Astronomy
&
Astrophysics

Global prominence oscillations
(Research Note)

U. Anzer

Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85740 Garching, Germany
e-mail: ula@mpa-garching.mpg.de

ABSTRACT

Aims. The question of the different oscillation periods for global modes of quiescent prominences is discussed.
Methods. Simple 1D prominence configurations are used to describe the magnetohydrostatic equilibrium and their oscillations for small amplitudes.
Results. Three basic modes of oscillations were found and their periods as a function of the magnetic field configuration and the assumed geometry are given.

Key words. Sun: prominences

1. Introduction

Oscillatory motions of solar prominences have been observed for a long time, going back to the early investigations of Harvey (1969). The observed velocity amplitudes of the oscillations range from a few $km\,s^{-1}$ up to around $100\ km\,s^{-1}$. Many theoretical investigations which try to model these oscillations have been presented over the years. A summary of these theoretical models can be found in the review of Ballester (2006), which also gives in some detail the relevant observations.

As is well known, prominences are massive cool structures in the solar corona which have to be supported against gravity by a sufficiently strong upward force. The most obvious force is the Lorentz force of the associated magnetic field. This aspect has to be taken into account in the basic equilibrium models. Another important point for the oscillation modelling is the fact that the prominence can interact with its coronal surroundings; this will have a non-negligible effect on the the different modes.

Most of the existing models do not fulfill all these requiremnts simultaneously. There is for example a paper by Oliver et al. (1992) where the full magneto-hydrostatic (MHS) equilibrium is properly treated. But only internal modes are considered by prescribing vanishing velocities at the prominence-corona interface. This then means that the effects of the surrounding corona are ignored. On the other hand a study by Joarder & Roberts (1992) allows both for internal and global oscillations (i.e. modes where the prominence and the corona are involved). But in this investigation gravity is completely neglected and the magnetic support not treated correctly.

There are also a few models which aim at including the effects of gravity and of the surrounding corona simultaneously, e.g. an investigation by Joarder & Roberts (1993a) and another by Oliver et al. (1993). We postpone the comparison of these models to our new results to Sect. 6. In the present investigation we look at oscillations of prominences which are in MHS equilibrium and which also can interact with the surrounding coronal magnetic field. In order to make this rather complex problem tractable for analytical investigations we have to make some rather strong simplifications which will be described in

the following sections. In Sect. 2 we present our basic prominence model and calculate the different oscillatory modes. In Sect. 3 we mention some other types of modelling and in Sect. 4 we describe possible applications of our results. Sect. 5 briefly outlines the consequences of magnetic shear. Sect. 6 gives the conclusions.

2. Global oscillations

For our modelling we use the simplest configuration of a prominence in MHS equilibrium. We take a 1D slab model of the type proposed earlier by Kippenhahn & Schlüter (1957); more details of such slab models can be found in the paper by Heinzel & Anzer (2005) and models that allow for vertical variations have been discussed by Hood & Anzer (1990). But in the present investigation we adopt the most simple configuration. For the description we use Cartesian coordinates with x normal to the slab, y along its axis and z pointing vertically upward. In this model all physical quantities are assumed to vary only in the x-direction. For the magnetic field inside the prominence we take the relation

$$\boldsymbol{B} = (B_x, 0, B_z(x)), \tag{1}$$

with $B_x = $ const. and $B_z(x)$ being antisymmetric with respect to $x = 0$. The prominence boundaries are placed at the positions $x = \pm D/2$. If we then denote by $\rho(x)$ the density and by g the solar gravity we obtain the following equation for vertical equilibrium:

