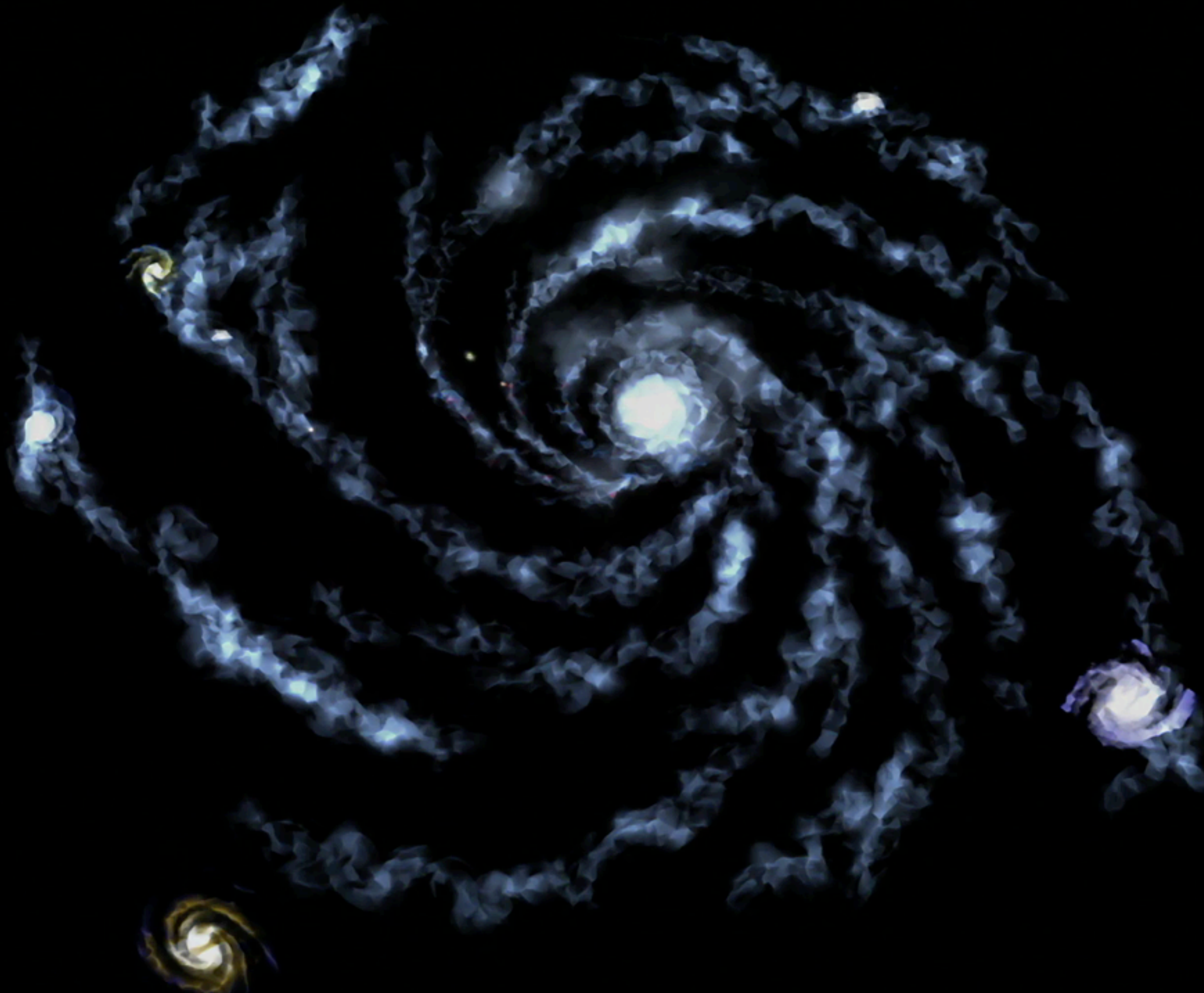
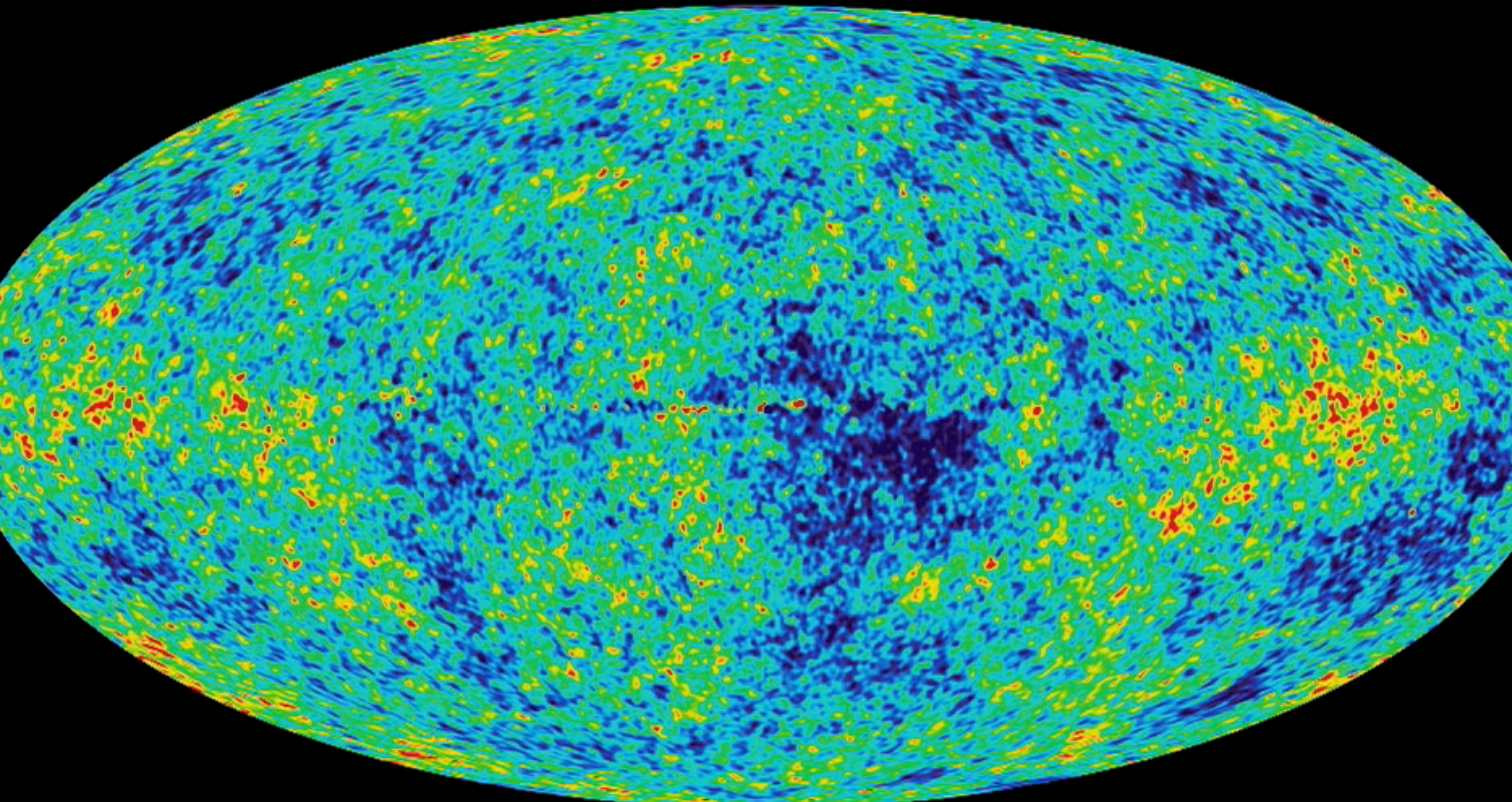
From Initial Conditions to Structure Formation, and Back

Eiichiro Komatsu (Max Planck Institute for Astrophysics)
COSMO-16, Univ. of Michigan, August 8, 2016





Fluctuations in photons: how about the matter distribution?

A Remarkable Story

- Observations of CMB taught us that **galaxies, stars, planets, and ourselves originated from tiny fluctuations in the early Universe**

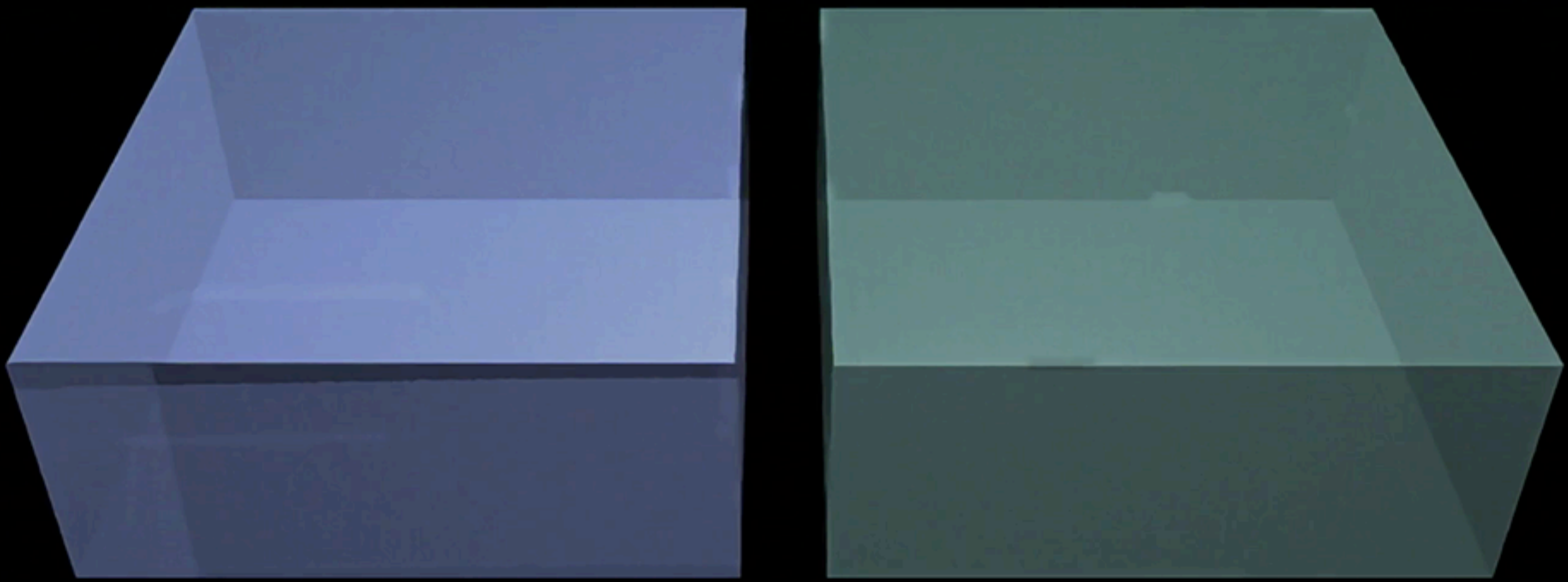
But, how confident are we?

Cosmic Miso Soup

- When matter and radiation were hotter than 3000 K, matter was completely ionised. The Universe was filled with plasma, which behaves just like a soup

- Think about a Miso soup (if you know what it is). Imagine throwing Tofus into a Miso soup, while changing the density of Miso

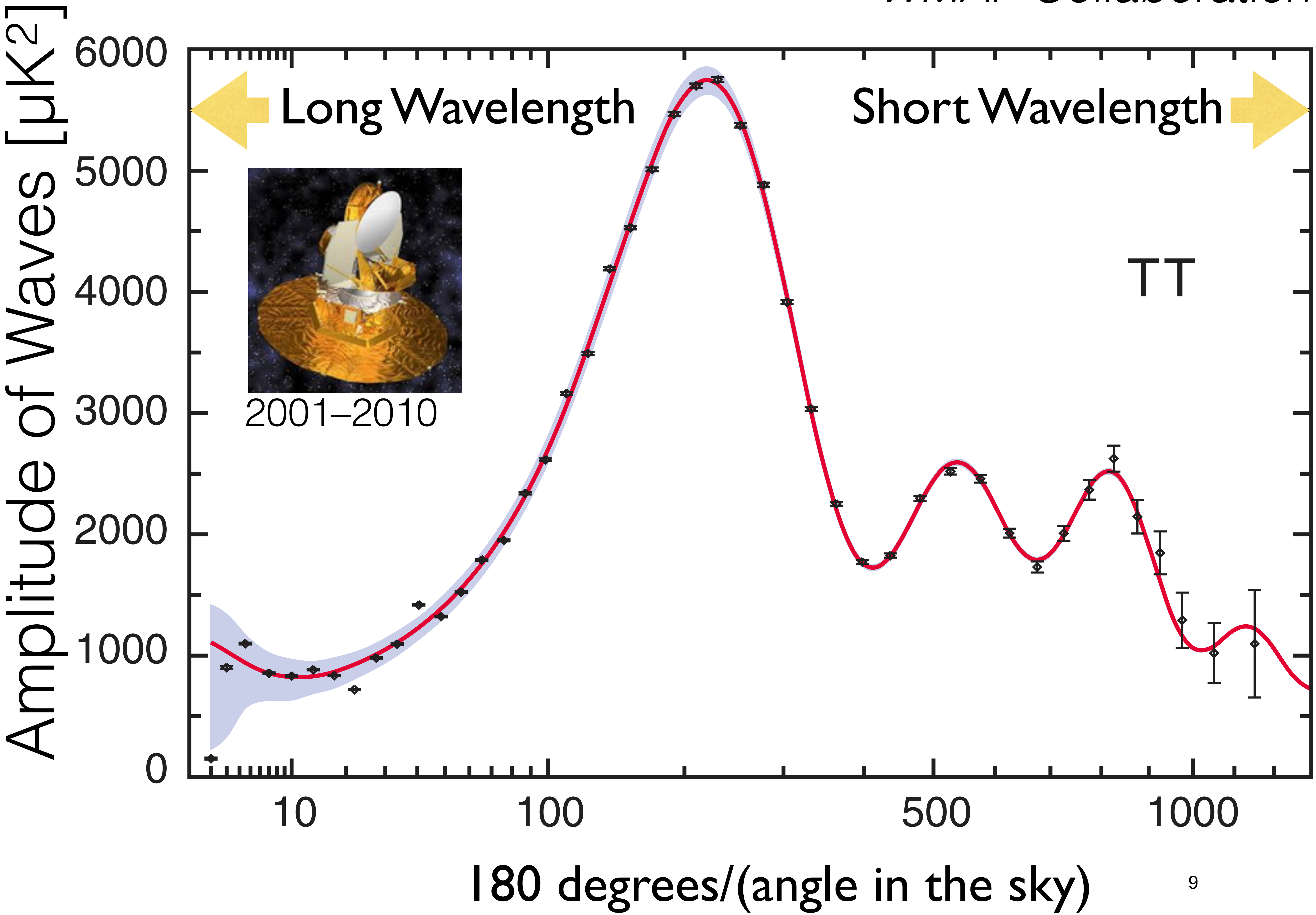
- And imagine watching how ripples are created and propagate throughout the soup

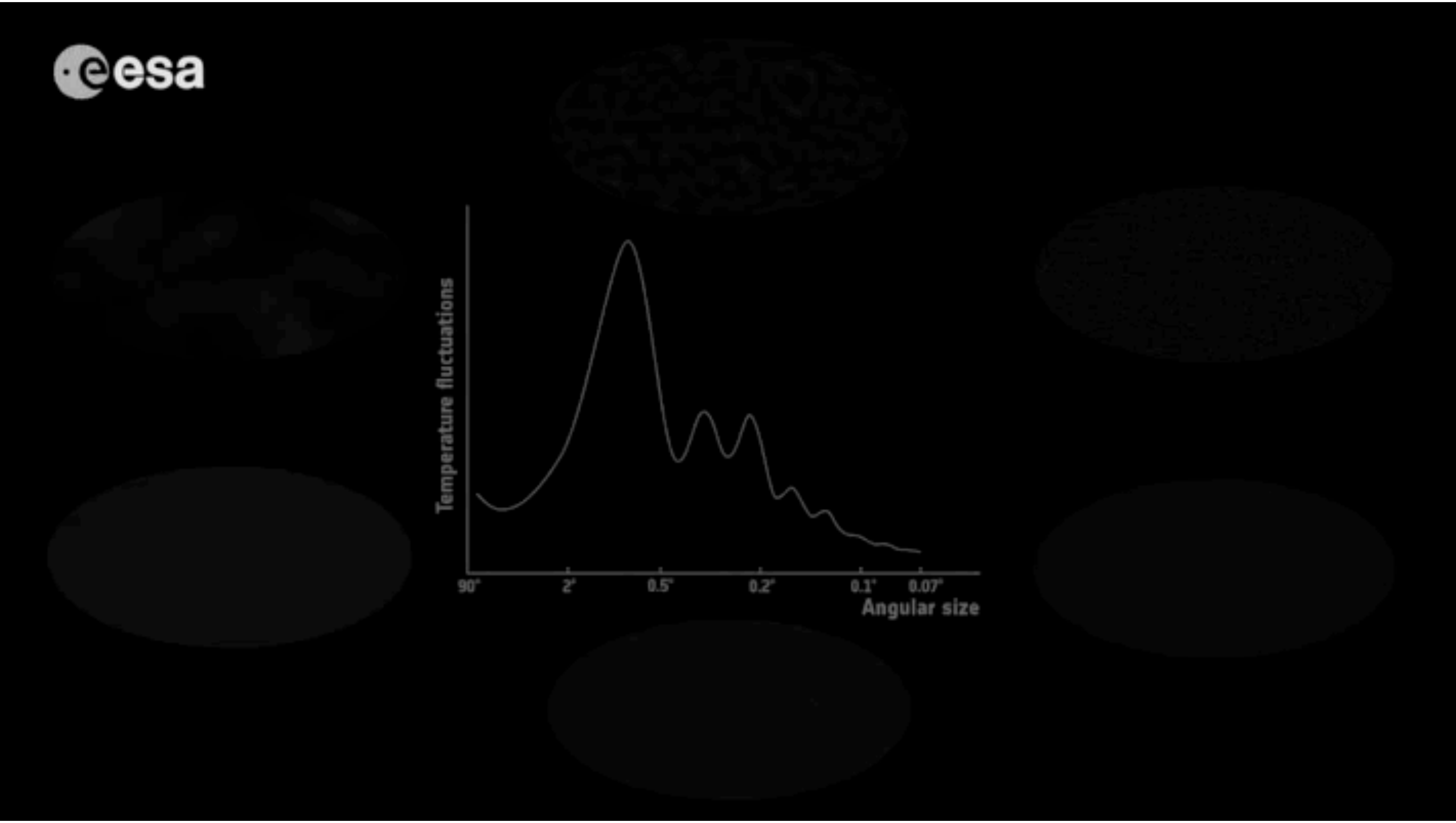


Power Spectrum Analysis

- Decompose temperature fluctuations in the sky into a set of waves with various wavelengths
- Make a diagram showing the strength of each wavelength

WMAP Collaboration

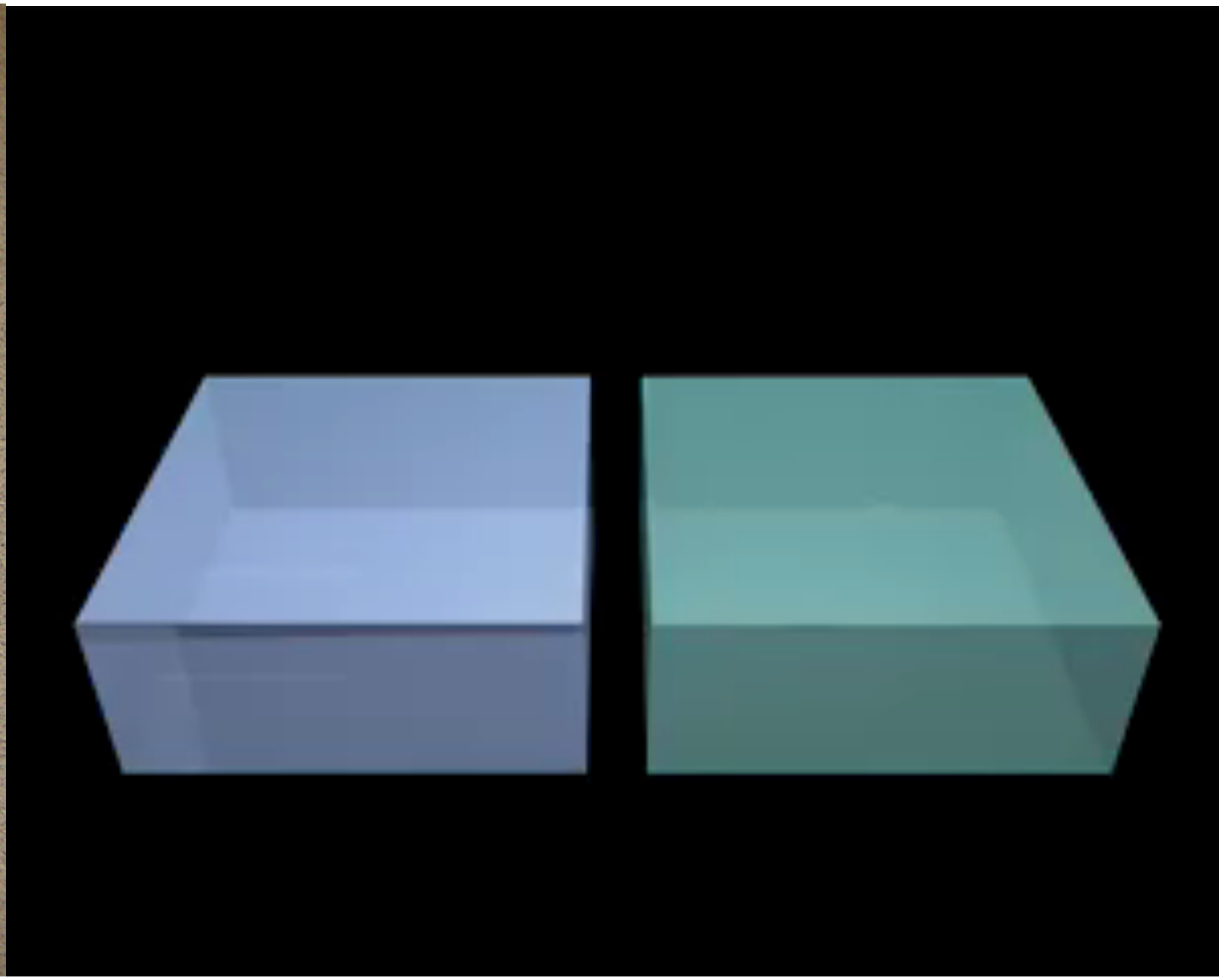


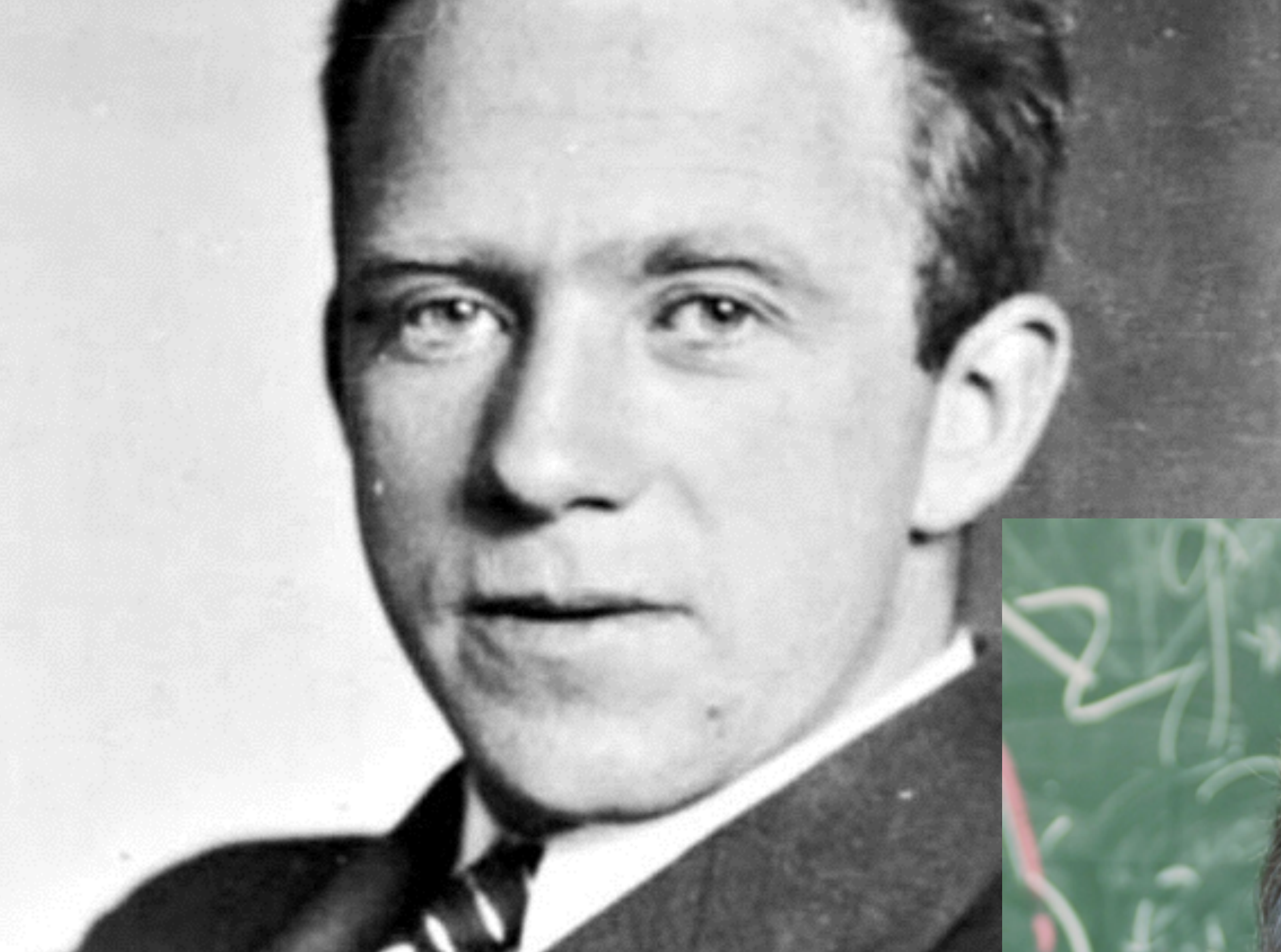


Temperature fluctuations
90°
2°
0.5°
0.2°
0.1°
0.07°
Angular size

Origin of Fluctuations

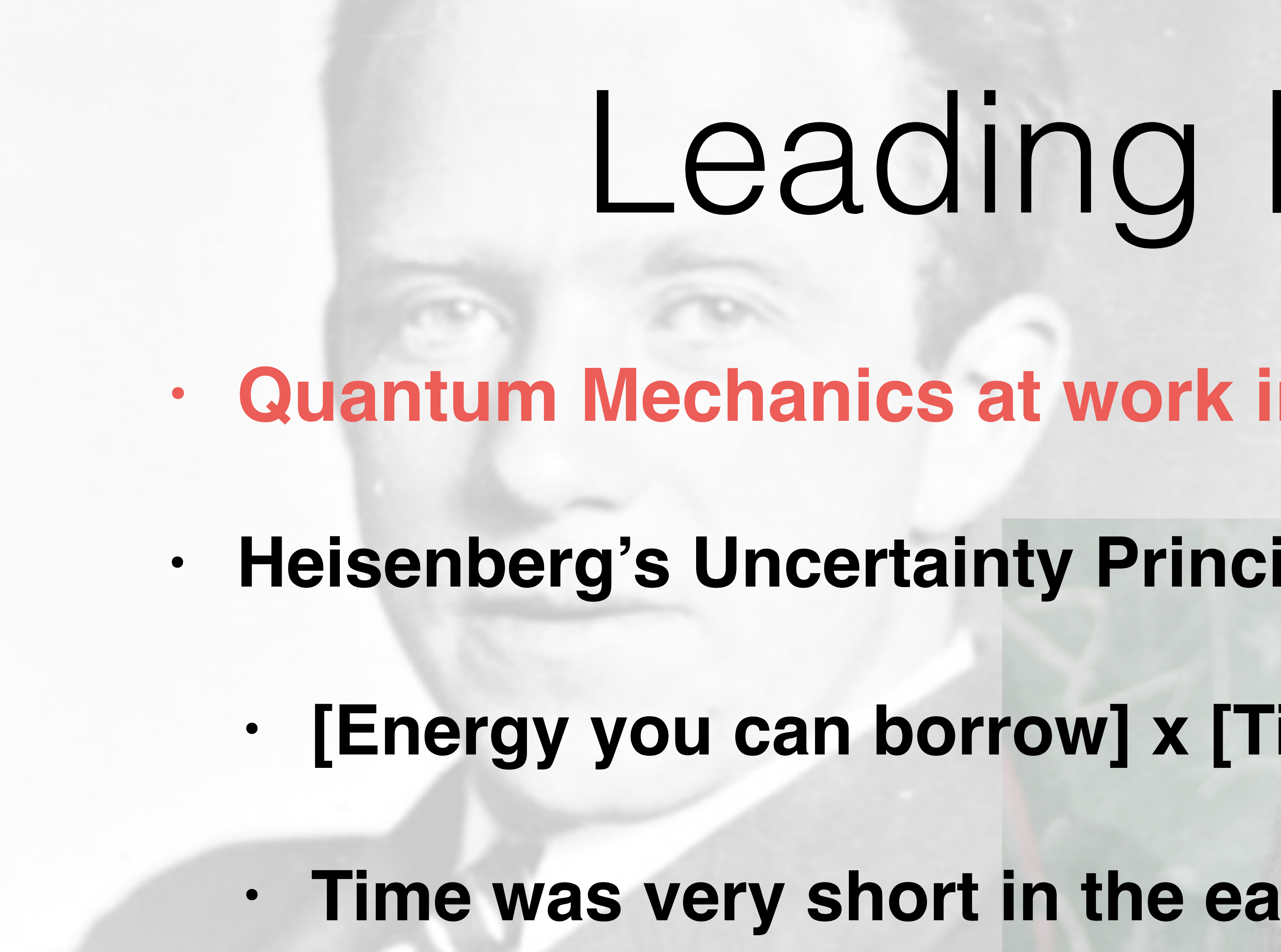
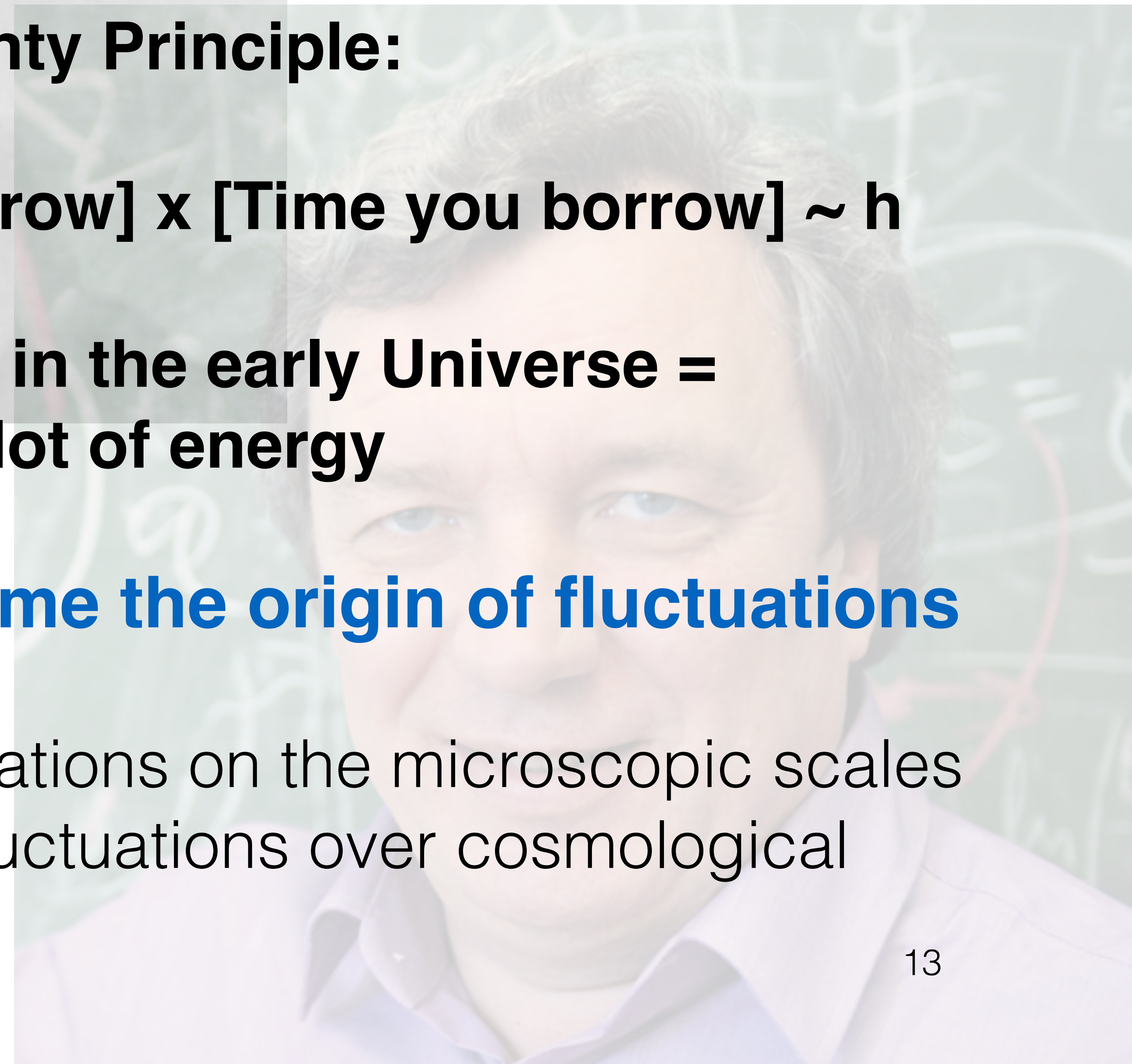
- **Who dropped** those Tofus into the cosmic Miso soup?





Werner Heisenberg
(1901–1976)

Slava Mukhanov
(Munich Univ.)

Leading Idea

- **Quantum Mechanics at work in the early Universe**
  (Mukhanov & Chibisov, 1981)
- **Heisenberg's Uncertainty Principle:**
  - **[Energy you can borrow] x [Time you borrow] ~ h**
  - **Time was very short in the early Universe = You could borrow a lot of energy**
- **Those energies became the origin of fluctuations**
- How did quantum fluctuations on the microscopic scales become macroscopic fluctuations over cosmological sizes?

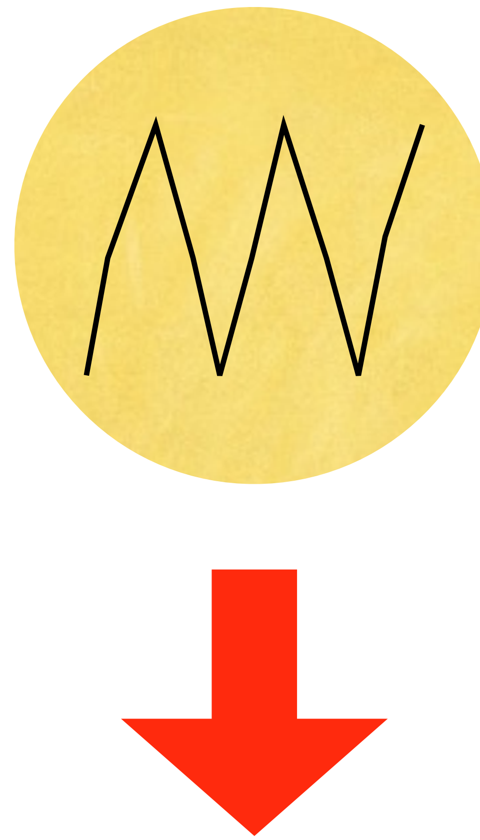
Starobinsky (1980); Sato (1981); Guth (1981); Linde (1982); Albrecht & Steinhardt (1982)

Cosmic Inflation

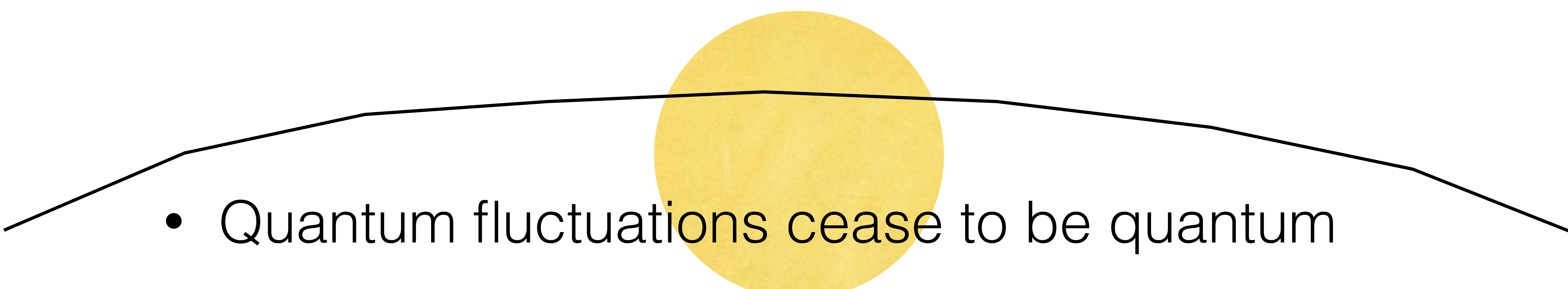
- In a tiny fraction of a second, the size of an atomic nucleus became the size of the Solar System
  - In 10^{-36} second, space was stretched by at least a factor of 10^{26}

Stretching Micro to Macro

Quantum fluctuations on microscopic scales



Inflation!



