

Determining the initial helium abundance of the Sun

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ABSTRACT

We determine the dependence of the initial helium abundance and the present-day helium abundance in the convective envelope of solar models (Y_{ini} and Y_{surf} respectively) on the parameters that are used to construct the models. We do so by using reference standard solar models to compute the power-law coefficients of the dependence of Y_{ini} and Y_{surf} on the input parameters. We use these dependencies to determine the correlation between Y_{ini} and Y_{surf} and use this correlation to eliminate uncertainties in Y_{ini} from all solar model input parameters except the microscopic diffusion rate. We find an expression for Y_{ini} that depends only on Y_{surf} and the diffusion rate. By adopting the helioseismic determination of solar surface helium abundance, $Y_{\odot}^{\text{surf}} = 0.2485 \pm 0.0035$, and an uncertainty of 20% for the diffusion rate, we find that the initial solar helium abundance, Y_{\odot}^{ini} , is 0.278 ± 0.006 independently of the reference standard solar models (and particularly on adopted the solar abundances) used in the derivation of the correlation between Y_{ini} and Y_{surf} . When non-standard solar models with extra mixing are used, then we derive $Y_{\odot}^{\text{ini}} = 0.273 \pm 0.006$. In both cases, the derived Y_{\odot}^{ini} value is higher than that directly derived from solar model calibrations when the low metallicity solar abundances (e.g. by Asplund et al.) are adopted in the models.

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1. INTRODUCTION

The primordial helium abundance, Y_P , can be determined to a high accuracy thanks to the measurement of cosmological parameters, particularly of the baryon-to-photon ratio η_{10} , by WMAP (Spergel et al. 2007). The most recent WMAP results (Larson et al. 2010) give $Y_P = 0.2486 \pm 0.0006$ (using the analytic formula presented by Steigman 2007 relating Y_P and η_{10}), but rely on the assumption of Standard Big Bang Nucleosynthesis (SBBN). On the other hand, empirical determinations of Y_P , that can be used to test the SBBN scenario, rely on the determination of helium abundances in metal poor HII regions and extrapolations to zero metallicity, assuming a linear relation between helium and metallicity (or oxygen) with a given slope $\Delta Y/\Delta Z$ (see Peimbert et al. 2007 for a detailed account of the method and uncertainties involved). The results are not exempt of controversy. For example, Peimbert et al. (2007) find $Y_P = 0.2477 \pm 0.0029$, in excellent agreement with WMAP+SBBN results, while other authors (e.g., Izotov & Thuan 2010), find higher values, $Y_P = 0.2565 \pm 0.0010$ (stat) ± 0.0050 (syst), and suggest deviations from SBBN. Discrepancies are not confined to Y_P , but are also found in the given values of $\Delta Y/\Delta Z$. Other attempts to determine $\Delta Y/\Delta Z$, both observationally and theoretically, include the use of the broadening of the lower main sequence (see e.g. Casagrande et al. 2007 and Gennaro et al. 2010 for recent works), the study of galactic stellar populations (Renzini 1994), modeling eclipsing binaries (Ribas et al. 2000), construction of galactic chemical evolution models (Timmer et al. 1995; Chiappini et al. 2003; Carigi & Peimbert 2008). The values obtained for $\Delta Y/\Delta Z$ range approximately between 1.4 and 2.5 for different authors and methods.

The initial composition of stars determines their structure and evolution and, by extension, the characteristics of stellar populations. The initial helium abundance of stars is in most cases impossible to determine directly, hence indirect methods have to be used. A common approach used in stellar and stellar populations modeling is to assume a value for $\Delta Y/\Delta Z$ and use the solar helium and metallicity as an anchor point for the relation. This is a routine procedure used in, for example, libraries of stellar evolutionary tracks to define the initial composition used in the stellar models (e.g., Schaller et al. 1992; Yi et al. 2001; Pietrinferni et al. 2004). The initial solar helium abundance, however, can not be determined directly. It is well established that element diffusion takes place in the Sun and that the present-day photospheric abundances of helium and metals are lower than the initial ones. Therefore, the initial solar helium abundance is usually obtained from cal-

ibrating solar models, which then introduces model dependencies on the determined values. An illustrative example is the effect of solar photospheric metallicity determinations on the inferred initial solar helium abundance. Solar models that use older solar abundances like those of Grevesse & Noels (1993) or Grevesse & Sauval (1998) lead to models with initial helium values in the models in the range $Y_{\text{ini}} \approx 0.272 - 0.275$ (Bahcall et al. 2001; Boothroyd & Sackmann 2003), while models that incorporate newer determinations of solar metallicity (Asplund et al. 2005; Lodders et al. 2009; Asplund et al. 2009, e.g.) give $Y_{\text{ini}} \approx 0.260 - 0.263$ (Bahcall et al. 2005; Guzik et al. 2005; Serenelli et al. 2009). While the models with the older abundances have present-day surface helium abundances Y_{surf} consistent with helioseismic determinations of the helioseismology determinations of the present-day solar convection-zone helium abundance, the models with the lower metal abundances do not. It is interesting to note also that if we adopt $Y_{\text{ini}} \approx 0.260 - 0.263$ values together with initial solar metallicities derived from the same solar models, i.e. $Z_{\text{ini}} \approx 0.014 - 0.015$, then values as low as 0.8 are obtained for $\Delta Y/\Delta Z$ (assuming Y_{p} from SBBN). This is in disagreement with all accepted determinations of $\Delta Y/\Delta Z$ as we have summarized above.

