

An Examination of Recent Transformations to the $BV(RI)_C$ Photometric System From the Perspective of Stellar Models for Old Stars

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ABSTRACT

Isochrones for ages $\gtrsim 4$ Gyr and metallicities in the range $-2.5 \lesssim [\text{Fe}/\text{H}] \lesssim +0.3$ that take the diffusion of helium and recent advances in stellar physics into account are compared with observations in the Johnson-Cousins $BV(RI)_C$ photometric system for several open and globular star clusters. The adopted color- T_{eff} relations include those which we have derived from the latest MARCS model atmospheres and the empirical transformations for dwarf and subgiant stars given by Casagrande et al. (2010, A&A, 512, 54; hereafter CRMBA). Those reported by Vandenberg & Clem (2003, AJ, 126, 778) have also been considered, mainly to resolve some outstanding questions concerning them. Indeed, for the latter, $V - I_C$ colors should be corrected by ≈ -0.02 mag, for all metal abundances, in order to obtain consistent interpretations of the observed $(B - V, V)$ -, $(V - R_C, V)$ -, and $(V - I_C, V)$ -diagrams for M 67 and the Hyades, as well as for local subdwarfs. Remarkably, when the subdwarfs in the CRMBA data set that have $\sigma_\pi/\pi \leq 0.15$ are superimposed on a set of 12 Gyr isochrones spanning a

wide range in $[\text{Fe}/\text{H}]$, the inferred metallicities and effective temperatures agree, in the mean, with those given by CRMBA to within ± 0.05 dex and ± 10 K, respectively. Thus the hot T_{eff} scale derived by CRMBA is nearly identical with that predicted by stellar models; and consequently, there is excellent consistency between theory and observations on the H-R diagram *and* the different color-magnitude diagrams considered in this investigation. To obtain similar consistency, the colors obtained from the MARCS and Vandenberg & Clem ($B-V$)– T_{eff} relations for metal-poor dwarf stars should be adjusted to the red by 0.02–0.03 mag. In general, isochrones that employ the CRMBA transformations provide reasonably consistent fits to our $BV(RI)_C$ photometry for main-sequence stars in the globular clusters 47 Tuc, M 3, M 5, M 92, and NGC 1851 — but not the cluster giants (when adopting the synthetic MARCS colors). We speculate that differences between the actual heavy-element mixtures and those assumed in the theoretical models may be the primary cause of this difficulty.

Subject headings: color-magnitude diagrams — globular clusters: general — globular clusters: individual (M 3, M 5, M 92, NGC 1851, 47 Tuc) — Hertzsprung-Russell diagram — open clusters and associations: general — open clusters and associations: individual (Hyades, M 67, NGC 6791) — stars: fundamental parameters (temperatures) — stars: evolution

1. Introduction

The empirical stellar T_{eff} scale is still uncertain by $\gtrsim 100$ K in most parts of the H-R diagram despite painstaking spectroscopic and photometric work by many investigators for many years (e.g., Gratton, Carretta, & Castelli 1996; Alonso, Arribas, & Martinez-Roger 1996; Barklem et al. 2002; Ramírez & Meléndez 2005; Nissen et al. 2007). Such uncertainties have important consequences for the determination of other fundamental properties of stars — notably their chemical abundances. For instance, the metallicities of solar neighborhood stars derived by Gratton et al. tend to be 0.1–0.25 dex more metal-rich than those reported by Cenarro et al. (2007) because, in part, the temperatures adopted by the latter are up to 150 K cooler than those estimated by the former (see Vandenberg 2008, his Figs. 1, 2). Even when the temperature is known to very high accuracy, as in the case of the Sun, absolute abundances can vary by ~ 0.2 dex when 3D hydrodynamical model atmospheres are employed instead of the classical 1D hydrostatic models, departures from LTE are taken into account, improved atomic and molecular data are incorporated into the analyses, etc. (Asplund et al. 2005). Adding to the confusion is the fact that stellar models

appear to have considerable difficulty matching the properties of globular cluster (GC) giants as derived by Carretta & Gratton (1997) when the same models reproduce quite well the T_{eff} and $[m/H]$ values derived by the same researchers (Carretta et al. 2000) for the Population II subdwarf standards (see, e.g., Bergbusch & Vandenberg 2001, their Figs. 11–15; Vandenberg, Bergbusch, & Dowler 2006, their Fig. 13).

[As shown later in this paper, significantly improved agreement between theory and observations is obtained on the assumption of the recently revised metallicity scale for GCs given by Carretta et al. (2009). It should be appreciated, however, that their revision to lower $[Fe/H]$ values (by typically ~ 0.2 dex) is due, in part (i.e., along with improvements to the spectra and $\log gf$ values), to their adoption of lower temperatures to be consistent with the Alonso, Arribas, & Martinez-Roger (1999) T_{eff} scale for giants. These temperatures may be too low. According to Casagrande et al. (2010), the main difference between their relatively high temperatures and those determined by Alonso, Arribas, & Martinez-Roger (1996) for dwarf and subgiant stars is the underlying absolute calibration of the Infrared Flux Method. Since a different calibration will mainly cause a zero-point offset, we would expect that Alonso et al. would obtain warmer T_{eff} values for both dwarfs and giants were they to adopt the Casagrande et al. calibration. Thus, it is quite possible that Carretta et al. *should* assume higher temperatures, in which case their $[Fe/H]$ estimates would also increase, thereby moving them closer to the values originally published by Carretta & Gratton (1997).]

One obvious way of constraining the stellar T_{eff} scale is to obtain photometry in many different bandpasses and then to examine the extent to which a consistent interpretation of the data can be obtained on all of the possible color-magnitude diagrams (CMDs) that can be constructed. This approach motivated the studies by, in particular, Vandenberg & Clem (2003; hereafter VC03) and Clem et al. (2004) of the $BV(RI)_C$ and the Strömrgren *uvby* photometric systems, respectively. Using theoretical color indices derived from MARCS model atmospheres as the starting point, these investigations determined the corrections that should be applied to the synthetic colors in order to satisfy a variety of observational constraints. Not surprisingly, the inferred corrections generally increased with decreasing T_{eff} and they tended to be larger for colors involving ultraviolet or blue magnitudes. Calamida et al. (2007) and Dotter et al. (2008), among others, have used the resultant semi-empirical color transformations in their analyses of observed CMDs with apparently quite favorable results.

However, it is very difficult to avoid small zero-point or systematic errors in *any* color– T_{eff} relations. For instance, as discussed by VC03, isochrones employing their transformations provide a good match to the Hyades $[(B - V)_0, M_V]$ - and $[(V - R), M_V]$ -diagrams, on the assumption of well-determined estimates of $(m - M)_0$, $[Fe/H]$, and Y , but they tend to be ≈ 0.02 mag redder than the cluster observations on the $[(V - I)_0, M_V]$ -plane. [Note that,

R and I are used interchangeably with R_C and I_C ; i.e., all of the R and I photometry that is mentioned in this paper is in the Cousins system, as defined by the standard stars of Graham (1982) and (Landolt 1983, 1992).] It was not at all clear to VC03 how best to explain this conundrum because no such difficulty was apparent when they fitted isochrones to BV and VI data for the open clusters M 67 and NGC 6791, or the very metal-deficient GC M 68. The subsequent study of M 67 by VandenBerg & Stetson (2004) showed, in fact, that the Montgomery, Marschall, & Janes (1993) photometry used in VC03’s analysis agreed well with the CMDs produced by most, but not all, other workers. Only the Sandquist (2004) VI observations were clearly different, though (curiously) they provided the best match to the $(B - V)$ – $(V - I)$ diagram given by Caldwell et al. (1993), based on their standardization of the Cousins system. According to Sandquist, the main sequence of M 67 is 0.01–0.03 mag bluer on the $[(V - I)_0, M_V]$ -plane than the determination by Montgomery et al. If correct (which we now believe to be the case, see § 3), this would imply that the VC03 $(V - I)$ – T_{eff} relations should be adjusted to the blue by ≈ 0.02 mag in order to achieve consistency with their $B - V$ and $V - R$ transformations (and thereby also solve the Hyades problem).

In the meantime, one of us (PBS) has made considerable progress in his endeavor (see Stetson 2000) to collect, reduce, and carefully calibrate to the Landolt (1992) system a significant fraction of the world’s photometry for open and globular star clusters. As shown later in this paper, his BVI data for M 67 are in good agreement with those published by Sandquist (2004), thereby reinforcing our suspicion that the VC03 $(V - I)$ – T_{eff} relations are too red for near solar-abundance stars. Moreover, he found that his current photometric data (which are used here) for some GCs differ at the level of 0.01–0.03 mag from published CMDs, sometimes in a systematic sense. Besides the availability of these very homogeneous data, there are two other recent developments that make a further examination of color– T_{eff} relations worthwhile.

First, new and significantly improved grids of MARCS model atmospheres and synthetic spectra have been published by Gustafsson et al. (2008). In §2, we describe how the latter have been processed into synthetic $BV(RI)_C JHK_S$ magnitudes. (Note that a thorough study of the Strömgren color indices derived from the new MARCS models is provided by Önehag et al. 2009.) Second, Casagrande et al. (2010; hereafter CRMBA) have used the Infrared Flux Method (IRFM) to produce a new calibration of the T_{eff} scale for dwarf and subgiant stars spanning a wide range in $[\text{Fe}/\text{H}]$ for which the zero-point should be much more accurate than previous calibrations because it is based on a number of solar twins. Their results are presented in the form of analytic expressions that relate many different photometric indices to T_{eff} and $[\text{Fe}/\text{H}]$. As these color-temperature relations are based on field stars, it is of some interest to examine how well they can reproduce the main-sequence (MS) fiducials of star clusters when coupled with up-to-date stellar evolutionary computations.

(The main advantage of such systems over field stars is that their CMDs provide well-defined loci which connect stars of the *same* metallicity. In view of the mounting evidence for multiple stellar populations in some GCs, one should focus on only those clusters with very narrow fiducial sequences and, even in the most favorable cases, be wary of the possibility that chemical abundance peculiarities may affect the fluxes in some bandpasses more than others.) § 3 presents an analysis of cluster and field-star data using the MARCS, CRMBA, and VC03 transformations. Finally, a brief summary of the main conclusions of this investigation are given in §4. [Because a companion paper by Brasseur et al. (2010) focusses on the color– T_{eff} relations for the infrared, the present paper has been restricted, with one exception, to a consideration of the $BV(RI)_C$ transformations. In the case of M 67 we compare isochrones with VK_S observations in order to demonstrate the advantages of having $V - K_S$ colors to complement those at optical wavelengths.]

2. Synthetic $BV(RI)_C$ Magnitudes Derived from MARCS (2008) Model Atmospheres

In the following, synthetic colors and bolometric corrections¹ in the Johnson-Cousins $BV(RI)_C$ system have been computed following the formalism described in Casagrande, Portinari, & Flynn (2006). The only difference is the reference spectrum of Vega, now based on the updated absolute spectrophotometry of Bohlin (2007), which intermingles *HST* observations with model fluxes and provides the best accuracy available to date, at least in the optical region. Note that, despite the complications posed by the pole-on and rapidly rotating nature of Vega, the effects on the blue part of the spectrum are expected to be small or negligible (e.g., Casagrande, Portinari, & Flynn 2006; Bohlin 2007, and references therein), though some fine-tuning in the infrared may be necessary (see CRMBA). Briefly, the spectrum of Vega has been convolved with the $BV(RI)_C$ filter transmission curves of Bessell (1990b) and the results forced to match its observed magnitudes (Bessell 1990a) in order to determine the corresponding zero-points for each band. The latter are needed to place onto the standard Johnson-Cousins system colors that are derived by convolving spectral libraries with the aforementioned filter transmission curves.

The new grid of MARCS synthetic spectra with “standard” chemical abundances (i.e., $[\alpha/\text{Fe}] = 0.0$ for $[\text{Fe}/\text{H}] \geq 0.0$, a linear increase of $[\alpha/\text{Fe}]$ from 0.0 at $[\text{Fe}/\text{H}] = 0.0$ to 0.4 at $[\text{Fe}/\text{H}] = -1.0$, and $[\alpha/\text{Fe}] = 0.4$ for $[\text{Fe}/\text{H}] \leq -1.0$) has been used (Gustafsson et al. 2008). This choice is well suited for the purpose of the present investigation, since the α –

¹The usual definition of bolometric correction has been adopted: $BC_V = M_{\text{bol}} - M_V$, where $M_{\text{bol},\odot} = 4.75$.

enhancement measured in the majority of field stars and clusters follows this relation quite closely. It is known that most of the GCs seem to exhibit abundance variations and/or anomalies (see e.g. Gratton, Sneden, & Carretta 2004), which, in principle, could also affect the predicted colors. This would require a case-by-case study, tailoring synthetic models to the detailed chemical composition of each cluster, which is not always possible and clearly outside the scope of the present investigation. (As shown later in this paper, the inability of isochrones to reproduce the full CMD features of *some* GCs may be telling us that the use of synthetic color– T_{eff} relations for the standard mix of heavy elements might not be appropriate for these systems.)

For the sake of completeness, we generated synthetic colors using the (as yet, smaller) set of MARCS models having $[\alpha/\text{Fe}] = 0$ for all metallicities below solar. Differences in the resultant $B - V$, $V - R$, and $V - I$ colors amount to a few millimagnitudes (at most) for $T_{\text{eff}} \gtrsim 5000$ K, though they steadily increase with decreasing temperature, mostly because of the formation of molecules. Thus, a fine-tuning of the α -enrichment could have a limited impact along the RGB or the lower MS, but the bulk of the CMD morphologies discussed in this paper is unaffected by this choice. The full MARCS library is given for a microturbulent velocity $\xi = 2$ km/s. Also, the geometry of the models (plane-parallel for $\log g \geq 3.0$ or spherical for $\log g \leq 3.5$) have no significant impact on our broad-band colors in the overlap region, as we have found from our tests that the two geometries produce a nearly constant offset of a few millimagnitudes. (Tables of synthetic magnitudes in various photometric systems for all of these models will be published in a forthcoming paper.)

[Interestingly, we found from two solar-like spectra for $\xi = 1$ and 2 km/s that this choice has a non-negligible ($\lesssim 0.02$ mag) effect on the calculated $B - V$ color, while indices at longer wavelengths appear to be considerably less affected. This presumably occurs because microturbulence will partly redistribute the flux in spectral regions that are crowded with lines (i.e., mainly in the blue). This clearly introduces an additional degree of freedom which can be avoided only by hydrodynamical simulations that treat the velocity field in a consistent manner (Collet, Asplund, & Trompedach 2007), though the impact of 3D model atmospheres on synthetic colors is still largely unexplored (see Casagrande 2009; Kučinskas et al. 2009). Notwithstanding the fact that a microturbulent velocity $\xi = 1$ km/s is usually adopted for the Sun, the generally good agreement between the synthetic (MARCS) and empirical (CRMBA) color– T_{eff} relations reported in this paper suggests that $\xi = 2$ km/s is probably a good choice, at least for MS stars.]

Differently from other stars, the Sun does not provide a robust benchmark for testing synthetic colors. In fact, it cannot be directly observed with the same instrumentation used for stellar photometry; consequently, its colors can be derived only indirectly. Recent

advances using photometry of solar twins and solar spectrophotometry have improved upon this situation and suggest that the latest MARCS model provides a decent fit to observations, especially in terms of broad-band colors (always within 0.02 mag), while larger discrepancies at the bluest wavelengths and in intermediate-band filters appear to be present (Edvardsson 2008; Meléndez et al. 2010, CRMBA). The comparison presented in CRMBA also suggests that our adopted zero-points are indeed appropriate for generating synthetic colors: the tendency of the MARCS solar spectrum to return bluer than the “observed” $B - V$ color index does, in fact, stem from the model atmosphere for the Sun.

There are a few other subtleties that are usually neglected when generating synthetic colors and we would like to comment on them since they could be relevant in the context of the present investigation (see also Bessell, Castelli, & Plez 1998 and Bessell 2005 for a more detailed discussion). In principle, to mimic photometric measurements, synthetic photometry should reproduce the instrumental system used and the same transformation equations determined from observations should then be applied to replicate the standard system under investigation. In practice, this can hardly be achieved, especially if measurements from different instruments are used; whilst the zero-points of observational photometry are defined over an ensemble of well-measured stars, the common practice in the case of synthetic photometry (as in this study) is to set the zero-points using one reference star (usually Vega), for which its spectrophotometry and observed apparent magnitudes or color indices are well established.

Fortunately, there is a general good agreement between the highly standardized and homogeneous photometry used in this paper for open and globular clusters and other independent measurements, despite the known small differences between the Landolt (1992) and Cousins standards (Bessell 1995). This means that transformations from the instrumental to the standard system are indeed accurate and reproducing the latter using only Vega returns meaningful synthetic colors. Nevertheless, a contemporary standard system, although well defined by a list of standard stars, might not represent a real linear system anymore, implying that is impossible to realize it with a unique passband and a linear transformation. Therefore, other than the passband, when trying to reproduce a given set of observations, the same linear and non-linear transformations used to place observations onto the standard system should be adopted. In practice, this task is very difficult to achieve; as a result, corrections of a few percent to the synthetic colors cannot be totally excluded across the whole temperature range of the models. Despite this discouraging scenario, the general good agreement between observed and synthetic colors shown in the next section of this paper suggests that both are well standardized, and hence that the comparison between observed stars and theoretical isochrones can be used to gain insights concerning synthetic color- T_{eff} relations.

Summarizing: from the above discussion, it should be kept in mind that model inaccuracies as well as detailed chemical composition and photometric uncertainties may all play a role in the final results and it is generally quite difficult to disentangle them; consistency in different bands at the level of 0.01-0.02 mag can therefore be regarded as excellent. In the following, these possibilities have been taken into account and they are discussed when it is relevant to do so.

3. Star Cluster and Field Subdwarf Constraints on the $BV(RI)_C$ Transformations

In the following analysis, theoretical isochrones will be transformed to the observed plane using the CRMBA, MARCS, and VC03 color- T_{eff} relations and then compared with the CMDs for a few open and globular star clusters and field subdwarfs that span a wide range in [Fe/H]. Most of the models are taken from the new Victoria-Regina grids that have been generated by D. VandenBerg et al. (2010, in preparation; hereafter VR2010), though some of them have been computed specifically for this or other projects currently underway. Nevertheless, the same version of the Victoria evolutionary code has been used in all instances.

Because this code has undergone substantial revisions since the last presentation of Victoria-Regina isochrones (VandenBerg et al. 2006), we provide a brief summary of the main modifications that have since been made to it. First, the latest rates for the pp -chain and the CNO-cycle (see, e.g., Weiss 2008), including, in particular, that for the important $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction (Marta et al. 2008) have been adopted. Second, the gravitational settling of helium (and lithium), as well as turbulent mixing below envelope convection zones are now treated using methods very similar to those described by Proffitt & Michaud (1991). (The observed solar Li abundance was used to constrain the amount of mixing in a Standard Solar Model, and thereby to determine the value of the free parameter in our adopted formulation of this additional mixing; see the VR2010 paper for details.) Third, we have implemented the improved conductive opacities reported by Cassisi et al. (2007). Fourth, the model atmospheres that are needed to define the outer boundary conditions for the stellar interior models were obtained by integrating the hydrostatic equation in conjunction with the scaled empirical Holweger & Müller (1974) T - τ relation given by VandenBerg & Poll (1989). As shown by VandenBerg et al. (2008), this choice provides a very good approximation to the use of scaled-solar, differentially corrected MARCS model atmospheres as boundary conditions over wide ranges in T_{eff} , $\log g$, and metallicity. (Indeed, this paper contains quite a thorough discussion of not only the impact of different treatments of the atmospheric layers

on the predicted temperature scale of stellar models, but also of the associated uncertainties. Interested readers are encouraged to refer to this work.)

