

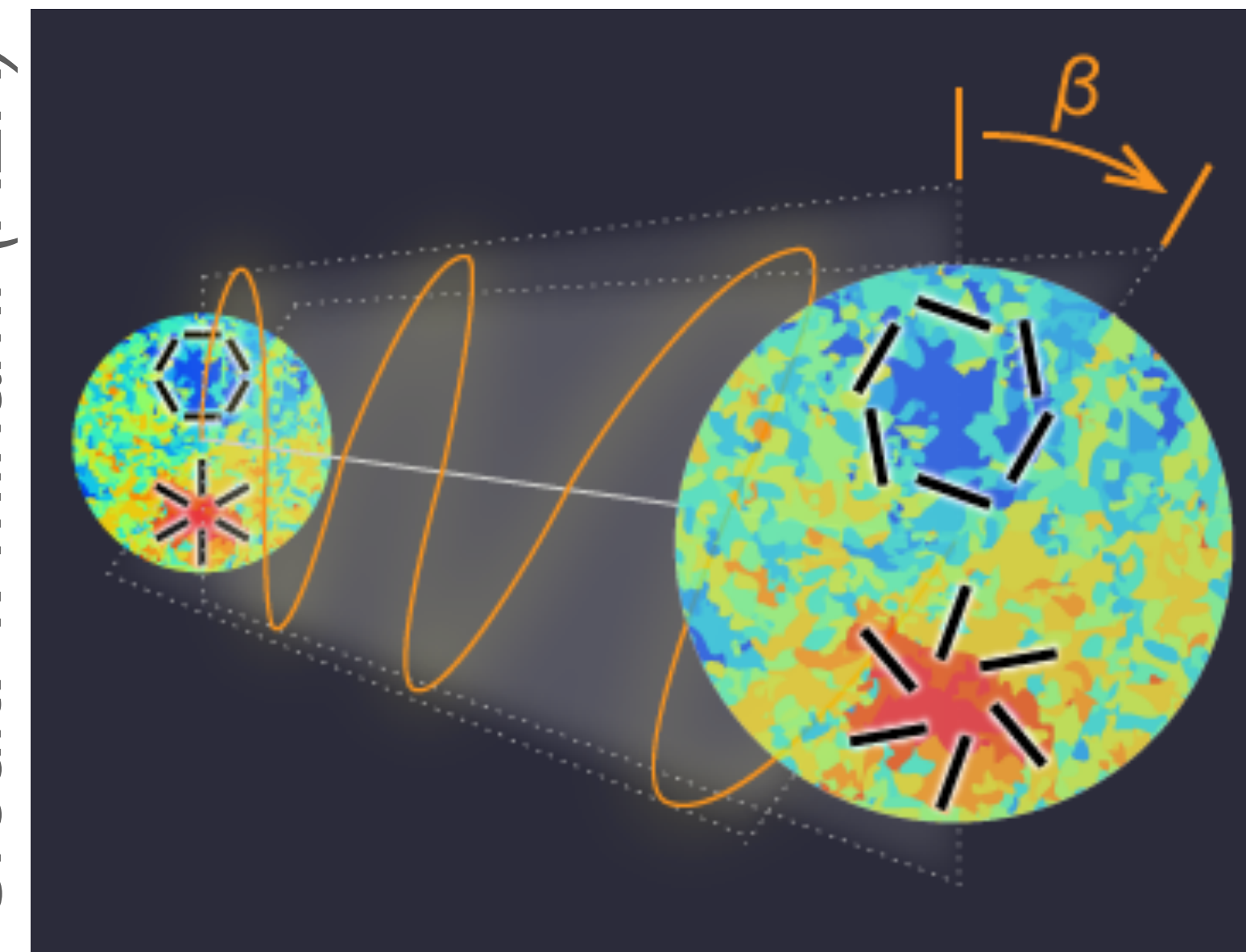
Hunting for parity-violating physics in polarisation of the cosmic microwave background

a.k.a. “Cosmic Birefringence”

Yuto Minami (KEK -> Osaka University)
Eiichiro Komatsu (Max-Planck-Institut für Astrophysik)

CEICO Seminar, November 26, 2020

Credit: Y. Minami (KEK)



Featured in Physics

Editors' Suggestion

New Extraction of the Cosmic Birefringence from the Planck 2018 Polarization Data

Yuto Minami and Eiichiro Komatsu

Phys. Rev. Lett. **125**, 221301 – Published 23 November 2020

Physics See synopsis: [Hints of Cosmic Birefringence?](#)

Article

References

No Citing Articles

PDF

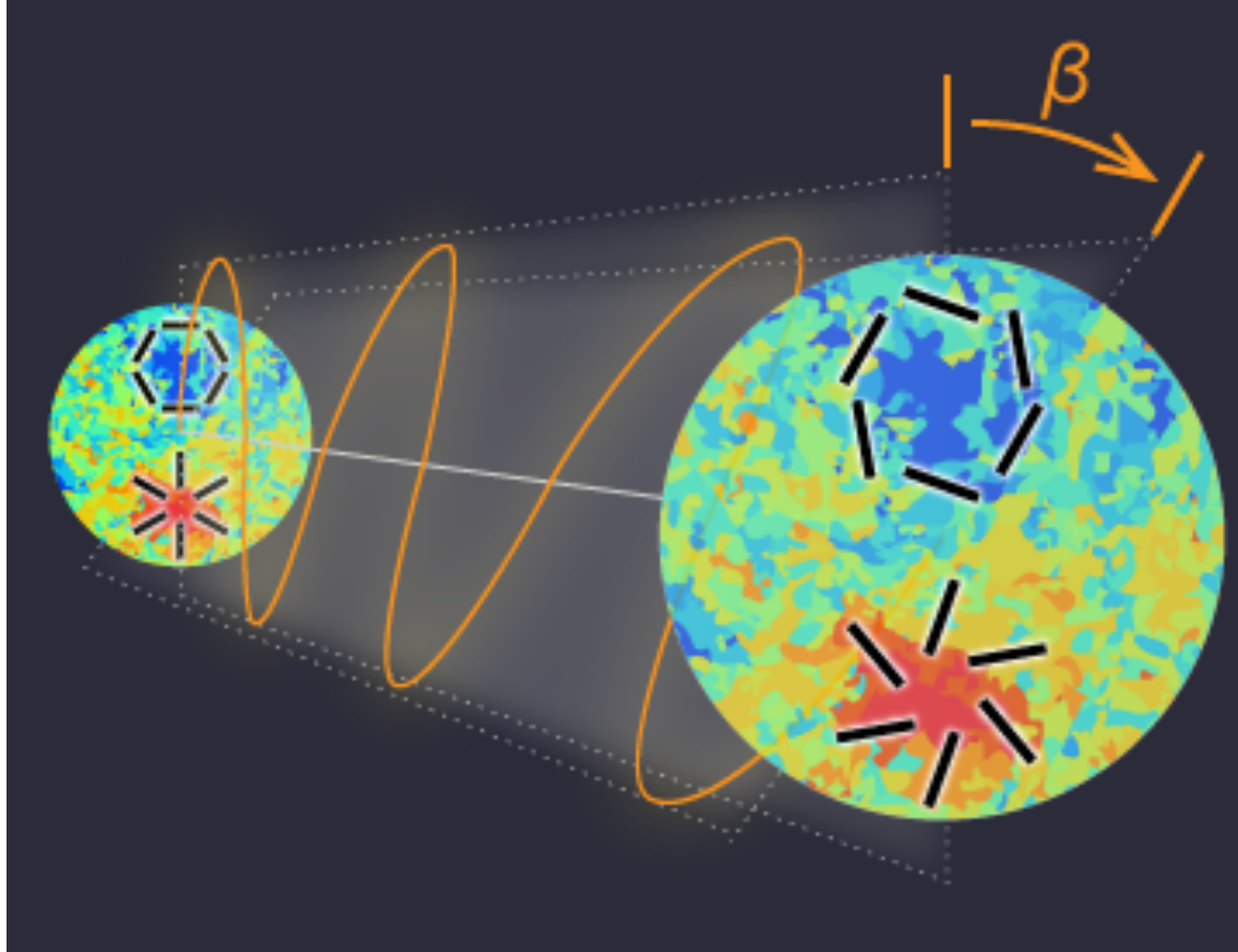
HTML

Export Citation

ABSTRACT

We search for evidence of parity-violating physics in the Planck 2018 polarization data and report on a new measurement of the cosmic birefringence angle β . The previous measurements are limited by the systematic uncertainty in the absolute polarization angles of the Planck detectors. We mitigate this systematic uncertainty completely by simultaneously determining β and the angle miscalibration using the observed cross-correlation of the E - and B -mode polarization of the cosmic microwave background and the Galactic foreground emission. We show that the systematic errors are effectively mitigated and achieve a factor-of-2 smaller uncertainty than the previous measurement, finding $\beta = 0.35 \pm 0.14$ deg (68% C.L.), which excludes $\beta = 0$ at 99.2% C.L. This corresponds to the statistical significance of 2.4σ .

Credit: Y. Minami (KEK)



Yuto Minami
(KEK -> Osaka U.)

The methodology papers that led to this measurement

We have been working on this for ~2 years

1. **Minami**, Ochi, Ichiki, Katayama, Komatsu & Matsumura, “*Simultaneous determination of the cosmic birefringence and miscalibrated polarization angles from CMB experiments*”, PTEP, 083E02 (2019)

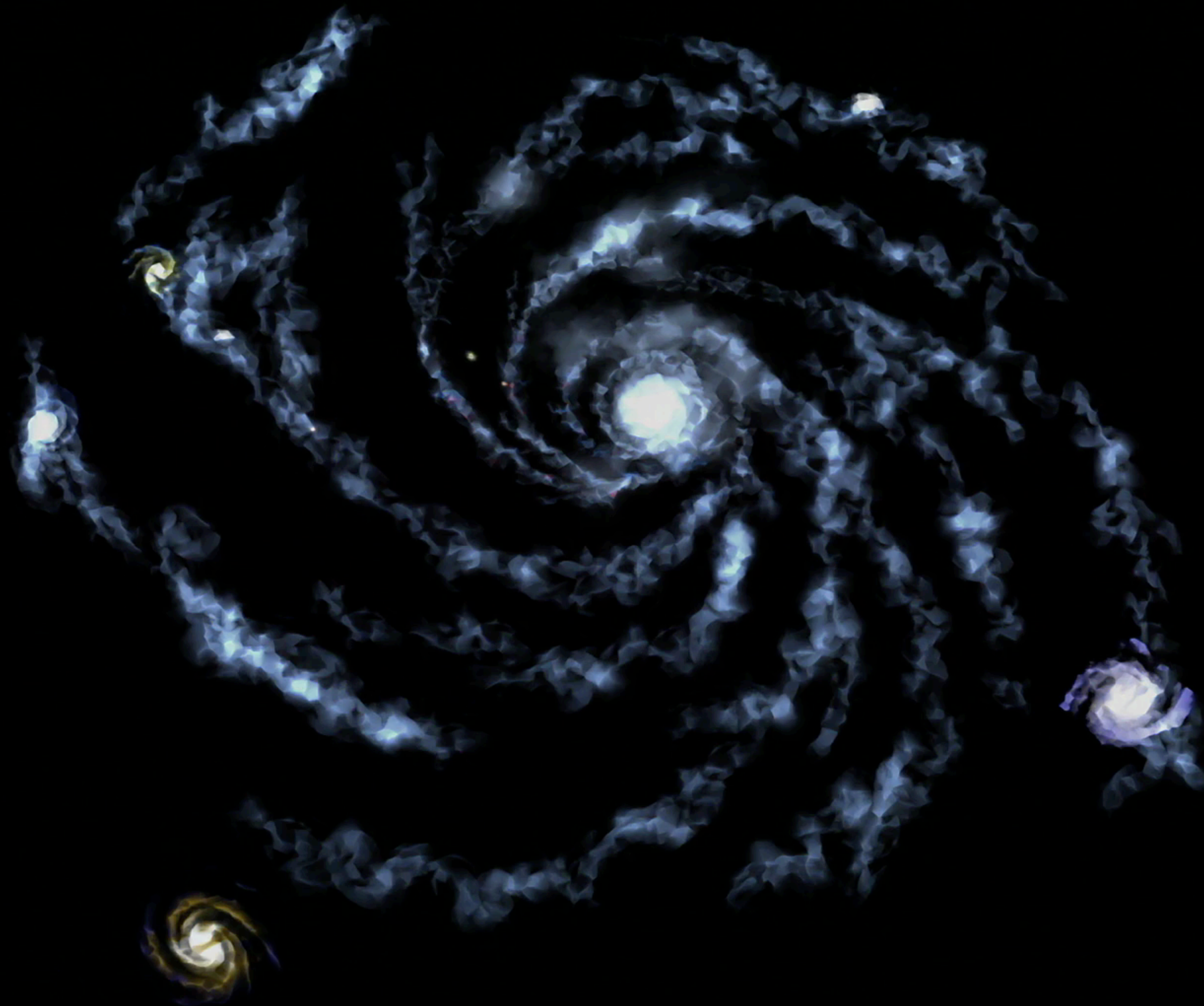
- The original paper to describe the basic idea, methodology, and validation
- Assumed full-sky data

2. **Minami**, “*Determination of miscalibrated polarization angles from observed CMB and foreground EB power spectra: Application to partial-sky observation*”, PTEP, 063E01 (2020)

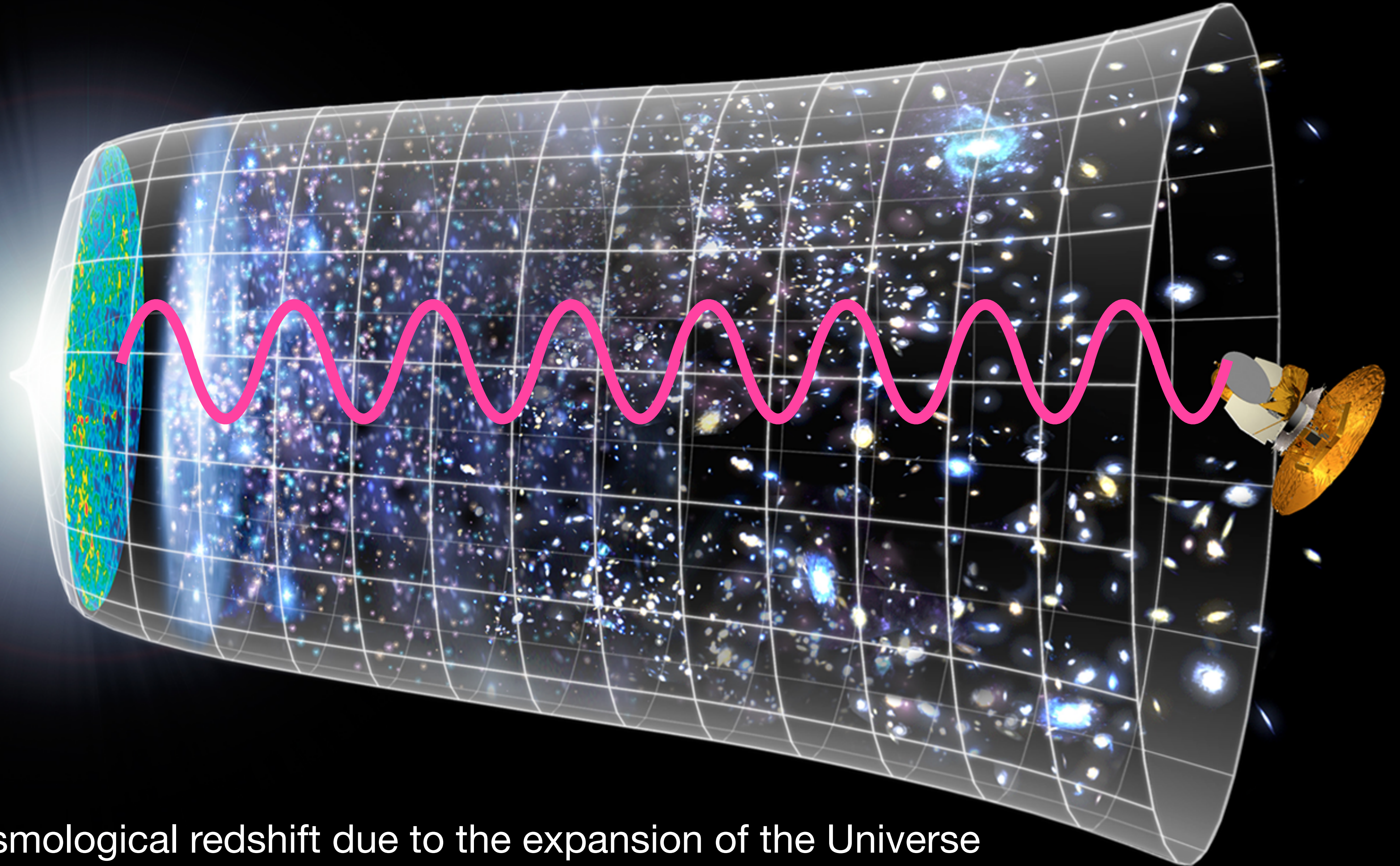
- Extension to partial-sky data

3. **Minami** & Komatsu, “*Simultaneous determination of the cosmic birefringence and miscalibrated polarization angles II: Including cross-frequency spectra*”, PTEP, 103E02 (2020)

