Lecture 6: Acoustic Oscillation

Part I: Hydrodynamics of Photon-baryon Fluid

Creation of sound waves in the fireball Universe

Basic equations

- Conservation equations (energy and momentum)
- Equation of state, relating pressure to energy density of the α component

$$P_\alpha = P_\alpha(\rho_\alpha)$$

- General relativistic version of the "Poisson equation", relating gravitational potential to energy density

$$\nabla^2 \Phi(t, \boldsymbol{x}) = 4\pi G a^2(t) \delta\rho_M(t, \boldsymbol{x})$$

- Evolution of the "anisotropic stress" (viscosity)

↑ This is still the Newtonian expression, which must be extended to GR.

Energy Conservation

- **Total energy conservation:** α = baryon, photon, neutrino, dark matter

$$\sum_\alpha \left\{ \delta\dot\rho_\alpha + \frac{\dot a}{a}(3\delta\rho_\alpha + 3\delta P_\alpha + \nabla^2\pi_\alpha) - 3(\bar\rho_\alpha + \bar P_\alpha)\dot\Psi + \frac{1}{a^2}(\bar\rho_\alpha + \bar P_\alpha)\nabla^2\delta u_\alpha \right\} = 0\,,$$

anisotropic stress:
[or, viscosity]

$$\Delta T_{ij} = a^2\partial_i\partial_j\pi$$

velocity potential

$$\mathbf{v}_\alpha = \frac{1}{a}\nabla\delta u_\alpha$$

(
- C.f., Total energy conservation [unperturbed]

$$\sum_\alpha \left[\dot{\bar\rho}_\alpha + \frac{3\dot a}{a}(\bar\rho_\alpha + \bar P_\alpha)\right] = 0$$
)

- C.f., Newtonian result

$$\delta\dot\rho + \bar\rho\nabla^2\delta u = 0$$

$$\bar\rho\delta\dot u = -\delta P - \bar\rho\Phi$$

Energy Conservation

- **Total energy conservation:** α = baryon, photon, neutrino, dark matter

$$\sum_\alpha \left\{ \delta\dot{\rho}_\alpha + \frac{\dot{a}}{a}(3\delta\rho_\alpha + 3\delta P_\alpha + \nabla^2\pi_\alpha) - 3(\bar{\rho}_\alpha + \bar{P}_\alpha)\dot{\Psi} \right.$$
$$\left. + \frac{1}{a^2}(\bar{\rho}_\alpha + \bar{P}_\alpha)\nabla^2\delta u_\alpha \right\} = 0\,,$$

- **Again, this is the effect of locally-defined inhomogeneous scale factor**, i.e.,
  - The spatial metric is given by $ds^2 = a^2(t)\exp(-2\Psi)d\mathbf{x}^2$
  - Thus, locally we can define a new scale factor: $\tilde{a}(t,\mathbf{x}) = a(t)\exp(-\Psi)$

Energy Conservation

- **Total energy conservation:** α = baryon, photon, neutrino, dark matter

$$\sum_\alpha \left\{ \delta\dot\rho_\alpha + \frac{\dot a}{a}(3\delta\rho_\alpha + 3\delta P_\alpha + \nabla^2\pi_\alpha) - 3(\bar\rho_\alpha + \bar P_\alpha)\dot\Psi + \boxed{\frac{1}{a^2}(\bar\rho_\alpha + \bar P_\alpha)\nabla^2\delta u_\alpha} \right\} = 0\,,$$

- **Momentum flux going outward (inward) -> reduction (increase) in the energy density**

(- C.f., Newtonian result

$$\boxed{\delta\dot\rho + \bar\rho\nabla^2\delta u = 0}$$

$$\bar\rho\delta\dot u = -\delta P - \bar\rho\Phi$$)

Momentum Conservation

- **Total momentum conservation** $\mathbf{v}_\alpha = \frac{1}{a}\nabla\delta u_\alpha$

$$\sum_\alpha \left\{ \frac{\partial}{\partial t}[(\bar{\rho}_\alpha + \bar{P}_\alpha)\delta u_\alpha] + \frac{3\dot{a}}{a}(\bar{\rho}_\alpha + \bar{P}_\alpha)\delta u_\alpha + (\bar{\rho}_\alpha + \bar{P}_\alpha)\Phi + \delta P_\alpha + \nabla^2 \pi_\alpha \right\} = 0\,,$$

