

Why the fundamental plane of black hole activity is not simply a distance driven artifact

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Abstract

The fundamental plane of black hole activity is a non-linear correlation among radio core luminosity, X-ray luminosity and mass of all accreting black holes, both of stellar mass and supermassive, found by Merloni et al. [Merloni, A., Heinz, S., di Matteo, T. 2003. MNRAS 345, 1057 (MHD03)] and, independently, by Falcke et al. [Falcke, H., Körding, E., Markoff, S., 2004. A & A 414, 895]. Here we further examine a number of statistical issues related to this correlation. In particular, we discuss the issue of sample selection and quantify the bias introduced by the effect of distance in two of the correlated quantities. We demonstrate that the fundamental plane relation cannot be a distance artifact, and that its non-linearity must represent an intrinsic characteristic of accreting black holes. We also discuss possible future observational strategies to improve our understanding of this correlation.

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1. Introduction

The search for statistical associations between the X-rays and radio core emission in Quasars and AGN is about as old as X-ray astronomy itself. Very early on, a number of statistical issues related to the search of correlations between radio and X-ray luminosities in actively accreting black holes was already under discussion. In fact, the questions that arose in this discussion stimulated the

formulation and the wider recognition of a set of statistical methods specifically targeted to astrophysical problems (see e.g. Feigelson and Berg, 1983; Kembhavi et al., 1986; for a comprehensive discussion of statistical methods and problems in astrophysics, see Babu and Feigelson, 1996, and references therein).

In particular, the fundamental question was raised (see e.g. Elvis et al., 1981; Feigelson and Berg, 1983) of whether correlations are more accurately measured by comparing observed flux densities or intrinsic luminosities, as it is obvious that in flux limited samples spanning large ranges in redshift (i.e. distance) spurious correlations can be inferred in luminosity–luminosity plots if only detected

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points are considered. On the other hand, as clearly discussed in Feigelson and Berg (1983) and in Kembhavi et al. (1986), flux–flux correlations can themselves lead to spurious results, whenever there exists any non-linear intrinsic correlation between luminosities. The most statistically sound way to deal with the aforementioned biases has been formalized in terms of partial correlation analysis capable to handle censored data (upper limits), as discussed in Akritas and Siebert (1996). With such a method not only a correlation coefficient can be calculated for any luminosity–luminosity relationship in flux limited samples, but also a significance level can be assigned to it.

1.1. The fundamental plane of black hole activity

Black holes as mathematical entities are extremely simple, being fully described by just three quantities: mass, spin and charge. For astrophysical black holes, necessarily uncharged, little is known so far about their spin distribution. However, it is well established observationally that black holes do span a wide range in masses, from the $\sim 10M_{\odot}$ ones in X-ray binaries (XRB) to the supermassive ($\sim 10^6 - 10^9 M_{\odot}$) ones in the nuclei of nearby galaxies and in active galactic nuclei (AGN). In Merloni et al. (2003) (MHD03), we posed the following question: is the mathematical simplicity of black holes also manifest in their observational properties? More specifically: which observed black hole characteristics do scale with mass?

To answer such a question, we searched for a common relation between X-ray luminosity, radio core luminosity and black hole masses among X-ray binaries and AGN. This necessarily imposes a set of complications for any statistical analysis. These are essentially twofold. On the one hand, as already pointed out in the original papers on the subject (MHD03; Falcke et al., 2004, hereafter FKM04), there is a vastly differing distance scale between the two populations that should at some level induce spurious (linear) correlations between the observed luminosities even for a *completely* random distribution of fluxes.

Moreover, the inclusion of black hole mass in the analysis imposes some complex selection criterion on any sample: mass is estimated in a number of different ways, implying different levels and types of uncertainties linked to the specific observational strategy. It is therefore virtually impossible, at least with the current data, to estimate the degree of incompleteness of any black hole mass sample.