$$\frac{1}{4\pi} B_x \frac{\mathrm{d}B_z}{\mathrm{d}x} - \rho g = 0, \tag{2}$$

and the column mass, M, is obtained as

$$M = \int_{-D/2}^{+D/2} \rho(x)\mathrm{d}x. \tag{3}$$

Equation (2) can be integrated and leads to

$$\frac{B_x B_{z_1}}{2\pi} = Mg, \tag{4}$$

where $B_{z_1} = B_z(D/2)$ has been taken.
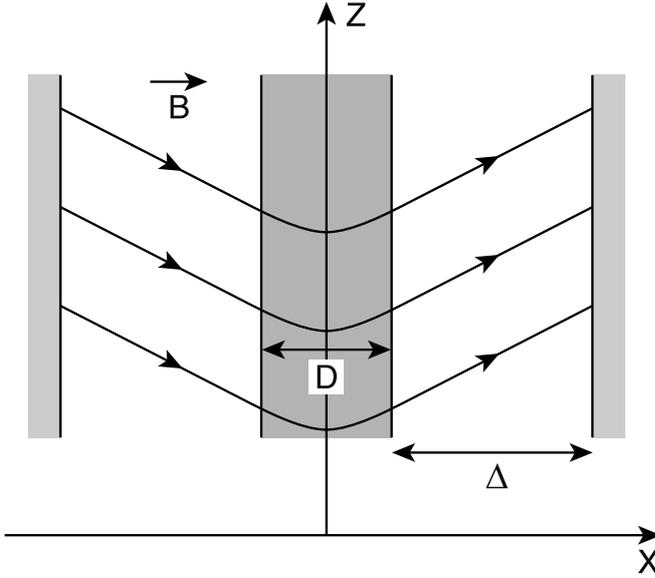
Fig. 1. Sketch of the prominence configuration used for the present modelling. Note that the true width of the prominence is much smaller than drawn here.

Note: Eq. (4) holds for any temperature distribution present inside the prominence. More details about the internal structure of the prominence can be found in Heinzel & Anzer (2005).

Outside of the prominence we have a low β plasma for which we assume a potential magnetic field which is anchored in the solar photosphere. In order to make the equilibrium configuration tractable for an analytical investigation we replace the photosphere by two vertical solid boundaries placed at $x = \pm\Delta$. In this case the coronal field is given by

$$\boldsymbol{B} = (B_x, 0, \pm B z_1). \tag{5}$$

This configuration is shown in Fig.1.

For this type of slab model one has 3 fundamental oscillatory modes: horizontal motion in x-direction, horizontal motion in y-direction and vertical oscillations. For all of the fundamental modes the prominence will move as a solid body without any internal deformations. We now denote by ξ the displacement in each of these modes and then obtain the general equation of motion:

$$M\frac{\mathrm{d}^2\xi}{\mathrm{d}t^2} = F(\xi), \tag{6}$$

where $F(\xi)$ is the restoring force which has to be determined for each of the 3 modes individually.

2.1. Oscillations in the x-direction

The restoring force in this case is the difference in magnetic pressure on the right and left side of the prominence. Because of flux conservation one has the relation

$$B_z = B_{z_1}(1 \pm \xi/\Delta). \tag{7}$$

With this one then obtains for F the expression

$$F(\xi) = -\frac{B_{z_1}^2}{2\pi\Delta}\xi. \tag{8}$$

Making use of Eqs. (4) and (6) we arrive at the oscillation equation given by

$$\frac{\mathrm{d}^2\xi}{\mathrm{d}t^2} = -\frac{B_{z_1}}{B_x}\frac{g}{\Delta}\xi \tag{9}$$

and with this we obtain the period of the oscillations as

$$P = 2\pi\sqrt{\frac{B_x}{B_{z_1}}\frac{\Delta}{g}}\ . \tag{10}$$

2.2. Oscillations in the y- and z-direction

For the oscillations in the y-direction the restoring force results from the magnetic tension of the stretched field lines. We now have

$$F(\xi) = -\frac{B_x^2}{2\pi\Delta}\xi, \tag{11}$$

leading to a period of

$$P = 2\pi\sqrt{\frac{B_{z_1}}{B_x}\frac{\Delta}{g}}\ . \tag{12}$$

For the oscillations in the z-direction the restoration is again achieved by field line stretching. In this case we obtain

$$B_z = B_{z_1} + B_0\frac{\xi}{\Delta}\ . \tag{13}$$

Taking the MHS equilibrium as given in Eq. (4) into account we then arrive for the restoring force at the same expression as for the y-oscillations (i.e. $F(\xi)$ is also given by Eq. (11)) and therefore the periods are the same. It is interesting to note that although the physical situation is quite different for y- and z-oscillations, the periods turn out to be identical. But for configurations with magnetic shear this is no longer the case (see the discussion in Sect. 5).