In this work, we propose a simple method to determine the initial solar helium abundance based on the use of solar models to calibrate the relation between the initial and the present-day surface helium abundances. Although derived with the help of solar models, we then show that the predicted initial solar helium abundance (Y_{\odot}^{ini} ; not to be confused by the initial helium abundance Y_{ini} resulting from solar model calibrations) is a robust result, i.e. a very wide range of solar models (standard and non-standard) lead to the same Y_{\odot}^{ini} value. The structure of the paper is as follows: in § 2 we determine the effect of changes in the input parameters used in solar models (including metal abundances, diffusion rate, nuclear cross sections, etc.) on the initial and present-day helium abundances of the solar models; in § 3 we derive a simple analytic relation between Y_{ini} and Y_{surf} that minimizes the uncertainties originating in the inputs to solar model calculations; in § 4 we present our main results and discuss their range of applicability; finally, we summarize our findings and conclusions in § 5.

2. POWER-LAW DEPENDENCE OF Y_{surf} AND Y_{ini}

For this paper, we use the same technique that has been used earlier for analyzing neutrino fluxes (Bahcall 1989, see Chap.7). We express the dependence of the initial (Y_{ini}) and present-day (Y_{surf}) helium abundances of solar models as a product of powers of the different input parameters:

$$\left(\frac{Y_{\text{ini}}}{Y_{\text{ini},0}}\right) = \prod_{i=1,N} \left(\frac{p_i}{p_{i,0}}\right)^{\alpha_i} \quad (1)$$

$$\left(\frac{Y_{\text{surf}}}{Y_{\text{surf},0}}\right) = \prod_{i=1,N} \left(\frac{p_i}{p_{i,0}}\right)^{\beta_i}. \quad (2)$$

In these equations, the input parameters in solar models (nuclear cross sections, abundances of individual elements, age, etc.) are represented by a set of parameters $\{p_i\}_{i=1,N}$ with central, or reference, values denoted with the sub-index “0”. These parameters comprise: metal abundances, nuclear reaction rates, element diffusion rate, solar surface parameters (luminosity, age); a detailed list is given in Table 1. The exponents α_i and β_i respectively determine the dependence of Y_{ini} and Y_{surf} on the input parameters. They are defined as

$$\alpha_i \equiv \frac{\partial \log Y_{\text{ini}}}{\partial \log p_i}, \quad \beta_i \equiv \frac{\partial \log Y_{\text{surf}}}{\partial \log p_i}. \quad (3)$$

The indices in Equations (3) were calculated for standard solar models (SSM) using different determinations of photospheric solar composition. We chose the compilations by Grevesse & Sauval (1998) and Asplund et al. (2005), hereafter GS98 and AGS05 respectively, and *conservative uncertainties*, according to the definition by Bahcall et al. (2006, hereafter BSB06). A discussion of the central values for all parameters, as well as their uncertainties, is presented in that work. To derive the indices we proceeded as follows: for each input parameter p_i we computed a set of 50 SSMs where the value of p_i is drawn from a Gaussian distribution, with standard deviation equal to the assumed uncertainty of the parameter, while all other input parameters are kept fixed and equal to their central value. The logarithmic derivatives defined in Equations 3 are then obtained from fits to each of the small Monte Carlo simulations described above. Some examples are shown in Figure 1. The values of these dependencies are tabulated in Table 1. The exponents derived for both GS98 and AGS05 solar models are similar. This is so because relative changes in input parameters produce relative changes in solar model properties that do not, to the first order, depend on the reference solar model. Even more important for this work is that the ratios α_i/β_i are almost independent of the reference solar models used. Differences in the values obtained for GS98 and AGS05 models will be used later as a way to determine the systematic uncertainty of the method presented in this work.

Radiative opacities can not be represented by a single parameter. Hence to account for the influence of opacities on Y_{ini} and Y_{surf} we have proceeded in the following way: for both AGS05 and GS98 compositions we have computed two solar models, one with OPAL

opacities and the other one with OP opacities, while keeping all other input physics the same in both models. For each composition we define the 1σ uncertainties in Y_{ini} and Y_{surf} as half the difference in the values obtained for models with the same composition but the two different opacity sources. We find that for Y_{surf} the uncertainty from opacity is 0.23%(0.20%), while for Y_{ini} is 0.30%(0.28%) for the AGS05(GS98) composition. The uncertainties between Y_{ini} and Y_{surf} are found to be fully correlated. We account for opacity uncertainties by adding Gaussian dispersions with the above defined standard deviations to the distributions of Y_{ini} and Y_{surf} that we generate using Equations 1 and 2.

We have verified that Equations 1 and 2 describe the behavior of Y_{ini} and Y_{surf} in solar models very well by comparing distributions generated with these equations against results obtained with full solar models. In particular, we have used the large set of solar models computed for the Monte Carlo simulation done by BSB06, where different assumptions for the reference solar composition and abundance uncertainties were adopted. Results of the comparison are presented in Figure 2. The panel on the left shows results for the GS98 composition with conservative (large) abundance uncertainties (see BSB06 for details); while the panel on the right shows results for models using AGS05 composition and much smaller uncertainties, as originally given by AGS05. In both cases, results from the full solar models obtained from the BSB06 Monte Carlo simulations and the resulting distributions for the Y_{ini} and Y_{surf} obtained by means of Equations 1 and 2 are shown. We see in both cases that results from full solar models are reproduced very closely by the power-law expressions.

