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Formation and Evolution of Cataclysmic Variables

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Abstract. This article summarizes the basic facts and ideas concerning the formation and evolution of cataclysmic variables (CVs). It is shown why the formation of CVs must involve huge losses of mass and orbital angular momentum, very likely via a common envelope evolution. A brief discussion of the principles of the long-term evolution of semi-detached binaries follows. Finally a brief sketch of CV evolution is given.

Key words. Stars: evolution – Stars: binaries: close – Stars: novae, cataclysmic variables

1. Introduction

Cataclysmic variables (CVs) are short-period semi-detached binary systems in which a white dwarf (WD) primary accretes matter from a low-mass companion star (Warner 1995). CVs are intrinsically variable and that on a wide range of time scales (from seconds to $\gtrsim 10^6$ yr) and with a huge range of amplitudes (of up to 10^6 and possibly even more). The rich phenomenology of CV variability which includes, among other things, phenomena like flickering, dwarf nova and classical nova outbursts, can to a large extent be understood as either immediate or long-term consequences of the mass transfer process. Interesting as all these phenomena are, they are of no particular interest here. Rather, in the following I shall concentrate on evolutionary aspects, i.e. on the formation and evolution of CVs. Readers who are mainly interested in CVs as variable stars should instead turn to the monographs by Warner (1995) or Hellier (2002).

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2. Very basic facts about CVs and stellar evolution

2.1. Generic properties of CVs

From the perspective of stellar evolution a CV is a semi-detached binary in which a WD primary of mass M_1 accretes from a low-mass secondary star of mass M_2 which fills its critical Roche lobe. From Roche geometry it follows that the secondary's radius can be written as $R_2 = a\, f_2(q)$. Here a is the orbital separation, $q = M_1/M_2$ the mass ratio, and f_2 the fractional Roche radius of the donor star. For typical values of q found in CVs, i.e. $1 \lesssim q \lesssim 10$, Eqs.(2) or (3), given below, yield $0.2 \lesssim f_2 \lesssim 0.4$.

In principle, the mass of the WD component can be anywhere between the lowest possible value resulting from stellar evolution ($\sim 0.15 M_\odot$) and the Chandrasekhar mass $M_{\rm CH} \approx 1.4 M_\odot$. Observed masses are mostly in the range $0.5 M_\odot \lesssim M_1 \lesssim 1 M_\odot$. As to the mass distribution there are reasons to believe that intrinsically it is not unlike that of single WDs which have a mean mass of $< M_{WD} > \approx 0.6 M_\odot$.

From the observed mass transfer rates one can infer that mass transfer in CVs is stable. This, in turn, requires that the mass of the donor is typically less than that of the WD component, i.e. $M_2 \lesssim M_1$, or $q \gtrsim 1$, and thus that the donor is a low-mass star. Observations show that in more than 95% of all cases the donor star is on the main sequence (MS), though not necessarily close to the zero age main sequence (ZAMS). In rare cases the donor star is either a giant, or a WD of very low-mass ($M_2 \lesssim 0.05 M_\odot$).

For later comparison it is useful to keep in mind the resulting typical system parameters of a CV with a MS donor:

- **total mass:** $\mathcal{M} = M_1 + M_2 \approx M_\odot$
- **orbital separation:** $a \approx$ few $R_2 \approx R_\odot$
- **orbital period:** 80 min $\lesssim P_{\rm orb} \lesssim 10^{\rm h}$
- **orbital angular momentum:**

$$\begin{aligned} J_{\rm orb} &= G^{1/2} M_1\, M_2\, (M_1 + M_2)^{-1/2}\, a^{1/2} \quad (1)\\ &\approx J_0 = G^{1/2}\, {M_\odot}^{3/2}\, {R_\odot}^{1/2}\,. \end{aligned}$$

2.2. Evolution of single and binary stars

In the following I summarize the basic facts which characterize single star and binary evolution, and which are of relevance in the context of our considerations. These facts are:

1. *Stars grow considerably as they age* (by factors up to $\gtrsim 10^2$). Because this growth is not strictly monotonic one can distinguish distinct evolutionary phases during which a star grows. These phases are:
   - central hydrogen burning, i.e. on the MS
   - for intermediate mass and massive stars ($M \gtrsim 2.2 M_\odot$) the post-MS evolution towards He-ignition including the evolution through the Hertzsprung gap
   - for low-mass stars ($M \lesssim 2.2 M_\odot$) evolution on the first giant branch up to the He-flash
   - for low and intermediate mass stars ($M \lesssim 10 M_\odot$) evolution on the asymptotic giant branch (AGB)
2. *The more massive a star, the faster it ages.* Stars on the main sequence obey a mass luminosity relation. On the upper MS ($1\,M_\odot \lesssim M \lesssim 10\,M_\odot$) the luminosity L scales roughly as $L \propto M^{3.5}$. Hence the nuclear time scale is $\tau_{\rm nuc} \propto M/L \propto M^{-2.5}$. The immediate consequence of this is that *of two stars with the same age (as in a binary) but different mass the more massive star grows faster, i.e. is the bigger of the two.*
3. *In a binary the presence of a companion limits the size up to which a star can grow (Roche limit) without losing mass to its companion.* The maximum radii corresponding to the Roche limit are the critical Roche radii $R_{1,\rm R} = a\, f_1(q)$ and $R_{2,\rm R} = a\, f_2(q)$ for respectively the primary and the secondary, where $f_1(q) = f_2(1/q)$, and according to Paczyński (1971) and Eggleton (1983) for $1 \leqslant q \lesssim 10$

$$f_2(q) \approx 0.462\,(1+q)^{-1/3}, q \gtrsim 1.25 \qquad (2)$$
$$f_1(q) \approx 0.38 + 0.2 \log q \approx q^{0.45}\, f_2(q)\,. \quad (3)$$

As a consequence of this, stellar evolution in a binary of not too large an orbital separation a results sooner or later in the formation of a so-called semi-detached binary in which the more massive component reaches its Roche limit first and starts transferring mass to its companion.

2.3. Prerequisites for white dwarf formation

WDs are the end product of the evolution of stars of low and intermediate initial mass. Thereby the chemical composition of a WD reflects the evolutionary state of the star when it loses its hydrogen-rich envelope. Depending on when this happens along the evolution the result is either a WD consisting mainly of helium (He-WD), of carbon and oxygen (CO-WD), or oxygen and neon (ONe-WD).