In what follows, we will show that the VR2010 models satisfy all of the observational constraints that have been considered rather well. It is, of course, quite possible that the good consistency we have found between theory and observations is fortuitous to some extent; i.e., errors in one or more aspects of the models or observations have compensated for errors in other factors that play a role to give a misleadingly rosy picture. The models still employ the crude mixing-length theory of convection and a very *ad hoc* prescription for turbulent mixing, for instance; consequently, the physics incorporated in them can certainly be improved upon. Nevertheless, the Victoria-Regina models, coupled with the CRMBA or MARCS color– T_{eff} relations, appear to pass the tests to which they have been subjected (so far). Accordingly, one can have considerable confidence in the results that are obtained when these models are used, say, to interpret stellar populations data.²

Turning to the observations: where possible, distances derived from *Hipparcos* parallax measurements are assumed, along with current best estimates of the basic stellar/cluster parameters. However, even when (for instance) the adopted distance moduli are uncertain by ~ 0.1 – 0.2 mag, as in the case of most of the star clusters considered here, such uncertainties are of little importance. Even if the isochrones do not fit the observations particularly well in an absolute sense (for whatever reason), any discrepancies that are found *should* be apparent in the many different CMDs that can be constructed from $BV(RI)_C$ photometry *if* the color transformations that are used lead to a similar and consistent interpretation of the data on all color-magnitude planes. This does require, of course, that the observations are themselves free of systematic errors and that the extinction in each of the filter passbands is accurately determined if there is significant foreground reddening. (As already noted, the possibility that chemical abundance anomalies may affect some color indices more than others is also a

²The referee queried how well the isochrones computed by other workers fare in similar comparisons, which is tantamount to asking how well the VR2010 models agree with those published by other groups. We have not attempted to carry out such an analysis, which is beyond the scope of the present work, and which is worth doing only if it is first demonstrated that the evolutionary tracks produced by the different codes in use today are in good agreement when very similar physics is assumed. As shown by Weiss et al. (2007), who carried out such experiments, it is not straightforward to obtain the level of agreement that one would like (and expect) for, in particular, the predicted lifetime of a star of a given mass and chemical composition. In any case, we can report that both the computed track and the predicted age at the RGB tip from the Victoria code for the $1.0M_{\odot}$, $Y = 0.28$, $Z = 0.02$ test case considered by Weiss et al. are within 1–2% of those obtained from the BASTI code (Pietrinferni et al. 2004). Similar good agreement has been found when comparisons are made with the tracks produced by the MESA code (B. Paxton et al., in preparation; also see <http://mesa.sourceforge.net>) using more up-to-date physics. These and other tests of the reliability of the Victoria-Regina models are reported in much greater detail in the VR2010 study.

concern.)

Our examination of color– T_{eff} relations for stars having $[\text{Fe}/\text{H}] \gtrsim 0.0$ will focus on the Hyades, M 67, and NGC 6791 open clusters. Those appropriate to metal-deficient stars will be assessed using ~ 100 nearby subdwarfs having $[\text{Fe}/\text{H}] \lesssim -0.5$ (of which nearly three dozen have $\sigma_{\pi}/\pi \lesssim 0.15$), as well as the globular clusters 47 Tucanae, NGC 1851, M 5, M 3, and M 92, which have metallicities in the range $-0.8 \gtrsim [\text{Fe}/\text{H}] \gtrsim -2.4$. Unless noted otherwise, the most recent calibrations of the cluster photometry in the Stetson (2000) database are used in this study. Unfortunately, R_C photometry is available only for the Hyades, NGC 1851, M 5, M 92, and most of the field subdwarfs. For the other objects, our analysis is necessarily restricted to BVI_C photometry.

3.1. The Hyades ($[\text{Fe}/\text{H}] \approx +0.14$)

The Hyades provide an especially powerful constraint on stellar models because all of its basic parameters are known to high precision. It has $E(B - V) = 0.0$, $(m - M)_0 = 3.334 \pm 0.024$ from *Hipparcos* (van Leeuwen 2009), $[\text{Fe}/\text{H}] = +0.14 \pm 0.03$ from high-resolution spectroscopy (e.g., Cayrel de Strobel, Crifo, & Lebreton 1997; also see Paulson, Sneden, & Cochran 2003), and $Y \approx 0.26 \pm 0.005$ from the cluster binaries (Lebreton, Fernandez, & Lejeune 2001; VC03). As discussed in the VR2010 study, a Standard Solar Model — one that reproduces the properties of the Sun at its present age — requires $Y_{\odot, \text{initial}} = 0.26575$ if $Z_{\odot} = 0.01652$, assuming the solar heavy-element abundances given by Grevesse & Sauval (1998), and a value of 2.01 for the mixing-length parameter, α_{MLT} .³ This value of Z_{\odot} , together with the derived values of Y and $[\text{Fe}/\text{H}]$ for the Hyades, imply that the cluster stars have $Z \approx 0.023$. In fact, the VR2010 models provide a superb fit to the mass– M_V relation for the binaries (not shown here because it is essentially identical to that shown by VC03; see their Fig. 21)

³Although the VR2010 investigation also provides evolutionary tracks and isochrones for the solar distribution of the metals determined by Asplund et al. (2005), these models are not used in this investigation. This is mainly for the reason that recent revisions to nuclear reaction rates [especially for $^{14}\text{N}(p, \gamma)^{15}\text{O}$ — see Marta et al. 2008] imply a significant increase ($\gtrsim 0.06 M_{\odot}$) in the mass of the lowest mass star that has a convective core at central H exhaustion (Christensen-Dalsgaard 2009), thereby reducing the age of the oldest isochrone that predicts the existence of a gap near the turnoff. While it may still be possible to produce models for the Asplund et al. metal abundances that possess a gap at the observed luminosity by allowing for sufficient convective core overshooting, fine-tuning of the treatment of overshooting seems to be necessary to achieve this, especially if diffusive processes are also treated (see the careful and thorough analysis of this problem by Magic et al. 2010). In view of the additional difficulties presented by the Asplund et al. metals mixture for helioseismology (e.g., Bahcall et al. 2005), it seems advisable to use stellar models that assume the relative abundances of the heavy elements tabulated by Grevesse & Sauval (1998).

if they assume $Y = 0.257$, $Z = 0.023$, and an age of ≈ 700 Myr.

Figure 1 illustrates how well the main-sequence segment of an isochrone for these parameter values reproduces the Hyades CMDs (see VC03) constructed from BV , VR , and VI photometry reported by Taylor & Joner (1985), Joner & Taylor (1988), Reid (1993), and de Bruijne, Hoogerwerf, & de Zeeuw (2001), when the three different sets of color– T_{eff} relations considered in this investigation are assumed. Stars having $B - V \lesssim 0.9$, $V - R \lesssim 0.7$, and $V - I \lesssim 1.5$ (which corresponds, in turn, to values of $T_{\text{eff}} \gtrsim 5100$, 4450, and 4200 K) are well-fitted by the models and there is very good consistency between the three loci, except on the $(V - I), M_V$ diagram, where the VC03-transformed isochrone is ≈ 0.02 mag too red. At faint magnitudes, the CRMBA colors tend to be slightly too red, while the MARCS transformations yield $B - V$ and $V - R$ colors, but not $V - I$ indices, that are too blue. The fact that the observed $V - I$ colors are so well reproduced using the MARCS transformations while the colors derived from bluer bandpasses (notably B) tend to become discrepant at lower values of T_{eff} suggests that the MARCS atmospheres for cool, super-metal-rich stars have insufficient blanketing at shorter wavelengths. If this suspicion is correct, then it is the synthetic $B - V$ and (to a lesser extent) $V - R$ colors that need to be corrected in order to achieve better consistency between the three color planes at faint magnitudes — more so than the isochrone temperatures. The only obvious problem with the VC03 transformations appears to be the aforementioned offset in the $V - I$ colors. Whether the discrepancies between the solid curve and the observations is an artefact of the analytic expressions used by CRMBA to represent their color– T_{eff} relations⁴ or an indication of, say, a small problem with the model temperatures is not clear.

3.2. M 67 ($[\text{Fe}/\text{H}] \approx 0.0$)

According to the results of high-resolution spectroscopy, M 67 has $[\text{Fe}/\text{H}] = 0.0 \pm 0.03$, with very close to solar m/H number abundance ratios for all of the most important heavy elements (Tautvaišienė et al. 2000; Randich et al. 2006). Current best estimates of the foreground reddening favor values in the range $0.03 \lesssim E(B - V) \lesssim 0.04$ (Nissen, Twarog, & Crawford 1987; Schlegel, Finkbeiner, & Davis 1998; Sarajedini et al. 1999).

⁴Although the equations presented by CRMBA are in the form that is traditionally used to describe such results, a different functional relationship appears to be needed for M dwarfs (see Casagrande, Flynn, & Bessell 2008), which could alleviate the problem discussed here. However, except for solar abundance stars, it is not yet possible to extend the CRMBA calibrations to very red colors primarily because of the limited metallicity range encompassed by nearby M dwarfs. Indeed, the analytic expressions given by CRMBA are valid only for the color ranges that are specified in their paper.

In the course of examining the BVI photometry reduced by one of us (PBS), along with 2MASS near-infrared observations (Skrutskie et al. 2006), we found that it was possible to obtain very nice consistency of a 3.7 Gyr isochrone for the solar metallicity with all of the available data if $E(B - V) = 0.03$ was adopted. Since $E(V - K_S) = 2.72 E(B - V)$ (McCall 2004), even a change of 0.01 mag can have noticeable consequences for the consistency, or not, of optical and near-IR CMDs. If this reddening is assumed, together with an apparent distance modulus $(m - M)_V = 9.67$, we obtain the comparison between theory and observations shown in Figure 2. As in the case of the Hyades, the different line types are used to represent the different color transformations that have been used. Note that the true distance modulus assumed here, $(m - M)_0 \approx 9.58$, is in excellent agreement with the values of 9.57 and 9.60 derived by Sarajedini et al. and Sandquist (2004), respectively.

The isochrone which appears in this figure is the same one that was fitted to the CMD of M67 by Michaud et al. (2004). It assumes the heavy-element abundances given by Grevesse & Sauval (1998) and it takes the gravitational settling and radiative accelerations of helium and the metals into account. As also demonstrated by Michaud et al., this isochrone reproduces the morphology of the cluster CMD in the vicinity of the turnoff, including the location of the gap, particularly well. (Michaud et al. did not need to assume any convective core overshooting in their models, though some overshooting would almost certainly be required to compensate for the recent revisions to nuclear reactions; see footnote 3.) Because that isochrone terminated just above the base of the red-giant branch (RGB), the evolution to higher luminosities has been represented by the giant branch segment from a non-diffusive evolutionary track for 1.40 solar masses and the same chemical abundances. This locus, which was computed using the VR2010 code, had to be shifted by only $\delta \log T_{\text{eff}} = 0.0022$ in order to achieve continuity with the Michaud et al. isochrone on the theoretical plane.

Fig. 2 shows that the CRMBA, MARCS, and VC03 $B - V$ and $V - I$ transformations are in very good agreement from ~ 2 mag below the turnoff up to the base of the RGB, except that the VC03 $V - I$ colors apparently suffer from a small, nearly constant, offset. Only on the $[(B - V)_0, M_V]$ -diagram are there some noticeable differences in a systematic sense. The CRMBA $B - V$ colors for cool stars ($T_{\text{eff}} \lesssim 4950$ K, $(B - V)_0 \gtrsim 0.92$) appear to be slightly too red (possibly a reflection of the limitations of the analytic expression used to describe these color transformations, as already mentioned in footnote 4), while those based on MARCS model atmospheres cause the isochrone to deviate to the blue side of both the observed RGB and the lower main sequence, reminiscent of our findings in the case of the Hyades. The best fit to the cluster BV photometry is obtained using the VC03 transformations (by design, since the colors predicted by previous generations of MARCS model atmospheres were corrected so that isochrones would reproduce the *slopes* of cluster main sequences on

the different color planes). For the latter to provide a fully consistent explanation of the VI data, the VC03 $(V - I)$ - T_{eff} relations for dwarf stars should be adjusted to the blue by the amounts needed to bridge the gap between the dot-dashed curve and the observed MS of M 67.

Figure 3 has been included in this study to show that the same isochrones provide very similar fits to the CMDs of M 67 reported by Sandquist (2004), which indicates that the Stetson and Sandquist data sets are in close agreement. This implies, in turn, that the (Cousins) I magnitudes and $V - I$ colors determined for the cluster MS and turnoff stars in most of the CCD surveys of M 67 prior to Sandquist’s investigation (see Vandenberg & Stetson 2004) are too bright/red by up to a few hundredths of a magnitude. Such discrepancies are inherent to the VC03 $(V - I)$ - T_{eff} relations for solar metallicity stars because these transformations relied on the empirical constraints provided by pre-2003 photometry of M 67. (In the case of solar-type stars having $[\text{Fe}/\text{H}] \approx 0.0$, the model T_{eff} scale is not a significant source of uncertainty because it is precisely tied to the Sun through a calibration of α_{MLT} .) M 67 thus provides a sobering example of how difficult it is to obtain reliable CCD photometry to within 0.01 mag (Stetson 2005).

The CRMBA and MARCS transformations to $V - K_S$ are evidently almost identical for solar abundance stars (see the right-hand panel of Fig. 2), as both result in a nearly perfect superposition of the isochrone onto the observed CMD from at least $M_V = 7.5$ to the base of the RGB. Indeed, if the MARCS colors for low gravities were just ~ 0.04 mag redder, so that the dashed curve lined up with the solid curve along the lower RGB, the models would provide a very good match to the cluster giants as well. A particularly compelling illustration of the excellent consistency between theory and observations is shown in Figure 4. In this plot, the $(V - K_S)_0$ colors of M 67 stars were converted to $\log T_{\text{eff}}$ values using the CRMBA transformations, which are valid only for dwarfs and subgiants, and then the same isochrone that appears in the two previous figures was overlaid onto the resultant $(\log T_{\text{eff}}, M_V)$ -diagram. The solid curve reproduces the observed morphology so well that it looks more like an estimate of the mean cluster fiducial than a totally independent prediction from stellar evolutionary theory.

3.3. NGC 6791 ($[\text{Fe}/\text{H}] \approx +0.3$)

It can be expected that it will be difficult to use NGC 6791 to assess the accuracy of the color- T_{eff} relations for super-metal-rich stars because even an uncertainty of 0.1 dex in its $[\text{Fe}/\text{H}]$ value, or in the m/H number abundance ratios of many of the other metals, may affect some colors (notably $B - V$) more than others (e.g., $V - K_S$). The relatively

high and uncertain foreground reddening ($0.1 \lesssim E(B - V) \lesssim 0.2$; see the discussion by Stetson, Bruntt, & Grundahl 2003) is another complication, as is the cluster helium content, which is generally quite hard to determine in old, metal-rich open clusters. Fortunately, however, eclipsing binaries have been found in this system, and the mass–radius diagram derived from such stars can be used to determine the dependence of Y on $[\text{Fe}/\text{H}]$ through comparisons with stellar models (Grundahl et al. 2008).

In fact, a thorough study of NGC 6791 and its binary stars is being carried out by K. Brogaard et al. (2010, in preparation). Very preliminary results from this investigation suggest that, among other possibilities, the cluster stars have $Y \approx 0.30$ if the adopted $[\text{Fe}/\text{H}]$ value is $+0.30$ (Boesgaard, Jensen, & Deliyannis 2009), assuming that the metals are in the proportions given by Grevesse & Sauval (1998). Moreover, an age near 8.0 Gyr is required to obtain consistent fits to both the mass–radius and color–magnitude diagrams. Since these particular findings are based on models that were computed using the Victoria stellar evolution code, we have chosen to compare just this one isochrone that appears to provide a good fit to the observed properties of the cluster binaries with our BVI photometry for NGC 6791.

As shown in Figure 5, this isochrone provides a rather good fit to the cluster MS stars on the $[(V - I)_0, M_V]$ -plane if $E(B - V) = 0.15$, $E(V - I)/E(B - V) = 1.356$ (McCall 2004), $(m - M)_V = 13.57$, and either the CRMBA or MARCS color transformations are assumed. (The VC03 $V - I$ colors are too red, for reasons that now appear to be understood; see § 3.2.) Given the likelihood that some fraction of the bluest stars at the top of the main sequence are binaries, the isochrone faithfully reproduces the morphology of the CMD from ~ 2.5 mag below the turnoff through to the lower RGB, where the models seem to be ≈ 0.05 mag too red. The adopted reddening agrees well with the determinations from the Schlegel, Finkbeiner, & Davis (1998) dust maps, from the properties of the cluster sdB stars (Kaluzny & Rucinski 1995), and from a fit of the NGC 6791 CMD to the *Hipparcos* CMD for solar neighborhood stars (Sandage, Lubin, & Vandenberg 2003). Indeed, Brasseur et al. (2010) have found that the same isochrone, on the assumption of the same distance and $E(B - V)$ value, provides a fully consistent fit to VJK_S photometry of NGC 6791. To obtain such consistency when the reddening corrections that are applied to the different color indices vary so much — since $E(V - J)/E(B - V) = 2.251$ and $E(V - K_S)/E(B - V) = 2.72$ (McCall 2004) — provides an especially strong argument in support of the adopted reddening.

The fact that the CRMBA- and MARCS-transformed isochrones represent the $V - I$, $V - J$, and $V - K_S$ observations of NGC 6791 so well leads one to suspect that inadequacies in the $(B - V) - T_{\text{eff}}$ relations are responsible for the discrepancies that are apparent in the left-hand panel of Fig. 5. $B - V$ colors that are derived from model atmospheres and synthetic

spectra are bound to be more problematic than those photometric indices that involve redder filter bandpasses than B , especially when the metallicity is high. It is, however, somewhat surprising that the slope of the solid curve is appreciably shallower than that of the observed MS, given that no such problems are apparent in any of the comparisons of isochrones with observations at longer wavelengths.

As a check of how well the CRMBA transformations reproduce the properties of super-metal-rich stars, we have plotted in Fig. 5 the dwarfs from their paper having $0.15 \leq [\text{Fe}/\text{H}] \leq 0.45$, $M_V > 5.4$, and $\sigma_{M_V} \leq 0.15$ (based on parallaxes given in the New Hipparcos Catalogue by van Leeuwen 2007). Although there are relatively few stars, they define quite a tight sequence — especially in the right-hand panel, where the field stars provide a good match to the lower main-sequence of NGC 6791, thereby offering strong support for the adopted distance modulus (if the foreground reddening is $E(B - V) \approx 0.15$). The mean metallicity of the selected stars is $[\text{Fe}/\text{H}] = 0.23$ and, as it should, their mean locus on the $[(B - V)_0, M_V]$ -diagram is slightly to the blue of the solid curve, which assumes $[\text{Fe}/\text{H}] = 0.30$. Thus, the slope of the CRMBA-transformed isochrone in the left-hand panel of Fig. 5 is consistent with that implied by field dwarfs of high metallicity.

Since the reddening of NGC 6791 is fairly high, we also checked whether a scaling of the reddening correction with the intrinsic color of the stars, which is more correct than assuming a constant ratio between different bands (see, e.g., Bessell et al. 1998), could introduce any significant differential shift. For the parameter space relevant to the MS of NGC 6791, the differential effects between $E(B - V)$ and $E(V - I)$ amount to $\lesssim 0.01$ mag, and the consequences for M_V appear to be no more than ~ 0.02 mag. Hence, the steeper MS in the left-hand panel of Fig. 5 cannot be explained in terms of reddening effects. Perhaps chemical abundance peculiarities are responsible for the apparent difficulty of matching the BV observations, or there are some systematic errors in the photometry, or maybe the correct explanation is something entirely different. Hopefully the forthcoming paper by K. Brogaard et al. will shed some light on this issue. As found for the other open clusters that have been considered, the VC03 $(B - V)$ - T_{eff} relations produce the best overall match of the isochrone to the BV photometry for lower MS stars in NGC 6791, though they are also 0.01–0.02 mag too blue in the vicinity of the turnoff. The MARCS transformations appear to be the most realistic ones for super-metal-rich giants.