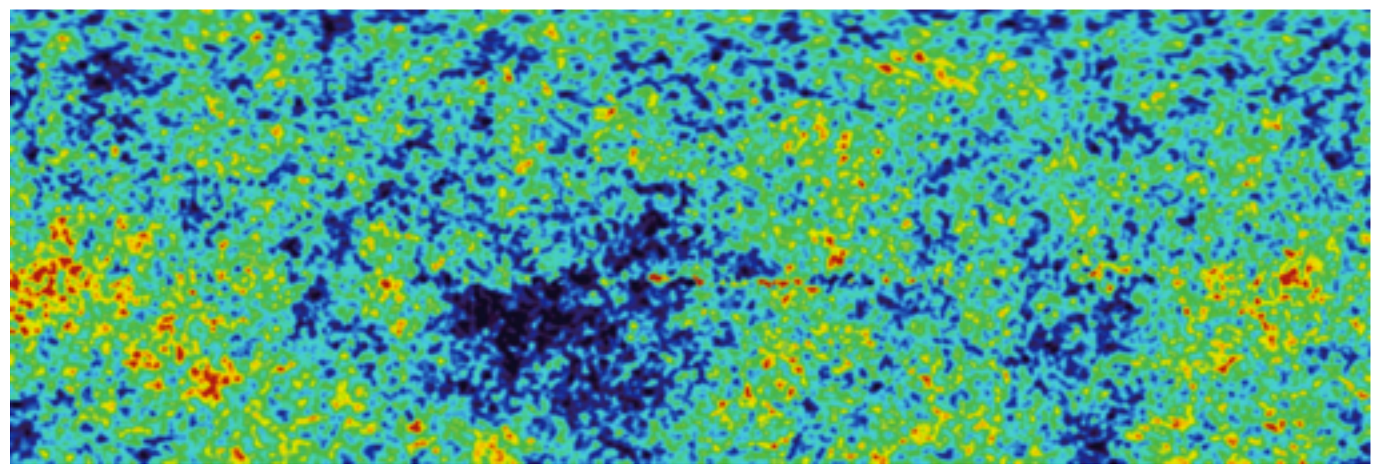
- Quantum fluctuations cease to be quantum
- Become macroscopic, classical fluctuations

Key Predictions of Inflation

ζ

scalar mode

- Fluctuations we observe today in CMB and the matter distribution originate from quantum fluctuations generated during inflation



h_{ij}

tensor mode

- There should also be ***ultra-long-wavelength*** **gravitational waves** generated during inflation

Starobinsky (1979)

We measure distortions in space

- A distance between two points in space

$$d\ell^2 = a^2(t)[1+2\zeta(\mathbf{x},t)][\delta_{ij}+h_{ij}(\mathbf{x},t)]dx^i dx^j$$

- ζ: “curvature perturbation” (scalar mode)
    - Perturbation to the determinant of the spatial metric
- h_{ij}: “gravitational waves” (tensor mode)
    - Perturbation that does not change the determinant (area)

$$\sum_i h_{ii} = 0$$

Heisenberg's Uncertainty Principle

- [Energy you can borrow] x [Time you borrow] = constant

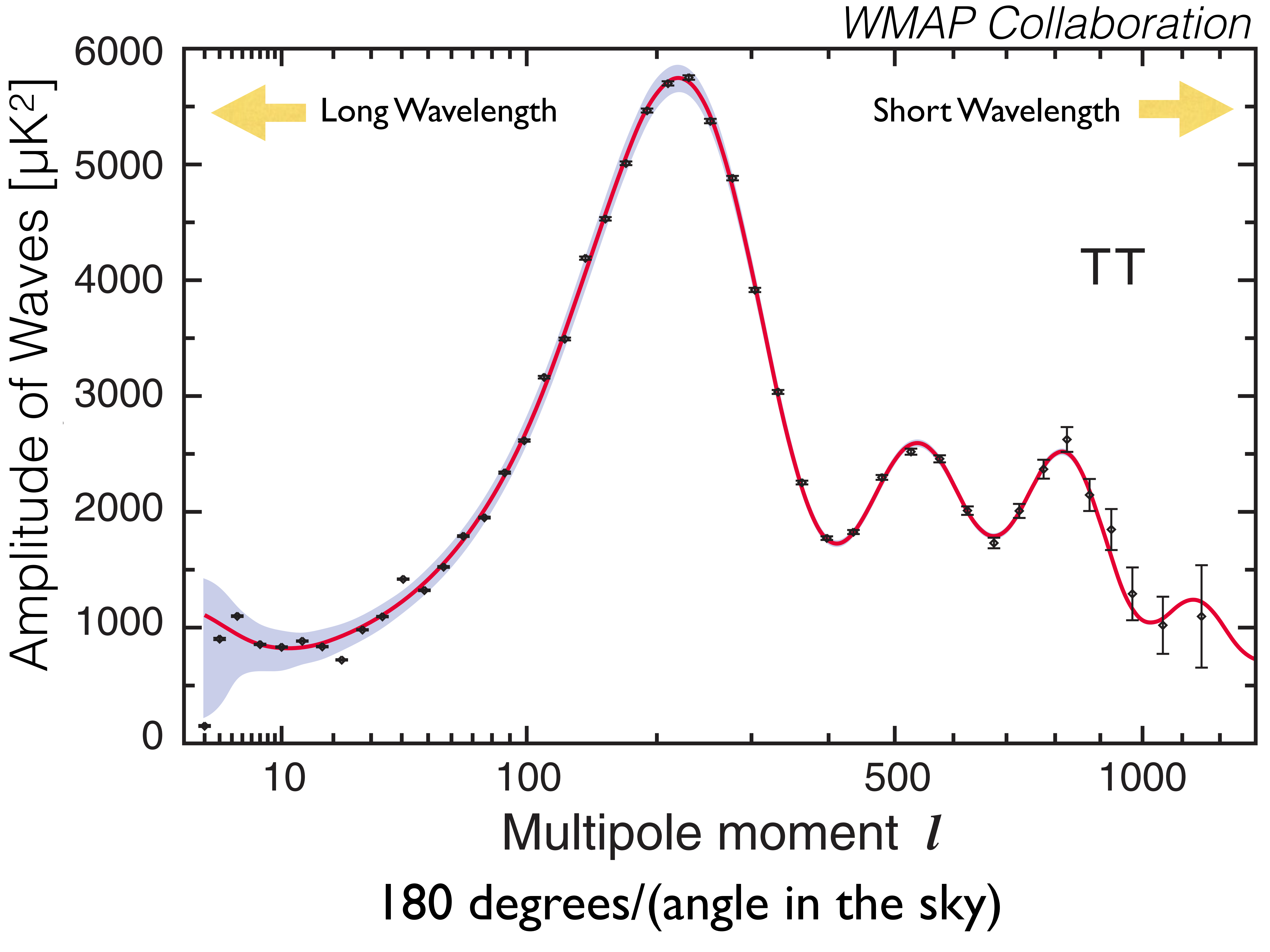
- Suppose that the distance between two points increases in proportion to **a(t)** [which is called the scale factor] by the expansion of the universe

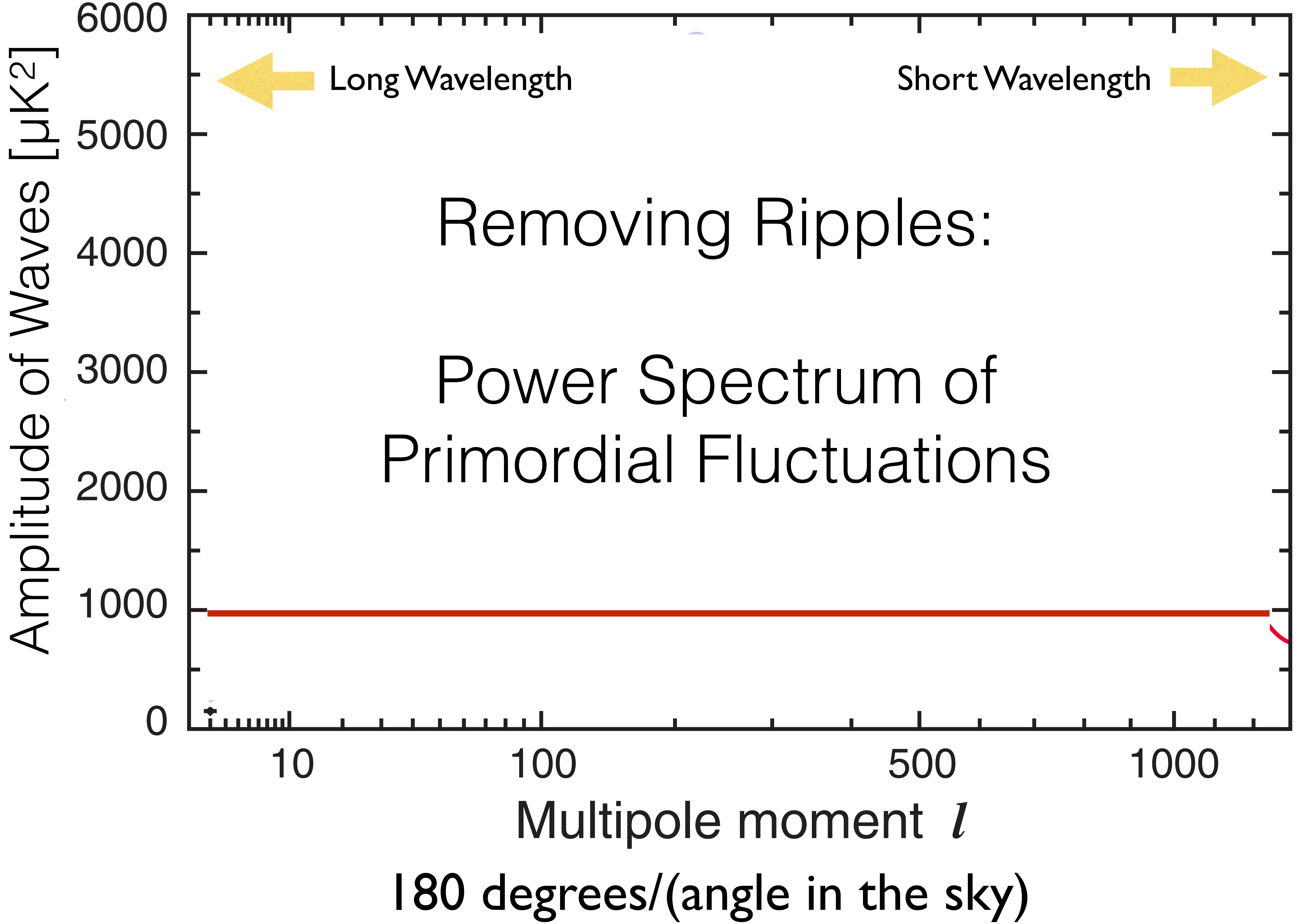
- Define the "expansion rate of the universe" as

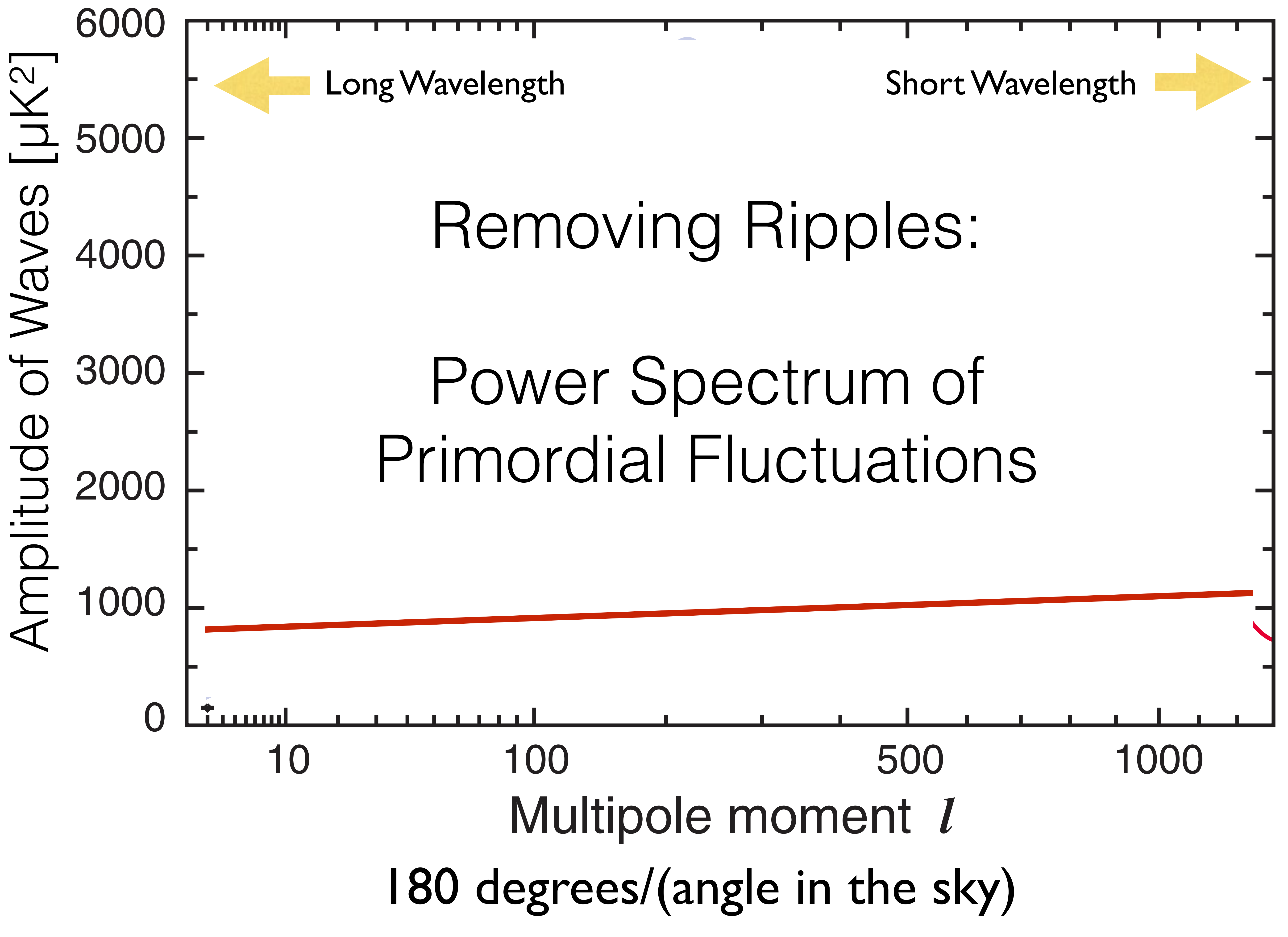
$$H \equiv \frac{\dot{a}}{a}$$ [This has units of 1/time]

Fluctuations are proportional to H

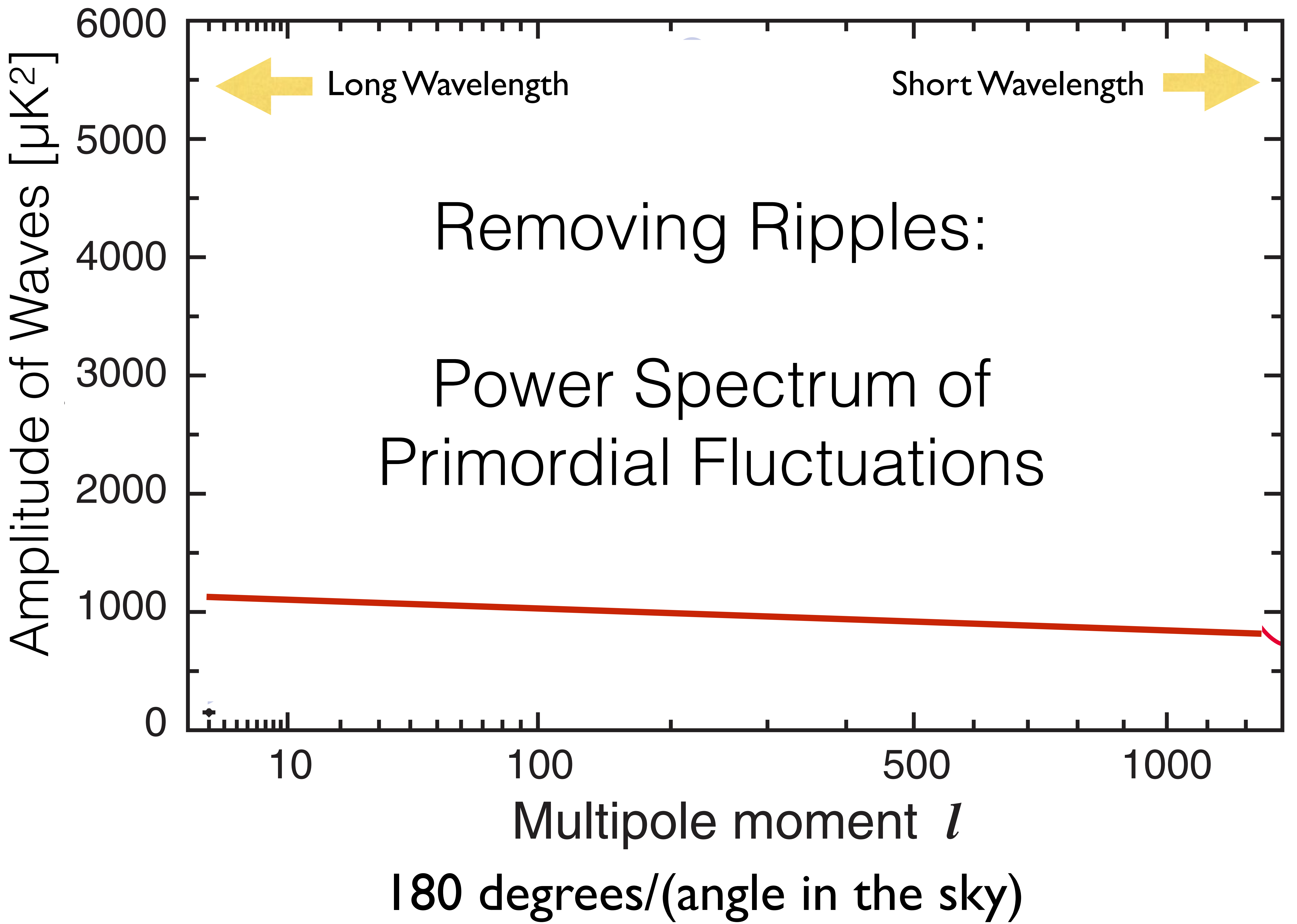
- [Energy you can borrow] x [Time you borrow] = constant
- $H \equiv \frac{\dot{a}}{a}$ [This has units of 1/time]
- Then, **both ζ and h_{ij} are proportional to H**
- Inflation occurs in 10^{-36} second - this is such a short period of time that you can borrow a lot of energy! **H during inflation in energy units is 10^{14} GeV**


WMAP Collaboration
Amplitude of Waves [μK²]
6000
5000
4000
3000
2000
1000
0
Long Wavelength
Short Wavelength
TT
10
100
500
1000
Multipole moment l
180 degrees/(angle in the sky)

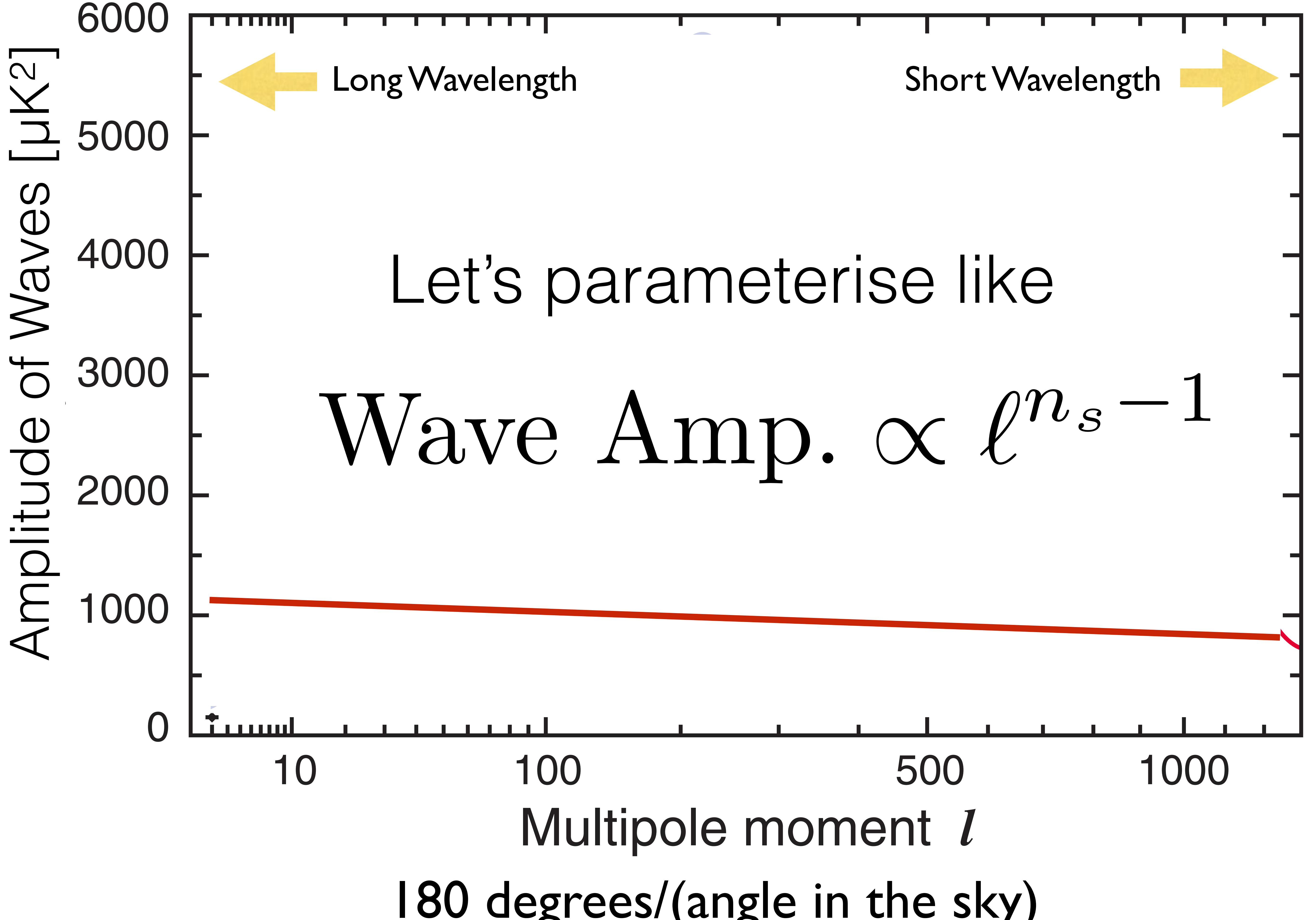

Removing Ripples:
Power Spectrum of Primordial Fluctuations
Long Wavelength
Short Wavelength
Amplitude of Waves [μK²]
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
Multipole moment l
180 degrees/(angle in the sky)



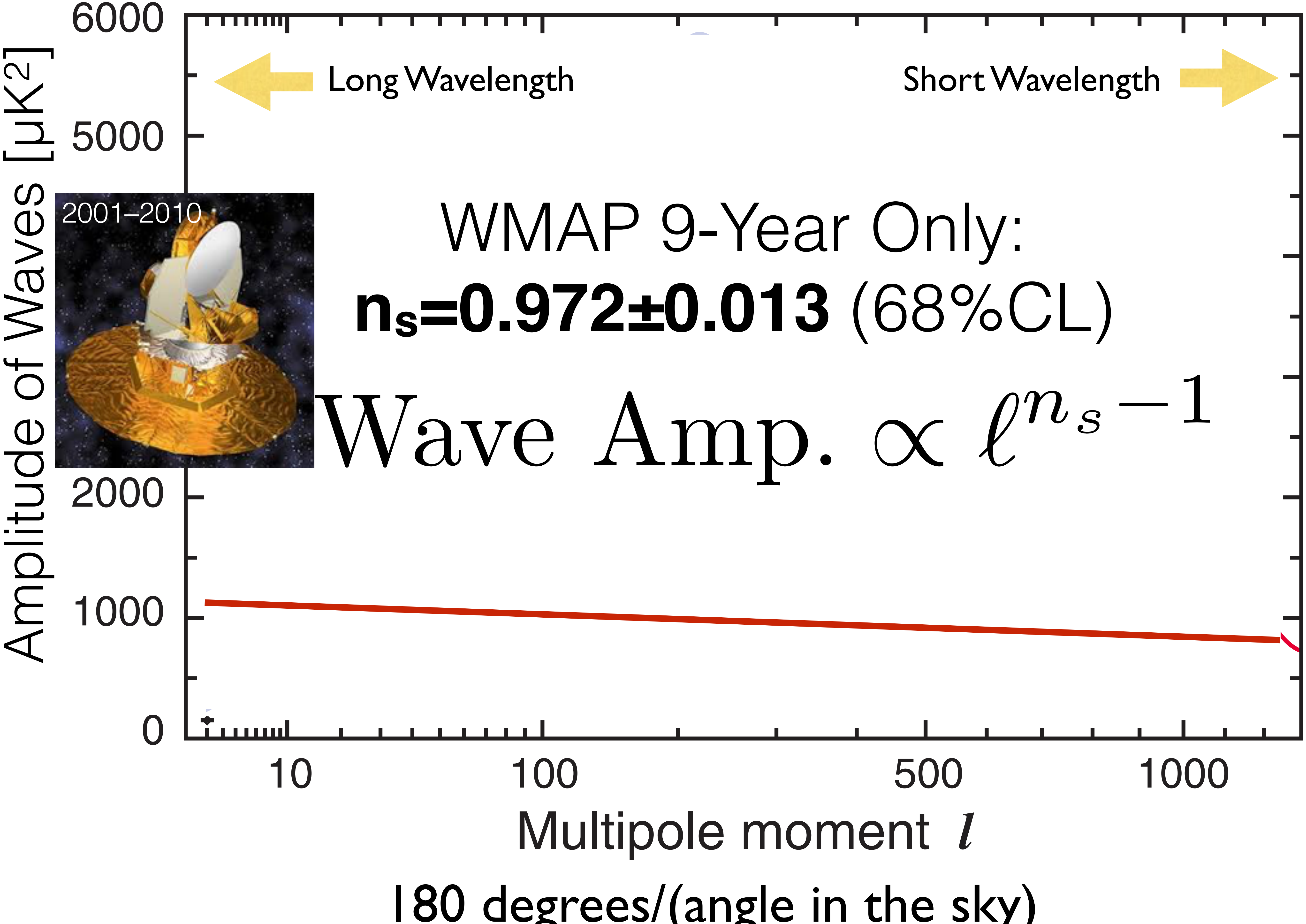
Removing Ripples:
Power Spectrum of
Primordial Fluctuations
Long Wavelength
Short Wavelength
Amplitude of Waves [μK^2]
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
Multipole moment l
180 degrees/(angle in the sky)

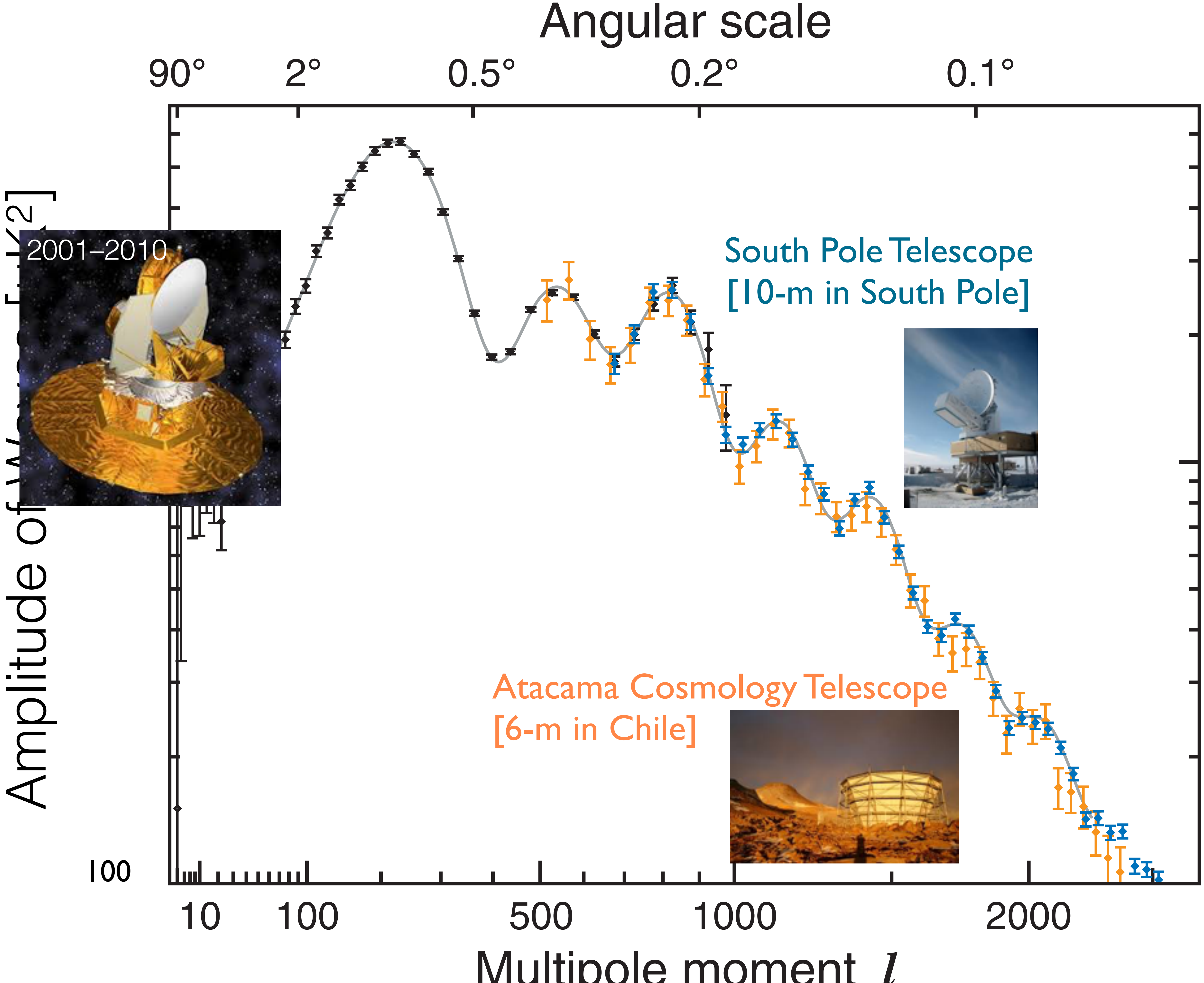


Long Wavelength
Short Wavelength
Removing Ripples:
Power Spectrum of
Primordial Fluctuations
Amplitude of Waves [μK²]
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
Multipole moment l
180 degrees/(angle in the sky)

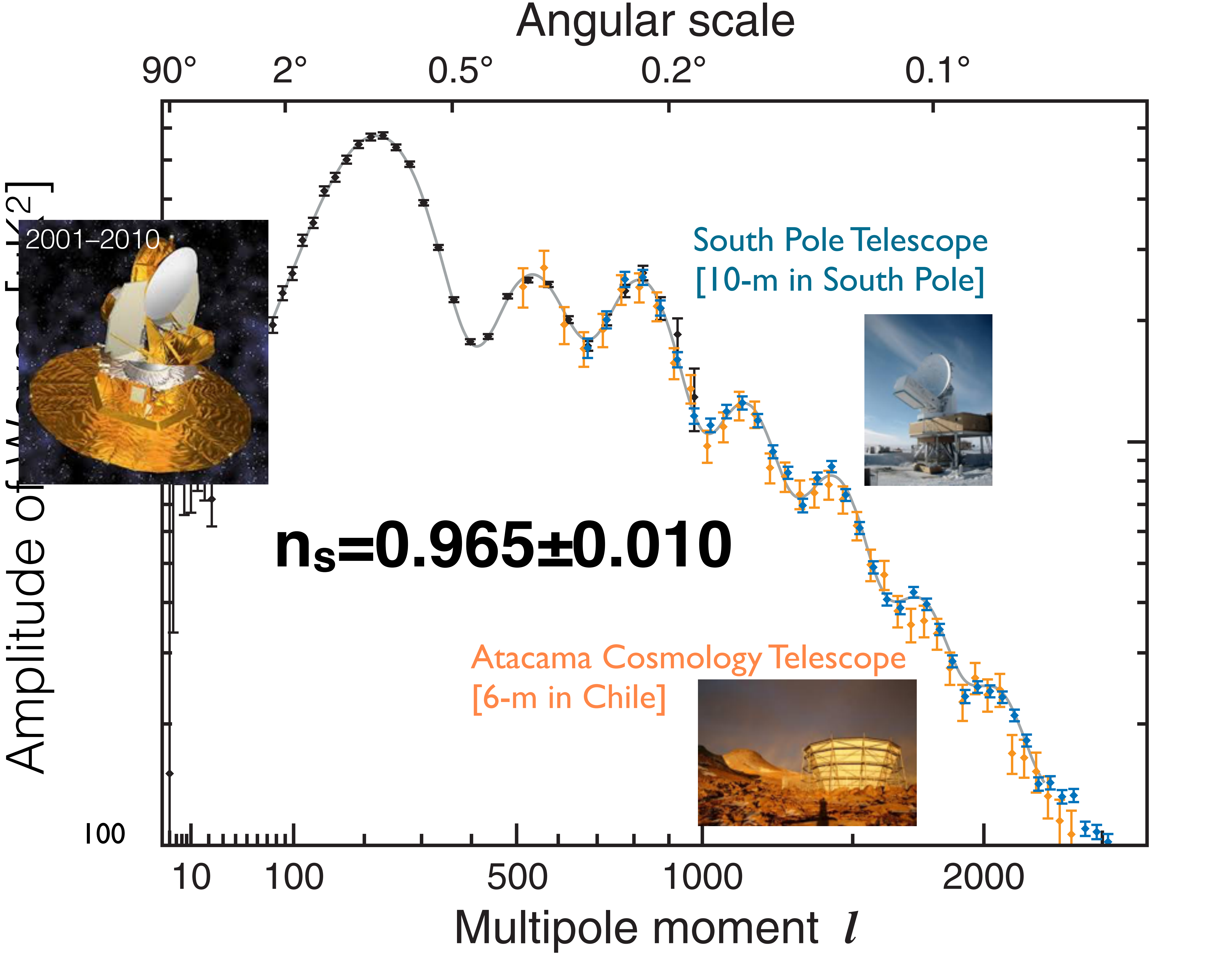


6000
5000
4000
3000
2000
1000
0
Amplitude of Waves [μK²]
Long Wavelength
Short Wavelength
Let's parameterise like
$$\text{Wave Amp.} \propto \ell^{n_s - 1}$$
10
100
500
1000
Multipole moment l
180 degrees/(angle in the sky)