- **Cosmological redshift of the momentum**
- **Gravitational force given by potential gradient**
- **Force given by pressure gradient**
- **Force given by gradient of anisotropic stress**

(
- C.f., Newtonian result

$$\delta\dot{\rho} + \bar{\rho}\nabla^2\delta u = 0$$

$$\boxed{\bar{\rho}\delta\dot{u} = -\delta P - \bar{\rho}\Phi}$$
)

Equation of State

- Pressure of non-relativistic species (i.e., baryons and cold dark matter) can be ignored relative to the energy density. Thus, we set them to zero: $\mathbf{P_B=0=P_D}$ and $\mathbf{\delta P_B=0=\delta P_D}$

- Unperturbed pressure of relativistic species (i.e., photons and relativistic neutrinos) is given by the third of the energy density, i.e., $\mathbf{P_\gamma=\rho_\gamma/3}$ and $\mathbf{P_\nu=\rho_\nu/3}$

- Perturbed pressure involves contributions from the bulk viscosity:

$$\delta P_\gamma = (\delta\rho_\gamma - \nabla^2\pi_\gamma)/3$$

$$\delta P_\nu = (\delta\rho_\nu - \nabla^2\pi_\nu)/3$$

- Pres cold dark sity. Thu

- Unp tons and energy den

If you know a bit of GR:
The reason for this is that the trace of the stress-energy of relativistic species vanishes:
$\sum_{\mu=0,1,2,3} T_{\mu}^{\mu} = 0$

$$T_0^0 + \sum_{i=1}^{3} T_i^i = -\rho + 3P + \nabla^2 \pi = 0$$

- Perturbed pressure involves contributions from the bulk viscosity: $\delta P_\gamma = (\delta\rho_\gamma - \nabla^2 \pi_\gamma)/3$

$$\delta P_\nu = (\delta\rho_\nu - \nabla^2 \pi_\nu)/3$$

Two remarks

Do we need to sum over α?

- In the standard scenario that we shall assume throughout this lecture,
  - Energy densities are conserved separately; thus we do not need to sum over all species.
  - Momentum densities of photons and baryons are NOT conserved separately but they are coupled via **Thomson & Coulomb scattering**. This must be taken into account when writing down separate momentum conservation equations.
- Next, we solve the conservation equations to derive the sound wave propagating in the fireball Universe.

Conservation Equations for Photons and Baryons

- Fourier transformation replaces $\nabla^2 \to -q^2$

$$X(t,\boldsymbol{x}) = (2\pi)^{-3}\int d^3q\; X_{\boldsymbol{q}}(t)\exp(i\boldsymbol{q}\cdot\boldsymbol{x})$$

$$\frac{\partial}{\partial t}(\delta\rho_\gamma/\bar{\rho}_\gamma) - \frac{4q^2}{3a^2}\delta u_\gamma = 4\dot{\Psi}$$

$$\frac{\partial}{\partial t}(\delta\rho_B/\bar{\rho}_B) - \frac{q^2}{a^2}\delta u_B = 3\dot{\Psi}$$

momentum transfer via scattering

$$a\frac{\partial}{\partial t}(\delta u_\gamma/a) + \Phi + \frac{\delta\rho_\gamma}{4\bar{\rho}_\gamma} - \frac{q^2\pi_\gamma}{2\bar{\rho}_\gamma} = \sigma_T\bar{n}_e(\delta u_B - \delta u_\gamma)$$

$$\delta\dot{u}_B + \Phi = -\frac{\sigma_T\bar{n}_e}{R}(\delta u_B - \delta u_\gamma)$$

$$R \equiv 3\bar{\rho}_B/4\bar{\rho}_\gamma$$

Conservation Equations for Photons and Baryons

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what about photon's viscosity?

$$a\frac{\partial}{\partial t}(\delta u_\gamma/a) + \Phi + \frac{\delta\rho_\gamma}{4\bar\rho_\gamma} - \frac{q^2\pi_\gamma}{2\bar\rho_\gamma} = \sigma_T\bar n_e(\delta u_B - \delta u_\gamma)$$

$$\delta\dot u_B + \Phi = -\frac{\sigma_T\bar n_e}{R}(\delta u_B - \delta u_\gamma)$$

$$R \equiv 3\bar\rho_B/4\bar\rho_\gamma$$

Peebles & Yu (1970); Sunyaev & Zeldovich (1970)