3.4. Field Subdwarfs ($-2.2 \lesssim [\text{Fe}/\text{H}] \lesssim -0.5$)

Metal-poor dwarfs in the solar neighborhood provide stronger constraints on stellar models than their counterparts in globular clusters because many of the former have well-

determined distances (from *Hipparcos* observations) and their temperatures and metallicities have been determined from high-resolution spectroscopy. Consequently, it makes sense to examine how well our isochrones are able to reproduce the properties of the field subdwarfs before turning our attention to GCs. (Given the possibility of systematic errors in the derived [Fe/H] values of field stars, which are often taken from different spectroscopic studies, one might expect that the slope of the main sequence on the various color-magnitude planes would be more reliably given by GCs. However, photometric calibrations may also suffer from such errors.)

Except for 4 stars, the sample of nearly 100 subdwarfs considered in this section has been drawn from the paper by CRMBA (their Table 8). We have opted to select those stars for which $BV(RI)_C$ photometry is given that have $\sigma_\pi/\pi \lesssim 0.15$ and metallicities in the range $-2.2 \lesssim [\text{Fe}/\text{H}] \lesssim -0.5$. A few stars that did not satisfy these criteria were included either to augment the number of stars with $[\text{Fe}/\text{H}] \lesssim -1.5$ or to increase the number in common to the studies by CRMBA and R. Gratton and collaborators (hereafter referred to as the “Gratton” sample — from Gratton et al. 1996, Clementini et al. 1999, or by private communication; for details, see Bergbusch & Vandenberg 2001). Four additional subdwarfs, not considered by CRMBA, but which satisfied the above constraints, were added to the sample: their basic parameters were derived by Clementini et al. As far as their distribution with metal abundance is concerned, the CRMBA sample is composed of 39, 25, 21, and 8 stars in the four 0.5 dex intervals of [Fe/H] between -0.50 and -2.50 , in turn, whereas the Gratton sample consists of 18, 13, 13, and 4 stars in the same metallicity bins.

It has already been mentioned (see § 1) that isochrones provide a good fit to the classic Population II subdwarfs on the $(\log T_{\text{eff}}, M_V)$ -diagram *if* their temperatures are close to those determined by Gratton et al. (We refer here to the 6–10 stars that have been used for many years to derive the distances to GCs via the main-sequence-fitting technique; see, e.g., Richer, Fahlman, & Vandenberg 1988; Sandquist et al. 1996.) As shown by Vandenberg (2008; see his Fig. 1), these temperature estimates are ~ 75 – 100 K warmer, in the mean, than those derived by, e.g., Alonso, Arribas, & Martinez-Roger (1996); Meléndez et al. (2006); and Cénarro et al. (2007). In fact, CRMBA favor even higher temperatures.

Figure 6 compares the T_{eff} values given by CRMBA and Gratton et al. for the 45 stars in our sample that were studied by both groups. The Gratton temperatures are cooler than those of CRMBA by 27 K, on average, so the former values were increased by this amount prior to being plotted. There is clearly some systematic variation in the scatter of the points about the diagonal “line of equality”, which indicates that the temperature differences actually range from near 0 K for the coolest stars to ≈ 70 K for the warmest ones. Curiously, the [Fe/H] values adopted by CRMBA are an average of 0.07 dex *less*

than the Gratton determinations (see Figure 7), despite the expectation that the warmer temperatures of CRMBA would require higher, not lower, metal abundances in order to match the observed strengths of spectral lines. However, the $[\text{Fe}/\text{H}]$ values given by CRMBA were collected from the literature and, although they are accurate enough for the purpose of their IRFM calculations, which depend only mildly on metallicity, it is not surprising to find the aforementioned differences. *Independent* determinations of a star’s metal abundance can easily, and often do, vary by at least 0.1 dex, which is typically taken to be the uncertainty of such measurements. In any case, such small differences in $[\text{Fe}/\text{H}]$ have only minor effects on the colors that are derived from color– T_{eff} relations, which are mainly a function of temperature (especially with decreasing metallicity).

Since the CRMBA transformations are based on their estimates of the basic properties of the same stars considered here (along with many more), they will necessarily yield color indices that, in the mean, agree very well with those observed. How well, then, will the MARCS and VC03 color– T_{eff} relations be able to reproduce the observed subdwarf colors if the assumed temperatures, gravities, and $[\text{Fe}/\text{H}]$ values are as given by CRMBA? The answer to this question is given in Figure 8. This shows that the predicted $B - V$ indices are too blue by $\sim 0.02\text{--}0.03$ mag (as noted in the uppermost panels), while the offsets in the $V - R$ and $V - I$ colors are $\lesssim 0.01$ mag (see the middle and bottom row of panels). Indeed, the model-atmosphere-based $B - V$ colors, in particular, tend to become more discrepant for redder stars. (It is somewhat disconcerting to find that the MARCS and VC03 transformations to $B - V$ appear to have more difficulty reproducing the observed colors of metal-poor stars, even relatively warm ones, than of those having close to the solar metallicity: recall our analyses of the M 67 and Hyades CMDs. Nevertheless, this is apparently the case.)

If, however, we consider the Gratton sample instead — i.e., we interpolate in the MARCS and VC03 transformations to derive colors on the assumption of the T_{eff} , $\log g$, and $[\text{Fe}/\text{H}]$ values derived by Gratton et al. — the results are quite different. As shown in Figure 9, the predicted $B - V$ indices are now able to reproduce the observed $B - V$ colors quite well, but only at the cost of worsening the agreement in the case of $V - R$ and (especially) $V - I$. [Assuming lower temperatures than those given by Gratton et al. by ≈ 100 K, so as to be closer to the Alonso et al. (1996) and Cennaro et al. (1997) T_{eff} scales, would result in $B - V$, $V - R$, and $V - I$ colors from the MARCS transformations that are an average of 0.017, 0.022, and 0.040 mag too red, respectively. This shows that the relatively low temperatures in such studies as Alonso et al. and Cennaro et al. are not consistent with the photometric T_{eff} scale implied by the CRMBA IRFM zero-points and absolute calibration, together with the MARCS model atmospheres.] If the predicted $V - R$ and $V - I$ colors are more trustworthy than $B - V$, we would conclude that Fig. 8 is closer to the truth than Fig. 9; i.e., the CRMBA T_{eff} scale is more realistic than the one derived by Gratton et al.

As there is already reasonably good agreement between the CRMBA, MARCS, and (to a lesser extent) VC03 $(V - R)$ - T_{eff} and $(V - I)$ - T_{eff} relations, very good consistency of those for $B - V$ can be obtained as well if the MARCS and VC03 transformations to $B - V$ were adjusted to the red by ≈ 0.02 – 0.03 mag.

The next step in our analysis is to determine how well the empirically derived temperatures of the subdwarfs agree with those predicted by stellar models. Usually (see, e.g., VandenBerg 2008) this involves the construction of a so-called “mono-metallicity” subdwarf sequence, whereby isochrones are used differentially to correct the measured T_{eff} of each subdwarf to the temperature it would have, at its observed M_V , if it had a particular (reference) $[\text{Fe}/\text{H}]$ value. Once such adjustments have been made to all of the subdwarfs, thereby compensating for metallicity differences, the latter are superimposed on an isochrone for the reference metallicity and some assessment made of the level of agreement between the two. To ensure that the results are essentially independent of the age of the isochrone, subdwarfs brighter than, say, $M_V = 5.0$ are generally excluded from such comparisons.

A different approach is adopted here. To be specific, those subdwarfs with well-determined M_V values are superimposed directly onto a set of isochrones for a fixed age (12 Gyr) and helium abundance ($Y = 0.25$) and a range in $[\text{Fe}/\text{H}]$ from -2.4 to -0.6 (with $[\alpha/\text{Fe}] \approx 0.4$), when plotted on the $(\log T_{\text{eff}}, M_V)$ -diagram (see the bottom panel of Figure 10). The extent to which isochrones for the *observed* metallicities are able to reproduce the subdwarf luminosities and temperatures clearly provides a powerful test of the models in an absolute sense. As there is a large number of metal-rich stars in the CRMBA sample with precise parallaxes (from van Leeuwen 2007), we have opted to use only those subdwarfs with $-1.0 \leq [\text{Fe}/\text{H}] \leq -0.5$ and $M_V \geq 5.2$ that have $\sigma_{M_V} \leq 0.1$. The majority of the more metal-deficient stars considered here also satisfy these constraints, though any such star with $M_V \geq 4.4$ and $\sigma_{M_V} \leq 0.15$ was included because of the paucity of good subdwarf standards with $[\text{Fe}/\text{H}] < -1.0$. No star that fulfilled these criteria was rejected, unless it is known to be a binary, even though some of them (e.g., BD +41 3306, HD 145417) appear to have anomalous locations on the H-R diagram relative to those of most of the stars that have similar metal abundances. The resultant data set consists of 33 stars, of which 11 have $[\text{Fe}/\text{H}] < -1.2$ (the filled circles in Fig. 10), and the rest are more metal rich (the open circles).

To demonstrate that age uncertainties do not play a significant role in this comparison of theory with observations, 10 Gyr isochrones for $[\text{Fe}/\text{H}] = -2.40$, -1.40 , and -0.60 have also been plotted (as dashed curves). Only at $M_V \lesssim 5.2$ are the differences in T_{eff} given by isochrones that differ in age by 2 Gyr larger than ~ 30 K. Even though a few of the subdwarfs are brighter than this absolute magnitude, their ages are probably closer to 12 Gyr than to 10 Gyr, if they are coeval with GCs having similar metallicities (see the fits of isochrones

to GC CMDs presented below). Consequently, the differences between the solid and dashed curves may be an overestimate of the actual uncertainties in the isochrone locations due to age. (As most of the subdwarfs are fainter than $M_V = 5.2$, age uncertainties are clearly of no concern for them.)

At the observed M_V of each subdwarf, the isochrones define a relationship between $\log T_{\text{eff}}$ and $[\text{Fe}/\text{H}]$ that can be readily interpolated (or extrapolated) to yield the metal abundance corresponding to the temperature given by CRMBA for that star. The difference between the observed $[\text{Fe}/\text{H}]$ and that implied by the isochrones is plotted as a function of $\log T_{\text{eff}}$ in the middle panel, which indicates that there is rather good consistency between the observations and the isochrones. As noted in this panel, and indicated by the arrow, the mean value of $\delta[\text{Fe}/\text{H}]$ is -0.03 , in the sense that the subdwarfs are slightly more metal-poor than one would infer from their locations relative to the isochrones (in the lower panel). There is no obvious trend in the $\delta[\text{Fe}/\text{H}]$ values with temperature, except in the case of the coolest stars, though unrecognized binaries may be responsible for some fraction of asymmetry. It has long been a puzzle, for instance, why HD 145417 and HD 25329 lie above the mono-metallicity subdwarf defined by HD 64090, HD 103095, HD 134439 and HD 134440 (e.g., see Bergbusch & Vandenberg 2001). Only one star (HD 193901) has a value of $\delta[\text{Fe}/\text{H}]$ larger than ± 0.6 ; hence, the metallicity given by CRMBA for this star is quite inconsistent with its location on the H-R diagram.

One can also interpolate within the isochrones to determine the correction to the temperature of each subdwarf that would be required to place it on the isochrone which has the same metal abundance as the subdwarf. The differences in T_{eff} so derived are plotted in the upper panel of Fig. 10 as a function of $\log T_{\text{eff}}$. Here, as well, the majority of the points lie slightly below the dashed line, which indicates that the temperatures given by CRMBA for these stars are less than those implied by the isochrones: the mean value of δT_{eff} is just -8 K (as noted within the panel and indicated by the arrow). A few stars, including BD +41 3306, HD 193901, HD 145417, and HD 25329, have δT_{eff} offsets that are larger than ± 100 K.⁵ Nevertheless, the temperature scale predicted by the isochrones is clearly very similar to that favored by CRMBA. (Even though there is a tendency for the models to be somewhat too warm relative to the observations of the faintest stars, it must be kept in mind that the comparisons presented in Fig. 10 are *very* sensitive to the adopted $[\text{Fe}/\text{H}]$ values. If the metallicities of the faintest, and hence coolest, stars have been underestimated by as

⁵It is easy to identify the most discrepant points, should anyone wish to do so. Since the same abscissa applies to all three panels, a vertical line through a star’s position in the bottom panel will pass through the points representing that star in the uppermost panels if its $\delta[\text{Fe}/\text{H}]$ and/or δT_{eff} values are within the ranges plotted.

little as 0.15 dex, the temperatures inferred for them from the isochrones would be much more consistent with those tabulated by CRMBA. To be sure, it is also possible that the systematic errors in the model T_{eff} scale are responsible for the noted tendency.)

(As far as we have been able to determine, Gratton et al. have not studied most of the subdwarfs that are identified in Fig. 10. They do provide metallicities and temperatures for all of stars that have been plotted as filled circles, except HD 97320, as well as for HD 201891. A similar analysis of just those 11 stars — not shown here — reveals that their estimates of $[\text{Fe}/\text{H}]$ are 0.02 dex more metal rich and their T_{eff} values warmer by 6 K, in the mean, than those implied by the isochrones. Because the Gratton sample is so small, the remainder of this section will consider only the subdwarfs plotted in Fig. 10 for which CRMBA provide $BV(RI)_C$ photometry as well as their estimates of T_{eff} , $\log g$, and $[\text{Fe}/\text{H}]$.)

Figure 11 repeats the comparisons shown in the previous figure, except that the isochrones have been transformed to the three color planes using the MARCS color– T_{eff} relations. For each subdwarf, the $[\text{Fe}/\text{H}]$ value given by CRMBA minus the $[\text{Fe}/\text{H}]$ value of the isochrone that intersects its location on the CMD is plotted in the middle panel, whereas the uppermost panel plots the observed subdwarf color minus that of the isochrone for the star’s metallicity (at the same absolute magnitude). It is quite clear, for instance, that the isochrones are too blue on the $[(B - V)_0, M_V]$ -diagram, since most of the points have negative $\delta[\text{Fe}/\text{H}]$ values and positive $\delta(\text{color})$ values. The mean offsets are -0.18 dex and 0.031 mag, respectively. That is, if the isochrone $B - V$ colors were adjusted redward by 0.031 mag, the mean residuals would be zero and the consistency between theory and observations would be about as good as one could possibly get without culling stars from the sample.

Because both $V - R$ and $V - I$ are much less sensitive to metallicity than $B - V$, especially at low $[\text{Fe}/\text{H}]$ values (compare the bottom three panels of Fig. 11), moderately large values of $\delta[\text{Fe}/\text{H}]$ translate to small values of $\delta(\text{color})$, as shown in the respective middle and uppermost panels. As a consequence, the apparent trends of the $\delta[\text{Fe}/\text{H}]$ values with $V - R$ and $V - I$ are misleading, except in the case of the reddest stars, which are discrepant in the same sense as found in the previous figure. They simply reflect the fact that the upper MS segments of the isochrones for metal-poor stars are close together. Indeed, the upper panels show that, bluer than $(V - R)_0 = 0.55$ or $(V - I)_0 = 1.0$, the stars are all quite close to the dashed lines, and hence that there is excellent consistency between the predicted and the observed colors (i.e., they differ by $\lesssim 0.02$ mag). Insofar as the reddest stars are concerned, it seems more likely that the discrepancies are indicative of a problem with the subdwarf $[\text{Fe}/\text{H}]$ values than with the model temperatures because the same isochrones provide excellent fits to lower-main-sequence observations of GC stars on the $[(V - I)_0, M_V]$ -diagram to at least $(V - I)_0 = 1.20$ (see the plots presented in the following sections).

The consequences of using the VC03 color– T_{eff} relations instead of the MARCS transformations are shown in Fig. 12. Although the former predict somewhat redder $B - V$ colors than the latter, the observed colors of the subdwarfs are still 0.018 mag redder, on average, than those inferred from the isochrones, reflecting the fact that the observed and predicted $[\text{Fe}/\text{H}]$ values differ by the mean value of -0.13 dex. Interestingly, there is little to distinguish the $\delta[\text{Fe}/\text{H}]$ and $\delta(V - R)$ plots when either the MARCS or VC03 transformations are assumed, but just as was found in our consideration of metal-rich open clusters (see § 3.1 and 3.2), the subdwarfs indicate that the VC03 transformations to $V - I$ are too red by about 0.02 mag. It is also worth noting that the open circles, which represent stars having $[\text{Fe}/\text{H}] > -1.2$, exhibit much more scatter than the filled circles, which represent lower metallicity stars. Metal abundance uncertainties will translate into a larger scatter at higher $[\text{Fe}/\text{H}]$ values simply because the dependence of $V - I$ on $[\text{Fe}/\text{H}]$ increases as the metallicity increases.

Figure 13 shows that VR2010 isochrones, together with the CRMBA empirical color– T_{eff} relations, provide the best overall match to the local subdwarfs on the three color planes considered in this investigation. For the $B - V$, $V - R$, and $V - I$ panels, in turn, the mean values of $\delta[\text{Fe}/\text{H}]$ are -0.04 , $+0.10$, and $+0.04$ dex, whereas the mean values of $\delta(\text{color})$ are $+0.007$, -0.003 , and -0.004 mag. Such small differences in the mean offsets indicate that there is very good consistency between the models and the best-observed subdwarf standards if their properties (i.e., temperatures, gravities, and metallicities) are close to those adopted by CRMBA. The $V - R$ and $V - I$ observations of the most metal-deficient subdwarfs, in particular (i.e., the filled circles), are especially well reproduced by the isochrones. It is also worth pointing out that some stars, which appeared anomalous in the $[\log T_{\text{eff}}, M_V]$ -diagram presented in Fig. 10 (notably BD +41 3306 and HD 25329) are well matched by the isochrones for the observed metallicities on the $V - R$ and $V - I$ color planes, in particular (see Fig. 13). The latter also shows that some stars (e.g., HD 144579, HD 216777) are matched by isochrones having quite different $[\text{Fe}/\text{H}]$ values on different CMDs. It would be worth the time and effort to study such stars (indeed, all of the subdwarfs) further to improve our understanding of these very important calibrators of GC distances. We now turn to a consideration of a small number of GCs with $[\text{Fe}/\text{H}]$ values spanning the range in $[\text{Fe}/\text{H}]$ from -0.8 (47 Tucanae) to -2.4 (M 92).

3.5. 47 Tucanae ($[\text{Fe}/\text{H}] \approx -0.8$)

Using an innovative statistical analysis of new, extensive photometry for 47 Tucanae, Bergbusch & Stetson (2009) have produced particularly tight and well-defined fiducial se-

quences for this GC on the $B - V$, V and $V - I, V$ planes. They showed that Victoria-Regina isochrones (VandenBerg et al. 2006) for $[\text{Fe}/\text{H}] = -0.83$ and $[\alpha/\text{Fe}] = 0.3$ reproduced these sequences very well from several magnitudes below the turnoff to the RGB tip if $E(B - V) = 0.04$ and $(m - M)_V = 13.375$. The assumed distance modulus was obtained from a fit of a theoretical zero-age horizontal branch (ZAHB) locus for the aforementioned metallicity to the lower bound of the distribution of cluster HB stars, as well as from a fit of the cluster MS to local subdwarfs having a similar metal abundance. Guided by these results, we have chosen to compare VR2010 isochrones for $[\text{Fe}/\text{H}] = -0.80$, which is close to the latest estimates from high-resolution spectroscopy (Koch & McWilliam 2008; Carretta et al. 2009), and $[\alpha/\text{Fe}] \approx 0.4$ to the Bergbusch-Stetson CMDs. (The assumed enhancements in the abundances of the individual α -elements range from 0.25 to 0.50, with a mean value of about 0.4; see the VR2010 paper.)

