

# Redshift degeneracy in the $E_{\text{iso}}-E_{\text{peak}}$ relation of gamma-ray bursts

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## ABSTRACT

In this Letter we show that there is a redshift degeneracy in the  $E_{\text{iso}}-E_{\text{peak}}$  relation of gamma-ray bursts (GRBs). If a GRB has a redshift solved from the  $E_{\text{iso}}-E_{\text{peak}}$  relation that lies in the range of  $0.9 < z < 20$ , a GRB that has the same observed fluence and peak spectral energy but is at a different redshift in that range also satisfies the  $E_{\text{iso}}-E_{\text{peak}}$  relation within a  $1\sigma$  error, implying an extremely large error in the calculated redshift. Even if the data scatter in the  $E_{\text{iso}}-E_{\text{peak}}$  relation is reduced by a factor of 2, the error in the predicted redshift is still large enough to prevent one from constraining the redshift meaningfully. Hence, the  $E_{\text{iso}}-E_{\text{peak}}$  relation is not useful for determining the GRB distance.

**Key words:** cosmology: theory – gamma-rays: bursts – gamma-rays: observations.

## 1 INTRODUCTION

It has been found that the isotropic equivalent energy of long-duration gamma-ray bursts (GRBs) is correlated with the peak energy of their integrated spectra, with only a few outliers (Amati et al. 2002; Friedman & Bloom 2005; Amati 2006);

$$E_{\text{iso}} = AE_{\text{peak}}^m. \quad (1)$$

The peak energy  $E_{\text{peak}}$  is defined to be the photon energy at the peak of the GRB spectrum,  $\nu F_{\nu}$ , converted to the rest frame of the GRB. The isotropic equivalent energy  $E_{\text{iso}}$  is defined in the 1–10 000 keV band in the GRB frame.

With a sample of 41 GRBs with firmly determined redshifts and peak spectral energy, Amati (2006) has found that  $m \approx 2$  and  $A \approx 0.92 \times 10^{48}$  erg ( $E_{\text{peak}}$  in keV). The spread of the GRBs in the sample ( $1\sigma$  width) around the best-fitting model is given by  $\sigma_{\log E_{\text{peak}}} \approx 0.15$ , or equivalently,  $\sigma_{\log E_{\text{iso}}} \approx 0.3$ . That is, the uncertainty in the isotropic equivalent energy predicted by the relation in (1) is about a factor of 2.

The  $E_{\text{iso}}-E_{\text{peak}}$  correlation (1) is often called the Amati relation. A statistical analysis based on the correlation between the intensity and the peak spectral energy of GRBs with unknown redshifts carried out by Lloyd, Petrosian & Malozzi (2000) before the discovery of the  $E_{\text{iso}}-E_{\text{peak}}$  relation (Amati et al. 2002) has predicted the existence of such a relation and has indicated that it is a physical relation rather than a superficial relation.

The correlation is even better when the correction to the GRB energy from the jet collimation is included (Ghirlanda, Ghisellini & Lazzati 2004; Ghirlanda et al. 2005; Friedman & Bloom 2005). However, this is achieved at a great cost: the GRB jet opening angle is involved and hence the application of the relation requires multiband

observations on the afterglow as well as knowledge of the jet model and the GRB environment parameters.

X-ray flashes (XRFs), a subclass of GRBs that are characterized by a large fluence in the X-ray energy band relative to the gamma-ray energy band (Sakamoto et al. 2004), are found to satisfy the  $E_{\text{iso}}-E_{\text{peak}}$  relation. They include the extremely soft GRB 020903 (Sakamoto et al. 2004) and the recently discovered supernova-connected GRB 060218 (Campana et al. 2006; Amati et al. 2006).

Outliers to the  $E_{\text{iso}}-E_{\text{peak}}$  relation do exist. Short-duration GRBs are often found to violate the relation. Among long-duration bursts, famous outliers are GRB 980425 and GRB 031203 (Amati 2006); both are subenergetic and have spectroscopically confirmed supernovae accompanying them. Nakar & Piran (2005) and Band & Preece (2005) have argued that a significant fraction of the long-duration GRBs detected by the Burst and Transient Source Experiment (BATSE) on the *Compton Gamma-Ray Observatory* (CGRO) must violate the Amati relation.

