Model-independent tests of modified gravity models

or: Can we ever rule out dark energy?

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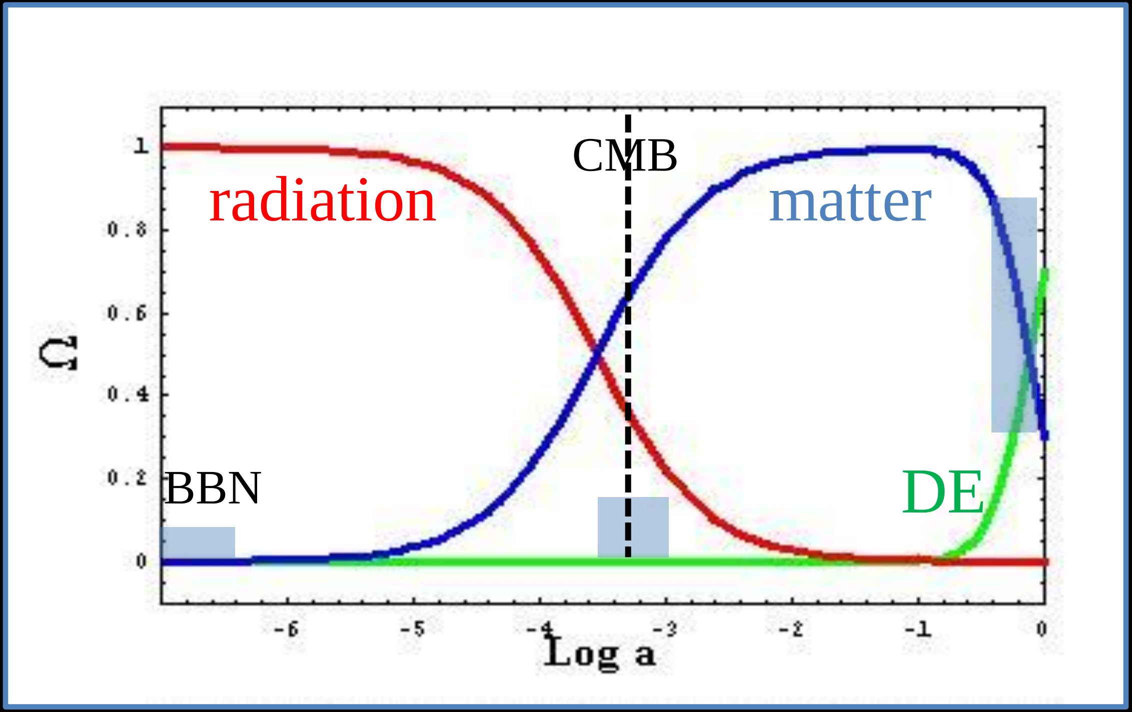
Time view



The two main problems of testing modified gravity

1) Problem of initial conditions

e.g, how do we know if the shape of the power spectrum we observe is due to dark energy or to initial conditions?

2) Problem of design

If our model parameter space is sufficiently large, we can design a model to fit any observation

Prolegomena zu einer jeder künftigen Metaphysik

jeder künftigen Dark Energy physik

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Observational requirements:

A) Isotropy

B) Large abundance

C) Slow evolution

D) Weak clustering

The past ten years of DE research

$$\int dx^4 \sqrt{-g}\left[R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter} \right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter} \right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter} \right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi,\frac{1}{2}\phi_{,\mu}\phi^{,\mu})R + G_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter} \right]$$

A quintessential scalar field

The most general 4D scalar field theory with second order equation of motion

$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)\, R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \\
\mathcal{L}_5 &= G_5(\phi, X)\, G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6} G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)\right].
\end{aligned}$$

- First found by Horndeski in 1975
- rediscovered by Deffayet et al. in 2011
- no ghosts, no classical instabilities
- it modifies gravity!
- it includes f(R), Brans-Dicke, k-essence, Galileons, clustering DE etc etc etc

Simplest modified gravity: f(R)

The simplest Horndeski model which still produces
a modified gravity: f(R)

$$\int dx^4 \sqrt{g}\left[f(R) + L_{matter}\right]$$

- equivalent to a Horndeski Lagrangian without kinetic terms
- easy to produce acceleration (first inflationary model)
- high-energy corrections to gravity likely to introduce higher-order terms
- particular case of scalar-tensor and extra-dimensional theory

The next ten years of DE research

Combine observations of background, linear and non-linear perturbations to reconstruct as much as possible the Horndeski model

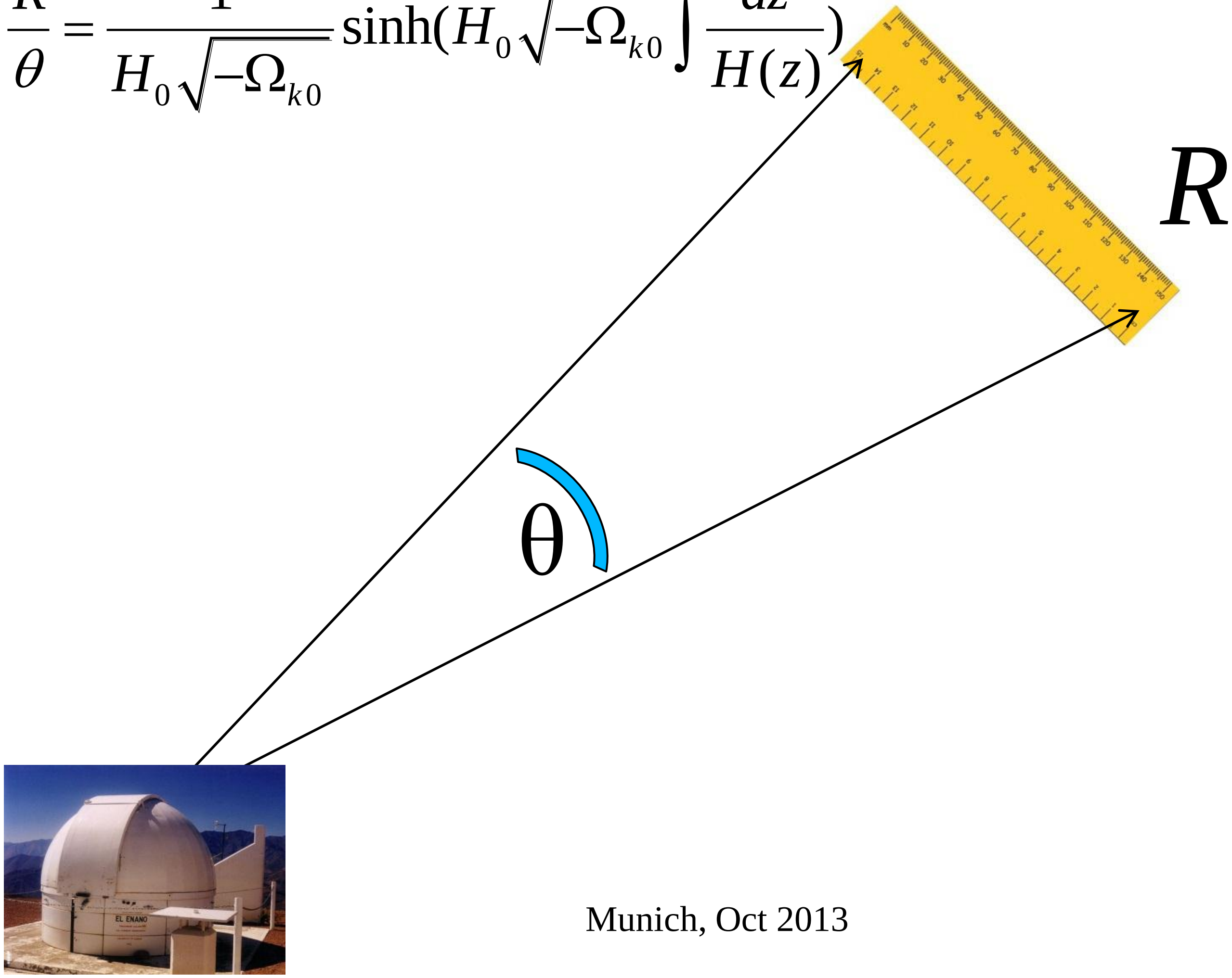
… or to rule it out!