As shown in Figure 14, fits of the cluster fiducials to the CRMBA-transformed isochrones for these abundances at $M_V \gtrsim 6$ yield $(m - M)_V \approx 13.40$ if $E(B - V) = 0.032$ (Schlegel et al. 1998) and $E(V - I)/E(B - V) = 1.356$ (McCall 2004). (This is equivalent to performing main-sequence fits to the local subdwarfs, in view of the fact that the CMD locations of such stars are quite well represented by isochrones that employ the CRMBA color- T_{eff} relations; as shown in the previous section.) The apparent distance modulus which is derived in this way is clearly a compromise since the observations are offset from the model loci in different directions in the two panels: the solid curve lies along the blue edge of the 47 Tuc fiducial on the $[(V - I)_0, M_V]$ -plane, while it coincides with the red edge of the MS observations in the $[(B - V)_0, M_V]$ -diagram. In order for the models to match the luminosity of the cluster subgiant branch, an age near 11 Gyr is required. (Our age estimate is 1 Gyr less than the age inferred from the same data set by Bergbusch & Stetson. We note that, in addition to minor differences in the adopted chemical abundances and the derived distance, only the isochrones used in the present study take the gravitational settling of helium into account. This has the effect of reducing the age at a given turnoff luminosity by ~ 8 –10%.)

The largest discrepancies between theory and observations occur along the RGB in the left-hand panel of Fig. 14, which suggests that the model-atmosphere-based transformations to $B - V$ for low-gravity stars having $[\text{Fe}/\text{H}] \approx -0.80$ are up to ~ 0.1 mag too blue (though this is not necessarily the correct explanation; see the next section). If a higher metal abundance were assumed, the corresponding isochrones would undoubtedly provide a better match to the observed RGB on the $[(B - V)_0, M_V]$ -diagram, but at the expense of worsening the fit of the models to the VI photometry. The main mismatch in the right-hand panel occurs near the turnoff, where the isochrones are too blue independently of the color transformations that are used. We are not able to provide a good explanation for this problem, as the photometric calibrations appear to be robust, and anything that alters the model T_{eff}

scale would work in opposite directions in the two color planes.

It is not impossible that the CRMBA transformations to $B - V$ for $[\text{Fe}/\text{H}] \approx -0.80$ and temperatures appropriate to the MS and turnoff stars of 47 Tuc are too red by ~ 0.02 mag. Indeed, the comparison of the CRMBA-transformed isochrones to the local subdwarfs with $[\text{Fe}/\text{H}] \geq -1.2$ (the open circles in the left-hand panels of Fig. 13) would be improved if the model colors were adjusted to the blue by ≈ 0.02 mag. Such an adjustment would also result in better consistency of the fits to the BV and VI observations, even though some difficulties would persist in an absolute sense. (In this and some of the other figures contained in this paper, there is a tendency for the CRMBA-transformed isochrones to be too red just at the base of the giant branch. This is possibly a consequence of the fact that there is no explicit gravity dependence in these color– T_{eff} relations and/or reflect the tendency of the adopted functional form to diverge at the coolest temperatures. Further work is clearly needed to resolve these issues.) We note, finally, that isochrones using the VC03 semi-empirical transformations provide a better fit to the *slope* of the lower MS on both color planes than isochrones that employ the MARCS transformations — as found in our consideration of metal-rich open clusters as well.

3.6. NGC 1851 and M 5 ($[\text{Fe}/\text{H}] \approx -1.4$)

Unfortunately, NGC 1851 was not one of the 19 globular clusters that was used by Carretta et al. (2009) to calibrate their new metallicity scale, nor was it among the sample that defined the original Carretta & Gratton (1997) scale. Although the former provide updated metallicities for most of the Galactic GCs, by performing weighted averages of data from different sources, their estimate of the metal abundance of NGC 1851, which is $[\text{Fe}/\text{H}] = -1.18$, may be too high by ~ 0.2 dex — notwithstanding the fact that a similar value was obtained by Kraft & Ivans (2003) from their spectroscopic analysis of Fe II lines. According to Zinn & West (1984), whose metallicity scale is still widely used, NGC 1851 has $[\text{Fe}/\text{H}] = -1.36$, which is only 0.04 dex greater than their value for M 5. Interestingly, the Carretta-Gratton determination of -1.11 for M 5 has been revised to -1.35 in the 2009 study by Carretta et al. (who used M 5 as one of the primary calibrating clusters). Also worth noting is the fact that stellar models have tended to favor $[\text{Fe}/\text{H}] \approx -1.4$ for both M 5 and NGC 1851 (e.g., VandenBerg 2000) for the following reason.

If the $E(B - V)$ values given by the Schlegel et al. (1998) dust maps are assumed, and the dereddened CMDs for these two clusters are overlaid in such a way that their red horizontal branch populations have the same luminosities, then their main sequences superimpose on one another almost perfectly. What does differ, as discussed recently by

Stetson (2009), is the location of their giant branches: the RGB of NGC 1851 is significantly redder than that of M 5 at the same M_V (on all CMDs). The cause of this color shift is not known, though differences in the $[m/Fe]$ ratios of some elements could well be the explanation given that recent evolutionary computations (see Dotter et al. 2007; VR2010) have shown that variations in the abundances of some metals (notably Mg and Si) will have important consequences for the temperatures, and therefore the colors, of giant stars, even at low $[Fe/H]$ values.

What is important for the present investigation is that (i) if $E(B - V) = 0.034$ (Schlegel et al. 1998) and $(m - M)_V \approx 15.50$, which is obtained from MS fits of the cluster fiducials to the CRMBA-transformed isochrones (and hence to the subdwarf standards), then both the BV and VI observations *for the lower main-sequence stars* in NGC 1851 can be matched quite well by theoretical isochrones for $[Fe/H] = -1.40$ and $[\alpha/Fe] \approx 0.4$ (see Figure 15), and (ii) the same isochrones provide an equally good match to the MS of M 5 if its reddening and distance are similarly derived (see Figure 16). Indeed, as already mentioned, this is to be expected if the assumed distance moduli are such that the red HB populations in the two GCs are made coincident. If the $[Fe/H]$ values of NGC 1851 and M 5 truly do differ by ≈ 0.2 dex (and if their reddenings have been accurately determined), then it must be the case that variations in the abundances of some other elements compensate for the difference in iron content in such a way as to produce CMDs for their MS stars that are nearly identical. (VR2010 models will be used to explore this possibility.)

As our R photometry for NGC 1851 is not of the same quality as B , V , and I , little can be said about the predicted $V - R$ colors for MS stars except that they appear to be too red by ~ 0.03 mag in the vicinity of the turnoff. The main point that can be made about the $(V - R) - T_{\text{eff}}$ relations is that the cluster giants are consistently fitted by the isochrones on both the $V - R$ and $V - I$ color planes. Fortunately, the fiducial sequence for dwarf stars is much better defined in M 5 than in NGC 1851, and the middle panel of Fig. 16 indicates that the isochrones actually provide a good fit to the observations at $M_V \gtrsim 5$. To have a completely consistent interpretation of the data at $3 \lesssim M_V \lesssim 5$ on all three color planes, the isochrone $V - R$ colors should be corrected to the blue by ≈ 0.02 mag, or less, if some fraction of this offset can be attributed to the calibration of the photometry. In fact, the observed $V - R$ colors have an uncertainty of at least ± 0.01 mag. Because the same isochrones reproduce the entire CMD of M 5 on the $[(B - V)_0, M_V]$ -diagram, including the RGB, the failure of the models to match the observed giants in the left-hand panel of Fig. 15 cannot be due to problems with just the color transformations. (Whatever is causing the mismatch between theory and the $B - V$ colors may also be occurring in 47 Tuc, given that similar discrepancies are evident for this system — see Fig. 14).

Indeed, one cannot say very much about the reliability of different color transformations using consistency arguments (i.e., from how well isochrones reproduce observations on different color planes) when GCs of apparently quite similar metallicities and ages, like NGC 1851 and M 5, can have significant variations of $\Delta(\text{color})_{\text{TO,RGB}}$, which is the difference in color between the turnoff and the RGB when measured at a fixed value of δV brighter than the turnoff. Because M 5 has a smaller value of $\Delta(\text{color})_{\text{TO,RGB}}$ than NGC 1851, it will not be possible to obtain satisfactory fits of isochrones to the RGBs of both clusters on different CMDs. It is surprising, however, that the models appear to reproduce the RGB of NGC 1851 so well on the $[(V - I)_0, M_V]$ -plane while providing a comparably good fit to the giant branch of M 5 on the $(B - V)_0, M_V]$ -plane (and vice versa for the cluster MS fiducials). Although Figs. 15 and 16 provide good support for the various color- T_{eff} relations applicable to dwarf stars having $[\text{Fe}/\text{H}] \approx -1.4$, the color transformations that are applicable to giants of the same metallicity cannot be assessed until we have a good understanding of the cause(s) of the difference in $\Delta(\text{color})_{\text{TO,RGB}}$ between NGC 1851 and M 5.

3.7. M 3 ($[\text{Fe}/\text{H}] \approx -1.6$)

M 3 is often considered to be the prototype of the intermediate-metal-poor group of GCs because it has a normal HB for its metallicity (relative to the expectations from stellar models) and its giants do not show the same degree of chemical abundance anomalies that are seen in many other systems of similar metallicity (e.g., giants in M 13, but not in M 3, have super-low oxygen abundances; see Kraft et al. 1992). However, estimates of its iron content have varied considerably over the years, from $[\text{Fe}/\text{H}] = -1.66$ (Zinn & West 1984) to -1.34 (Carretta & Gratton 1997) to -1.50 (Kraft & Ivans 2003; Carretta et al. 2009). As in the case of NGC 1851, comparisons of isochrones with the CMD of M 3 tend to favor a value near the low end of this range (e.g., Vandenberg 2000). This has not changed, despite on-going improvements to both the photometric data and the theoretical models.

In Figure 17, a VR2010 isochrone for $[\text{Fe}/\text{H}] = -1.60$ is compared with BVI observations for M 3, on the assumption of $E(B - V) = 0.013$ (Schlegel et al. 1998) and $(m - M)_V = 15.00$, which is derived from a main-sequence fit of the cluster photometry to the isochrones on the $[(V - I)_0, M_V]$ -diagram, and which agrees well with recently published estimates (see, e.g., Rood et al. 1999, Cacciari, Corwin, & Carney 2005). Our B photometry is not as deep as those for V and I ; consequently, the cluster’s principal sequence is not as well defined in the left-hand panel as in the right-hand panel. Still, it is a little disconcerting that the CRMBA-transformed isochrone is ~ 0.02 mag redder in the left-hand panel than it should be to provide a completely consistent interpretation of both the BV and VI observations.

According to the comparison of our isochrones with the most metal-deficient subdwarfs in Fig. 13 (the filled circles), the $B - V$ colors of the models appear to be, if anything, slightly too blue (not too red). Whether this discrepancy is due, in part, to the assumption of incorrect metal abundances for M 3 or to calibration errors is not known.

The fact that the isochrones provide such a good match to the entire RGB of M 3 on both color planes reinforces our contention that the difficulties that were encountered when analyzing observations of NGC 1851 and M 5 are not due simply to problems with the color- T_{eff} relations, but instead suggest that the models themselves are lacking in some fundamental way (see the discussion in the previous section). In fact, NGC 1851 has recently been discovered to have a double SGB (Milone et al. 2008), which is independently confirmed by our data (see Stetson 2009; Milone et al. 2009). The second (fainter) SGB is not evident in our plots because, to maximize the clarity of the comparison between the observations and the theoretical loci, we have plotted only a small, representative sample of photometric measurements selected to have the highest accuracy and precision. The relatively sparse population on the second SGB is not numerous enough to be evident in this sample. It has also been discovered recently that NGC 1851 shows chemical abundance anomalies, which are consistent with the hypothesis that the present stellar populations in this GC formed out of gas ejected by the asymptotic-giant-branch stars from a previous generation (Yong et al. 2009). Thus, there is some justification for believing that the assumed mix of heavy elements in the isochrones which we have fitted to the CMDs of (at least) NGC 1851 is not realistic.

3.8. M 92 ($[\text{Fe}/\text{H}] \approx -2.4$)

M 92 presents us with a somewhat different puzzle. As shown in Figure 18, isochrones for $[\text{Fe}/\text{H}] = -2.40$, with the usual enhancement in the abundances of the α -elements, are able to reproduce the detailed CMD morphology of this cluster on the $B - V$ and $V - R$ color planes *reasonably* well if canonical estimates of the reddening, $E(B - V) = 0.023$ (Schlegel et al. 1998), and distance modulus, $(m - M)_V = 14.62$, are assumed. (The latter is obtained by matching the SGB of M 92 to the field subgiant, HD 140283, which has close to the same metallicity as the GC; see VandenBerg et al. 2002.) However, to obtain a fully consistent fit of the isochrones to the cluster observations on the $[(V - I)_0, M_V]$ -plane as well, a significant blueward correction to the isochrone $V - I$ color indices is required (as indicated). Of all the star clusters considered in this investigation, this is the only one where the models fail to match the observed $V - I$ colors for MS stars, on the assumption of what we consider to be best estimates of the metallicity, reddening, and distance, without having to correct the isochrones in some way.

It is worth noting that the adopted metallicity agrees well with recent determinations of the $[\text{Fe}/\text{H}]$ value of M 92 from high-resolution spectroscopy, which lie in the range of -2.35 to -2.40 (Kraft & Ivans 2003; Carretta et al. 2009). Had models for $[\text{Fe}/\text{H}] \approx -2.2$ been selected, to be in better agreement with the original Carretta & Gratton (1997) and Zinn & West (1984) estimates of -2.16 and -2.24 , respectively, they would have been offset to the red by even larger amounts. What makes M 92 especially intriguing is that (i) excellent consistency on all three color planes is obtained if a small zero-point correction is applied to the model $V - R$ colors (an amount that is well within calibration uncertainties) and a constant offset of ≈ -0.025 mag is applied to the $V - I$ colors, and (ii) even larger adjustments (in the same direction) appear to be necessary to match the RGB segments of the same isochrones to the M 92 giants on the $[(V - J)_0, M_V]$ - and $[(V - K)_0, M_V]$ -diagrams (Brasseur et al. 2010). In the case of the near-IR CMDs, the MS stars can be fitted quite well by the isochrones, but the giants are as much as ~ 0.12 mag bluer in $V - K$ than the isochrones.

Careful inspection of Fig. 18 reveals another anomaly; namely, that the MARCS and CRMBA transformations for $[\text{Fe}/\text{H}] = -2.4$ yield fairly similar $B - V$ colors but different $V - I$ colors (by ~ 0.02 mag) along the MS, which is opposite to what was found at higher metallicities and opposite to what was inferred from the subdwarfs. (However, there are no subdwarfs in our sample with $[\text{Fe}/\text{H}] \approx -2.4$, and only four stars with $[\text{Fe}/\text{H}] < -1.5$. The most metal-poor one is HD 19445, which has $[\text{Fe}/\text{H}] \approx -2.0$; consequently, these stars do not provide any constraint on the color- T_{eff} relations for stars as metal-deficient as those in M 92.) Figure 19 compares the isochrones for $[\text{Fe}/\text{H}] = -2.40$ and -1.40 that have been used in this study. At $[\text{Fe}/\text{H}] = -1.40$, the CRMBA- and MARCS-transformed isochrones predict nearly the same $V - I$ and $V - R$ colors, but the $B - V$ indices differ by ~ 0.02 – 0.03 mag at the same absolute magnitude. At $[\text{Fe}/\text{H}] = -2.40$, there are only small differences between the predicted $B - V$ colors, whereas the $V - R$ and $V - I$ colors for dwarf stars are, in turn, ≈ 0.01 and ≈ 0.02 mag bluer, at a given M_V , than those obtained from the MARCS color- T_{eff} relations. Given the many uncertainties at play, it is difficult to determine whether these findings are trustworthy.

The bottom line is that we are unable to obtain a fully consistent explanation of the optical photometry of M 92 (or the near-IR data, judging from the work of Brasseur et al. 2010). Isochrones for the current best estimate of the cluster metallicity are able to reproduce the observed $[(B - V)_0, M_V]$ -diagram quite well, and aside from a zero-point offset of ≈ 0.025 mag, they provide a good match to the entire $[(V - I), M_V]$ -diagram. However, the same models apparently suffer from systematic errors when compared with $V - J$, V and $V - K_S$, V observations (see Brasseur et al.), insofar as they provide a reasonable fit to the MS stars, but not to the giants. There is no way in which the evolutionary calculations could be modified

to reconcile these conflicting indications. Consequently, errors from several sources — the models, the color– T_{eff} relations, the photometric data themselves, and perhaps the basic cluster parameters (reddening, distance, and chemical composition) — must be conspiring to cause the problems described above. A resolution of the M 92 conundrum must be left to future work.

4. Summary

This investigation has examined how well up-to-date theoretical isochrones that take the diffusion of helium into account are able to satisfy various observational constraints when they are transformed from the theoretical to the $(B - V, V)$ -, $(V - R, V)$ -, and $(V - I, V)$ -diagrams using the CRMBA, MARCS, and VC03 color transformations. In fact, the differences between the three sets of color– T_{eff} relations that have been considered are relatively minor, especially in the case of stars having close to the solar metallicity. Our consideration of the Hyades and M 67 has shown that, aside from the need to apply a blueward correction of ~ 0.02 mag to the $V - I$ colors given by VC03, isochrones are able to reproduce the observed CMDs very well, independently of which of the three color transformations are used. Only at $M_V \gtrsim 6.5$ (or fainter in the case of $V - I$) do the models fail to match the observed fiducial sequences: the MARCS-transformed isochrones deviate to the blue, possibly because of insufficient blanketing in the model atmospheres for cool stars on which they are based, while the opposite is found in the case of the CRMBA-transformed isochrones, which is likely due to the limitations of the analytic expressions used to present these empirical transformations. However, we were unable to obtain perfectly consistent fits of isochrones to BV and VI photometry for NGC 6791. Whether this is indicative of a problem with the color– T_{eff} relations for super-metal-rich stars, the assumed chemical abundances, or any of the other factors that play a role in such comparisons is not known.

One of the most striking results of this work is that the hot T_{eff} scale derived by CRMBA is in remarkable agreement with that predicted by stellar models. Comparisons of isochrones with 33 nearby subdwarfs having $[\text{Fe}/\text{H}]$ values between -2.0 and -0.5 , with well-determined M_V values from *Hipparcos*, have shown that the mean metallicities and temperatures that are inferred for the stars from their locations relative to the models on the $(\log T_{\text{eff}}, M_V)$ -plane agree with those given by CRMBA to within $\delta[\text{Fe}/\text{H}] = 0.05$ dex, and $\delta T_{\text{eff}} = 10$ K, respectively (see Fig. 10), which is obviously well within the uncertainties. Not surprisingly, because the CRMBA color– T_{eff} relations are based on a large sample of stars that includes the 33 subdwarfs, similar consistency is found on the $B - V$, $V - R$, and $V - I$ color planes. When the same comparisons are made using isochrones that employ the MARCS

transformations, the predicted $B - V$ colors are found to be too blue by about 0.03 mag, while the inferred $V - R$ and $V - I$ colors agree quite well with those observed. Why the MARCS $(B - V) - T_{\text{eff}}$ relations would be more problematic for metal-deficient stars than for those having $[\text{Fe}/\text{H}] \gtrsim 0.0$ is not clear, but if the MARCS transformations to $V - R$ and $V - I$ are more trustworthy, then the T_{eff} scale implied by the MARCS model atmospheres is not significantly different from the empirical one derived by CRMBA. Both give warmer temperatures by ~ 75 – 120 K than, e.g., Alonso et al. (1996) and Cennaro et al. (2007).

While these results depend quite critically on the adopted $[\text{Fe}/\text{H}]$ values of the subdwarfs (something that should be kept in mind), color– T_{eff} relations are not, by themselves, very dependent on the metal abundance (especially at lower metallicities). Consequently, it is reassuring to find that similar conclusions are reached regarding the MARCS transformations when the colors of ~ 100 local subdwarfs and subgiants are compared with those obtained by interpolating in the MARCS color tables for the values of T_{eff} , $\log g$, and $[\text{Fe}/\text{H}]$ given by CRMBA for those stars. As shown in Fig. 8, the predicted $B - V$ indices are ≈ 0.03 mag too blue, in the mean, while the predicted $V - R$ and $V - I$ colors agree very well with those observed. When subjected to the same tests, the VC03 transformations fare comparably, or less, well (depending on the color index considered), though they do enable models to provide satisfactory matches to the lower-MS slopes of observed CMDs. However, except for providing some guidance concerning the variation of the colors of cool MS stars with temperature, the VC03 color– T_{eff} relations are no longer very useful: they have effectively been superseded by the new MARCS transformations. Because they provide very good consistency on *all* color planes, the empirical transformations of CRMBA are the preferred ones to use for dwarf and SGB, but not necessarily lower RGB, stars.