- The complete methodology for multi-frequency data, used for analysing PR3

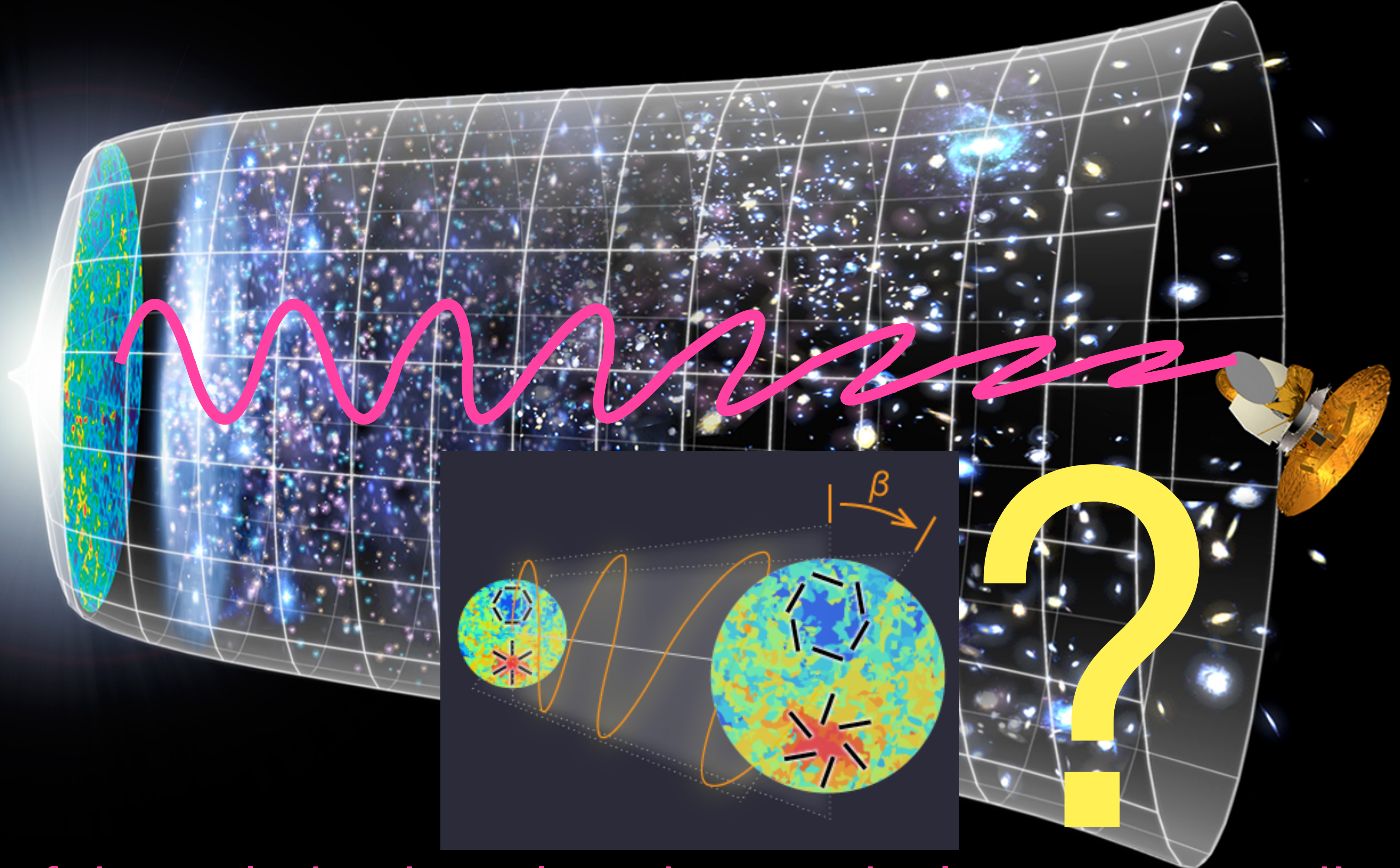


How does the electromagnetic wave of the CMB reach us?



Now shown: The cosmological redshift due to the expansion of the Universe

How does the electromagnetic wave of the CMB reach us?



Note: rotation of the polarisation plane is massively exaggerated!

Cosmic Birefringence

The Universe filled with a “birefringent material”

- If the Universe is filled with a pseudo-scalar field (e.g., an axion field) coupled to the electromagnetic tensor via a Chern-Simons coupling:

Turner & Widrow (1988)

the effective Lagrangian for axion electrodynamics is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\theta\partial^\mu\theta - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{g_a\theta F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\text{Chern-Simons term}}, \quad (3.7)$$

$$\tilde{F}^{\mu\nu} = \sum_{\alpha\beta} \frac{\epsilon^{\mu\nu\alpha\beta}}{2\sqrt{-g}} F_{\alpha\beta}$$

where g_a is a coupling constant of the order α , and the vacuum angle $\theta = \phi_a / f_a$ ($\phi_a =$ axion field). The equations

$$\sum_{\mu\nu} F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B} \cdot \mathbf{B} - \mathbf{E} \cdot \mathbf{E}) \quad \underline{\text{Parity Even}}$$

$$\sum_{\mu\nu} F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\mathbf{B} \cdot \mathbf{E} \quad \underline{\text{Parity Odd}}$$

- The axion field, θ , is a “pseudo scalar”, which is parity odd; thus, the last term in Eq.3.7 is parity even as a whole.

Cosmic Birefringence

The Universe filled with a “birefringent material”

- If the Universe is filled with a pseudo-scalar field (e.g., an axion field) coupled to the electromagnetic tensor via a Chern-Simons coupling:

Turner & Widrow (1988)

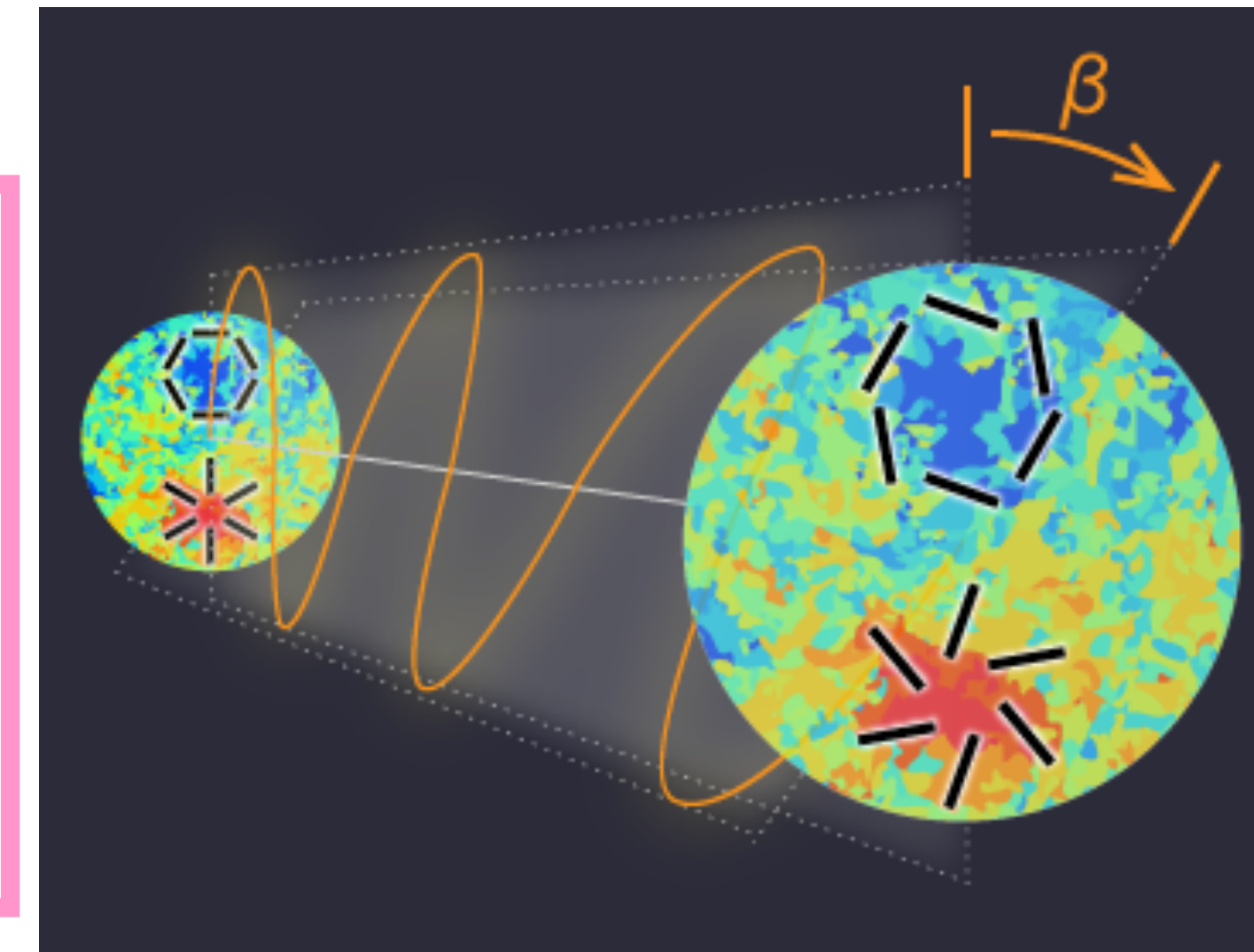
the effective Lagrangian for axion electrodynamics is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\theta\partial^\mu\theta - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \boxed{g_a\theta F_{\mu\nu}\tilde{F}^{\mu\nu}}, \quad (3.7)$$

Chern-Simons term

$$\tilde{F}^{\mu\nu} = \sum_{\alpha\beta} \frac{\epsilon^{\mu\nu\alpha\beta}}{2\sqrt{-g}} F_{\alpha\beta}$$

where g_a is a coupling constant of the order α , and the vacuum angle $\theta = \phi_a / f_a$ ($\phi_a =$ axion field). The equations



The “Cosmic Birefringence” (Carroll 1998)

This term makes the phase velocities of right- and left-handed polarisation states of photons different, leading to **rotation of the linear polarisation direction.**

Cosmic Birefringence

The effect accumulates over the distance

- If the Universe is filled with a pseudo-scalar field (e.g., an axion field) coupled to the electromagnetic tensor via a Chern-Simons coupling:

Turner & Widrow (1988)

the effective Lagrangian for axion electrodynamics is

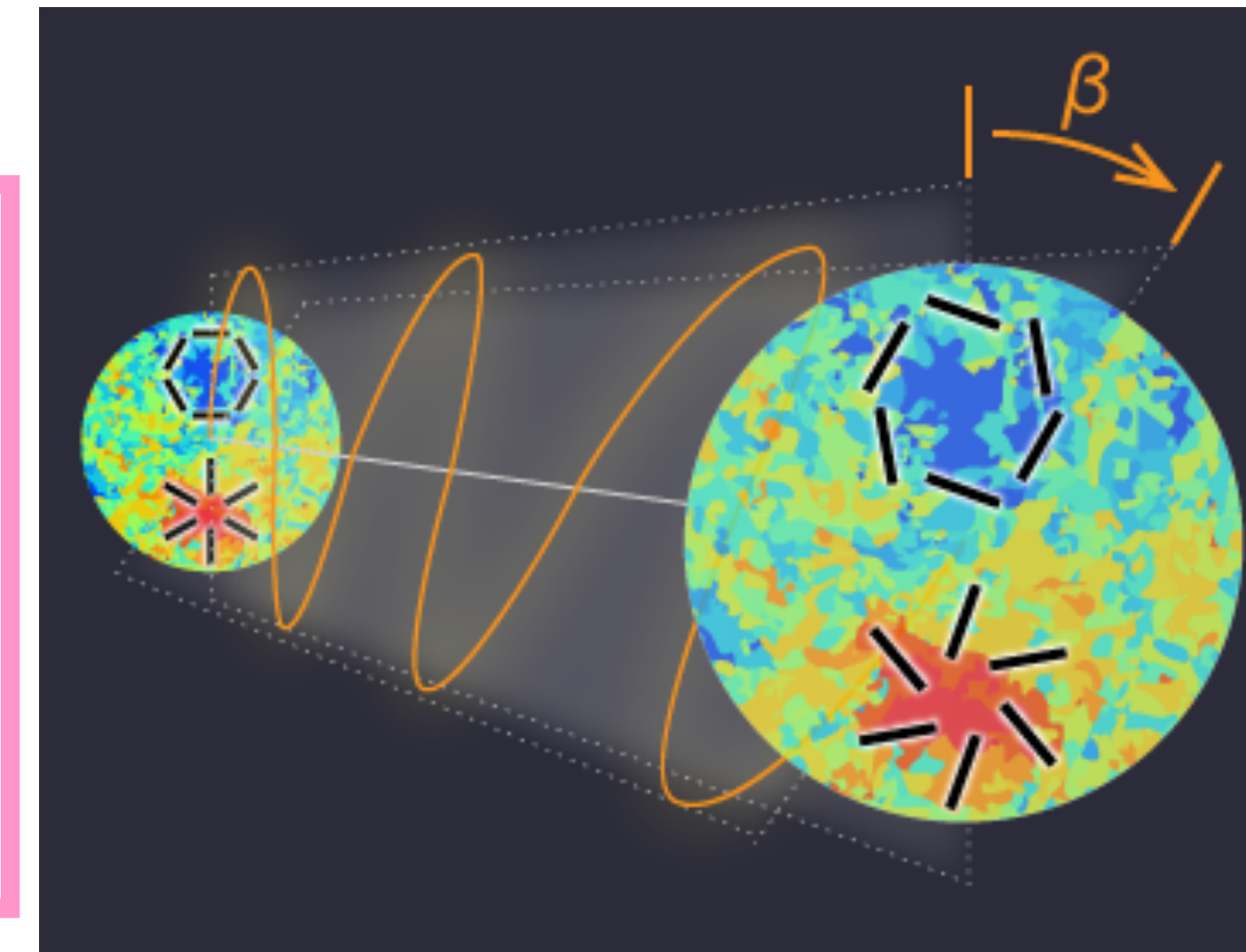
$$\mathcal{L} = -\frac{1}{2}\partial_\mu\theta\partial^\mu\theta - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{g_a\theta F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\text{Chern-Simons term}}, \quad (3.7)$$

$$\tilde{F}^{\mu\nu} = \sum_{\alpha\beta} \frac{\epsilon^{\mu\nu\alpha\beta}}{2\sqrt{-g}} F_{\alpha\beta}$$

where g_a is a coupling constant of the order α , and the vacuum angle $\theta = \phi_a / f_a$ ($\phi_a =$ axion field). The equations

$$\beta = 2g_a \int_{t_{\text{emission}}}^{t_{\text{observed}}} dt \dot{\theta}$$

The larger the distance the photon travels, the larger the effect becomes.



Motivation

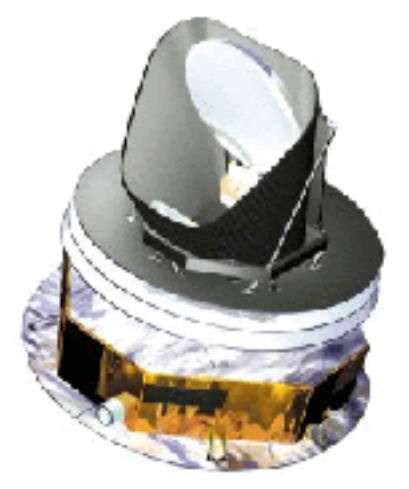
Why study the cosmic birefringence?

- The Universe's energy budget is dominated by two dark components:
 - Dark Matter
 - Dark Energy
- Either or both of these can be an axion-like field!
 - See Marsh (2016) and Ferreira (2020) for reviews.
- Thus, detection of parity-violating physics in polarisation of the cosmic microwave background can transform our understanding of Dark Matter/Energy.

(Simpler) Motivation

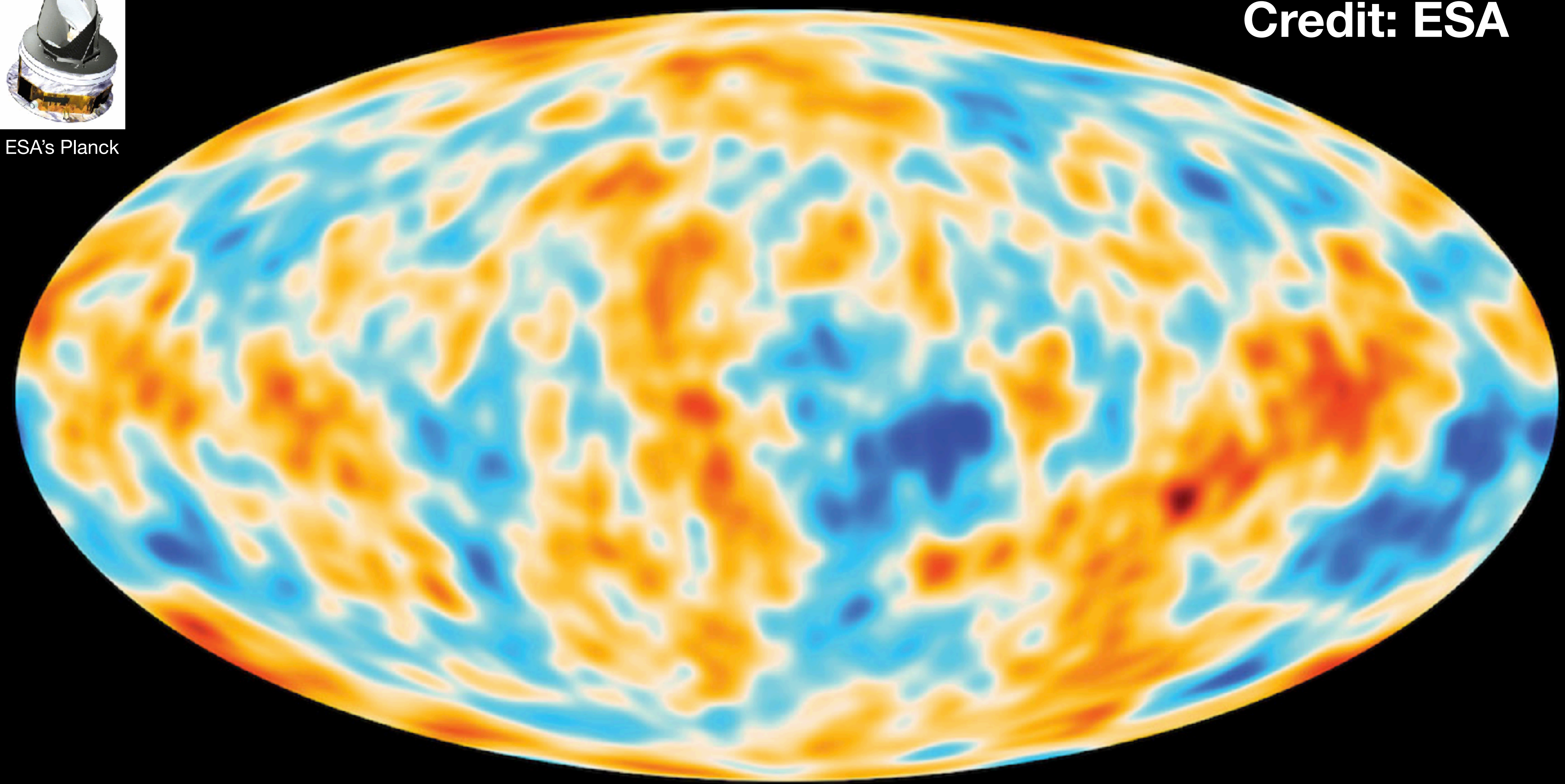
Why study the cosmic birefringence?