- **He-WDs** result from the complete loss of the hydrogen-rich envelope of a low-mass star (with an initial mass $M_{\rm i} \lesssim 2.2 M_\odot$) on the first giant branch, i.e. before reaching the He-flash. Accordingly, the mass of He-WDs is in the range $0.15 M_\odot \lesssim M_{\rm He-WD} \leqslant M_{\rm He-Fl}$, where $M_{\rm He-Fl} \approx 0.45 -$

$0.50 M_\odot$ is the mass of the He core at the onset of the He-flash. Because wind mass loss of single stars on the first giant branch is not strong enough for complete envelope loss isolated He-WDs are not formed. However, they can result from mass transfer in a close binary (see e.g. Kippenhahn, Kohl & Weigert (1967)).

- **CO-WDs** result from the complete loss of the hydrogen-rich envelope of intermediate mass stars on the AGB, i.e. before the onset of carbon burning. For single stars this happens if the initial mass is $M_\mathrm{i} \lesssim 6-8 M_\odot$. In binary stars this can happen for initial masses up to $\sim 10 M_\odot$. Accordingly, the resulting WD masses are in the range $M_\mathrm{He-Fl} \lesssim M_\mathrm{CO-WD} \leqslant M_\mathrm{C-ign}$, where $M_\mathrm{C-ign} \approx 1.1 M_\odot$ is the core mass at the onset of carbon ignition.
- **ONe-WDs** originate from stars which undergo off-center carbon ignition and subsequent envelope loss during the so-called super-AGB phase. For single stars this is possible for initial masses in the range $9 M_\odot \lesssim M_\mathrm{i} \lesssim 10 M_\odot$, whereas in binaries the mass range is $9 M_\odot \lesssim M_\mathrm{i} \lesssim 12 M_\odot$ (see e.g. Gil-Pons & García-Berro (2001), Gil-Pons et al. (2003)). The resulting WDs have masses in the range $1.1 M_\odot \lesssim M_\mathrm{ONe-WD} \lesssim 1.38 M_\odot$.

In the context of our considerations, one of the most important properties of stars which have a degenerate core of mass M_c is that they obey by and large a core mass-luminosity relation $\mathcal{L}(M_\mathrm{c})$, and to the extent that these stars have a sufficiently massive hydrogen-rich envelope and thus are close to the Hayashi-line, also a core mass-radius relation $\mathcal{R}(M_\mathrm{c})$ (see e.g. Paczyński (1970), Kippenhahn (1981), Joss, Rappaport & Lewis (1987)). This relation shows that the radius of such a star is a steeply increasing function of core mass and that, in particular, AGB stars and stars on the super-AGB are very large with radii of up to $\sim 10^3 R_\odot$. In other words: *the formation of a WD requires a lot of space*, the more massive the WD the more space. This is not a problem for single stars. But in a binary, as a consequence of the Roche limit, the orbital separation a sets an upper limit to the mass of the WD that can be formed: $M_\mathrm{WD} \lesssim \mathcal{R}^{-1}(a\, f_1(q))$.

2.4. Single star evolution versus binary star evolution

The task of calculating the structure and evolution of a single star consists of solving a well-known set of differential equations with appropriate boundary conditions and initial values (e.g. Kippenhahn & Weigert (1990)).

For calculating the evolution of a binary system (or of one of its components) the task is in principle the same as for single stars. The difference is that in a binary one has an additional boundary condition which derives from the presence of the companion star, i.e. from the Roche limit.

Consider for simplicity a system consisting of a "real " star, say the primary, and a point mass secondary. The simplest boundary condition that one could impose in this case is that $R_1 \leqslant a\, f_1(q)$. A more realistic approach would take into account that the surface of a star is not arbitrarily sharp, but rather is characterized by a finite scale height $H \ll R$ over which pressure, density etc. drop off, by expressing the mass loss rate $-\dot{M}_1$ as an explicit function of binary and stellar parameters (e.g. Ritter (1988)). What one finds is that $-\dot{M}_1$ is a steeply increasing function of $(R_1 - R_{1,\mathrm{R}})/H$ and that the primary suffers significant mass loss as $R_1 \to R_{1,\mathrm{R}}$.

The real problem when dealing with mass transfer consists of answering two questions: 1.) Where does the mass lost from the donor go? and 2.) How much angular momentum does it take with it? On the formal level this can be dealt with as follows: let us assume that a fraction η of the transferred mass is accreted by the secondary, i.e.

$$\dot{M}_2 = -\eta\, \dot{M}_1\,. \tag{4}$$

Accordingly, the mass loss rate from the system is $\dot{\mathcal{M}} = (1-\eta)\,\dot{M}_1$. The angular momentum loss rate associated with this mass loss can be written as

$$\dot{J}_\mathrm{orb} = \nu\, \dot{\mathcal{M}}\, J_\mathrm{orb} / \mathcal{M}\,, \tag{5}$$

where ν is a dimensionless factor measuring the angular momentum leaving the system.

What is known about the two parameters η and ν? In general not more than 1.) $0 \leqslant \eta \leqslant 1$, and 2.) $\nu \geqslant 0$. Otherwise η and ν are (almost) free functions of the problem. Therefore, *calculating binary evolution involves (at least) two almost free functions*. And the art of performing such calculations very often consists of making creative use of this freedom!

2.5. Generic properties of CV progenitors

We are now in a position to define the necessary criteria which a binary consisting initially of two ZAMS stars of mass $M_{1,\rm i}$ and $M_{2,\rm i}$ has to meet in order to later become a CV which, at the onset of mass transfer, i.e. at the beginning of its life as a CV, consists of a WD of mass $M_{\rm WD}$ and a donor star of mass M_2.

1. *$M_{1,\rm i}$ has to have sufficient mass to allow for the formation of a WD of mass $M_{\rm WD}$.*
   In theoretical calculations of the evolution of single stars with a fixed set of physical assumptions (such as initial chemical composition, equation of state, opacities, nuclear reaction rates, convection theory, wind mass loss, etc.) there is a one to one relation between the initial mass $M_{\rm i}$ and the mass $M_{\rm f}$ of the white dwarf produced. This relation is known as the *initial mass-final mass relation*, i.e. $M_{\rm WD} = M_{\rm f}(M_{\rm i})$. And, within the observational uncertainties, there is also ample observational evidence for this $M_{\rm i}$-$M_{\rm f}$-relation (see e.g. Salaris et al. (2008) and references therein).
   In binary evolution things are a little different: because mass transfer sets a premature end to the donor's nuclear evolution the mass of the resulting white dwarf is smaller than what single star evolution of the primary would yield, i.e. $M_{\rm WD} < M_{\rm f}(M_{1,\rm i})$. In other words: for the formation of a WD of mass $M_{\rm WD}$ the necessary condition is $M_{1,\rm i} > M_{\rm f}{}^{-1}(M_{\rm WD})$.
2. Because of the core mass-radius relation $\mathcal{R}(M_{\rm c})$ which holds for the giant primary when it reaches its Roche limit, the initial separation of the binary must be $a_{\rm i} = \mathcal{R}(M_{\rm WD})/f_1(q_{\rm i})$, where $q_{\rm i}$ is the initial mass ratio. For this estimate of $a_{\rm i}$ we have implicitely assumed that after the onset of (the first) mass transfer $M_{\rm WD}$ = const.
3. Finally for the secondary's mass we assume $M_{2,\rm i} = M_2$. A justification for this will be given below.