Although we are able to obtain reasonably consistent fits of the same isochrones to the dwarf and SGB populations of GCs on different color-magnitude planes — when well constrained estimates of reddening, distance, and metallicity are assumed — the cluster RGBs are much more problematic. In the case of M 3 (see Fig. 17), the models reproduce the observed $(B - V)_0$ and $(V - I)_0$ colors of the cluster giants quite well, but more often than not, isochrones are able to reproduce the VI photometry, but not the BV observations, along the cluster giant branch (e.g., see Fig. 14 for 47 Tucanae and Fig 15 for NGC 1851), *or vice versa* (e.g., see Figs. 16 and 18 regarding M 5 and M 92, respectively). In view of recent work which has shown that the location of the RGB on the H-R diagram is a sensitive function of the mix of heavy elements (Dotter et al. 2007; VR2010), we suspect that differences between the assumed and actual metal abundances may be the main cause of the noted difficulties. Indeed, the temperatures of MS stars can also be affected by variations in the abundances of such elements as Mg and Si, though to a lesser extent, which clearly complicates the interpretation of GC CMDs. Follow-up studies must be undertaken

to explore how color- T_{eff} relations are modified by such chemical abundance variations and to determine whether or not the resultant transformations lead to improved fits of theoretical isochrones to observations of GCs (on all color planes) compared with those presented here.

We note, finally, that it appears to be impossible to reconcile stellar models with all of the available photometric data for M 92. The $BV(RI)_C$ observations alone do not pose a serious problem, as isochrones for $Y = 0.25$, $[\text{Fe}/\text{H}] = -2.40$, and $[\alpha/\text{Fe}] \approx 0.4$ match the entire CMD rather well on the $B - V$, $V - R$, and $V - I$ color planes, provided that the predicted $V - I$ colors are adjusted to the blue by a small, constant amount (0.025 mag). While it is odd that the $V - I$ indices appear to be more problematic than the $B - V$ colors, what is really unexpected is that the same isochrones show large, and systematic, discrepancies when compared with $V - J$ and $V - K_S$ observations of M 92 — which are from the 2MASS catalogue in the case of the cluster giants (see Brasseur et al. 2010). The isochrones fit the MS and the upper RGB satisfactorily, but they are too red by $\delta(V - K_S) \approx 0.12$ mag just above the base of the giant branch. Is it possible that the MARCS model atmospheres for very metal-deficient upper MS and RGB stars are too bright in the near-IR? It seems unlikely, but this and other possible explanations need to be investigated. As a footnote to the main results of this study, it is worth pointing out that we find a significant dependence of GC ages on metallicity. Isochrones that faithfully reproduce the properties of local subdwarfs with accurate distances from *Hipparcos* predict that the ages of these systems vary from ≈ 13.5 Gyr at $[\text{Fe}/\text{H}] \approx -2.4$ (M 92) to ≈ 11 Gyr at $[\text{Fe}/\text{H}] \approx -0.8$ (47 Tuc). A similar variation was found by VandenBerg (2000), who used empirically constrained HB luminosities to establish the GC distance scale.

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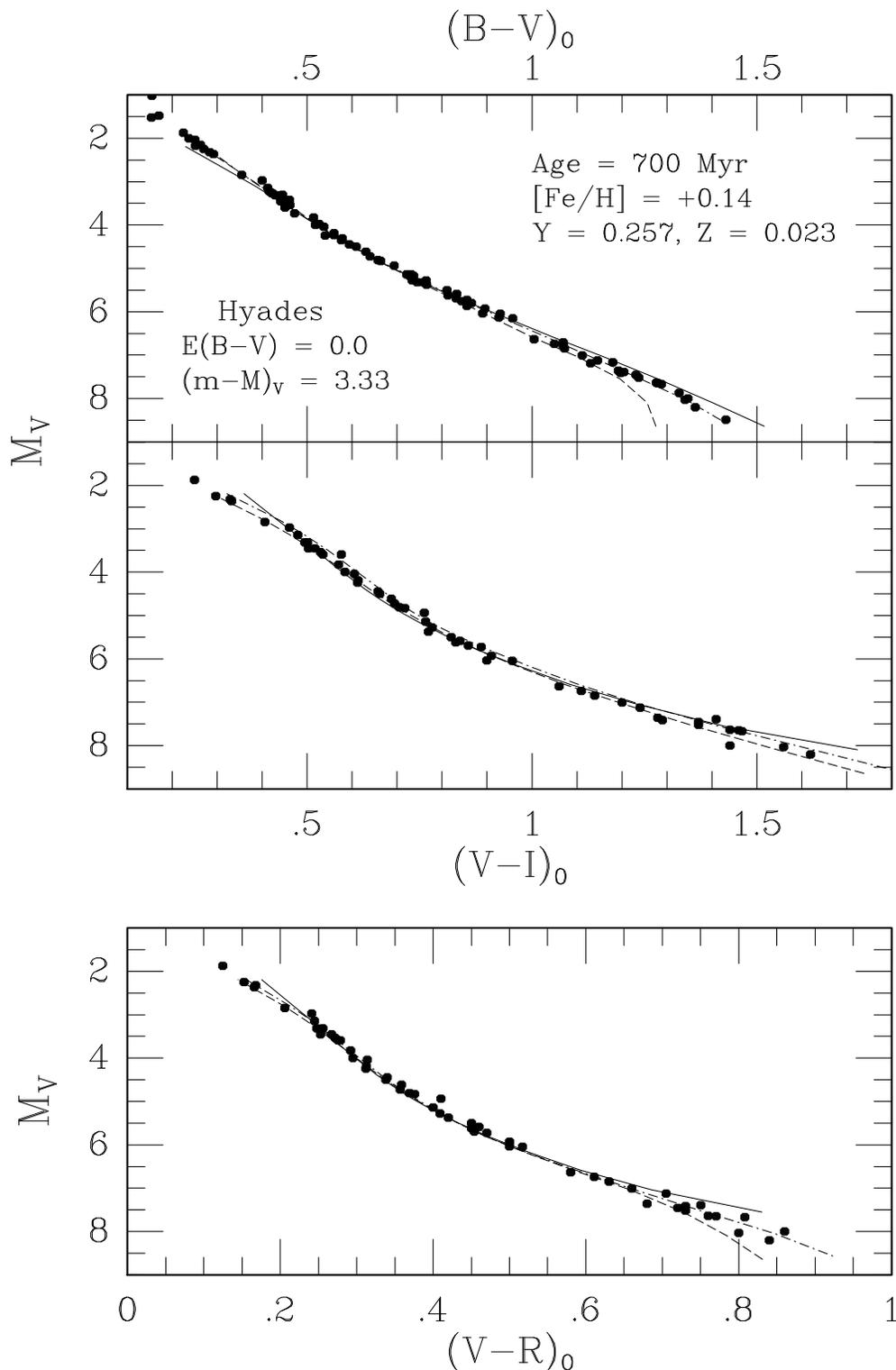


Fig. 1.— Comparison of an isochrone for the indicated age and chemical abundances with photometry of the Hyades on three different color- M_V diagrams assuming the CRMBA, MARCS, and VC03 color transformations (solid, dashed, and dot-dashed curves, respectively) and $(m - M)_V = 3.33$. The sources of the photometry are mentioned in the text.

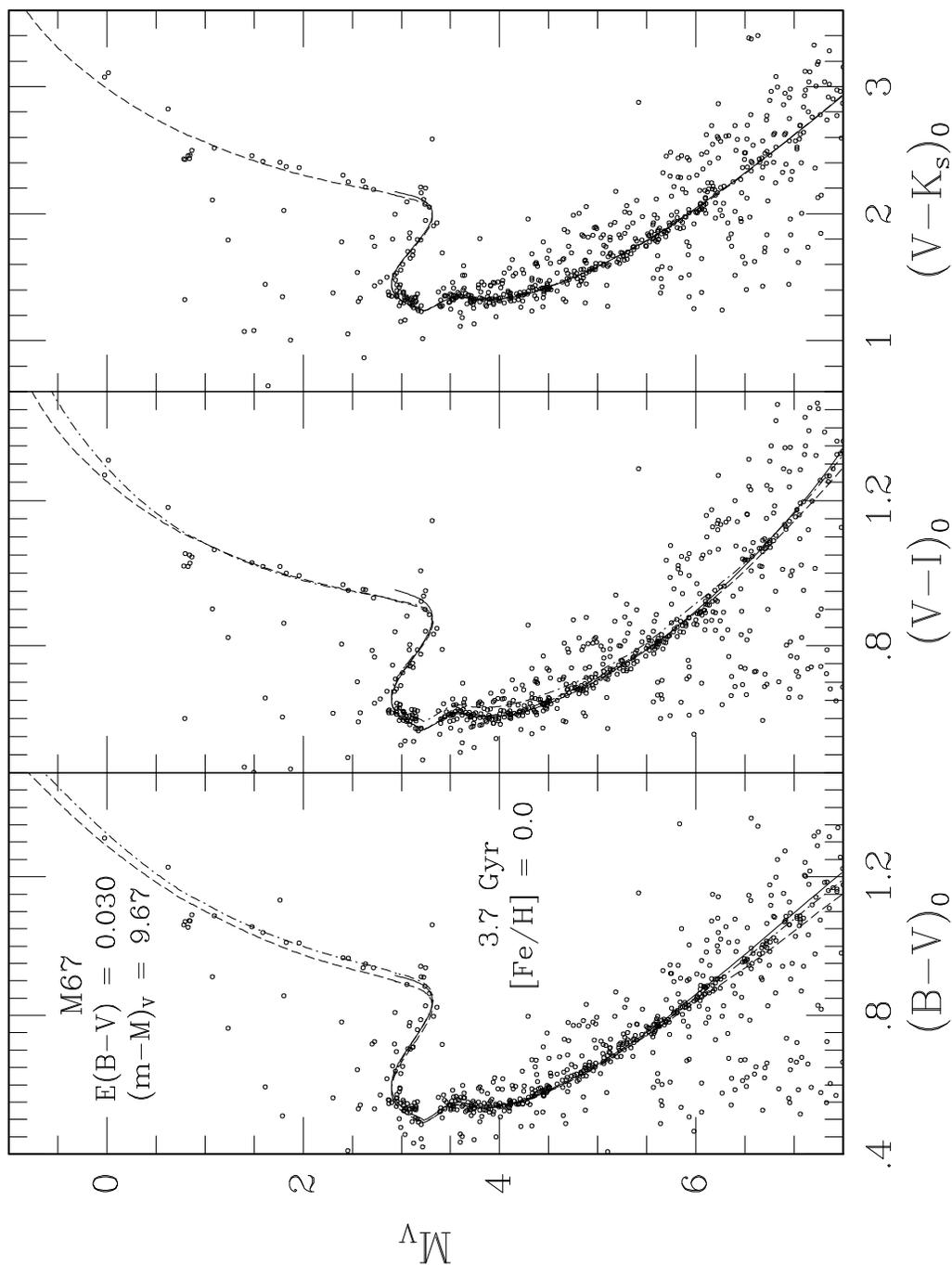


Fig. 2.— Similar to the previous figure, except that a 3.7 Gyr isochrone for the solar metallicity (from Michaud et al. 2004) has been overlaid onto three different CMDs for M67. The solid, dashed, and dot-dashed curves represent, in turn, the assumption of the CRMBA, MARCS, and VC03 color transformations. The latter predict $V - I$ colors that are ~ 0.02 mag too red. Note that the BVI photometry plotted here was newly reduced by one of us (PBS).

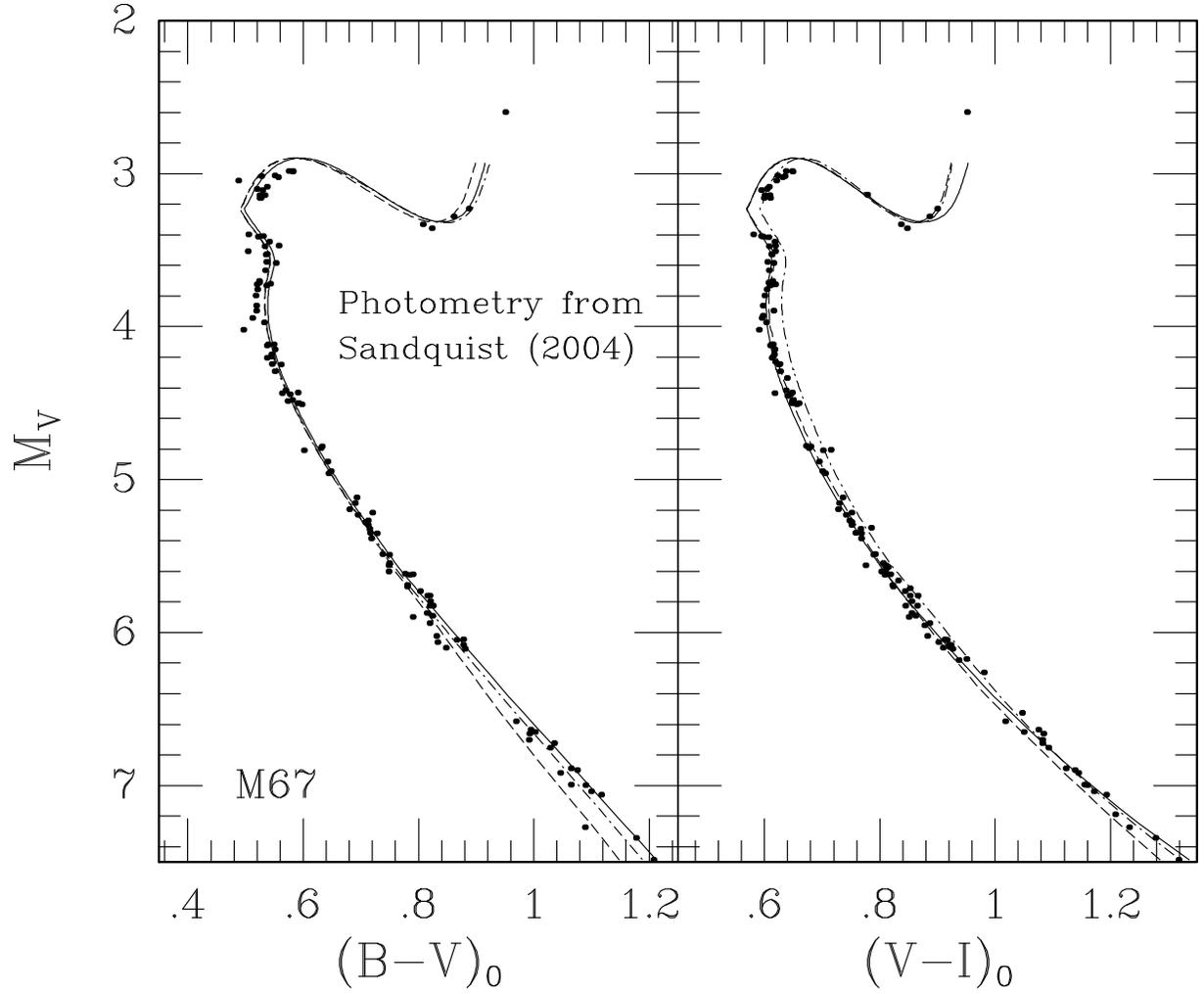


Fig. 3.— Similar to the middle and left-hand panels of Fig. 2 except that the photometric observations are from Sandquist (2004).

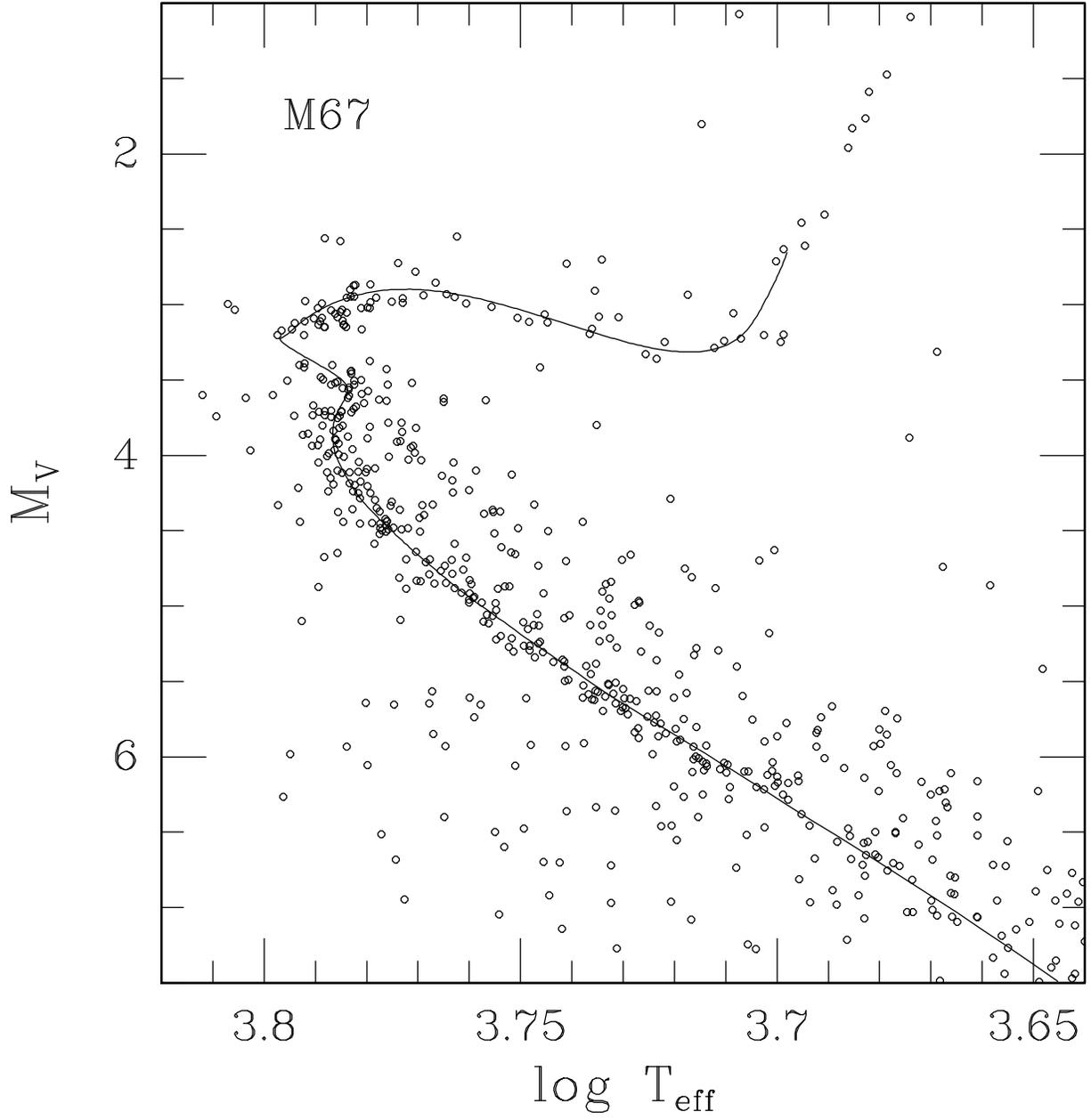


Fig. 4.— Similar to Fig. 2 except that the comparison is made on the $(\log T_{\text{eff}}, M_V)$ -diagram. The temperatures of M67 stars have been derived from their $(V - K_S)_0$ indices using the CRMBA color transformations.

