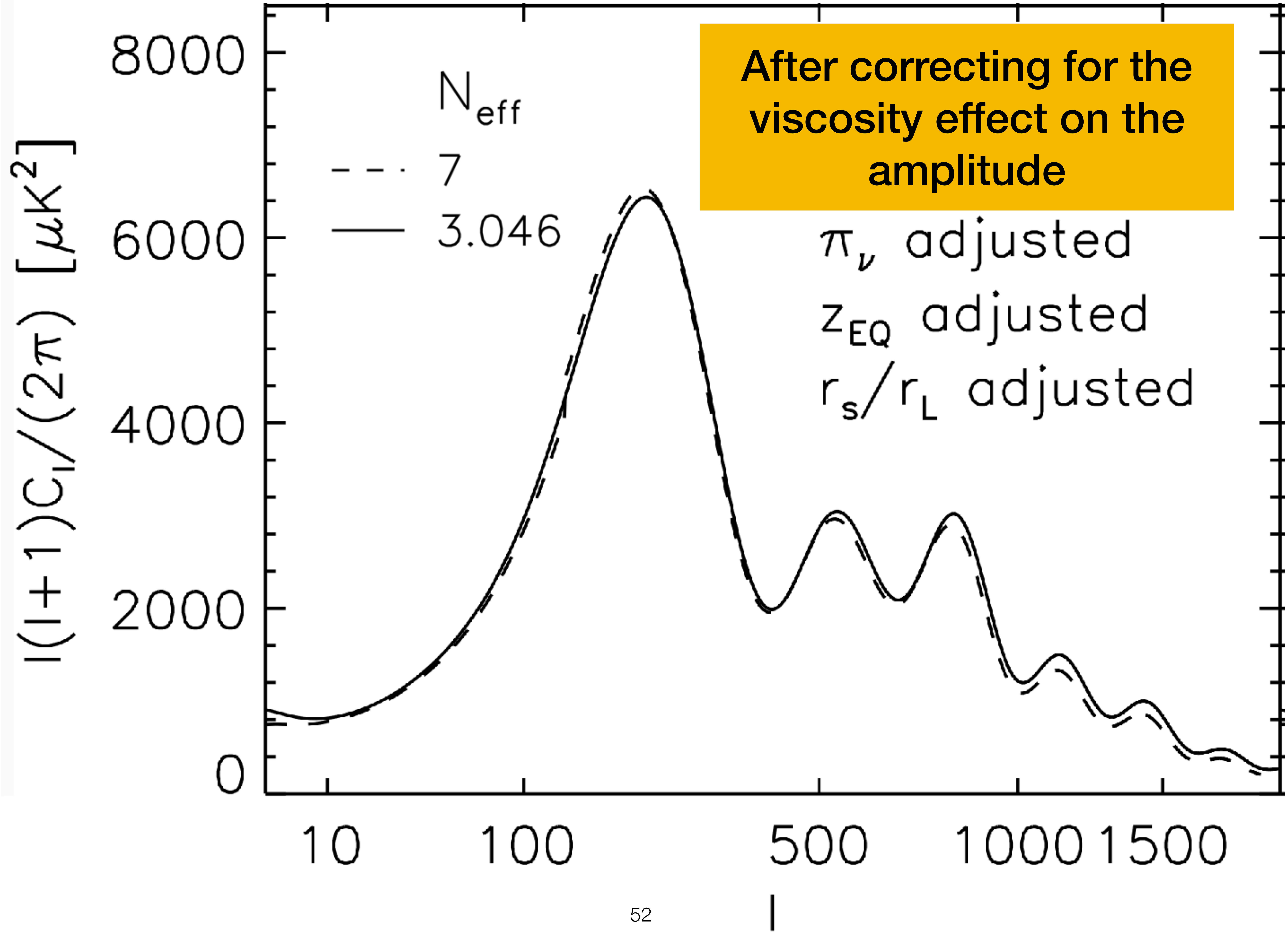
Hu & Sugiyama (1996)

$$R_\nu \equiv \bar{\rho}_\nu/(\bar{\rho}_\gamma + \bar{\rho}_\nu)$$
$$\approx 0.409$$

$$\tan\theta = -\frac{\Delta B_\nu}{-\zeta + \Delta A_\nu} \approx 0.063\pi$$

Phase shift!