The great Horndeski Hunt

Let us assume we have only
1) a perturbed FRW metric
2) pressureless matter
3) the Horndeski field
and

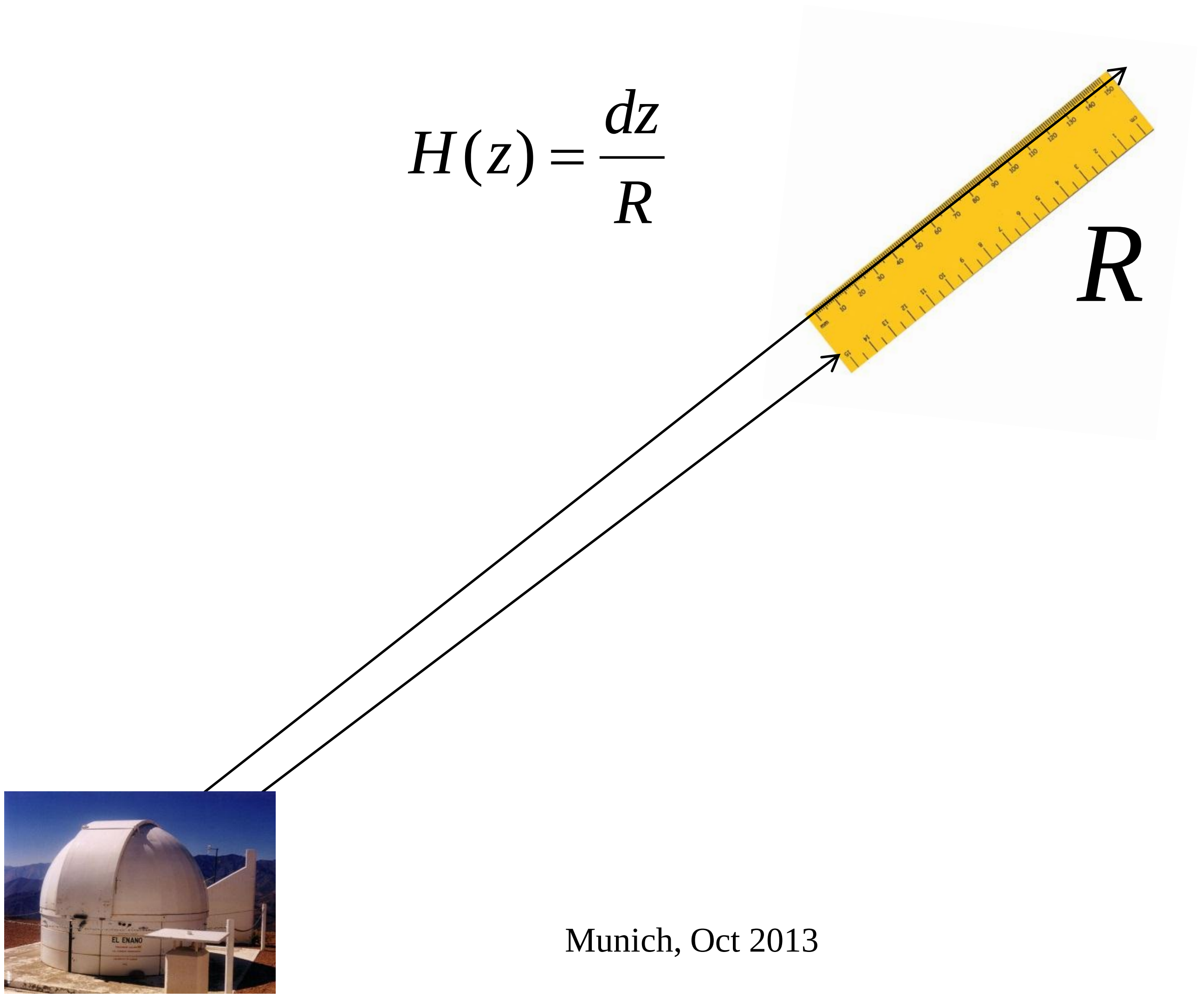
Standard rulers

$$D(z) = \frac{R}{\theta} = \frac{1}{H_0\sqrt{-\Omega_{k0}}}\sinh\left(H_0\sqrt{-\Omega_{k0}}\int\frac{dz}{H(z)}\right)$$