- We know that the weak interaction violates parity (Lee & Yang 1956; Wu et al. 1957).
 - Why should the laws of physics governing the Universe conserve parity?
- Let's look!



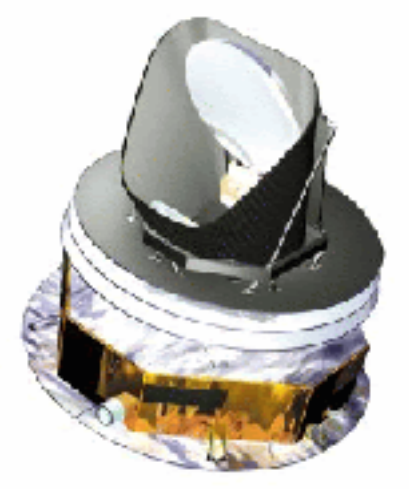
ESA's Planck

Credit: ESA



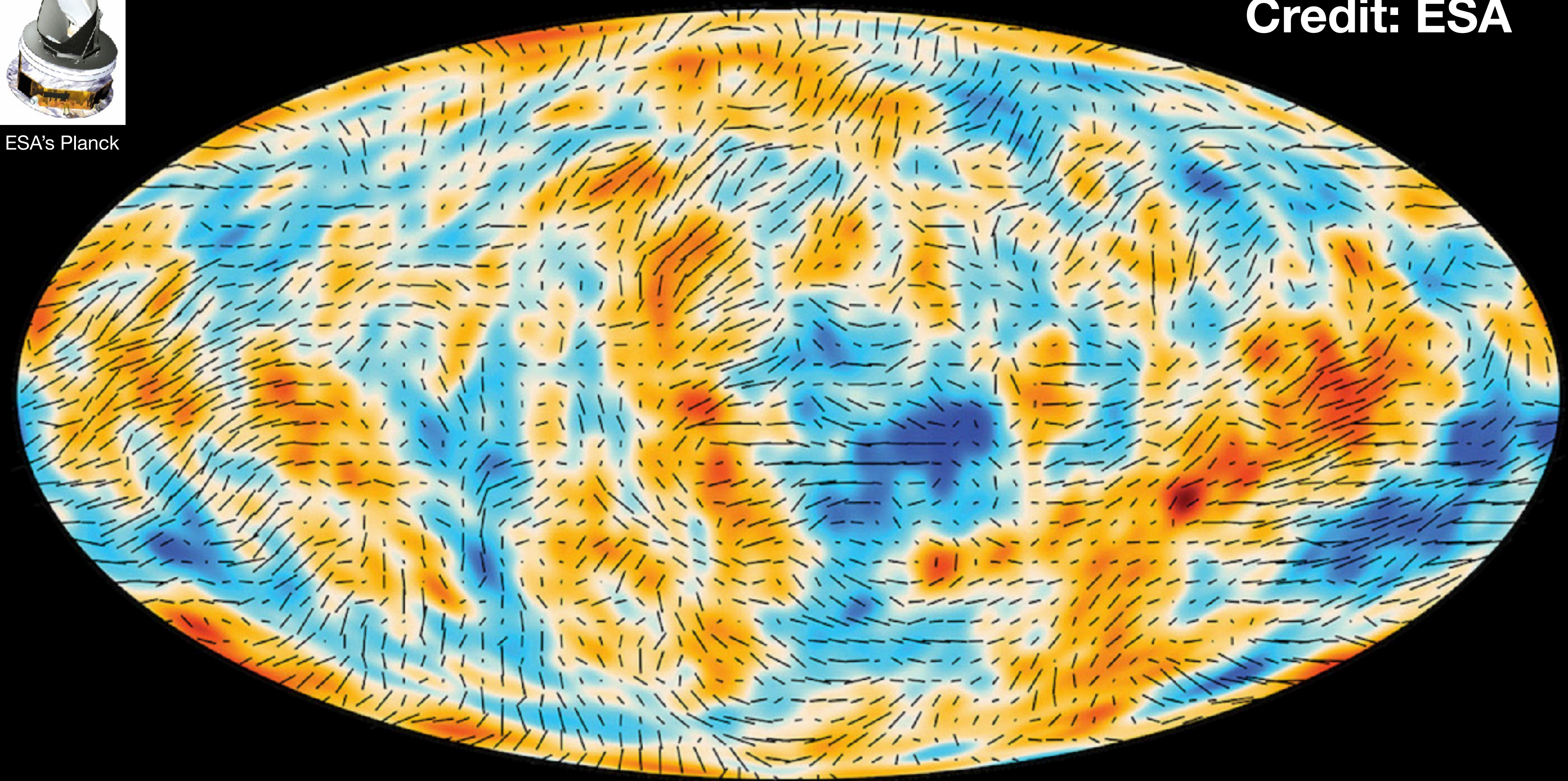
Foreground-cleaned Temperature (smoothed)

Emitted 13.8 billions years ago



ESA's Planck

Credit: ESA

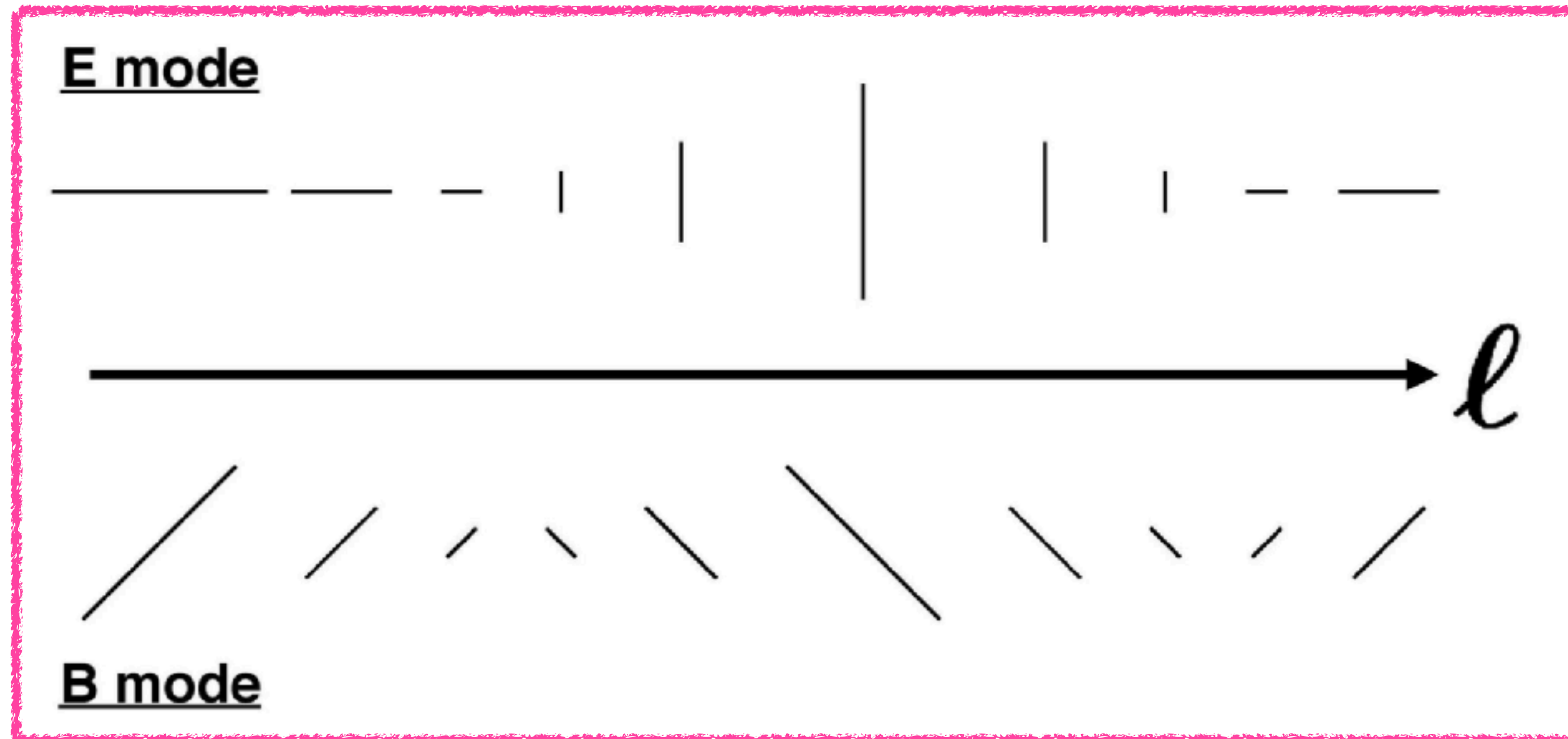


Foreground-cleaned Temperature (smoothed) + Polarisation

Emitted 13.8 billions years ago

E- and B-mode decomposition of linear polarisation

Concept defined in Fourier space



Direction of the Fourier wavenumber vector

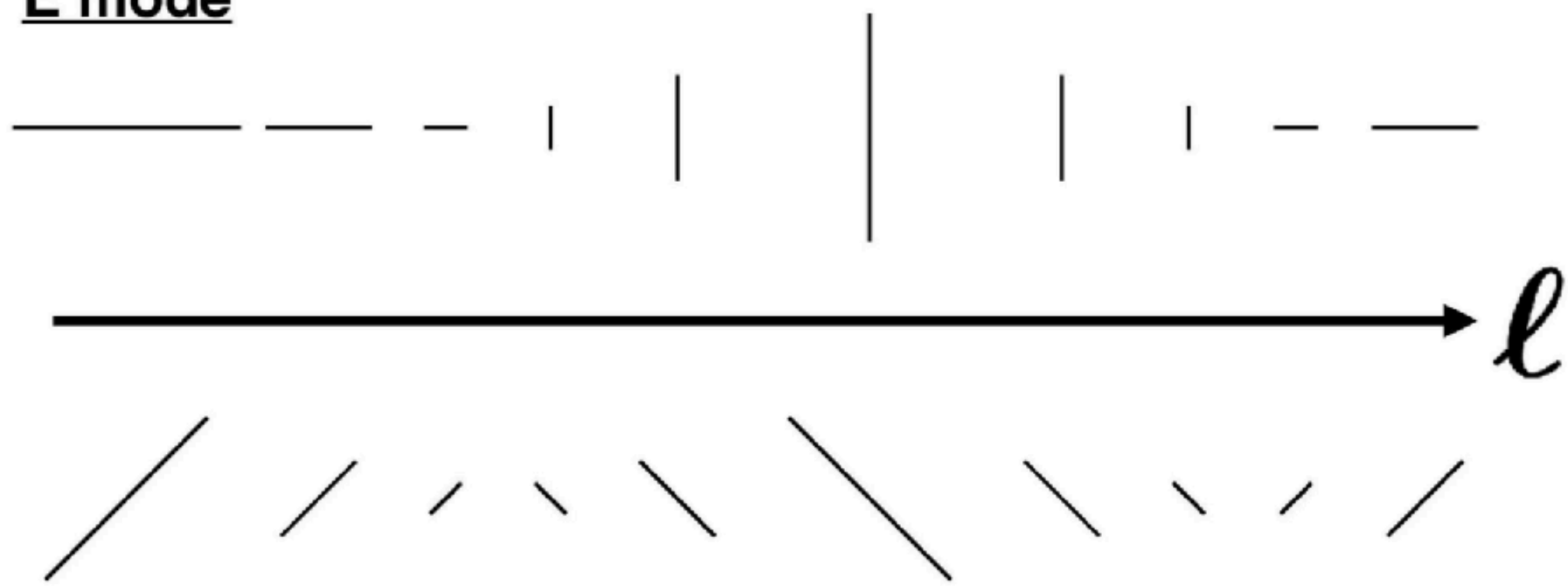
- **E-mode** : Polarisation directions are **parallel or perpendicular** to the wavenumber direction
- **B-mode** : Polarisation directions are **45 degrees tilted** w.r.t the wavenumber direction

IMPORTANT: These “E and B modes” are jargons in the CMB community, and completely unrelated to the electric and magnetic fields of the electromagnetism!!

Parity Flip

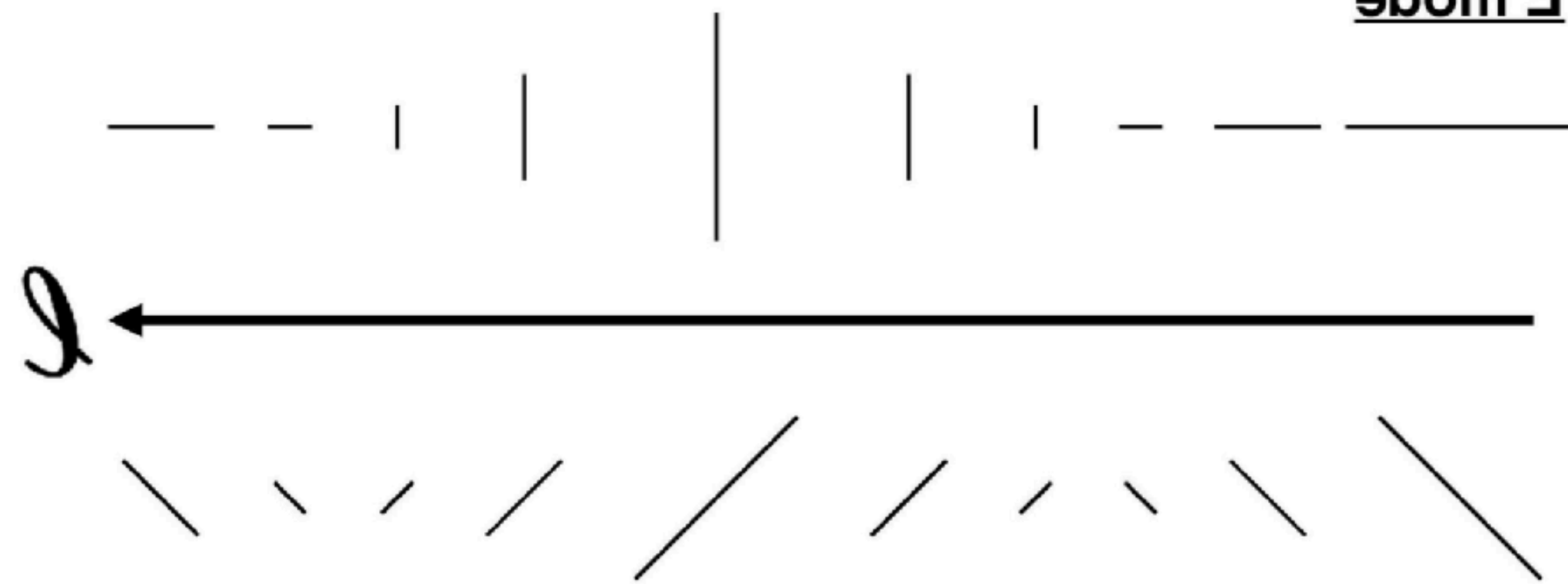
E-mode remains the same, whereas B-mode changes the sign