Formation of the "Photon-baryon Fluid"

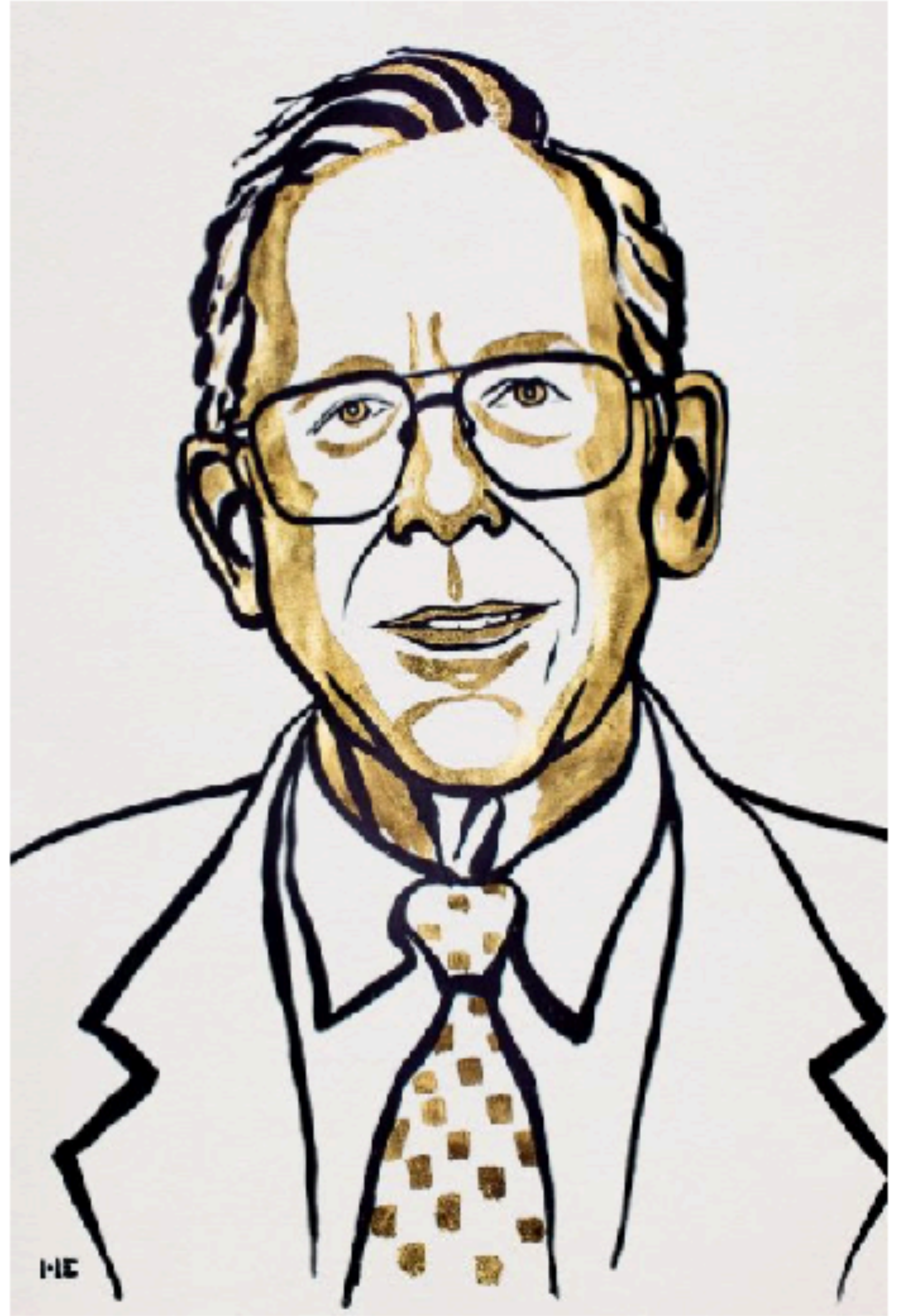
Nobel Prize in Physics (2019)

- *Photons are not an ideal fluid.* Photons free-stream at the speed of light.
  - The energy and momentum conservation equations are not enough because we need to specify the evolution of viscosity.
  - Solving for viscosity requires information of the phase-space distribution function of photons: **Boltzmann equation**.
- However, frequent scattering of photons with baryons(*) can make photons behave as a fluid: **Photon-baryon fluid.**

(*)Photons scatter with electrons via Thomson scattering. Protons scatter with electrons via Coulomb scattering. Thus we can say, effectively, photons scatter with baryons

Sound waves in the fireball Universe, predicted in 1970

James Peebles
Facts



Ill. Niklas Elmedhed. © Nobel Media.