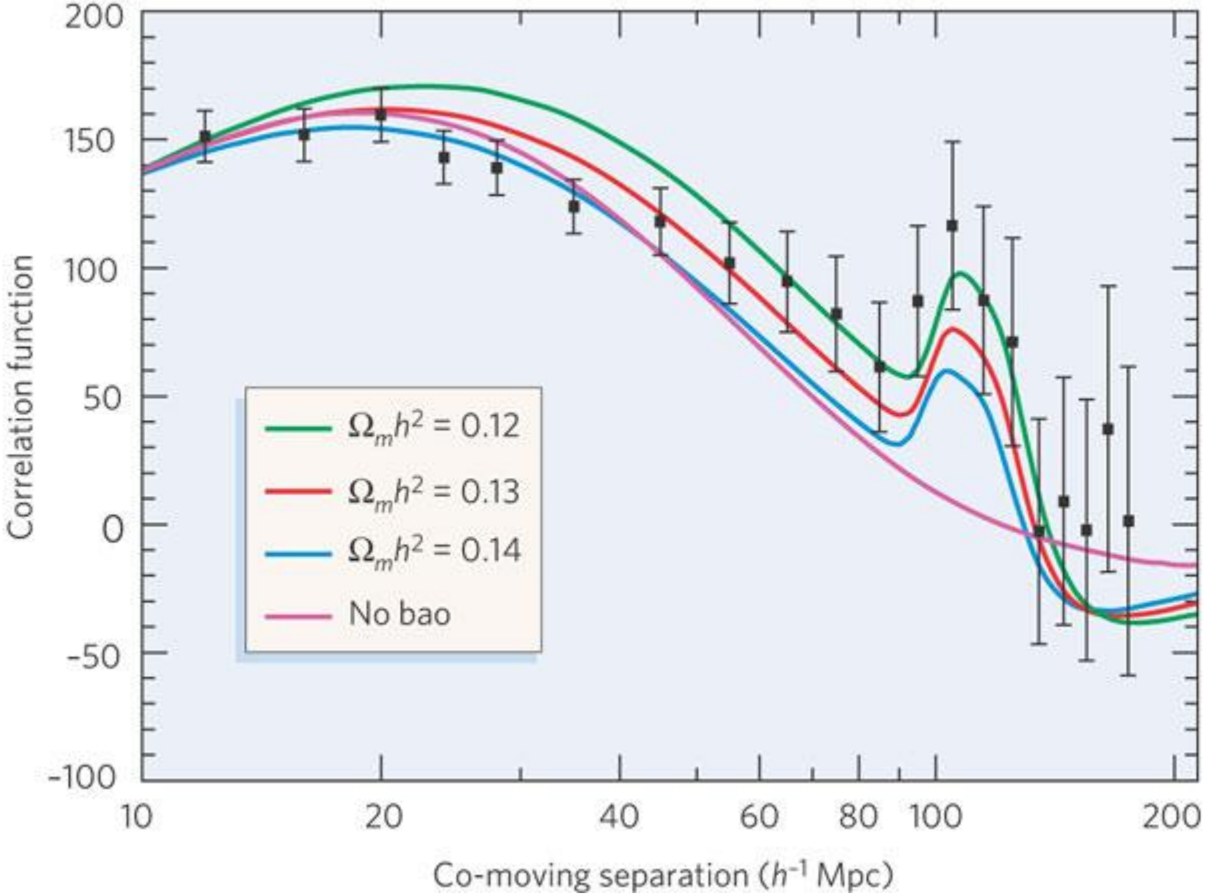


Standard rulers

$$H(z) = \frac{dz}{R}$$



BAO ruler



Charles L. Bennett

Nature 440, 1126-1131(27 April 2006)

Background: SNIa, BAO, …

Then we can measure H(z) and

$$D(z) = \frac{1}{H_0\sqrt{-\Omega_{k0}}}\sinh(H_0\sqrt{-\Omega_{k0}}\int\frac{dz}{H(z)})$$

and therefore Ω_{k0}

$$\Omega_x = 1 - \Omega_k - \Omega_m = 1 - \frac{H_0^2}{H^2}(\Omega_{k0}a^{-2} + \Omega_{m0}a^{-3})$$

Then we can measure everything up to Ω_{m0}

Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation $$\nabla^2\Psi = 4\pi G\rho_m\delta_m$$

- anisotropic stress $$1 = -\frac{\Psi}{\Phi}$$

Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation $$\nabla^2\Psi = 4\pi G\, Y(k,a)\rho_m \delta_m$$
- anisotropic stress $$\eta(k,a) = -\frac{\Psi}{\Phi}$$

Modified Gravity at the linear level

▪ standard gravity	$Y(k,a)=1$ $\eta(k,a)=1$	
▪ scalar-tensor models	$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{2(F+F'^2)}{2F+3F'^2}$ $\eta(a)=1+\frac{F'^2}{F+F'^2}$	Boisseau et al. 2000 Acquaviva et al. 2004 Schimd et al. 2004 L.A., Kunz &Sapone 2007
▪ f(R)	$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{1+4m\frac{k^2}{a^2R}}{1+3m\frac{k^2}{a^2R}},\quad \eta(a)=1+\frac{m\frac{k^2}{a^2R}}{1+2m\frac{k^2}{a^2R}}$	Bean et al. 2006 Hu et al. 2006 Tsujikawa 2007
▪ DGP	$Y(a)=1-\frac{1}{3\beta};\quad \beta=1+2Hr_c w_{DE}$ $\eta(a)=1+\frac{2}{3\beta-1}$	Lue et al. 2004; Koyama et al. 2006
▪ massive bi-gravity	$Y(a)=....$ $\eta(a)=....$	see L. A.,et al in prep. 2013

Modified Gravity at the linear level

In the quasi-static limit, every Horndeski model is characterized at linear scales by the two functions

$$\eta(k,a) = h_2 \left(\frac{1+k^2 h_4}{1+k^2 h_5} \right)$$

$$Y(k,a) = h_1 \left(\frac{1+k^2 h_5}{1+k^2 h_3} \right)$$

k = wavenumber

h_i = time-dependent functions

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

Modified Gravity at the linear level

$$h_1 \equiv \frac{w_4}{w_1^2} = \frac{c_T^2}{w_1}, \qquad h_2 \equiv \frac{w_1}{w_4} = c_T^{-2},$$

$$h_3 \equiv \frac{H^2}{2XM^2} \frac{2w_1^2 w_2 H - w_2^2 w_4 + 4w_1 w_2 \dot{w}_1 - 2w_1^2(\dot{w}_2 + \rho_m)}{2w_1^2},$$

$$h_4 \equiv \frac{H^2}{2XM^2} \frac{2w_1^2 H^2 - w_2 w_4 H + 2w_1 \dot{w}_1 H + w_2 \dot{w}_1 - w_1(\dot{w}_2 + \rho_m)}{w_1},$$