Now, let us take typical parameters for a CV, say $M_{\rm WD} \approx 1M_\odot$ and $M_2 \lesssim 1M_\odot$, in order to see where this leads us: with $M_{\rm WD} \approx 1M_\odot$ it follows from the $M_{\rm i}$-$M_{\rm f}$-relation that $M_{1,\rm i} \gtrsim 5M_\odot$, hence $\mathcal{M}_{\rm i} \gtrsim 6M_\odot$, and from the core mass-radius relation $\mathcal{R}(M_{\rm WD}) \approx 10^3 R_\odot$, and with $f_1(q_{\rm i}) \approx 0.5$, $a_{\rm i} \sim 2\,10^3 R_\odot$. Therefore, the initial orbital angular momentum of the binary is

$$J_{\rm orb,i} = J_0 \left(\frac{M_{1,\rm i}}{M_\odot}\right)\left(\frac{M_{2,\rm i}}{M_\odot}\right)\left(\frac{\mathcal{M}_{\rm i}}{M_\odot}\right)^{-1/2}\left(\frac{a_{\rm i}}{R_\odot}\right)^{1/2} \approx 10^2 J_0 \,. \tag{6}$$

Comparing now the total mass and orbital angular momentum of a CV (cf. Sect. 2.1) with the corresponding values of its progenitor system we find that $\mathcal{M}_{\rm i}/\mathcal{M}_{\rm CV} \approx 5 - 10$ and $J_{\rm orb,i}/J_{\rm CV} \approx 10^2$. In other words: the formation of a CV invokes a binary evolution in which the progenitor system has to lose $\sim 80\% - 90\%$ of its initial mass and up to $\sim 99\%$ of its initial orbital angular momentum (Ritter 1976), and that after the onset of mass transfer from the primary.

3. Mass transfer and its consequences

Since the primary of a CV progenitor does not stop growing when approaching its Roche limit, onset of mass transfer is unavoidable. And, because the subsequent formation of a CV involves huge losses of mass and orbital angular momentum from the binary system, it is necessary to examine the consequences of mass transfer for the ensuing evolution in more detail.

3.1. Stability of mass transfer

A detailed discussion of the stability of mass transfer is rather complex and beyond the scope of this article. For this the reader is referred to e.g. Ritter (1988) or Ritter (1996). Here I shall keep the presentation as simple as possible.

Let us assume for the moment that the primary star has a sharp outer boundary and that it has just reached its Roche limit, i.e. that $R_1 = R_{1,R}$. What happens if at that moment, which we denote by t_0, a small amount of mass δm is taken away from the primary and transferred to the secondary, i.e. if $M_1 \rightarrow M_1 - \delta m$ and $M_2 \rightarrow M_2 + \delta m$? As a consequence of this small mass transfer, not only the mass ratio q and the critical Roche radii $R_{1,R}$ and $R_{2,R}$ will change but also the stellar radii R_1 and R_2. Let us for the moment treat the secondary as a point mass. Then we have to deal only with the radii $R_1(t > t_0)$ and $R_{1,R}(t > t_0)$. Thereby, three different situations can arise:

1. $R_1(t > t_0) < R_{1,R}(t > t_0)$: In this case *mass transfer is stable*, because after a small mass transfer δm the donor underfills its critical Roche volume and mass transfer stops.
2. $R_1(t > t_0) > R_{1,R}(t > t_0)$: In this case *mass transfer is unstable*, because if $R_1(t > t_0) - R_{1,R}(t > t_0) > 0$ even more mass flows over.
3. $R_1(t > t_0) = R_{1,R}(t > t_0)$: In this case *mass transfer is marginally stable*.

In order to decide which of the three above cases arises we must know how R_1 and $R_{1,R}$ react to mass transfer. For all practical purposes $R_{1,R}$ adjusts instantaneously (actually on the orbital time scale) to changes in M_1, M_2 and $J_{\rm orb}$. Although, in principle, calculating $R_{1,R}$ is straighforward, for this it is still necessary to precisely specify where the transferred mass goes and, if the system loses mass, how much angular momentum it takes with it, i.e. one has to specify the parameters η and ν. The change of $R_{1,R}$ is conveniently expressed in terms of the mass radius exponent

$$\zeta_{R,1} = \left(\frac{\partial \ln R_{1,R}}{\partial \ln M_1}\right)_*, \tag{7}$$

where the subscript $*$ is a reminder that for its calculation η and ν need to be specified.

On the other hand, the reaction of the donor's radius R_1 to mass loss is more complicated: besides hydrostatic equilibrium which readjusts on the orbital time scale, mass loss disturbes also the thermal equilibrium of a star. Therefore, its reaction depends on the ratio of the mass loss time scale τ_M to the time scale $\tau_{\rm th}$ on which the star can readjust to thermal equilibrium. If $\tau_M/\tau_{\rm th} \ll 1$ the star reacts essentially adiabatically, and the radius change is expressed in terms of the adiabatic mass radius exponent

$$\zeta_{\rm ad,1} = \left(\frac{\partial \ln R_1}{\partial \ln M_1}\right)_{\rm ad}. \tag{8}$$

If, on the other hand, mass loss is very slow, i.e. $\tau_M/\tau_{\rm th} \gg 1$, the star has time to adjust to near thermal equilibrium in which case the radius change is expressed by the thermal equilibrium mass radius exponent

$$\zeta_{\rm th,1} = \left(\frac{\partial \ln R_1}{\partial \ln M_1}\right)_{\rm th}. \tag{9}$$

Accordingly, there are two criteria for the stablity of mass transfer:

1. Mass transfer is *adiabatically* stable if

$$\zeta_{\rm ad,1} - \zeta_{R,1} > 0 \tag{10}$$

2. Mass transfer is *thermally* stable if

$$\zeta_{\rm th,1} - \zeta_{R,1} > 0\,. \tag{11}$$

What does all that mean for the CV progenitor system at the onset of mass transfer? In order to tell one has to know the values of $\zeta_{R,1}$, $\zeta_{\rm ad,1}$, and $\zeta_{\rm th,1}$. Because $M_{1,i} > M_{2,i}$ one invariably finds that $\zeta_{R,1} > 0$ even in the most favourable case where no orbital angular momentum is lost. The values of $\zeta_{\rm ad,1}$, and $\zeta_{\rm th,1}$, on the other hand, depend on the internal structure of the star in question. In our case the donor is a star with a degenerate core and a deep outer convective envelope. For such stars one typically finds $-1/3 \lesssim \zeta_{\rm ad} \lesssim 0$ and $\zeta_{\rm th} \lesssim 0$ (Hjellming & Webbink 1987). Taken together this means that mass transfer in such a system is adiabatically and thermally unstable. And as a consequence of the adiabatic instability mass transfer quickly accelerates to the point where the mass transfer rate reaches values of order of $-\dot{M}_{1,\mathrm{ad}} \sim M_1/\tau_{\mathrm{conv}} \sim M_\odot \mathrm{yr}^{-1}$, where $\tau_{\mathrm{conv}} \sim$ yr is the convective turnover time scale (Paczyński & Sienkiewicz 1972).