James Peebles
The Nobel Prize in Physics 2019

Born: 1935, Winnipeg, Canada

Affiliation at the time of the award: I
Princeton, NJ, USA

Prize motivation: "for theoretical dis
cosmology."

Prize share: 1/2

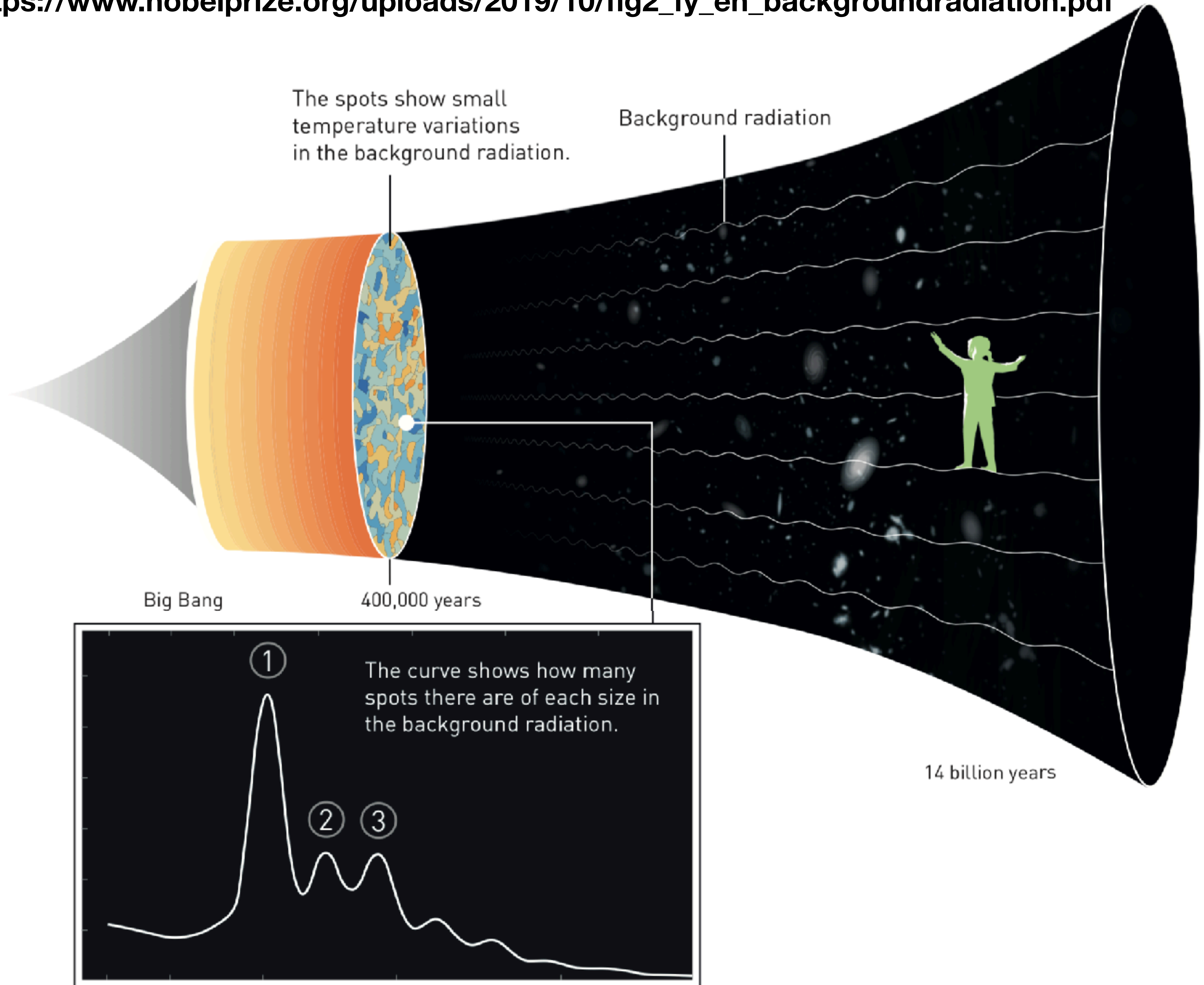
https://www.nobelprize.org

THE ASTROPHYSICAL JOURNAL, 162:815–836, December 1970

PRIMEVAL ADIABATIC PERTURBATION IN AN EXPANDING UNIVERSE*

P. J. E. PEEBLES†
Joseph Henry Laboratories, Princeton University
AND
J. T. YU‡
Goddard Institute for Space Studies, NASA, New York
Received 1970 January 5; revised 1970 April 1