E mode



B mode

E mode



B mode

- Two-point correlation functions invariant under the parity flip are

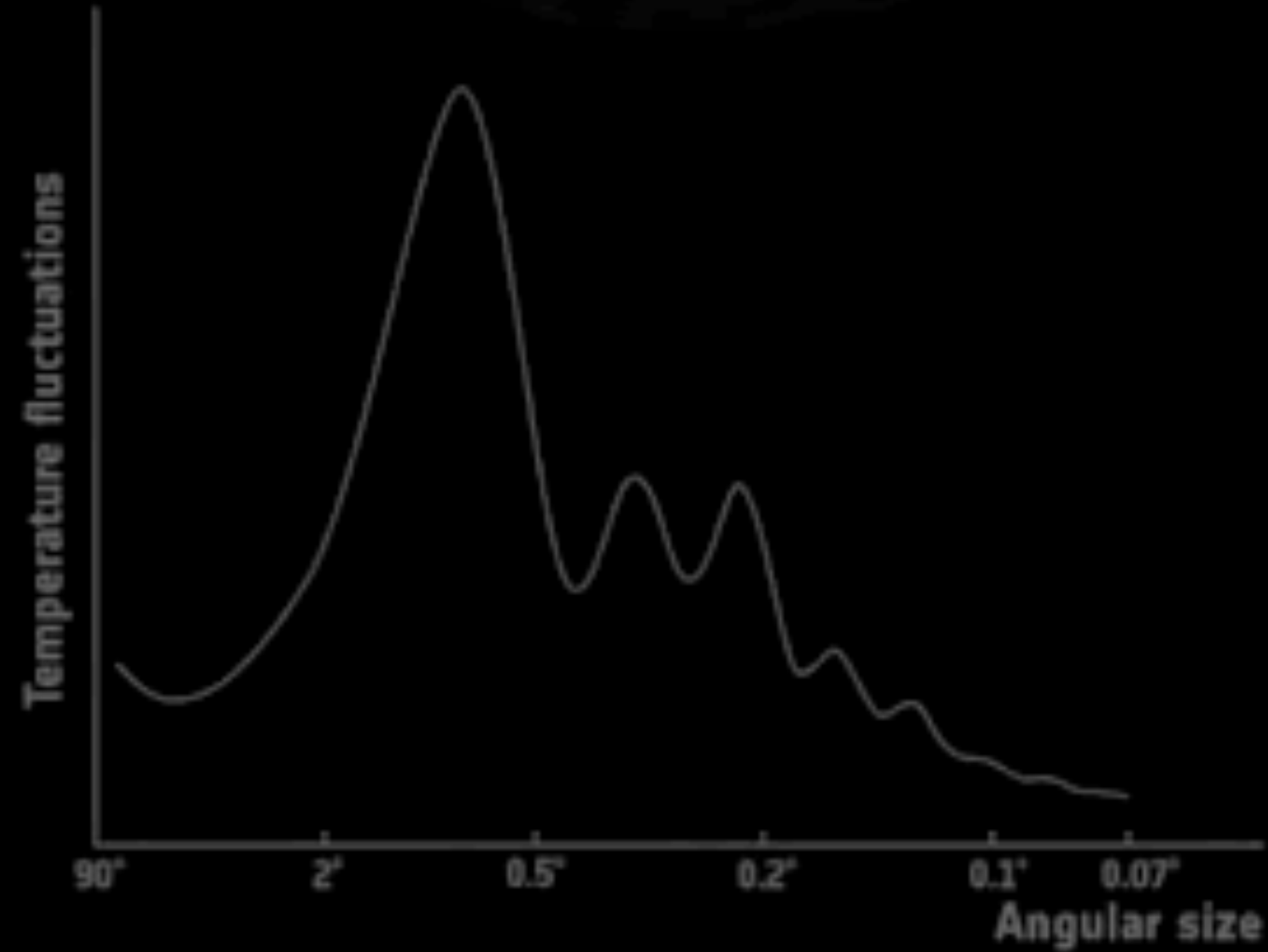
$$\langle E_{\ell} E_{\ell'}^* \rangle = (2\pi)^2 \delta_D^{(2)}(\ell - \ell') C_{\ell}^{EE}$$

$$\langle B_{\ell} B_{\ell'}^* \rangle = (2\pi)^2 \delta_D^{(2)}(\ell - \ell') C_{\ell}^{BB}$$

$$\langle T_{\ell} E_{\ell'}^* \rangle = \langle T_{\ell'}^* E_{\ell} \rangle = (2\pi)^2 \delta_D^{(2)}(\ell - \ell') C_{\ell}^{TE}$$

- The other combinations $\langle TB \rangle$ and $\langle EB \rangle$ are not invariant under the parity flip.

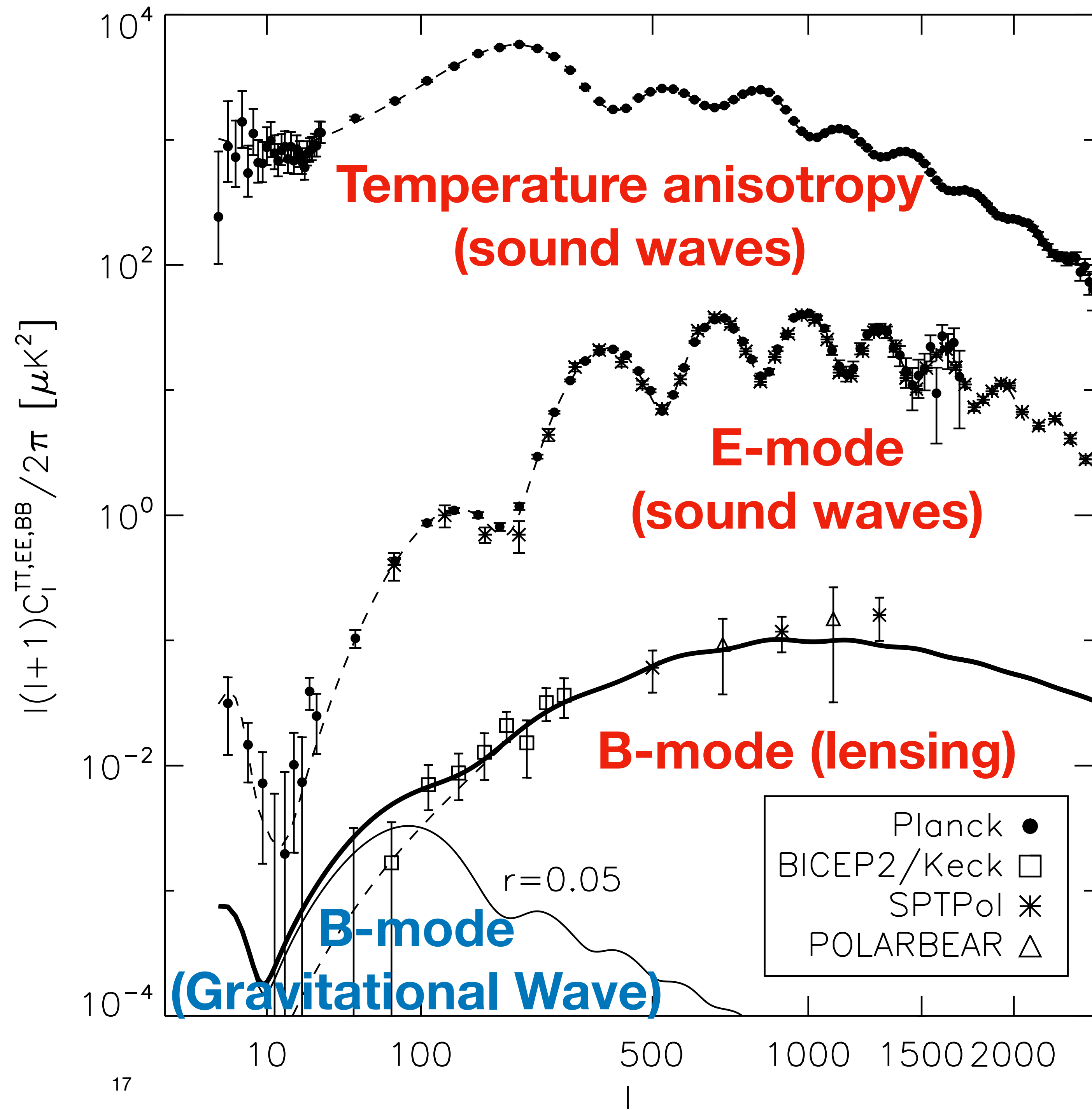
- **We can use these combinations to probe parity-violating physics (e.g., axions)**



Power Spectra

A lot have been measured

- This is the typical figure that you find in many talks on CMB.
- The temperature power spectrum and the E- and B-mode polarisation power spectra have been measured well.
- Our focus is the EB power spectrum, which is not shown here.

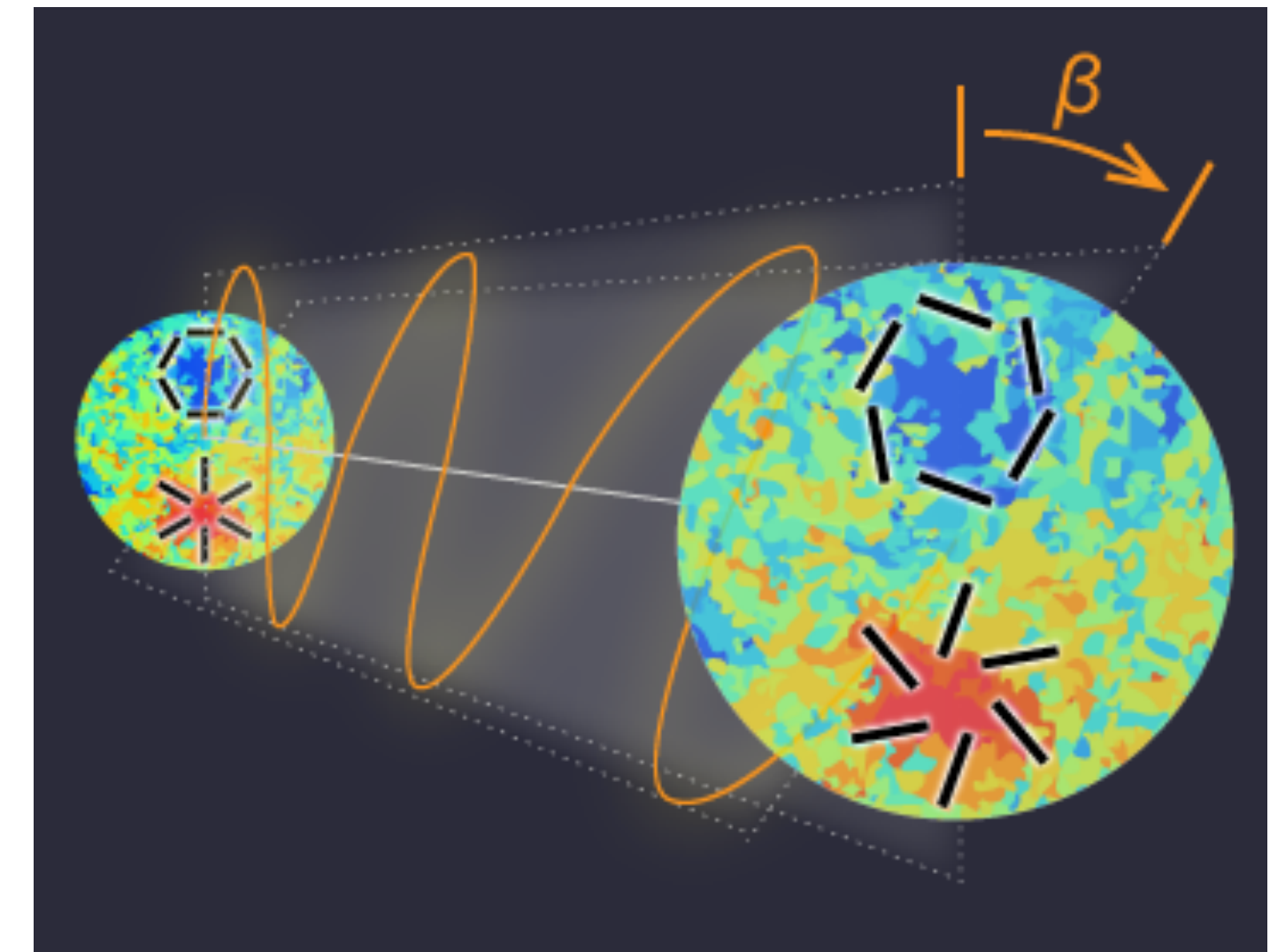


EB correlation from the cosmic birefringence

E \leftrightarrow B conversion by rotation of the linear polarisation plane

- The intrinsic EE, BB, and EB power spectra 13.8 billion years ago would yield the observed EB as

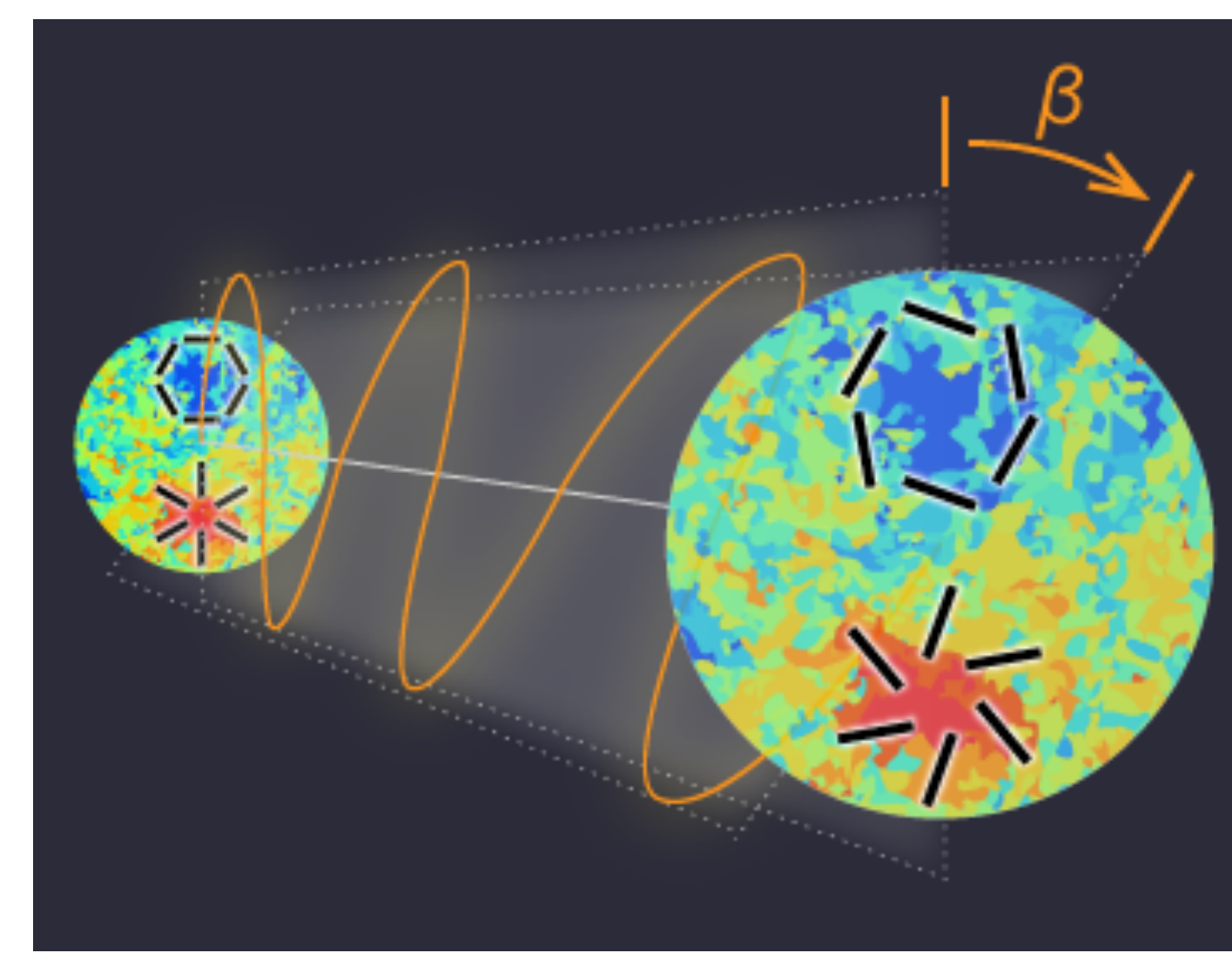
$$C_{\ell}^{EB, \text{obs}} = \frac{1}{2} (C_{\ell}^{EE} - C_{\ell}^{BB}) \sin(4\beta) + C_{\ell}^{EB} \cos(4\beta)$$



- **How do we infer β from the observational data?**
- Traditionally, one would find β by fitting $C_{\ell}^{EE, \text{CMB}} - C_{\ell}^{BB, \text{CMB}}$ to the observed $C_{\ell}^{EB, \text{obs}}$ using the best-fitting CMB model, and assuming the intrinsic EB to vanish, $C_{\ell}^{EB} = 0$.

Searching for the birefringence

Improvement #1 (Zhao et al. 2015)



- If we look at how EE and BB spectra are also modified,

$$C_{\ell}^{EE, \text{obs}} = C_{\ell}^{EE} \cos^2(2\beta) + C_{\ell}^{BB} \sin^2(2\beta) - C_{\ell}^{EB} \sin(4\beta)$$

$$C_{\ell}^{BB, \text{obs}} = C_{\ell}^{EE} \sin^2(2\beta) + C_{\ell}^{BB} \cos^2(2\beta) + C_{\ell}^{EB} \sin(4\beta)$$

- We find

$$C_{\ell}^{EE, \text{obs}} - C_{\ell}^{BB, \text{obs}} = (C_{\ell}^{EE} - C_{\ell}^{BB}) \cos(4\beta) - 2C_{\ell}^{EB} \sin(4\beta)$$

- Thus,

$$C_{\ell}^{EB, \text{obs}} = \frac{1}{2} (C_{\ell}^{EE} - C_{\ell}^{BB}) \sin(4\beta) + C_{\ell}^{EB} \cos(4\beta)$$

$$= \frac{1}{2} \boxed{(C_{\ell}^{EE, \text{obs}} - C_{\ell}^{BB, \text{obs}})} \tan(4\beta) + \frac{C_{\ell}^{EB}}{\cos(4\beta)}$$

No need to assume a model

The Biggest Problem: Miscalibration of detectors

Impact of miscalibration of polarisation angles

Cosmic or Instrumental?



- Is the plane of linear polarisation rotated by the genuine cosmic birefringence effect, or simply because the polarisation-sensitive directions of detectors are rotated with respect to the sky coordinates (and we did not know it)?
- If the detectors are rotated by α , it seems that we can measure only the **sum $\alpha + \beta$** .