For the specific example we are interested in, a relationship is posited between the radio core luminosity (at 5 GHz) L_R of a black hole, its X-ray luminosity L_X , and its mass M . L_R and L_X are derived quantities, each carrying, in addition to the respective flux, a factor of D^2 , where D is the luminosity distance to the source.

As discussed in Section 1, statistical tools exist to test whether a correlation is, in fact, an artifact of distance, or whether it reflects an underlying luminosity–luminosity

relation, even in flux limited samples (Feigelson and Berg, 1983). In MHD03 (Section 3) a partial correlation analysis was performed, including all upper limits in the sample using the algorithm for performing Kendall’s τ test in the presence of censored data proposed by Akritas and Siebert (1996). Such an analysis showed unequivocally that, even after the large range of distances in the sample was taken into account, the radio core luminosity was correlated with both X-ray luminosity and mass, both for the entire sample (XRB plus AGN) and for the sample of supermassive black holes only.²

Motivated by the findings of the partial correlation analysis, MHD03 performed a linear regression fit to the data and found them to be well described by the following expression:

$$\text{Log}L_R = 0.6\text{Log}L_X + 0.78\text{Log}M + 7.33, \quad (1)$$

with a substantial residual scatter ($\sigma \approx 0.88$). A very similar result was obtained independently, from a different but largely congruent sample of sources, at essentially the same time by FKM04.

In the following, we review some of the original arguments presented in MHD03 that address the following question: is the multivariate correlation of Eq. (1) a spurious result due to the effect of plotting distance vs. distance in flux limited samples? In doing so, we present further evidence that a strong *non-linear* correlation among L_R , L_X and M indeed exists, which is *not* affected by the range of distance and the heterogeneity of the sample selection criteria.

2. Fundamental plane vs. distance driven artifact

Besides the formal partial correlation analysis, other rather straight-forward tests can be easily carried out to check to what extent distance bias in our sample may be responsible for inducing the observed correlation. For example, one can randomize the observed fluxes in any one band, and compare the correlation strengths of the original and the randomized (“scrambled”) data.³ The reason for this is obvious: if the observed correlation is just an artifact introduced by the range of distances in a sample of otherwise uncorrelated luminosities, then the randomized datasets (the fluxes of which are guaranteed to be intrinsically non-correlated) should show the same degree of correlation as the real dataset from which the fundamental plane was derived. Below, we will present a thorough, comparative statistical analysis of the original sample with the randomized ones.

² The partial correlation analysis carried on in MHD03 further demonstrated that the radio core luminosity is correlated with black hole mass after the common dependence on X-ray luminosity is taken into account, and vice versa, thus not only justifying, but statistically *mandating* the multivariate linear regression, rather than just a bivariate one.

³ This specific test was proposed by Bregman (2005).