https://www.nobelprize.org/uploads/2019/10/fig2_fy_en_backgroundradiation.pdf



At the *ICGC2011* conference, Goa, India

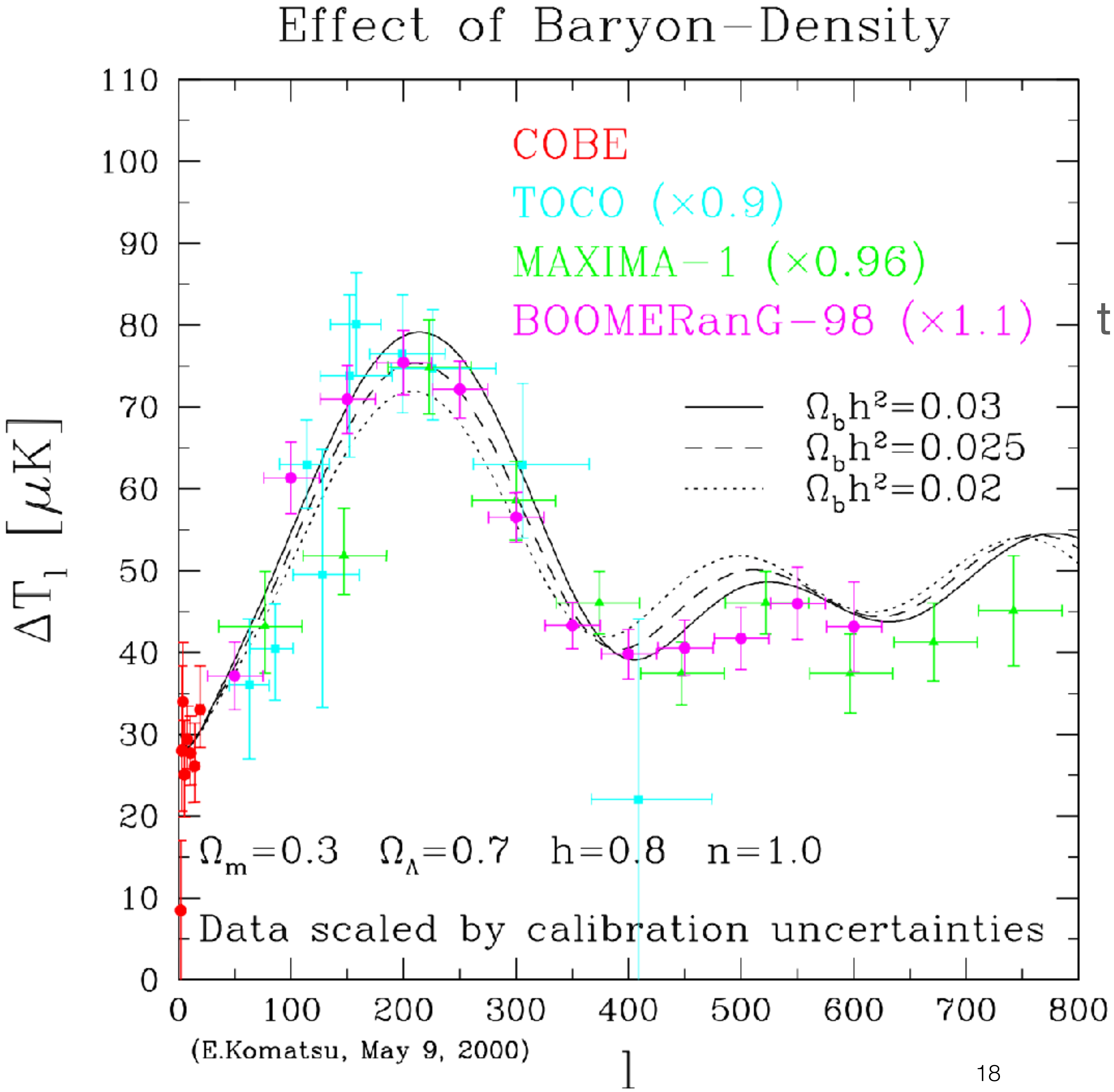
Sound waves in the fireball Universe, predicted in 1970

Astrophysics and Space Science 7 (1970) 3–19.