3.2. Fast accretion onto a main sequence star

So far we have treated the MS secondary as a point mass. Whereas before the onset of mass transfer this is an adequate approximation, this is not always true afterwards. Numerical calculations (e.g. Kippenhahn & Meyer-Hofmeister (1977), Neo et al. (1977)) show that the low-mass secondary, exposed to the prodigious mass inflow rates associated with the adiabatic mass transfer instability, starts expanding rapidly to giant dimensions. The reason for this behaviour is that the thermal time scale of the accreted envelope around the secondary is much longer than the mass accumulation time. As a consequence, the accreted matter can not cool efficiently and, therefore, forms a deep and very extended convective envelope of high entropy material around the secondary. The star thus attains a structure similar to that of a giant/AGB star which, however, derives its luminosity mainly from accretion rather than from nuclear burning.

3.3. Formation of a common envelope

The situation of a CV progenitor at the onset of mass transfer can now be characterized as follows: because mass transfer occurs from the more massive star, the orbital separation a as well as the critical Roche radii $R_{1,\mathrm{R}}$ and $R_{2,\mathrm{R}}$ shrink. At the same time, the mass losing donor star has the tendency to expand (negative ζ_{ad} and ζ_{th}). But forced by dynamical constraints to essentially follow $R_{1,\mathrm{R}}$ the donor must lose mass at rates approaching $\sim M_\odot \mathrm{yr}^{-1}$. And the secondary, in turn, exposed to such enormous accretion rates, reacts by rapid expansion. The consequence of all this is that within a very short time after the onset of mass transfer the system evolves into deep contact. An attempt to model this very complicated process has been made by Webbink (1979). Accordingly, the immediate result of this evolution can then be roughly chracterized as follows: A binary system consisting of the primary's core (the future WD) of mass M_c and the original secondary of mass $M_{2,\mathrm{i}}$ finds itself deeply immersed in a common envelope (CE) of mass $M_{\mathrm{CE}} = M_{1,\mathrm{i}} - M_\mathrm{c}$ and a size which must be of order of or even larger than the radius given by the core mass-radius relation, i.e. $R_{\mathrm{CE}} \gtrsim \mathcal{R}(M_\mathrm{c})$.

4. Common envelope evolution and CV formation

Common envelope evolution is the name of a generic process which arises as a consequence of dynamical time scale mass transfer and as a result of which a detached short-period binary is formed in which one of its components is the core of the former primary (in our case a pre-WD). Because of its importance for the formation of all sorts of compact binaries the subject has generated a vast literature. For lack of space I am unable to give a detailed review here. Rather I shall concentrate on sketching a few key aspects of this process and for more details refer the reader to recent reviews by Taam & Sandquist (2000) and Webbink (2008).

4.1. The Darwin instability

Let us now consider the following idealized situation: a binary consisting of the original primary's core of mass M_c and the secondary of mass M_2 with orbital separation a and orbital frequency ω_{**} is embeded in an envelope of mass M_E, radius R_E, moment of inertia I_E which is in solid body rotation with an angular frequency Ω_E. If $\omega_{**} > \Omega_\mathrm{E}$ tidal interaction and friction between the binary and envelope lead to energy dissipation and angular momentum transport from the binary to the envelope with $\dot{J}_{**} = -\dot{J}_\mathrm{E} < 0$. As a consequence, the envelope, initially rotating slower than the binary, is spun up. But according to Kepler's third law also the binary's orbital frequency increases due to the loss of orbital angular momentum. The question of interest is thus whether through this spin-up the difference $\omega_{**} - \Omega_{\rm E}$ increases or decreases.

If $\omega_{**} - \Omega_{\rm E} > 0$ and $\dot{\omega}_{**} - \dot{\Omega}_{\rm E} < 0$ the envelope is synchronized, i.e. $\Omega_{\rm E} \to \omega_{**}$.

If, on the other hand, $\omega_{**} - \Omega_{\rm E} > 0$ and $\dot{\omega}_{**} - \dot{\Omega}_{\rm E} > 0$, runaway friction results, and the binary spirals in. The condition for this to happen is easily derived: the binary's orbital angular momentum is

$$J_{**} = G^{2/3} \frac{M_{\rm c} M_2}{(M_{\rm c} + M_2)^{1/3}} \, \omega_{**}{}^{-1/3} \tag{12}$$

$$= I_{**} \omega_{**} \,, \tag{13}$$

where

$$I_{**} = \frac{M_{\rm c} M_2}{M_{\rm c} + M_2} \, a^2 \tag{14}$$

is the orbital moment of inertia. The envelope's spin angular momentum is

$$J_{\rm E} = I_{\rm E} \, \Omega_{\rm E} \,. \tag{15}$$

With (12) and (15) angular momentum conservation, i.e. $\dot{J}_{**} + \dot{J}_{\rm E} = 0$, yields

$$\dot{\omega}_{**} - \dot{\Omega}_{\rm E} = \dot{\omega}_{**} \left(1 - \frac{1}{3} \frac{I_{**}}{I_{\rm E}} \right) . \tag{16}$$

From (16) it is seen that the envelope can be synchronized only if $I_{\rm E} < 1/3 I_{**}$. If, on the other hand,

$$I_{\rm E} > \frac{1}{3} I_{**} \tag{17}$$

the envelope cannot be synchronized and spiral-in of the binary is unavoidable. The impossibility of synchronizing the envelope results from a variant of an instability which is actually long known: discovered by Darwin (1879), though in a different context, it is commonly called Darwin instability.

Whether the Darwin instability is of relevance for our problem, i.e. whether the criterion (17) is met with the formation of a CE after the onset of adiabatically unstable mass transfer, needs of course first to be checked. Since adequate model calculations of the formation of a CE are still not feasible, simple estimates must do. And these indicate indeed that for typical parameters of CV progenitor systems the forming CE systems are Darwin unstable.