The past measurements

The quoted uncertainties are all statistical only (68%CL)

- $\alpha + \beta = -6.0 \pm 4.0$ deg (Feng et al. 2006) first measurement
- $\alpha + \beta = -1.1 \pm 1.4$ deg (WMAP Collaboration, Komatsu et al. 2009; 2011)
- $\alpha + \beta = 0.55 \pm 0.82$ deg (QUaD Collaboration, Wu et al. 2009)
- ...
- $\alpha + \beta = 0.31 \pm 0.05$ deg (Planck Collaboration 2016)
- $\alpha + \beta = -0.61 \pm 0.22$ deg (POLARBEAR Collaboration 2020)
- $\alpha + \beta = 0.63 \pm 0.04$ deg (SPT Collaboration, Bianchini et al. 2020)
- $\alpha + \beta = 0.12 \pm 0.06$ deg (ACT Collaboration, Namikawa et al. 2020)
- $\alpha + \beta = 0.09 \pm 0.09$ deg (ACT Collaboration, Choi et al. 2020)



Why not yet discovered?

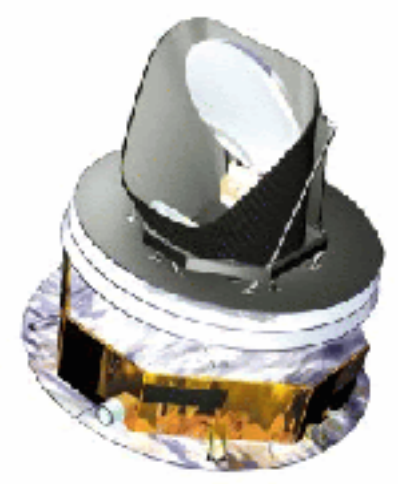
The past measurements

Now including the estimated systematic errors on α

- $\beta = -6.0 \pm 4.0 \pm ??$ deg (Feng et al. 2006)
- $\beta = -1.1 \pm 1.4 \pm 1.5$ deg (WMAP Collaboration, Komatsu et al. 2009; 2011)
- $\beta = 0.55 \pm 0.82 \pm 0.5$ deg (QUaD Collaboration, Wu et al. 2009)
- ...
- $\beta = 0.31 \pm 0.05 \pm 0.28$ deg (Planck Collaboration 2016)
- $\beta = -0.61 \pm 0.22 \pm ??$ deg (POLARBEAR Collaboration 2020)
- $\beta = 0.63 \pm 0.04 \pm ??$ deg (SPT Collaboration, Bianchini et al. 2020)
- $\beta = 0.12 \pm 0.06 \pm ??$ deg (ACT Collaboration, Namikawa et al. 2020)
- $\beta = 0.09 \pm 0.09 \pm ??$ deg (ACT Collaboration, Choi et al. 2020)

Uncertainty in the calibration of α has been the major limitation

The Key Idea: The polarised Galactic foreground emission as a calibrator



ESA's Planck

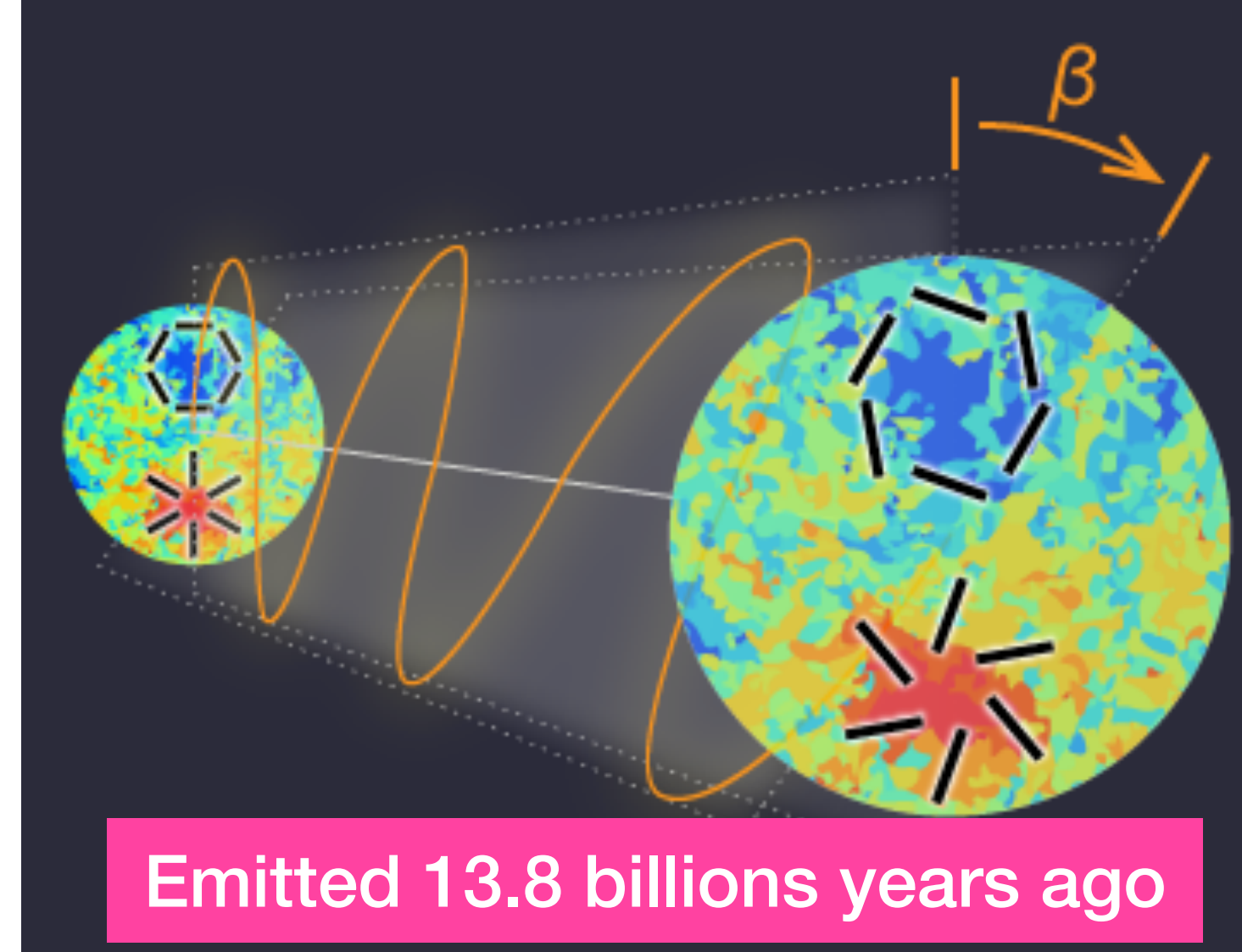
Polarised dust emission within our Milky Way!

Emitted “right there” - it would not be affected by the cosmic birefringence.

Directions of the magnetic field inferred from polarisation of the thermal dust emission in the Milky Way

Searching for the birefringence

Improvement #2 (Minami et al. 2019)



But the source of foreground is much closer!

- **Idea:** Miscalibration of the polarization angle α rotates both the foreground and CMB, but β affects only the CMB.

$$E_{\ell,m}^o = E_{\ell,m}^{\text{fg}} \cos(2\alpha) - B_{\ell,m}^{\text{fg}} \sin(2\alpha) + E_{\ell,m}^{\text{CMB}} \cos(2\alpha + 2\beta) - B_{\ell,m}^{\text{CMB}} \sin(2\alpha + 2\beta) + E_{\ell,m}^{\text{N}}$$

$$B_{\ell,m}^o = E_{\ell,m}^{\text{fg}} \sin(2\alpha) + B_{\ell,m}^{\text{fg}} \cos(2\alpha) + E_{\ell,m}^{\text{CMB}} \sin(2\alpha + 2\beta) + B_{\ell,m}^{\text{CMB}} \cos(2\alpha + 2\beta) + B_{\ell,m}^{\text{N}}$$

- Thus,

$$\langle C_{\ell}^{EB,o} \rangle = \frac{\tan(4\alpha)}{2} \left(\underbrace{\langle C_{\ell}^{EE,o} \rangle - \langle C_{\ell}^{BB,o} \rangle}_{\text{measured}} \right) + \frac{\sin(4\beta)}{2 \cos(4\alpha)} \left(\underbrace{\langle C_{\ell}^{EE,\text{CMB}} \rangle - \langle C_{\ell}^{BB,\text{CMB}} \rangle}_{\text{known accurately}} \right) + \frac{1}{\cos(4\alpha)} \langle C_{\ell}^{EB,\text{fg}} \rangle + \frac{\cos(4\beta)}{\cos(4\alpha)} \langle C_{\ell}^{EB,\text{CMB}} \rangle.$$

Key: No explicit modelling of the foreground EE and BB is necessary

Assumption for the baseline result

What about the intrinsic EB correlation of the foreground emission?

$$\langle C_{\ell}^{EB,o} \rangle = \frac{\tan(4\alpha)}{2} \left(\langle C_{\ell}^{EE,o} \rangle - \langle C_{\ell}^{BB,o} \rangle \right) + \frac{\sin(4\beta)}{2 \cos(4\alpha)} \left(\langle C_{\ell}^{EE,CMB} \rangle - \langle C_{\ell}^{BB,CMB} \rangle \right) + \frac{1}{\cos(4\alpha)} \langle C_{\ell}^{EB,fg} \rangle + \frac{\cos(4\beta)}{\cos(4\alpha)} \langle C_{\ell}^{EB,CMB} \rangle.$$

- For the baseline result, we ignore the intrinsic EB correlations of the foreground $\langle C_{\ell}^{EB,fg} \rangle$ and the CMB $\langle C_{\ell}^{EB,CMB} \rangle$.
- The latter is justifiable but the former is not. We will revisit this important issue at the end.

Likelihood for the simplest case

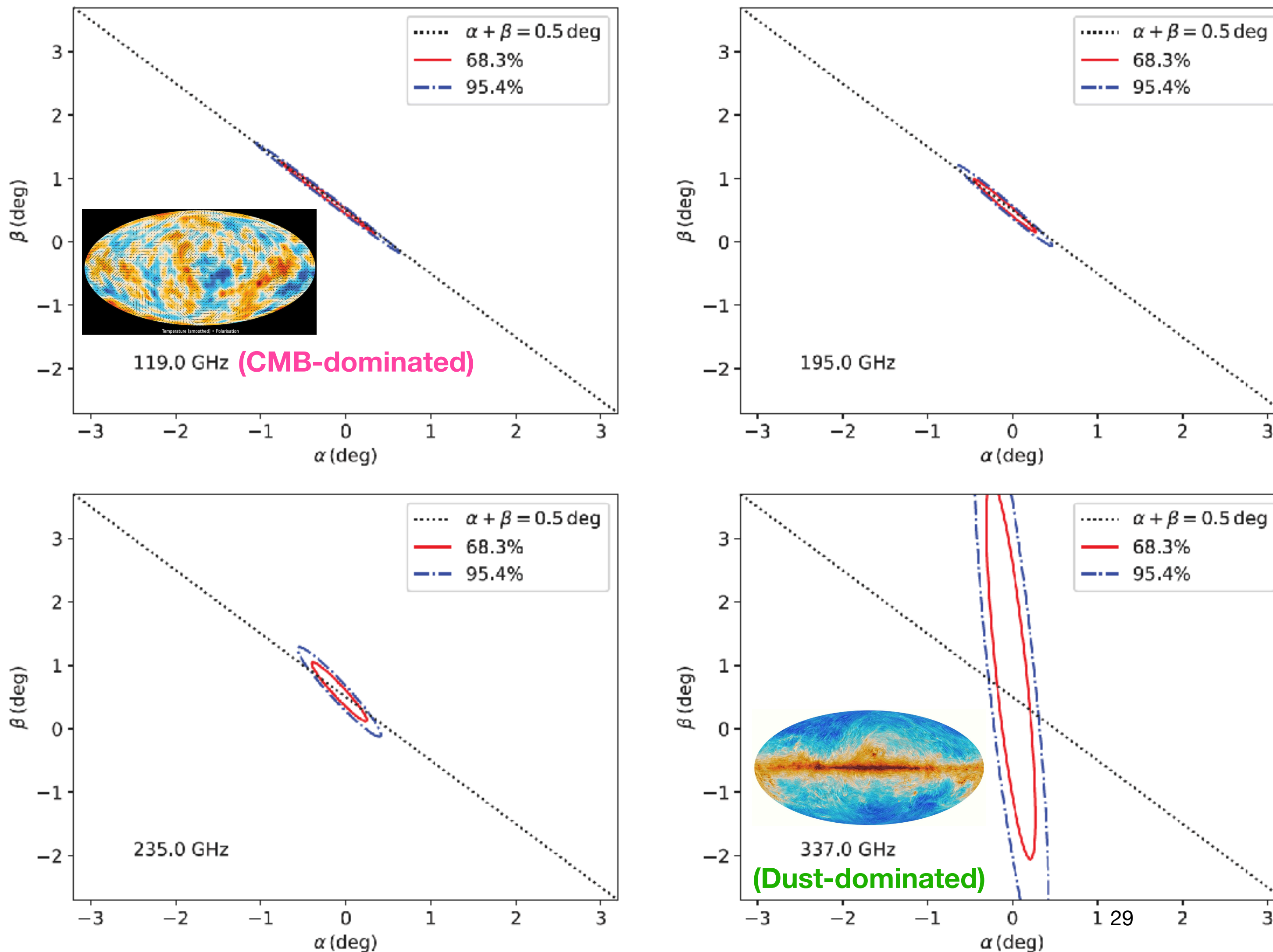
Single-frequency case, full sky data

$$-2 \ln \mathcal{L} = \sum_{\ell=2}^{\ell_{\max}} \frac{\left[C_{\ell}^{EB,o} - \frac{\tan(4\alpha)}{2} \left(C_{\ell}^{EE,o} - C_{\ell}^{BB,o} \right) - \frac{\sin(4\beta)}{2 \cos(4\alpha)} \left(C_{\ell}^{EE,\text{CMB}} - C_{\ell}^{BB,\text{CMB}} \right) \right]^2}{\text{Var} \left(C_{\ell}^{EB,o} - \frac{\tan(4\alpha)}{2} \left(C_{\ell}^{EE,o} - C_{\ell}^{BB,o} \right) \right)}$$

- We determine α and β simultaneously from this likelihood.
- We first validate the algorithm using simulated data.
- For analysing the Planck data, we use the multi-frequency likelihood developed in Minami and Komatsu (2020a).

How does it work?

Simulation of future CMB data (LiteBIRD)



- When the data are dominated by CMB, the sum of two angles, $\alpha + \beta$, is determined precisely.
 - This is the diagonal line.
- The foreground determines α with some uncertainty, breaking the degeneracy. Then $\sigma(\beta) \sim \sigma(\alpha)$ because $\sigma(\alpha + \beta) \ll \sigma(\alpha)$.
- When the data are dominated by the foreground, it can determine α but not β due to the lack of sensitivity to the CMB.

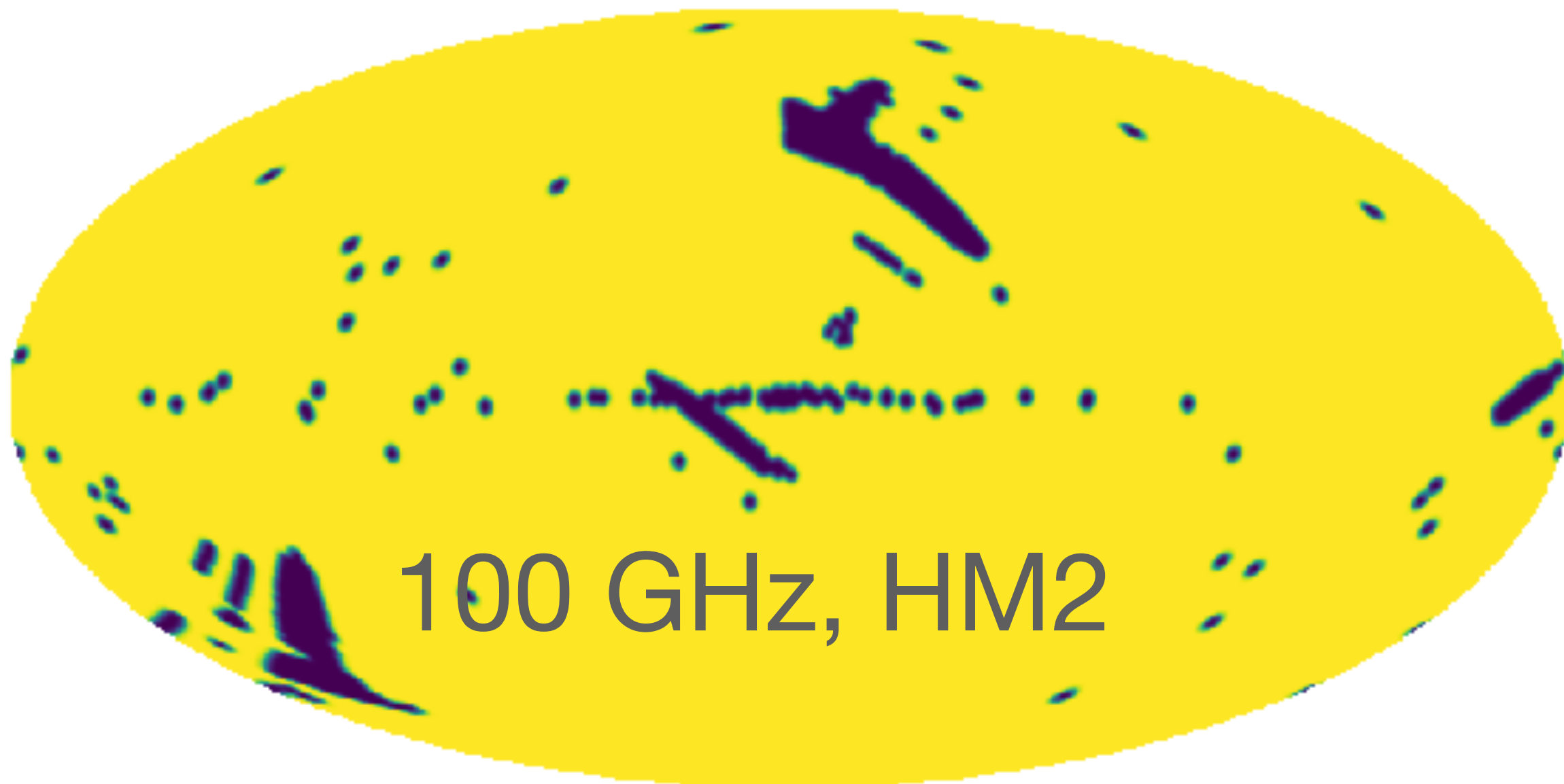
Application to the Planck Data (PR3)