Having verified the validity of power-law expressions in reproducing the correct distributions of Y_{ini} and Y_{surf} , in what follows, we concentrate on using these expressions to constrain the solar initial abundance, Y_{\odot}^{ini} , in a model independent manner. We do this by using the helioseismically determined solar convection-zone helium abundance.

3. CORRELATION BETWEEN Y_{surf} AND Y_{ini}

In this paper we assume, unless specified otherwise, that only microscopic diffusion of helium and heavy elements modify the composition of the outer layers of the Sun¹. Therefore, in the absence of diffusion, Y_{surf} would exactly match Y_{ini} . The most evident effect of diffusion, through gravitational settling, is to decrease Y_{surf} with respect to Y_{ini} . Thus Y_{surf} and the diffusion rate are anticorrelated. Diffusion also modifies the composition in the solar core. In particular, gravitational settling of metals in the core leads to a higher radiative opacity

¹The surface abundance of lithium and beryllium are also modified by nuclear burning but that is not relevant given the scope of the present work.

and hence, to a steeper temperature profile. The slightly increased temperature in turn increases the rates of nuclear energy production. However, the solar luminosity is fixed. Thus, the increase in energy production due to the larger temperatures has to be compensated by decreasing the amount of available hydrogen or, equivalently, by increasing the helium abundance in the core. As a consequence, Y_{ini} is positively correlated with the diffusion rate. We call these two combined effects, the negative correlation of Y_{surf} and the positive correlation of Y_{ini} with the diffusion rate, the ‘direct influence’ of diffusion.

As can be seen from the coefficients listed in Table 1, the direct influence of diffusion is the only effect that modify Y_{surf} and Y_{ini} in opposite ways. For this reason, in what follows we treat the effect of diffusion separately. Thus Equations. 1 and 2 are modified to:

$$\left(\frac{Y_{\text{ini}}}{Y_{\text{ini},0}} \right) = \left(\frac{p_{\text{Diff}}}{p_{\text{Diff},0}} \right)^{\alpha_{\text{Diff}}} \left[\prod_{i=1, N; i \neq \text{Diff}} \left(\frac{p_i}{p_{i,0}} \right)^{\alpha_i} \right] \quad (4)$$

$$\left(\frac{Y_{\text{surf}}}{Y_{\text{surf},0}} \right) = \left(\frac{p_{\text{Diff}}}{p_{\text{Diff},0}} \right)^{\beta_{\text{Diff}}} \left[\prod_{i=1, N; i \neq \text{Diff}} \left(\frac{p_i}{p_{i,0}} \right)^{\beta_i} \right] \quad (5)$$

Using the expressions in brackets in the equations above, we obtained distributions for Y_{surf} and Y_{ini} that exclude the direct effect of diffusion. The correlation between these two quantities is now very tight as can be seen in Figure 3. Naively, we would expect no dispersion to be present, since the diffusion rates are kept fixed. However, keeping the diffusion rates fixed does not imply the total amount of helium (and metals) diffused out of the convective envelope is constant. The effective rate of diffusion in the solar envelope is directly related to the extension of the convective envelope which acts as a reservoir of helium. Therefore, any changes to the model inputs that have an effect on the mass contained in the convective envelope will be reflected in a deviation of the behavior of Y_{surf} with respect to that of Y_{ini} (i.e., effectively changing the values of the power-law exponents α_i and β_i to a small extent), leading to the dispersion seen in Figure 3.

Following a procedure analogous to that used by Haxton & Serenelli (2008) to study solar neutrino fluxes, we take advantage of the small dispersion in the correlation between Y_{surf} and Y_{ini} to express the uncertainties in Y_{ini} in terms of those in Y_{surf} while minimizing the uncertainty in all parameters except for diffusion. To do this we combine Equations 4 and 5 to obtain:

$$\left(\frac{Y_{\text{ini}}}{Y_{\text{ini},0}} \right) = \left(\frac{Y_{\text{surf}}}{Y_{\text{surf},0}} \right)^{K_{(\text{I},\text{S})}} \left(\frac{p_{\text{Diff}}}{p_{\text{Diff},0}} \right)^{\gamma_{\text{Diff}}} \left\{ \prod_{i=1, N; i \neq \text{Diff}} \left(\frac{p_i}{p_{i,0}} \right)^{\gamma_i} \right\} \quad (6)$$

where

$$\gamma_i = \alpha_i - K_{(\text{I,S})}\beta_i. \quad (7)$$

The constant $K_{(\text{I,S})}$ is obtained by doing a linear fit to the simulated data for $\log(Y_{\text{ini}})$ and $\log(Y_{\text{surf}})$ using the power-law expressions while ignoring the direct dependence on diffusion. For the GS98-C case we obtain $K_{(\text{I,S})} = 0.836$. The residuals of the fit have a dispersion of 0.4% (left panels of Figure 3). For the exponent in the diffusion factor we obtain $\gamma_{\text{Diff}} = 0.088$. We can now write

$$\left(\frac{Y_{\text{ini}}}{Y_{\text{ini},0}}\right) = \left(\frac{Y_{\text{surf}}}{Y_{\text{surf},0}}\right)^{0.836} \left(\frac{p_{\text{Diff}}}{p_{\text{Diff},0}}\right)^{0.088} [1 \pm 0.4\%(\text{ND})], \quad (8)$$

where the uncertainty of 0.4% represents the total uncertainty due to all non-diffusion parameters (note that only abundance-uncertainties play an appreciable role). If we used optimistic abundance-uncertainties instead, the total non-diffusion uncertainty would be only 0.2%.