4.2. Common envelope evolution

Despite decades of heroic efforts to model common envelope evolution, for a review see e.g. Taam & Sandquist (2000), to this day it has not yet been possible to follow such an evolution from its beginning to its end with really adequate numerical computations. Therefore, it is still not possible for a given set of initial parameters to reliably predict the outcome of common envelope evolution. The expectation is that in many, but not necessarily all cases the frictional energy release will unbind the CE and leave a close binary consisting of the former primary's degenerate core and the secondary.

Clearly the ejection of the CE requires the release of the envelope's binding energy in a sufficiently short time, i.e. that the time scale of the spiral-in is short. However, there are limits to how short the spiral-in can be. From simplified one-dimensional hydrostatic model calculations Meyer & Meyer-Hofmeister (1979) found that there is a negative feedback between the frictional energy release and the resulting radiation pressure. An estimate of the duration of the spiral-in is obtained from the argument that because of this feedback the frictional luminosity $L_{\rm frict}$ can not exceed the Eddington luminosity

$$L_{\rm Edd} = \frac{4 \pi G c \mathcal{M}}{\kappa_{\rm es}} \tag{18}$$

by much. Here $\kappa_{\rm es}$ is the electron scattering opacity. The evolution of the binary with masses $M_{\rm c}$ and M_2 from an initial separation $a_{\rm i}$ to a final separation $a_{\rm f} \ll a_{\rm i}$ releases the orbital binding energy

$$\Delta E_{\rm B} \approx \frac{G M_{\rm c} M_2}{2 a_{\rm f}} \,. \tag{19}$$

This yields a rough estimate of the spiraling-in time scale

$$\tau_{\rm CE} \approx \frac{\Delta E_{\rm B}}{L_{\rm frict}} \gtrsim \frac{\Delta E_{\rm B}}{L_{\rm Edd}} \tag{20}$$

$$\gtrsim 400 {\rm yr} \frac{M_{\rm c} M_2}{(M_{\rm c} + M_2) M_\odot} \frac{a_{\rm f}}{R_\odot} \,. \tag{21}$$

Thus for the typical parameters of a CV (see Sect.2.1) $\tau_{\rm CE}$ is very short, so short indeed that the secondary star has no time to accrete a significant amount of mass during the CE phase (Hjellming & Taam 1991). This is the a posteriori justification for our assumption in Sect. 2.5 that $M_{2,i} = M_{2,f}$.

Because of the short duration of CE evolution chances of observing a binary system during this phase are extremely small, apart from the fact that it is not even quite clear what to look for. Worse, the spiraling-in binary is hidden from view as long as it is inside the CE. In view of our limited theoretical understanding of CE evolution in general and the ejection of the CE in particular, and the fact that this process is virtually unobservable, one has to ask why we can be sure that CE evolution really happens as described above. Beyond all the uncertainties, the concept of CE evolution does make at least one prediction that is testable: at the end of the CE process, if the envelope is ejected, we expect a binary inside the now more or less transparent envelope. And in this binary the primary's degenerate core emerges as a very hot pre-WD which, in turn, ionizes the surrounding gas, thereby transforming the ejected CE into a planetary nebula. The concept of CE evolution thus implies the existence of planetary nebulae with short-period binary central stars. And indeed, such objects are obeserved: currently we know of $\sim$ 20 short-period binary central stars of planetary nebulae (see e.g. De Marco, Hillwig & Smith (2008) and Ritter & Kolb (2003)).

4.3. Formal treatment of the CE phase

CE evolution, if it ends with the ejection of the CE, transforms a binary with initial parameters $(M_{1,i}, M_{2,i}, a_i)$ to one with final parameters $(M_{1,f}, M_{2,f}, a_f)$. With current theory it is not possible to precisely link these two sets of parameters. Therefore, in evolutionary studies and population synthesis calculations of compact binaries (e.g. de Kool (1990), de Kool (1992), de Kool & Ritter (1993), Politano (1996), Politano (2004), Politano (2007)), CE evolution is usually dealt with by means of a simple estimate (Webbink 1984) which derives from the assumption that a fraction $\alpha_{CE} \lesssim 1$ of the binary's binding energy which is released in the spiraling-in process, $\Delta E_{B,**}$, is used to unbind the CE.

Using $M_{1,f} = M_{c,i} = M_c$, $M_{2,f} = M_{2,i} = M_2$ we have

$$\Delta E_{B,**} = \frac{G\, M_c\, M_2}{2} \left(\frac{1}{a_i} - \frac{1}{a_f} \right) . \tag{22}$$

On the other hand, the binding energy of the CE can be written as

$$E_{B,CE} = -\frac{G\, M_{1,i}\, M_{CE}}{\lambda\, R_{1,i}} , \tag{23}$$

where $M_{CE} = M_{1,i} - M_c$ is the mass and $R_{1,i} = a_i\, f_1(q_i)$ the radius of the CE, and λ a dimensionless factor which can be determined from stellar structure calculations provided one knows exactly where the mass cut between core and envelope is. Unfortunately it turns out that λ depends rather sensitively on this (Tauris & Dewi 2001). The CE criterion, namely that

$$E_{B,CE} = \alpha\, \Delta E_{B,**} \tag{24}$$

is then equvalent to

$$a_f = a_i \left\{ \frac{2\, M_{1,i}\, M_{CE}}{\alpha_{CE}\, \lambda\, M_c\, M_2\, f_1(q_i)} - \frac{M_{1,i}}{M_c} \right\}^{-1} . \tag{25}$$

Eq. (25) provides the formal link between the pre-CE and the post-CE binary parameters. As can be seen from Eq. (25) when dealing with CE evolution in this way one introduces essentially one free parameter, namely $\alpha_{CE}\, \lambda$ (per CE phase). Since we do not have any a priori knowledge about α_{CE} and since also λ is not really well known, the degree of uncertainty introduced via $\alpha_{CE}\, \lambda$ is quite considerable.

Several recent investgations of binary evolution involving CE evolution have come to the conclusion that the energy criterion (24) is not always adequate and that in addition to the orbital binding energy possibly also other sources of energy such as the ionization energy have to be taken into account. For a comprehensive discussion of this point see Webbink (2008).

4.4. Evolution of post-common envelope binaries

The ejection of the CE leaves a detached short-period binary inside a planetary nebula which is excited by the hot pre-WD component. Once the planetary nebula disappears, either because it dissolves or because of lack of ionizing radiation from the pre-WD, what remains is a binary consisting of a WD and an essentially unevolved companion. And because the lifetime of a typical planetary nebula of $\sim 10^4$ yr is much shorter than the lifetime of a typical post-CE binary in the detached phase, the intrinsic number of detached post-CE systems lacking a visible planetary nebula must be vastly larger than that of post-CE systems with a planetary nebula. And although such systems are intrinsically rather faint (both the WD and its low-mass companion are faint), because of their rather high space density quite a number of such systems are known (currently $\gtrsim 50$, see Ritter & Kolb (2003) for a compilation). They are collectively refrerred to as *precataclysmic binaries*, hereafter pre-CVs.