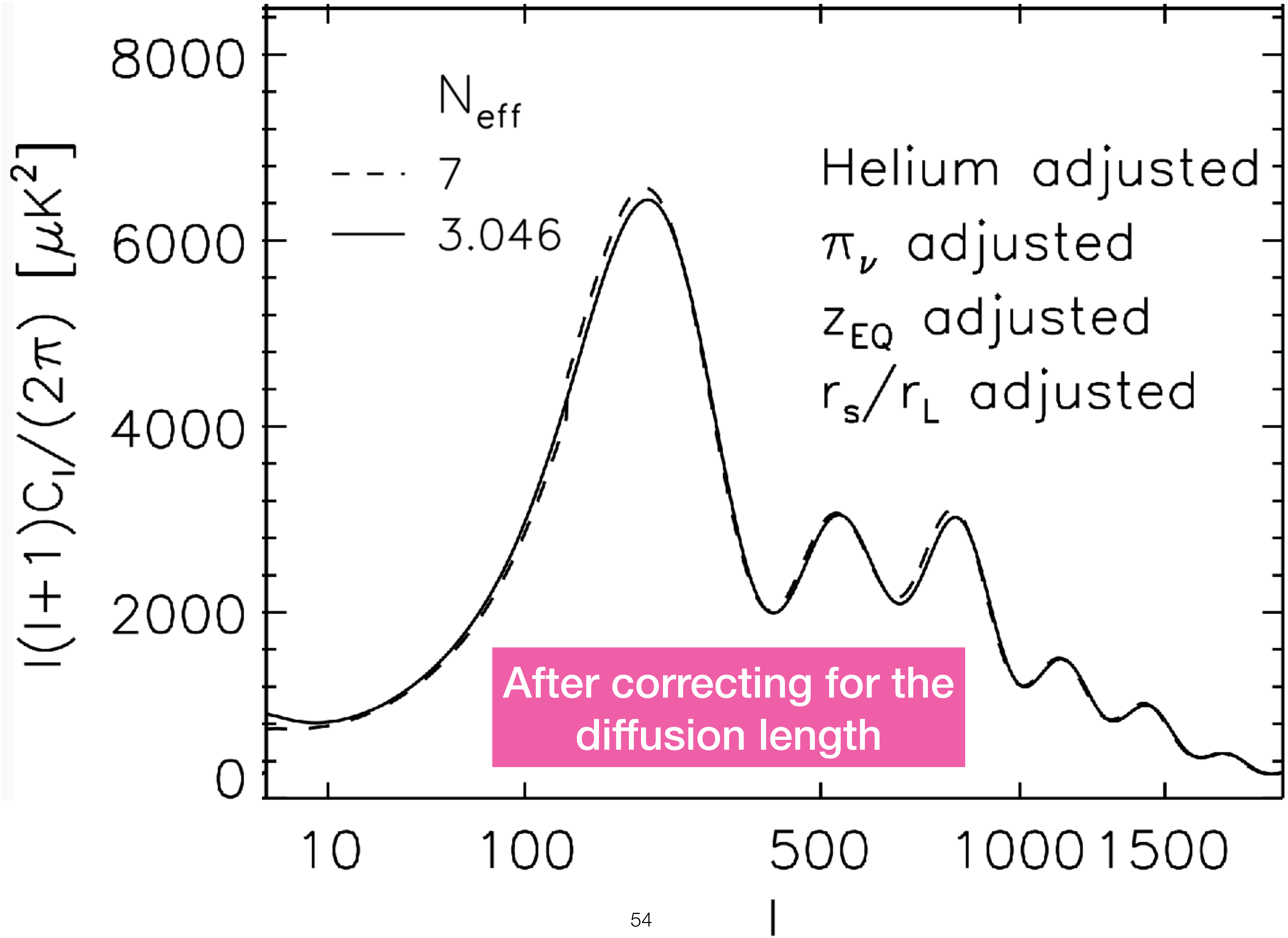
Bashinsky & Seljak (2004)


After correcting for the viscosity effect on the amplitude
N_{eff}
7
3.046
π_ν adjusted
z_{EQ} adjusted
r_s/r_L adjusted
$l(l+1)C_l/(2\pi)$ [μK^2]
8000
6000
4000
2000
0
10
100
500
1000
1500
l