2.3. Interpretations

Equations (10) and (12) are actually those of a swinging pendulum which has a length of $\Delta(B_{z_1}/B_x)$, respectively $\Delta(B_x/B_{z_1})$. At first sight this result might seem somewhat surprising since the restoring force in all cases is of magnetic origin and therefore one would expect the field strength to be the determing factor. But since in our MHS equilibrium the field is directly related to gravity these results are very plausible. An alternative interpretation based upon a model consisting of an isolated blob hanging in a magnetic string has been proposed by Roberts (1991). This modelling gives the same result as Eq. (12) for vertical oscillations. Much more detailed discussions of the so-called string modes can be found in Joarder & Roberts (1992, 1993a) and Roberts & Joarder (1994). In our modelling the quantity $\Delta(B_{z_1}/B_x)$ entering Eq. (12) can be interpreted as the total depth of the field line dips; the quantity $\Delta(B_x/B_{z_1})$ has no obvious physical meaning. Both observations and prominence modelling indicate that in general the field line inclination is rather small and therfore $B_{z_1}/B_x \ll 1$ will hold. From this we conclude that in most cases the periods of the x-oscillations will be longer than those of the other two modes. As a representative example we take $B_{z_1}/B_x = 0.1$ and $\Delta = 100\,000$ km and then obtain typical values for the periods of 200 min for x-oscillations and 20 min for the other cases. If the field lines are shorter, and Δ smaller, then the periods will be reduced correspondingly.

3. Pressure driven oscillations

One could also consider the alternative possibility that the coronal magnetic field is so strong that the prominence cannot distort this field by a large amount. If we further assume that the field inside the prominence is basically horizontal (i.e. has only negligible dips) then the only restoring force will be the difference in coronal gas pressure between the left and right side of the slab. This force is given by

$$F(\xi) = -2\frac{p_c}{\Delta}\xi \tag{14}$$

and therefore one has

$$P = 2\pi \sqrt{\frac{\Delta}{2p_c}M}, \tag{15}$$

where p_c is the unperturbed coronal pressure. Since in this particular case the pressure inside the prominence, and therefore also the density because of an assumed constant temperature, are uniform one obtains the relation

$$M = \rho_0 D, \tag{16}$$

with ρ_0 denoting the prominence density and D its full width. For the equilibrium one also needs

$$p_0 = p_c. \tag{17}$$

We also set

$$\rho_0 = \frac{p_0}{c_s^2}, \tag{18}$$

where c_s is the sound velocity inside the cool slab. Making use of these relations we arrive at the equation

$$P = 2\pi \sqrt{\frac{\Delta D}{2}}\frac{1}{c_s} \tag{19}$$

for the period.

Note: this period is identical to that found by Joarder & Roberts (1992) for the case of slow kink modes.

If we now take some typical values for the configuration with $p_c = 0.03$ dyn/cm^2, $M = 10^{-4}$ g/cm^2 and $T = 6000$ K we can derive the following numbers: $\rho_0 = 7 \times 10^{-14}$ g/cm^3, $c_s = 6.5$ km s^{-1} and $D = 14\,000$ km. Together with the value for Δ which we used in the previous section of $100\,000$ km we get in this case an oscillation period of $P = 430$ min. This value is much larger than any period observed in prominences. The values for the period could be reduced by assuming higher temperatures which will result in higher sound velocities and also smaller column masses which correspond to smaller values of D (e.g. a temperature of $10\,000$ K and a column mass of 5×10^{-5} g/cm^2 gives a period of 245 min.).