In the following, we need to discuss two questions: 1) how does a detached pre-CV become semi-detached, i.e. a CV, and 2) whether with the onset of mass transfer all pre-CVs really become CVs or perhaps follow a totally different evolutionary path.

Since in a detached system the future donor star underfills its Roche lobe, mass transfer can only be initiated if either the donor star grows (as a consequence of nuclear evolution) or if the orbital separation shrinks as a consequence of orbital angular momentum loss (AML). Which of the two possibilities is relevant for a particular binary system depends on the ratio of the nuclear time scale

$$\tau_{\mathrm{nuc},2} = \left(\frac{\partial t}{\partial \ln R_2}\right)_{\mathrm{nuc}} \tag{26}$$

on which the star grows to the AML time scale

$$\tau_{\mathrm{J}} = -\left(\frac{\partial t}{\partial \ln J_{\mathrm{orb}}}\right) = -2\left(\frac{\partial t}{\partial \ln a}\right) \tag{27}$$

on which the orbital separation a shrinks.

If $\tau_{\mathrm{J}} < 2\,\tau_{\mathrm{nuc},2}$ mass transfer is initiated by AML, otherwise by nuclear evolution. The typical future donor star of a pre-CV is a low-mass MS star. Thus $\tau_{\mathrm{nuc},2} > 10^9$ yr. AML in such binaries results either from the emission of gravitational waves (Kraft, Mathews & Greenstein 1962) or from *magnetic braking*, i.e. a magnetically coupled stellar wind from the tidally locked companion. In typical pre-CV systems AML is dominated by magnetic braking. Unfortunately, for that case there is as yet no theory which would allow computation of $\dot{J}_{\mathrm{orb}}$ from first principles. Again, simple semi-empirical estimates (e.g. Verbunt & Zwaan (1981)) or simplified theoretical approaches (e.g. Mestel & Spruit (1987)) must do. For the typical pre-CV with a low-mass MS companion, these estimates yield $\tau_{\mathrm{J}} \sim 10^8$ yr. Thus, for such systems mass transfer is typically initiated via AML (see e.g. Ritter (1986), Schreiber & Gänsicke (2003)). But the simple fact that we do observe a number of long-period CVs with a giant donor shows that mass transfer can also be initiated by nuclear evolution of the future donor star. However, the fraction of pre-CV systems ending up with a giant donor is small and, unfortunately, strongly model-dependent (de Kool 1992).

When the secondary reaches its Roche limit and mass transfer sets in stability of mass transfer becomes again an issue. Whether mass transfer is stable depends on whether the criteria which we had derived in Sect. 3.1, but now applied to the secondary star, are fulfilled. Why is this important? Observations and theoretical arguments show that in the vast majority of CVs mass transfer is thermally and adiabatically stable. In other words: only those pre-CVs for which $\zeta_{\mathrm{ad},2}-\zeta_{\mathrm{R},2} > 0$ and $\zeta_{\mathrm{th},2}-\zeta_{\mathrm{R},2} > 0$ can directly become CVs. What happens to the rest? That depends mainly on the evolutionary status of the donor and the binary's mass ratio. If we distinguish for simplicity MS stars and giants as possible donor stars, then the following cases can arise:

1. MS donor, mass transfer thermally and adiabatically stable → short-period CV ($P_{\mathrm{orb}} \lesssim 0.5$ d) with an unevolved donor.
2. MS donor, mass transfer adiabatically stable but thermally unstable → thermal time scale mass transfer, WD with stationary hydrogen burning, system

appears as a supersoft X-ray source (see e.g. van den Heuvel et al. (1992), Schenker et al. (2002)) → CV with an artificially evolved MS donor.

3. MS donor, mass transfer adiabatically unstable → very high mass transfer rates, second common envelope?, coalescence?
4. giant donor, mass transfer thermally and adiabatically stable → long-period CV ($P_{\rm orb} \gtrsim 1$ d).
5. giant donor, mass transfer either thermally or adiabatically unstable → very high mass transfer rates, second common envelope?, formation of an ultrashort-period detached WD+WD binary?

5. CV evolution

CV evolution is a complex subject. Yet, because of space constraints, here I can only present a brief outline of this topic. For readers wishing to learn more about it the reviews by King (1988) and Ritter (1996) are a good starting point.

5.1. Mass transfer in semi-detached binaries

If mass transfer in a binary is thermally and adiabatically stable, as in the majority of CVs, no mass transfer occurs unless some external force drives it. And in CVs the driving agents are the same as in pre-CVs (cf. Sect. 4.4), i.e. AML and nuclear evolution of the donor. Furthermore, if mass transfer is stable and the strength of the driving changes only on long time scales, mass transfer will be essentially stationary. In that case the donor's radius R_2 and its Roche radius $R_{2,\rm R}$ are equal to within very few atmospheric scale heights $H \ll R_2$ (Ritter 1988). Thus, to a very good accuracy we must have $\dot{R}_2 = \dot{R}_{2,\rm R}$, or, using $R_2 = R_{2,\rm R}$,

$$\frac{d\ln R_2}{dt} = \frac{d\ln R_{2,\rm R}}{dt}\,. \tag{28}$$

Now, the donor's radius can change because of mass loss, nuclear evolution, and thermal readjustment. As mentioned earlier (Sect. 3.1) mass loss (if nothing else) drives a star out of thermal equilibrium. If mass loss were stopped the star evolved back towards thermal equilibrium, thereby changing it radius initially at a relative rate

$$\left(\frac{\partial \ln R_2}{\partial t}\right)_{\rm th} = \frac{1}{\tau_{\rm th,2}}\,, \tag{29}$$

where $\tau_{\rm th,2}$ is the thermal time scale. Thus the rate of change of R_2 can be decomposed as follows:

$$\frac{d\ln R_2}{dt} = \frac{\dot{M}_2}{M_2}\zeta_{\rm ad,2} + \frac{1}{\tau_{\rm th,2}} + \frac{1}{\tau_{\rm nuc,2}} \tag{30}$$

On the other hand, the donor's Roche radius can change because of mass transfer and AML. With (27) we have

$$\frac{d\ln R_{2,\rm R}}{dt} = \frac{\dot{M}_2}{M_2}\zeta_{\rm R,2} - \frac{2}{\tau_{\rm J}}\,. \tag{31}$$

Eqs. (28), (30), and (31) finally yield the mass transfer rate

$$-\dot{M}_2 = \frac{1}{\zeta_{\rm ad,2} - \zeta_{\rm R,2}}\left(\frac{1}{\tau_{\rm th,2}} + \frac{1}{\tau_{\rm nuc,2}} + \frac{2}{\tau_{\rm J}}\right). \tag{32}$$