SMALL-SCALE FLUCTUATIONS OF RELIC RADIATION*

R. A. SUNYAEV and YA. B. ZELDOVICH
Institute of Applied Mathematics, Academy of Sciences of the U.S.S.R., Moscow, U.S.S.R.

(Received 11 September, 1969)



The predicted sound wave was found in 1999-2000.

No one (Peebles, Sunyaev, or Zeldovich) thought that this would ever be observed, because the effect seemed so tiny.

The golden lesson to learn

It does not matter how small the effect would seem to you now. Publish your calculation!

If the effect is worth measuring, it will be measured.

Part II: Tight-coupling approximation

Let’s solve them!

- Fourier transformation replaces $\nabla^2 \to -q^2$

$$X(t, \boldsymbol{x}) = (2\pi)^{-3} \int d^3q \; X_{\boldsymbol{q}}(t) \exp(i\boldsymbol{q} \cdot \boldsymbol{x})$$

$$\frac{\partial}{\partial t}(\delta\rho_\gamma/\bar{\rho}_\gamma) - \frac{4q^2}{3a^2}\delta u_\gamma = 4\dot{\Psi}$$

$$\frac{\partial}{\partial t}(\delta\rho_B/\bar{\rho}_B) - \frac{q^2}{a^2}\delta u_B = 3\dot{\Psi}$$

$$a\frac{\partial}{\partial t}(\delta u_\gamma/a) + \Phi + \frac{\delta\rho_\gamma}{4\bar{\rho}_\gamma} - \frac{q^2\pi_\gamma}{2\bar{\rho}_\gamma} = \sigma_\mathcal{T}\bar{n}_e(\delta u_B - \delta u_\gamma)$$

$$\delta\dot{u}_B + \Phi = -\frac{\sigma_\mathcal{T}\bar{n}_e}{R}(\delta u_B - \delta u_\gamma)$$

$$R \equiv 3\bar{\rho}_B/4\bar{\rho}_\gamma$$

Peebles & Yu (1970)

Tight-coupling Approximation

- When the Thomson scattering is efficient, photons and baryons "move together"; thus, their relative velocity is small. We write

$$\delta u_B - \delta u_\gamma = d/\sigma_{\mathcal{T}} \bar{n}_e$$

[d is an arbitrary dimensionless variable]

- And take $\sigma_{\mathcal{T}} \bar{n}_e \to \infty$ (*). We obtain

$$a\frac{\partial}{\partial t}(\delta u_\gamma / a) + \Phi + \frac{\delta\rho_\gamma}{4\bar{\rho}_\gamma} = d\,, \qquad \delta\dot{u}_\gamma + \Phi = -\frac{d}{R}$$

(*) In this limit, viscosity π_γ is exponentially suppressed. This result comes from the Boltzmann equation but we do not derive it here. It makes sense physically.

Peebles & Yu (1970)

Tight-coupling Approximation

- Eliminating *d* and using the fact that R is proportional to the scale factor, we obtain

$$a\frac{\partial}{\partial t}\left[(1+R)\delta u_\gamma/a\right]+(1+R)\Phi+\frac{\delta\rho_\gamma}{4\bar\rho_\gamma}=0$$

- Using the energy conservation to replace δu_γ with $\delta\rho_\gamma/\rho_\gamma$, we obtain

$$\frac{1}{a(1+R)}\frac{\partial}{\partial t}\left[a(1+R)\frac{\partial}{\partial t}(\delta\rho_\gamma/\bar\rho_\gamma-4\Psi)\right]+\frac{4q^2}{3a^2}\Phi+\frac{q^2}{a^2}\frac{\delta\rho_\gamma/\bar\rho_\gamma}{\boxed{3(1+R)}}=0$$

The wave equation, with the speed of sound of $c_s^2 = 1/3(1+R)$!