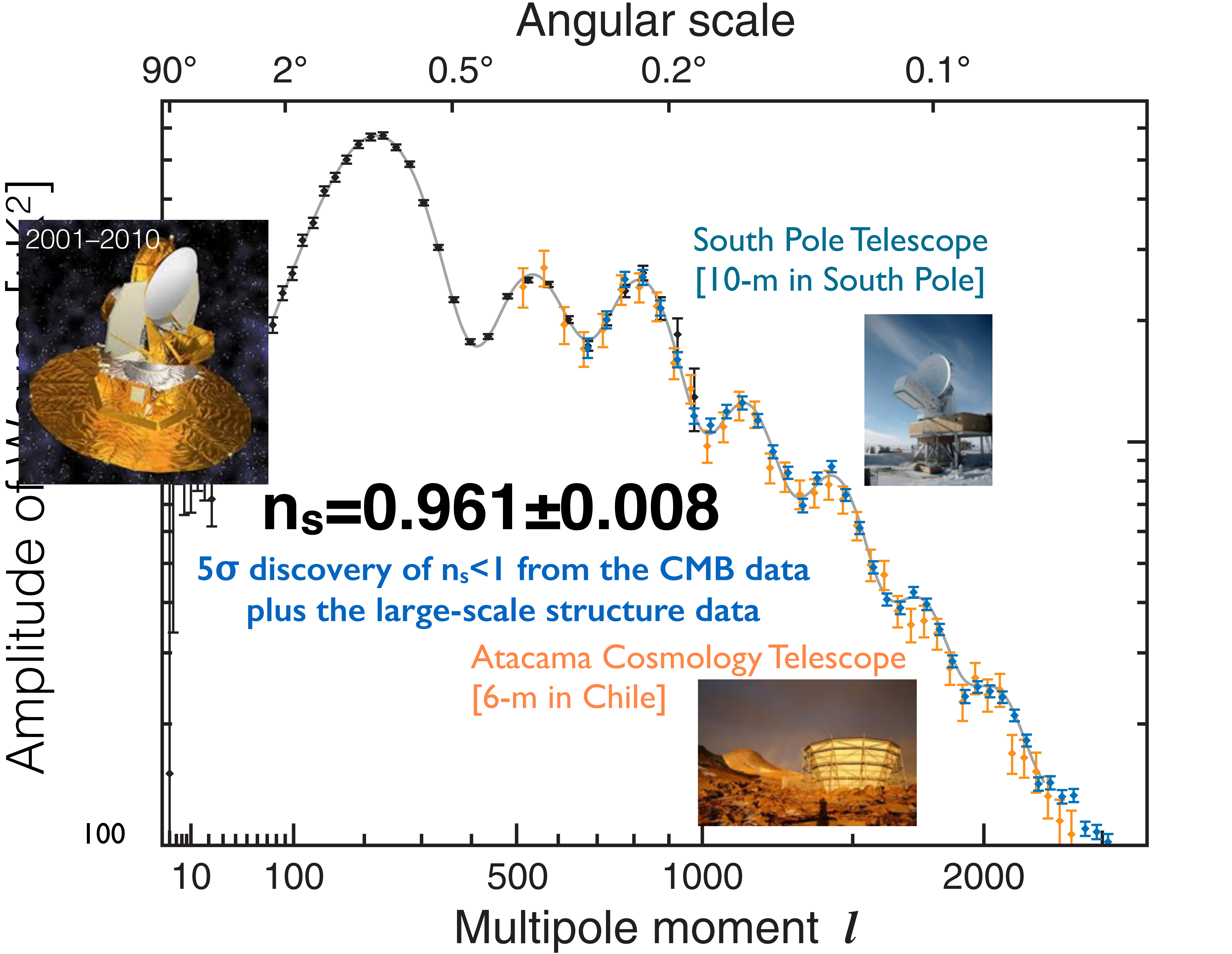

Long Wavelength
Short Wavelength
Amplitude of Waves [μK^2]
6000
5000
2000
1000
0
2001–2010
WMAP 9-Year Only:
n_s=0.972±0.013 (68%CL)
$$\text{Wave Amp.} \propto \ell^{n_s - 1}$$
10
100
500
1000
Multipole moment l
180 degrees/(angle in the sky)

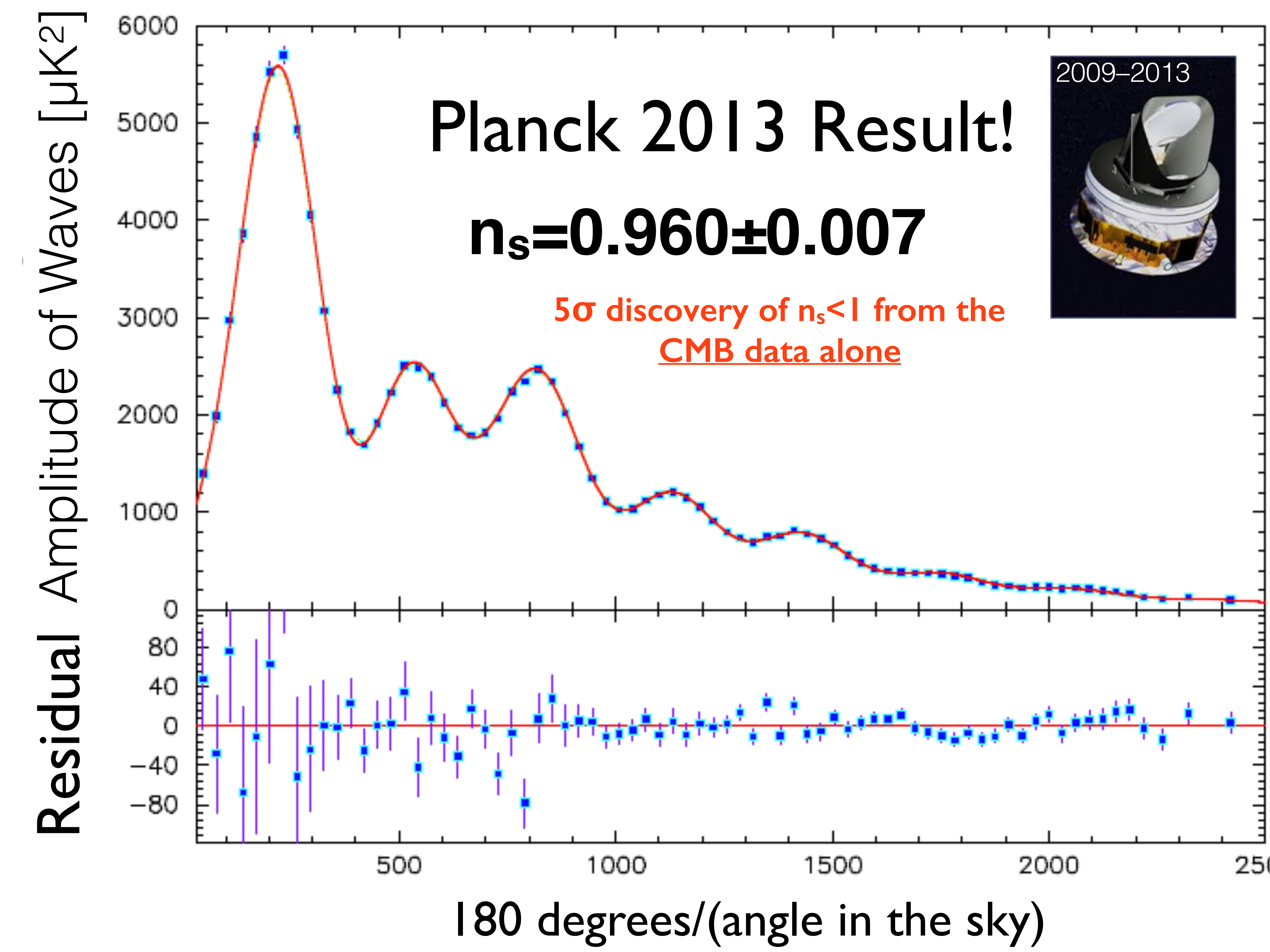


Angular scale
90°
2°
0.5°
0.2°
0.1°
Amplitude of Waves [μK²]
2001–2010
South Pole Telescope
[10-m in South Pole]
Atacama Cosmology Telescope
[6-m in Chile]
100
10
100
500
1000
2000
Multipole moment l



Angular scale
90°
2°
0.5°
0.2°
0.1°
Amplitude of Waves [μK²]
2001–2010
South Pole Telescope
[10-m in South Pole]
n_s=0.965±0.010
Atacama Cosmology Telescope
[6-m in Chile]
100
10
100
500
1000
2000
Multipole moment l


Angular scale
90°
2°
0.5°
0.2°
0.1°
2001–2010
South Pole Telescope
[10-m in South Pole]
n_s=0.961±0.008
5σ discovery of n_s<1 from the CMB data
plus the large-scale structure data
Atacama Cosmology Telescope
[6-m in Chile]
Amplitude of
100
10
100
500
1000
2000
Multipole moment l


Planck 2013 Result!
n_s=0.960±0.007
5σ discovery of n_s<1 from the CMB data alone
2009–2013
Amplitude of Waves [μK²]
Residual
180 degrees/(angle in the sky)
6000
5000
4000
3000
2000
1000
0
80
40
0
−40
−80
500
1000
1500
2000

Predicted in 1981. Finally discovered in 2013 by WMAP and Planck

- Inflation must end
- Inflation predicts $n_s \sim 1$, but not exactly equal to 1. Usually $n_s < 1$ is expected
- **The discovery of $n_s < 1$ has been a dream of cosmologists since 1992,** when the CMB anisotropy was first discovered and $n_s \sim 1$ (to within 30%) was indicated

Slava Mukhanov said in his 1981 paper that n_s should be less than 1

An implication of n_s<1

- Accelerated expansion during the early universe

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0 \quad \Rightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} < 1$$

- When ε<<1, the universe expands quasi-exponentially.

- If ε=0, space-time is exactly de Sitter:

$$ds^2 = -dt^2 + e^{2Ht}d\mathbf{x}^2$$

- *But, inflation never ends if ε=0.* When ε<<1, space-time is nearly, but not exactly, de Sitter:

$$ds^2 = -dt^2 + e^{2\int dt' H(t')}d\mathbf{x}^2$$

Symmetry of de Sitter Space

$$ds^2 = -dt^2 + e^{2Ht} d\mathbf{x}^2$$

- De Sitter spacetime is invariant under 10 isometries (transformations that keep ds^2 invariant):

- Time translation, followed by space dilation

$$t \to t - \lambda/H \,, \quad \mathbf{x} \to e^{\lambda} \mathbf{x}$$

- Spatial rotation, $\mathbf{x} \to R\mathbf{x}$

- Spatial translation, $\mathbf{x} \to \mathbf{x} + c$

- Three more transformations irrelevant to this talk

Symmetry of de Sitter Space

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discovered in 2012/13

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Rotational Invariance?

$$ds^2 = -dt^2 + e^{2Ht} d\mathbf{x}^2$$

- De Sitter spacetime is invariant under 10 isometries (transformations that keep ds^2 invariant):

- Time translation, followed by space dilation

$$t \to t - \lambda / H\,, \quad \mathbf{x} \to e^{\lambda} \mathbf{x}$$

discovered in 2012/13

- Spatial rotation, $\mathbf{x} \to R\mathbf{x}$

Is this symmetry valid?

- Spatial translation, $\mathbf{x} \to \mathbf{x} + c$

- Three more transformations irrelevant to this talk

Anisotropic Expansion

$$ds^2 = -dt^2 + e^{2Ht}\left[e^{-2\beta(t)}dx^2 + e^{2\beta(t)}(dy^2 + dz^2)\right]$$

- How large can $\dot{\beta}/H$ be during inflation?
- In single scalar field theories, Einstein's equation gives

$$\dot{\beta} \propto e^{-3Ht}$$

- But, the presence of anisotropic stress in the stress-energy tensor can source a **sustained** period of anisotropic expansion:

$$T^i_j = P\delta^i_j + \pi^i_j \quad \text{with} \quad \pi^1_1 = -\frac{2}{3}\mathcal{V}, \quad \pi^2_2 = \pi^3_3 = \frac{1}{3}\mathcal{V}$$

$$\Rightarrow \quad \ddot{\beta} + 3H\dot{\beta} = \frac{1}{3}\mathcal{V}$$

Inflation with a vector field

- Consider that there existed a vector field at the beginning of inflation:

$$A_\mu = (0, u(t), 0, 0)$$

A_1: Preferred direction in space at the initial time

- You might ask where A_μ came from. Well, if we have a scalar field and a tensor field (gravitational wave), why not a vector?

- The conceptual problem of this setting is not the existence of a vector field, but that it requires A_1 that is homogeneous over a few Hubble lengths before inflation

  - But, this problem is common with the original inflation, which requires ϕ that is homogeneous over a few Hubble lengths, in order for inflation in occur in the first place!

Ackerman, Carroll & Wise (2007); Watanabe, Kanno & Soda (2010)

Observational Consequence

- Broken rotational invariance makes **the power spectrum depend on a direction of wavenumber**

$$P(k) \to P(\mathbf{k}) = P_0(k)\left[1 + g_*(k)(\hat{k}\cdot\hat{E})^2\right]$$

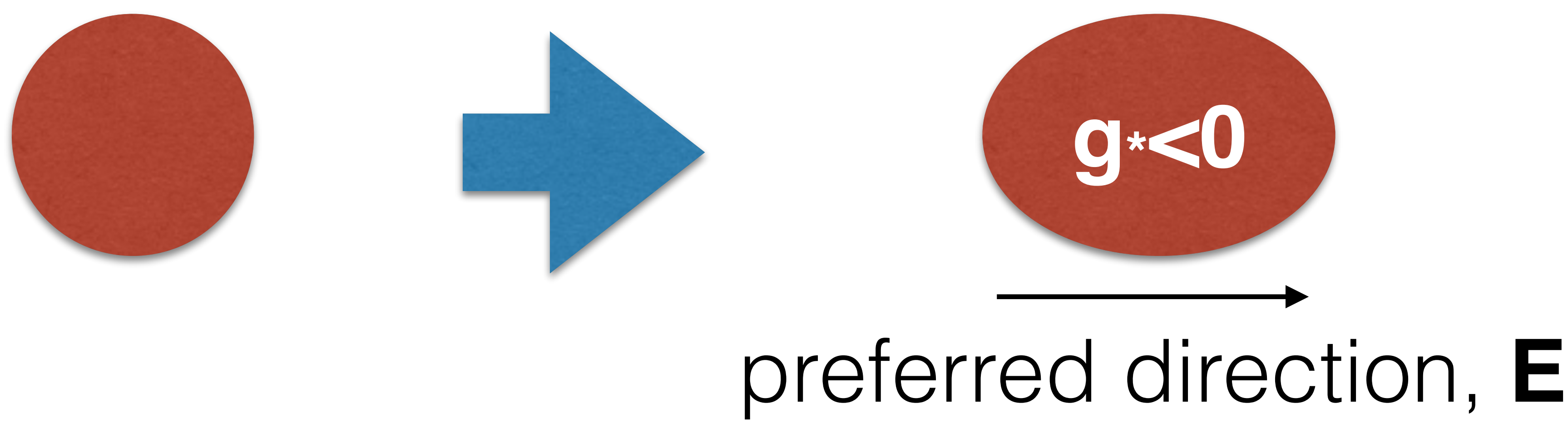
where $\hat{E}$ is a preferred direction in space

- A model predicts: $g_*(k) = -\mathcal{O}(1) \times 24 I_k N_k^2$

- I = [energy density of a vector]/[energy density of a scalar], divided by ε
- N~60 is the number of e-fold of inflation counted from the end of inflation

Signatures in CMB

- This "quadrupolar" modulation of the power spectrum turns a circular hot/cold spot of CMB into an elliptical one



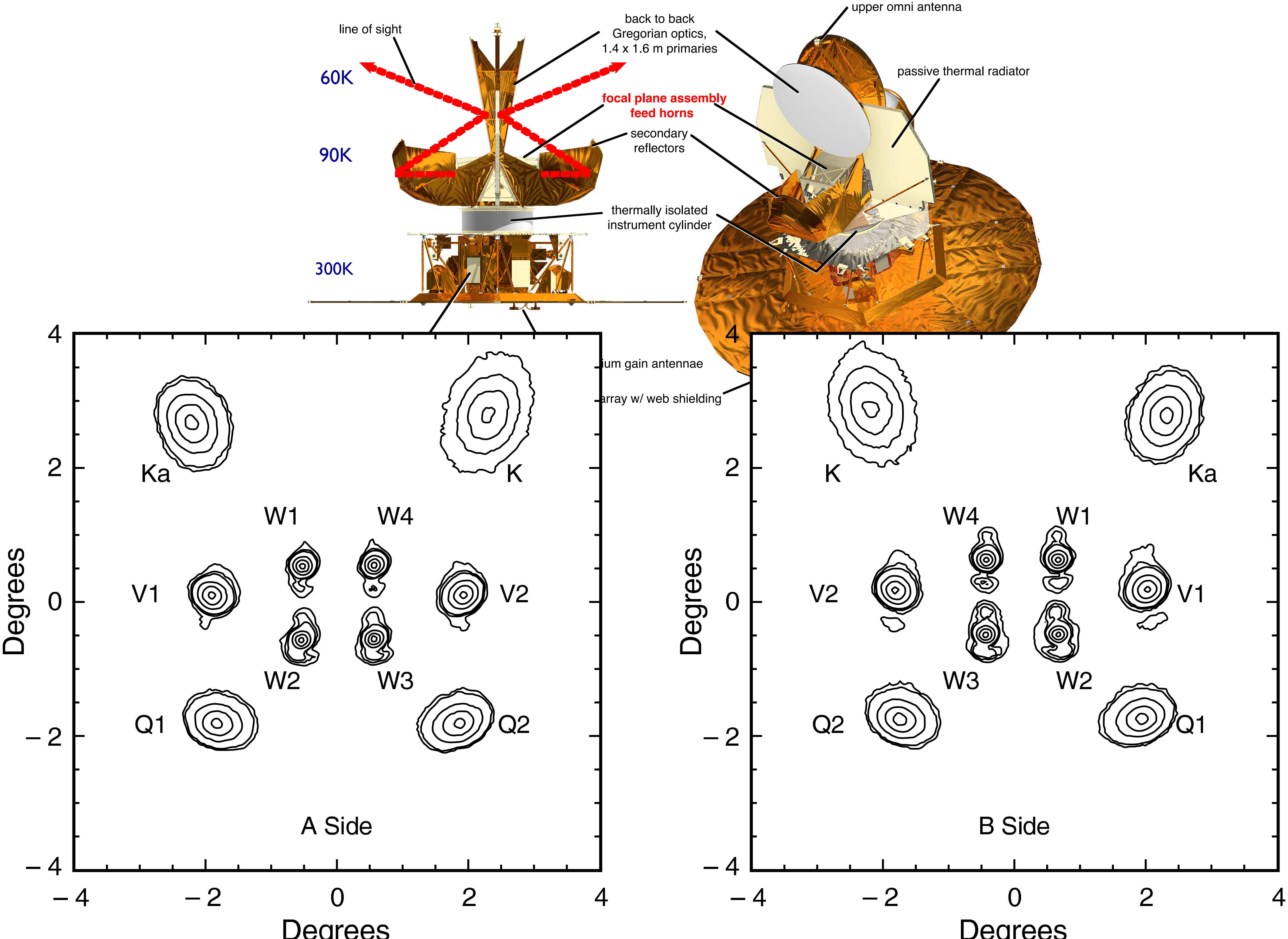
- This is a local effect, rather than a global effect: the power spectrum measured at *any location* in the sky is modulated by $(\hat{k} \cdot \hat{E})^2$

A Beautiful Story

- In 2007, Ackerman, Carroll and Wise proposed g_* as a powerful probe of rotational symmetry
- In 2009, Groeneboom and Eriksen reported a significant detection, $g_*=0.15\pm0.04$, in the WMAP data at 94 GHz
- **Surprise!** And a beautiful story - a new observable proposed by theorists was looked for in the data, and was found

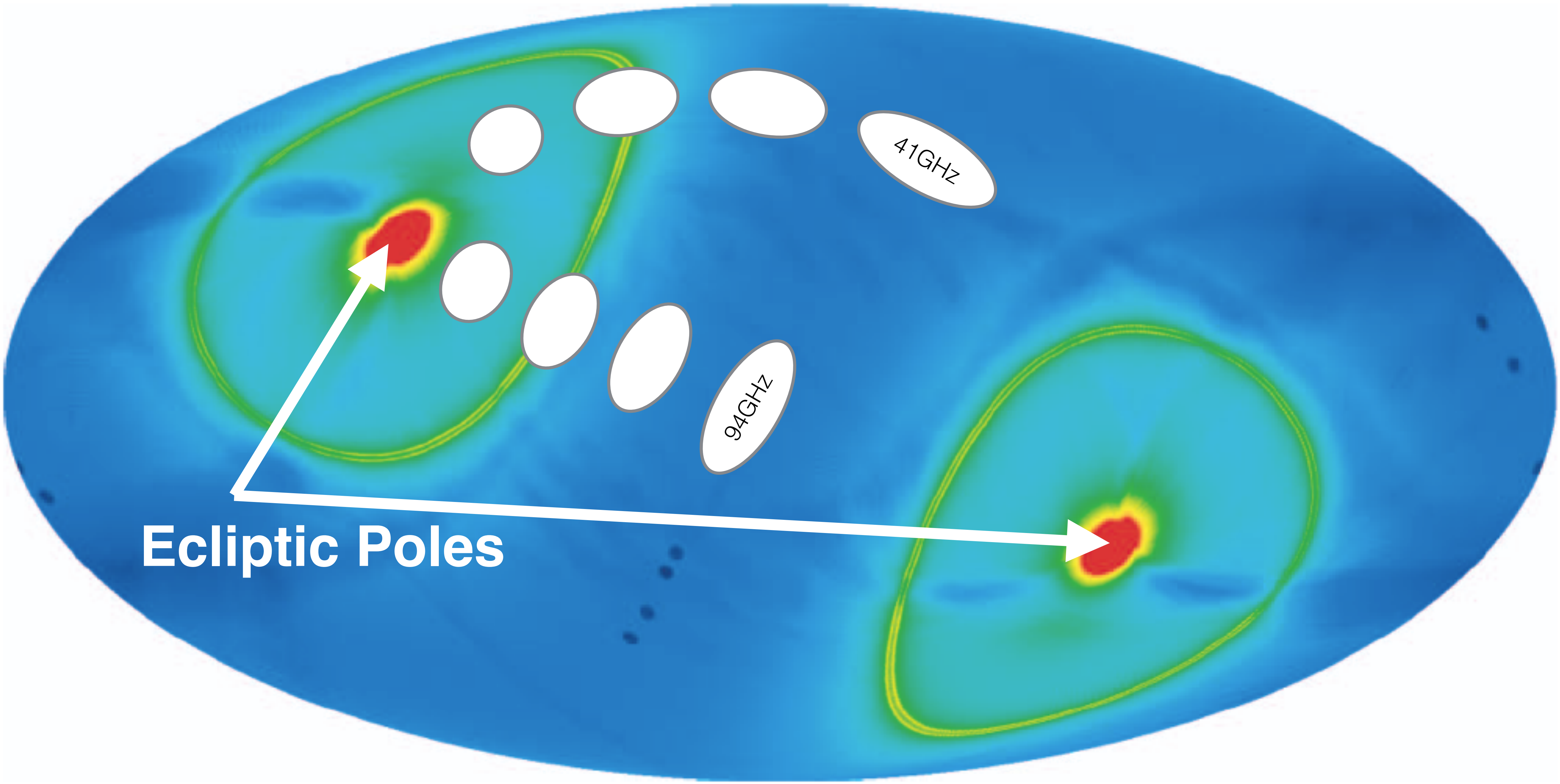
Subsequent Story

- In 2010, Groeneboom et al. reported that the WMAP data at 41 GHz gave the opposite sign: $g_* = -0.18 \pm 0.04$, suggesting instrumental systematics
- The best-fit preferred direction in the sky was the ecliptic pole
- Elliptical beam (point spread function) of WMAP was a culprit!



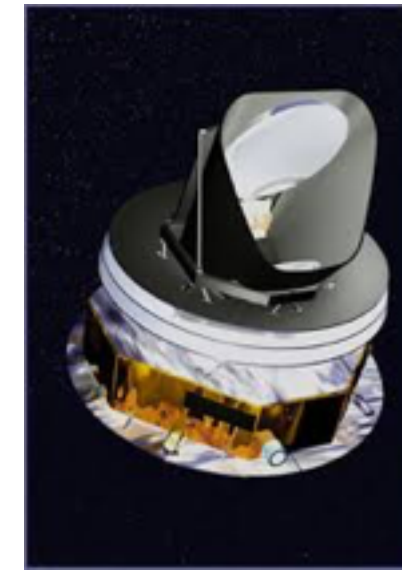
line of sight
back to back
Gregorian optics,
1.4 x 1.6 m primaries
upper omni antenna
passive thermal radiator
60K
focal plane assembly
feed horns
90K
secondary
reflectors
thermally isolated
instrument cylinder
300K
ium gain antennae
array w/ web shielding
Ka
K
W1
W4
V1
V2
W2
W3
Q1
Q2
A Side
K
Ka
W4
W1
V2
V1
W3
W2
Q2
Q1
B Side
Degrees
Degrees
Degrees
Degrees
4
2
0
−2
−4
−4
−2
0
2
4

# of observations in Galactic coordinates



- WMAP visits ecliptic poles from many different directions, circularising beams
- WMAP visits ecliptic planes with 30% of possible angles

Planck 2013 Data

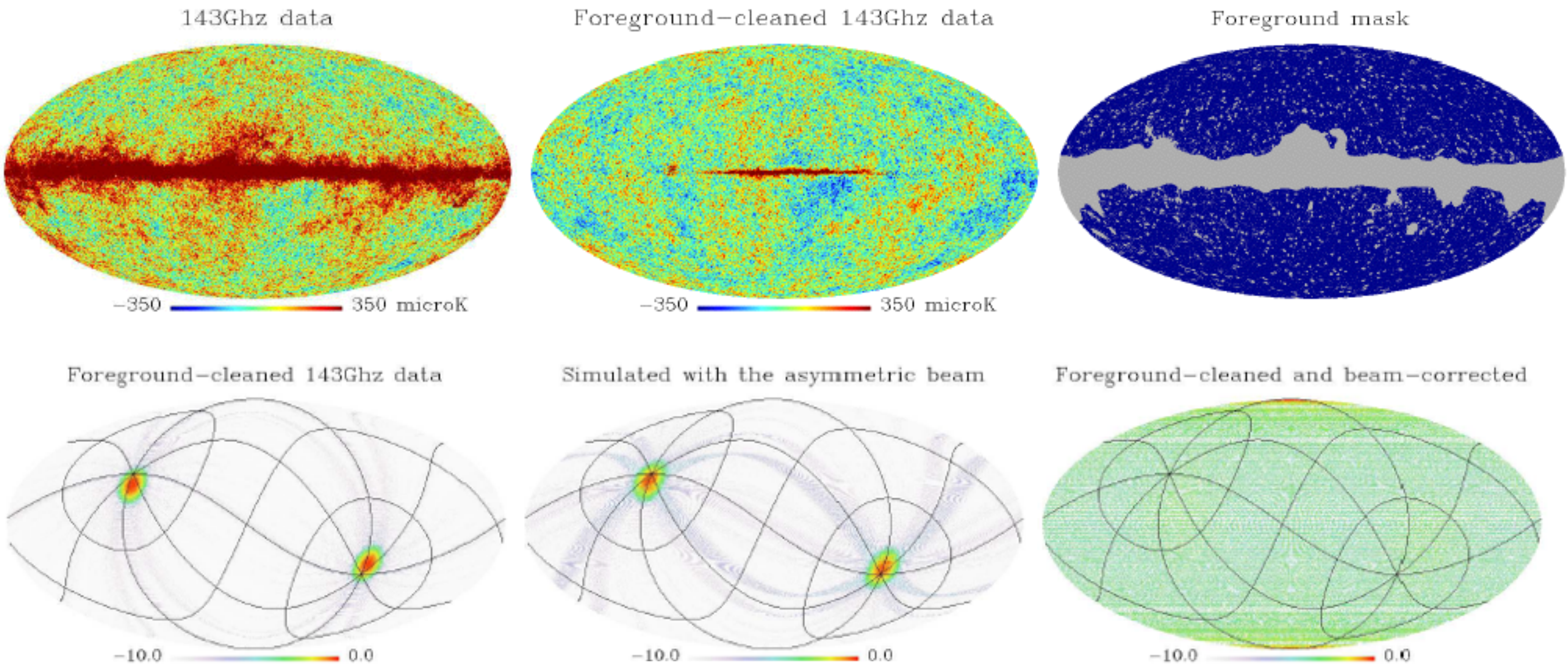


- We analysed the Planck 2013 temperature data at 143GHz, and found significant **$g_*=-0.111\pm0.013$** [after removing the foreground emission]

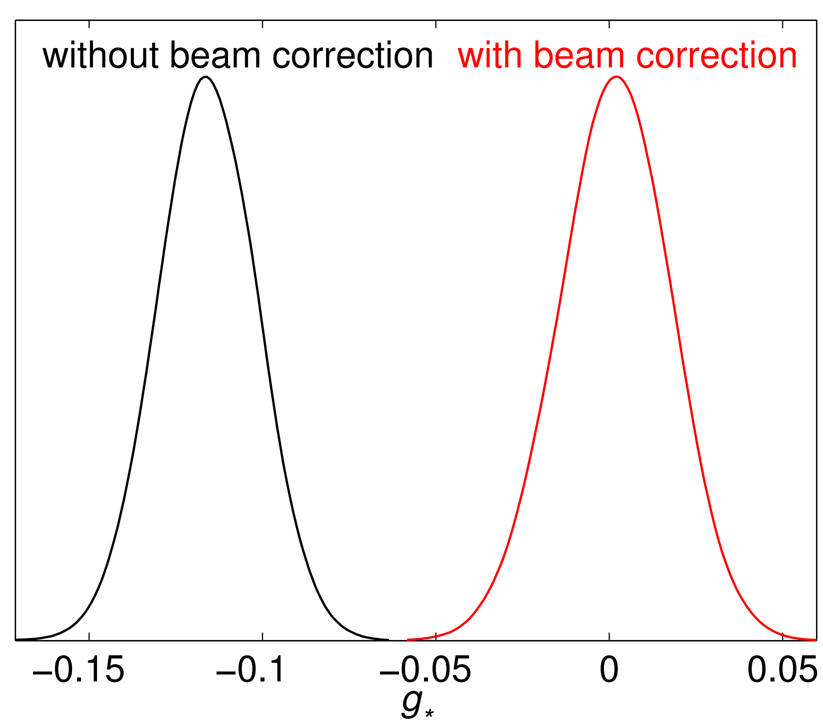
- This is consistent with what we expect from the beam ellipticity of the Planck data

- After subtracting the effect of beam ellipticities, no evidence for g_* was found

Kim & EK (2013)



g_*(raw)=–0.111±0.013
(68%CL)



g_*=0.002±0.016
(68%CL)

Naruko, EK & Yamaguchi (2015)

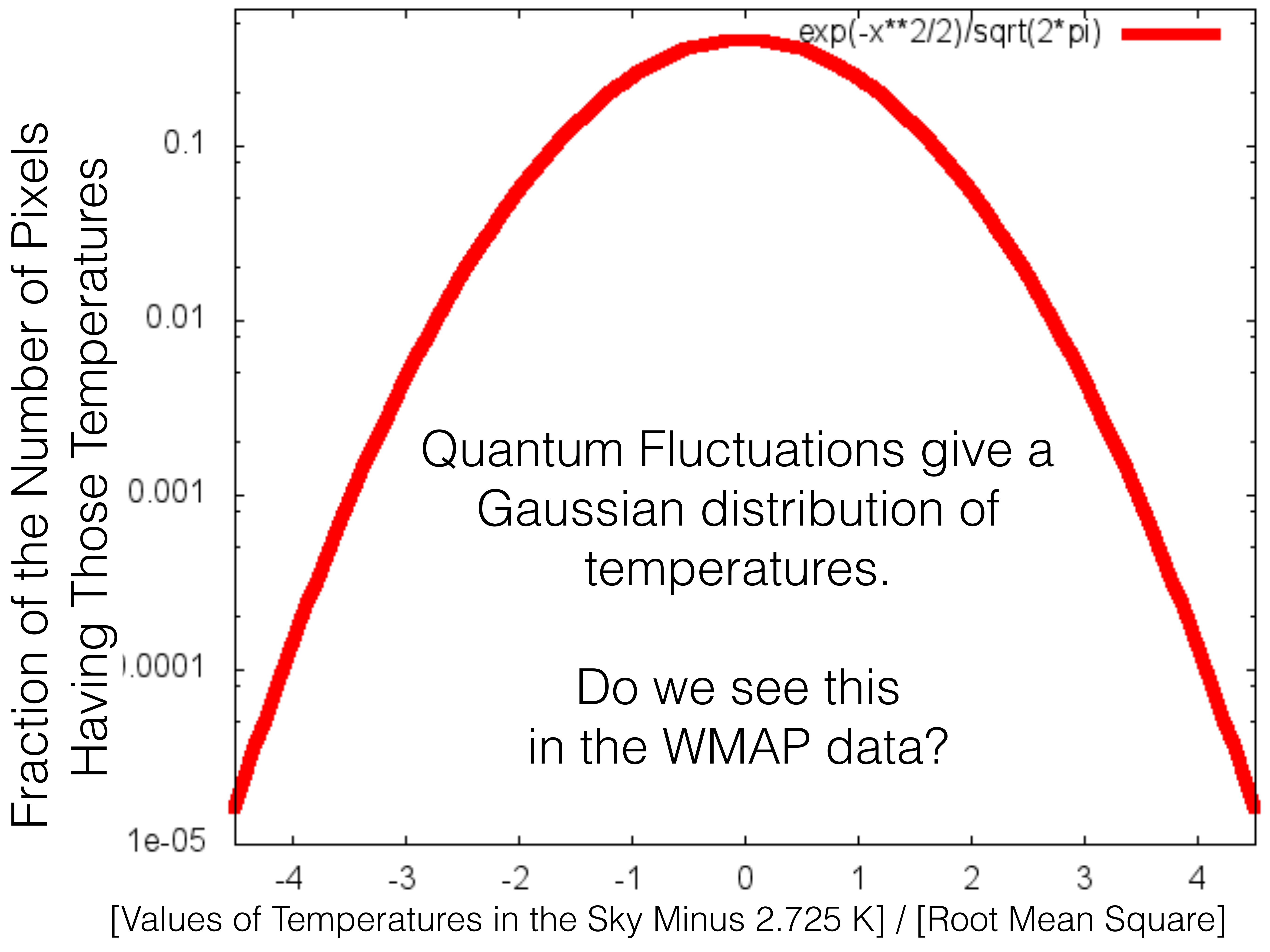
Implication for Rotational Symmetry

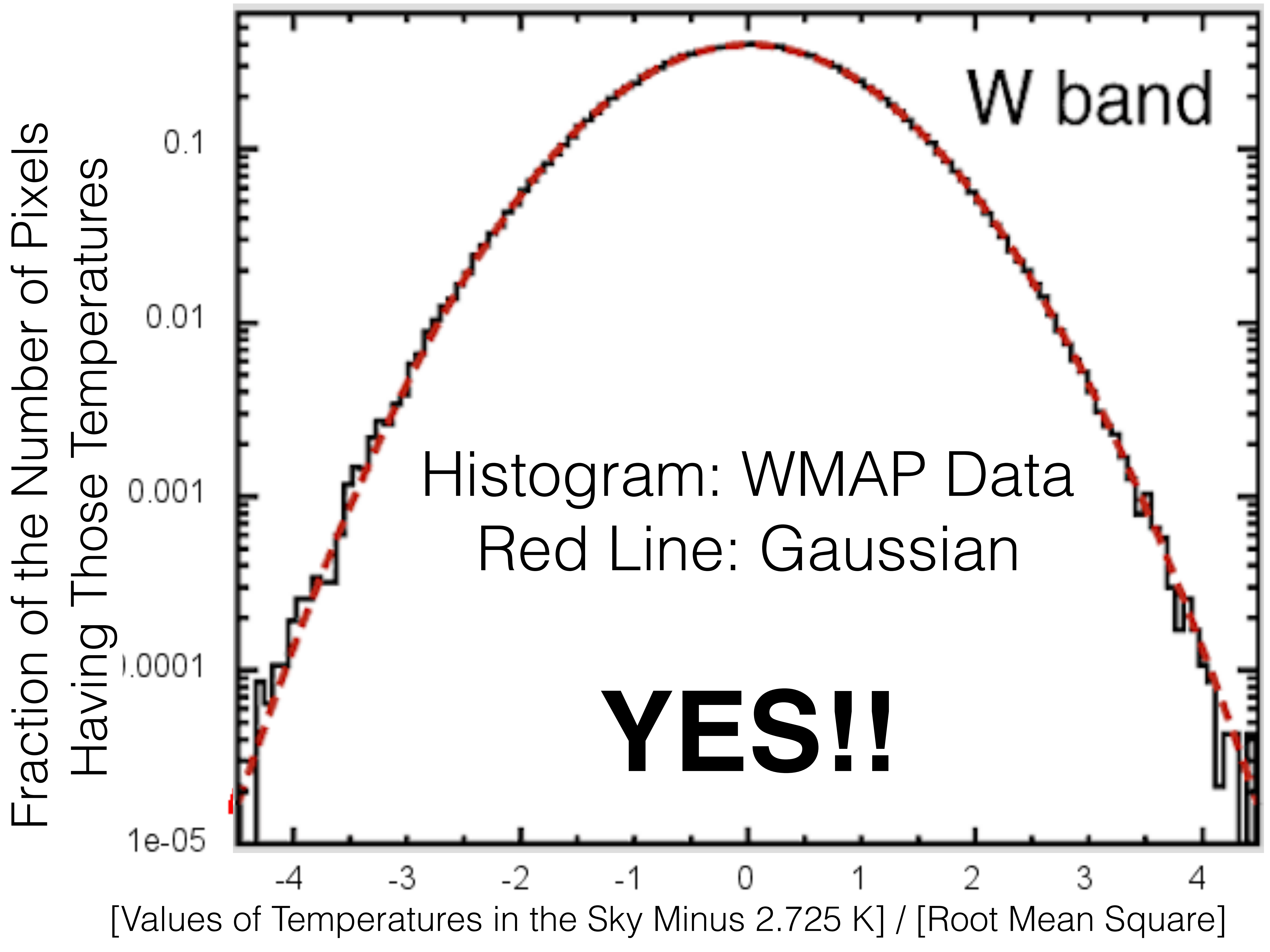
- g_* is consistent with zero, with 95%CL upper bound of $|g_*|<0.03$
- Comparing this with the model prediction, $|g_*| \sim 24IN^2$, we conclude $I<5\times10^{-7}$
- Thus, $$\frac{\dot{\beta}}{H} \approx \frac{\mathcal{V}}{U} \approx \epsilon I < 5\times10^{-9}$$

Breaking of rotational symmetry is tiny, if any!