If mass transfer is sufficienly slow such that the donor remains close to thermal equilibrium, its radius changes according to

$$\frac{d\ln R_2}{dt} = \frac{\dot{M}_2}{M_2}\zeta_{\rm th,2} + \frac{1}{\tau_{\rm nuc,2}}\,, \tag{33}$$

and together with (28) and (31) we can write

$$-\dot{M}_2 = \frac{1}{\zeta_{\rm th,2} - \zeta_{\rm R,2}}\left(\frac{1}{\tau_{\rm nuc,2}} + \frac{2}{\tau_{\rm J}}\right). \tag{34}$$

From what has been said so far, it is easily seen that for the sign of the mass transfer rate to be correct, i.e. for $-\dot{M}_2 > 0$, the denominator in (32) and (34) must be positive, i.e. that

$$\zeta_{\rm ad,2} - \zeta_{\rm R,2} > 0 \tag{35}$$

and

$$\zeta_{\rm th,2} - \zeta_{\rm R,2} > 0\,. \tag{36}$$

With Eqs. (35) and (36) we have thus recovered the stability criteria for mass transfer. What this implies is that for mass transfer to be stationary it has also to be stable.

5.2. Computing the evolution of a semi-detached binary

Can we use Eqs. (32) or (34) for calculating the evolution of a semi-detached binary? Unfortunately, this is in general not the case. The virtue of Eqs. (32) or (34) and the reason why we have derived them here is that they show clearly how the long-term evolution of a semi-detached binary works: mass transfer must be stable and be driven by some mechanism. The obvious ones are the growth of the donor star due to nuclear evolution or AML which shrinks the binary. A less obvious driving agent is the growth of the donor star as a consequence of thermal relaxation (cf. (32)). However, thermal relaxation, itself mainly being a consequence of mass loss, cannot maintain mass transfer for times long compared to $\tau_{\rm th}$ without external driving by one of the other mechanisms.

The reason why we cannot use Eqs. (32) or (34) for evolutionary computations is that most of the quantities appearing in these equations are not explicitly known. In partcular, $\zeta_{\rm ad}$, $\zeta_{\rm th}$, $\tau_{\rm nuc}$, and $\tau_{\rm th}$ require knowledge of the complete internal structure of the donor star, i.e. nothing less than the whole past history of the binary system. Worse, even if all that were known, the above quantities can only be determined numerically. Furthermore, computing $\zeta_{\rm R}$ requires specification of ν and η (Eqs. (5) and (4)). Finally, apart from gravitational radiation, the AML rate is not well known and in some cases only given as an implicit function of binary parameters (Mestel & Spruit 1987). Even more exotic effects such as irradiation of the donor star or the accretion disc can strongly affect the quantities appearing in (32) or (34) (see e.g. Ritter (1996), Büning & Ritter (2004), or Ritter (2008) for more).

Application of Eqs. (32) or (34) for evolutionary computations is therefore limited to cases where the donor star can either be approximated by a particularly simple stellar model, e.g. by a polytrope (Rappaport, Joss & Webbink 1982), a bipolytrope (e.g. Rappaport, Verbunt & Joss (1983), Kolb & Ritter (1992)), or where stellar structure data determined beforehand from single star evolution can be used (Webbink, Rappaport & Savonije (1983), Ritter (1999)).

In general, such simplifications are unsatisfactory. For a more realistic simulation the full stellar structure problem must be solved as described in Sect. 2.4. Stellar evolution is an initial value problem. So for setting up a simulation of a CV evolution one has first to decide at which moment of the evolution to start the calculation, e.g. at the onset of mass transfer from the secondary, and then to specify at least the masses of the components and the internal structure, i.e. the evolutionary status of the donor star, but as the case may be also the structure of the accreting WD. Furthermore, one has to adopt values or prescriptions for ν and η, and finally to decide what to do about AML, in particular about magnetic braking, i.e. which of the various prescriptions available in the lierature (e.g. Verbunt & Zwaan (1981), Mestel & Spruit (1987)) to use. When everything is set up calculating the evolution is in the simplest case just a single star evolution for the donor star with variable mass where the mass loss rate is an eigenvalue of the problem and is determined by the additional outer boundary condition , e.g. by $R_2 \leqslant R_{2,\rm R}$.

5.3. A sketch of CV evolution

The orbital period $P_{\rm orb}$ is the only physical quantity which is known with some precision for a large number of CVs, currently for over 700 objects (Ritter & Kolb 2003). Reliable masses, on the other hand, are known, if at all, only for a very small minority of CVs. Therefore, much of the work on CV evolution in the past 30 years has concentrated on understanding the observed period distribution of CVs. Broadly speaking, this distribution is bimodal with $\sim 45\%$ of the objects having periods in the range $3^{\rm h} \lesssim P_{\rm orb} \lesssim 16^{\rm h}$, another $\sim 45\%$ with $80\,\rm min \lesssim P_{\rm orb} \lesssim 3^{\rm h}$, and the remaining $\sim 10\%$ with $2^{\rm h} \lesssim P_{\rm orb} \lesssim 3^{\rm h}$. The dearth of objects in the period interval $2^{\rm h} \lesssim P_{\rm orb} \lesssim 3^{\rm h}$ is known in the literature as the *period gap*.

The maximum period of $\sim 16^{\rm h}$ is easily understood as a consequence of the facts that 1) the donor is a MS star, 2) the mass of the WD is $M_{\rm WD} < M_{\rm CH} \approx 1.4 M_\odot$ and 3) mass transfer must be stable.

The minimum period of ~ 80 min, in turn, is at least qualitatively understood as a consequence of mass transfer from a hydrogen-rich donor which is mainly driven by gravitational radiation (Paczyński & Sienkiewicz (1981), Paczyński & Sienkiewicz (1983), Rappaport, Joss & Webbink (1982)). Because of mass loss, of the order of a few $10^{-11} M_\odot {\rm yr}^{-1}$, the donor star becomes more and more degenerate when $M_2 \lesssim 0.1 M_\odot$ and its structure changes from that of a low-mass MS star to that of a brown dwarf. Thereby its effective mass radius exponent $\zeta_{\rm eff,2} = d \ln R_2 / d \ln M_2$ changes from ~ 0.8 on the MS to $-1/3$. $P_{\rm orb}$ is minimal when $\zeta_{\rm eff,2} = +1/3$. Whether mass transfer near the period minimum is really driven by gravitational radiation only is currently under dispute because of the mismatch between the corresponding theoretical prediction for the minimum period of ~ 70 min and the observed value of ~ 77 min (see e.g. Renvoizé et al. (2002) or Barker & Kolb (2003) for a discussion).