But a more serious problem with this type of models is the fact that they require extremely large coronal magnetic fields. In order to be able to neglect the effects of hydrostatic layering inside the slab the magnetic field has to be horizontal to a high degree of accuracy. If we take for example this constraint as $B_{z_1} \ll 0.1B_x$ then Eq. (4) together with a column mass of 10^{-4} g/cm^2 leads to the condition

$$B_x \gg 13 \text{ G}. \tag{20}$$

Therefore one has to conclude that only those prominences with such very large magnetic fields will allow this kind of simplified modelling.

4. Consequences for the diagnosing of prominences

The study of prominence oscillations can give us detailed insight into the physics of prominences. The present status of this field of so-called prominence seismology has been summarised in the reviews of Oliver (2004) and Ballester (2006). Interesting new observations can be found in the papers of Régnier et al. (2001) and Pouget et al. (2006). Our new modelling of of the global oscillations can then also be used to derive the basic parameters of solar prominences. The important point is that both the periods of normal and tangential oscillations must be measured. If we denote by P_1 the period of oscillations in the x-direction and by P_2 that of oscillations either in the y- or in z-direction we obtain from Eqs. (10) and (12) the relation

$$\frac{B_{z_1}}{B_x} = \frac{P_2}{P_1}. \tag{21}$$

We can determine in this way the field line inclination at the prominence boundary. One also gets

$$\Delta = 4\pi^2 g P_1 P_2, \tag{22}$$

which gives an estimate of the length of the coronal field lines.

In addition the observations of prominences on the limb seen in the Hα line allow a determination of the column mass, M (Gouttebroze et al. 1994). With this knowledge we can then also calculate the strength of the magnetic field as

$$B_x = \sqrt{2\pi M g \frac{P_1}{P_2}}. \tag{23}$$

This shows that for our models all global physical parameters can be derived, in principle. But it is important to note that for this procedure the knowledge of both P_1 and P_2 of the prominence under investigation will be needed.

5. Effects resulting from magnetic shear

So far we have considered only unsheared prominence configurations in order to make our analysis as simple and transparent as possible. But the observations of quiescent prominences indicate the presence of strong magnetic shear (see e.g. Leroy et al. 1984). Therefore we shall now briefly outline what consequences the magnetic shear has on our modelling.

We model this by adding a constant y-component, B_y, to the equilibrium field. With this then the horizontal field strength is given by

$$B_h = \sqrt{B_x^2 + B_y^2}, \tag{24}$$

and the shear is quantified by the parameter $\alpha = B_h/B_x$. With this definition the length of the horizontal field to the boundary now amounts to $\Delta_1 = \alpha\Delta$ and the column mass as measured along the field is given by $M_1 = \alpha M$. With this modified configuration we find the following three basic types of oscillations: motions along the horizontal field direction, those perpendicular to it, and verical ones. The first two modes correspond respecively to the x- and y-oscillation of the unsheared case. We found that the equations for the periods of the first two types of oscillations are the same as previously calculated but with B_x replaced by B_h and M replaced by M_1. In the equation for the vertical mode an additional factor $\sqrt{\alpha}$ has to introduced. If we keep the values of B_x and Δ fixed we get the following scaling laws for the periods: the mode along the field is proportional to α, the vertical

one to $\sqrt{\alpha}$, and the perpendicular mode is unchanged. This result implies that the periods of the first two modes are increased and the third one is not affected by the shear. Very long period oscillations resulting from a strong magnetic shearing were also predicted by Joarder & Roberts (1993b).

6. Conclusions

The prominence models considered in this paper are highly simplified configurations. Still, they can be taken as reasonable first order approximations to real prominence slabs. The assumption that the slab is very extended along the direction of the prominence axis is acceptable. But assuming also that the prominence extension in height is very large is less appropriate. There is in particular the aspect that the solar surface represents a solid lower boundary. This can influence the vertical oscillations. If we assume that the prominence does not extend down to the surface then the magnetic field which crosses the region below the prominence will provide some additional restoring force. This means that the periods calculated from our Eq. (12) are actually overestimates and the true periods of vertical oscillations will be smaller. The reduction of the period will depend on the details of the prominence geometry.