[cf: "natural" value is either 10^{-2} or $e^{-3N}=e^{-150}$!!]

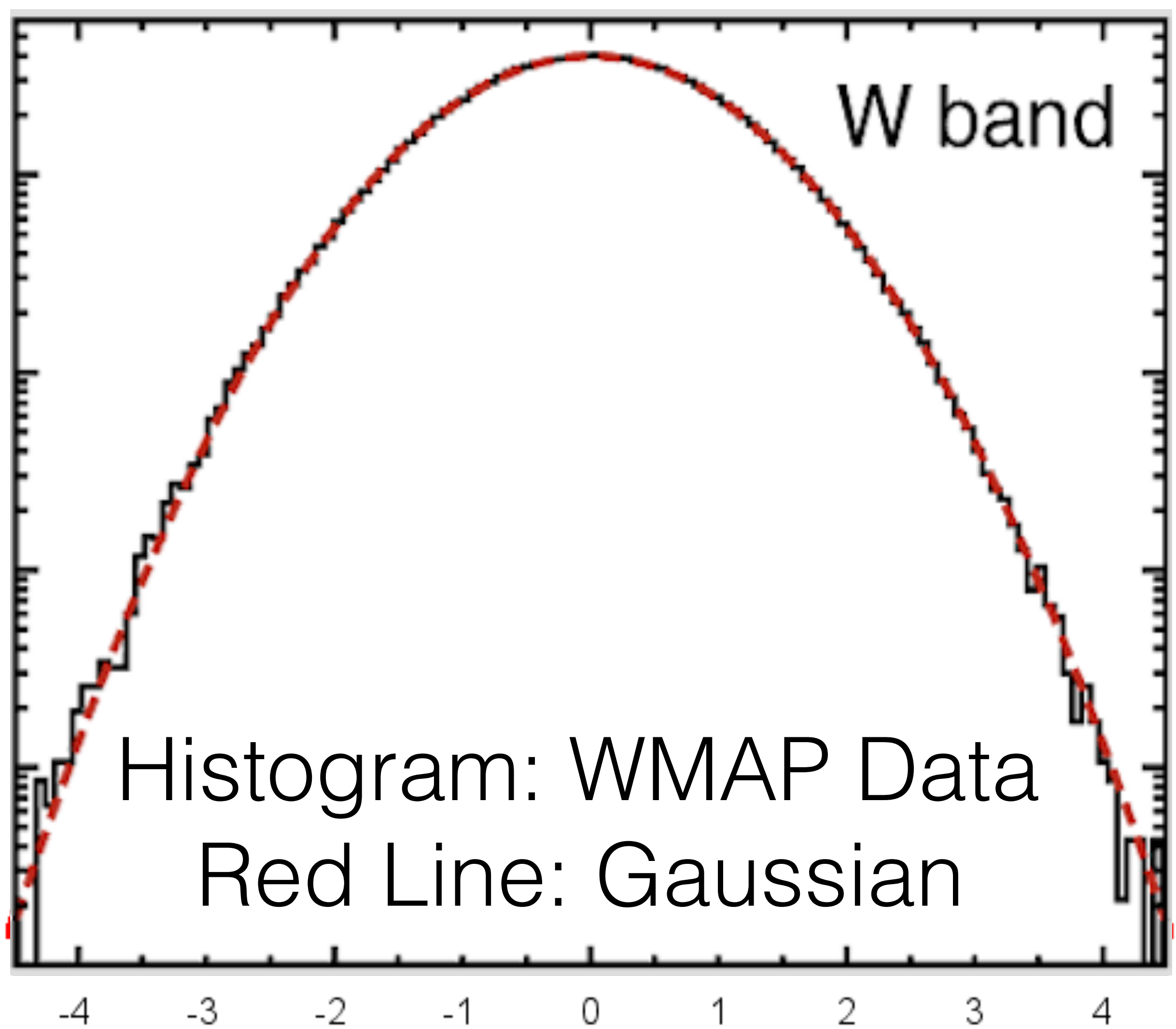
How do we know that primordial fluctuations were of *quantum mechanical origin*?


exp(-x**2/2)/sqrt(2*pi)
0.1
0.01
0.001
0.0001
1e-05
-4
-3
-2
-1
0
1
2
3
4
Fraction of the Number of Pixels
Having Those Temperatures
[Values of Temperatures in the Sky Minus 2.725 K] / [Root Mean Square]
Quantum Fluctuations give a
Gaussian distribution of
temperatures.
Do we see this
in the WMAP data?



W band
0.1
0.01
0.001
0.0001
1e-05
-4
-3
-2
-1
0
1
2
3
4
Histogram: WMAP Data
Red Line: Gaussian
YES!!
Fraction of the Number of Pixels
Having Those Temperatures
[Values of Temperatures in the Sky Minus 2.725 K] / [Root Mean Square]

Testing Gaussianity



Fraction of the Number of Pixels Having Those Temperatures

[Values of Temperatures in the Sky Minus 2.725 K]/ [Root Mean Square]

Since a Gauss distribution is symmetric, it must yield a vanishing **3-point function**

$$\langle \delta T^3 \rangle \equiv \int_{-\infty}^{\infty} d\delta T \; P(\delta T) \delta T^3$$

More specifically, we measure this using temperatures at three different locations and average:

$$\langle \delta T(\hat{n}_1) \delta T(\hat{n}_2) \delta T(\hat{n}_3) \rangle$$

Non-Gaussianity:

A Powerful Test of Quantum Fluctuations

- The WMAP data show that the distribution of temperature fluctuations of CMB is **very precisely** Gaussian
  - with an upper bound on a deviation of **0.2%**
- With improved data provided by the Planck mission, the upper bound is now **0.03%**

Secondary Anisotropies: Structure Formation seen in the CMB

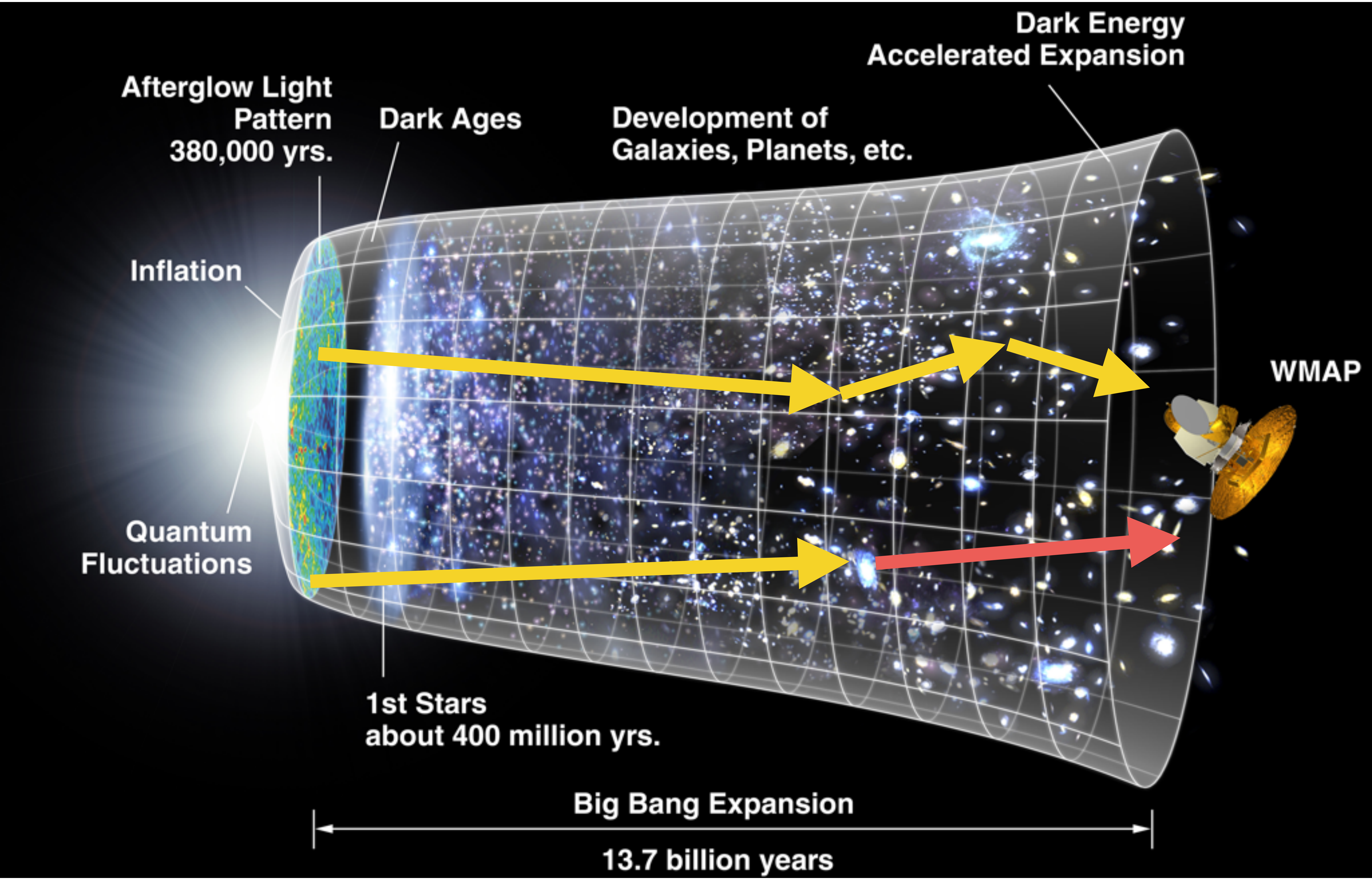
Gravitational Lensing

- Matter bends light of the CMB

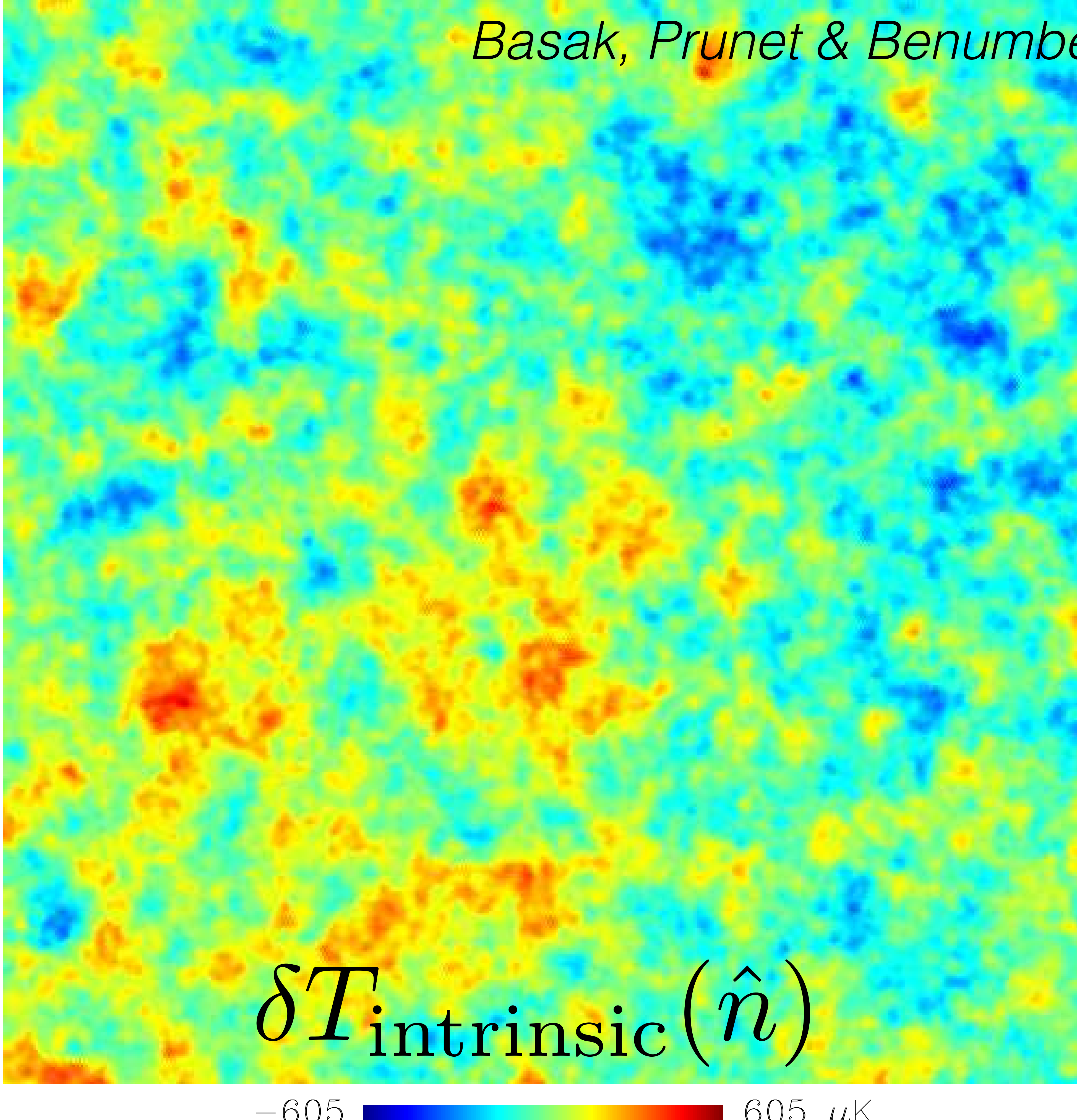
Sunyaev-Zel'dovich Effect

- Electrons in hot, collapsed gas up-scatter low-energy CMB photons, distorting the black-body spectrum of the CMB

- Both have been measured, providing the key insights into how the structures grew out of initial conditions. **Initial conditions to structure formation**, using the CMB data only!

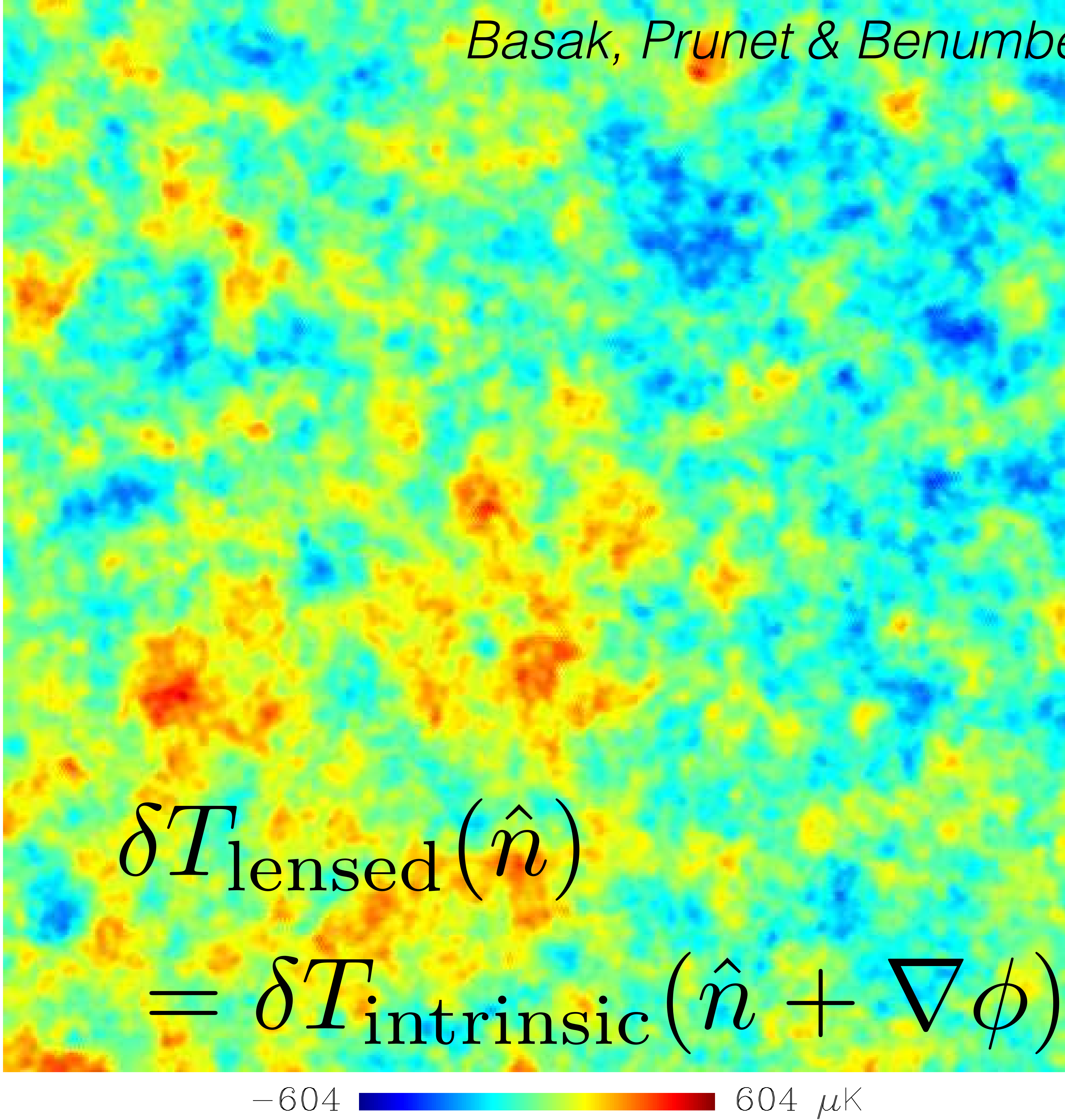

Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Basak, Prunet & Benumbed (2008)

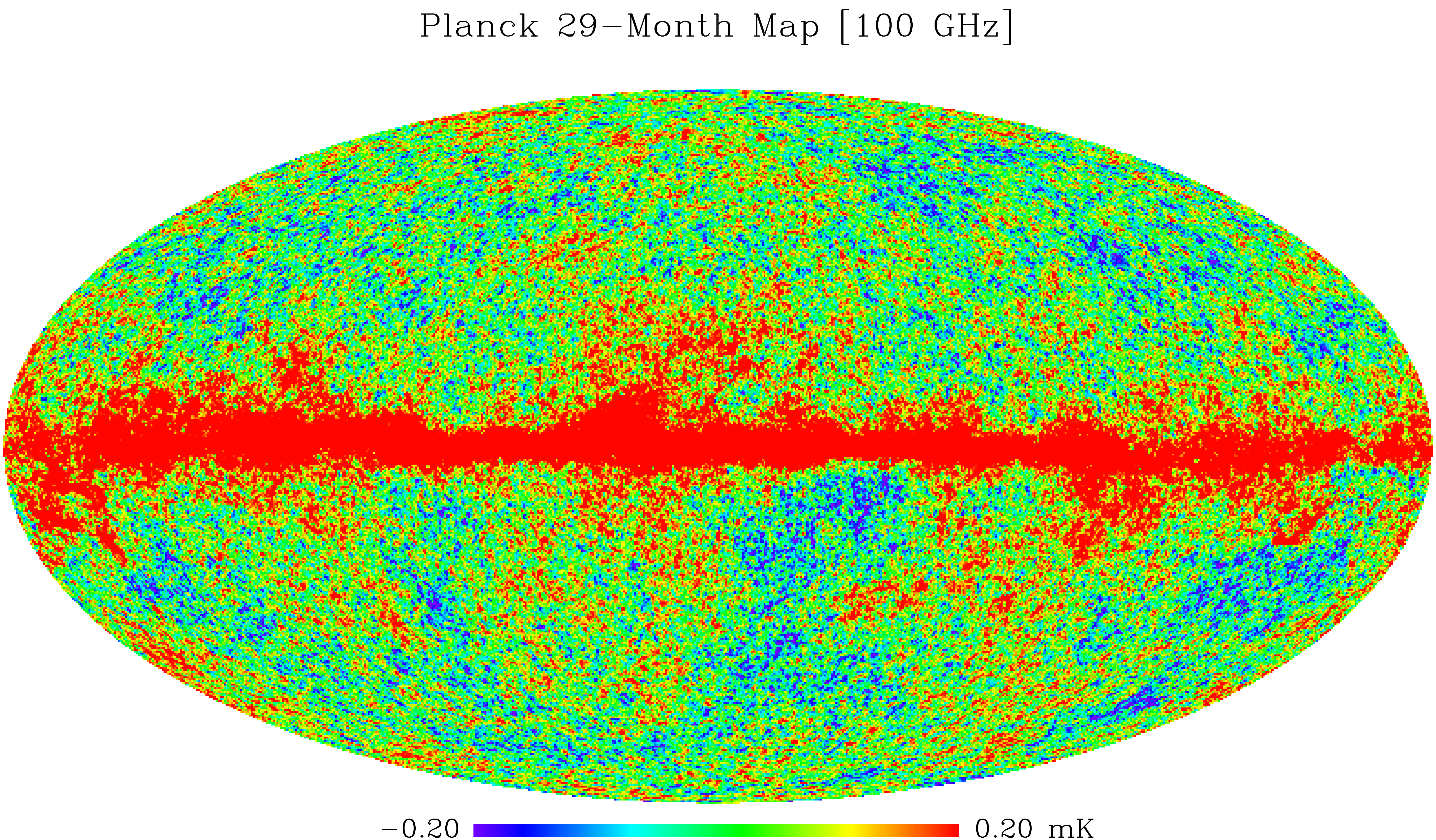


$$\delta T_{\mathrm{intrinsic}}(\hat{n})$$

Basak, Prunet & Benumbed (2008)

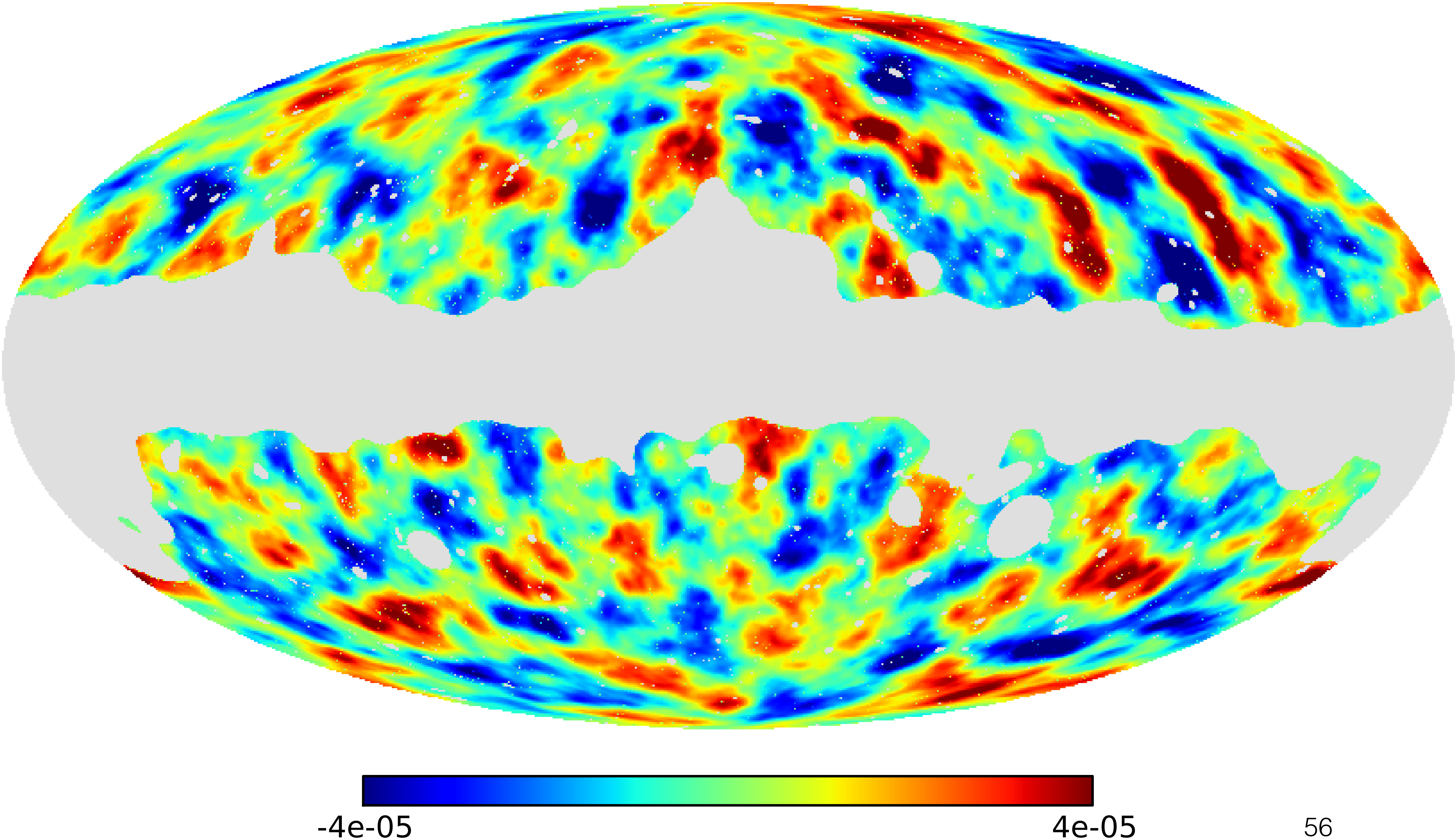


$$\delta T_{\mathrm{lensed}}(\hat{n}) = \delta T_{\mathrm{intrinsic}}(\hat{n} + \nabla\phi)$$

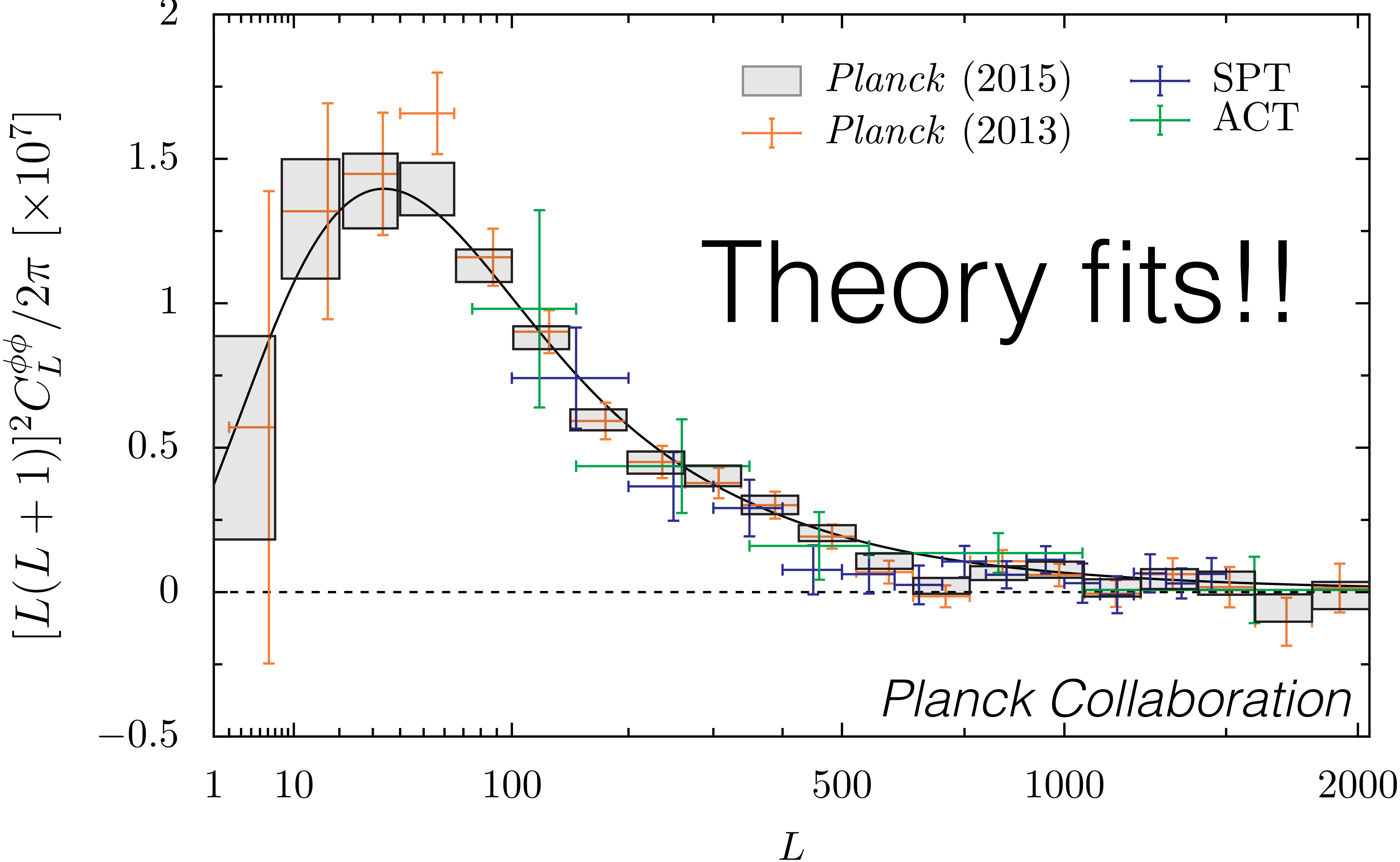


From full-sky temperature maps to…

Planck Collaboration



A full-sky lensing potential map!



...and our knowledge of the matter distribution improves

$\Omega_m = 0.315 \pm 0.013$

$\sigma_8 = 0.829 \pm 0.014$

Adding the lensing info

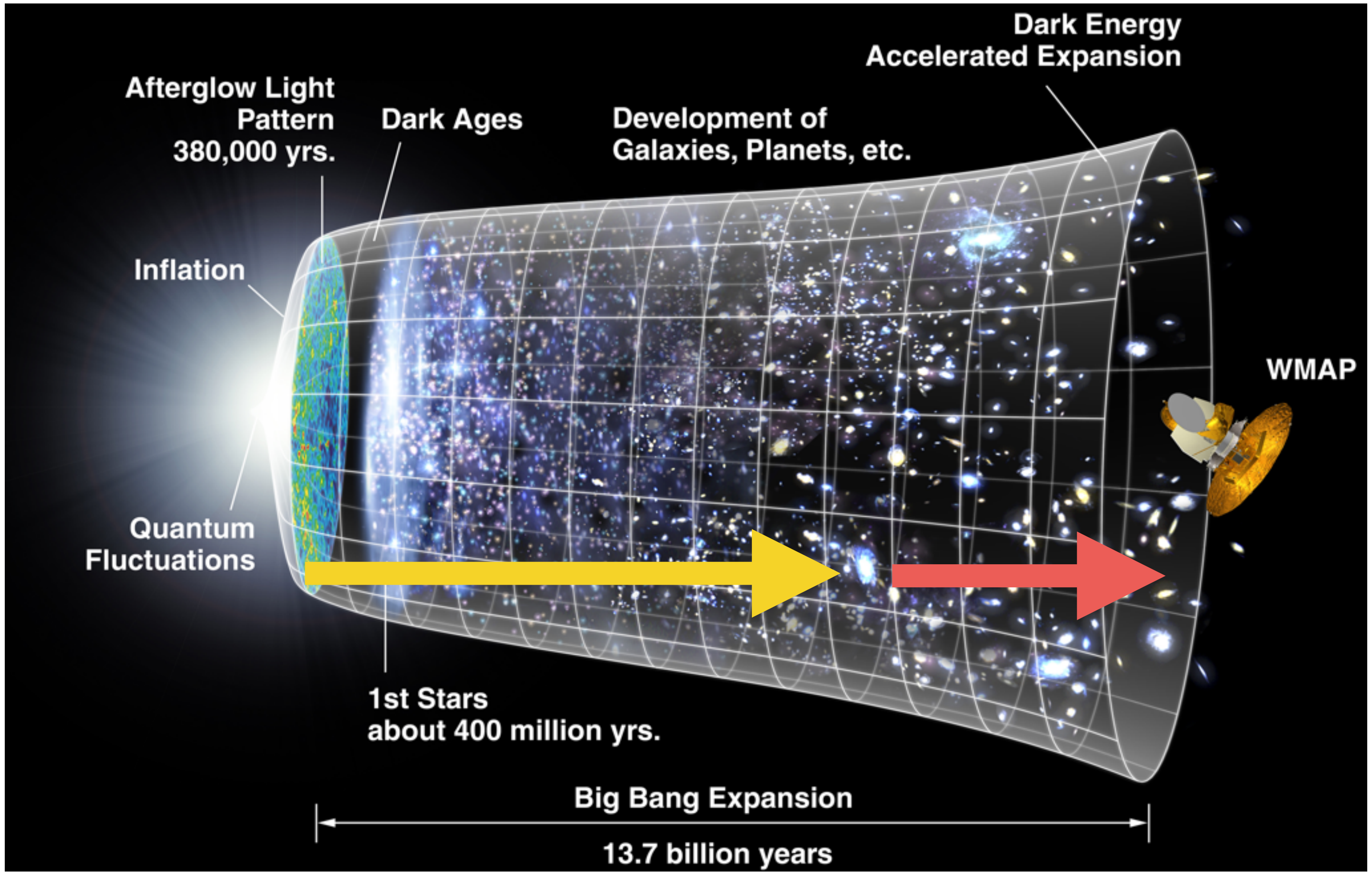
$\Omega_m = 0.308 \pm 0.012$

$\sigma_8 = 0.8149 \pm 0.0093$

The Sunyaev-Zel'dovich Effect

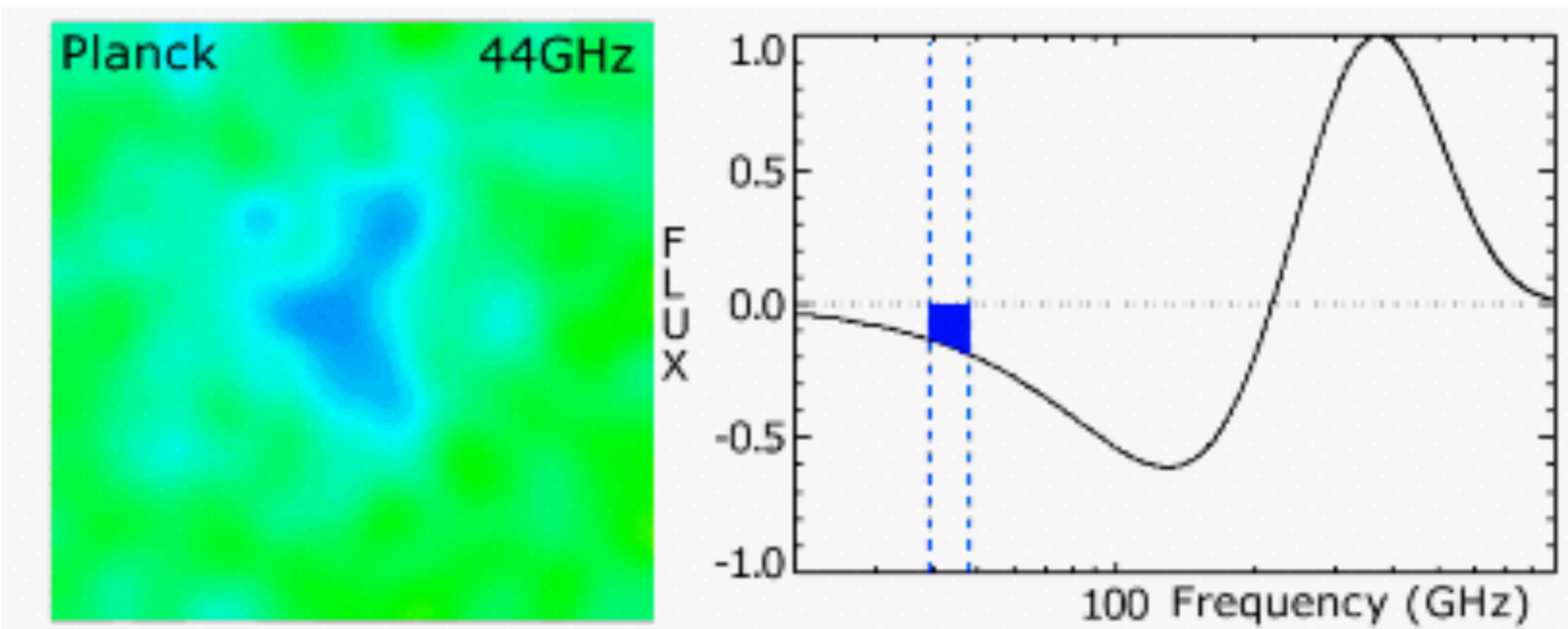
- The "thermal" SZ effect (in the non-relativistic limit) enables us to map thermal pressure in the universe

$$Y_{\mathrm{tSZ}}(\boldsymbol{\theta}) = \frac{k_B \sigma_T}{m_e c^2} \int \mathrm{d}l \, n_e(\boldsymbol{\theta}, l) \, T(\boldsymbol{\theta}, l)$$