However, it has been claimed that GRB 031203 had a significant soft X-ray component not seen by the *International Gamma-Ray Astrophysics Laboratory* (INTEGRAL), which may make GRB 031203 consistent with the Amati relation by reducing the value of its  $E_{\text{peak}}$  (Watson et al. 2004, 2006; Tiengo & Mereghetti 2006). This idea becomes particularly attractive after GRB 060218, which had a prompt emission directly detected in both gamma-ray and X-ray bands, lasting about 2000 s. It has motivated Ghisellini et al. (2006) to propose that GRB 980425 and GRB 031203 are twins of GRB 060218 and they indeed satisfy the Amati relation.

Given its universality and simplicity, the Amati relation might be considered as a good redshift or distance estimator for GRBs. In fact, it has already been claimed by Amati (2006) that the relation can be the most reliable redshift estimator. Because it only involves the prompt emission of the GRB, one may hope that the Amati relation can be applied to GRBs at very high redshift.

However, in this Letter, we show that it is impossible to use the  $E_{\text{iso}}-E_{\text{peak}}$  relation to estimate the redshift of a GRB at redshift

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$z \gtrsim 1$ . This is because of the fact that there is an intrinsic redshift degeneracy in the Amati relation, which causes more problems in determining the redshift than the dispersion in the relation. The degeneracy arises from the definition of the Amati relation and the fact that every empirical relation in astronomy has a non-zero intrinsic dispersion. The existence of the redshift degeneracy makes it impossible to distinguish GRB redshifts in the range of  $z = 0.9-20$ , if  $1\sigma$  error is allowed.

Throughout the Letter, we adopt a cosmology with  $\Omega_m = 0.3$ ,  $\Omega_\Lambda = 0.7$  and  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

## 2 THE REDSHIFT DEGENERACY IN THE $E_{\text{ISO}}-E_{\text{PEAK}}$ RELATION

The peak spectral energy of the GRB, measured in the GRB frame, is related to the peak spectral energy determined in the observer's frame by

$$E_{\text{peak}} = (1+z)E_{\text{obs,peak}}, \quad (2)$$

where  $z$  is the redshift of the GRB.

The isotropic equivalent energy of the GRB, defined in the GRB frame, is related to the observed bolometric fluence of the GRB by

$$E_{\text{iso}} = \frac{4\pi D_{\text{lum}}^2}{1+z} F_{\text{bol}} = 4\pi D_{\text{com}}^2 (1+z) F_{\text{bol}}, \quad (3)$$

where  $F_{\text{bol}}$  is the bolometric fluence,  $D_{\text{lum}}$  and  $D_{\text{com}}$  are, respectively, the luminosity distance and the comoving distance of the GRB.

$D_{\text{lum}}$  and  $D_{\text{com}}$  are calculated by

$$D_{\text{lum}} = D_{\text{com}}(1+z) = \frac{c}{H_0}(1+z)w(0,z), \quad (4)$$

where  $c$  is the speed of light,  $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$  is today's Hubble constant, and

$$w(z_1, z_2) \equiv \int_{z_1}^{z_2} \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_R(1+z)^2 + \Omega_\Lambda}}. \quad (5)$$

Here,  $\Omega_m$ ,  $\Omega_\Lambda$  and  $\Omega_R = 1 - \Omega_m - \Omega_\Lambda$  are, respectively, the ratio of today's mass density in dark and ordinary matter, in vacuum energy (cosmological constant or dark energy), and in the spatial curvature of the Universe to the critical mass density (Peebles 1993).

If a GRB has an observed bolometric fluence  $F_{\text{bol}}$  and an observed peak spectral energy  $E_{\text{obs,peak}}$  but an unknown redshift, we can solve for the redshift from the  $E_{\text{iso}}-E_{\text{peak}}$  relation (equation 1), by assuming that the GRB satisfies the relation. The dispersion of the  $E_{\text{iso}}-E_{\text{peak}}$  relation will lead to an error in the calculated redshift.

Submitting equations (2) and (3) into equation (1), we get

$$f(z) \equiv \frac{w(0,z)^2}{(1+z)^{m-1}} = \frac{A}{4\pi} \left( \frac{c}{H_0} \right)^{-2} \frac{E_{\text{obs,peak}}^m}{F_{\text{bol}}}. \quad (6)$$

All the redshift dependence is on the left-hand side.