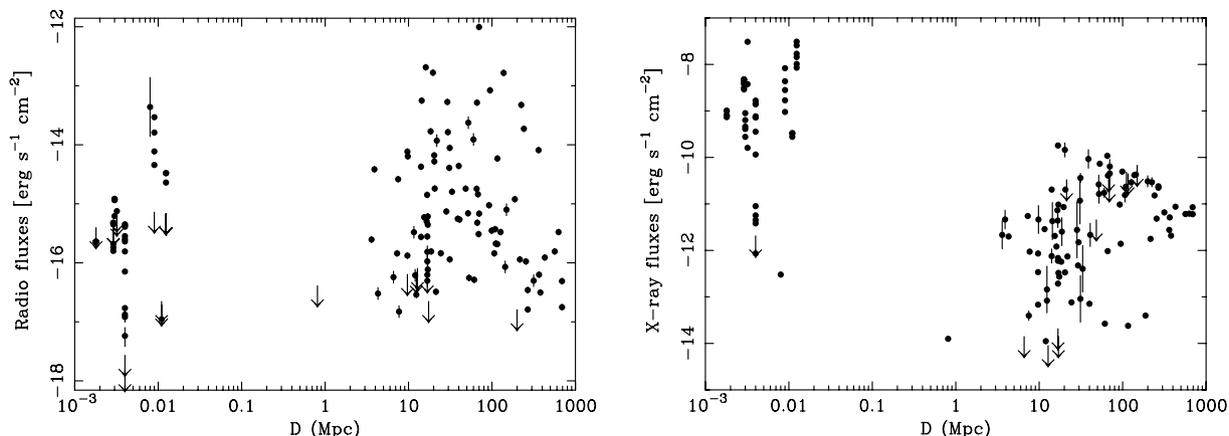


Fig. 1. Radio (left) and X-ray (right) fluxes for the sources in the original sample of MHD03 vs. distance (in Mpc). Upper limits are marked by green arrows.

2.1. The scrambling test I: SMBH only

We will first consider the extragalactic supermassive black holes (SMBH) in the sample.⁴ If we consider only the detections (79 objects) and exclude the upper and lower 5% in radio luminosity, the sample spans a 90% range of $\log F_{R,\max} - \log F_{R,\min} \approx 3.6$ orders of magnitude in radio luminosity and of $\log F_{X,\max} - \log F_{X,\min} \approx 3.3$ in X-ray luminosity (see Fig. 1). The range of distances spanned by the SMBH sample is also significant. The 90% range in the distances of the detected objects is 85, so that the factor distance squared, that enters in the luminosity has a range of about 7.2×10^3 , which is of the same order as the range in fluxes. As argued by Kembhavi et al. (1986), a comparable spread in distance should prevent a spurious luminosity–luminosity correlation from dominating a strong, underlying correlation signal. However, it is clear that care has to be taken when studying luminosity–luminosity correlations and that distances effects should always be accounted for.

To test whether distance bias dominates the correlation we take the radio fluxes of the detected sources and randomize them by assigning radio fluxes to objects in the sample via random permutations. To construct this Monte Carlo test, we repeat this procedure 10^6 times and calculate the Pearson correlation coefficient between L_R and $0.6L_X + 0.78M$ for each of the randomized datasets (using the code `slopes`, developed by M. Akritas and M. Bershady <http://astrostatistics.psu.edu/statcodes>). The upper left panel of Fig. 2 shows the distribution of the correlation coefficients obtained from the randomized datasets. For comparison, the correlation coefficient ($R \approx 0.7775$) of the *actual*, observed SMBH sample is marked by a vertical line.

The figure shows that, as expected, the range of distances in the sample does induce at some level a spurious

correlation, as the distribution of R is peaked at positive values. However, if the correlation seen in the real dataset were purely due to this spurious effect, its Pearson correlation coefficient would lie within the distribution of the scrambled data, which is clearly excluded by our Monte Carlo simulation. Out of a million realizations of the randomized radio flux distribution, only 3 had a larger correlation coefficient than the real data. Clearly, the real data are much more strongly correlated than the scrambled data.

We also performed a linear regression on the scrambled data, with slope b and intercept a , using a “symmetric” fitting algorithm (see MHD03, Section 3.1).⁵ The upper right hand panel of Fig. 2 shows the uncertainty in the derived value for the slope b , which can itself be regarded as a measure of the intrinsic scatter of the fitted data. Only in about 0.2% of the scrambled datasets was this uncertainty smaller than that obtained for the real sample. This confirms the statement made in MHD03 (derived from partial correlation analysis), that the degree of correlation among L_R , L_X and M cannot be dominated by the effect of distances.

2.2. The scrambling test II: SMBHs and XRBs

Next, we consider the entire sample of detected sources, including XRBs, bringing the sample up to 117 points in total. It is obvious that when the XRB in our own Galaxy are included the range of distances spanned by the sample increases dramatically. The 90% ranges in $\log F_R$, $\log F_X$ and D^2 are now, respectively, 4, 5.7 and 4.6×10^{10} .