$$h_5 \equiv \frac{H^2}{2XM^2} \frac{2w_1^2 H^2 - w_2 w_4 H + 4w_1 \dot{w}_1 H + 2\dot{w}_1{}^2 - w_4(\dot{w}_2 + \rho_m)}{w_4},$$

$$\begin{aligned}
w_1 \equiv& 1 + 2\left(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}XHG_{5,X}\right), \\
w_2 \equiv& -2\dot{\phi}\left(XG_{3,X} - G_{4,\phi} - 2XG_{4,\phi X}\right) + \\
&+ 2H\left(w_1 - 4X\left(G_{4,X} + 2XG_{4,XX} - G_{5,\phi} - XG_{5,\phi X}\right)\right) - \\
&- 2\dot{\phi}XH^2\left(3G_{5,X} + 2XG_{5,XX}\right), \\
w_3 \equiv& 3X\left(K_{,X} + 2XK_{,XX} - 2G_{3,\phi} - 2XG_{3,\phi X}\right) + 18\dot{\phi}XH\left(2G_{3,X} + XG_{3,XX}\right) - \\
&- 18\dot{\phi}H\left(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2 G_{4,\phi XX}\right) - \\
&- 18H^2\left(1 + G_4 - 7XG_{4,X} - 16X^2 G_{4,XX} - 4X^3 G_{4,XXX}\right) - \\
&- 18XH^2\left(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2 G_{5,\phi XX}\right) + \\
&+ 6\dot{\phi}XH^3\left(15G_{5,X} + 13XG_{5,XX} + 2X^2 G_{5,XXX}\right), \\
w_4 \equiv& 1 + 2\left(G_4 - XG_{5,\phi} - XG_{5,X}\ddot{\phi}\right).
\end{aligned}$$

De Felice et al. 2011; L.A. et al.,PRD, arXiv:1210.0439, 2012

Quasi-static approximation

$$c_s{}^2 k^2 \gg a^2 H^2$$

From a wave equation:

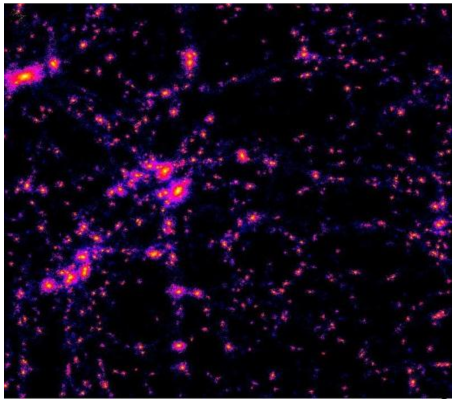
$$E_{\delta\phi} \equiv D_1\ddot{\Phi} + D_2\ddot{\delta\phi} + D_3\dot{\Phi} + D_4\dot{\delta\phi} + D_5\dot{\Psi} + D_6\frac{k^2}{a^2}\dot{\chi} + \left(D_7\frac{k^2}{a^2} + D_8\right)\Phi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + \left(D_{10}\frac{k^2}{a^2} + D_{11}\right)\Psi + D_{12}\frac{k^2}{a^2}\chi = 0\,,$$

To a "Poisson" equation:

$$B_7\frac{k^2}{a^2}\Phi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + A_6\frac{k^2}{a^2}\Psi \simeq 0\,,$$

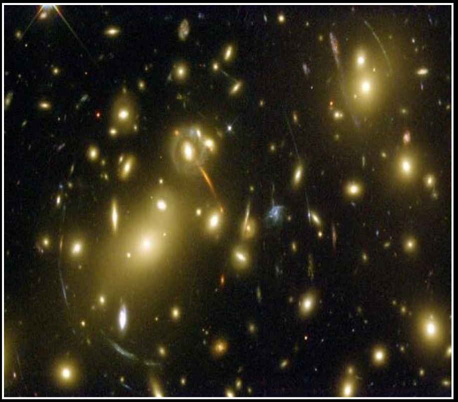
Reconstruction of the metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$



Non-relativistic particles respond to Ψ

$$\delta_m'' + (1 + \frac{\mathcal{H}'}{\mathcal{H}})\delta_m' = -k^2\Psi$$



Relativistic particles respond to Φ-Ψ

$$\alpha = \int \nabla_{perp}(\Psi - \Phi)dz$$

Deconstructing the galaxy power spectrum

Galaxy clustering

Redshift distortion

Line of sight angle

$$\delta_{gal}(k,z,\mu) = Gb\sigma_8(1+\frac{f}{b}\mu^2)\delta_{t,0}(k)$$

$$\left(f = \frac{\delta'}{\delta}\right)$$

Growth function

Galaxy bias

Present mass power spectrum

Three linear observables: A, R, L

clustering

μ=0
Amplitude
A

$$\delta_{gal}(k,z,0) = Gb\sigma_8\delta_{t,0}(k) \equiv A$$

μ=1
Redshift distortion
R

$$\delta_{gal}(k,z,1) = G\sigma_8 f\,\delta_{t,0}(k) \equiv R$$

lensing

Lensing
L

$$k^2\Phi_{lens} = k^2(\Psi - \Phi) = -\frac{3}{2}\Sigma G\Omega_m\sigma_8\delta_{t,0}(k) \equiv L$$

$$\Sigma = Y(1+\eta)$$

The only model-independent ratios

Redshift distortion/Amplitude

$$P_1 = \frac{R}{A} = \frac{f}{b}$$

Lensing/Redshift distortion

$$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1+\eta)}{f}$$

Redshift distortion rate

$$P_3 = \frac{R'}{R} = \frac{f'}{f} + f$$

Expansion rate

$$E = \frac{H}{H_0}$$

How to combine them to test the theory?

Summarizing....