To proceed further, we need to consider the uncertainty of the diffusion rate (hereafter σ_{Diff}). The best agreement between the structure of solar models and the helioseismically determined solar structure is obtained when helium and heavy-element diffusion is included in the models (Bahcall et al. 1995; Christensen-Dalsgaard et al. 1996). However, the actual diffusion rates of helium and metals and, in particular, the associated uncertainties are not well constrained. Thoul et al. (1994) have estimated an uncertainty of about 15% in the diffusion rates. A similar result was obtained by Fiorentini et al. (1999) by using conditions at the base of the convective zone to constrain the diffusion rates. A detailed study by Turcotte et al. (1998) shows that diffusion velocities depend on the different assumptions made in the calculation for different elements and, at different regions of the Sun, the variation ranges from a few percent to about 40%. Here, we adopt an intermediate fiducial value $\sigma_{\text{Diff}} = 20\%$ as the uncertainty in the diffusion rates. It should be noted that the uncertainty in Y_{ini} because of uncertainties in the diffusion rate are only one-twelfth that of uncertainties in the diffusion rate. Then, for $\sigma_{\text{Diff}} = 20\%$, the contribution of diffusion to the total uncertainty in Y_{ini} is only 1.7%. Thus,

$$\left(\frac{Y_{\text{ini}}}{Y_{\text{ini},0}}\right) = \left(\frac{Y_{\text{surf}}}{Y_{\text{surf},0}}\right)^{0.836} [1 \pm 1.7\%(\text{Diff}) \pm 0.4\%(\text{ND})]. \quad (9)$$

We now scale the relation to the Sun by adopting the present-day surface helium abundance determined using helioseismology. We adopt the value determined by Basu & Antia (2004), i.e., $Y_{\odot}^{\text{surf}} = 0.2485 \pm 0.0034$, where the uncertainty accounts for systematics, including those caused by uncertainties in the equation of state (see Basu & Antia 2008 for a recent discussion about the determination of Y_{\odot}^{surf}). Then, the solar initial helium abundance Y_{\odot}^{ini}

can be expressed as

$$Y_{\odot}^{\text{ini}} = Y_{\text{ini},0} \left(\frac{0.2485}{Y_{\text{surf},0}} \right)^{0.836} [1 \pm 1.1\%(\text{Helio}) \pm 1.7\%(\text{Diff}) \pm 0.4\%(\text{ND})]. \quad (10)$$

In the above expression, standard solar models (SSMs) play a two-folded role. First, a reference model is used to get the scaling factors $Y_{\text{ini},0}$ and $Y_{\text{surf},0}$. Second, models are used to determine the exponent that relate Y_{\odot}^{surf} and $Y_{\text{surf},0}$. However, as we show in the following section, Equation 10 has predictive power independent of the (standard) solar model used as a reference.

4. RESULTS

As a first test of Equation 10, we use values $Y_{\text{ini},0}$ and $Y_{\text{surf},0}$ obtained from SSM calculations of Serenelli et al. (2009). The results are summarized in Table 2 where we have identified the solar models by the solar composition adopted for each of them. The second and third columns give the SSM predictions for the initial and present-day surface helium mass fraction. The fourth column gives the Y_{\odot}^{ini} values estimated using Equation 10; the uncertainty in all cases is $\sigma_{Y_{\odot}^{\text{ini}}} = \pm 0.006$. Comparing results for the different standard solar models we find that the dispersion in Y_{\odot}^{ini} is about ten times smaller than $\sigma_{Y_{\odot}^{\text{ini}}}$ and hence we conclude that the estimated values of Y_{\odot}^{ini} are consistent with each other, and they are independent of the standard solar model used as reference.

If standard solar models based on AGS05 composition are used to derive the relation between Y_{ini} and Y_{surf} , we get $K_{(\text{I,S})} = 0.903$ instead of 0.836. We have used GS98 and AGS05 for this comparison in an attempt to bracket what the real solar abundance probably is. These two compilations represent the two ends of the currently accepted range of values for solar photospheric abundances ². Therefore, we also expect that our estimates of $K_{(\text{I,S})}$ for the GS98-C and AGS05-Op cases represent the two extremes of the possible range of values for this quantity. Replacing $K_{(\text{I,S})} = 0.903$ in Eq. 10 and again using the standard solar models from Serenelli et al. (2009), we get the Y_{\odot}^{ini} values listed in the last column of Table 2. A comparison of the Y_{\odot}^{ini} results obtained with the two values of $K_{(\text{I,S})}$ for each SSM shows that the uncertainty in the determination of $K_{(\text{I,S})}$ has a very minor effect in the

²In fact, newer determinations of solar abundances, e.g. by Asplund et al. (2009); Caffau et al. (2010), point towards abundances of individual elements in the solar photosphere that are systematically higher than those in Asplund et al. (2005) (although consistent within the quoted uncertainties), but lower than those in Grevesse & Sauval (1998).

predicted Y_{\odot}^{ini} values. The largest difference is found for the AGSS09ph model, for which $\Delta Y_{\odot}^{\text{ini}} = 0.0017$, a factor of ~ 4 smaller than $\sigma_{Y_{\odot}^{\text{ini}}}$ estimated above.