As for the SMBH sample discussed above, we performed a Monte Carlo simulation by randomizing the radio fluxes of the entire sample 10^6 times. The distribution of the resulting correlation coefficients for the scrambled

⁴ Unlike the original sample by MHD03, we remove the only genuinely beamed source (3 C 273) for consistency.

⁵ In particular, we have used here both the OLS bisector and the reduced major axis method as described in Isobe et al. (1990) and in Feigelson and Babu (1992), and implemented in the code `slopes`; Fig. 2 shows only the results for the reduced major axis method, but the results are consistent in the two cases.

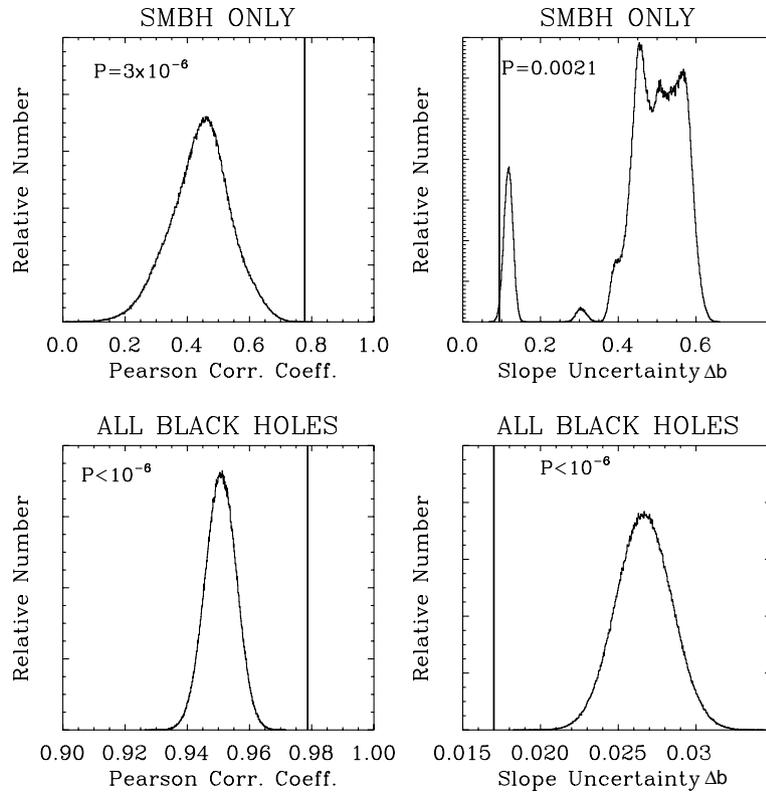


Fig. 2. Results of the Monte Carlo simulation of scrambled radio fluxes. *Upper panels*: extragalactic supermassive black holes only; *lower panels*: entire sample of detected sources, including XRBs; *left hand panels*: distributions of the Pearson's correlation coefficients for randomized fluxes (curve), compared to correlation coefficient of the original dataset (vertical line); *right hand panels*: distributions of the uncertainties in the regression slope for the randomized fluxes (curve), compared to the value for the original data (vertical line). Also shown are the Monte Carlo likelihood values, P , for the observed values as random chance realizations of the randomized sample (upper left corners). All plots show clearly that the randomized sample is not as strongly correlated as the real one.

dataset (including XRB) is shown in the lower right panel of Fig. 2.

As expected, this distribution is now peaked at very high values of R , demonstrating that indeed the large range in distances can induce a spuriously strong correlation. This effect is unavoidable when comparing SMBH and XRB, and it is not going to improve with any volume limited sample of extragalactic sources, as current telescope sensitivities are still far from what would be required in order to observe XRB down to low luminosities in nearby AGN hosts (see below).