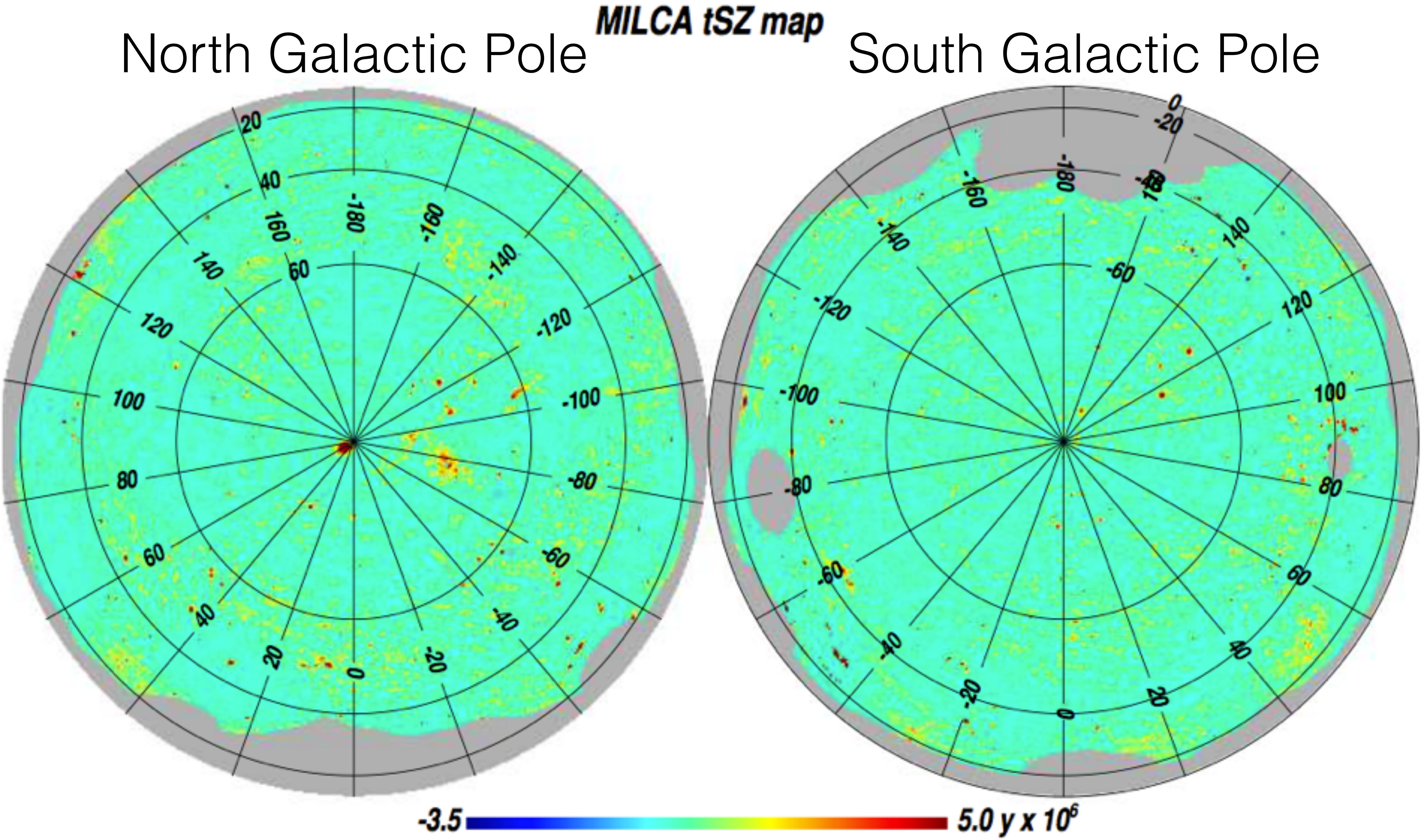


Thermal SZ (tSZ) Effect

- The unique frequency dependence of tSZ: we can make a map of Y_{tSZ}



Full-sky Thermal Pressure Map



Planck Collaboration

We can simulate this

Klaus Dolag (MPA/LMU)

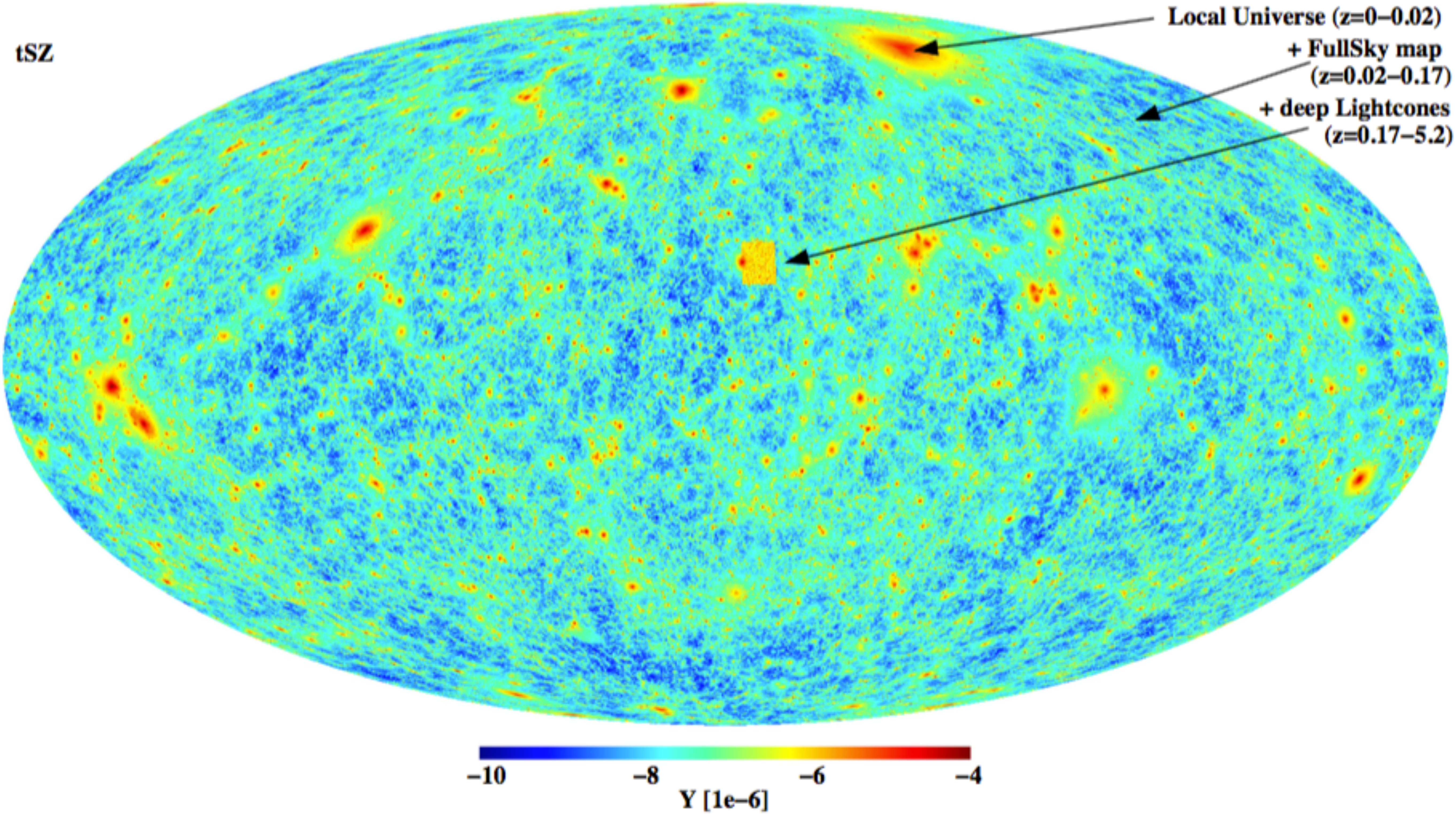
arXiv:1509.05134 [accepted for publication in MNRAS]

SZ effects in the Magneticum Pathfinder Simulation: Comparison with the Planck, SPT, and ACT results

K. Dolag[1,2*], E. Komatsu[2,3] and R. Sunyaev[2,4]
[1] *University Observatory Munich, Scheinerstr. 1, 81679 Munich, Germany*
[2] *Max-Planck-Institut für Astrophysik, Karl-Schwarzschild Strasse 1, 85748 Garching, Germany*
[3] *Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU, WPI), Todai Institutes for Advanced Study, the University of Tokyo, Kashiwa 277-8583, Japan*
[4] *Space Research Institute (IKI), Russian Academy of Sciences, Profsoyuznaya str. 84/32, Moscow, 117997 Russia*

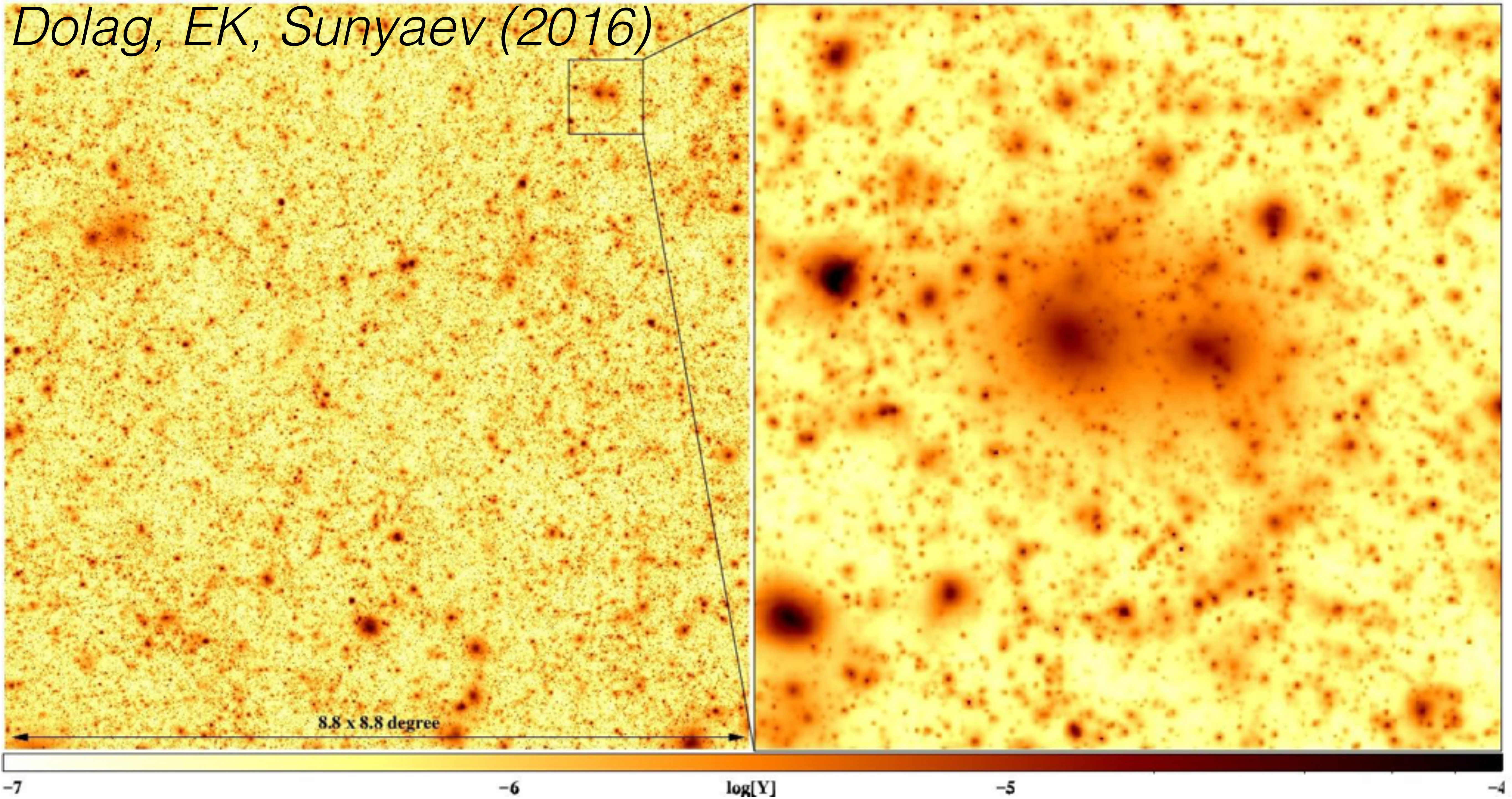
- Volume: (896 Mpc/h)^3
- Cosmological hydro (P-GADGET3) with star formation and AGN feed back
- 2 x 1526^3 particles (m_{DM}=7.5x10^8 M_{sun}/h)

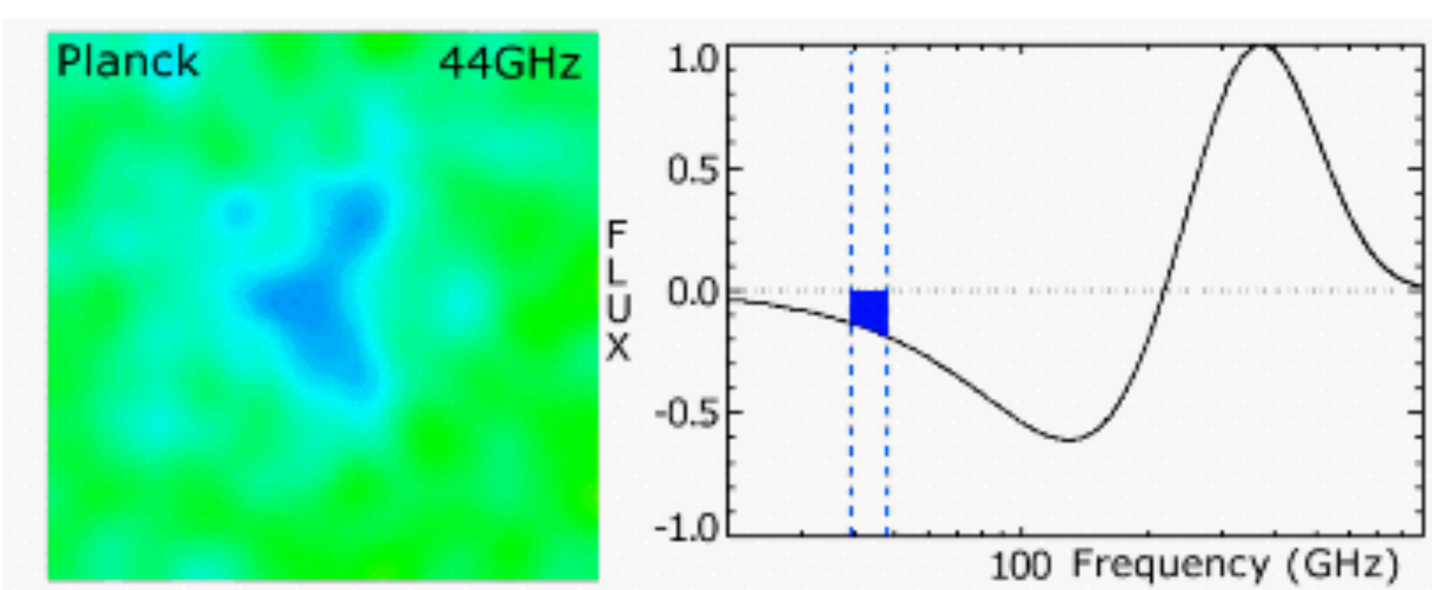
Dolag, EK, Sunyaev (2016)



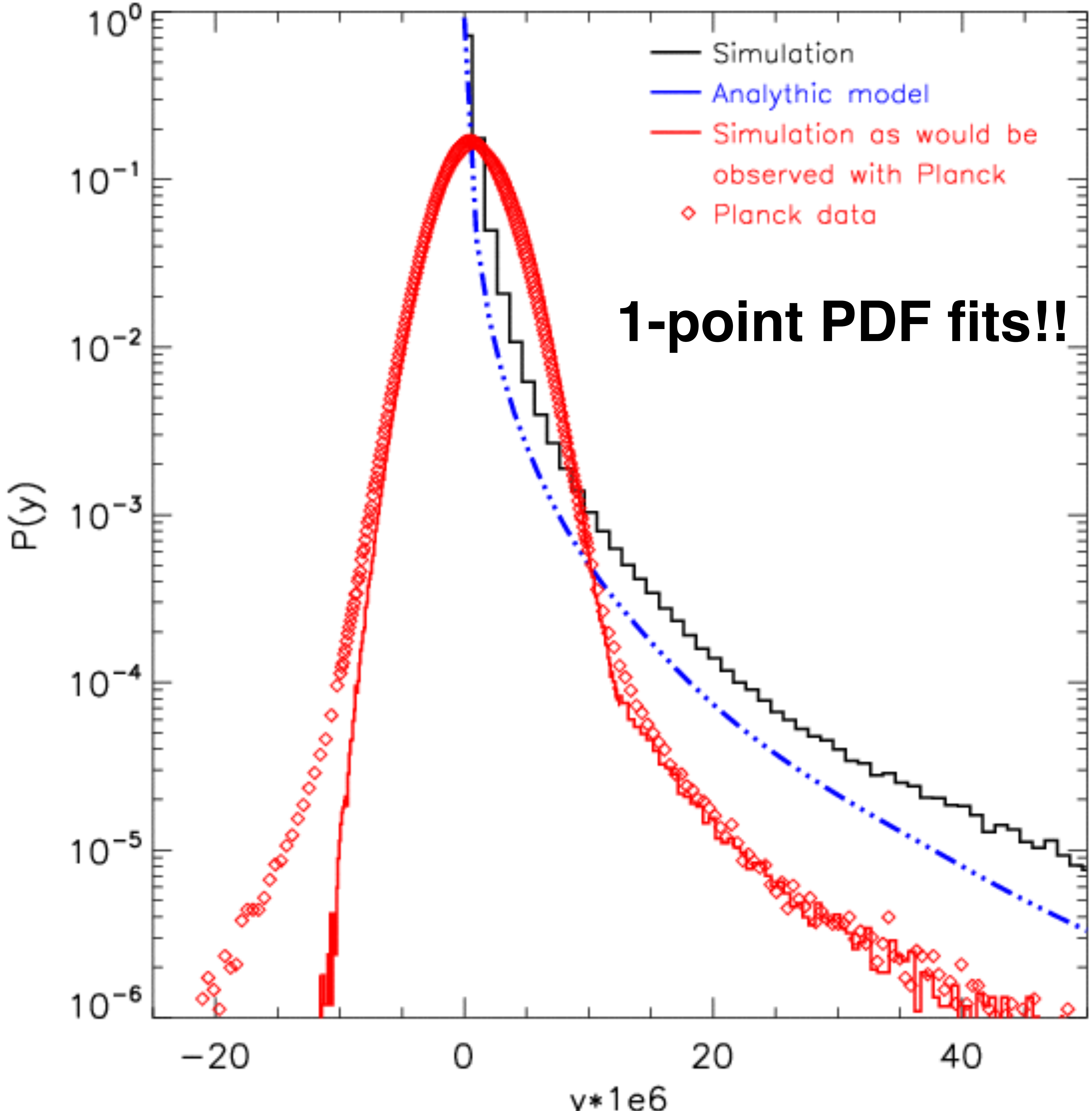
$$Y_{\mathrm{tSZ}}(\boldsymbol{\theta}) = \frac{k_B \sigma_T}{m_e c^2} \int \mathrm{d}l \, n_e(\boldsymbol{\theta}, l) \, T(\boldsymbol{\theta}, l)$$

Dolag, EK, Sunyaev (2016)



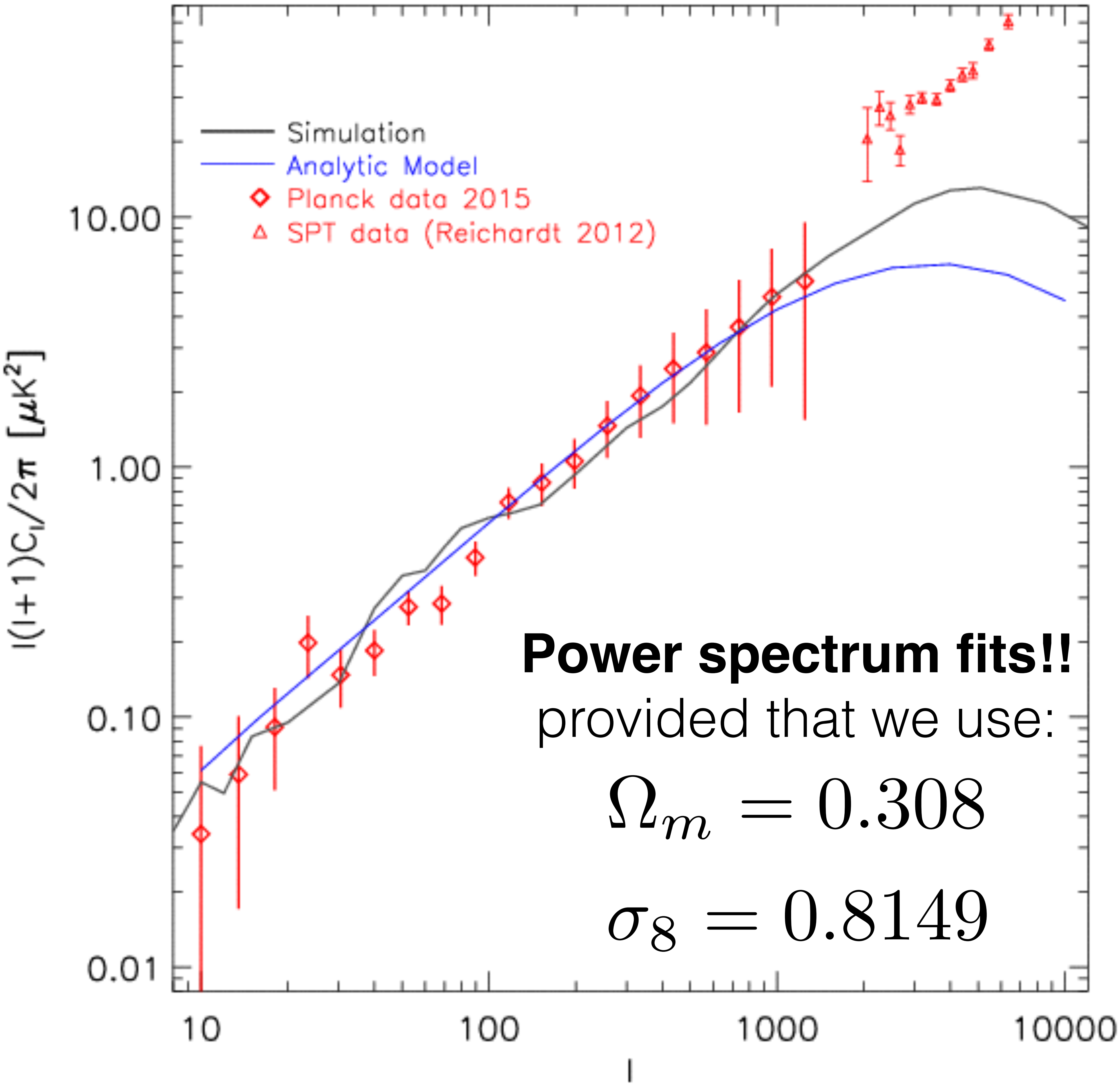


Dolag, EK, Sunyaev (2016)



1-point PDF fits!!

Dolag, EK, Sunyaev (2016)



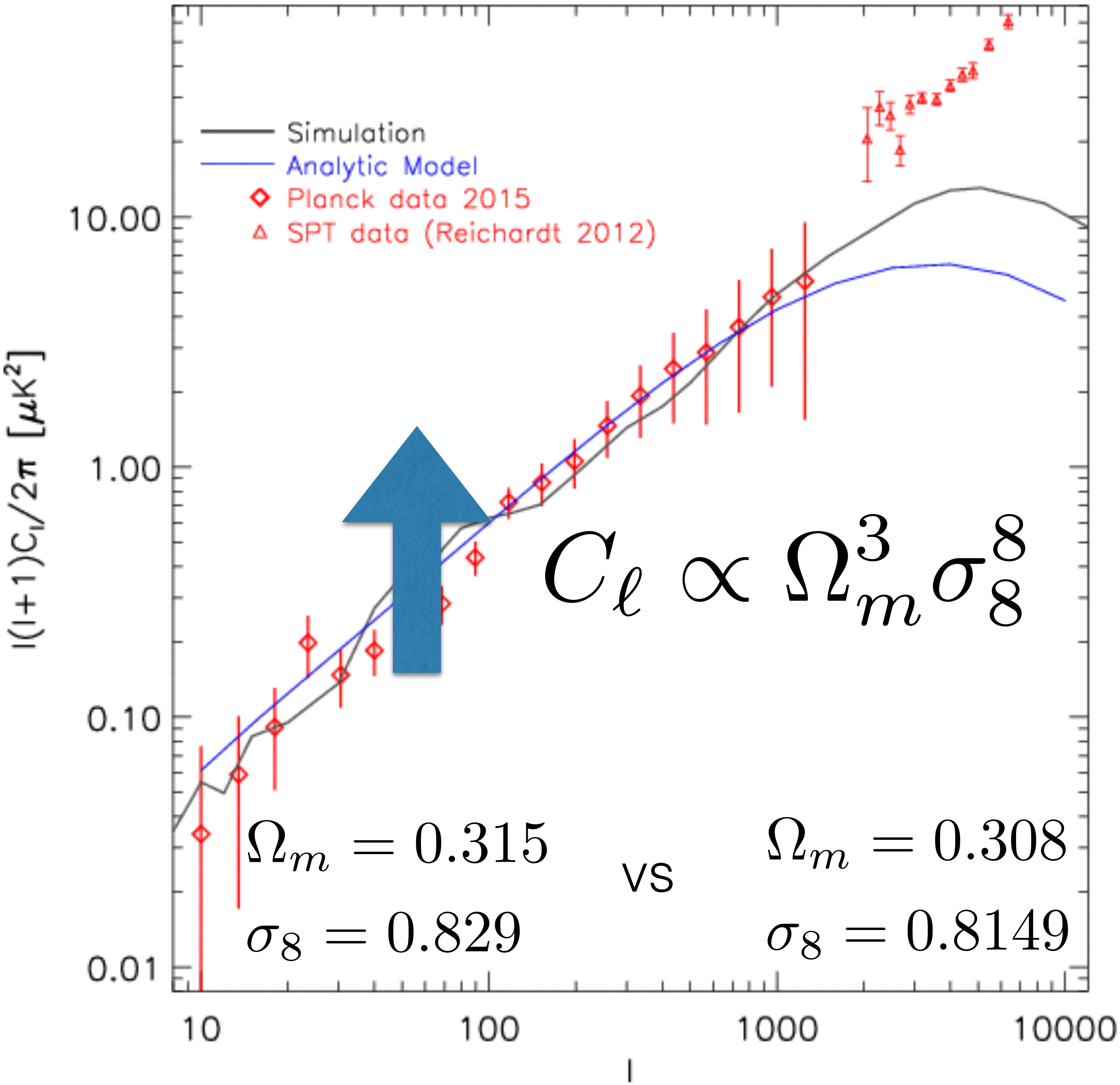
Power spectrum fits!!

provided that we use:

$$\Omega_m = 0.308$$

$$\sigma_8 = 0.8149$$

Dolag, EK, Sunyaev (2016)

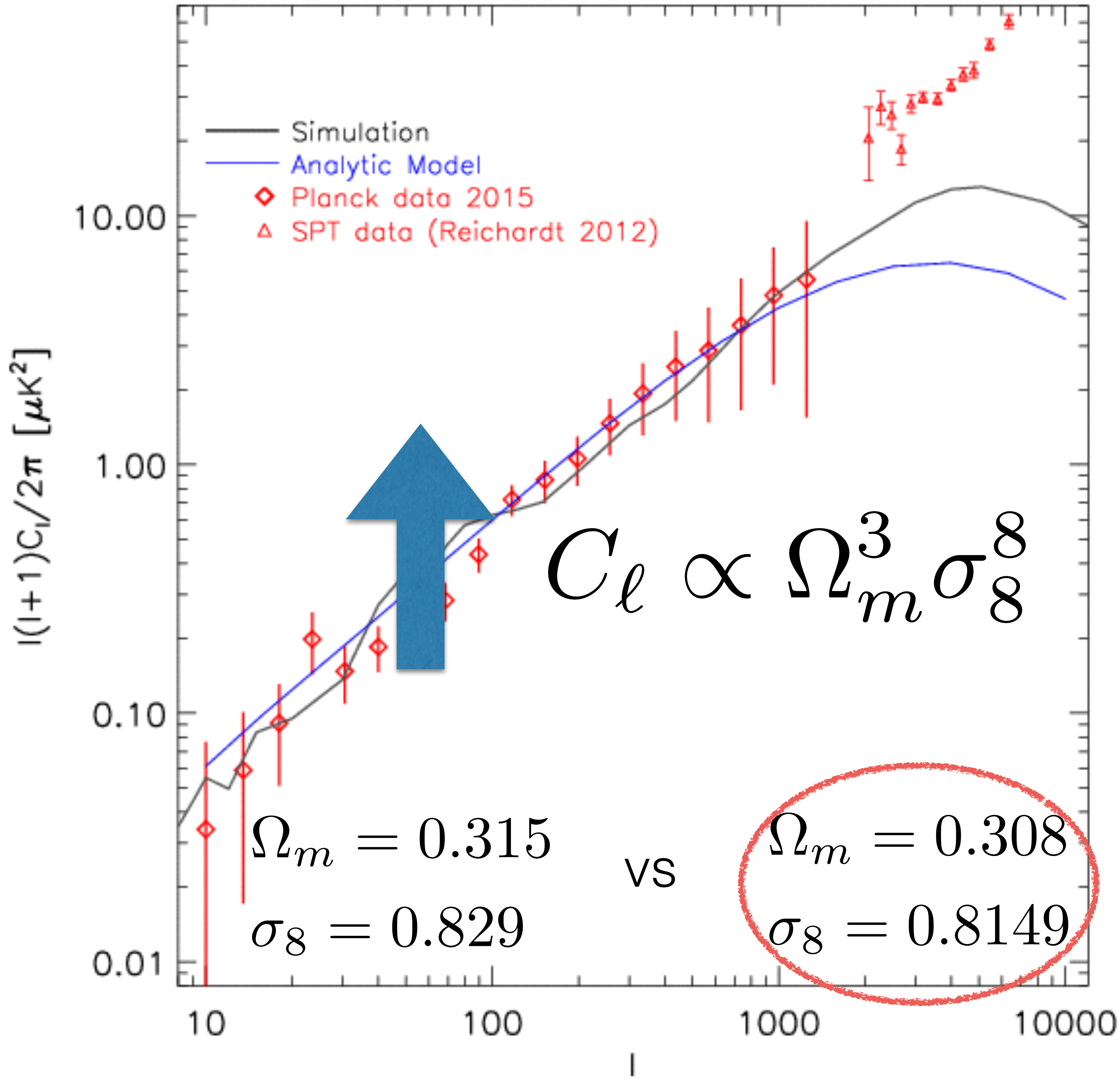


$$C_\ell \propto \Omega_m^3 \sigma_8^8$$

$\Omega_m = 0.315$ vs $\Omega_m = 0.308$

$\sigma_8 = 0.829$ $\sigma_8 = 0.8149$

Dolag, EK, Sunyaev (2016)

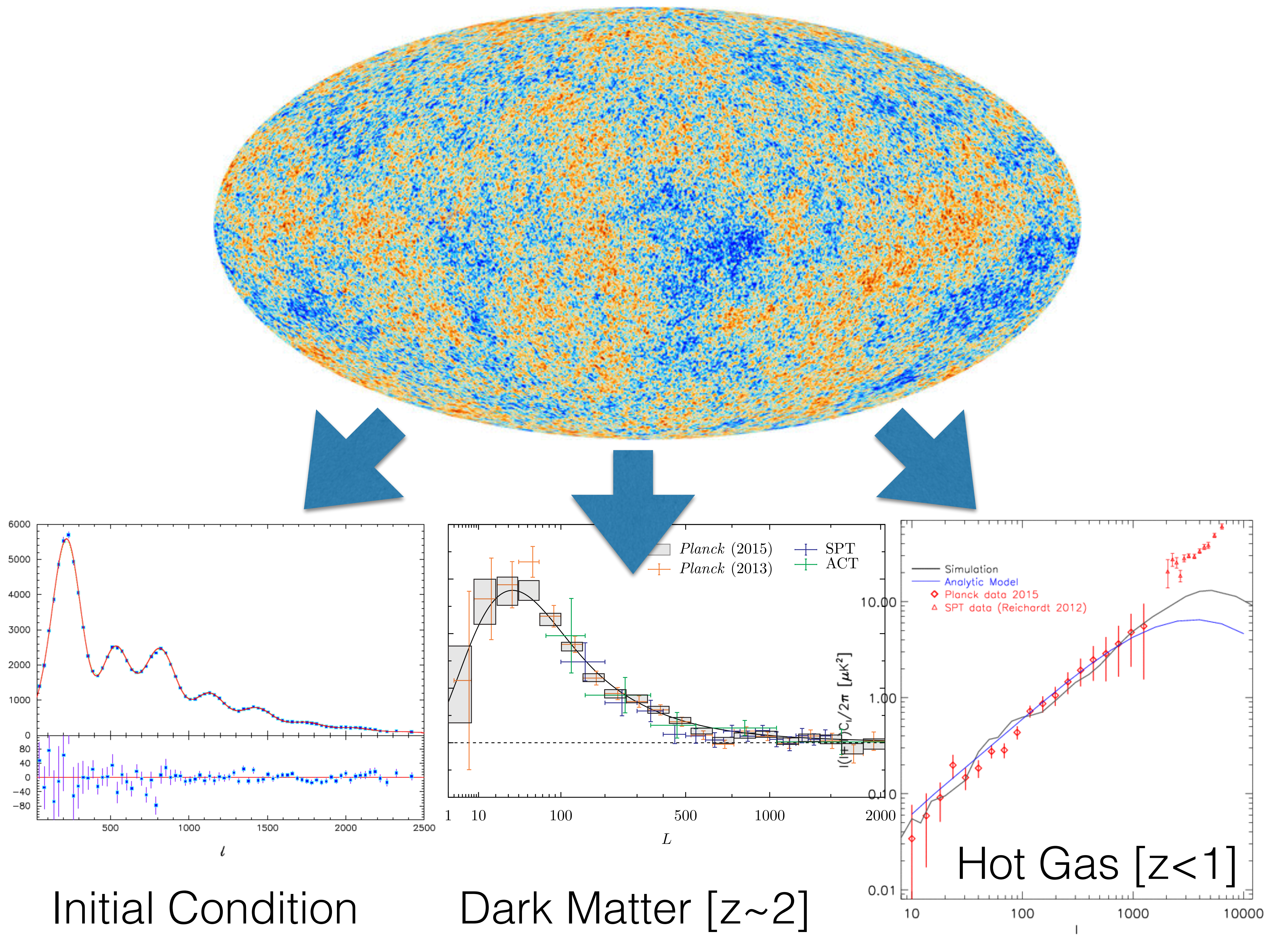


$$C_\ell \propto \Omega_m^3 \sigma_8^8$$

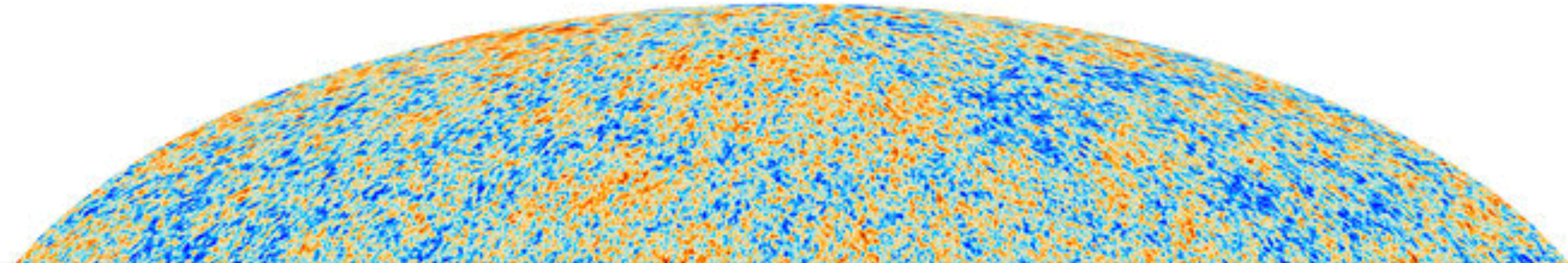
$\Omega_m = 0.315$
$\sigma_8 = 0.829$

vs

$\Omega_m = 0.308$
$\sigma_8 = 0.8149$

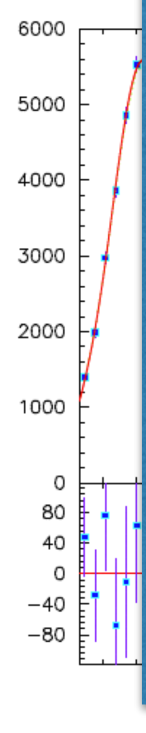


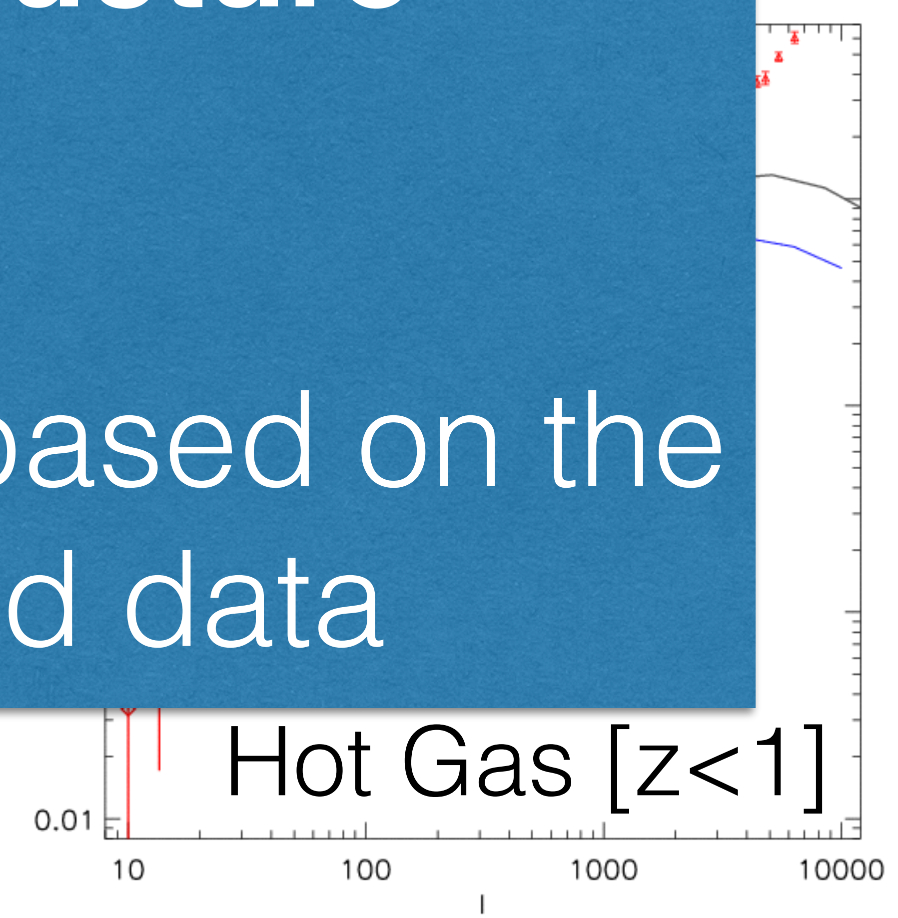
Initial Condition
Dark Matter [z~2]
Hot Gas [z<1]
6000
5000
4000
3000
2000
1000
0
80
40
0
−40
−80
500
1000
1500
2000
2500
ℓ
Planck (2015)
Planck (2013)
SPT
ACT
1
10
100
500
1000
2000
L
Simulation
Analytic Model
Planck data 2015
SPT data (Reichardt 2012)
10.00
1.00
0.10
0.01
l(l+1)C_l/2π [μK²]
10
100
1000
10000
l