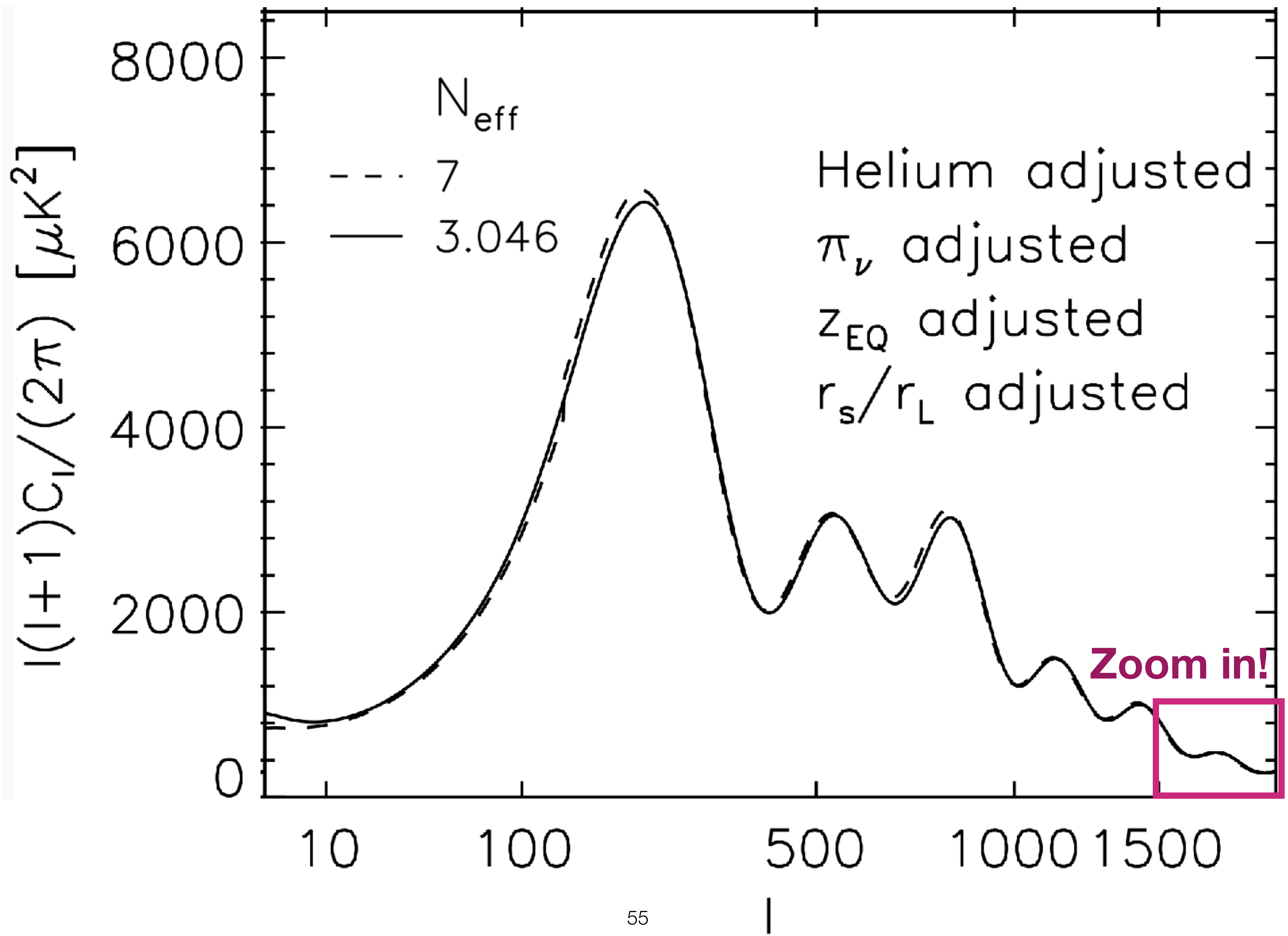
Bashinsky & Seljak (2004)