Equation (6) does not always have a solution. If  $m > 1$  ( $m \approx 2$  in the Amati relation), the function  $f(z)$  approaches zero as  $z \rightarrow 0$  or  $z \rightarrow \infty$ , and peaks at a redshift  $z_p$ . The maximum of  $f$  is then  $f_p = f(z_p)$ . If the right-hand side of equation (6), which we denote by  $f_s$ , is greater than  $f_p$ , a solution for  $z$  will not be found. Based on this kind of arguments, Nakar & Piran (2005) have shown that at least 25 per cent of the bursts in the BATSE sample are outliers to the Amati relation (see also Band & Preece 2005).

On the other hand, if  $f_s < f_p$ , we will find two solutions for the redshift, which we denote by  $z_-$  and  $z_+$ ,  $z_- < z_+$ . In the limiting case of  $f_s = f_p$ , we have  $z_- = z_+$ .

The error in  $z_\pm$  is determined by the spread in the Amati relation. It is assumed that the spread is given by a factor of  $\mu (> 1)$  in the calculated isotropic energy, i.e. within  $1\sigma$  the isotropic energy of the GRB is expected to be in the range of  $E_{\text{iso}}/\mu - \mu E_{\text{iso}}$ , where  $E_{\text{iso}}$  is the isotropic energy calculated with equation (1). Based on a sample of 41 long-duration GRBs, Amati (2006) obtained that  $\mu \approx 2$  (see the second paragraph in Section 1 of this Letter).

Imagine that the same observed GRB that satisfies equation (1) at  $z = z_\pm$  is assigned another redshift  $z'$ . Then, the GRB would have an isotropic equivalent energy

$$E'_{\text{iso}} = E_{\text{iso}} \frac{D_{\text{lum}}'^2}{D_{\text{lum}}^2} \frac{1+z}{1+z'},$$

and a peak spectral energy

$$E'_{\text{peak}} = E_{\text{peak}} \frac{1+z'}{1+z}.$$

Thus we have

$$E'_{\text{iso}} = A E_{\text{peak}}^m \frac{f(z')}{f(z)}. \quad (7)$$

The  $1\sigma$  ranges of  $z_-$  and  $z_+$ , denoted respectively by  $(z_{-,l}, z_{-,r})$  and  $(z_{+,l}, z_{+,r})$ , are hence determined by

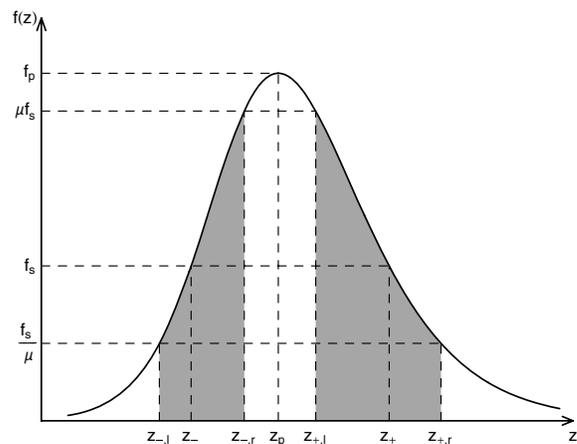
$$f(z_{-,l}) = f(z_-)/\mu, \quad f(z_{-,r}) = \mu f(z_-); \quad (8)$$

and

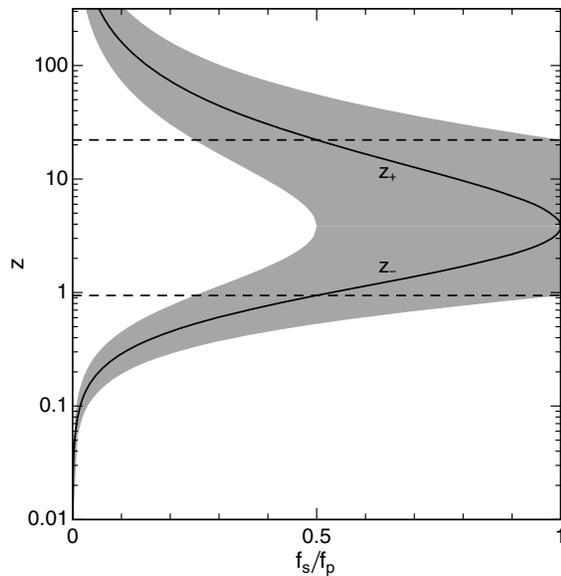
$$f(z_{+,l}) = \mu f(z_+), \quad f(z_{+,r}) = f(z_+)/\mu. \quad (9)$$

The solutions of  $z_\pm$ , as well as their  $1\sigma$  ranges, are sketched in Fig. 1. Obviously, the  $1\sigma$  regions of  $z_-$  and  $z_+$  overlap if  $f_s = f(z_\pm) \geq f_p/\mu$ .