Matter conservation equation independent of gravity theory

$$\delta_m'' + (1 + \frac{\mathcal{H}'}{\mathcal{H}})\delta_m' = -k^2\Psi$$

Observables

$$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1+\eta)}{f} \qquad P_3 = \frac{R'}{R} = \frac{f'}{f} + f \qquad E = \frac{H}{H_0}$$

The anisotropic stress is directly observable

A unique combination of model independent observables

$$\frac{3P_2(1+z)^3}{2E^2\left(P_3+2+\frac{E'}{E}\right)}-1=\eta$$

Observables

Theory

Testing the entire Horndeski Lagrangian

A unique combination of model independent observables

$$\frac{3P_2(1+z)^3}{2E^2\left(P_3+2+\frac{E'}{E}\right)}-1=\eta=h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

Observables

Theory

L.A. et al. 1210.0439

Horndeski Lagrangian: not too big to fail

$$g(z,k) \equiv \frac{(REa^2)'}{LEa^2}$$

$$2g_{,k}g_{,kkk} - 3(g_{,kk})^2 = 0$$

If this relation is falsified, the Horndeski theory is rejected*

L.A., M. Motta, I. Sawicki,
M. Kunz, I. Saltas, 1210.0439

Beyond the quasi-static condition

General consistency relation

$$\eta\Gamma' + \eta'' + \Gamma\left(\eta\Gamma + 2\eta' + \bar{\alpha}_1\eta - \bar{\alpha}_2\right) +$$
$$+ \bar{\alpha}_1\eta' + \bar{\alpha}_3\eta - \bar{\alpha}_5 + k^2\left(\bar{\alpha}_4\eta - \bar{\alpha}_6\right) = \bar{\alpha}_7\varpi \,.$$

$$\Gamma \equiv \frac{\Psi'}{\Psi} = \frac{L'}{L} - \frac{\eta'}{1+\eta} - 1\,,$$

$$\varpi = \frac{1+\eta}{P_2}$$

M. Motta, L.A, I. Sawicki,
M. Kunz, I. Saltas, 2013

Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy

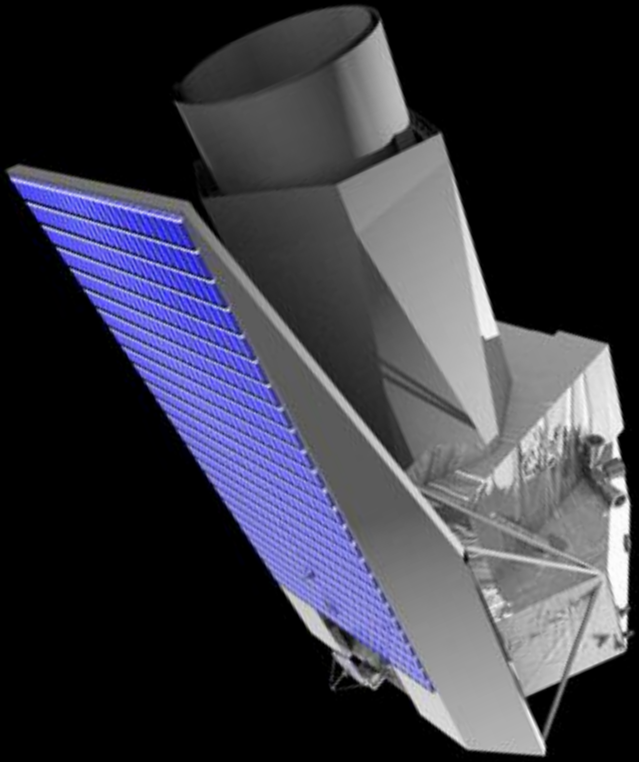
15,000 square degrees

100 million redshifts, 2 billion images

Median redshift z = 1

PSF FWHM ~0.18’’

>1000 peoples, >10 countries



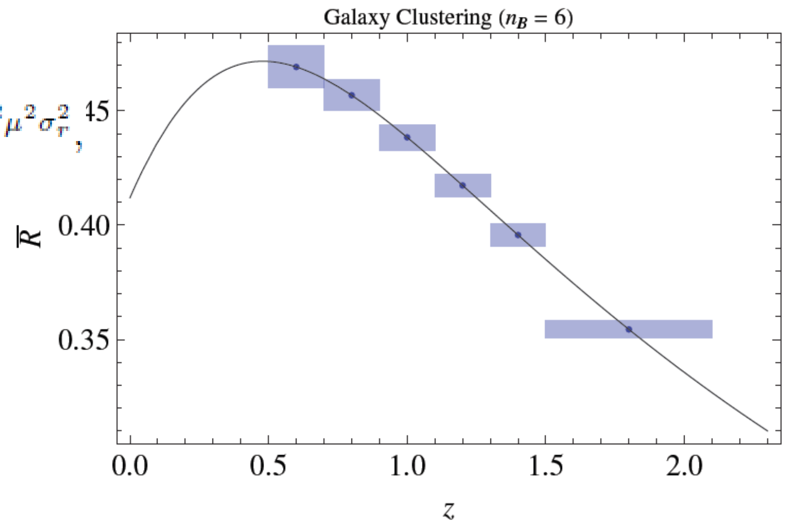
Euclid satellite

arXiv Red Book 1110.3193

arXiv Theory Review 1206.1225

Euclid forecasts…

$$P(k,\mu) = (A + R\mu^2)^2 e^{-k^2\mu^2\sigma_r^2} = (\bar{A} + \bar{R}\mu^2)^2 \delta_{t,0}^2(k) e^{-k^2\mu^2\sigma_r^2},$$



$$F_{\alpha\beta}^{\rm GC} = \frac{1}{8\pi^2}\int_{-1}^{1} {\rm d}\mu \int_{k_{\rm min}}^{k_{\rm max}} k^2 V_{\rm eff} D_\alpha D_\beta \, {\rm d}k,$$

$$D_\alpha \equiv \frac{d \log P}{dp_\alpha},$$

$$D_{\bar{A}} = \frac{2}{\bar{A} + \bar{R}\mu^2},$$

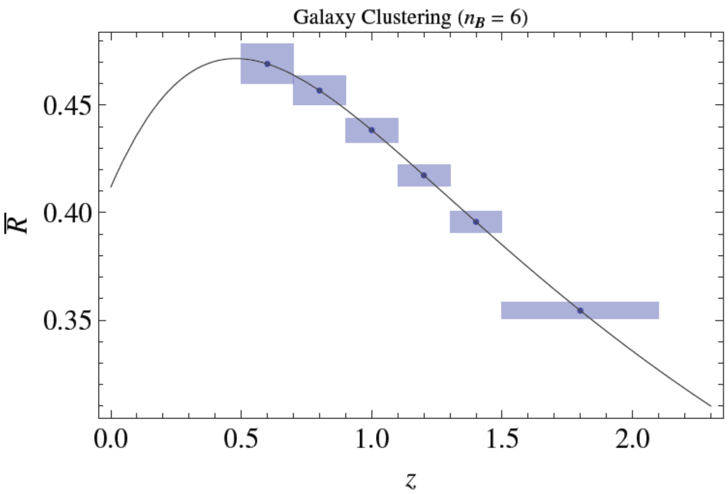
$$D_{\bar{R}} = \frac{2\mu^2}{\bar{A} + \bar{R}\mu^2},$$