(c.f.) $\delta\ddot\rho_{\mathbf{q}}+c_s^2q^2\delta\rho_{\mathbf{q}}=0$

Peebles & Yu (1970); Sunyaev & Zeldovich (1970)

Sound Wave!

$$\frac{1}{a(1+R)}\frac{\partial}{\partial t}\left[a(1+R)\frac{\partial}{\partial t}(\delta\rho_\gamma/\bar{\rho}_\gamma - 4\Psi)\right] + \frac{4q^2}{3a^2}\Phi + \frac{q^2}{a^2}\frac{\delta\rho_\gamma/\bar{\rho}_\gamma}{3(1+R)} = 0$$

- To simplify the equation, let's first look at the high-frequency solution
  - Specifically, we take q >> aH (the wavelength of fluctuations is much shorter than the Hubble length). Then we can ignore time derivatives of R and Ψ because they evolve in the Hubble time scale:

$$\frac{1}{a}\frac{\partial}{\partial t}\left[a\frac{\partial}{\partial t}(\delta\rho_\gamma/\bar{\rho}_\gamma)\right] + \frac{q^2c_s^2}{a^2}\left[\delta\rho_\gamma/\bar{\rho}_\gamma + 4(1+R)\Phi\right] = 0$$

The sound wave solution!

(c.f.)

$$\delta\rho_{\mathbf{q}}(t) = A_{\mathbf{q}}\cos(qc_st) + B_{\mathbf{q}}\sin(qc_st)$$

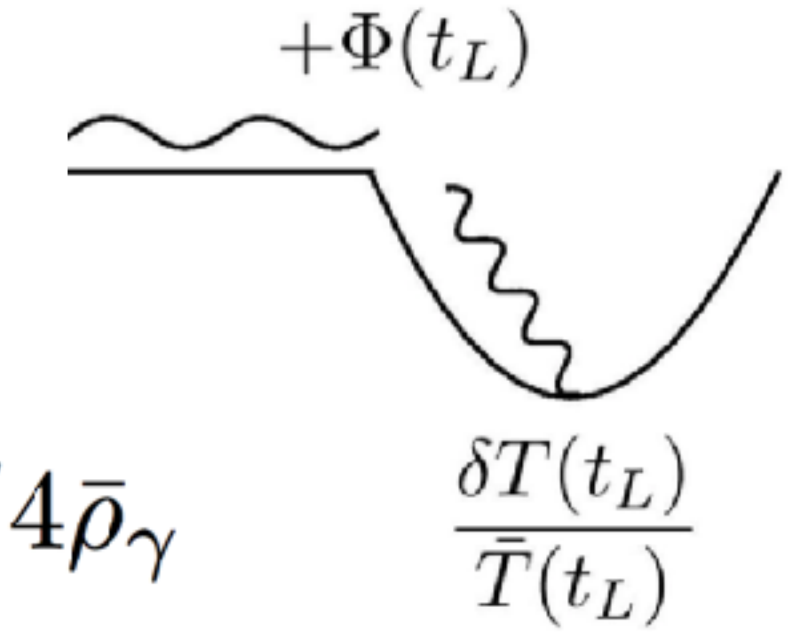
$$\frac{\delta\rho_\gamma}{4\bar{\rho}_\gamma} + \Phi = A\cos(qr_s) + B\sin(qr_s) - R\Phi$$

Recap
Focus on physics!

- Photons are not a fluid; but Thomson scattering couples photons to baryons, forming a **photon-baryon fluid.**
- The reduced sound speed, $c_s^2=1/3(1+R)$, emerges automatically. Beautiful!
- The relevant sound horizon is $r_s = \int_0^{t_L} \frac{dt'}{a(t')} c_s(t')$
- $\delta\rho_\gamma/4\rho_\gamma$ is the temperature anisotropy at the bottom of the potential well. Adding gravitational redshift, the observed temperature anisotropy is $\delta\rho_\gamma/4\rho_\gamma + \Phi$, which is

$$\frac{\delta\rho_\gamma}{4\bar{\rho}_\gamma} + \Phi = A\cos(qr_s) + B\sin(qr_s) - R\Phi$$