(3): Change in the Silk Damping

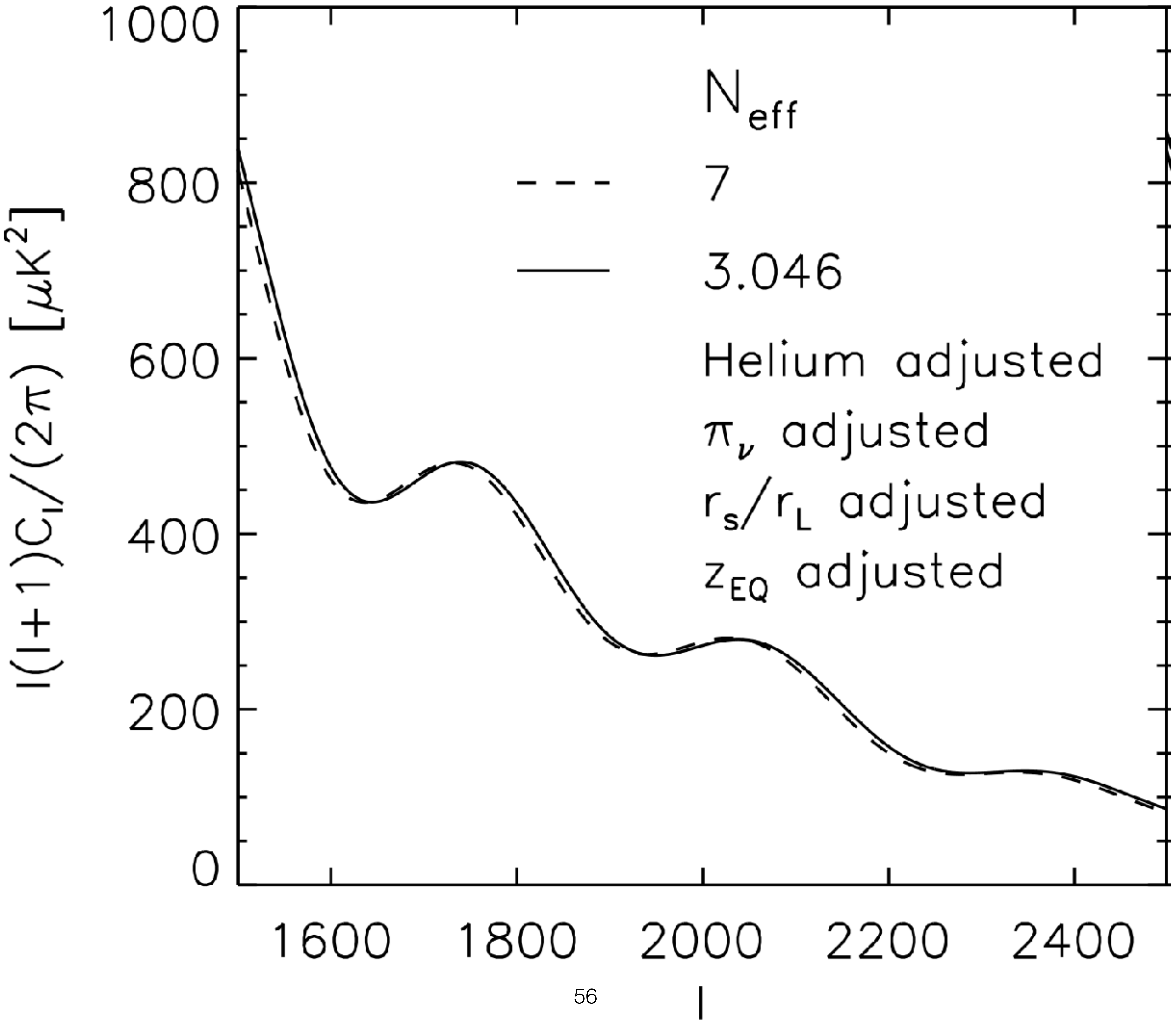
- Greater neutrino energy density implies greater Hubble expansion rate, $H^2 = 8\pi G \sum \rho_\alpha / 3$
- This reduces the sound horizon in proportion to H^{-1}, as $r_s \sim c_s H^{-1}$
- This also reduces the diffusion length, but in proportional to $H^{-1/2}$, as $a/q_{silk} \sim (\sigma_T n_e H)^{-1/2}$ **Consequence of the random walk!**
- As a result, l_{silk} decreases relative to the first peak position, enhancing the Silk damping



N_{eff}
7
3.046
Helium adjusted
π_ν adjusted
z_{EQ} adjusted
r_s/r_L adjusted
After correcting for the diffusion length
$l(l+1)C_l/(2\pi)$ [μK^2]
8000
6000
4000
2000
0
10
100
500
1000
1500
l



$l(l+1)C_l/(2\pi)$ [μK^2]
8000
6000
4000
2000
0
N_{eff}
7
3.046
Helium adjusted
π_ν adjusted
z_{EQ} adjusted
r_s/r_L adjusted
Zoom in!
10
100
500
1000
1500
l



N_{eff}
7
3.046
Helium adjusted
π_ν adjusted
r_s/r_L adjusted
z_{EQ} adjusted
$l(l+1)C_l/(2\pi)$ [μK^2]
1000
800
600
400
200
0
1600
1800
2000
2200
2400
l