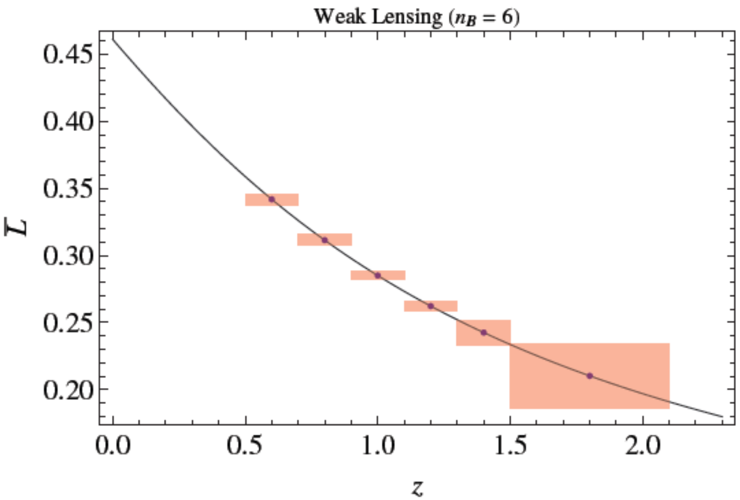
$\bar{z}$	$\bar{n}(\bar{z}) \times 10^{-3}$	$\bar{A}(\bar{z})$	$\Delta\bar{A}$	$\Delta\bar{A}(\%)$	$\bar{R}(\bar{z})$	$\Delta\bar{R}$	$\Delta\bar{R}(\%)$	$E(\bar{z})$	ΔE	$\Delta E(\%)$
0.6	3.56	0.612	0.0022	0.37	0.469	0.0092	2.0	1.37	0.12	8.5
0.8	2.42	0.558	0.0017	0.3	0.457	0.0068	1.5	1.53	0.073	4.8
1.0	1.81	0.511	0.0015	0.29	0.438	0.0056	1.3	1.72	0.058	3.4
1.2	1.44	0.47	0.0014	0.29	0.417	0.0049	1.2	1.92	0.05	2.6
1.4	0.99	0.434	0.0015	0.35	0.396	0.0047	1.2	2.14	0.051	2.4
1.8	0.33	0.377	0.0018	0.47	0.354	0.0039	1.1	2.62	0.061	2.3

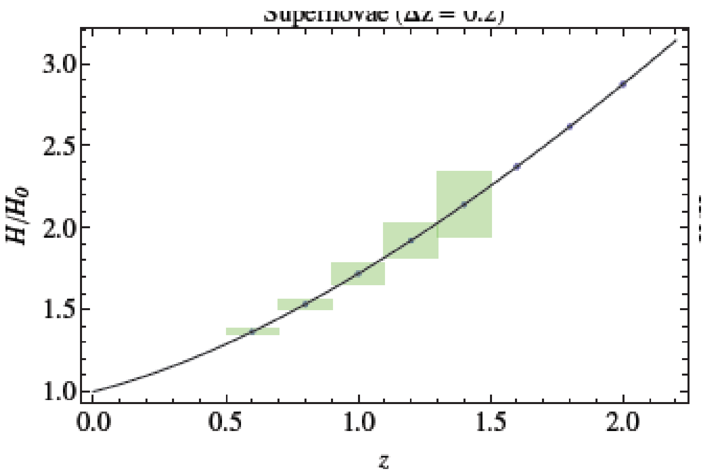
L.A, M. Kunz, A. Vollmer,
A. Trilleras, S. Fogli, work in progress, 2013

Euclid forecasts…

Combining galaxy clustering, weak lensing and SN….







100k SN (LSST)

Results..

Model 1: η constant for all z, k
Error on η around 5%

Model 2: η varies in z
Error on η

$\bar{z}$	$P_1(\bar{z})$	ΔP_1	$\Delta P_1(\%)$	$P_2(\bar{z})$	ΔP_2	$\Delta P_2(\%)$	$P_3(\bar{z})$	ΔP_3	$\Delta P_3(\%)$	$\bar{\eta}$	$\Delta\bar{\eta}$	$\Delta\bar{\eta}(\%)$
0.6	0.766	0.012	1.6	0.729	0.011	1.6	0.134	0.13	99.	1	0.43	43.
0.8	0.819	0.01	1.2	0.682	0.0088	1.3	0.317	0.12	38.	1	0.37	37.
1.	0.859	0.0093	1.1	0.65	0.0086	1.3	0.46	0.12	26.	1	0.36	36.
1.2	0.888	0.0092	1.	0.628	0.014	2.3	0.569	0.13	23.	1	0.39	39.
1.4	0.911	0.01	1.1	0.613	0.019	3.2	0.654	0.11	16.	1	0.31	31.

L.A, M. Kunz, A. Vollmer,
A. Trilleras, S. Fogli, work in progress, 2013

Results..

Model 3: η varies in z and k
Error on η

$\bar{z}$	k-bin	$P_1(\bar{z})$	ΔP_1	$\Delta P_1(\%)$	$P_2(\bar{z})$	ΔP_2	$\Delta P_2(\%)$	$P_3(\bar{z})$	ΔP_3	$\Delta P_3(\%)$	$\bar{\eta}(\bar{z})$	$\Delta\bar{\eta}$	$\Delta\bar{\eta}(\%)$
	1		0.14	18.		0.12	17.		1.2	920.		4.	400.
0.6	2	0.766	0.032	4.1	0.729	0.029	4.	0.134	0.29	210.	1	0.93	92.
	3		0.013	1.7		0.012	1.7		0.12	92.		0.4	40.
	1		0.11	13.		0.092	13.		1.	330.		3.2	320.
0.8	2	0.819	0.024	2.9	0.682	0.021	3.	0.317	0.24	74.	1	0.73	73.
	3		0.011	1.4		0.0096	1.4		0.11	36.		0.36	36.
	1		0.093	11.		0.076	12.		1.	220.		3.	300.
1.	2	0.859	0.02	2.3	0.65	0.018	2.7	0.46	0.25	53.	1	0.74	74.
	3		0.011	1.2		0.0093	1.4		0.16	35.		0.49	49.
	1		0.084	9.4		0.074	12.		1.1	190.		3.2	320.
1.2	2	0.888	0.017	2.	0.628	0.02	3.2	0.569	0.29	50.	1	0.84	84.
	3		0.011	1.2		0.015	2.4		0.24	42.		0.7	70.
	1		0.079	8.7		0.084	14.		0.79	120.		2.3	230.
1.4	2	0.911	0.017	1.9	0.613	0.025	4.	0.654	0.17	26.	1	0.49	49.
	3		0.013	1.4		0.022	3.6		0.14	21.		0.39	39.