$l_{\min} = 51$, $l_{\max} = 1500$ (the same as those used by the Planck team)

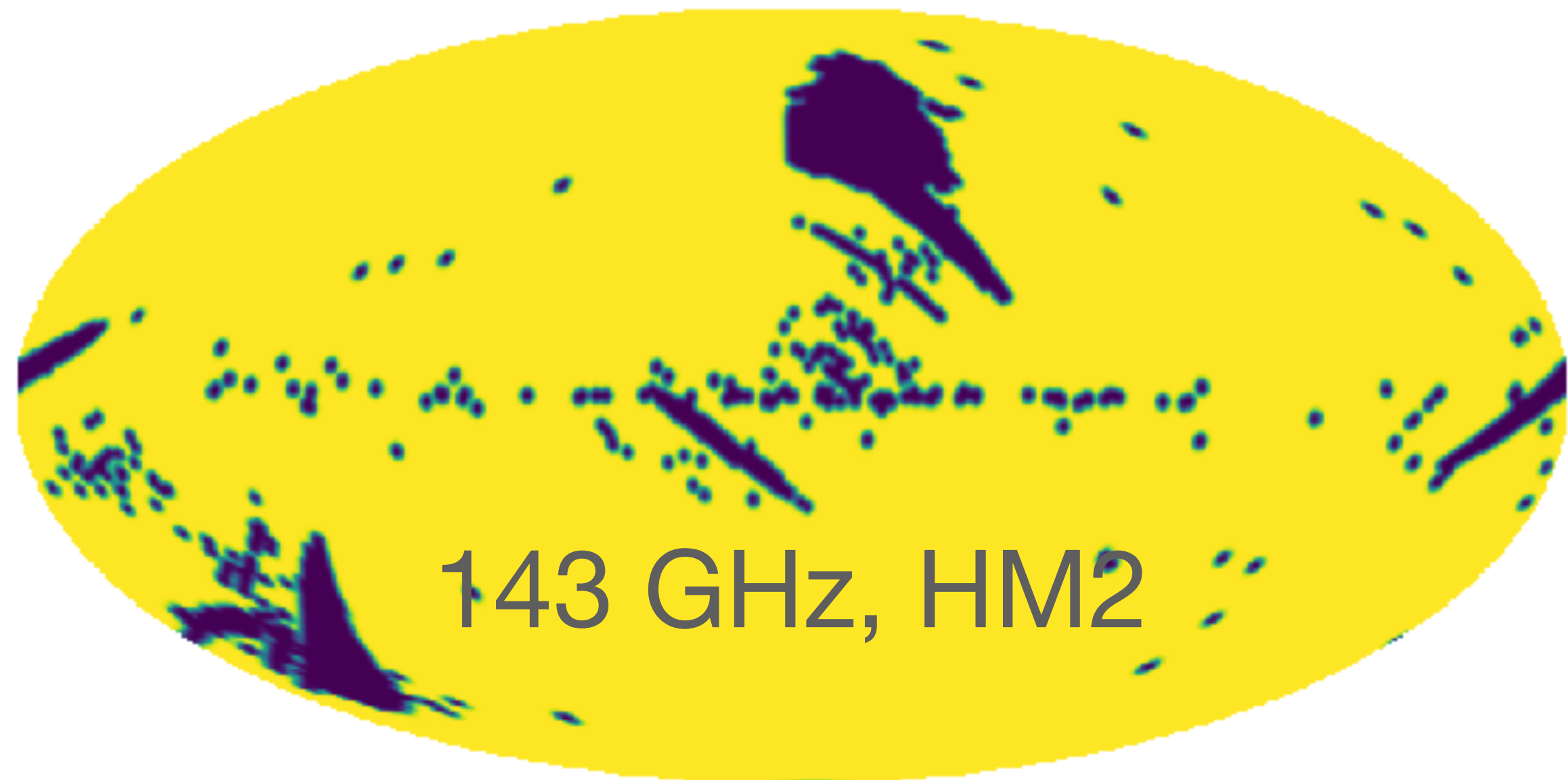
Information for experts

- Planck High Frequency Instrument (HFI) data (100, 143, 217, 353 GHz)
 - Measure power spectra from “Half Missions” (HM1, HM2)
- Mask (using NaMaster [Alonso et al.], apodization by “Smooth” with 0.5 deg)
 - Bright CO regions, Bright point sources, Bad pixels
- $l \rightarrow P$ leakage due to the beam is corrected using QuickPol
 - It does not change the result even if we ignore this correction: good news!

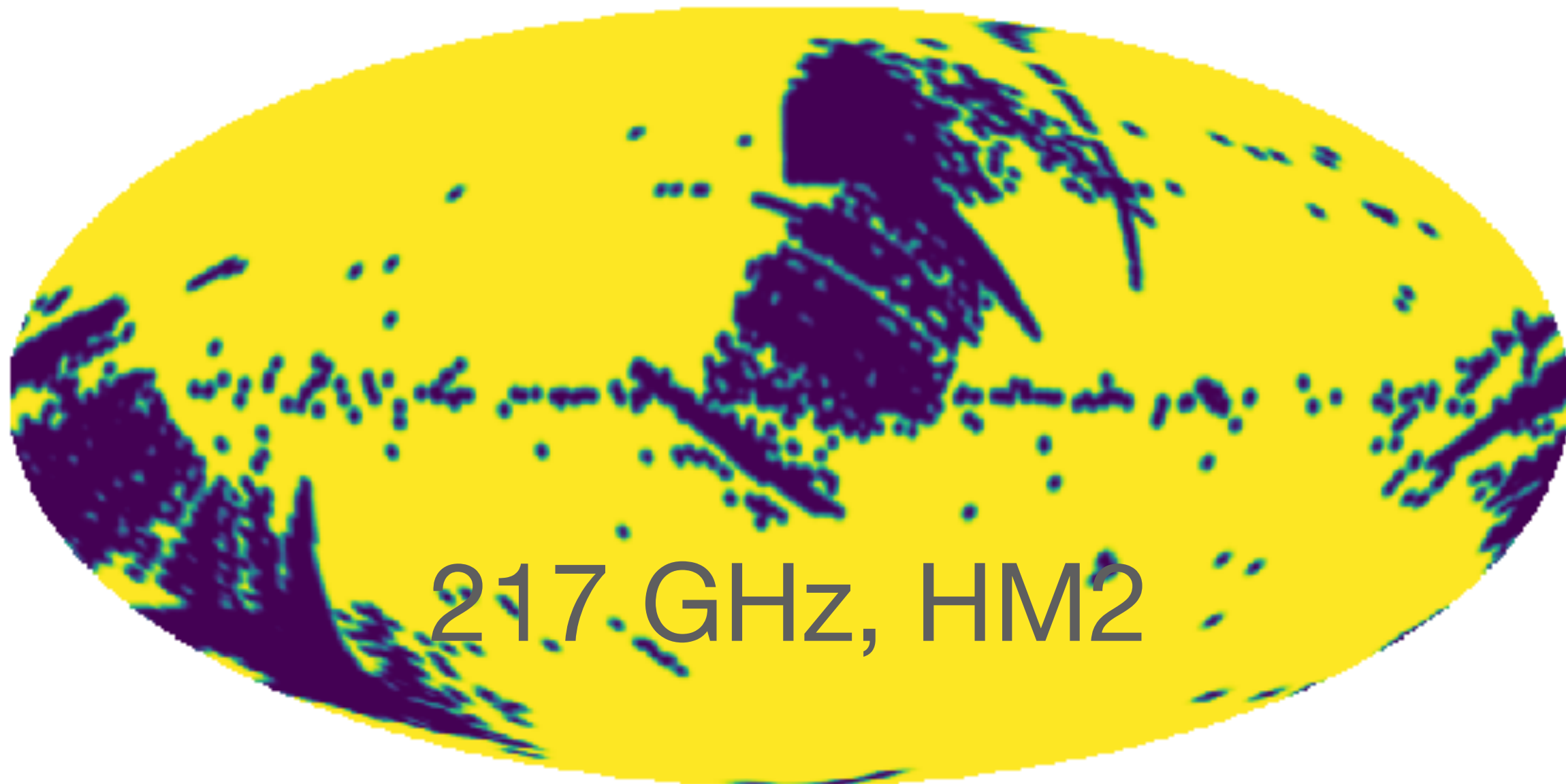
HFI_freq100_hm2_PSwithMasked_CO10p0_apo0p5deg.fits



HFI_freq143_hm2_PSwithMasked_CO10p0_apo0p5deg.fits



HFI_freq217_hm2_PSwithMasked_CO10p0_apo0p5deg.fits



HFI_freq353_hm2_PSwithMasked_CO10p0_apo0p5deg.fits



Validation by FFP10

FFP10 = Planck team's "Full Focal Plane Simulation"

- There are 4 α_ν 's and one β
- 10 simulations, no foreground is included because of the treatment of the beam
 - α -only fit: $\alpha_\nu = \{-0.008 \pm 0.047, 0.013 \pm 0.033, 0.017 \pm 0.065, 0.14 \pm 0.41\}$ deg
for $\nu \in \{100, 143, 217, 353\}$ GHz
 - β -only fit: $\beta = 0.010 \pm 0.030$ deg
- No bias found. The test passed.

Main Results

$\beta > 0$ at 2.4σ

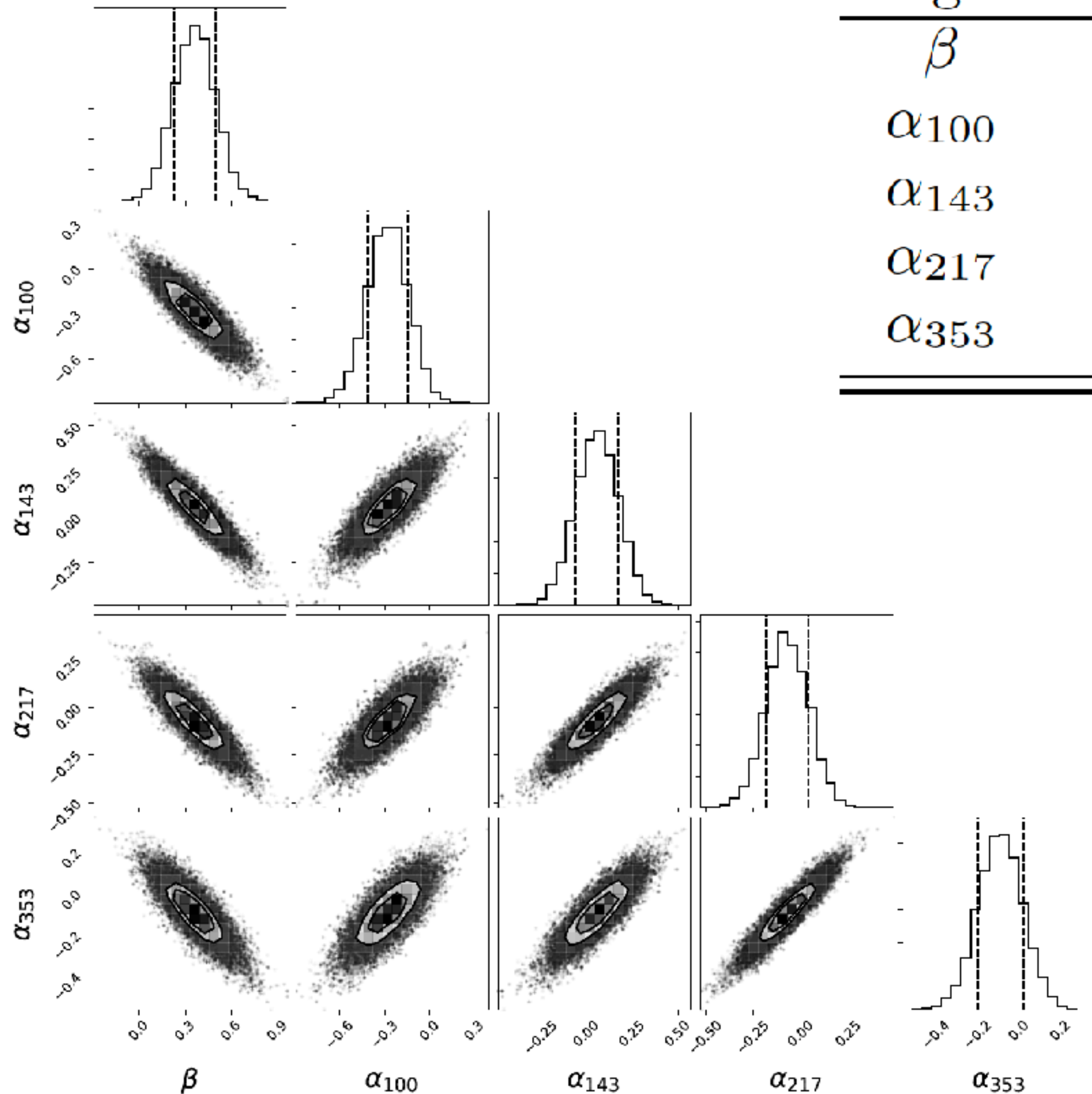
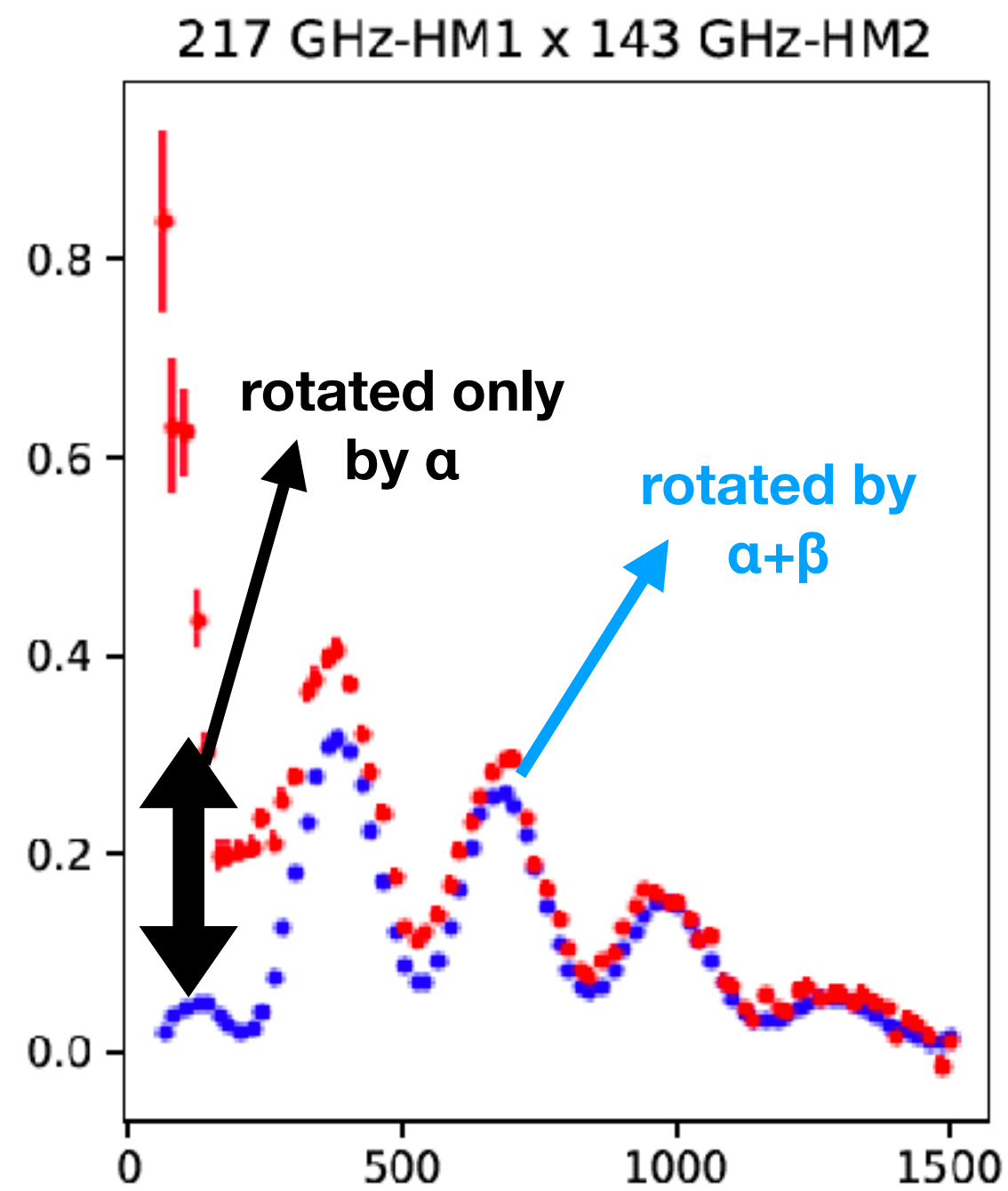
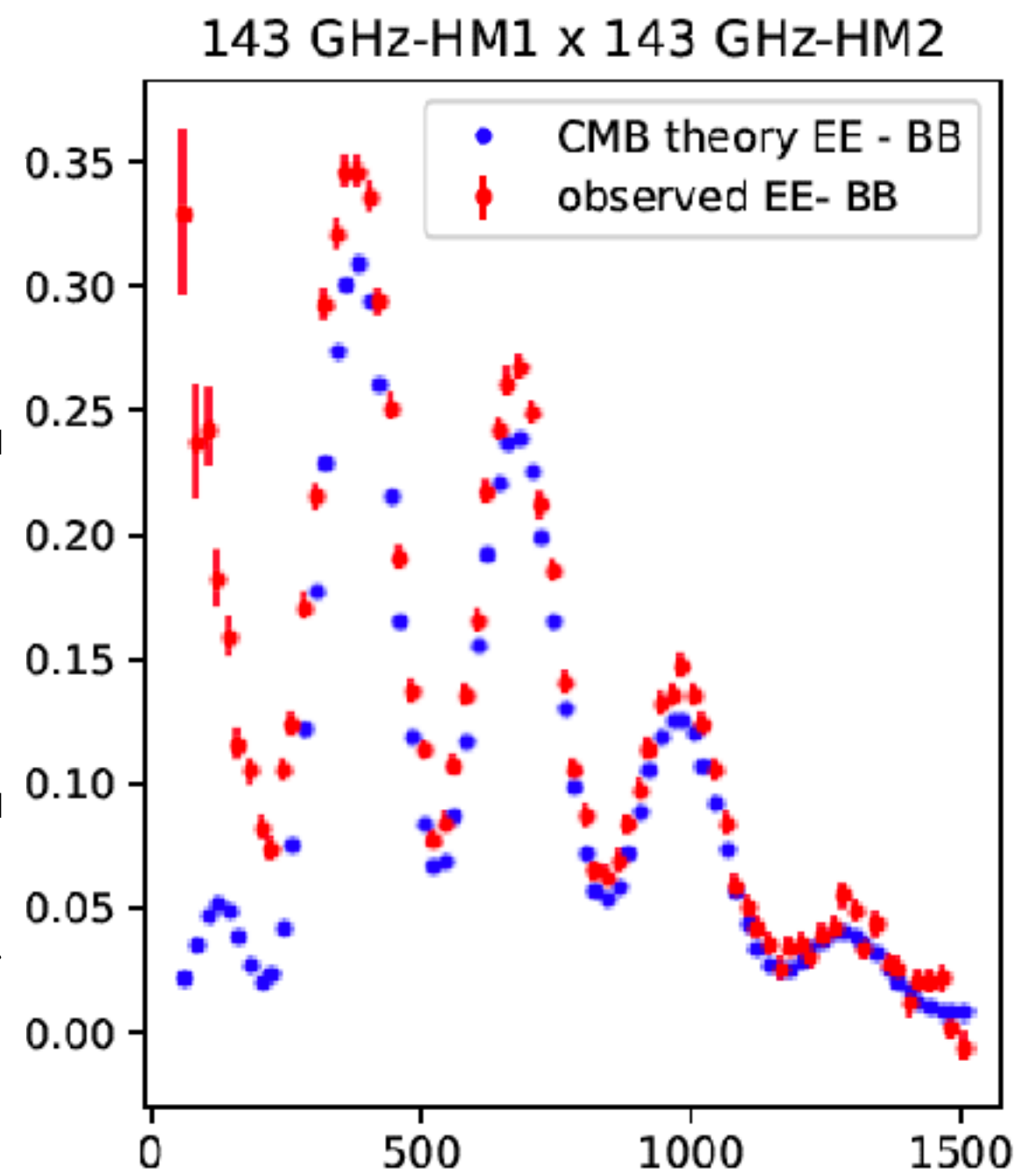