$R \equiv 3\bar{\rho}_B/4\bar{\rho}_\gamma$



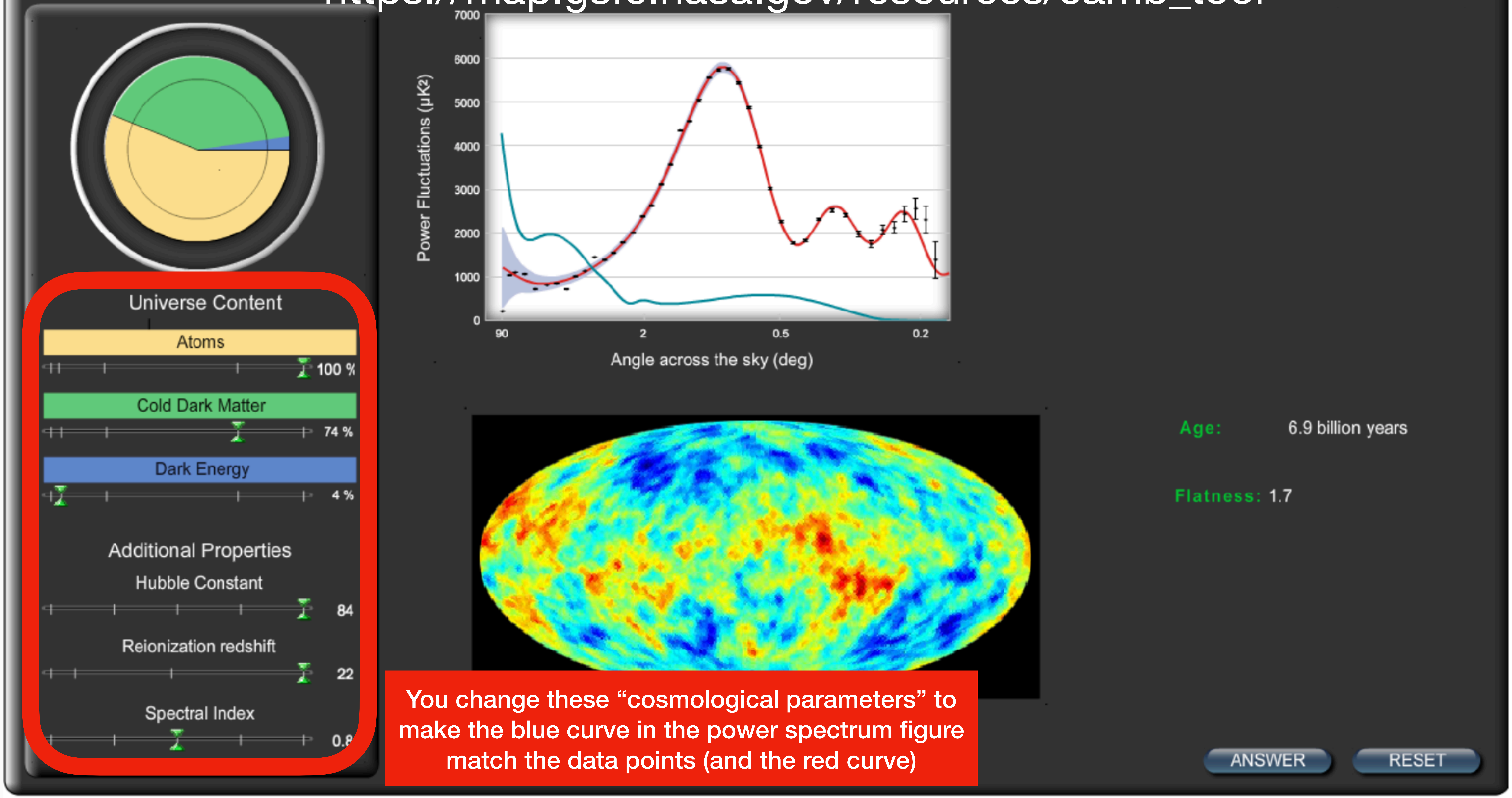
Part III: Build a Universe!

https://wmap.gsfc.nasa.gov/resources/camb_tool/index.html

Running this web tool requires Flash Player.
Enable it before using this tool.

CMB Analyzer

https://map.gsfc.nasa.gov/resources/camb_tool



You change these "cosmological parameters" to make the blue curve in the power spectrum figure match the data points (and the red curve)

Your Mission

“Fit” the data

1. **Find** the parameters that match the data points

2. **Record** the behaviour of the power spectrum, when you vary a parameter

   - For example: What happens when you reduce the “Spectral Index”? What happens when you increase “Atoms”?

   - **Tip**: Where to start? Start by varying one parameter away from the best-fitting parameter you found in (1)

   - Explore the behaviours of as many parameters as you have time to explore

3. **Document** your findings in the **shared note.**

   - I have not yet taught you how the power spectrum depends on the parameters. So, collect data yourself now; it helps you understand physics later.