What is striking about the Monte Carlo results derived from the combined sample is that the Pearson correlation coefficient of the actual dataset ($R = 0.9786$) is even *more* inconsistent with the randomized data than in the SMBH-only case. Out of a million realizations of the randomized datasets, *not even one* showed a stronger correlation than the real data. In other words, the probability that the correlation found by MHD03 is entirely due to distance effects is less than 10^{-6} . This statement is confirmed by the distribution of the uncertainties in the regression slope, shown in the lower right panel.

This is partly due to the fact that in the XRB sample, the radio and X-ray luminosities are correlated quite tightly, over a range of luminosities much larger than the range

in distances out of which they are observed (see e.g. Gallo et al., 2003). More importantly, the X-ray fluxes of the XRBs are systematically enhanced compared to the AGN X-ray fluxes, while the radio fluxes of both samples are comparable. In other words, the correlation is *non-linear* ($L_R \propto L_X^{0.7}$) and the *slope* of the XRB correlation is, within the errors, consistent with being *the same* as that derived from the best fit of the SMBH only sample. It is thus a fortiori consistent (within the uncertainties imposed by the significant residual scatter) with the correlation that is derived for the entire sample.

If the effect were purely distance driven, one would expect to find a correlation slope much closer to linear (see Section 3). The non-linearity between L_R and L_X and the fact that the power-law index is the same for XRBs and SMBHs produces a very strong signal in the correlation analysis, much stronger than the spurious one induced by the distance effects (only the latter can be recovered from a sample with scrambled radio fluxes), at greater than the 99.9999% level.

This simple test leaves little room for arguing that the “fundamental plane” correlation between radio luminosity, X-ray luminosity, and black hole mass does not exist and that instead is induced entirely by distance bias. These results are consistent with the partial correlation analysis

by MHD03, where non-parametric tests were used to handle censored data.

The fact that the correlation is stronger when XRB are included rather than in the SMBH sample alone, *even after the effect of distances is considered* can be hard to visualize when plotting the entire fundamental plane. Such a difficulty amounts to that of distinguishing two correlations, one with a Pearson correlation coefficient of $R \approx 0.94$, another with $R \approx 0.98$, extending over more than 12 orders of magnitude.⁶ We believe that the difficulty in visualizing this statistically significant difference may induce some concern on the fundamental plane correlation. As we have shown, however, an accurate statistical analysis can easily reveal this difference. This visualization difficulty also explains why the few upper limits in the MHD03 sample, when plotted against the entire fundamental plane, will follow the same correlation. A better test in this case would be to *quantify* the degree of such a correlation for the censored data in the sample. The scrambling tests suggest that they will indeed be correlated, but not as strongly as the real dataset. There are, however, too few upper limits in the SMBH sample of MHD03 to allow a meaningful statistical test.

2.3. On the effects of flux limits

Another way to test whether the fundamental plane is a pure distance artifact is to explore the flux selection effects using a Monte Carlo simulation under the null-hypotheses that the radio, X-ray luminosities and black hole masses are not correlated and assume we have a purely flux limited sample (however, see Section 2.5 below for a more accurate discussion of the actual sample selection).

Radio and X-ray luminosity functions of AGN evolve strongly with redshift (see e.g., [Hasinger et al., 2005](#), and [references therein](#); [Willott et al., 2001](#)), however, as the original samples of MHD03 and FKM04 were almost exclusively made of local sources, the inclusion of these effects is beyond the scope of this simple test. The observed luminosity functions are more complex than a simple power law, being usually described by a flat power law (index 0.5–0.9) at lower luminosities and steeper one towards higher luminosities (for the 2–10 keV hard X-rays luminosity function the transition takes place around 10^{44} erg s⁻¹, see [Ueda et al., 2003](#)). However, as the majority of our sources are nearby galaxies with low luminosities, we can simplify our model and describe the luminosity functions with a simple power law.