Standard ΛCDM Model, starting with **inflation** producing adiabatic, Gaussian, isotropic, n_s<1 primordial fluctuations fit all the data from the **initial condition to structure formation!**

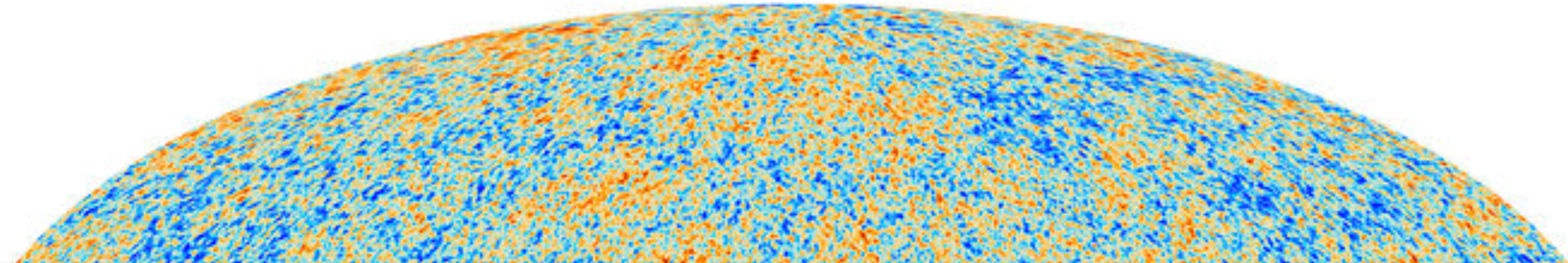
These results are all solely based on the microwave background data





Initial Condition

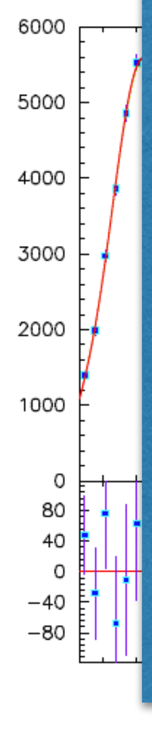
Dark Matter [z~2]



Standard ΛCDM Model, starting with **inflation** producing adiabatic,

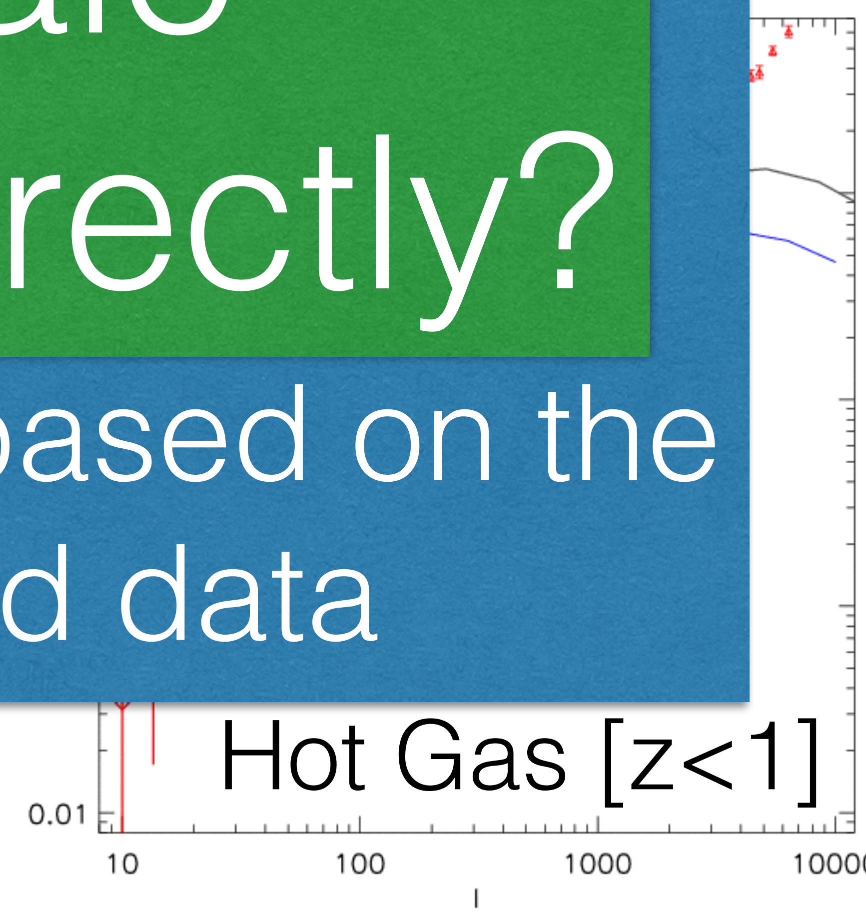
But, why not looking at the large-scale structure data directly?

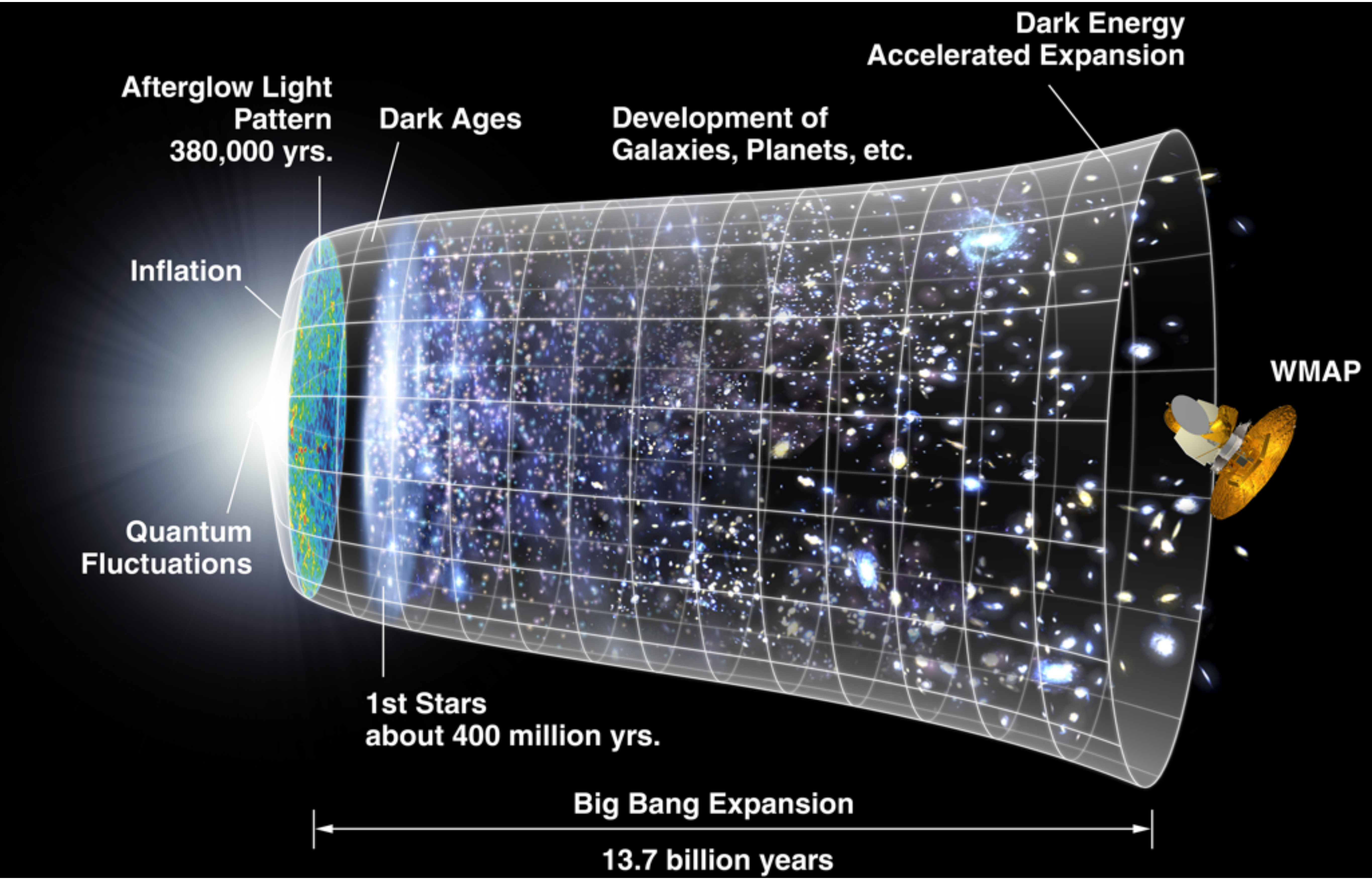
These results are all solely based on the microwave background data



Initial Condition

Dark Matter [z~2]




Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
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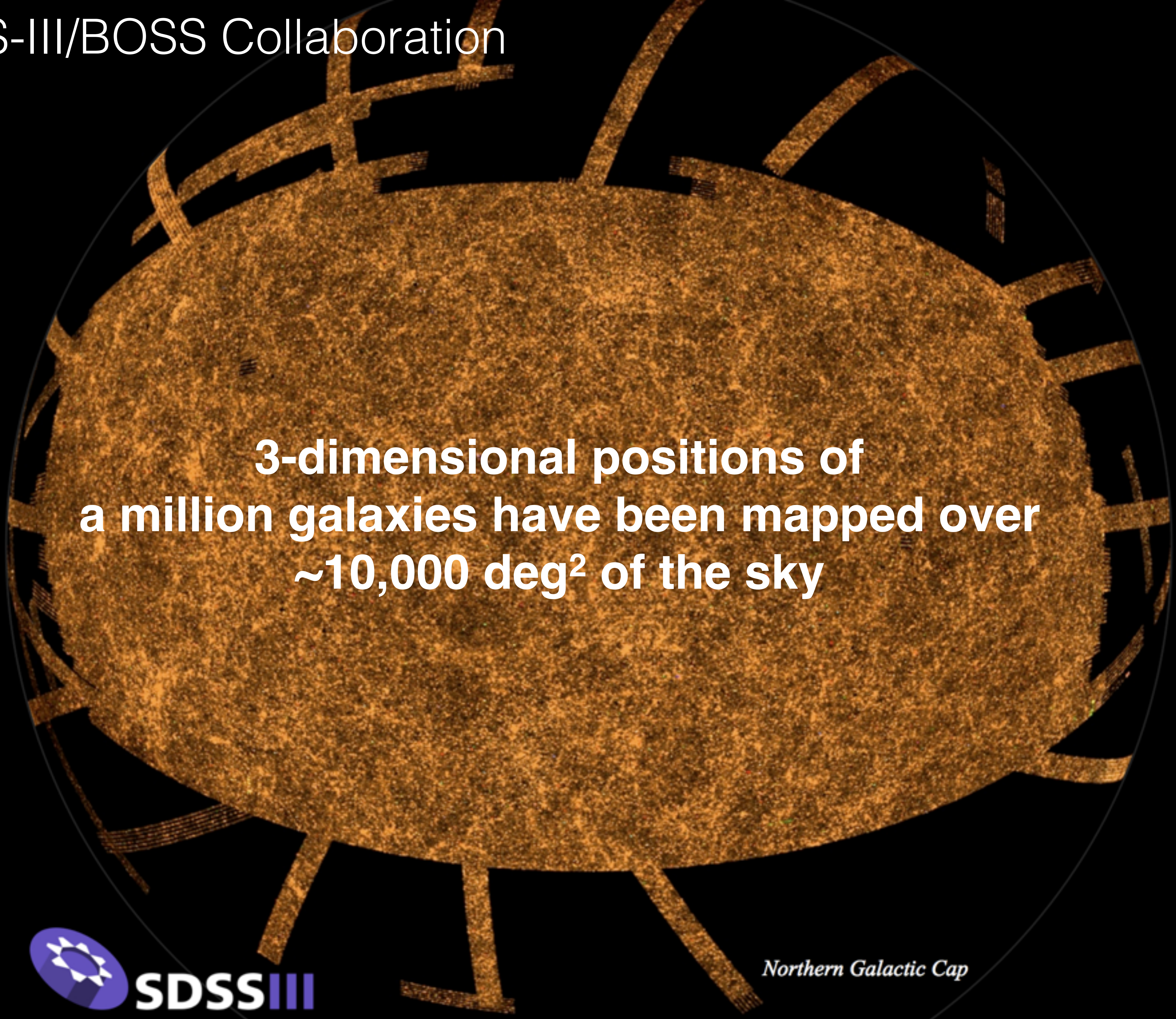
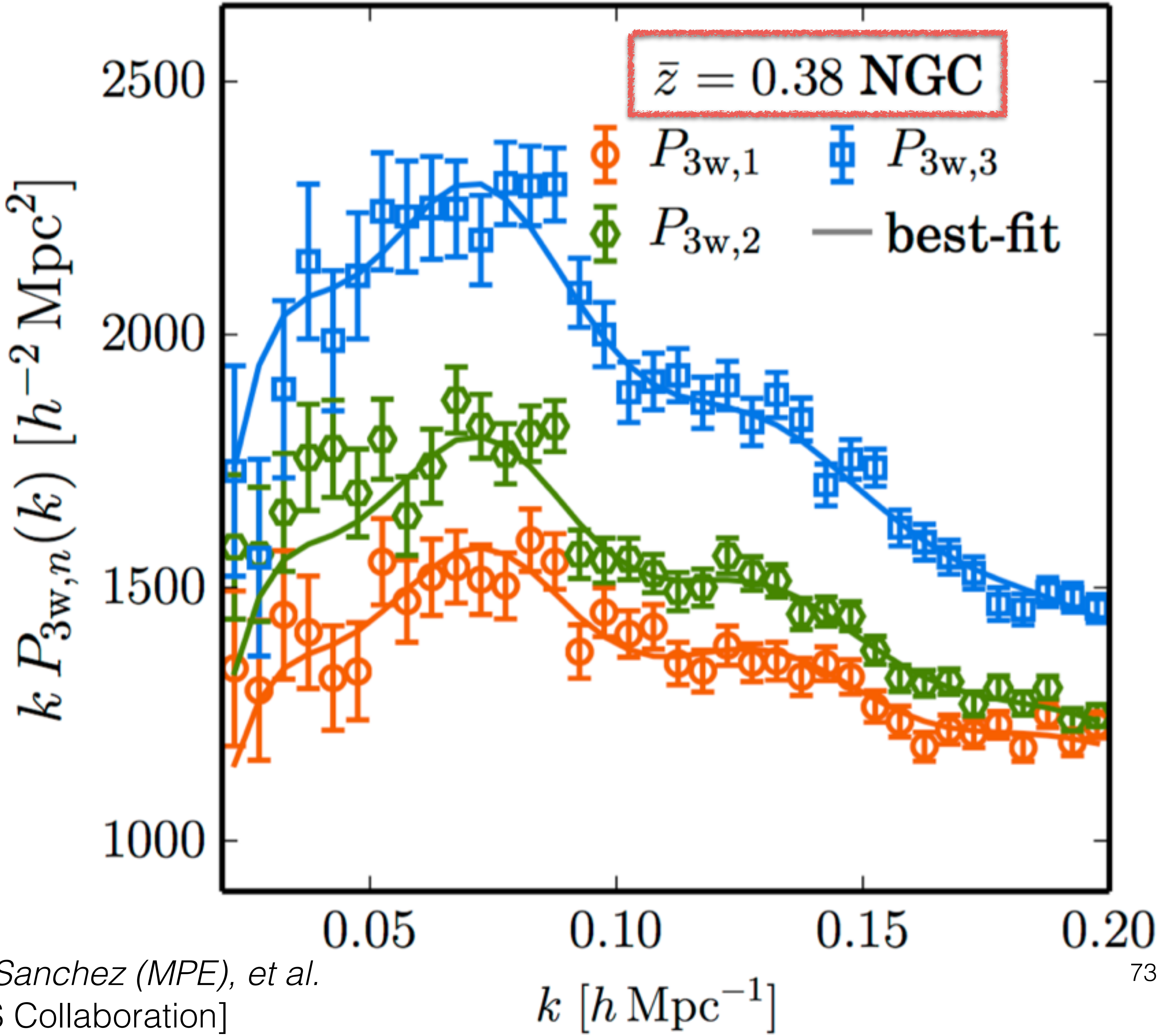
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

SDSS-III/BOSS Collaboration

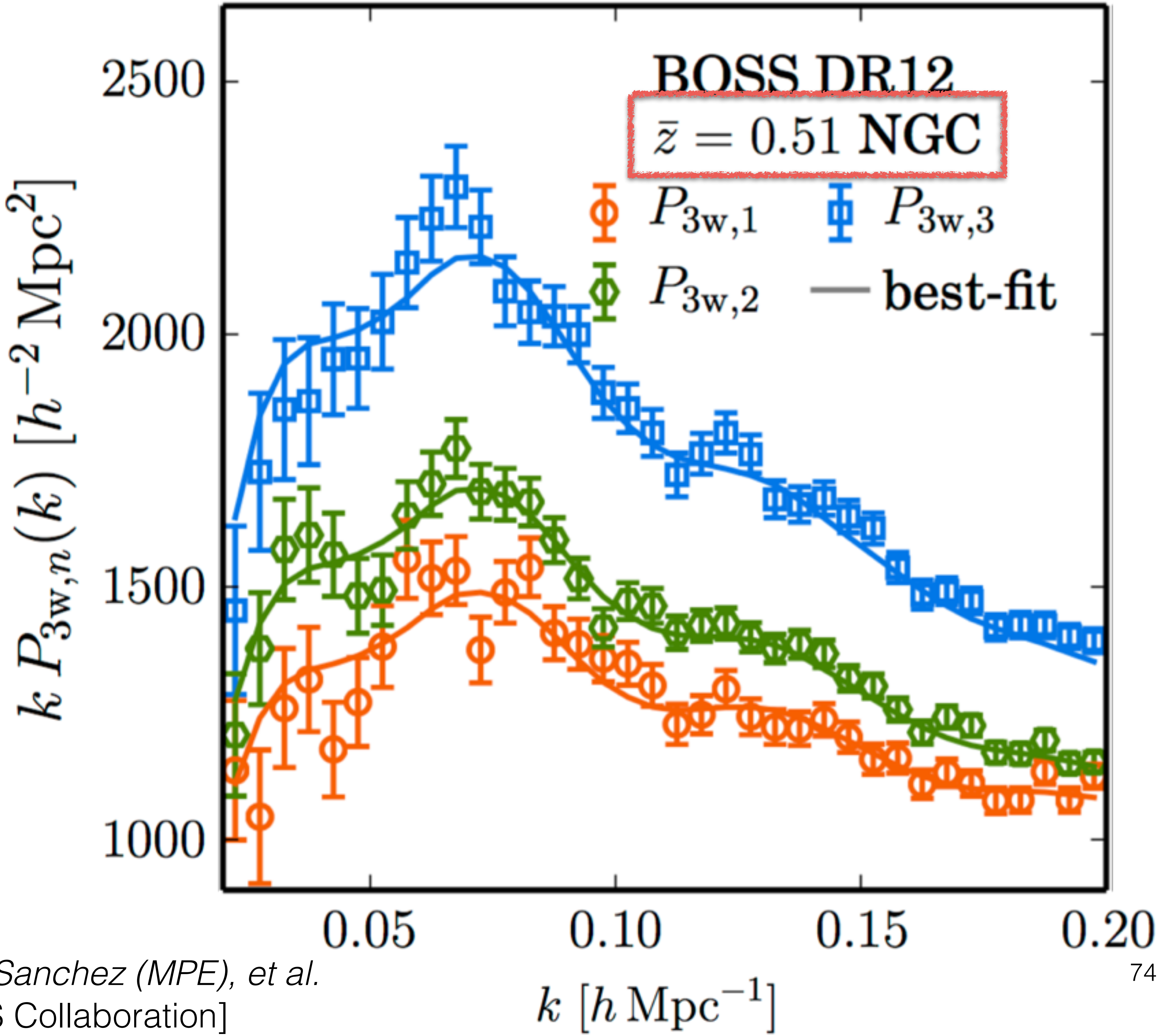


3-dimensional positions of a million galaxies have been mapped over ~10,000 deg^2 of the sky

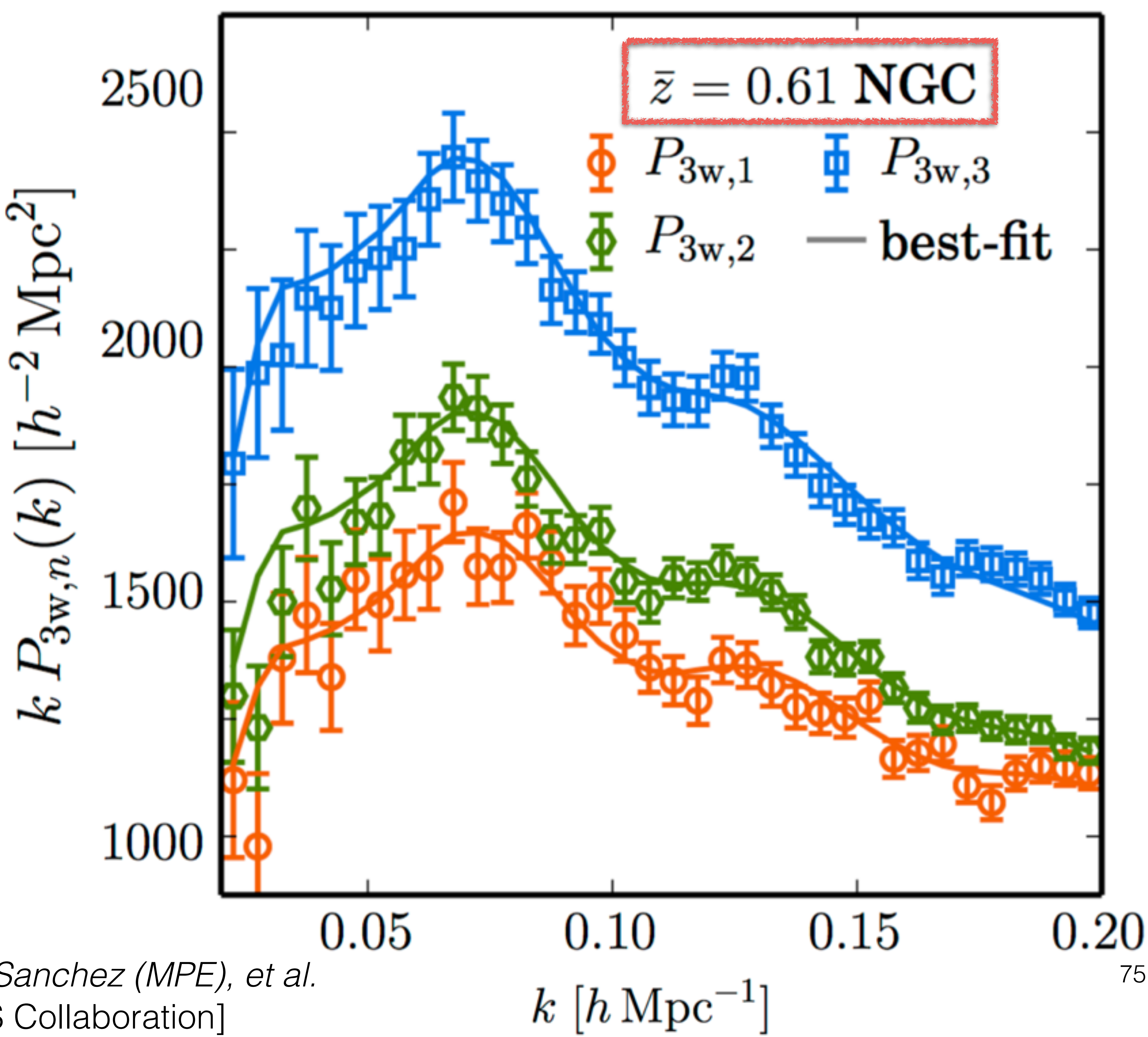
SDSSIII



Grieb, Sanchez (MPE), et al.
[BOSS Collaboration]



Grieb, Sanchez (MPE), et al.
[BOSS Collaboration]



Grieb, Sanchez (MPE), et al.
[BOSS Collaboration]

What have we learned?

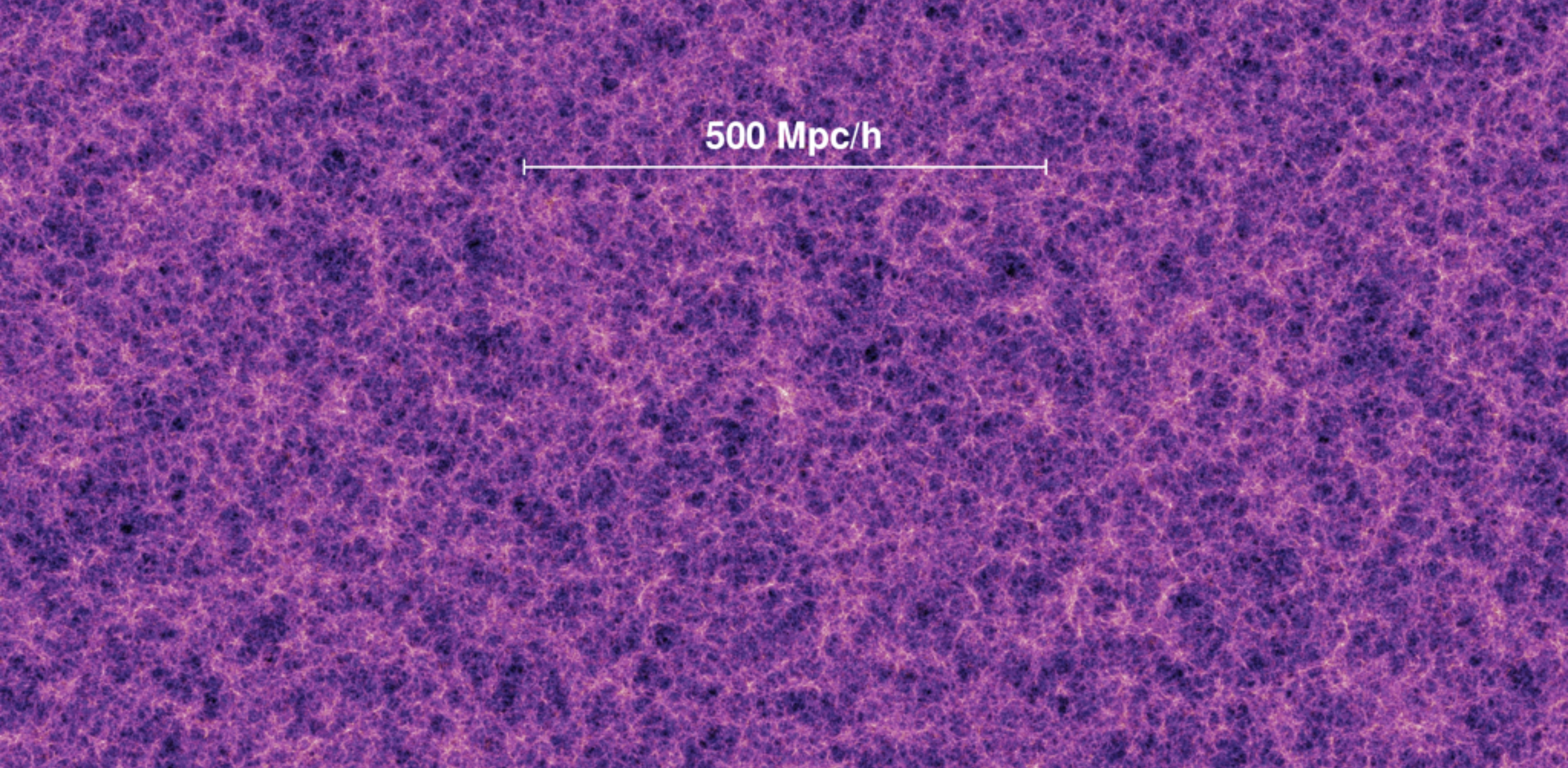
- **Theory continues to fit!** with the cosmological parameters consistent with CMB, lensing, and SZ effect

- What else can we learn?

A Simple Question

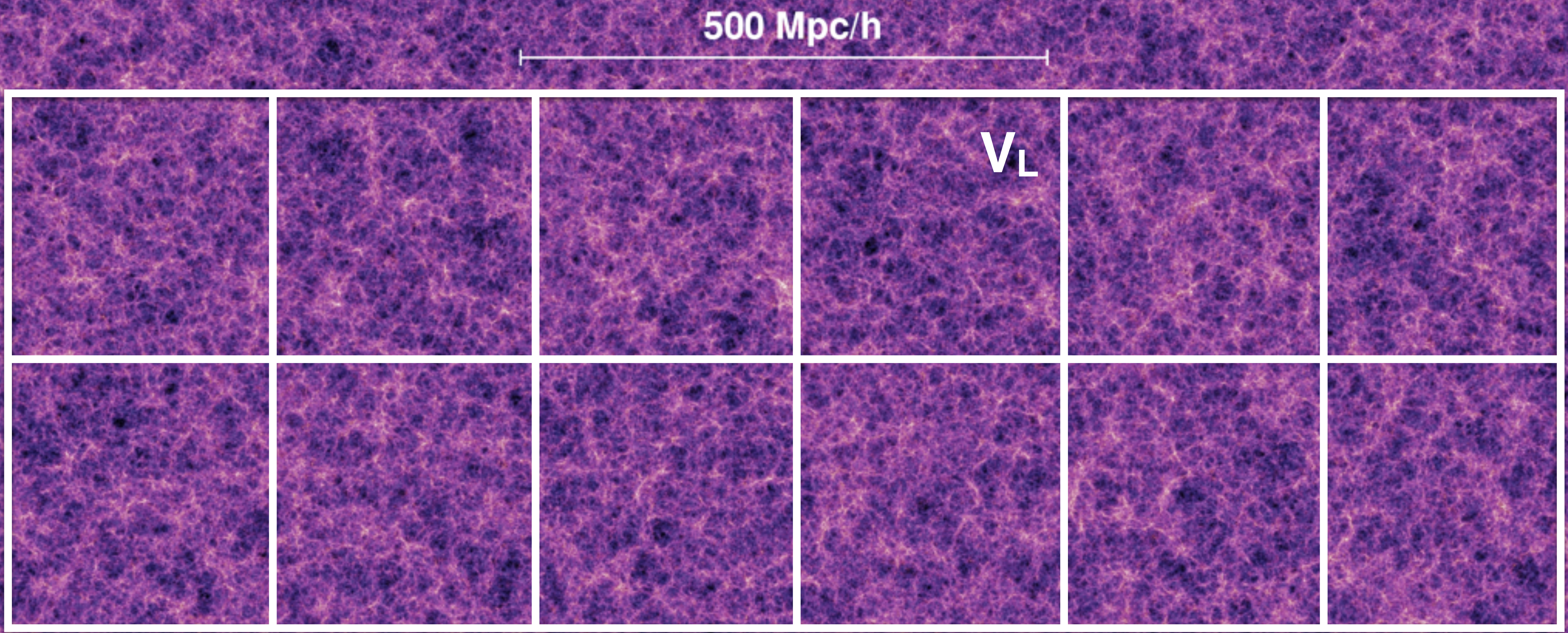
- How do the cosmic structures evolve in an over-dense region?