We have shown that the predicted Y_{\odot}^{ini} values have a very small dependence on the solar model used to determine it. The choice of standard solar models for deriving the exponent $K_{(\text{I,S})}$ also introduces a small scatter in the resulting Y_{\odot}^{ini} values. Therefore, we average all values Y_{\odot}^{ini} presented in Table 2 to determine the final result and also use the scatter to define the systematic uncertainty of the method. The final result that we obtain is

$$Y_{\odot}^{\text{ini}} = 0.278 \pm 0.006(\text{helio, param}) \pm 0.002(\text{syst}) \quad (11)$$

where the first part of the uncertainty includes contributions from helioseismology and solar input parameters, and the second one is the systematic uncertainty of the method as defined above.

We have tested the robustness of this result by applying Equation (10) to a variety of solar models, standard and non-standard, i.e. including additional physics not considered in SSMs. As a first consistency check, we have applied Equation 10 to the two large sets of SSMs computed by BSB06 used to assess the validity of the power-law expansions obtained in § 2. Averaging all results for the AGS05-Op Monte Carlo set presented in BSB06, we find $Y_{\odot}^{\text{ini}} = 0.2780 \pm 0.0043$ and for the GS98-Cons set we get $Y_{\odot}^{\text{ini}} = 0.2783 \pm 0.0041$, i.e. results fully consistent with the basic derivation of Y_{\odot}^{ini} and its uncertainty based on the four SSMs presented in Table 2. These uncertainties do not include errors in the helioseismic results.

To further test the robustness of our derivation of Y_{\odot}^{ini} we have resorted to results for standard and non-standard solar models from the literature. Non-standard models have been considered only when the modification to the input physics results in a model that is in better agreement with helioseismic results than its SSM counterpart. As a last test, we have computed additional non-standard models to test the influence of turbulent diffusion on the robustness of our predictions.

Here is a summary of our observations:

- Turcotte et al. (1998) have tested the effect of different diffusion schemes (including radiative levitation in some cases) and the use of monochromatic opacities. By applying Equation (10) to their models that include diffusion, we obtain an average $Y_{\odot}^{\text{ini}} = 0.2780 \pm 0.0017$, where the uncertainty refers to the scatter between models. From all their models we infer Y_{\odot}^{ini} consistent within 1σ of our result in Equation 11.
- Some standard and non-standard models have been presented in Bahcall et al. (2001). From these models (except their models with no diffusion and with a mixed core;

both clearly ruled out by helioseismology) we determine an average value $Y_{\odot}^{\text{ini}} = 0.2774 \pm 0.0018$ (scatter between models). The model that most deviates from this results includes rotation³ which, by introducing additional mixing, tends to inhibit the efficiency of diffusion. For this particular model we get $Y_{\odot}^{\text{ini}} = 0.2725$, still within 1σ of the predicted value in Equation 11.

- Boothroyd & Sackmann (2003) have computed a large set of different solar models with different variations in the input physics and parameters. For all these models we derive Y_{\odot}^{ini} consistent within 1σ of the central result shown in Eq. 11. The only exception is their model where the diffusion velocity of helium is increased by 20%, for which we derive $Y_{\odot}^{\text{surf}} = 0.2873$, a 1.5σ difference with our derived central value for Y_{\odot}^{surf} ; but this model shows a convective envelope too shallow compared to the helioseismic value, and hence, we do not consider the discrepancy to be important. Averaging all results, we get $Y_{\odot}^{\text{ini}} = 0.2783 \pm 0.0013$.
- We have computed solar models that include turbulent diffusion as given by Equation (2) of Proffitt & Michaud (1991), where the diffusion coefficient is $D_T = D_0 [\rho_{\text{CZ}}/\rho]^3$, and ρ is the density and ρ_{CZ} the density at the base of the convective envelope. As pointed out by Christensen-Dalsgaard & Di Mauro (2007), even a small value for D_0 ($200 \text{ cm}^2\text{s}^{-1}$) eliminates the bump in the sound-speed difference between models and the Sun below the convective envelope. We computed models for $D_0 = 300, 1000$, and, $3000 \text{ cm}^2\text{s}^{-1}$ with both GS98 and AGS05 compositions. Applying Equation (10) to these models, we find $Y_{\odot}^{\text{ini}} = 0.2725, 0.2690, 0.2665$ for increasing D_0 and irrespective of the composition adopted. It should be noted that all models with turbulent diffusion predict sound-speed profiles with average root-mean-square deviations from helioseismic results that are comparable to the rms deviations of SSMs. In fact, for models based on the GS98 composition we get an average rms sound speed deviation of 0.0009 for the SSM while for models with $D_0 = 300, 1000$, and, $3000 \text{ cm}^2\text{s}^{-1}$ we get 0.0009, 0.0009, and 0.0010 respectively. For models based on the AGS05 composition, results are 0.0047, 0.0053, 0.0054, and 0.0055 for the SSM and models with increasing D_0 as given above. This is a result of the fact that, although for models with turbulent diffusion the bump below the CZ disappears, the lower central helium abundance in the models lead to a slight worsening of the agreement with the solar sound speed towards the center. This is also reflected in the frequency separation ratios, particularly r_{02} (Basu et al. 2007), which in all three cases show worse agreement with the observed solar r_{02} than the corresponding SSM constructed with the same

³According to Pinsonneault et al. (1999), this model sets a reasonable upper limit to rotation-induced mixing, compatible with the observed lithium depletion in the Sun.

composition. Larger effects in this direction are found for increasing D_0 values. From these results, we conclude that non-SSMs accounting for the effect of extra mixing lead to the prediction:

$$Y_{\odot}^{\text{ini}} = 0.273 \pm 0.006(\text{helio, param}) \pm 0.002(\text{syst}), \quad (12)$$

consistent with the result from SSMs given in 12.