Thus, we assume that the density of X-ray emitting AGN, Φ_X , can be written as $\Phi_X(L_X) \propto L_X^{-\alpha_x}$. Similarly, the radio luminosity function is chosen to be

$\Phi_R(L_R) \propto L_R^{-\alpha_r}$. As a reference model, we fixed $\alpha_x = 0.85$ ([Ueda et al., 2003](#)) and $\alpha_r \approx 0.78$ ([Nagar et al., 2005](#)), and we will discuss below how our results depend on the exact slopes of the luminosity functions. We assume that the objects have a constant space density. For simplicity, we first assume that the luminosity functions for XRBs is the same as for AGN, though this will be corrected in a second step.

We then construct an artificial sample containing 50 XRBs and 51 nearby low luminosity AGN and 48 distant AGN and restrict the distances to the range observed: for XRBs 2–10 kpc, for LLAGN 3–50 Mpc, and for AGN 50–1000 Mpc. The average mass of an XRB is set to $10M_\odot$, with the masses normally distributed in log-space with a dispersion of 0.3 dex. For AGN, we assume an average mass of 10^8M_\odot and a dispersion of 1 dex. We always assume a distance measurement error of 10% and an error in the mass estimate for the supermassive black holes of 0.35 dex.

This artificial sample can now be observed with given flux limits. As $\alpha_x, \alpha_r > 0$, brighter objects are more likely to be detected at larger distances as the available volume is larger; obviously, fainter sources cannot be detected out to these distances due to the flux limits of the sample.

We have then performed a correlation analysis on these simulated samples, varying both the flux limits and the slopes of the luminosity functions. The results are shown in [Table 1](#). All samples show strong correlations (Pearson correlation coefficient ≈ 0.95), indeed consistent with the results of the scrambled samples discussed in [Section 2.2](#). However, not a single setup is able to yield the parameters similar to those of the fundamental plane, neither in term of correlation strength, nor in terms of linear regression slopes. In particular, as the correlation is only created by the flux limits, we find $\xi_{RX} \approx 1$, as it is expected given that we are simply plotting distance against distance ([Feigelson and Berg, 1983](#)). Also as expected, the flux limits have no effect on the mass term, and one therefore finds no mass dependence of the radio luminosity $\xi_{RM} \approx 0$.

When we perform on our artificial samples a partial correlation analysis, i.e. we study the strength of the observed correlation *taking distance into account*, as it was done in MHD03, we found that the partial correlation coefficient, $R_{RX,D}$ is always compatible with being zero, while the observed fundamental plane has a partial correlation coefficient of about 0.6. If one decreases the power law indexes for the luminosity functions, the perpendicular scatter σ_\perp increases, but the slope of the spurious correlation remains the same. We can therefore safely reject the null-hypotheses.

As discussed above, for XRBs it has been shown that in all low/hard state objects the radio and X-rays are correlated ([Gallo et al., 2003](#)). Thus, the question arises of whether the fundamental plane relation can be a spurious effect generated by combining the genuinely correlated sample of XRB with a flux limited, uncorrelated sample of nearby SMBH.

⁶ If two samples of 117 data each have two measured Pearson correlation coefficients of 0.94 and 0.98 respectively, then it is possible to show that the probability of the former being intrinsically a better correlation than the latter, is of the order of 10^{-5} , see Num. Rec., chapter 14.