TABLE I. Cosmic birefringence and miscalibration angles from the Planck 2018 polarization data with 1σ (68%) uncertainties

Angles	$\alpha_v=0$	Results (deg)
β	0.289 ± 0.048	0.35 ± 0.14
α_{100}		-0.28 ± 0.13
α_{143}		0.07 ± 0.12
α_{217}		-0.07 ± 0.11
α_{353}		-0.09 ± 0.11

- All α_v 's are consistent with zero either statistically, or within the ground calibration error of 0.28 deg.
- Removing 100 GHz did not change β
- $\beta=0.35$ deg also agrees well with the Planck determination assuming $\alpha_v=0$:
 - $\beta(\alpha_v=0) = 0.29 \pm 0.05$ (stat. from EB) ± 0.28 (syst.) [Planck Int. XLIX]

$\ell(C_\ell^{EE} - C_\ell^{BB}) [\mu\text{K}^2]$



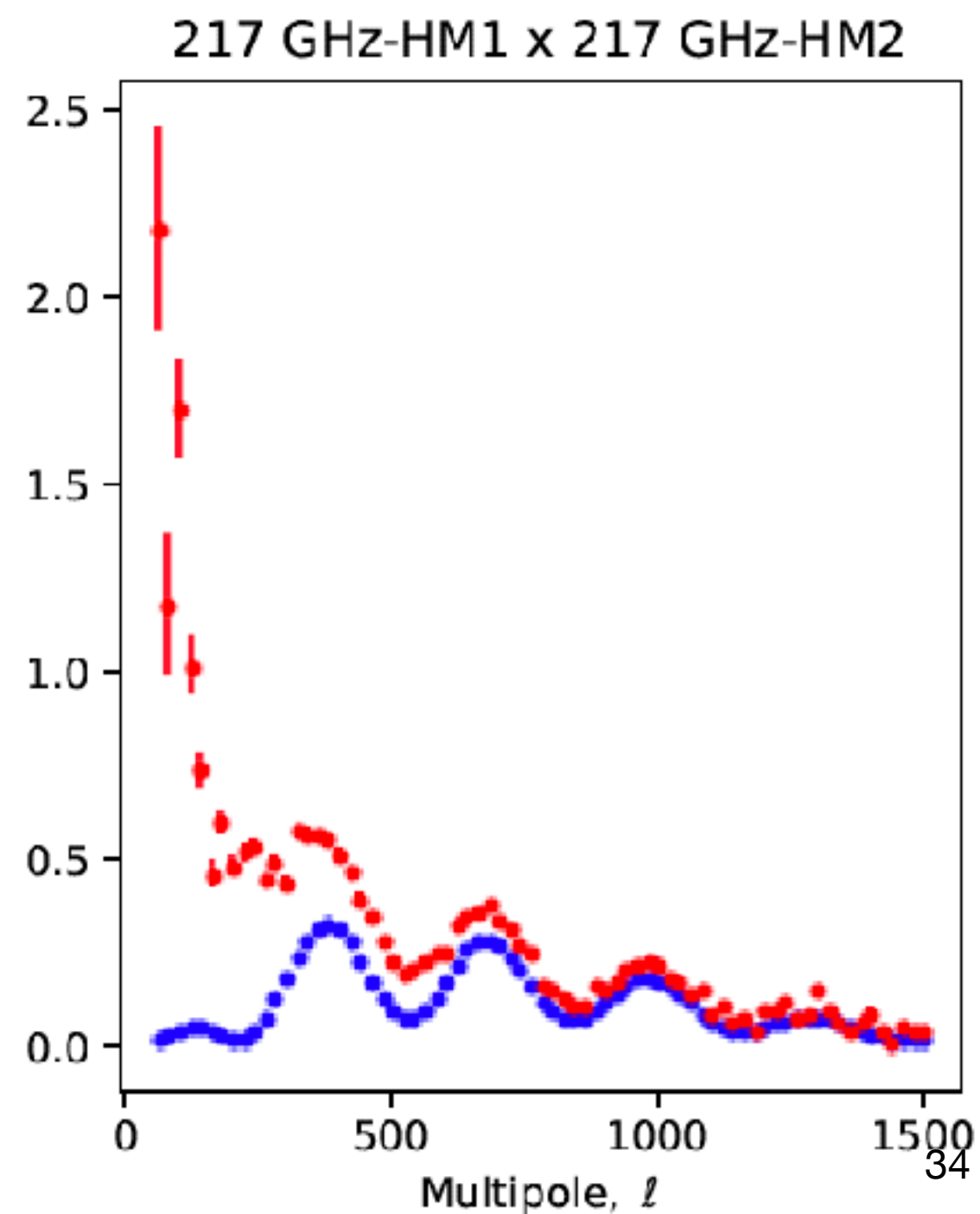
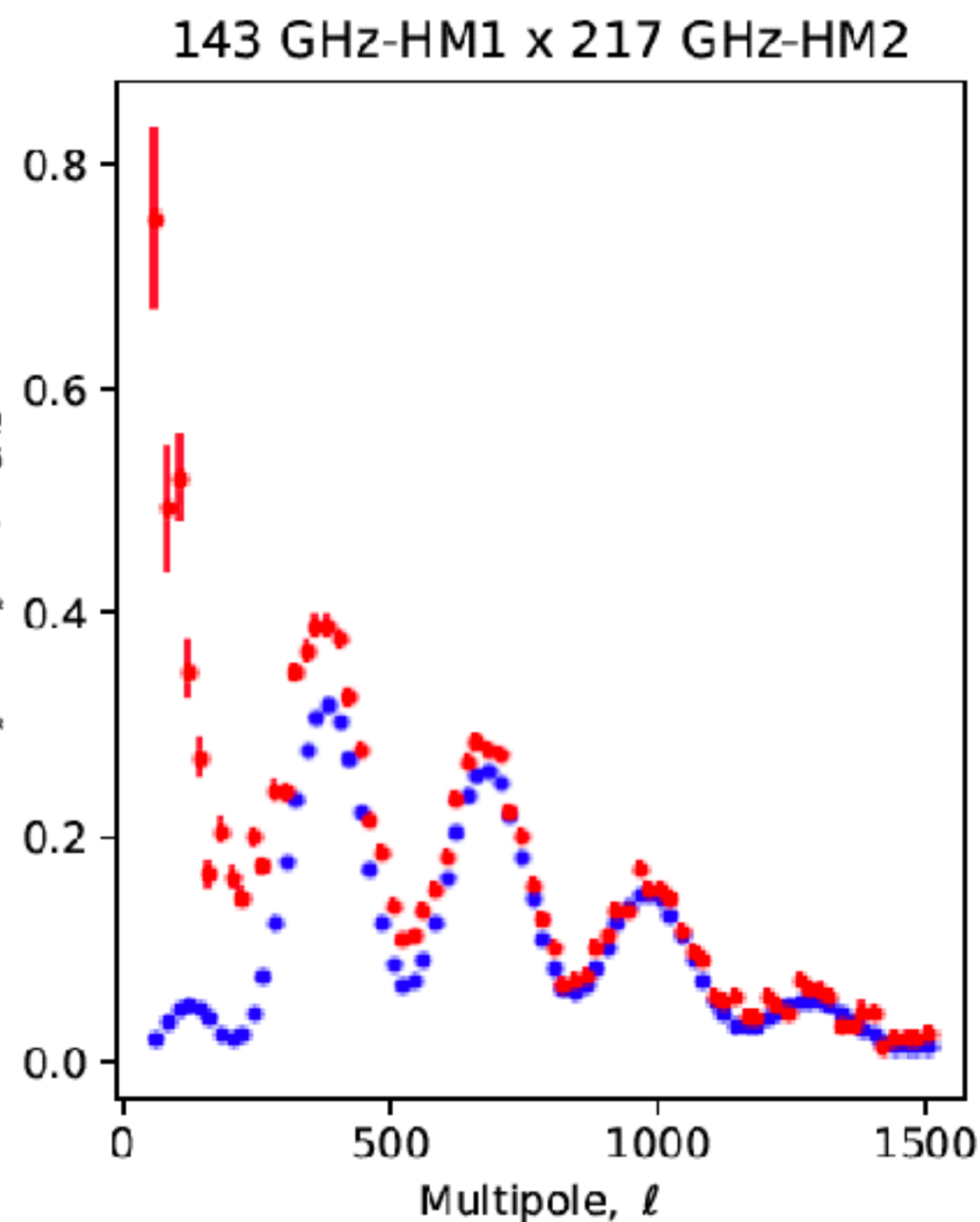
$$\langle C_\ell^{EB,o} \rangle = \frac{\tan(4\alpha)}{2} \left(\langle C_\ell^{EE,o} \rangle - \langle C_\ell^{BB,o} \rangle \right) + \frac{\sin(4\beta)}{2 \cos(4\alpha)} \left(\langle C_\ell^{EE,CMB} \rangle - \langle C_\ell^{BB,CMB} \rangle \right)$$

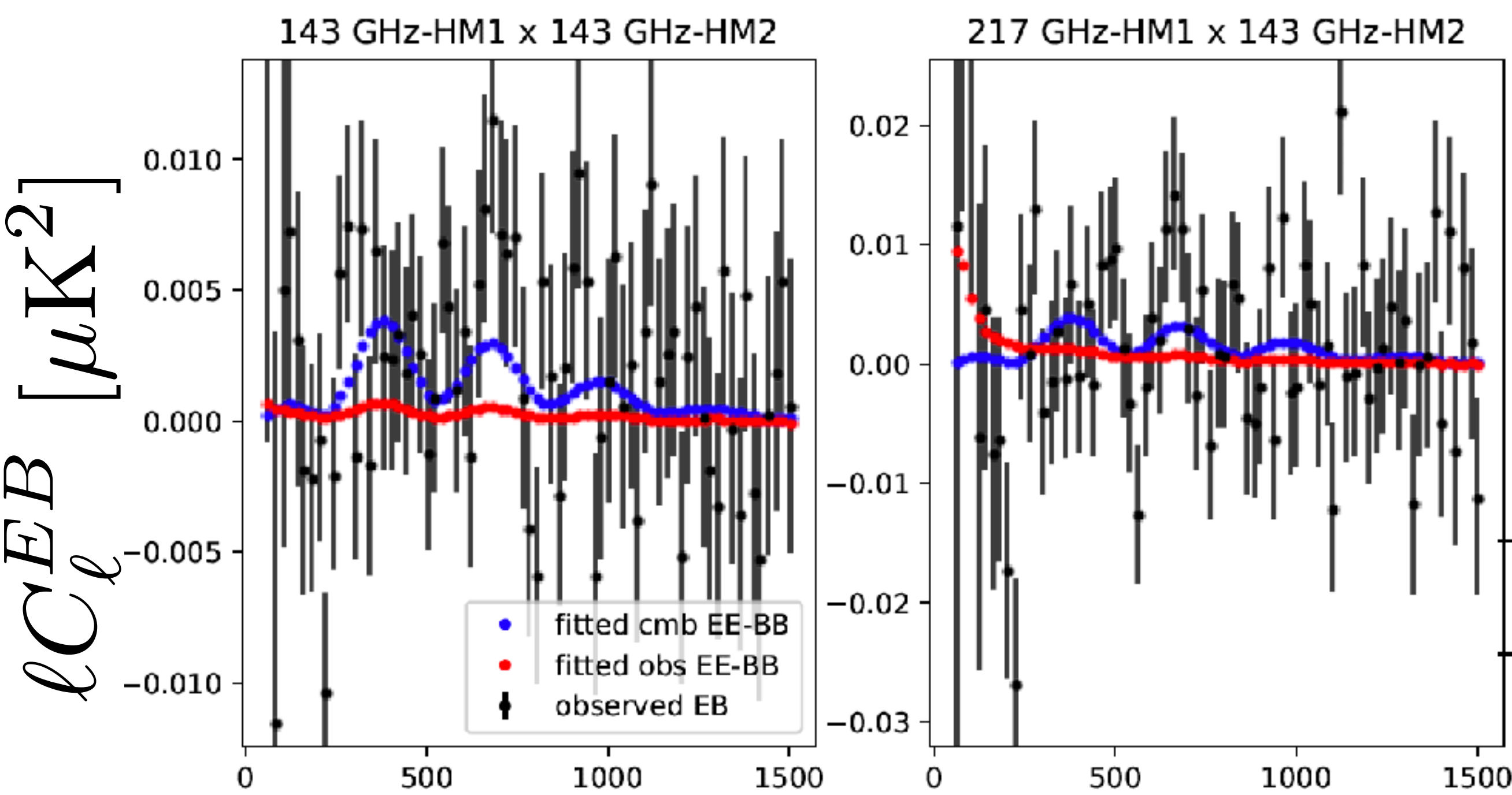
- Can we see $\beta = 0.35 \pm 0.14$ deg by eyes?
- First, take a look at the observed EE–BB spectra.