5. SUMMARY AND CONCLUSIONS

We have used standard solar models to study the dependence of the initial (Y_{ini}) and surface (Y_{surf}) helium abundances on different input physics and element abundances entering solar model calculations. As expected, there is a very tight correlation between these two quantities, particularly if direct effects of diffusion are not taken into account. We have used this tight correlation to express the initial solar helium abundance (Y_{\odot}^{ini} ; not to be confused with the initial value predicted by models) as a function of the solar surface helium abundance, Y_{\odot}^{surf} , determined from helioseismology, while minimizing the uncertainties from solar model input parameters. This result is expressed in a compact form in Equation 10. A slightly more general expression where the scaling of the helium and metals diffusion rate is still explicit is given in Equation 8.

We have applied Equation (10) to different, recently published standard solar models listed in Table 2, and derive $Y_{\odot}^{\text{ini}} = 0.278 \pm 0.006$ (Equation 11). The uncertainty is dominated by uncertainties in the diffusion rate of elements and those in the helioseismic determination of the solar surface helium abundance. The dispersion in the results for the different solar models is one order of magnitude smaller. The results are also robust in terms of the standard solar models used to derive Equation 10.

To test further the robustness of our results, we have compiled results from literature for standard and non-standard solar models (though only those non-standard solar models that improve agreement with helioseismic inferences of solar structure). In all cases, even for the non-standard ones, applying Equation 10 to results from solar models yield Y_{\odot}^{ini} results consistent to within 1σ of our central value of $Y_{\odot}^{\text{ini}} = 0.278 \pm 0.006$.

Finally, we have also computed solar models including turbulent diffusion as a phenomenological approach to eliminate the bump in the sound-speed difference between models and the Sun that is seen below the convective envelope. For these models, a small value of the free parameter D_0 , $D_0 = 300 \text{ cm}^2 \text{ s}^{-1}$, leads to $Y_{\odot}^{\text{ini}} = 0.2725$, consistent within 1σ with our prediction given in Equation 11. Larger values of D_0 predict systematically lower values for Y_{\odot}^{ini} but lead to slightly degraded agreement with helioseismology, as evinced by

frequency separation ratios (probably a result of the lower central helium abundance predicted by these models). Thus, we find no evidence, at least from helioseismology data, that support the need of larger D_0 values. It has to be kept in mind that this formulation of turbulent diffusion represents only a phenomenological approach to eliminating the bump in the sound-speed difference profile. More realistic models that account for rotationally-induced mixing, like that one derived by Pinsonneault et al. (1999), lead again to $Y_{\odot}^{\text{ini}} = 0.2725$.

We conclude that the initial solar helium abundance, as inferred from the present-day solar surface helium abundance determined by helioseismology, is $Y_{\odot}^{\text{ini}} = 0.278 \pm 0.006$. Although solar models are used to reach this conclusion, we find this result is almost independent of which solar models are used in its derivation. If Y_{\odot}^{ini} is determined from non-standard solar models that account for the effects of extra mixing, we obtain $Y_{\odot}^{\text{ini}} = 0.273 \pm 0.006$. In all cases, i.e., for both standard and non-standard models, we infer Y_{\odot}^{ini} values that are higher than the initial helium abundance obtained in solar models that use the solar abundances by Asplund et al. (2005) or the most recent determination by Asplund et al. (2009). This result points towards a deficit in solar models using these abundances; whether the problem lies in the abundance determinations or in the constitutive physics of the models is beyond the scope of this work.

Depending on the solar metallicity adopted, we derive a helium-to-metal enrichment of $\Delta Y/\Delta Z \sim 1.7 - 2.2$, in line with standard predictions of Galactic chemical evolution and other determinations of $\Delta Y/\Delta Z$ available in the literature.