Simple Statistics



- Divide the survey volume into many sub-volumes V_L, and compare locally-measured power spectra with the corresponding local over-densities

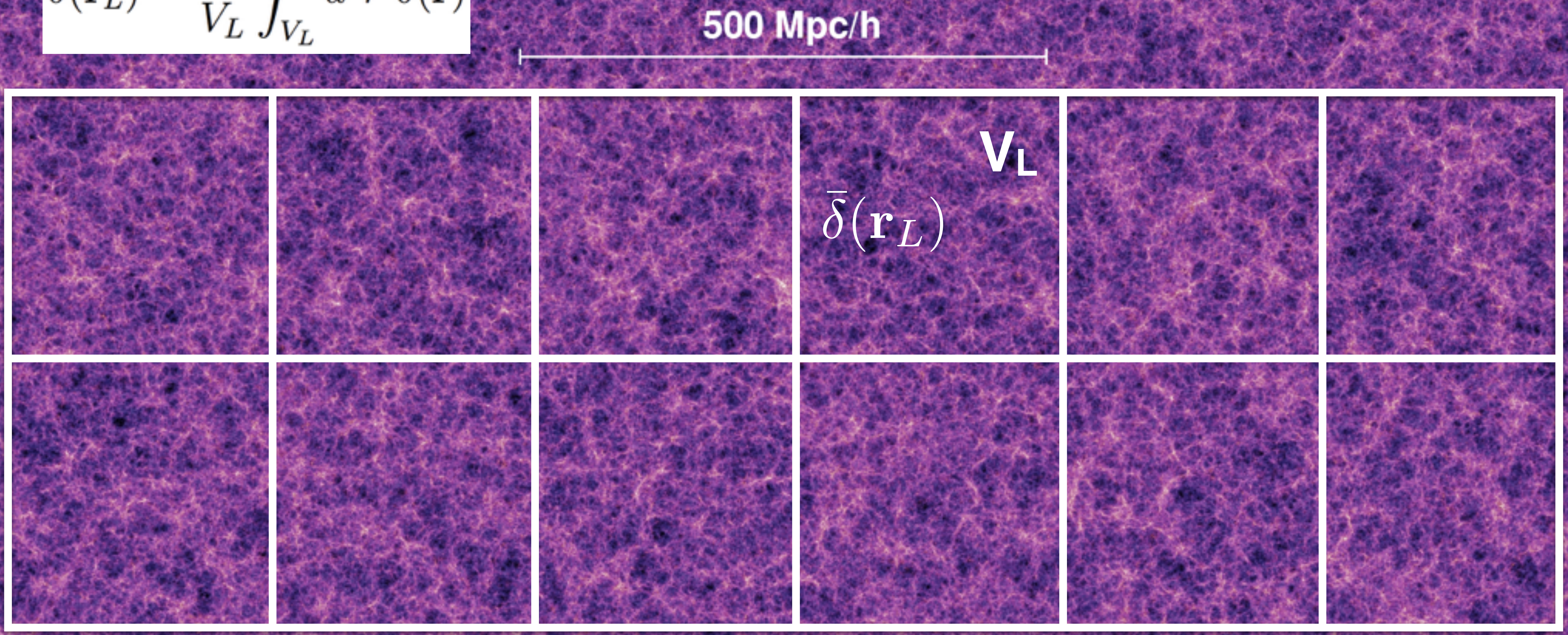
Simple Statistics



- Divide the survey volume into many sub-volumes V_L, and compare locally-measured power spectra with the corresponding local over-densities

Simple Statistics

$$\bar{\delta}(\mathbf{r}_L) = \frac{1}{V_L}\int_{V_L} d^3r\ \delta(\mathbf{r})$$



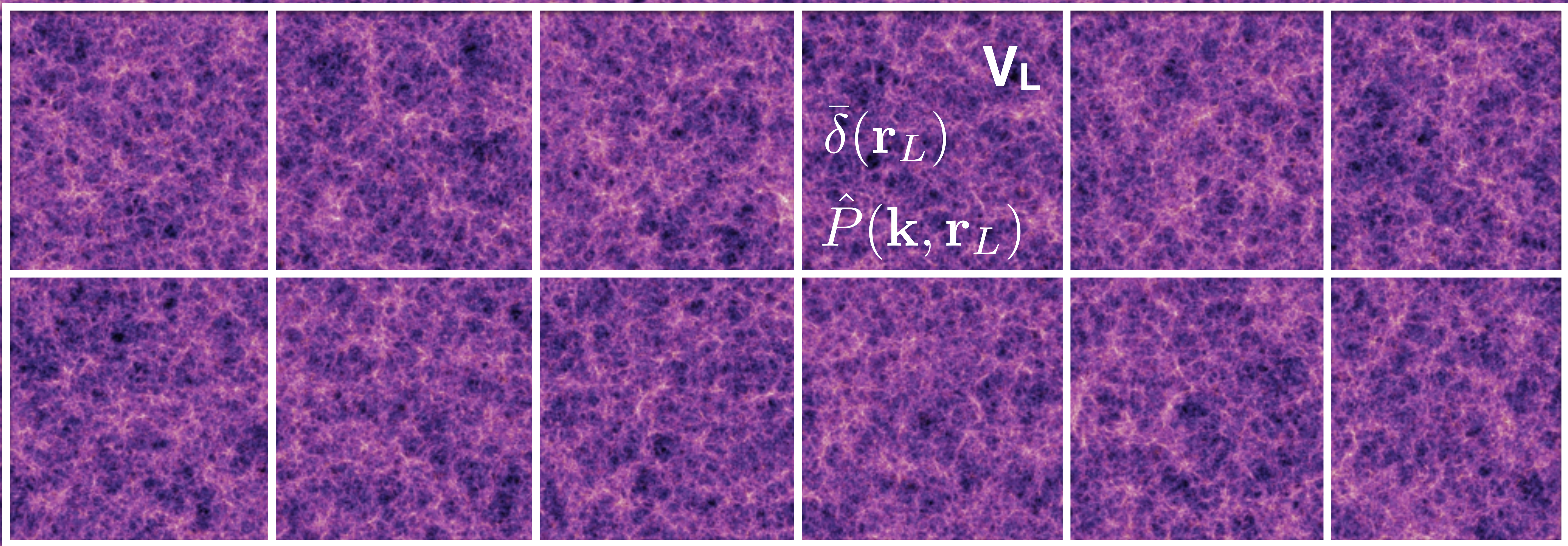
- Divide the survey volume into many sub-volumes V_L, and compare locally-measured power spectra with the corresponding local over-densities

Simple Statistics

$$\bar{\delta}(\mathbf{r}_L) = \frac{1}{V_L}\int_{V_L} d^3r\ \delta(\mathbf{r})$$

$$\hat{P}(\mathbf{k}, \mathbf{r}_L) \equiv \frac{1}{V_L}|\delta(\mathbf{k}, \mathbf{r}_L)|^2$$

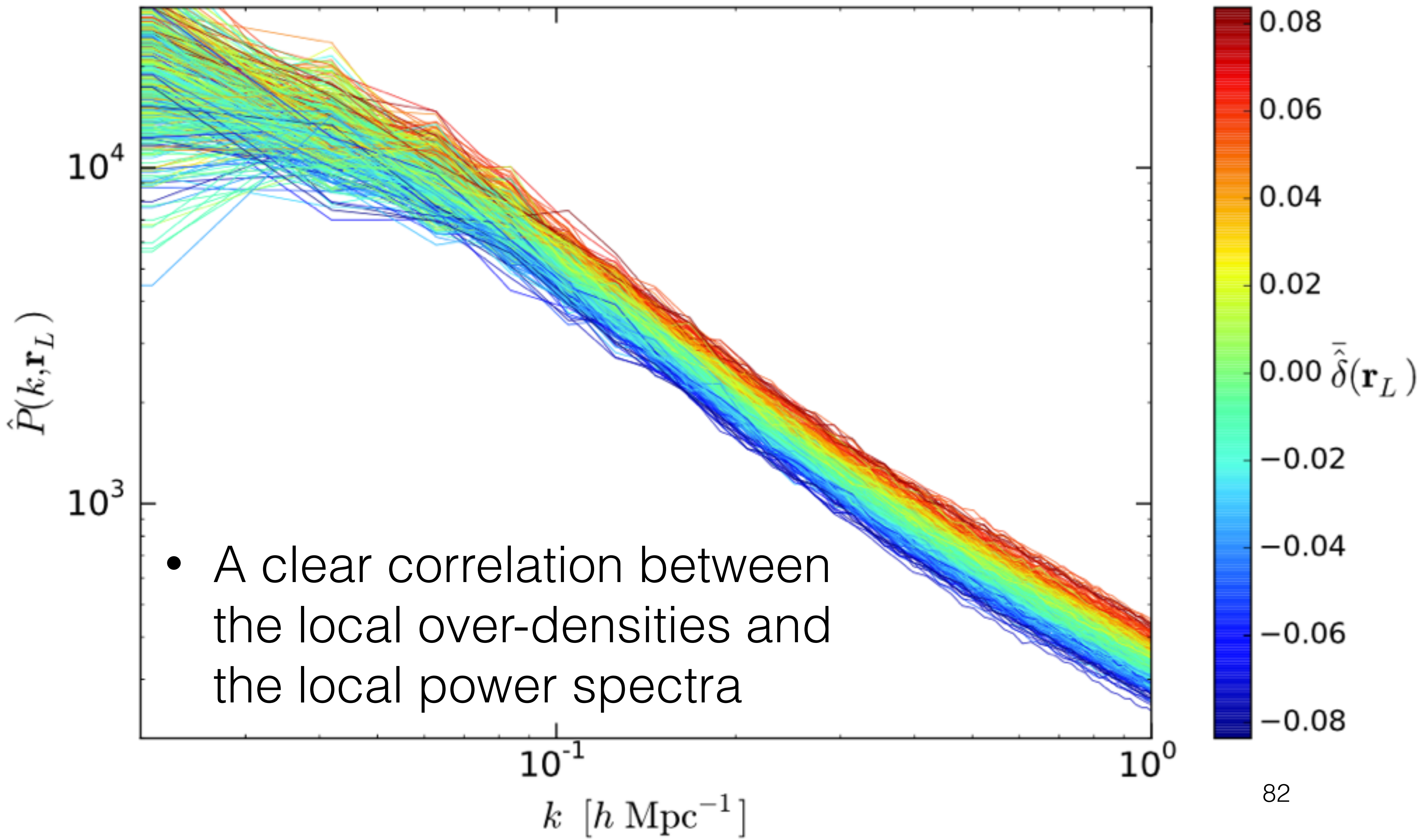
500 Mpc/h



- Divide the survey volume into many sub-volumes V_L, and compare locally-measured power spectra with the corresponding local over-densities

Chiang, Wagner, Schmidt, EK (2014)

Position-dependent $\hat{P}(k)$



- A clear correlation between the local over-densities and the local power spectra

Chiang, Wagner, Schmidt, EK (2014)

Integrated Bispectrum, *iB*(*k*)

- Correlating the local over-densities and power spectra, we obtain the "integrated bispectrum":

$$i\hat{B}_L(k) = \frac{1}{N_{\rm cut}^3} \sum_{i=1}^{N_{\rm cut}^3} \hat{P}(k, \mathbf{r}_{L,i}) \hat{\bar{\delta}}(\mathbf{r}_{L,i})$$

- This is a (particular configuration of) **three-point function**! The three-point function in Fourier space is the bispectrum, and is defined as

$$\langle \delta(\mathbf{q}_1)\delta(\mathbf{q}_2)\delta(\mathbf{q}_3)\rangle = B(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)(2\pi)^3\delta_D(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3)$$

Chiang, Wagner, Schmidt, EK (2014)

Integrated Bispectrum, *iB*(*k*)

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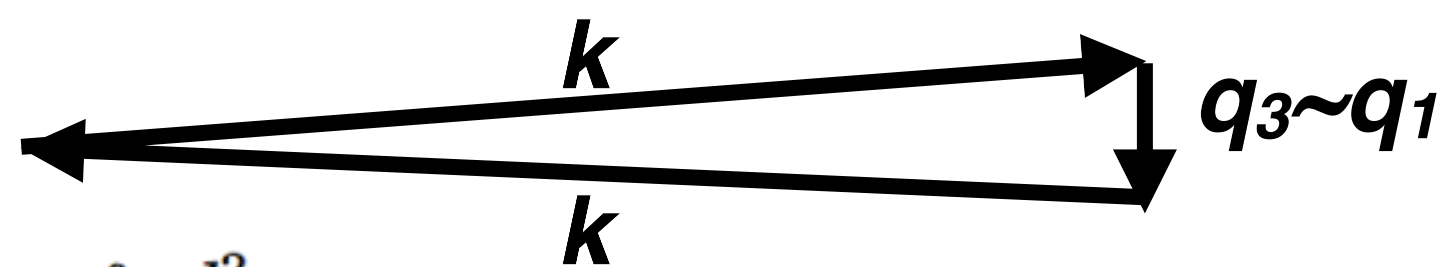
$$i\hat{B}_L(k) = \frac{1}{N_{\rm cut}^3}\sum_{i=1}^{N_{\rm cut}^3} \hat{P}(k,\mathbf{r}_{L,i})\hat{\bar{\delta}}(\mathbf{r}_{L,i})$$

- The expectation value of this quantity is an integral of the bispectrum that picks up the contributions **mostly from the squeezed limit**:



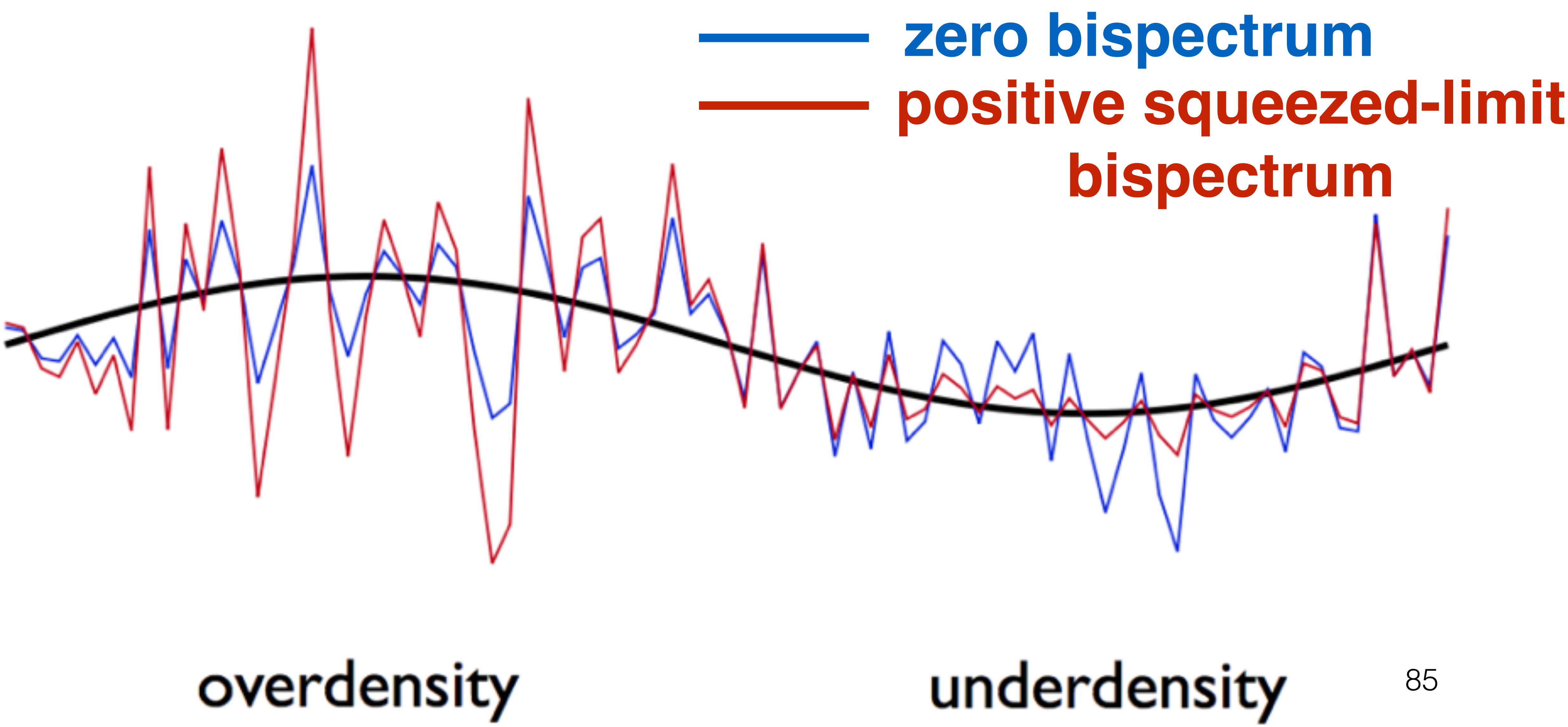
$$\begin{aligned} iB_L(k) &= \langle \hat{P}(k,\mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle \\ &= \frac{1}{V_L^2}\int \frac{d^2\hat{k}}{4\pi}\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_3}{(2\pi)^3}\, B(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_3, -\mathbf{q}_3) \\ &\qquad \times W_L(\mathbf{q}_1)W_L(-\mathbf{q}_1-\mathbf{q}_3)W_L(\mathbf{q}_3) \end{aligned}$$

“taking the squeezed limit and then angular averaging”

Chiang, Wagner, Schmidt, EK (2014)

Power Spectrum Response

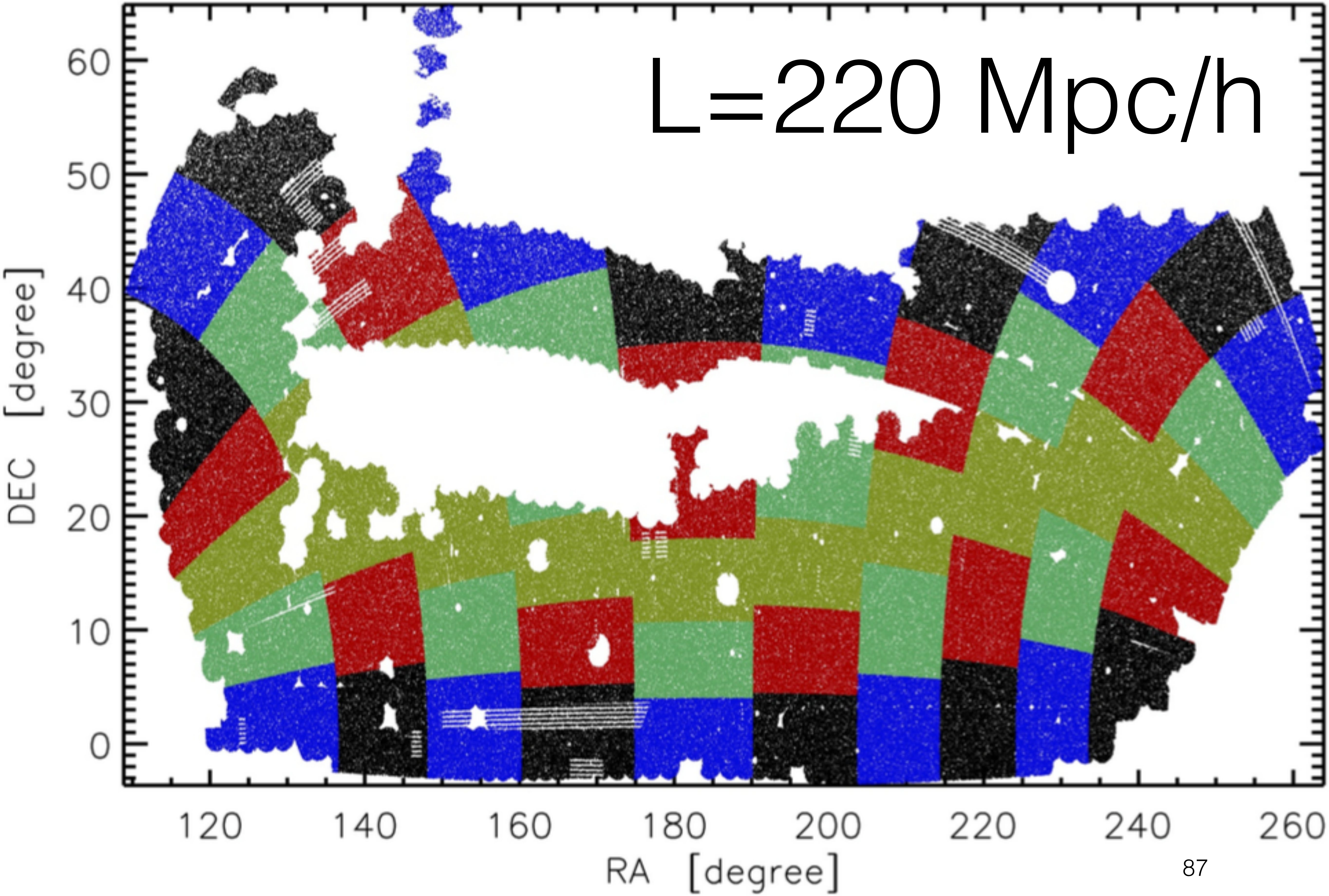
- The integrated bispectrum measures how the local power spectrum responds to its environment, i.e., a long-wavelength density fluctuation



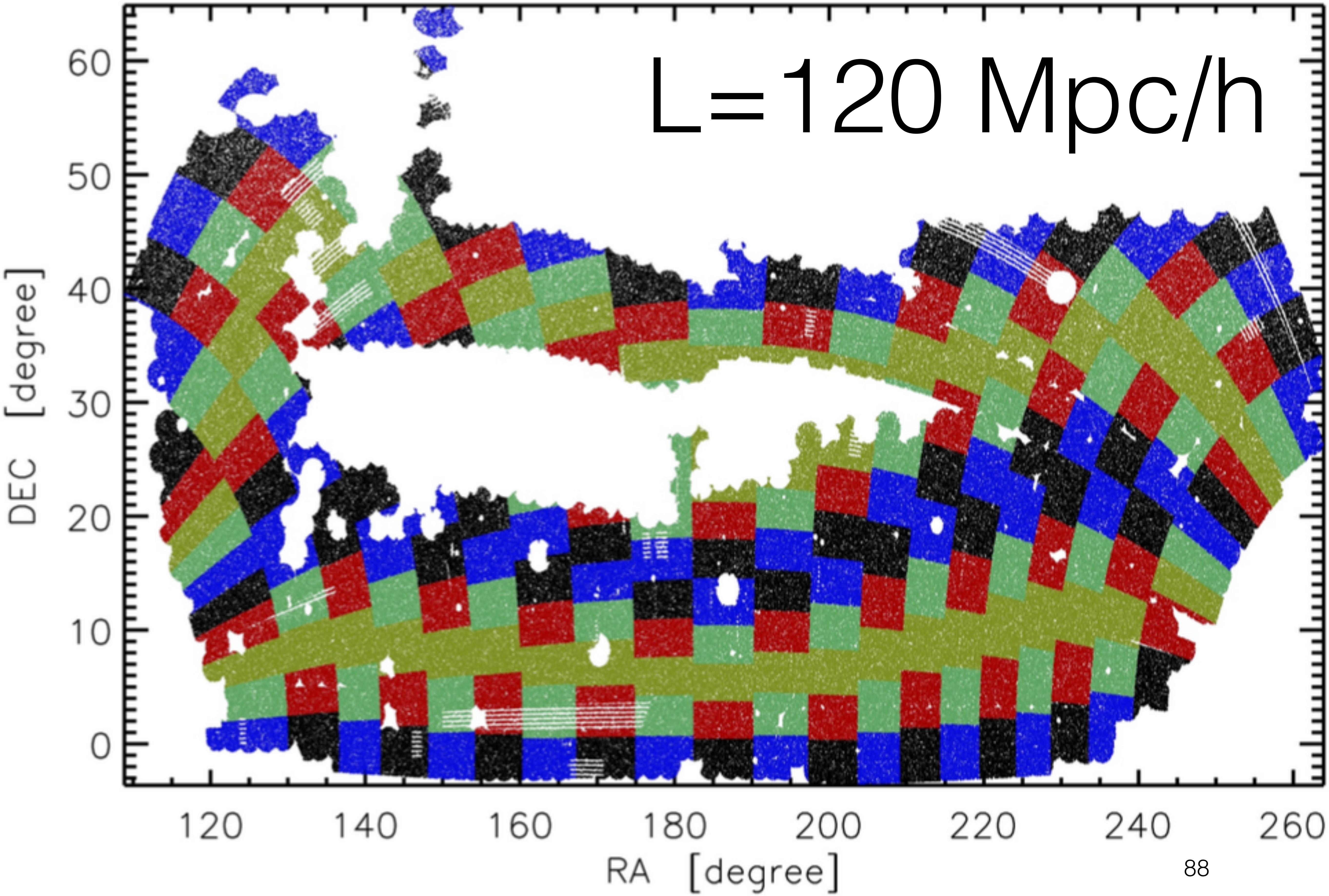
SDSS-III/BOSS DR10

- OK, now, let's look at the real data (BOSS DR10) to see if we can detect the expected influence of environments on the small-scale structure growth

Chiang, Wagner, Sanchez, Schmidt, EK (2015)

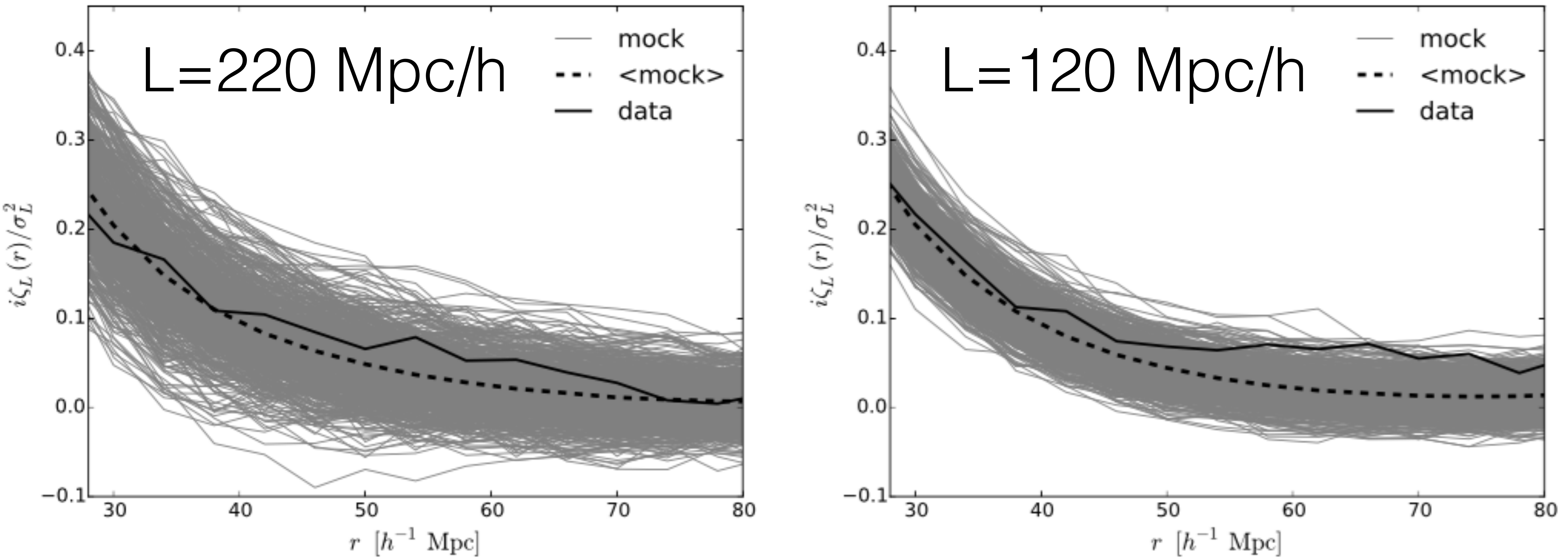


Chiang, Wagner, Sanchez, Schmidt, EK (2015)



Chiang, Wagner, Sanchez, Schmidt, EK (2015)

Results: χ^2/DOF = 46.4/38

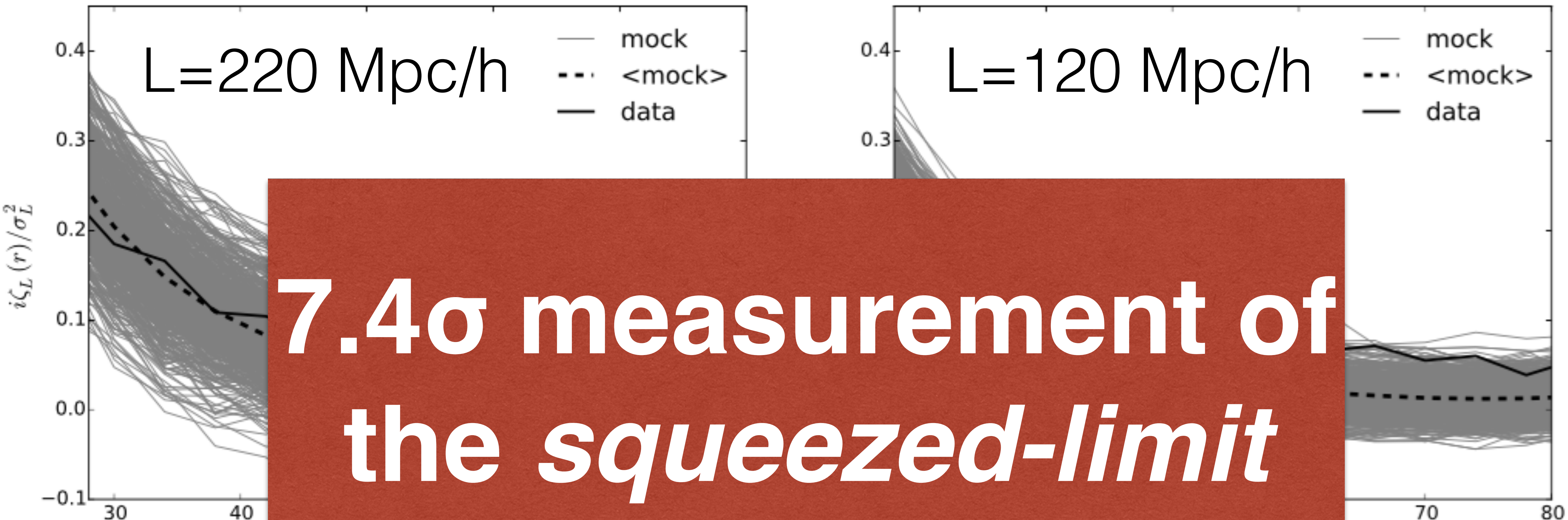


- Because of complex geometry of DR10 footprint, we use the local correlation function, instead of the power spectrum. Power spectrum will be presented using DR12 in the future

- Integrated three-point function, iζ(r), is just Fourier transform of iB(k): $$i\zeta_L(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\, iB_L(\mathbf{k})e^{i\mathbf{r}\cdot\mathbf{k}}$$

Chiang, Wagner, Sanchez, Schmidt, EK (2015)

Results: χ^2/DOF = 46.4/38



7.4σ measurement of the *squeezed-limit bispectrum*!!

- Bec … se the loca … m. Power spectrum will be presented using DR12 in the future

- Integrated three-point function, iζ(r), is just Fourier transform of iB(k): $$i\zeta_L(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\, iB_L(\mathbf{k}) e^{i\mathbf{r}\cdot\mathbf{k}}$$

Nice, but what is this good for?

- Primordial non-Gaussianity (“local-type f_{NL}”)
  - The constraint from BOSS is work in progress, but the Fisher matrix analysis suggests that the integrated bispectrum is a **nearly optimal estimator for the local-type f_{NL}**
  - We no longer need to measure the full bispectrum, if we are just interested in f_{NL}^{local}!

Nice, but what is this good for?

- Primordial non-Gaussianity ("local-type f_{NL}")
  - Th ... ,
    bu ... he
    in
    es

Back to the initial condition!

- We no longer need to measure the full bispectrum, if we are just interested in f_{NL}^{local}!

Nice, but what is this good for?

- We can also learn about galaxy bias
  - Local bias model:
    - $\delta_g(x)=b_1\delta_m(x)+(\mathbf{b_2}/2)[\delta_m(x)]^2+\ldots$
- **The bispectrum can give us b_2 at the leading (tree-level) order**, unlike for the power spectrum that has b_2 at the next-to-leading order

Result on b_2

- We use the simplest, tree-level SPT bispectrum in redshift space with the local bias model to interpret our measurements
    - [We also use information from BOSS's 2-point correlation function on $f\sigma_8$ and BOSS's weak lensing data on σ_8]
- We find: $\mathbf{b_2 = 0.41 \pm 0.41}$

CMB Research:
Next Frontier

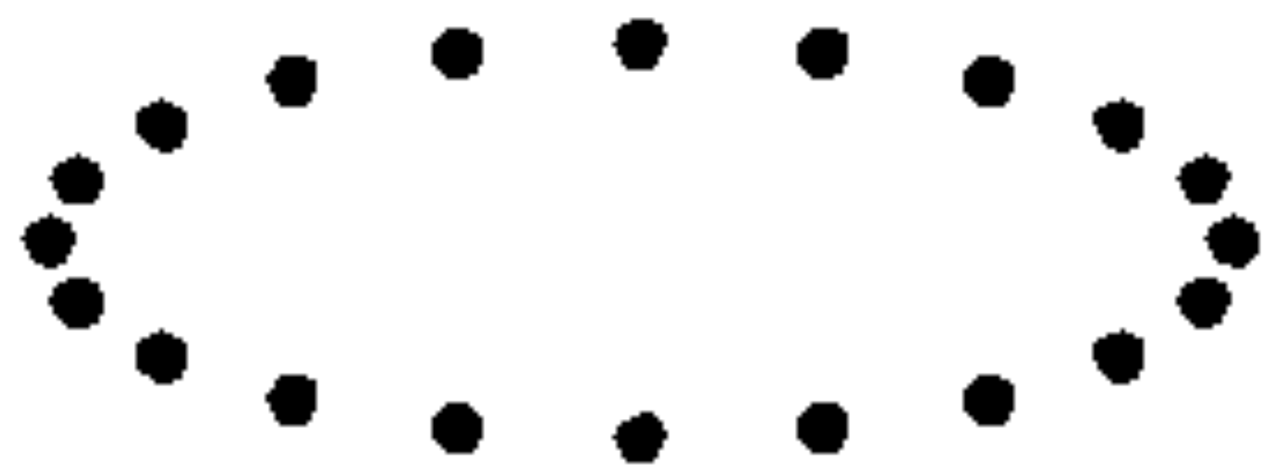
Primordial Gravitational Waves

Extraordinary claims require extraordinary evidence.
The same quantum fluctuations could also generate gravitational waves, and we wish to find them