- **Red: Total**

- **Blue: The best-fitting CMB model**

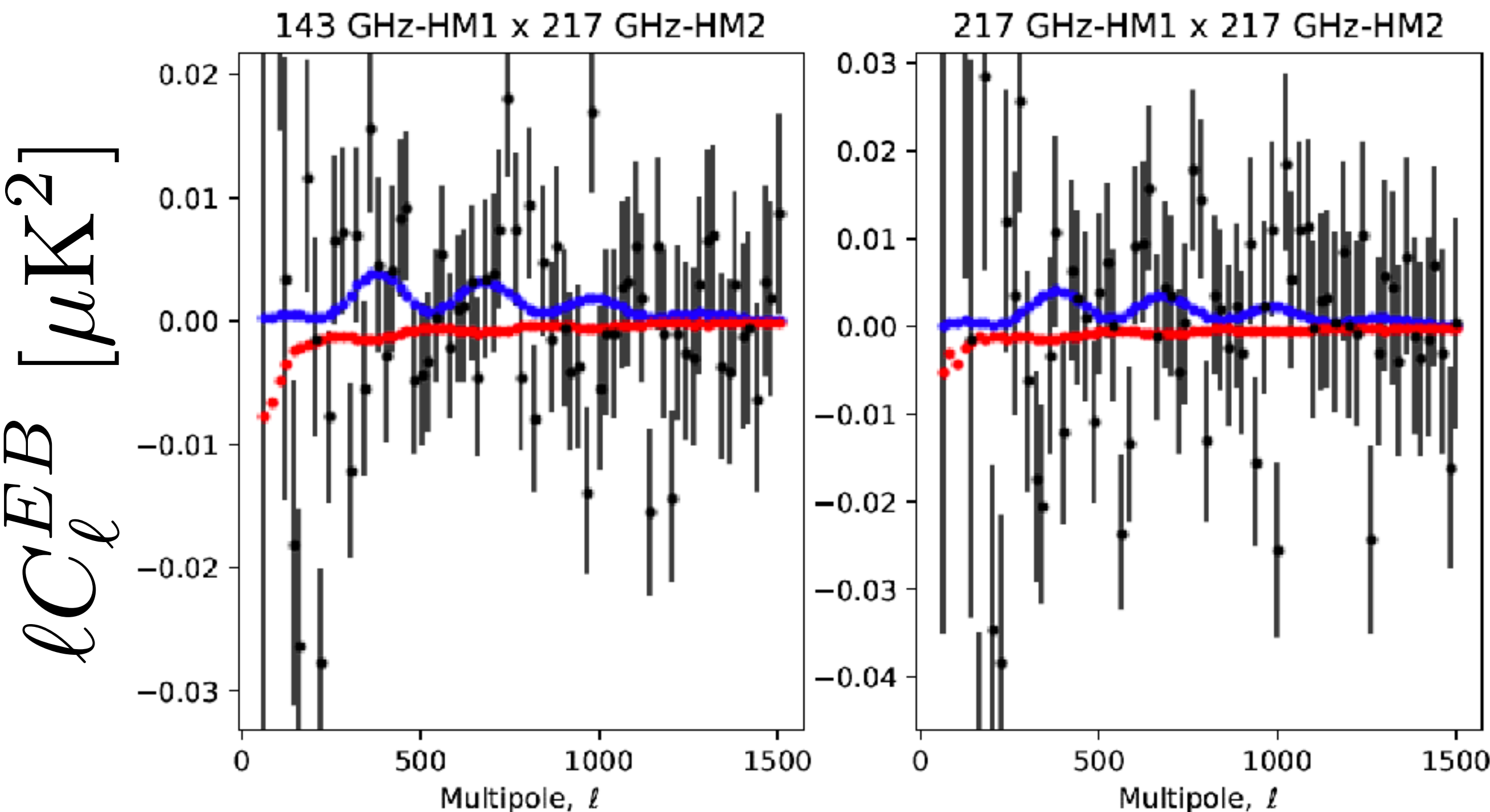
- *The difference is due to the FG (and potentially systematics)*





$$\langle C_\ell^{EB,o} \rangle = \frac{\tan(4\alpha)}{2} \left(\langle C_\ell^{EE,o} \rangle - \langle C_\ell^{BB,o} \rangle \right) + \frac{\sin(4\beta)}{2 \cos(4\alpha)} \left(\langle C_\ell^{EE,CMB} \rangle - \langle C_\ell^{BB,CMB} \rangle \right)$$

- Can we see $\beta = 0.35 \pm 0.14$ deg by eyes?
- **Red**: The signal attributed to the miscalibration angle, α_v
- **Blue**: The signal attributed to the cosmic birefringence, β
- **Red + Blue** is the best-fitting model for explaining the data points



How about the foreground EB?

- If the intrinsic foreground EB power spectrum exists, our method interprets it as a miscalibration angle α .
- Thus, $\alpha \rightarrow \alpha + \gamma$, where γ is the contribution from the intrinsic EB.
 - The sign of γ is the same as the sign of the foreground EB.
- From FG: $\alpha + \gamma$. From CMB: $\alpha + \beta$.
 - Thus, our method yields **$\beta - \gamma = 0.35 \pm 0.14$ deg.**
- There is evidence for the dust-induced $TE_{\text{dust}} > 0$ and $TB_{\text{dust}} > 0$. Then, we'd expect $EB_{\text{dust}} > 0$ (Huffenberger et al. 2020), i.e., $\gamma > 0$. If so, β increases further...

Implications

What does it mean for your models of dark matter and energy?

- When the Lagrangian density includes a Chern-Simons coupling between a pseudo scalar field and the electromagnetic tensor given by

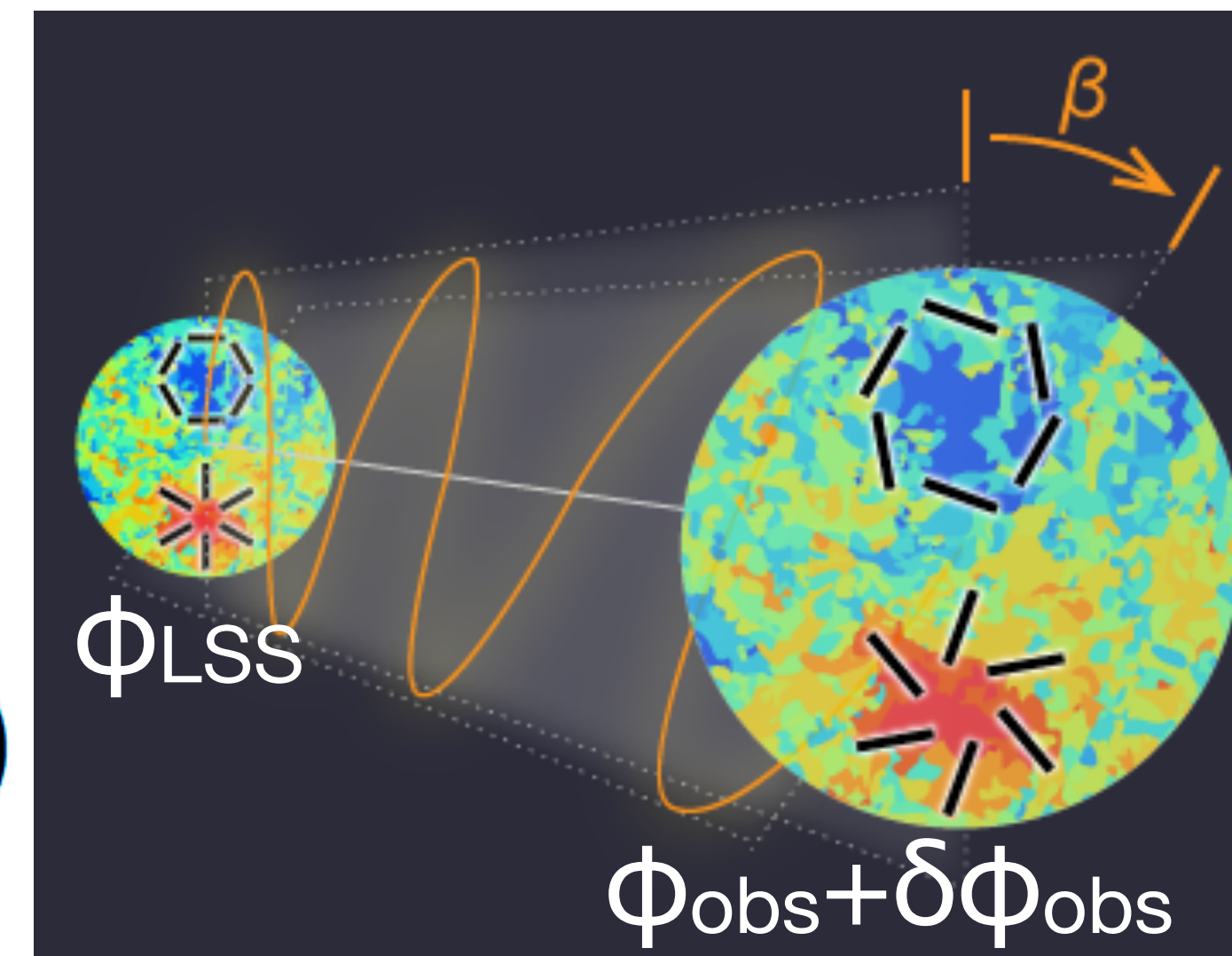
$$\mathcal{L} \supset \frac{1}{4} g_{\phi\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- The birefringence angle is

$$\beta = \frac{1}{2} g_{\phi\gamma} (\bar{\phi}_{\text{obs}} - \bar{\phi}_{\text{LSS}} + \delta\phi_{\text{obs}})$$

- Our measurement yields

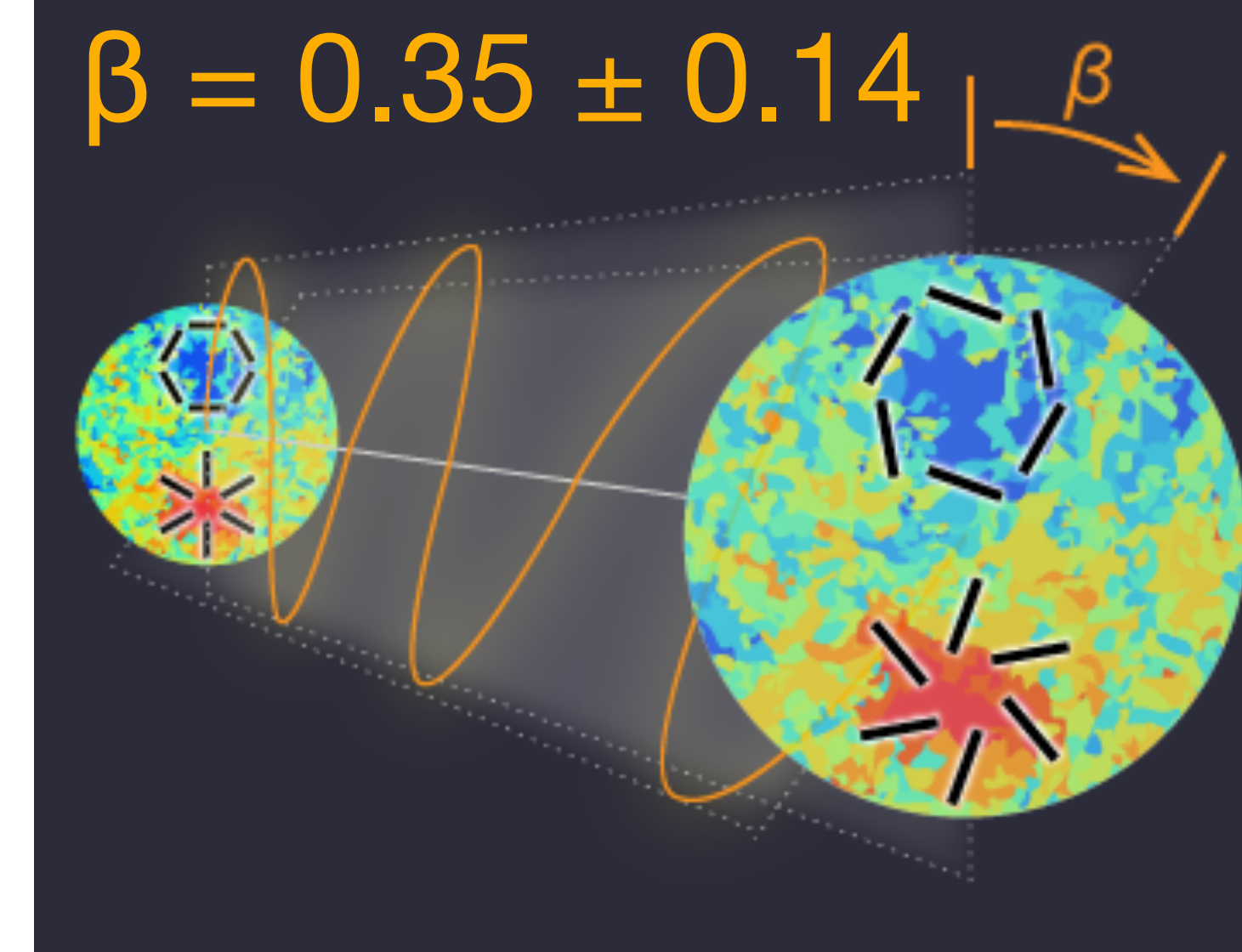
$$g_{\phi\gamma} (\bar{\phi}_{\text{obs}} - \bar{\phi}_{\text{LSS}} + \delta\phi_{\text{obs}}) = (1.2 \pm 0.5) \times 10^{-2} \text{ rad}.$$



Conclusion

$$\beta = 0.35 \pm 0.14 \text{ (68\%CL)}$$

- We perfectly understand what 2.4σ means!
 - Higher statistical significance is needed to confirm this signal.
- Our new method finally allowed us to make this “impossible” measurement, which may point to new physics.
 - Our method can be applied to any of the existing and future CMB experiments.
 - The confirmation (or otherwise) of the signal should be possible immediately.
- If confirmed, it would have important implications for dark matter/energy.



Back-up Slides

Likelihood

Partial-sky, Multi-frequency extension

$$\ln \mathcal{L} = -\frac{1}{2} \sum_{\ell=2}^{\ell_{\max}} \left(\mathbf{A} \vec{C}_{\ell}^0 - \mathbf{B} \vec{C}_{\ell}^{\text{CMB,th}} \right)^T \mathbf{C}^{-1} \left(\mathbf{A} \vec{C}_{\ell}^0 - \mathbf{B} \vec{C}_{\ell}^{\text{CMB,th}} \right),$$

- where

$$\vec{C}_{\ell}^0 = \left(C_{\ell}^{E_i E_j, 0} \quad C_{\ell}^{B_i B_j, 0} \quad C_{\ell}^{E_i B_j, 0} \right)^T$$

$$\vec{C}_{\ell}^{\text{CMB,th}} = \left(C_{\ell}^{E_i E_j, \text{CMB,th}} b_{\ell}^i b_{\ell}^j \quad C_{\ell}^{B_i B_j, \text{CMB,th}} b_{\ell}^i b_{\ell}^j \right)$$

$$\mathbf{C} = \mathbf{A} \text{Cov}(\vec{C}_{\ell}^0, \vec{C}_{\ell}^{0T}) \mathbf{A}^T \quad \text{Cov}(\vec{C}_{\ell}^{0,ij}, \vec{C}_{\ell}^{0,pqT})$$

$$= \begin{pmatrix} \text{Cov}(C_{\ell}^{E_i E_j, 0}, C_{\ell}^{E_p E_q, 0}) & \text{Cov}(C_{\ell}^{E_i E_j, 0}, C_{\ell}^{B_p B_q, 0}) & \text{Cov}(C_{\ell}^{E_i E_j, 0}, C_{\ell}^{E_p B_q, 0}) \\ \text{Cov}(C_{\ell}^{B_i B_j, 0}, C_{\ell}^{E_p E_q, 0}) & \text{Cov}(C_{\ell}^{B_i B_j, 0}, C_{\ell}^{B_p B_q, 0}) & \text{Cov}(C_{\ell}^{B_i B_j, 0}, C_{\ell}^{E_p B_q, 0}) \\ \text{Cov}(C_{\ell}^{E_i B_j, 0}, C_{\ell}^{E_p E_q, 0}) & \text{Cov}(C_{\ell}^{E_i B_j, 0}, C_{\ell}^{B_p B_q, 0}) & \text{Cov}(C_{\ell}^{E_i B_j, 0}, C_{\ell}^{E_p B_q, 0}) \end{pmatrix}$$

Likelihood

Partial-sky, Multi-frequency extension

$$\ln \mathcal{L} = -\frac{1}{2} \sum_{\ell=2}^{\ell_{\max}} \left(\mathbf{A} \vec{C}_{\ell}^0 - \mathbf{B} \vec{C}_{\ell}^{\text{CMB,th}} \right)^{\text{T}} \mathbf{C}^{-1} \left(\mathbf{A} \vec{C}_{\ell}^0 - \mathbf{B} \vec{C}_{\ell}^{\text{CMB,th}} \right),$$

- where

\mathbf{A} is a block diagonal matrix of $\begin{pmatrix} -\vec{R}^{\text{T}}(\alpha_i, \alpha_j) \mathbf{R}^{-1}(\alpha_i, \alpha_j) & 1 \end{pmatrix}$,

- with $\vec{R}(\theta_i, \theta_j) = \begin{pmatrix} \cos(2\theta_i) \sin(2\theta_j) \\ -\sin(2\theta_i) \cos(2\theta_j) \end{pmatrix}$

$$\mathbf{R}(\theta_i, \theta_j) = \begin{pmatrix} \cos(2\theta_i) \cos(2\theta_j) & \sin(2\theta_i) \sin(2\theta_j) \\ \sin(2\theta_i) \sin(2\theta_j) & \cos(2\theta_i) \cos(2\theta_j) \end{pmatrix}$$

Likelihood

Partial-sky, Multi-frequency extension

$$\ln \mathcal{L} = -\frac{1}{2} \sum_{\ell=2}^{\ell_{\max}} \left(\mathbf{A} \vec{C}_{\ell}^{\circ} - \mathbf{B} \vec{C}_{\ell}^{\text{CMB,th}} \right)^{\text{T}} \mathbf{C}^{-1} \left(\mathbf{A} \vec{C}_{\ell}^{\circ} - \mathbf{B} \vec{C}_{\ell}^{\text{CMB,th}} \right),$$

- where

\mathbf{B} is a block diagonal matrix of $[\vec{R}^{\text{T}}(\alpha_i + \beta, \alpha_j + \beta) - \vec{R}^{\text{T}}(\alpha_i, \alpha_j) \mathbf{R}^{-1}(\alpha_i, \alpha_j) \mathbf{R}(\alpha_i + \beta, \alpha_j + \beta)]$,

- with $\vec{R}(\theta_i, \theta_j) = \begin{pmatrix} \cos(2\theta_i) \sin(2\theta_j) \\ -\sin(2\theta_i) \cos(2\theta_j) \end{pmatrix}$
- $\mathbf{R}(\theta_i, \theta_j) = \begin{pmatrix} \cos(2\theta_i) \cos(2\theta_j) & \sin(2\theta_i) \sin(2\theta_j) \\ \sin(2\theta_i) \sin(2\theta_j) & \cos(2\theta_i) \cos(2\theta_j) \end{pmatrix}$