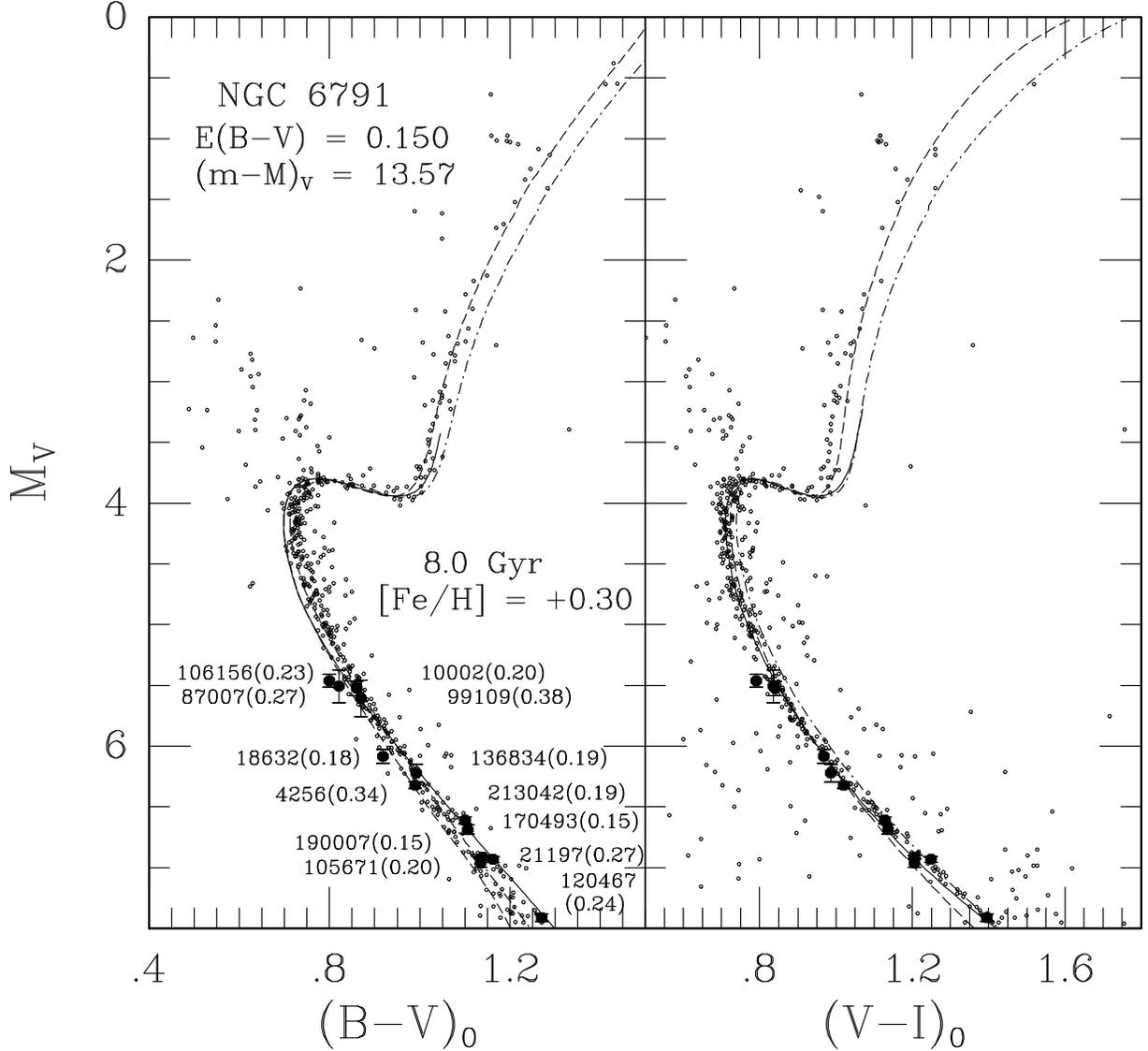


Fig. 5.— Comparison of an 8.0 Gyr isochrone for $[Fe/H] = +0.30$ and $Y = 0.30$ with current BV and VI observations for NGC 6791 from the on-going Stetson (2000) project. The adoption of the CRMBA, MARCS, and VC03 color transformations are represented, in turn, by the solid, dashed, and dot-dashed loci, respectively. Filled circles represent those stars in the sample studied by CRMBA that have $0.15 \leq [Fe/H] \leq 0.45$ and 1σ uncertainties in their M_V values of ≤ 0.15 mag based on *Hipparcos* parallaxes (van Leeuwen 2007). The stars, which all have $E(B - V) = 0.0$ according to CRMBA, are identified by their “HD” numbers: their $[Fe/H]$ values (from CRMBA) are given within the parentheses. In the right-hand panel, the observed turnoff is bluer than the values predicted by the CRMBA, MARCS, and VC03 loci by about 0.005, 0.015, and 0.03 mag, respectively. In the left-hand panel, the observed turnoff is ~ 0.03 mag redder than the turnoffs of the CRMBA- and MARCS-transformed isochrones, and ≈ 0.012 mag redder than the VC03-transformed isochrone.

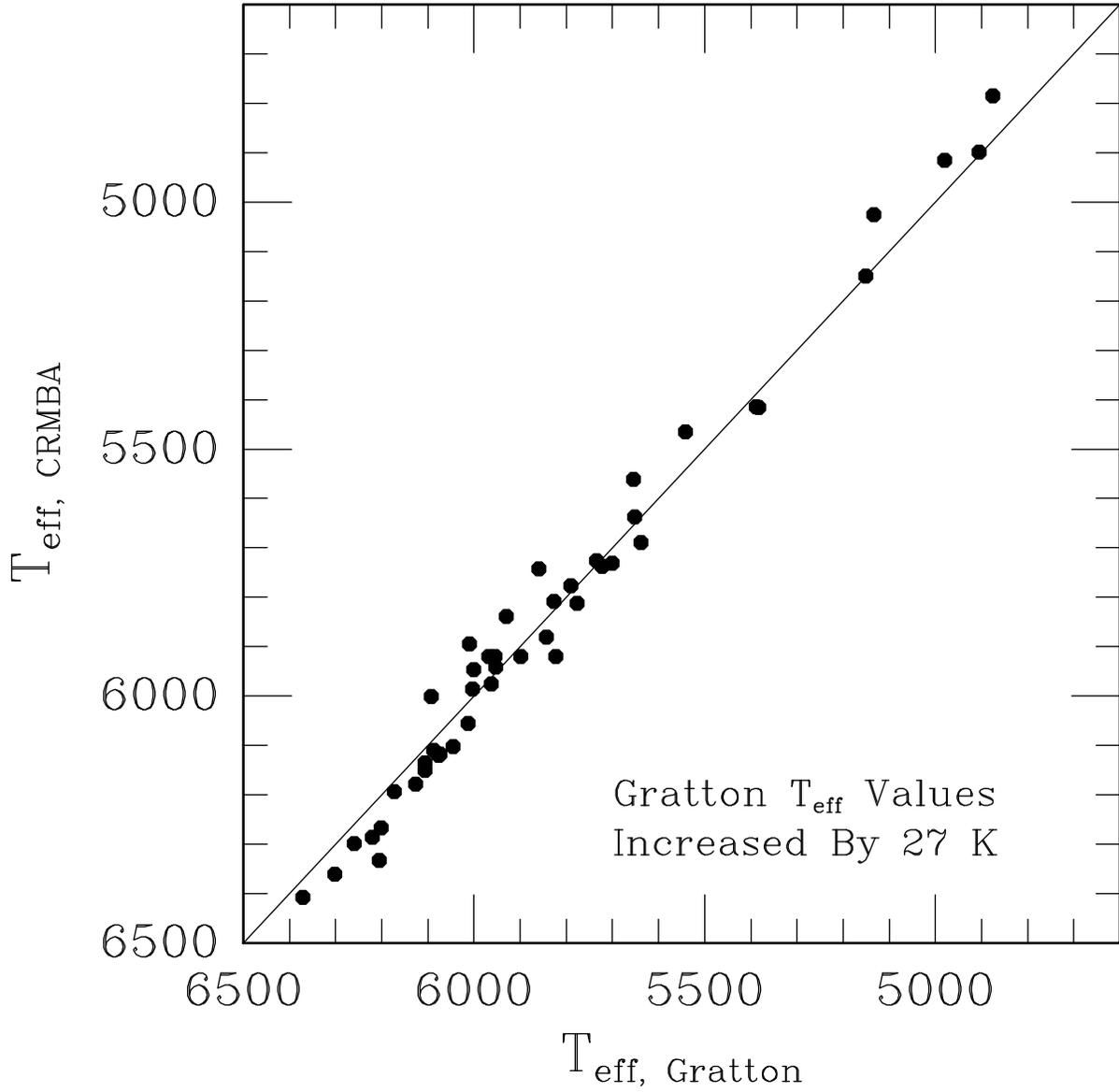


Fig. 6.— Comparison of the effective temperatures derived by CRMBA and by R. Gratton and colleagues (see the text) after the latter values were increased by 27 K (as noted), which is the mean difference between the two data sets.

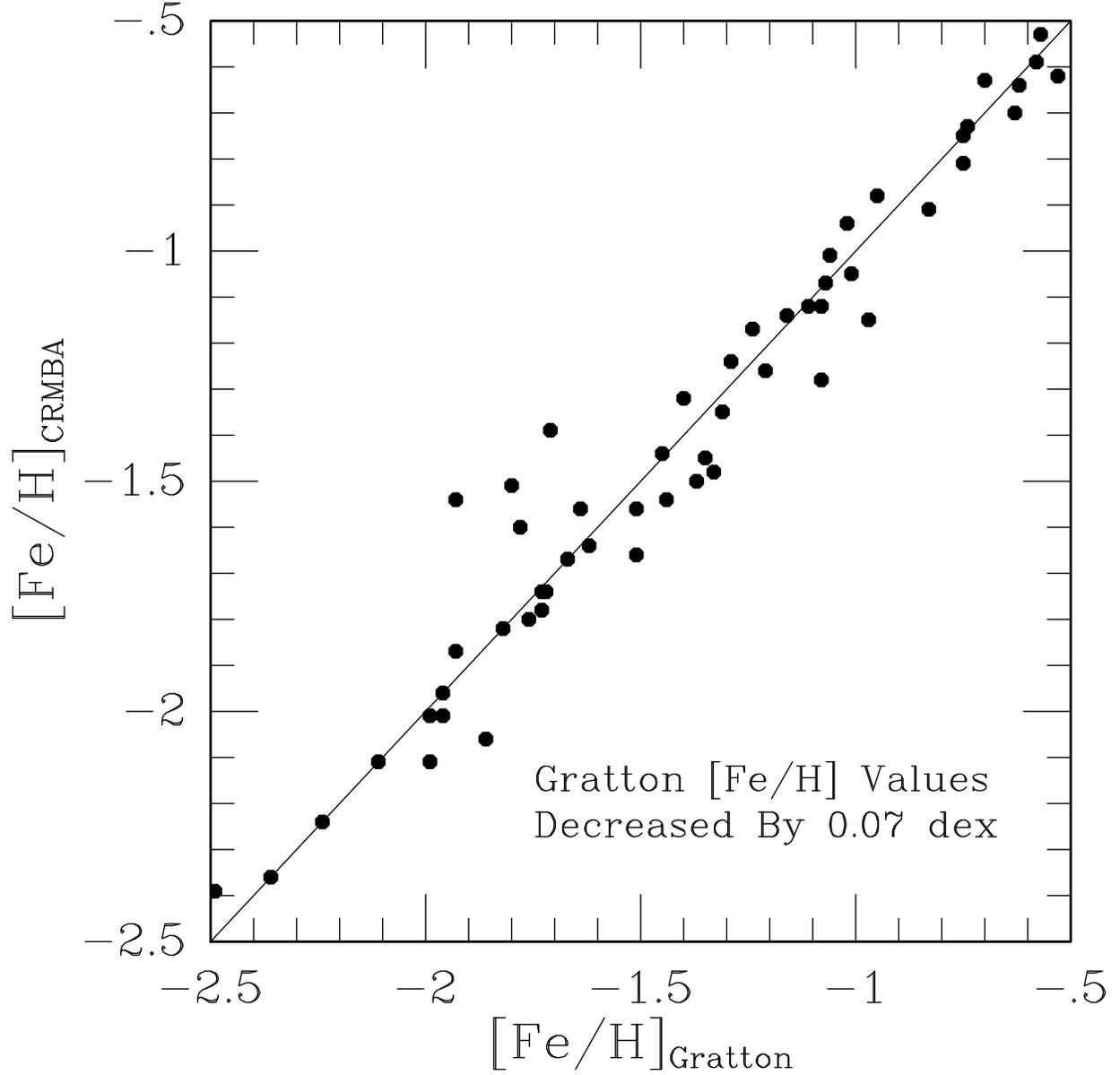


Fig. 7.— As in the previous figure, except that the $[\text{Fe}/\text{H}]$ values determined by the two groups are compared (after the indicated adjustment to the Gratton iron abundances was made). Note that the sample of stars considered here was restricted to $[\text{Fe}/\text{H}]$ values from -2.5 to -0.5 .

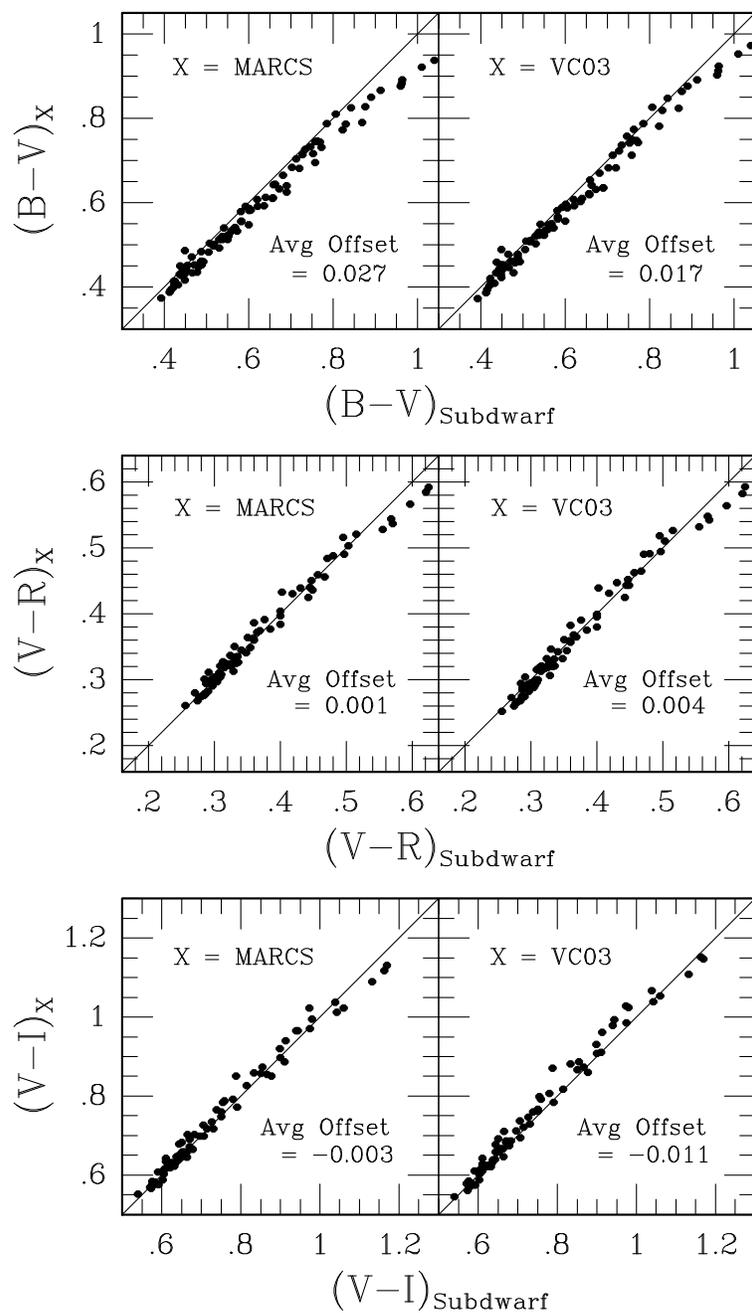


Fig. 8.— Comparison of the observed subdwarf colors with those predicted by MARCS and VC03 color transformations on the assumption of the subdwarf properties (i.e., T_{eff} , $\log g$, and $[\text{Fe}/\text{H}]$) determined by CRMBA. The noted offsets are in the sense “observed minus predicted”.

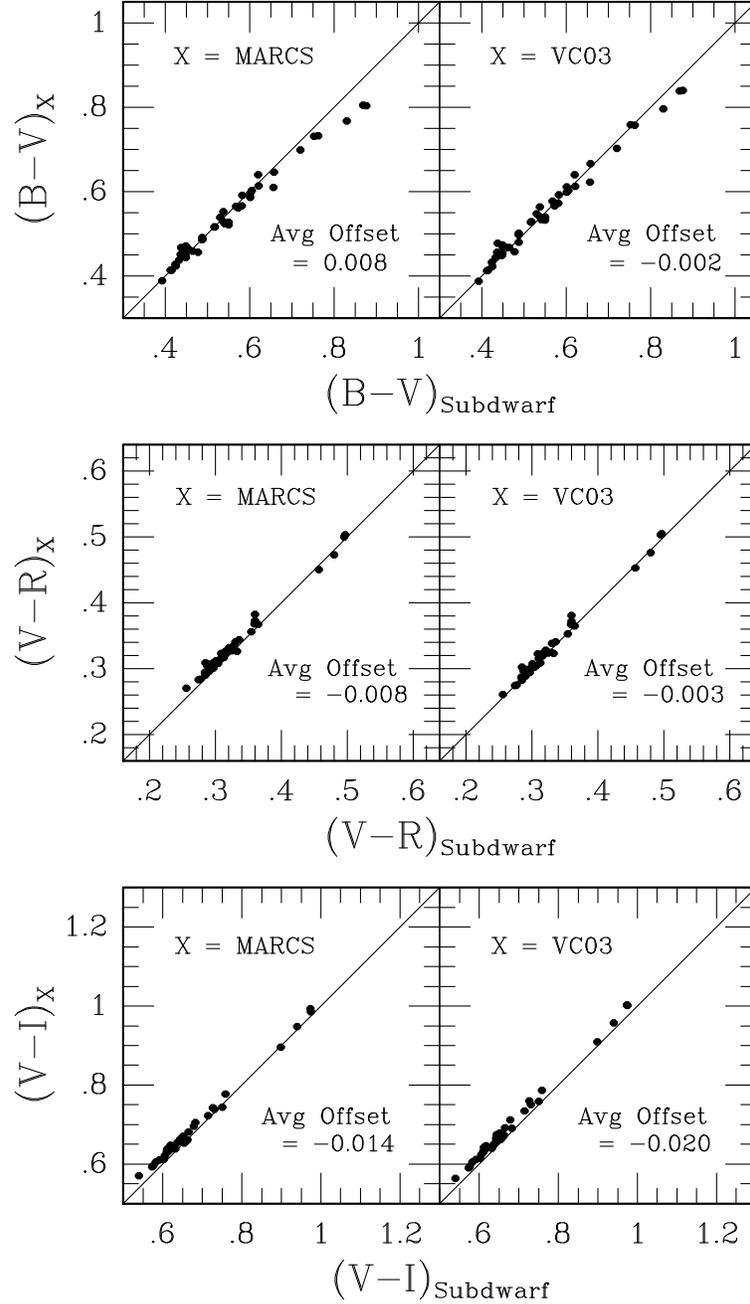


Fig. 9.— As in the previous figure, except that the T_{eff} , $\log g$, and $[\text{Fe}/\text{H}]$ values derived by Gratton et al. are assumed in calculating the corresponding MARCS and VC03 colors.