The period gap is more difficult to account for. Over the years a number of different hypotheses have been put forward to explain it. For lack of space I cannot review them all here. Rather I shall concentrate on the one hypothesis (Spruit & Ritter (1983), Rappaport, Verbunt & Joss (1983)) which, in my view, still provides the most plausible explanation for what we see, and which is known in the literature as the *disrupted (magnetic) braking* hypothesis. It postulates that as long as the donor star has a radiative core "magnetic braking" is effective and CV evolution is driven by a high AML rate due to "magnetic braking" and gravitational radiation, but that, as soon as the donor star becomes fully convective, "magnetic braking" becomes ineffective and thus the evolution is driven by AML from gravitational radiation only. In the following I shall try to explain step by step how the gap arises in the framework of this hypothesis.

First it is important to note that the evolution of CVs with a MS donor driven by AML leads from longer to shorter orbital periods. And MS donor stars with a mass $\lesssim 1 M_\odot$ have a convective envelope and a radiative core. With decreasing mass, i.e. $P_{\rm orb}$, the mass of the radiative core shrinks until at a particular mass $M_{\rm 2,conv}$, i.e. orbital period $P_{\rm orb} = P_{\rm u}$, the donor becomes fully convective. According to the above hypothesis, at this point the AML rate drops from a high value which is mainly due to "magnetic braking" to a small value due to graviational radiation only.

If "magnetic braking" is sufficiently strong, then for periods $> P_{\rm u}$ mass loss from the donor occurs on a timescale much shorter than its thermal time scale. As a result the donor is significantly driven out of thermal equilibrium and, therefore, oversized compared to its thermal equilibrium radius, i.e. $R_2(P_{\rm orb} > P_{\rm u}) > R_{\rm 2,e}$, and the faster the mass loss, the larger the difference $R_2 - R_{\rm 2,e}$. Suppose now that the driving AML rate drops by a large factor on a short time scale. What will happen? The donor will detach from its Roche lobe because initially it will continue losing mass and shrink at the same rate as before while its Roche radius, because of the reduced AML rate will shrink much more slowly. So mass transfer stops and the star, being oversized because of previous high mass loss but now without mass loss contracts towards its thermal equilibrium radius $R_{\rm 2,e}$, and that on its thermal time scale which is initially shorter than the time scale on which its Roche radius shrinks. Mass transfer can only resume when the shrinking Roche radius reaches the stellar radius, i.e. the latest when $R_{\rm 2,R} = R_{\rm 2,e}$. Once mass transfer resumes the binary's orbital period is $P_{\rm l} < P_{\rm u}$. In other words: the binary has crossed the period range $P_{\rm l} \leqslant P_{\rm orb} \leqslant P_{\rm u}$ as a detached system. And because of lacking accretion luminosity, such systems are intrinsically very faint, fainter even than pre-CVs, and, therefore, virtually unobservable. A gap in the period distribution can thus arise if a) a sudden drop of the AML rate causes CVs to detach, b) if that happens to most of the CVs evolving from $P_{\rm orb} > P_{\rm u} \rightarrow P_{\rm orb} < P_{\rm u}$, and if c) the values of $P_{\rm u}$ and $P_{\rm l}$ are practically the same for all systems going through a detached phase.

So far I have not yet addressed the question why the AML rate should drop by a large factor and that also on a sufficiently short time scale. The idea behind this proposition is that effective amplification of magnetic flux via a dynamo and thus efficient AML loss via "magnetic braking" is strongly tied to the presence of a convective envelope and a radiative core in the donor star (e.g. Spruit & Ritter (1983)). Accordingly, it is proposed that AML via "magnetic braking" decreases rapidly when, as a consequence of ongoing mass loss, the donor eventually becomes fully convective. The questions of whether that really happens and whether AML via "magnetic braking" stops completely or only partially when the donor becomes fully convective have remained somewhat controversial to this day. Qualitative theoretical arguments in favour of the above proposition have, however, been presented by Taam & Spruit (1989).

In order for the *disrupted magnetic braking* proposition to work quantitatively the following requirements must be met: AML above the gap must drive mass transfer at a level of $-\dot{M}_2 \sim 10^{-9} M_\odot \mathrm{yr}^{-1}$. As a result, the donor becomes fully convective when $M_{\mathrm{conv}} \sim 0.2 M_\odot$ and $P_\mathrm{u} \sim 3^\mathrm{h}$. At that moment, as a consequence of previous high mass loss, the stellar radius is larger by about 30% than in thermal equilibrium. With the disappearance of the AML from "magnetic braking" the AML loss rate drops by a factor of $\sim 10-20$ to essentially the value due to gravitational radiation alone. After the detached phase which lasts $\sim 10^9$ yr mass transfer resumes with $M_2 = M_{\mathrm{conv}} \sim 0.2 M_\odot$, $R_2 = R_{2,\mathrm{e}} \sim 0.2 R_\odot$ and $P_{\mathrm{orb}} = P_\mathrm{l} \sim 2^\mathrm{h}$ at a level of $-\dot{M}_2 \sim 5\,10^{-11} M_\odot \mathrm{yr}^{-1}$. Explaining the gap as a collective phenomenon of CV evolution requires furthermore that the majority of the donor stars are all of the same type, i.e. MS stars, and that AML via "magnetic braking" yields similar mass transfer rates in different systems at the same orbital period. This guarantees that P_u and P_l are more or less the same for all systems and thus the coherence of the phenomenon.

The fact that the period range of the gap is not empty already indicates that not all CVs follow the above-described evolution strictly. There are several reasons for why there may be CVs in the gap. The most important ones are: 1) a donor mass such that at the end of the detached pre-CV evolution the orbital period is $2^\mathrm{h} \lesssim P_{\mathrm{orb}} \lesssim 3^\mathrm{h}$ (e.g. Kolb (1993), Davis et al. (2008)); 2) a donor star which initially was close to the terminal age MS (see e.g. Ritter (1994)), or which is the artificially evolved remnant of earlier thermal time scale mass transfer (Schenker & King 2002); 3) reduced "magnetic braking" because of the presence of a strongly magnetized WD (for details see Li, Wu & Wickramasinghe (1994)).

At the end of CV evolution the donor star is a very faint brown dwarf. The WD, in turn, with an effective temperature of typically $< 10^4$K is also very faint. And because the mass transfer rate resulting from gravitational radiation is very small as well, i.e. $-\dot{M}_2 \lesssim 10^{-11} M_\odot \mathrm{yr}^{-1}$, so is the resulting accretion luminosity. Thus, such CVs are extremely faint and inconspicuous objects, and correspondingly difficult to detect. And though intrinsically about 90% of all CVs are in this late phase (Kolb 1993) so far only one convincing candidate beyond and far from the period minimum is known (Littlefair et al. 2006). The CV graveyard, as this evolutionary branch is sometimes referred to, is thus largely hidden from our view.

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