For  $m = 2$  (then  $z_p = 3.825$ ,  $f_p = 0.561$ , for  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$ ),  $\mu = 2$  (Amati 2006), and  $f_s \leq f_p$ , the solutions of  $z_\pm$  and their errors are shown in Fig. 2. A striking feature is that for  $f_p/\mu \leq f_s \leq f_p$ , the  $1\sigma$  regions of  $z_\pm$  merge to form a single  $1\sigma$  region that spans a very large interval in redshift, covering the entire range from  $z = 0.94$  to  $z = 22$  (the region bounded by the two dashed lines). This means that, if the redshift of a GRB calculated by the Amati relation is in the range of 0.94–22, then within  $1\sigma$  error the redshift can have any value in the range from 0.94 to 22. Thus,  $0.94 <$



**Figure 1.** The function  $f(z)$  and the solution for the GRB redshift ( $z_\pm$ ) determined by equation (6).  $f_s = f(z_-) = f(z_+)$ , which is equal to the right-hand side of equation (6). The solutions of  $f(z) = \mu f_s$  and  $f(z) = f_s/\mu$  determine the  $1\sigma$  regions of  $z_\pm$  (equations 8 and 9; shaded regions).  $f_p$  is the maximum of  $f(z)$ , occurring at  $z = z_p$ . When  $f_s > f_p$ , equation (6) has no solution. (The redshift  $z$  is in logarithmic scale.)



**Figure 2.** The redshift determined by the Amati relation (i.e. the solutions to equation 6, with  $m = 2$ ) and  $1\sigma$  error.  $f_s$  is equal to the right-hand side of equation (6), and  $f_p$  is the peak of  $f(z)$  (Fig. 1). When  $f_s > f_p$ , equation (6) has no solution. When  $f_s < f_p$ , equation (6) has two solutions:  $z_-$ , the lower branch of the solid curve, and  $z_+$ , the upper branch. The two solutions merge at  $f_s = f_p$ . The shaded region shows the  $1\sigma$  error of  $z_{\pm}$ , arising from the spread in the Amati relation (a factor of  $\mu = 2$  in the isotropic energy). When  $f_s \geq f_p/\mu = 0.5 f_p$ , the  $1\sigma$  region of  $z_-$  merges with that of  $z_+$ , leading to a single  $1\sigma$  region in  $z_{\pm}$  that includes the entire region of  $z$  bounded by the two horizontal dashed lines (defined by  $z = 0.94$  and  $z = 22$ ).

$z < 22$  is a *redshift degeneracy* region because in it the redshift of a GRB cannot be determined by the Amati relation if the  $1\sigma$  error is allowed. If a  $2\sigma$  error ( $\mu = 2^2 = 4$ ) is allowed, then the redshift degeneracy region is  $0.53 < z < 55$ . Even if the scatter in the Amati relation is reduced by a factor of 2, i.e.  $\mu = 2^{1/2} = 1.4$ , the redshift-degeneracy region is  $1.4 < z < 12$  in  $1\sigma$ , still quite large.

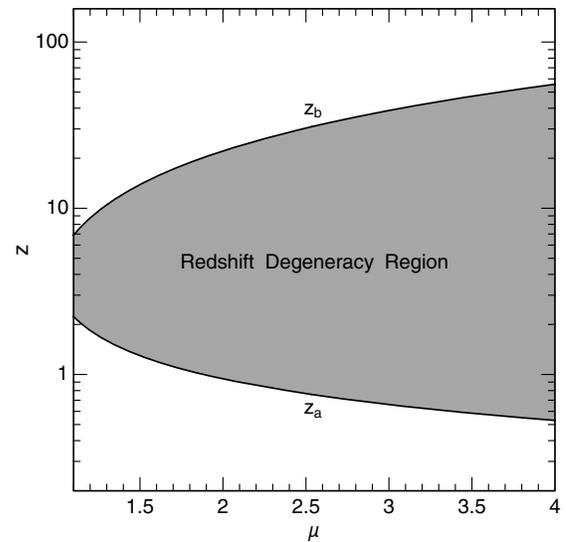
The merged  $1\sigma$  region is bounded by  $z_{-,l}$  and  $z_{+,r}$ , and is minimized when  $f_s = f_p$ . Thus, with a  $1\sigma$  error allowed, the redshift degeneracy region is bounded by the two roots of the equation