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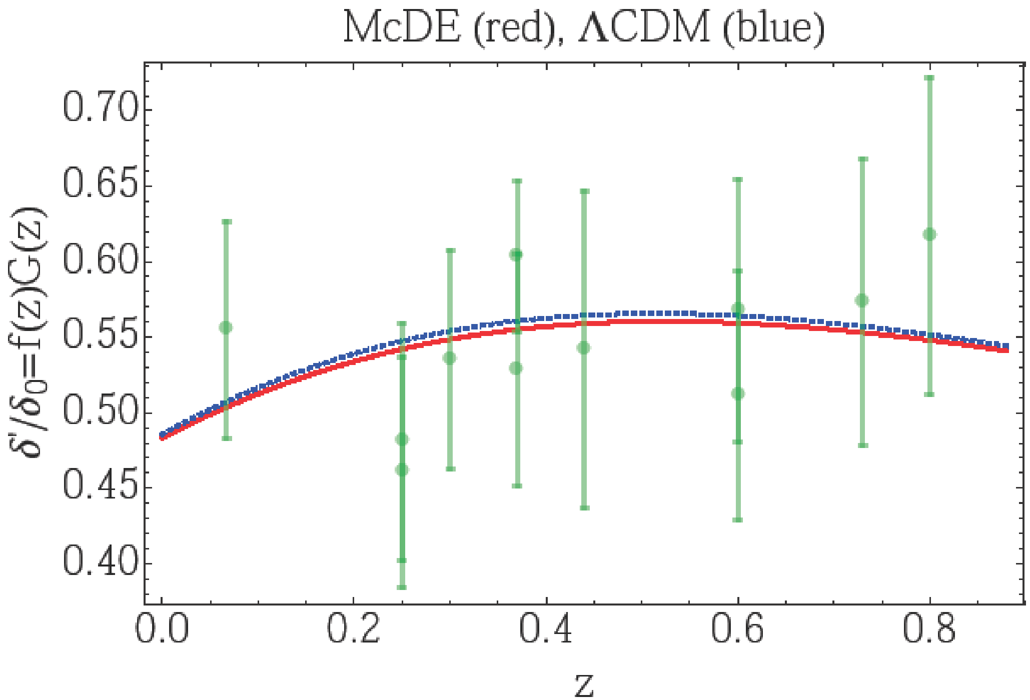
A. Trilleras, S. Fogli, work in progress, 2013

Importance of k-binned data

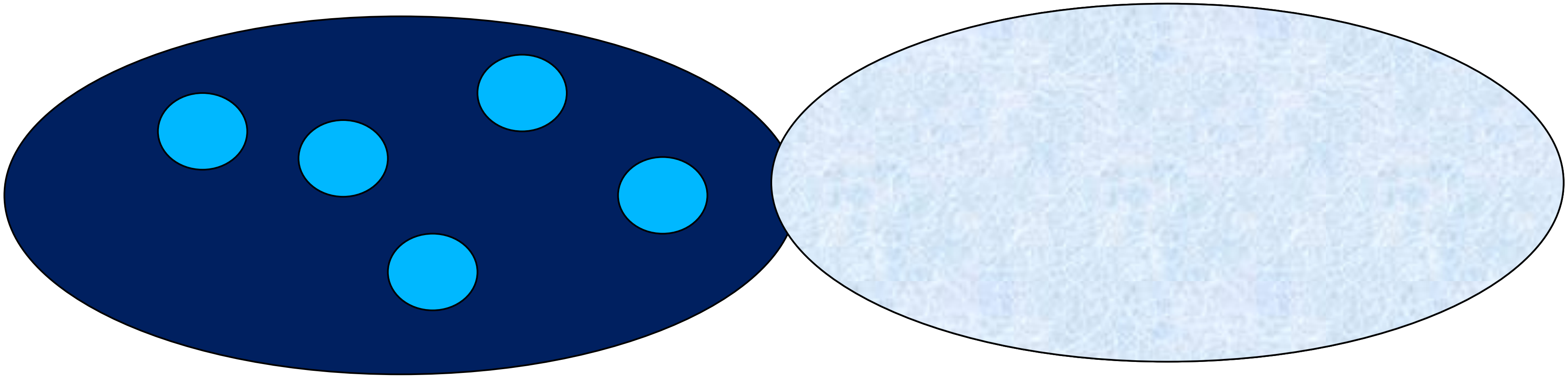


growth data compilation
Macaulay et al 2013

Under the carpet..

Problem of non-linearity:
screening effects mix linear and non-linear scales

same density contrast



different physics

Three Messages

1

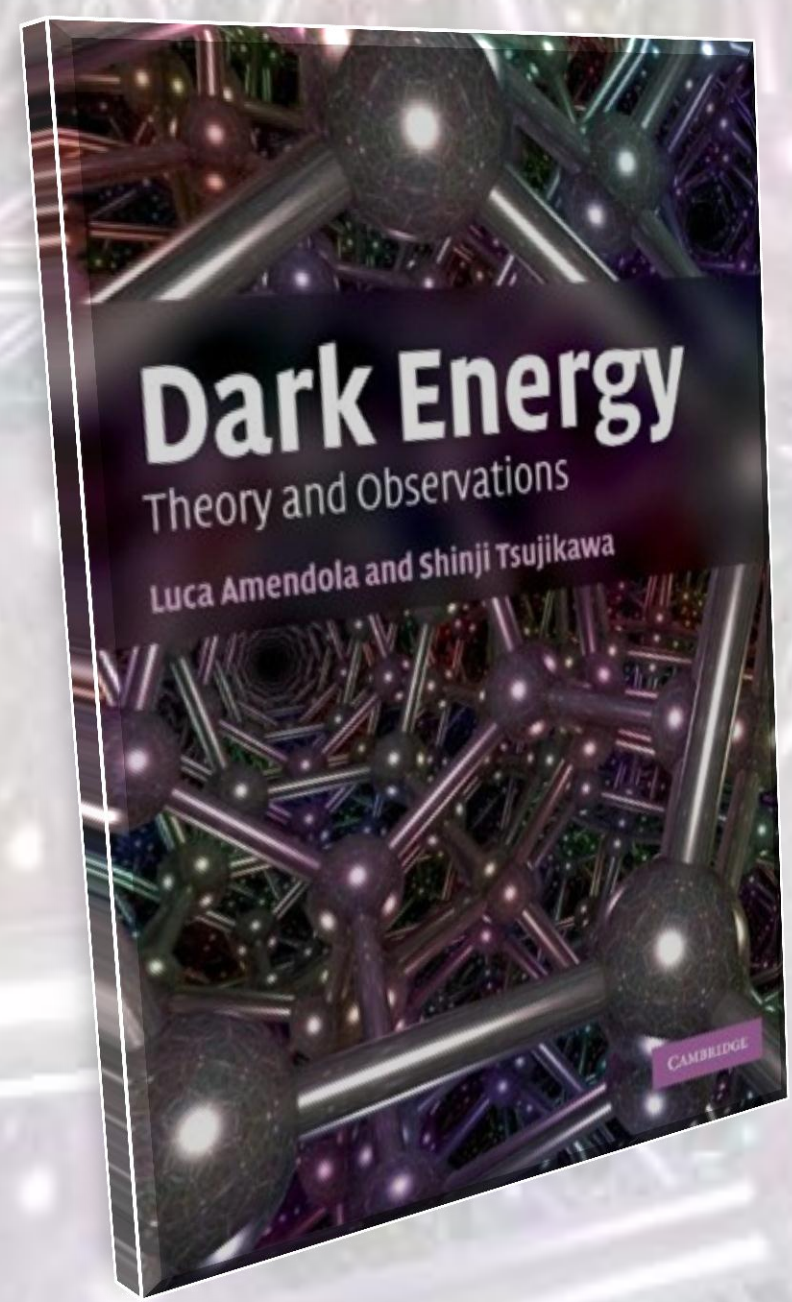
If DE is not a Horndeski field, then
we don't know what could be