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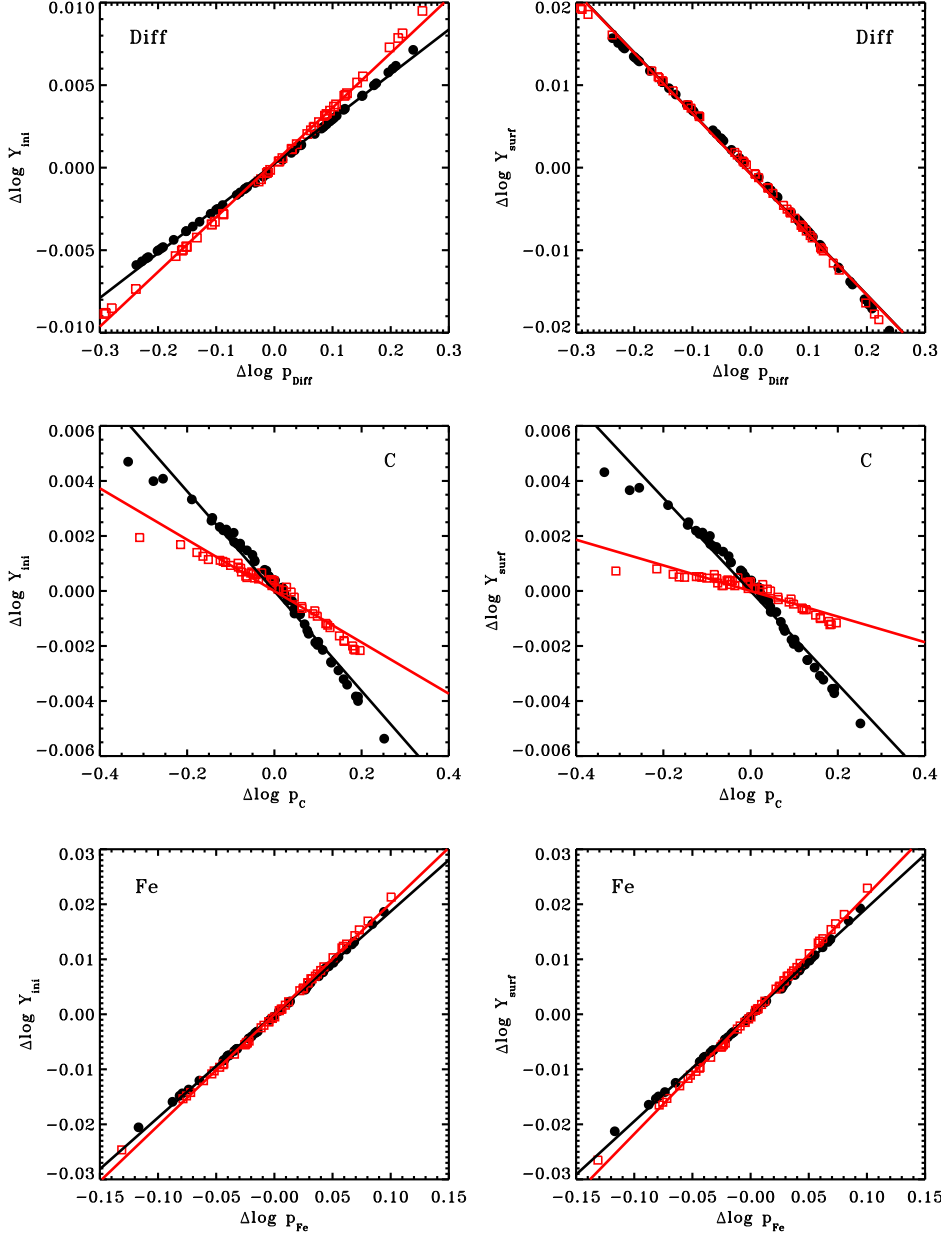


Fig. 1.— Some examples of the derivation of power-law exponents. Left panels show results for Y_{ini} and right panels for Y_{surf} . From top to bottom, results correspond to changes in the diffusion rate, carbon, and iron abundances. Black filled circles: results for GS98 composition; red empty squares: results for AGS05 composition. In each panel, lines denote the linear fits to the model results; the slope is the power-law exponent (see Equations 3).

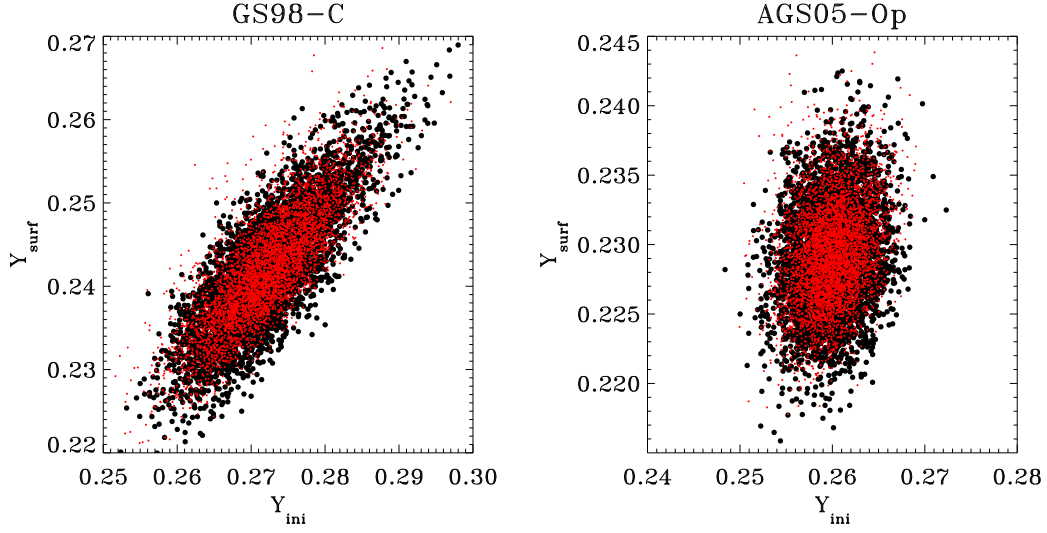


Fig. 2.— Comparison between Y_{surf} vs. Y_{ini} relation obtained with full solar models (big black dots) with the obtained using power-law expressions given in Eqs. 1 and 2 and Table 1 (small red dots). Results for GS98 composition and conservative uncertainties and AGS05 composition with optimistic uncertainties are shown.