$$f(z) = f_p/\mu. \quad (10)$$

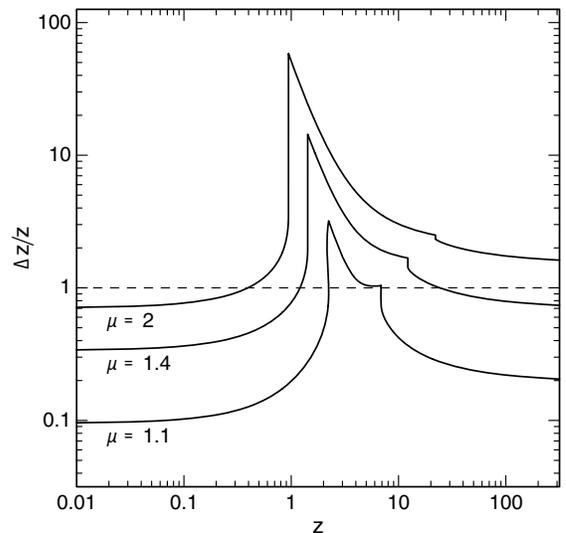
The two roots,  $z_a$  and  $z_b$  as functions of  $\mu$ , as well as the redshift degeneracy region bounded by them, are shown in Fig. 3.

The width of the  $1\sigma$  region of the solution  $z = z_{\pm}$  is  $\Delta z = z_r - z_l$ , where  $z_l$  and  $z_r$  define the boundary of the  $1\sigma$  region. If  $f_s < f_p/\mu$ , the  $1\sigma$  region of  $z_-$  and that of  $z_+$  are separated, so that  $z_l = z_{-,l}$ ,  $z_r = z_{-,r}$  for  $z = z_-$  and  $z_l = z_{+,l}$ ,  $z_r = z_{+,r}$  for  $z = z_+$ . If  $f_p/\mu \leq f_s \leq f_p$ , the two  $1\sigma$  regions merge to form a single one with  $z_l = z_{-,l}$  and  $z_r = z_{+,r}$  for both  $z = z_-$  and  $z = z_+$ , which causes a jump in  $\Delta z$  at  $f_s = f_p/\mu$ .

If  $\Delta z > z$ , the redshift solved from the Amati relation will not be very useful because the error in it is too large. In Fig. 4 we plot the ratio  $\Delta z/z$  against  $z$ , for different choices of the spread parameter  $\mu$ . For  $\mu = 2$ , inferred from the current data (Amati 2006),  $\Delta z/z > 1$  when  $z > 0.395$  and up to at least  $z \approx 300$  – the maximum  $z$  plotted in the figure. When  $\mu = 1.4$  (i.e. the spread in  $\log E_{\text{iso}} - \log E_{\text{peak}}$  is reduced by a factor of 2 compared to  $\mu = 2$ ),  $\Delta z/z > 1$  when  $1.21 < z < 24$ . When  $\mu = 1.1$  (i.e. the spread in  $\log E_{\text{iso}} - \log E_{\text{peak}}$  is reduced by a factor of 7.3 compared to  $\mu = 2$ ),  $\Delta z/z > 1$  when  $2.2 < z < 6.9$ .



**Figure 3.** The  $1\sigma$  redshift degeneracy region (the shaded region) bounded by  $z_a$  and  $z_b$ , obtained by solving equation (10). Given the  $1\sigma$  spread in the  $E_{\text{iso}} - E_{\text{peak}}$  relation – a factor of  $\mu = 2$  in  $E_{\text{iso}}$  obtained with the current data – the redshift degeneracy region spans from  $z = 0.94$  to  $z = 22$ .



**Figure 4.** The fractional error in the GRB redshift determined by the Amati relation, as a function of the solved redshift. Different curves correspond to different choice of  $\mu$ , as labelled. The horizontal dashed line marks the position of  $\Delta z = z$ .