(4): Viscosity Effect on the Phase of Sound Waves

The solution is

$$X = -C\cos(\varphi + \theta)$$

$$R_\nu \equiv \bar{\rho}_\nu/(\bar{\rho}_\gamma + \bar{\rho}_\nu) \approx 0.409$$

where

$$C \equiv \sqrt{(-\zeta + \Delta A_\nu)^2 + \Delta B_\nu^2}$$

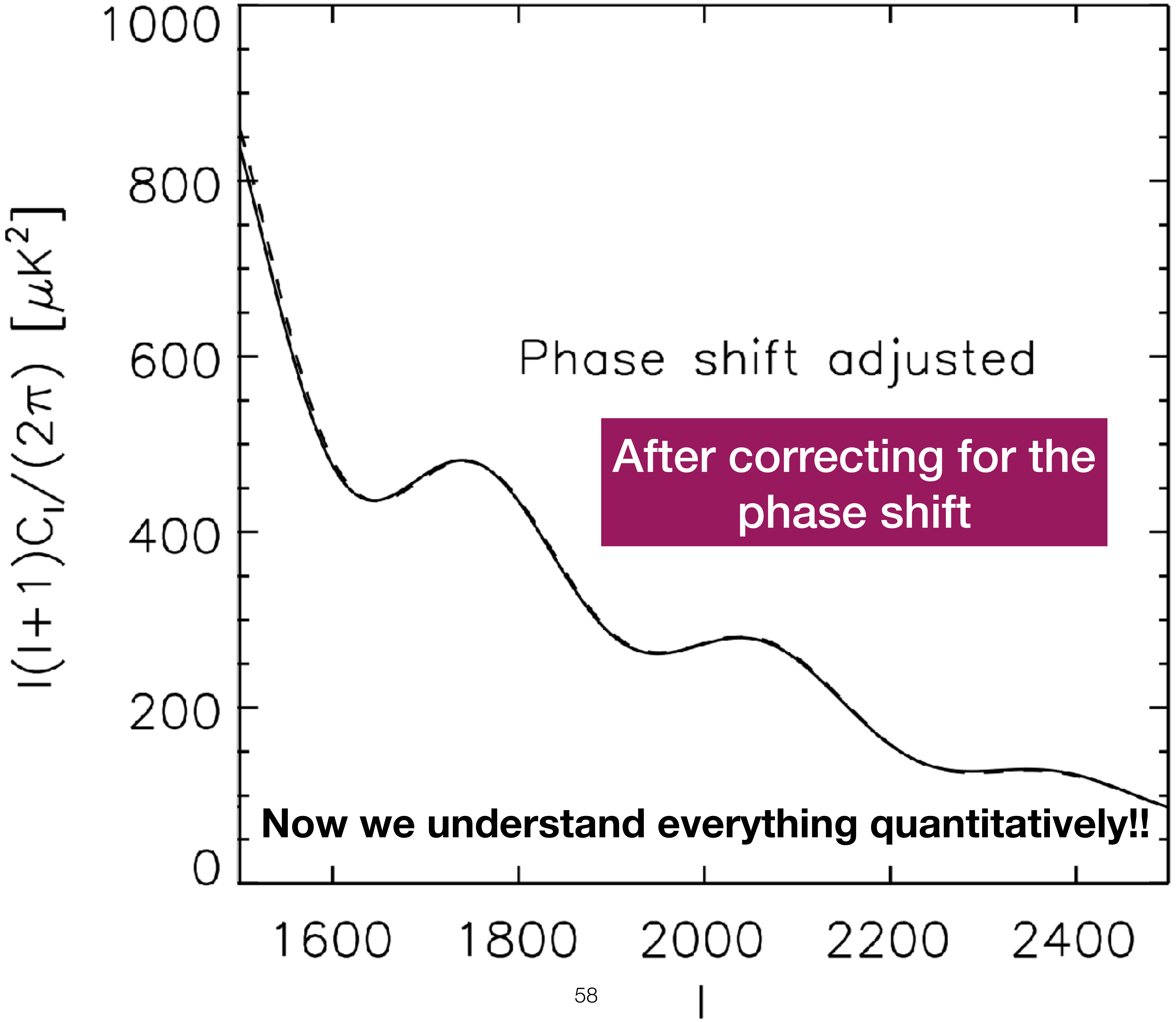
$$\approx \zeta(1 + 4R_\nu/15)^{-1}$$

Hu & Sugiyama (1996)

$$\tan\theta = -\frac{\Delta B_\nu}{\zeta + \Delta A_\nu} \approx 0.063\pi$$

Phase shift!

Bashinsky & Seljak (2004)



1000
800
600
400
200
0
l(l+1)Cl/(2π) [μK²]
Phase shift adjusted
After correcting for the phase shift
Now we understand everything quantitatively!!
1600
1800
2000
2200
2400
l