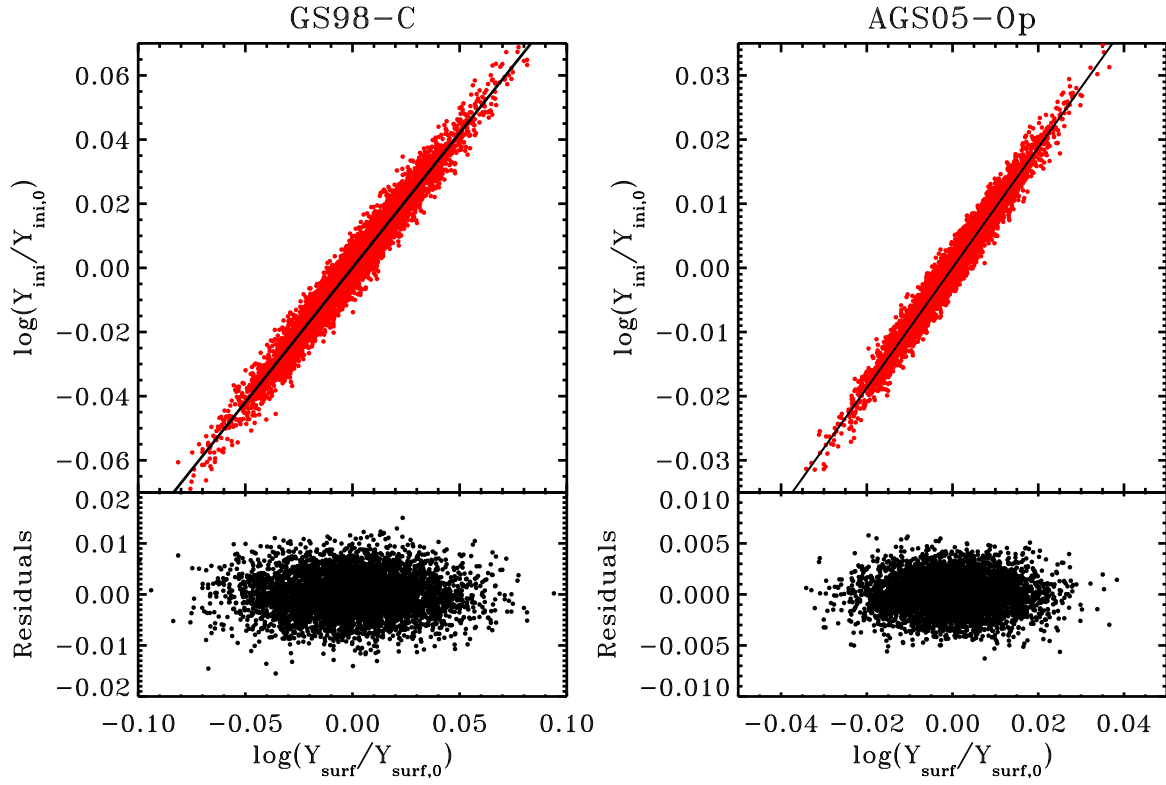


Fig. 3.— Y_{ini} vs. Y_{surf} distributions obtained using power-law expansions (Equations 4 and 5) but neglecting variations in the diffusion rates (first factor in the equations).

Table 1. Power-law exponents for solar model input parameters.

| Solar param. | GS98 | | AGS05 | |
|--------------|------------------|-------------------|------------------|-------------------|
| | Y_{ini} | Y_{surf} | Y_{ini} | Y_{surf} |
| | α_i | β_i | α_i | β_i |
| s11 | 0.0606 | 0.1290 | 0.0603 | 0.1393 |
| s33 | -0.0037 | -0.0047 | -0.0034 | -0.0044 |
| s34 | 0.0079 | 0.0098 | 0.0075 | 0.0095 |
| s17 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| sbe7e | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| shep | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| sn14p | 0.0010 | 0.0011 | 0.0006 | 0.0007 |
| age | -0.1360 | -0.1954 | -0.1408 | -0.2002 |
| difu | 0.0271 | -0.0733 | 0.0331 | -0.0733 |
| lumi | 0.3770 | 0.3522 | 0.4070 | 0.3810 |
| C | -0.0079 | -0.0073 | -0.0040 | -0.0020 |
| N | -0.0017 | -0.0009 | -0.0004 | 0.0010 |
| O | 0.0083 | 0.0183 | 0.0140 | 0.0285 |
| Ne | 0.0285 | 0.0355 | 0.0237 | 0.0313 |
| Mg | 0.0277 | 0.0317 | 0.0314 | 0.0366 |
| Si | 0.0593 | 0.0582 | 0.0682 | 0.0696 |
| S | 0.0439 | 0.0414 | 0.0506 | 0.0480 |
| Ar | 0.0123 | 0.0112 | 0.0075 | 0.0070 |
| Fe | 0.0813 | 0.0842 | 0.0877 | 0.0943 |

Table 2. Initial solar helium abundance predictions from different standard solar models

| SSM | $Y_{\text{ini},0}$ | $Y_{\text{surf},0}$ | Y_{\odot}^{ini} $K_{(\text{I},\text{S})} = 0.836$ | Y_{\odot}^{ini} $K_{(\text{I},\text{S})} = 0.903$ |
|----------|--------------------|---------------------|---|---|
| GS98 | 0.2721 | 0.2423 | 0.2779 | 0.2782 |
| AGS05 | 0.2593 | 0.2292 | 0.2774 | 0.2785 |
| AGSS09 | 0.2617 | 0.2314 | 0.2777 | 0.2787 |
| AGSS09ph | 0.2653 | 0.2349 | 0.2771 | 0.2788 |