### 3 CONCLUSIONS

We have shown that the  $E_{\text{iso}} - E_{\text{peak}}$  relation has a serious redshift degeneracy which prohibits one from using the  $E_{\text{iso}} - E_{\text{peak}}$  relation with the spread given by the current data to determine the redshift of a GRB in the range of  $z = 0.9 - 20$ . Even if the spread in the relation is reduced by a factor of 2, the redshift degeneracy region is still quite large,  $z = 1.4 - 12$ . The numbers cited above are obtained under the assumption that a  $1\sigma$  error is allowed. If a  $2\sigma$  error is allowed, the redshift degeneracy region in the current  $E_{\text{iso}} - E_{\text{peak}}$  relation is  $z = 0.5 - 50$ .

Mathematically, the existence of the redshift degeneracy is caused by the fact that in the  $\log E_{\text{iso}} - \log E_{\text{peak}}$  plane, the trajectory of a GRB is almost parallel to the  $E_{\text{iso}} - E_{\text{peak}}$  relation, if the redshift

of the GRB varies from  $z \sim 1$  to  $z \sim 20$  but the observed fluence and peak energy remain unchanged (Nakar & Piran 2005; Donaghy et al. 2006). Thus, if a GRB satisfies the Amati relation, as its redshift varies in this range the GRB remains in the neighbourhood of the relation within a  $1\sigma$  error. This is most clearly described by the case of GRB 060121, the redshift of which is constrained to lie between  $z = 1.5$  and  $4.6$  by its optical and near-infrared afterglow observations (Donaghy et al. 2006). As the redshift of GRB 060121 varies from  $z = 1.5$  to  $z = 5$ , the burst always satisfies the Amati relation (Donaghy et al. 2006, fig. 19).

Although the degeneracy described in this Letter does not exist for GRBs at low redshift, the error in the low redshift determined by the Amati relation is still large. For example, if the GRB is at  $z = 0.1$ , the redshift solved from the Amati relation is  $0.1_{-0.03}^{+0.05}$  ( $\mu = 2$ ). Because of this and the additional fact that the corresponding relation for Type Ia supernovae has a much smaller scatter, GRBs are not very useful for cosmology at low redshift.

A big problem that has hindered the development in understanding the nature of GRBs has been the determination of the distance of GRBs. Various redshift and distance estimators for GRBs have been proposed, based on the empirical correlations that have been discovered (Mészáros 2006). The results in this Letter indicate that the  $E_{\text{iso}}-E_{\text{peak}}$  relation cannot be used to determine the redshift and distance of GRBs.

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#### REFERENCES

- Amati L., 2006, MNRAS, 372, 233  
 Amati L. et al., 2002, A&A, 390, 81  
 Amati L., Della Valle M., Frontera F., Malesani D., Guidorzi C., Montanari E., Pian E., 2006, A&A, submitted (astro-ph/0607148)  
 Band D., Preece R. D., 2005, ApJ, 627, 319  
 Campana S. et al., 2006, Nat, 442, 1008  
 Donaghy T. Q. et al., 2006, ApJ, submitted (astro-ph/0605570)  
 Friedman A. S., Bloom J. S., 2005, ApJ, 627, 1  
 Ghirlanda G., Ghisellini G., Lazzati D., 2004, ApJ, 616, 331  
 Ghirlanda G., Ghisellini G., Lazzati D., Firmani C., 2005, Nuovo Cimento C., 28, 303  
 Ghisellini G., Ghirlanda G., Mereghetti S., Bosnjak Z., Tavecchio F., Firmani C., 2006, MNRAS, 372, 1699  
 Lloyd N. M., Petrosian V., Mallozzi R. S., 2000, ApJ, 534, 227  
 Mészáros P., 2006, Rept. Prog. Phys., 69, 2259  
 Nakar E., Piran T., 2005, MNRAS, 360, L73  
 Peebles P. J. E., 1993, Principles of Physical Cosmology. Princeton Univ. Press, Princeton  
 Sakamoto T. et al., 2004, ApJ, 602, 875  
 Tiengo A., Mereghetti S., 2006, A&A, 449, 203  
 Watson D. et al., 2004, ApJ, 605, L101  
 Watson D. et al., 2006, ApJ, 636, 967

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