2

k-binned data are crucial!
e.g. growth factor, redshift distortion parameter

3

Only by combining galaxy clustering and lensing
can DE be constrained (or ruled out!) in a model-independent way



Cambridge University Press

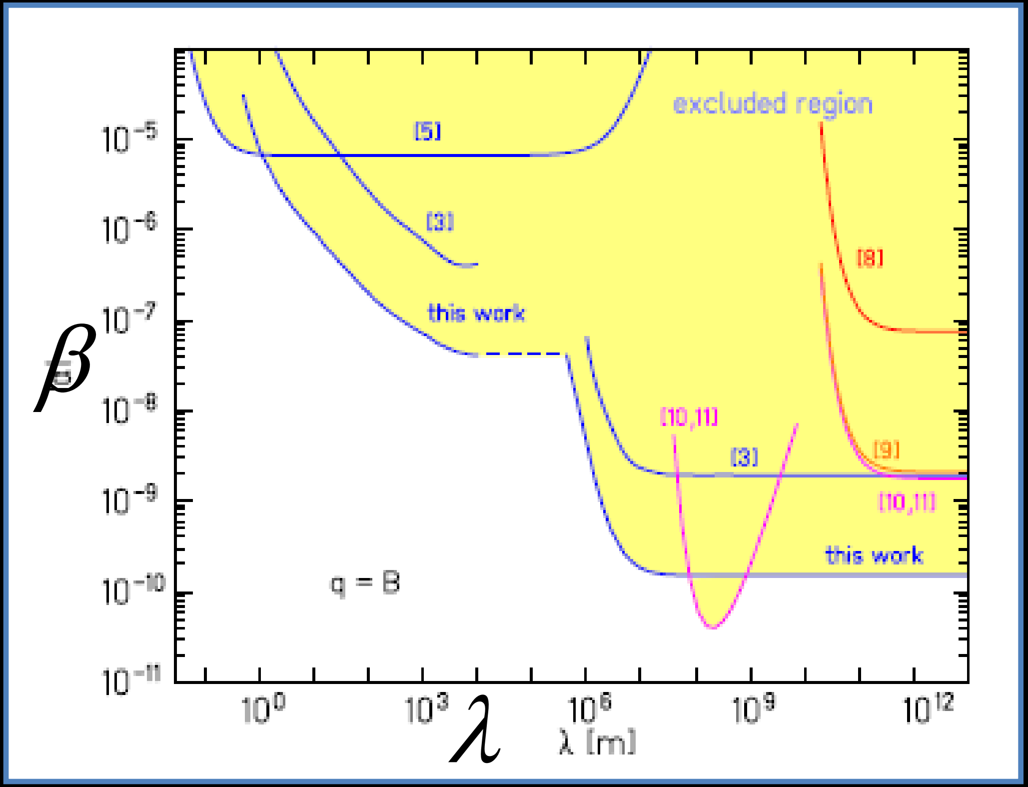
Dark Force

Limits on Yukawa coupling are strong but local!

$$\frac{GM}{r} \rightarrow \frac{GM}{r}(1+\beta e^{-r/\lambda})$$

$$\lambda^2 = m_\varphi^{-2} = V''(\varphi)$$

$$\varphi = \varphi(r)$$



Schlamminger et al 2008

The Yukawa correction

Every Horndeski model induces at linear level, on sub-Hubble scales, a Newton-Yukawa potential

$$\Psi(r) = -\frac{GM}{r}(1+\alpha e^{-r/\lambda})$$

where α and λ depend on space and time

Quasi-static approximation

$$c_s^{\,2} k^2 \gg a^2 H^2$$

From a wave equation:

$$E_{\delta\phi} \equiv D_1\ddot{\Phi} + D_2\ddot{\delta\phi} + D_3\dot{\Phi} + D_4\dot{\delta\phi} + D_5\dot{\Psi} + D_6\frac{k^2}{a^2}\dot{\chi} + \left(D_7\frac{k^2}{a^2} + D_8\right)\Phi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + \left(D_{10}\frac{k^2}{a^2} + D_{11}\right)\Psi + D_{12}\frac{k^2}{a^2}\chi = 0\,,$$

To a "Poisson" equation:

$$B_7\frac{k^2}{a^2}\Phi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + A_6\frac{k^2}{a^2}\Psi \simeq 0\,,$$

Final remarks

Few things I learned

- String theory is not dead…and can save cosmology
- SN Ia progenitors still a mystery
- Euclid will fly in a different landscape…
- Industrial approach to model comparison

One thing I am trying to learn

How to make a perfect Bavarian toast

Few things to improve

- Faster N-body simulations
- New standards:
  GW sirens, gal ages, AGN, real-time observables
- Effects of inhomogeneities and anisotropies on
  parameter estimation

Few things to understand

- Does vacuum energy gravitate?
- Is the coincidence problem(s) to be taken seriously?
- Does a "linear regime" really exists?

  And can it be identified by observations alone?

Few wishes for the future

- More theory-lite observational results
- More observation-lite theoretical results
- Meet again soon for deSitter III !