Our simple analytical results given by Eqs. (10) and (12) can also be compared with other approaches given in the literature. Joarder & Roberts (1993a) investigated oscillations that occur in the prominence model of Menzel. They obtained periods for the different modes ranging from 4 min up to 48 min. But these results have to be taken with caution since the Menzel model is not a realistic model for prominences and a comparison with our results is not so meaningful. The only other investigation which takes both gravity and the coronal surrounding into account is that of Oliver et al. (1992). Because of the large complexity they had to treat the problem numerically and thus could only calculate the oscillations for one prominence. For the fundamental modes they found $P = 70$ min for horizontal oscillations and $P = 13$ min for vertical ones. Using our Eqs. (10) and (12) and taking their prominence model we get 78 min, resp. 15 min for the basic modes. Therefore our simple, but analytical, model gives similar results. From this we conclude that if one is interested in the oscillations of the prominence as a whole our simple treatment is fully adequate.

In addition we have the problem of prominence fine structure. But as long as these fine structures are small compared to the prominence extensions one can consider all the quantities used here as mean values. With this then the global aspects of the oscillations will be described appropriately. But if one considers localised modes of oscillations then the fine structures are of primary interest. Such a fine structure modelling has recently be done by Joarder et al. (1997).

The other crucial point is that the coronal magnetic field is assumed to have straight field lines which are anchored in two vertical rigid boundaries. In reality the field has to bend over to connect to the solar surface. This bending is completely ignored in our modelling, whereas the photospheric line-tying is included. Therefore our estimates for the magnetic restoring forces are only approximations. Unfortunately the modelling based upon more realisic curved field configurations cannot be done in a simple analytic way.

We also want to point out that our modelling concentrates on the global modes and therefore it will give us information on the overall magnetic field structure as well as the total prominence mass. For complementary information concerning the interior of the prominence the internal modes of oscillation which were briefly lined out in the introduction should be used.

References

Ballester, J. L. 2006, Phil. Trans. R. Soc. A 364, 405
Gouttebroze, P., Heinzel, P., & Vial, J. C. 1994, A&AS, 99, 513
Harvey, J. 1969, Ph.D. Thesis, University of Colorado, USA
Heinzel, P., & Anzer, U. 2005, in Solar Magnetic Phenomena, ed. A. Hanslmeier, A. Veronig, & M. Messerotti (Springer) ASSL, 320 115
Hood, A. W., & Anzer, U. 1990, SP, 126, 117
Joarder, P. S., & Roberts, B. 1992, A&A, 261, 625
Joarder, P. S., & Roberts, B. 1993a, A&A, 273, 642
Joarder, P. S., & Roberts, B. 1993b, A&A, 277, 225
Joarder, P. S., Nakariakov, V. M., & Roberts, B. 1997, SP, 173, 81
Kippenhahn, R., & Schlüter, A. 1957, Z. Astrophys., 43, 36
Leroy, J. L., Bommier, V., & Sahal-Bréchot, S. 1984, A&A, 131, 33
Oliver, R. 2004, in Proc. of SOHO 13, Palma de Mallorca, ESA SP-547, 175
Oliver, R., Ballester, J. L., Hood, A. W., & Priest, E. R. 1992, ApJ, 400, 369
Oliver, R., Ballester, J. L., Hood, A. W., & Priest, E. R. 1993, ApJ, 409, 809
Pouget, G., Bocchialini, K., & Solomon, J. 2006, A&A, 450, 1189
Régnier, S., Solomon, J., & Vial, J. C. 2001, A&A, 376, 292
Roberts, B. 1991, Geophys. Astrophys. Fluid Dyn., 62, 83
Roberts, B., & Joarder, P. S. 1994, in Proc. of 7th ESP meeting on Solar Physics, ed. G. Belvedere, M. Rodonò, & G. M. Simnett, LNP, 432, 173