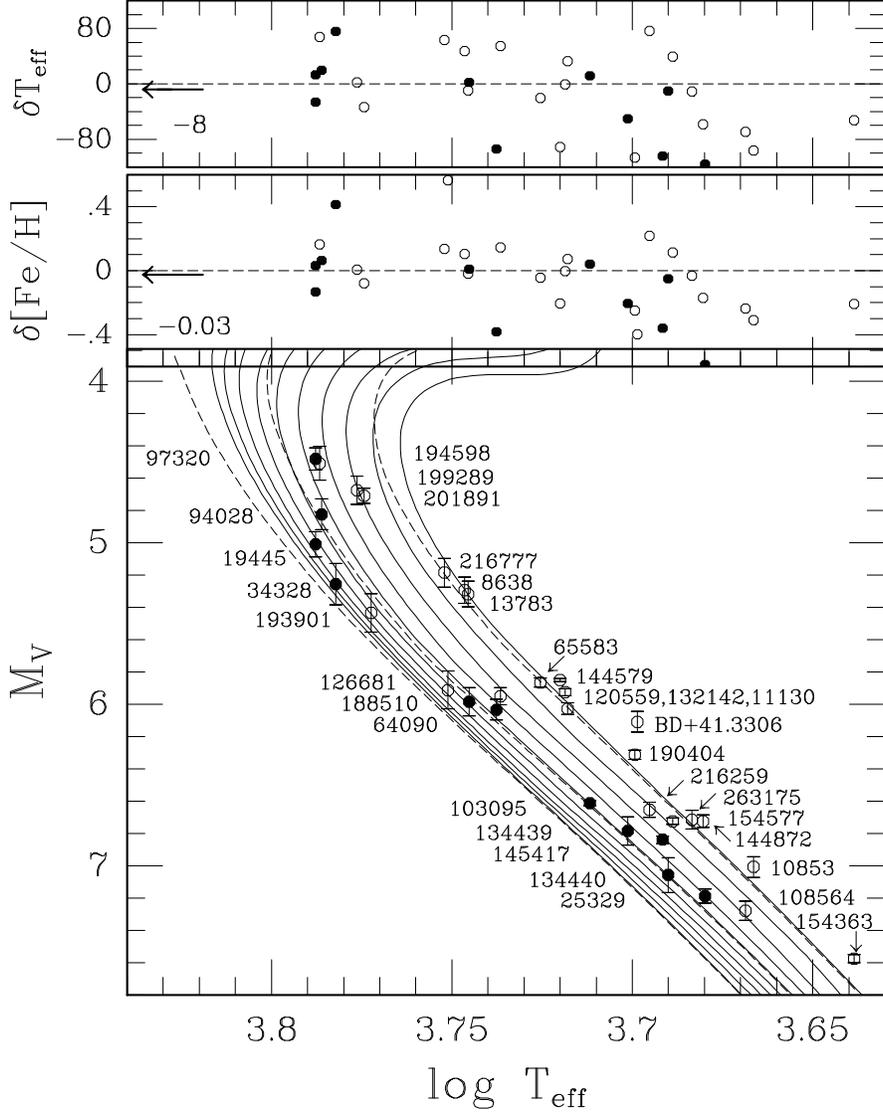


Fig. 10.— *Lower panel:* Superposition of the properties of local subdwarfs having well determined M_V values onto a set of 12 Gyr isochrones for $[\text{Fe}/\text{H}]$ values from -2.40 to -0.60 , in 0.2 dex steps (from left to right). Isochrones for -2.4 , -1.4 , and -0.60 , but for an age of 10 Gyr, are plotted as dashed curves. Filled and open circles represent subdwarfs having $[\text{Fe}/\text{H}] \leq -1.2$ and > -1.2 (according to CRMBA), respectively. The error bars on the points depict the 1σ uncertainties in the M_V values as derived from *Hipparcos* parallaxes (van Leeuwen 2007). The stars are identified by their “HD” numbers, except for BD+41 3306. *Middle panel:* Plotted as a function of $\log T_{\text{eff}}$, the difference between the CRMBA estimate of $[\text{Fe}/\text{H}]$ for each star and that inferred from the interpolated (or extrapolated) isochrone that matches its location on the $(\log T_{\text{eff}}, M_V)$ -diagram in the lower panel. *Upper panel:* The difference in T_{eff} that would need to be applied to each subdwarf in order to achieve perfect consistency of its position in the lower panel. The numbers and arrows in the middle and upper panels give the mean values of $\delta[\text{Fe}/\text{H}]$ and δT_{eff} , respectively, that were computed using all stars.

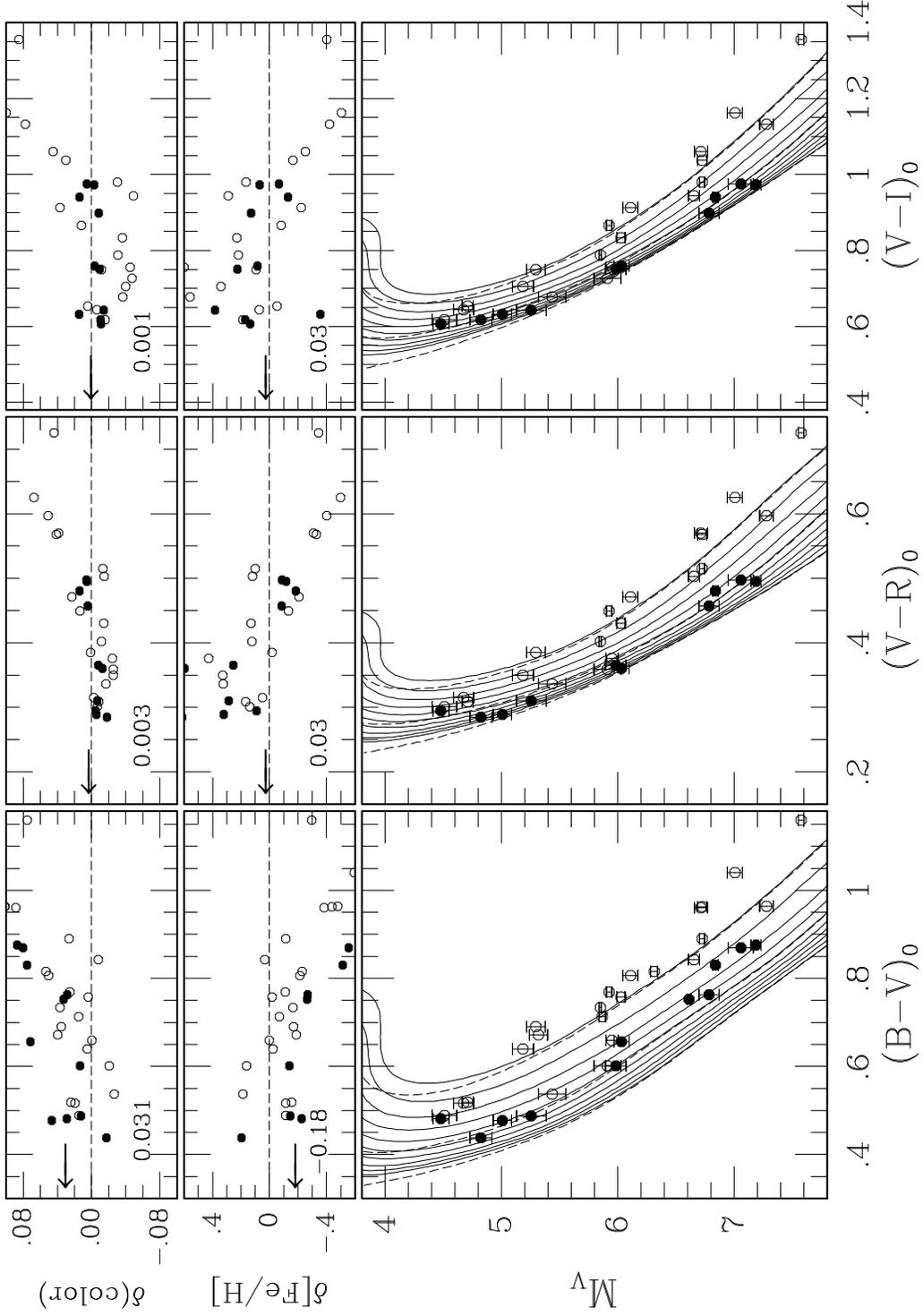


Fig. 11.— Similar to the previous figure, except that the subdwarfs are compared with the isochrones that have been transposed to the $B - V$, $V - R$, and $V - I$ color planes using the MARCS color- T_{eff} relations. The subdwarf $[\text{Fe}/\text{H}]$ values and colors are taken from the study by CRMBA.

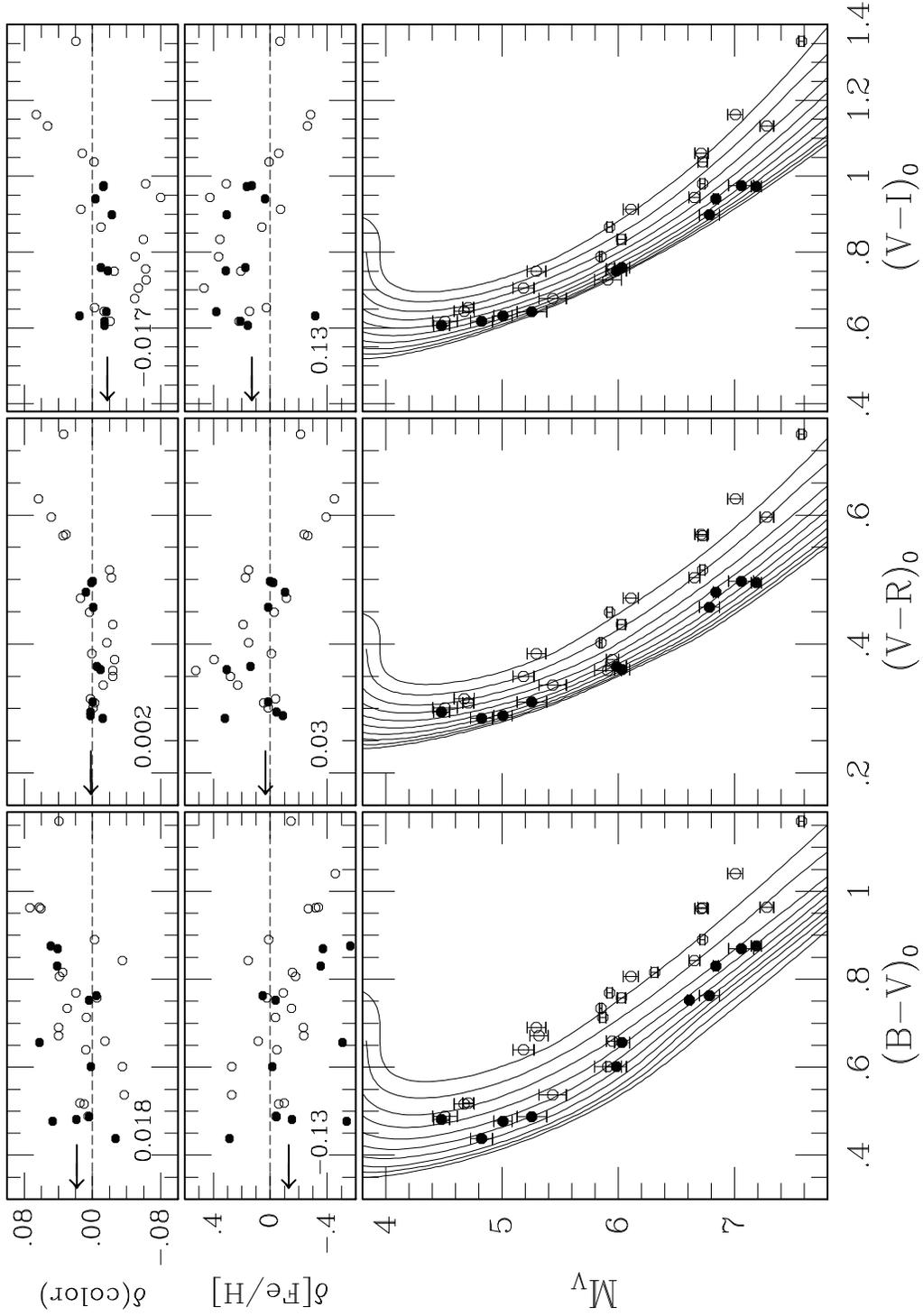


Fig. 12.— As in the previous figure, except that the VC03 color- T_{eff} relations have been used to transpose the isochrones to the various observed planes and the dashed curves have been omitted.

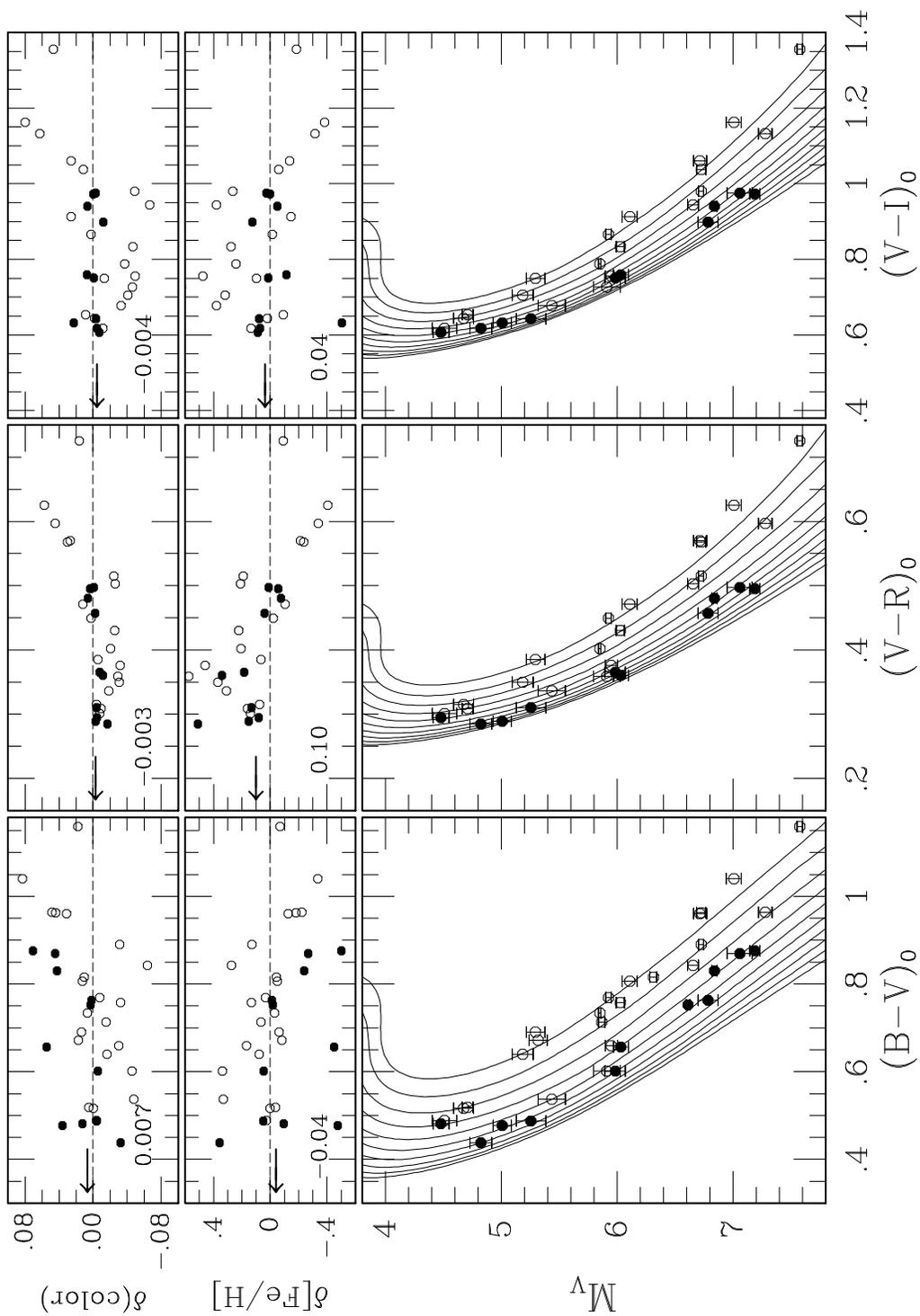


Fig. 13.— As in the previous figure, except that the empirical color- T_{eff} relations given by CRMBA for dwarf and subgiant stars have been used to transpose the isochrones to the various observed planes.

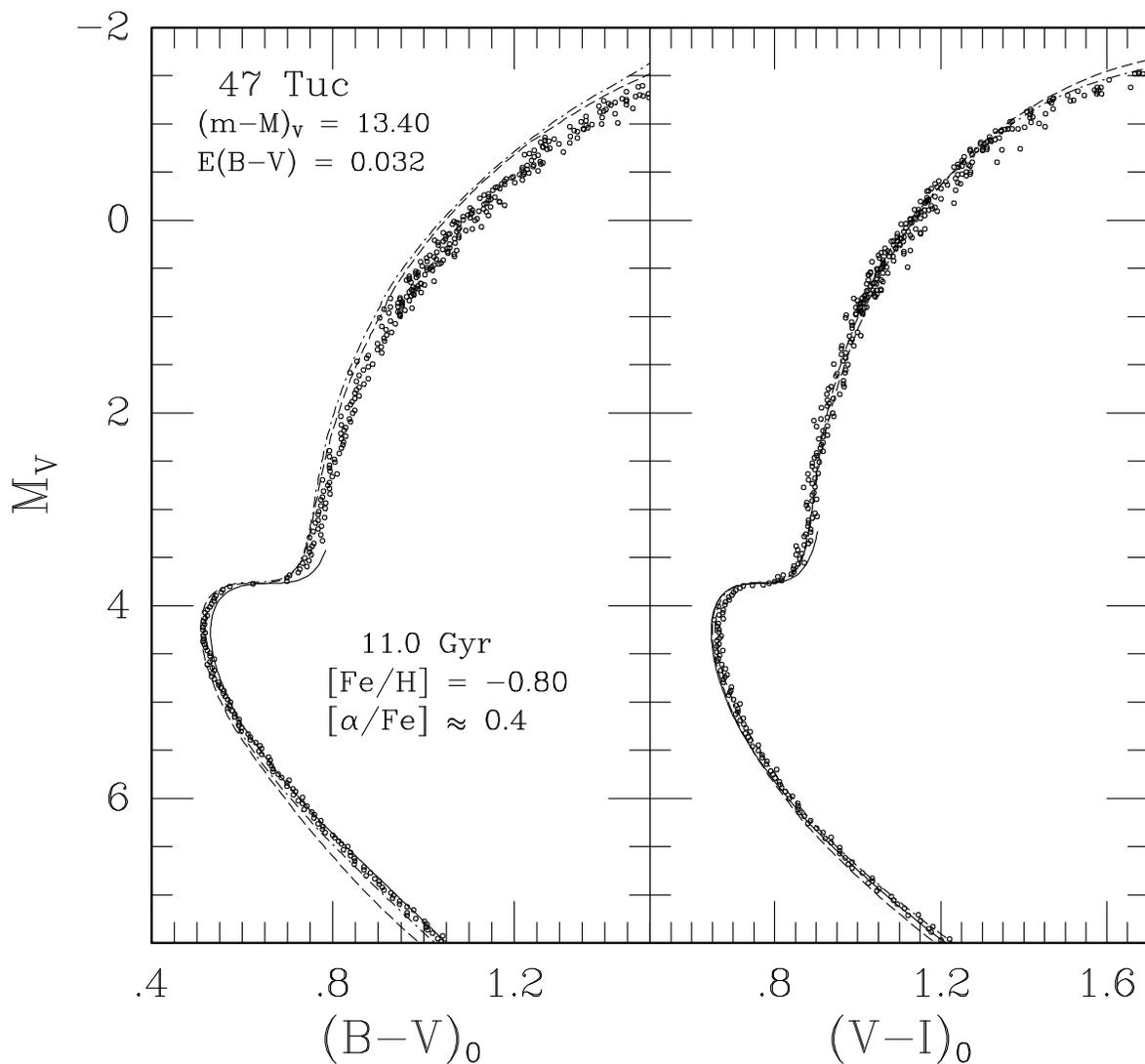


Fig. 14.— Comparison of an 11.0 Gyr isochrone for $[Fe/H] = -0.8$, $[\alpha/Fe] \approx 0.4$, and $Y = 0.25$ with photometry for 47 Tucanae from Bergbusch & Stetson (2009). The solid, dashed, and dot-dashed curves assume, in turn, the CRMBA, MARCS, and VC03 color transformations. The predicted and observed turnoff colors agree to within 0.02 mag, with the solid curves showing the largest blueward and redward offsets in both the right- and left-hand panels, respectively.