Measuring GW

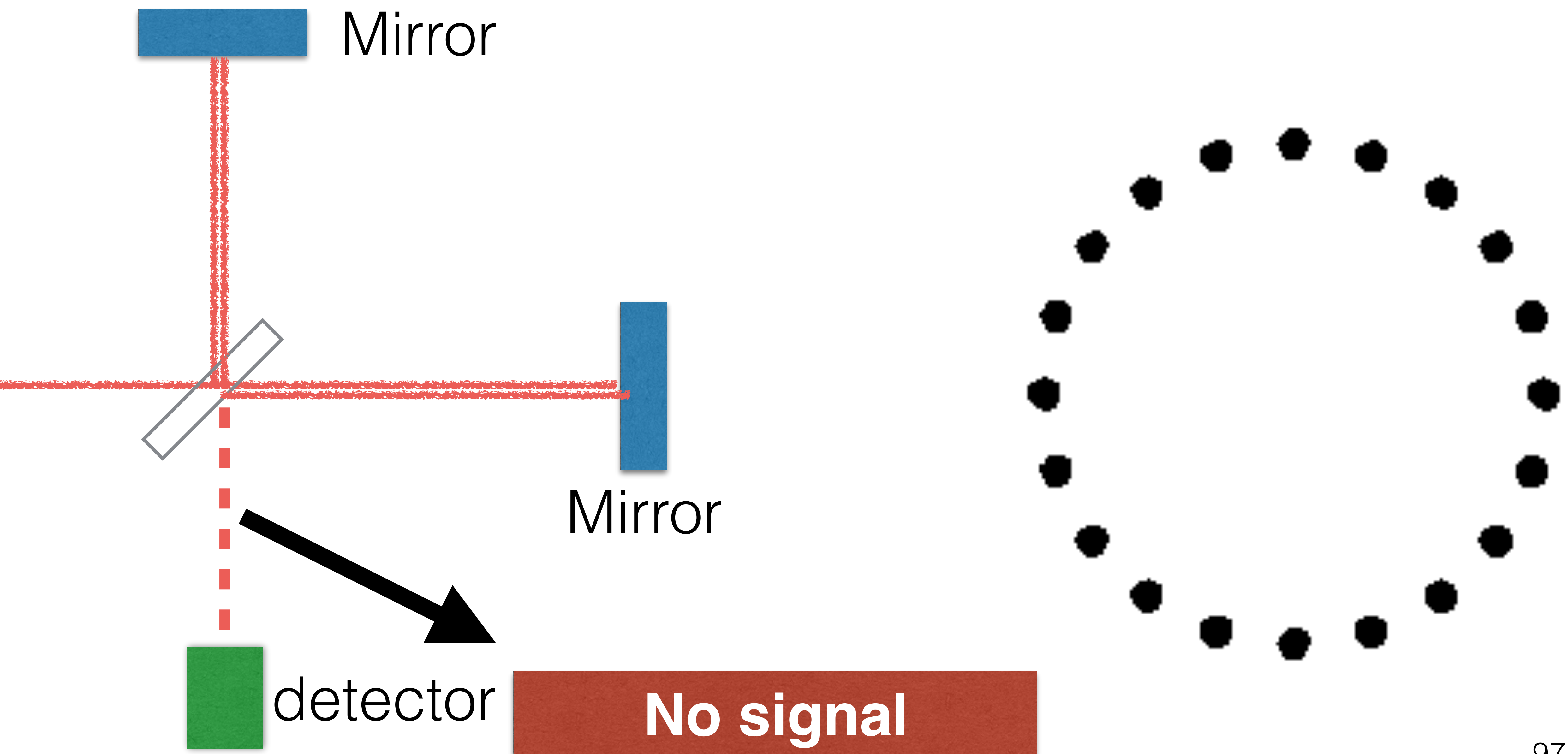
- GW changes the distances between two points

$$d\ell^2 = d\mathbf{x}^2 = \sum_{ij} \delta_{ij} dx^i dx^j$$

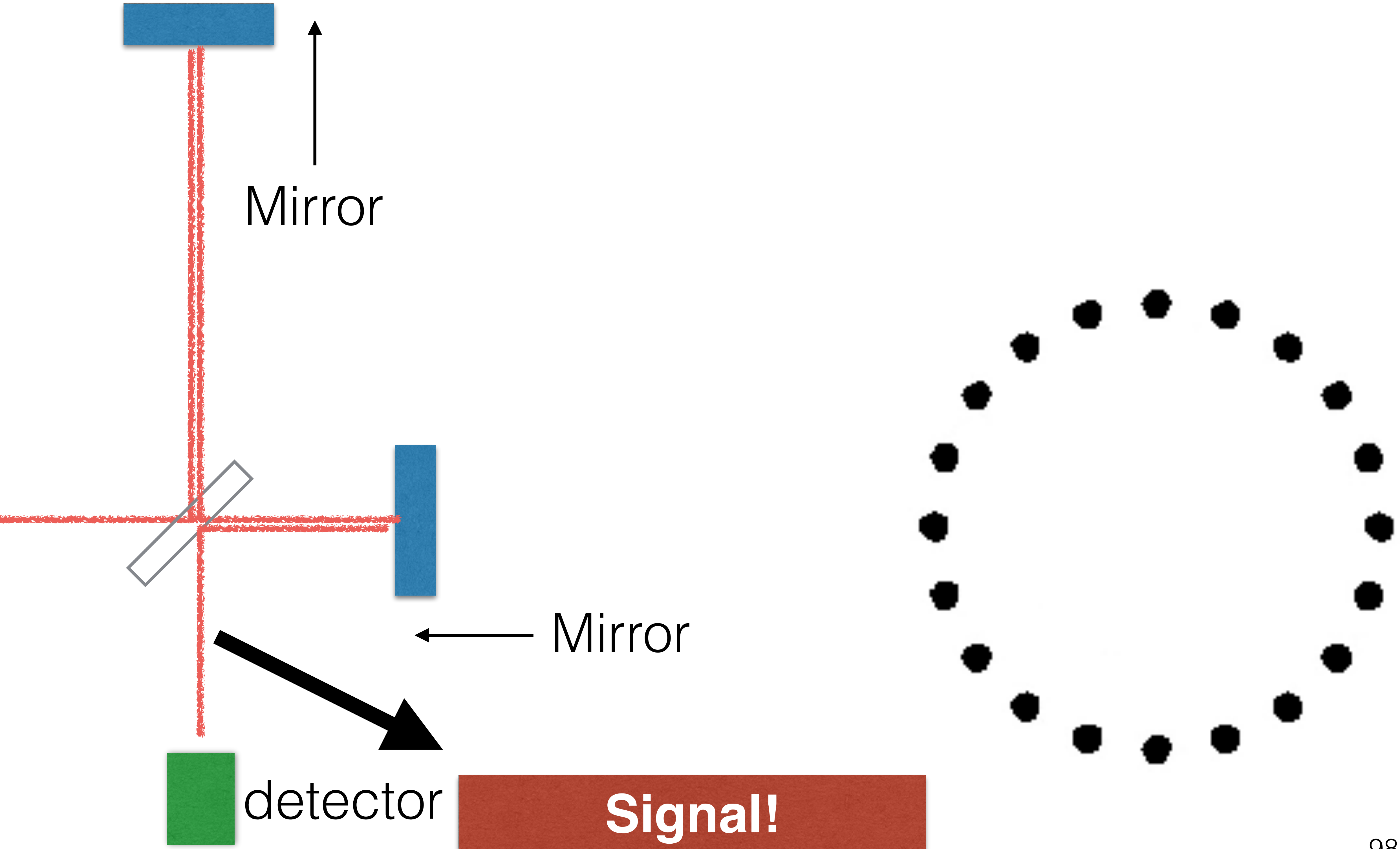


$$d\ell^2 = \sum_{ij} (\delta_{ij} + \underline{h_{ij}}) dx^i dx^j$$

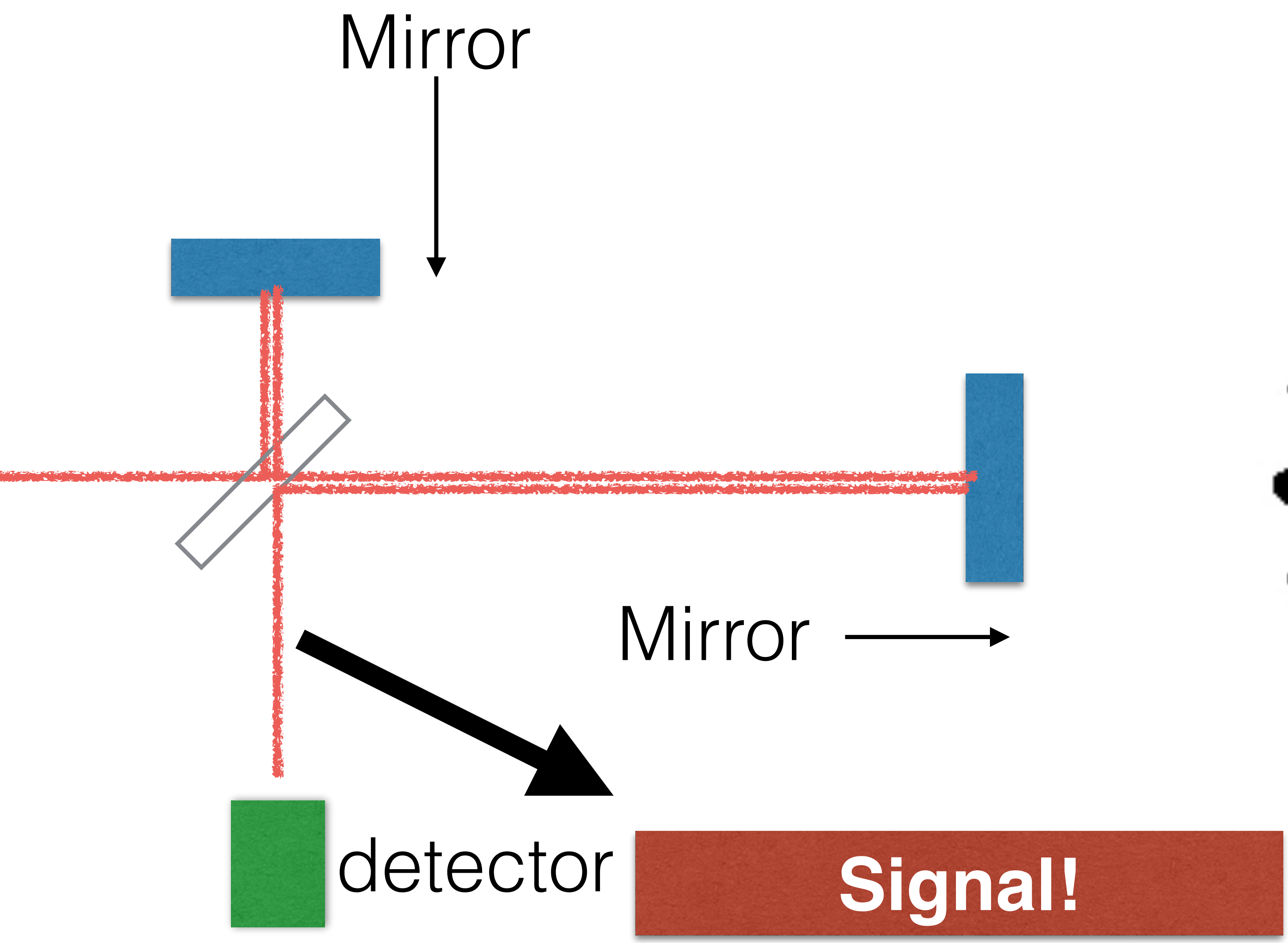
Laser Interferometer

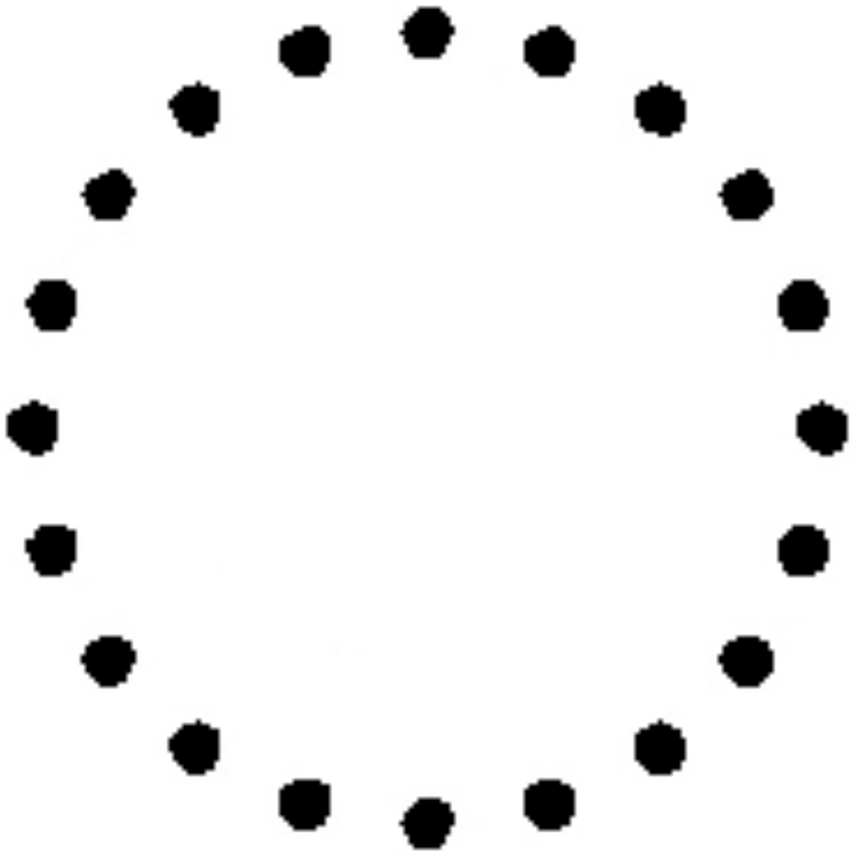


Laser Interferometer



Laser Interferometer





LIGO detected GW from a binary blackholes, with the wavelength of thousands of kilometres

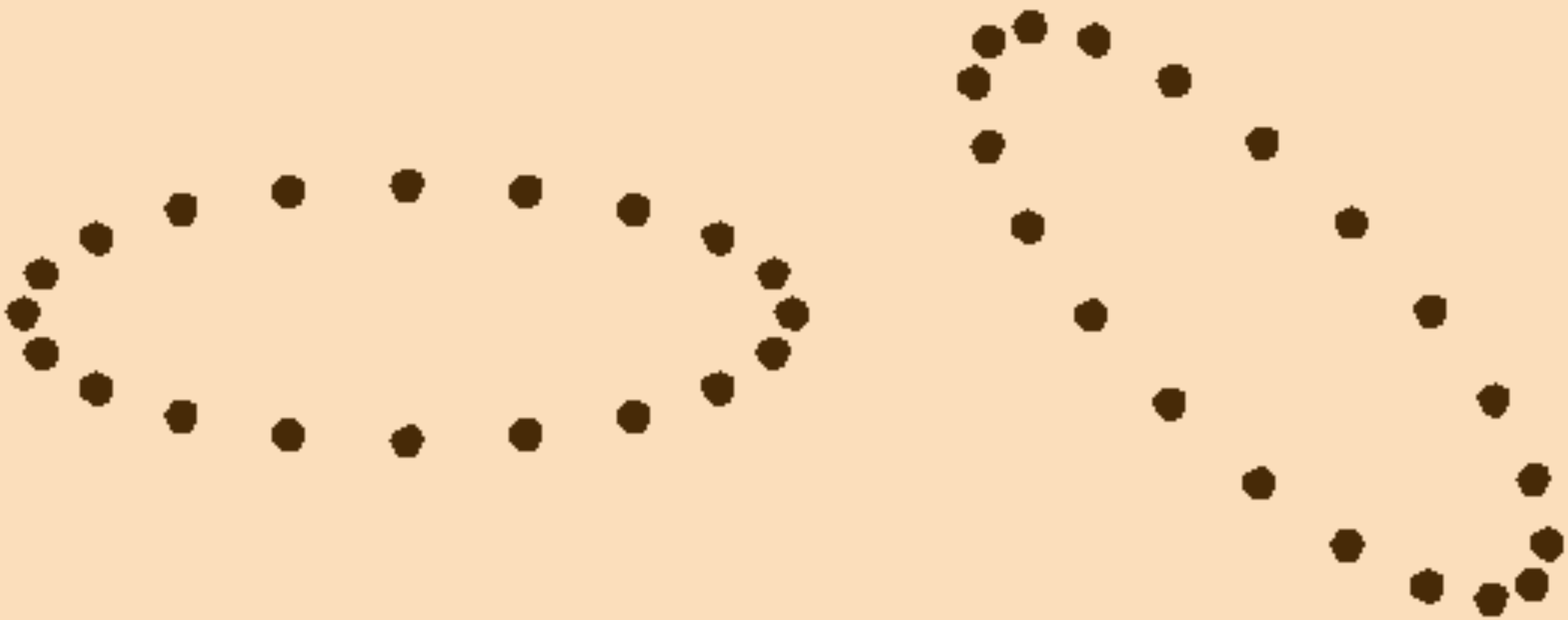
But, the primordial GW affecting the CMB has a wavelength of **billions of light-years**!! How do we find it?

Detecting GW by CMB

Isotropic electro-magnetic fields

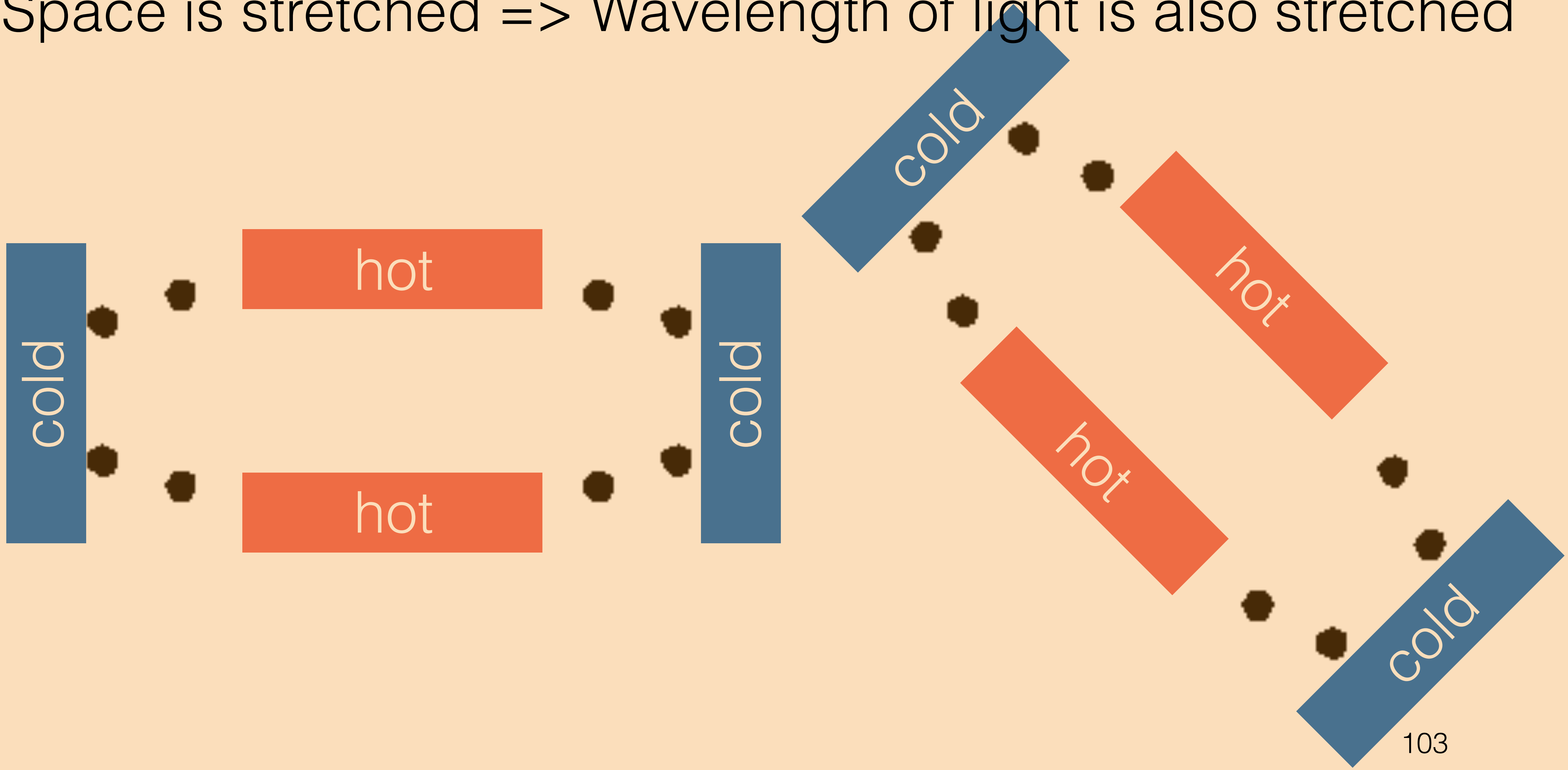
Detecting GW by CMB

GW propagating in isotropic electro-magnetic fields



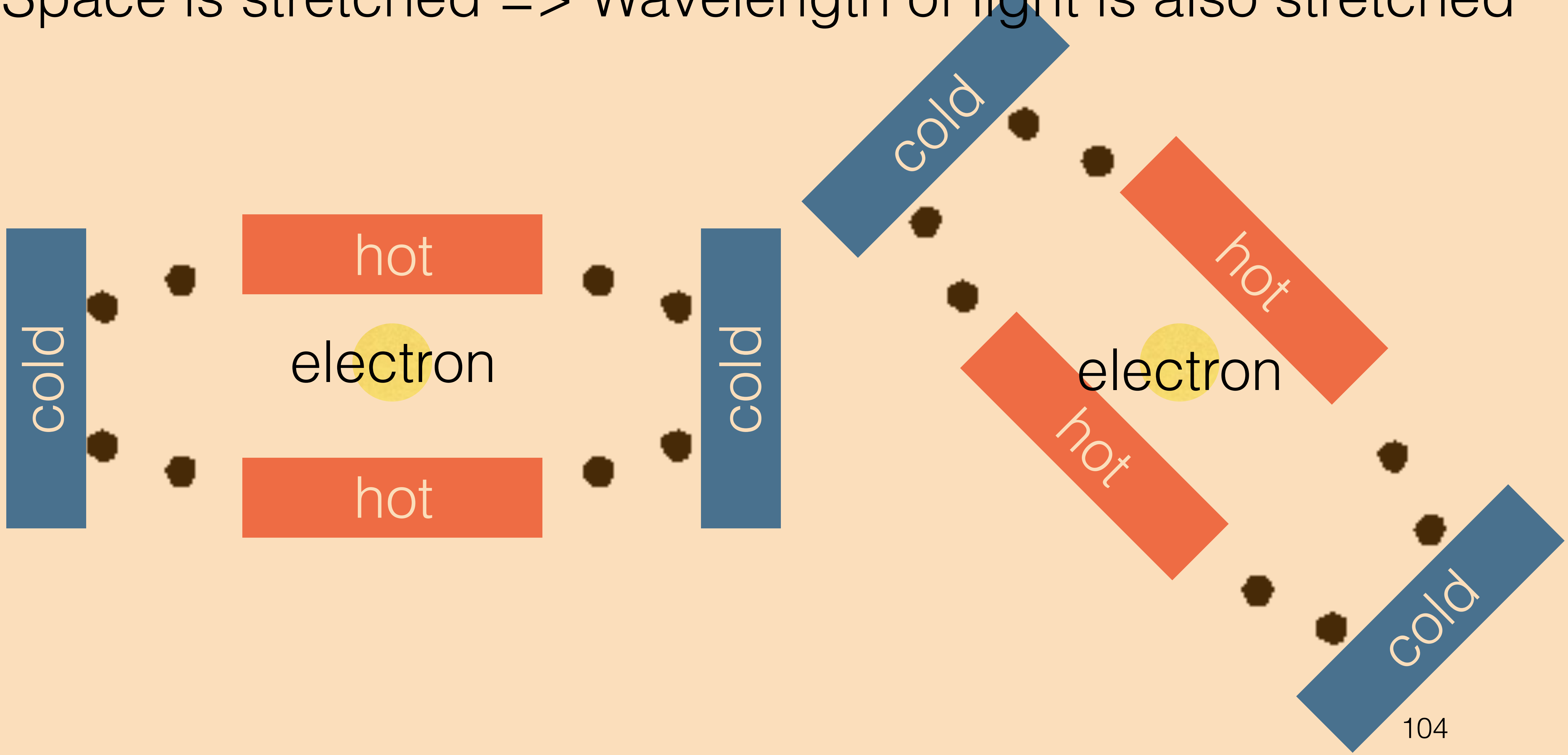
Detecting GW by CMB

Space is stretched => Wavelength of light is also stretched



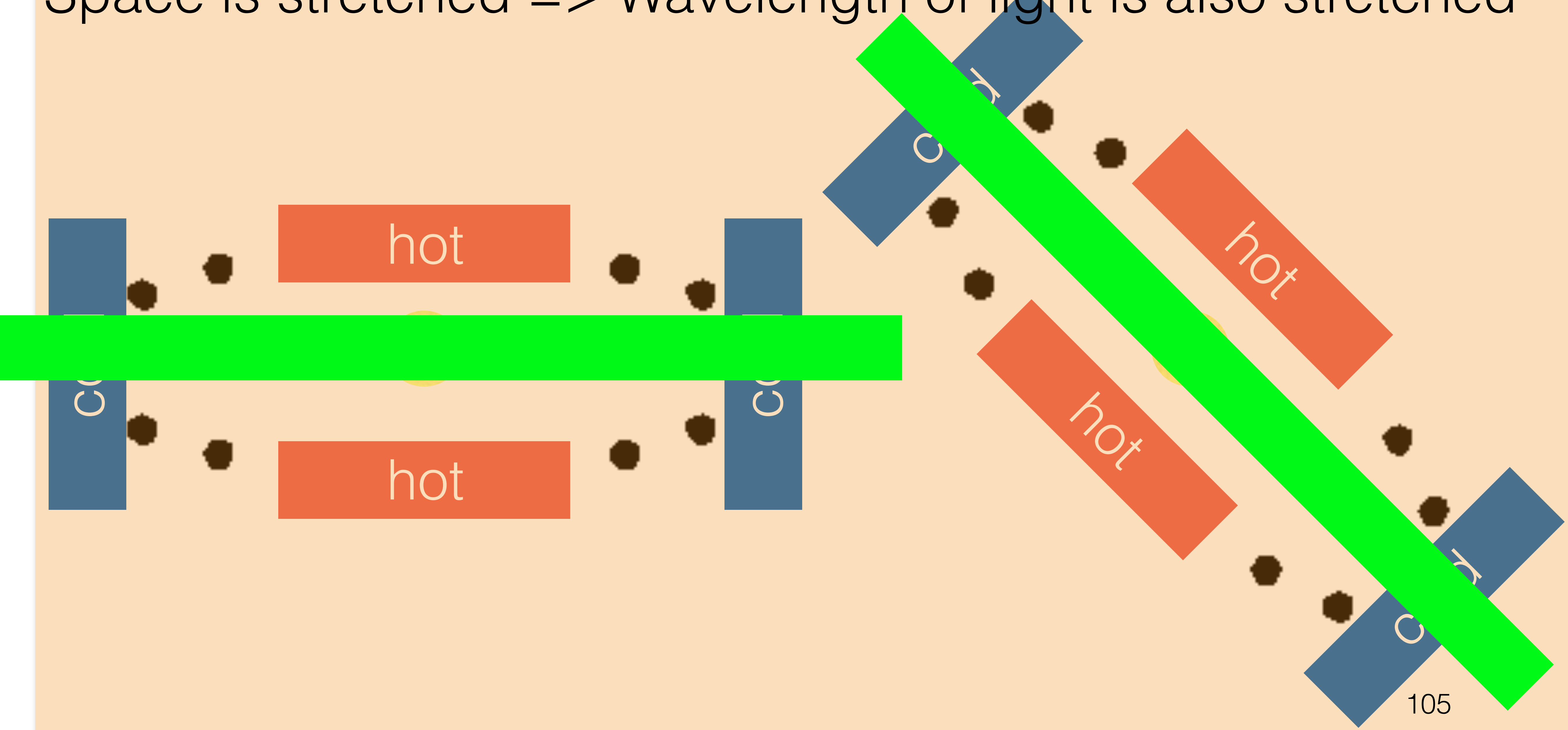
Detecting GW by CMB
Polarisation

Space is stretched => Wavelength of light is also stretched



Detecting GW by CMB **Polarisation**

Space is stretched => Wavelength of light is also stretched

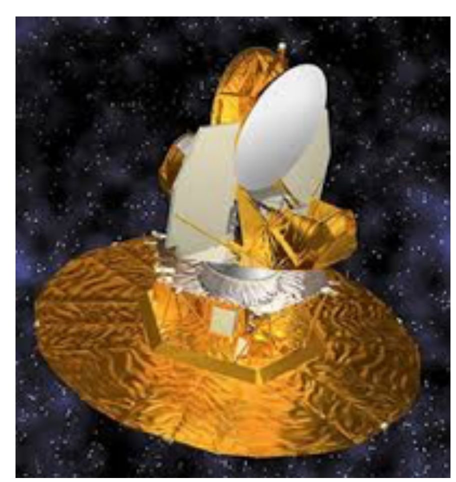


Current Situation

- No detection of polarisation from primordial GW yet
- Many ground-based and balloon-borne experiments are taking data now

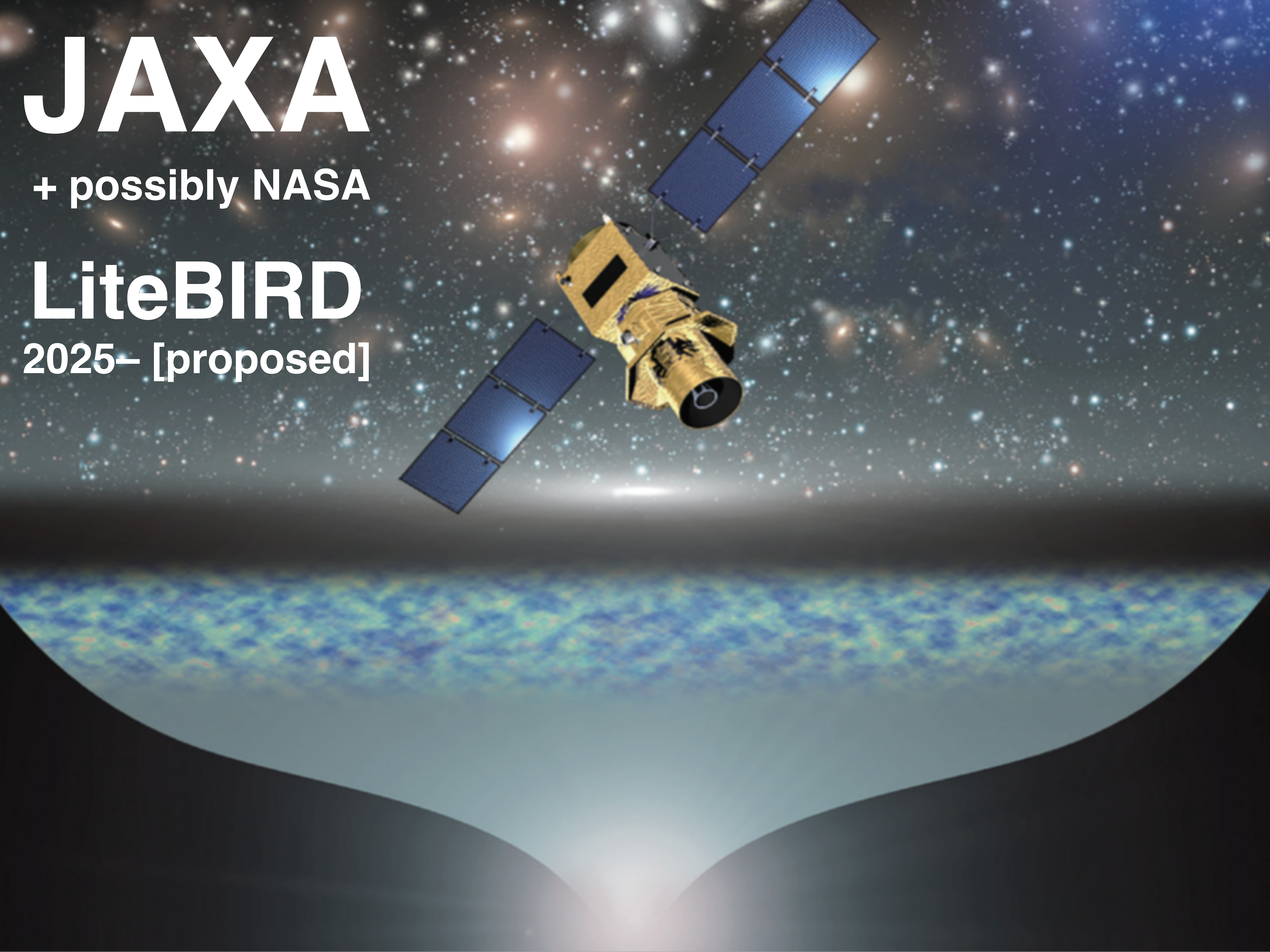
The search continues!!


1989–1993


2001–2010


2009–2013

202X–

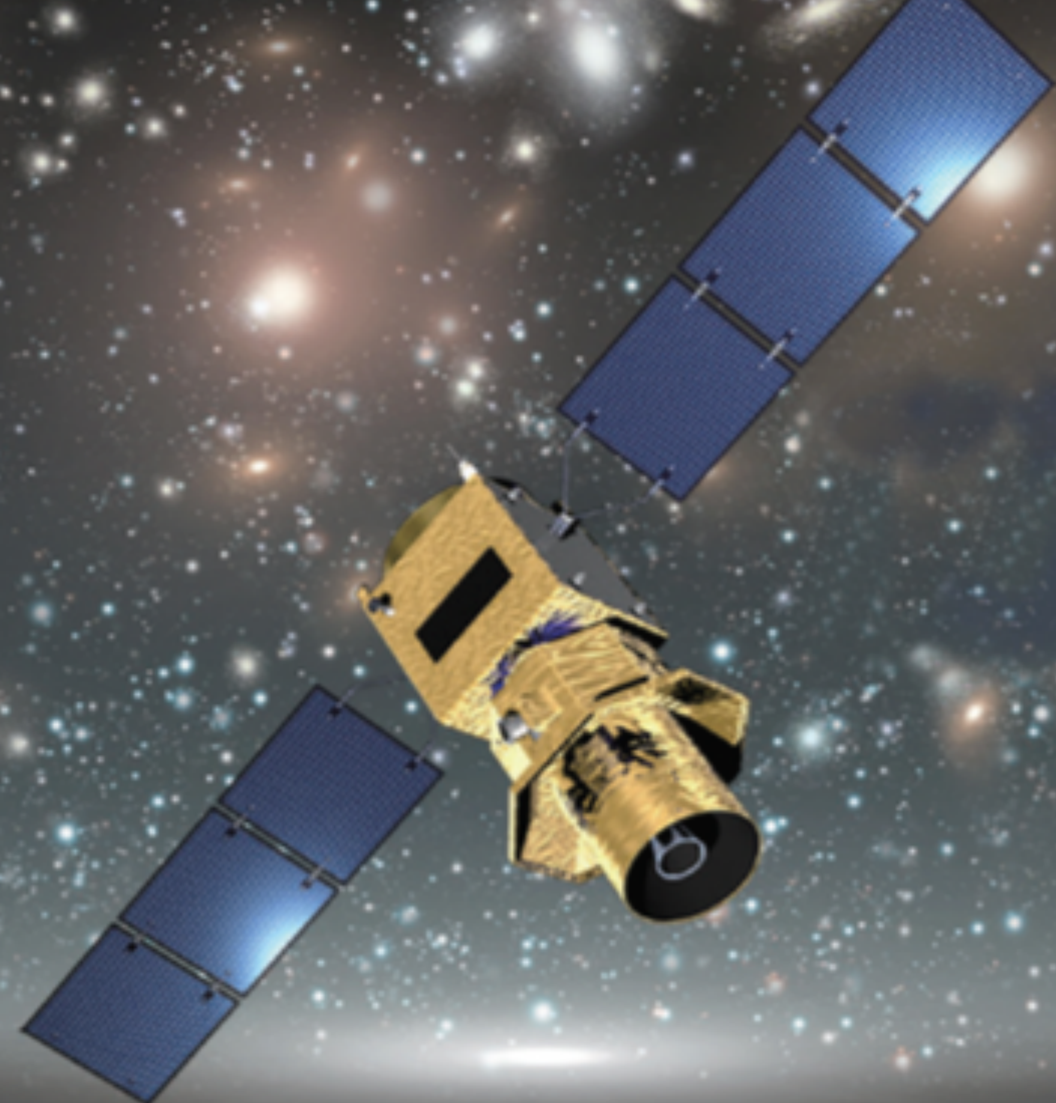

JAXA
+ possibly NASA
LiteBIRD
2025– [proposed]

JAXA

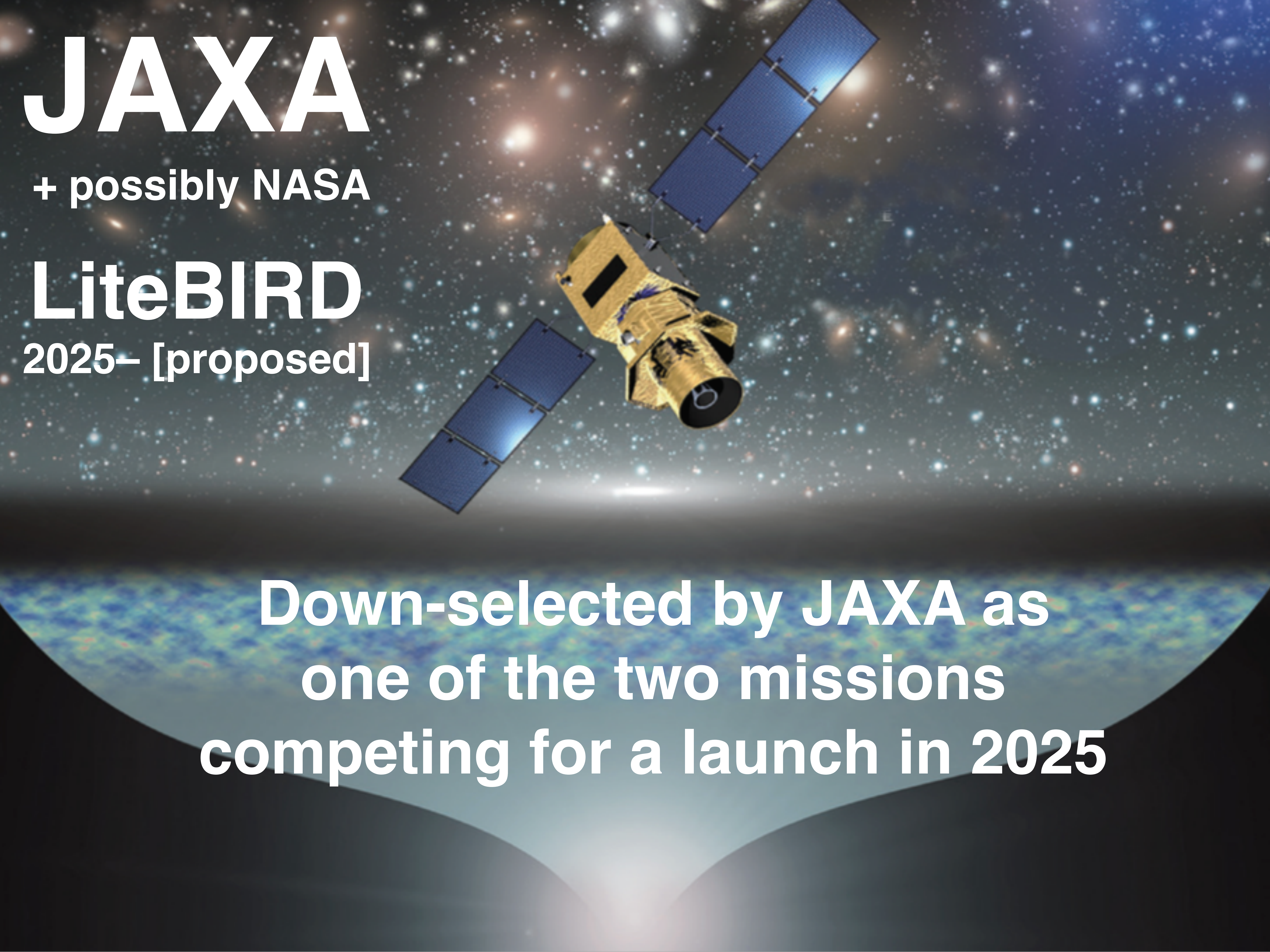
+ possibly NASA

LiteBIRD

2025– [proposed]



Polarisation satellite dedicated to measure CMB polarisation from primordial GW, with a couple thousand super-conducting detectors in space


JAXA
+ possibly NASA
LiteBIRD
2025– [proposed]
Down-selected by JAXA as
one of the two missions
competing for a launch in 2025

Conclusions:
From Initial Conditions…

- **n_s<1 finally discovered**. Time-translation symmetry of space-time is broken during inflation, as predicted
  - **No evidence for breaking of rotational symmetry**, consistent with the simplest model
- Strong evidence for inflation, but not a final word
  - Vigorous searches for the **primordial GW from inflation** are underway. Fingers crossed for the final selection of LiteBIRD by JAXA

Conclusions:
to Structure Formation…

- **Secondary anisotropy of the CMB** tells us about the subsequent gravitational and baryonic evolution of initial fluctuations
    - Theory fits!
- The large-scale structure of the universe traced by the distribution of galaxies also tests our theory for the subsequent evolution
    - Theory fits!

Conclusions:
…and Back!!

- Measurements of the three-point function (**bispectrum**) **of the large-scale structure** are not as scary as before

  - They can finally be used to test properties of the primordial fluctuations, potentially better than CMB studies could

  - **The position-dependent power spectrum is a powerful, easy, and intuitive way to get this job done**

[Review: Chi-Ting Chiang, arXiv:1508.03256]