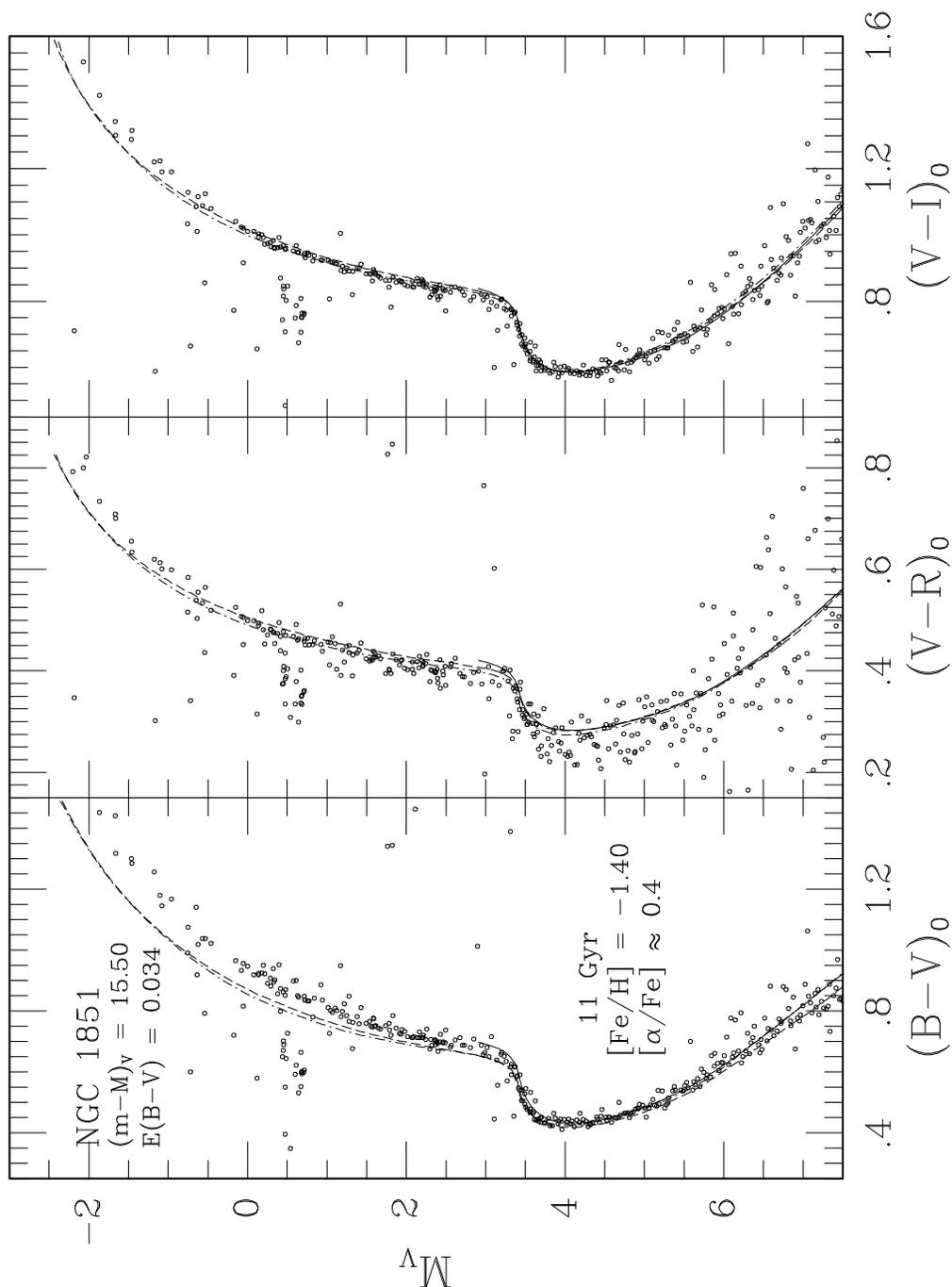


Fig. 15.— Comparison of an 11.0 Gyr isochrone for $[\text{Fe}/\text{H}] = -1.40$, $[\alpha/\text{Fe}] \approx 0.4$, and $Y = 0.25$ with the latest calibration of photometry for NGC 1851 from the Stetson (2000) database. The solid, dashed, and dot-dashed curves assume, in turn, the CRMBA, MARCS, and VC03 color transformations. In general, the predicted and observed turnoff colors agree to within 0.015 mag.

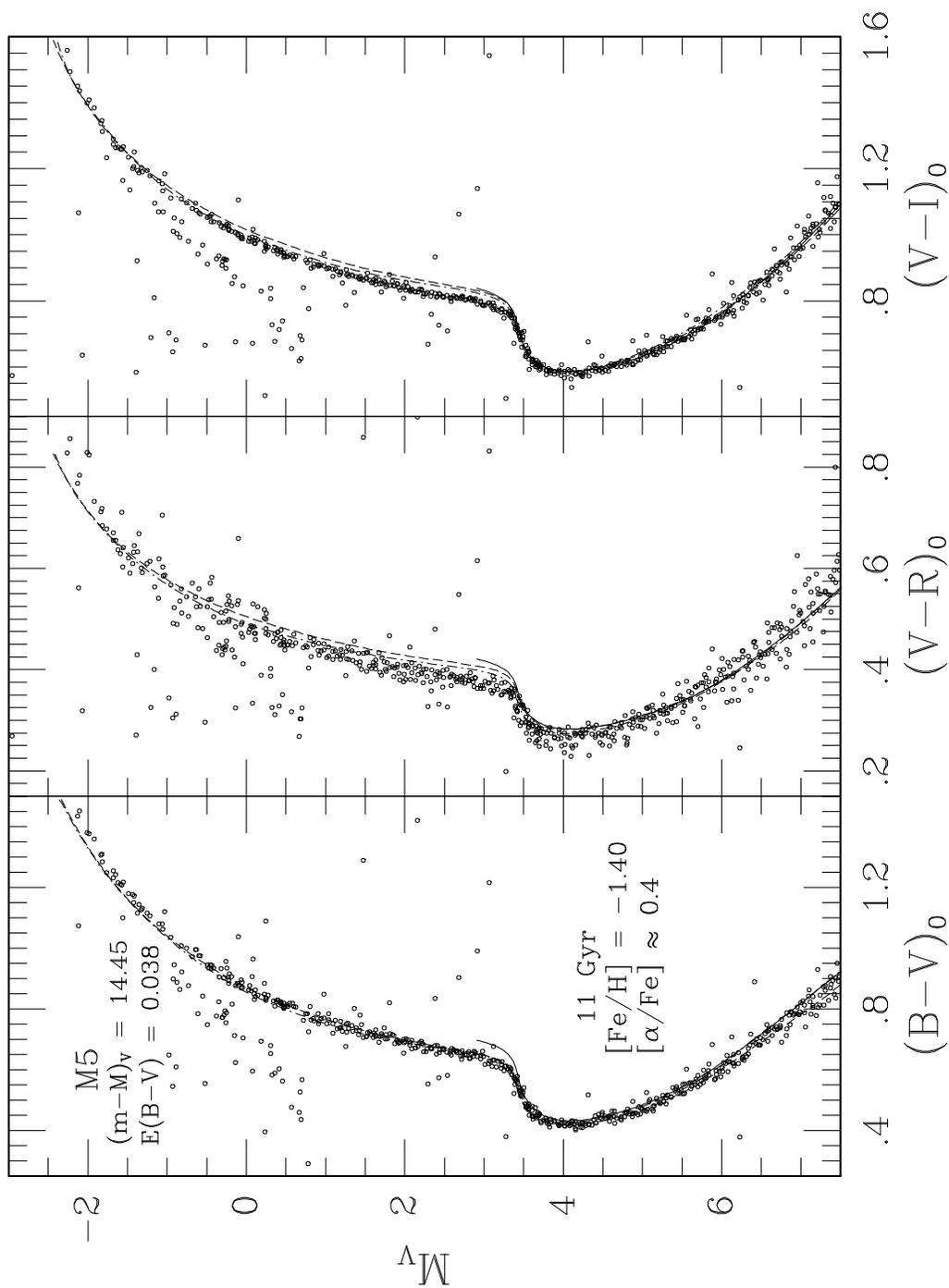


Fig. 16.— As in the previous figure, except that the same isochrones are compared with our photometry for M5. Here as well, the predicted and observed turnoff colors agree to within 0.015 mag.

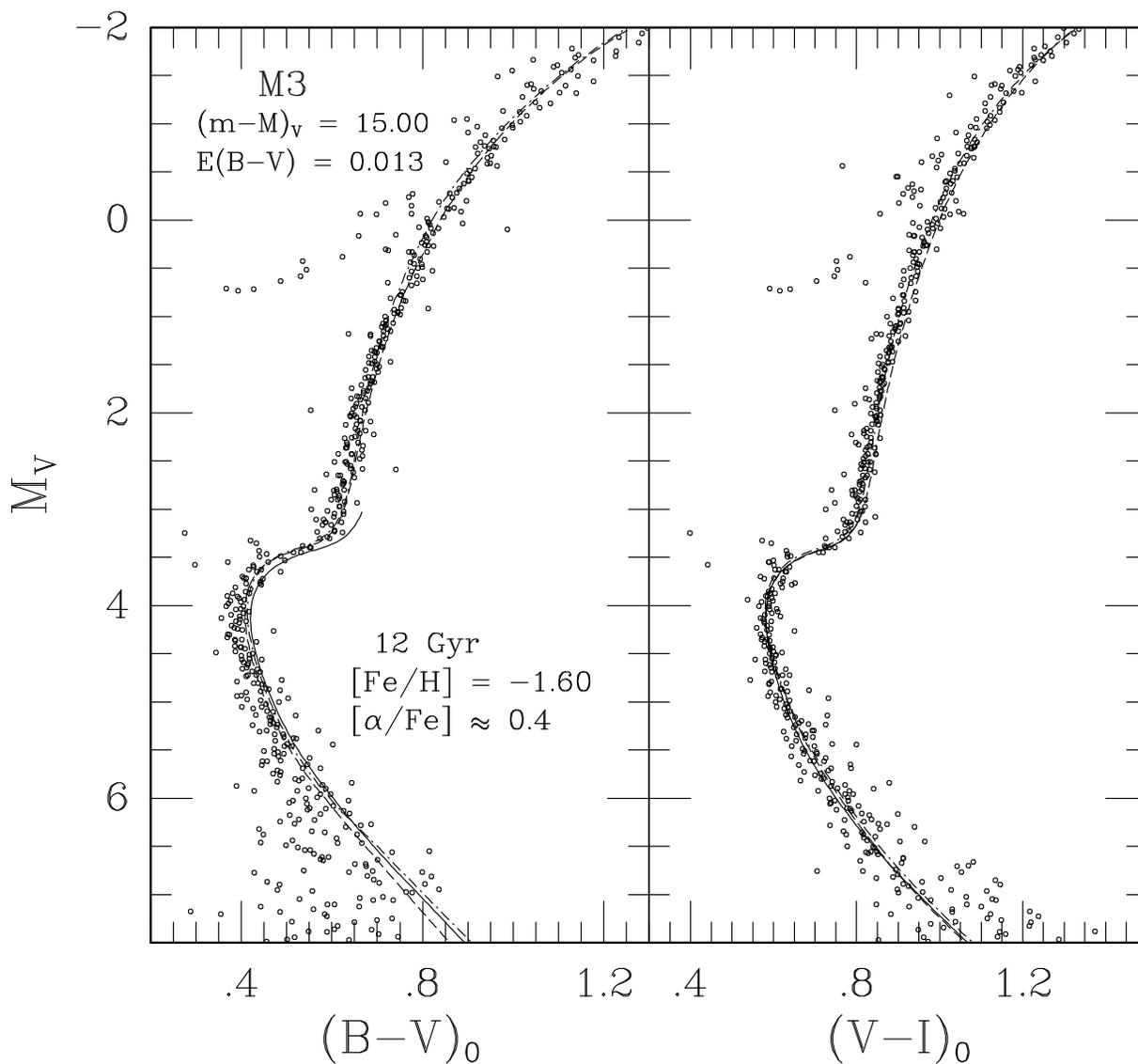


Fig. 17.— As in the previous figure, except that a 12.0 Gyr isochrone for a slightly lower metallicity (as noted) is compared with observations of M3. Whereas there is excellent consistency of the predicted and turnoff colors in the right-hand panel, the observed turnoff is 0.02–0.025 mag bluer than that predicted by the solid curve, with somewhat smaller offsets in the case of the other isochrones, though in the same sense.

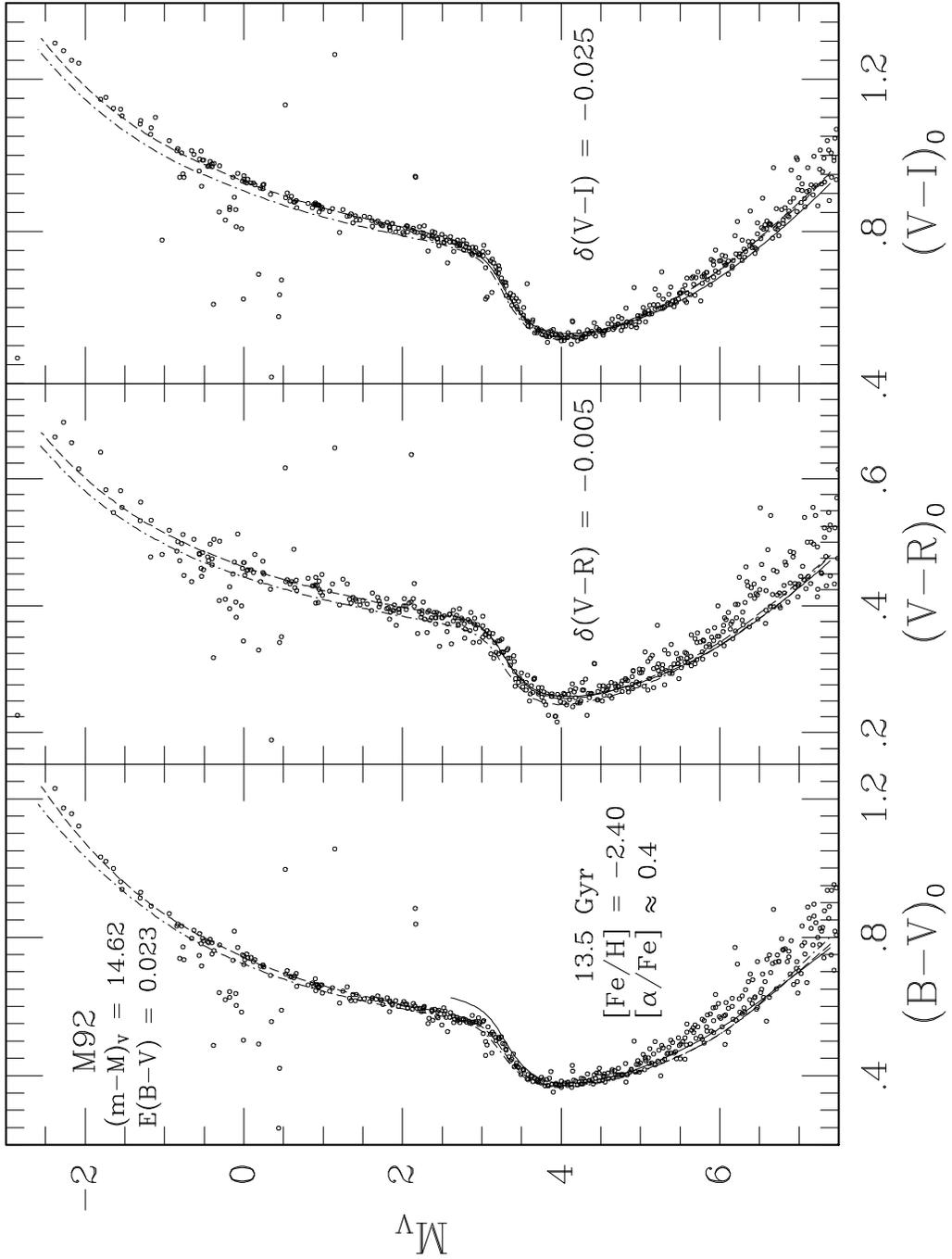


Fig. 18.— As in the previous figure, except that a 13.5 Gyr isochrone for $[\text{Fe}/\text{H}] = -2.40$ is compared with observations of M92. Aside from the large zero-point offset that has been applied to the observations in the right-hand panel, as indicated, there is good consistency of the predicted and observed turnoff colors (i.e., to within 0.015 mag).

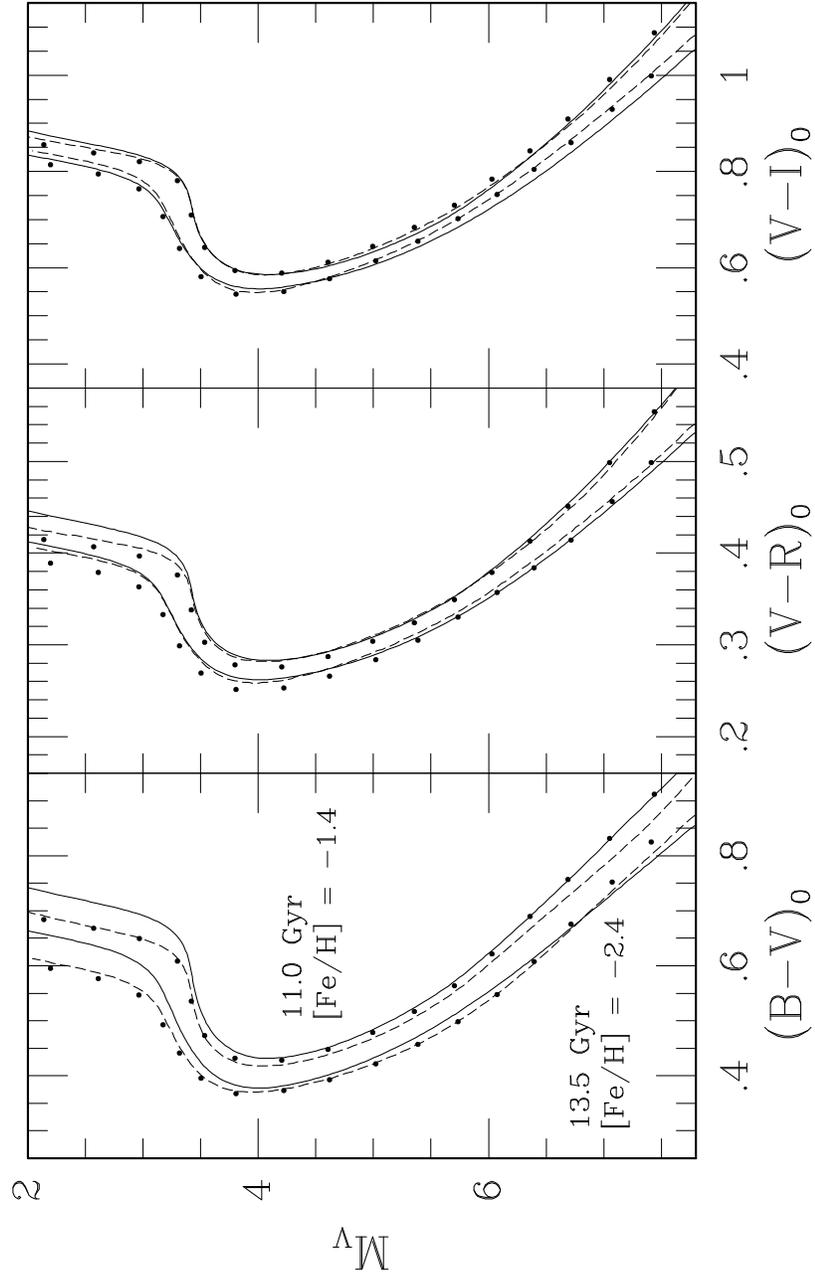


Fig. 19.— Comparisons of isochrones for the indicated ages and $[Fe/H]$ values using the CRMBA (solid curve), MARCS (dashed curve), and VC03 (dotted curve) color transformations.