Table 1
Effects of the observing flux limits on uncorrelated data

f_r	f_x	ζ_{RX}	ζ_{RM}	b	σ_{\perp}	$R_{RX,D}$
<i>Luminosity function: $\alpha_r = 0.78$ and $\alpha_x = 0.85$</i>						
0.1	0.1	1.09 ± 0.07	-0.05 ± 0.09	-6.7 ± 2.2	0.54 ± 0.04	0.04 ± 0.09
0.5	1	1.01 ± 0.05	0.0 ± 0.06	-3.9 ± 1.6	0.53 ± 0.05	0.05 ± 0.08
0.5	10	1.04 ± 0.09	-0.05 ± 0.10	-5.9 ± 2.8	0.53 ± 0.05	0.05 ± 0.09
5	10	1.03 ± 0.05	-0.02 ± 0.06	-4.5 ± 1.6	0.51 ± 0.05	0.04 ± 0.08
<i>Luminosity function: $\alpha_r = 0.55$ and $\alpha_x = 0.65$</i>						
0.1	0.1	1.12 ± 0.11	-0.09 ± 0.13	-7.3 ± 3.7	0.73 ± 0.06	0.01 ± 0.08
<i>Luminosity function: $\alpha_r = 1.5$ and $\alpha_x = 1.5$</i>						
0.1	0.1	1.14 ± 0.05	-0.06 ± 0.04	-8.5 ± 1.5	0.34 ± 0.02	0.09 ± 0.07
<i>Luminosity function: $\alpha_r = 0.78$ and $\alpha_x = 0.85$ XRBs correlated</i>						
0.5	1	1.06 ± 0.06	0.32 ± 0.04	-8.7 ± 2.1	0.50 ± 0.04	0.26 ± 0.09
5	0.1	1.22 ± 0.09	0.51 ± 0.05	-14.5 ± 3.3	0.54 ± 0.05	0.09 ± 0.08
0.1	10	1.0 ± 0.05	0.12 ± 0.04	-6.6 ± 1.8	0.46 ± 0.05	0.35 ± 0.08
5	10	1.1 ± 0.07	0.30 ± 0.05	-10.1 ± 2.4	0.48 ± 0.04	0.02 ± 0.09

The first column is the radio flux limit in mJy, the second the X-ray flux limit in units of $10^{-13} \text{ erg s}^{-1} \text{ cm}^2$. Third, fourth and fifth column show the fitting parameters of the artificial sample with a linear relationship $\log L_R = \zeta_{RX} \log L_X + \zeta_{RM} \log M + b$ (see Eq. (1)). σ_{\perp} is the scatter perpendicular to the fitted plane. The partial correlation coefficient, $R_{RX,D}$ measures the degree of intrinsic correlation of radio and X-ray luminosity once the effects of distance are taken into account. α_r and α_x indicate the slope of the AGN luminosity functions in the radio and X-ray bands, respectively, assumed in order to create artificial uncorrelated samples. The last set of results are for a sample of uncorrelated AGN together with XRB following the observed radio–X-ray correlation.

The results are also shown in Table 1. Again, the parameters are not compatible with those of the fundamental plane. The partial correlation coefficient is now bigger than zero, as the XRBs are indeed correlated (but not the AGN), but it is still much lower than for the real MHD03 sample. Thus, as the exact shape of the luminosity functions does not seem to change our results, this result further supports the idea that the radio and X-ray luminosities of accreting black holes, as well as their masses are genuinely correlated.

2.4. Distance independent plots

Obviously, when studying the relationship between accreting black holes luminosity and mass, it is possible to remove the distance effect entirely from the analysis of any sample, for example by studying the correlation between the ratio L_R/L_X and M . However, such an analysis implicitly assumes that L_R and L_X are themselves linearly correlated. In fact, as was explained at length in MHD03, and as should be apparent from the well known radio/X-ray relation in XRBs, we do *not* expect a priori that the relation between the two is simply linear. Rather, it is reasonable to expect that, to lowest order, the two will follow a non-linear relation of the form $L_R \propto L_X^{\zeta_{RX}}$.

This suggests that a better correlation to study would be $L_R/L_R^{\zeta_{RX}}$ vs. M . Using the best fit value of $\zeta_{RX} = 0.6$ from MHD03, this is shown in Fig. 3, where it is compared to the same plot if a linear relation between L_R and L_X is assumed. Clearly, adopting $\zeta_{RX} = 0.6$ gives a much stronger correlation with black hole mass than what one would obtain by assuming a simple linear relationship between luminosities. This statement can be quantified: The correla-

tion coefficient for the two variables $L_R/L_X^{\zeta_{RX}}$ and M has a maximum of $R = 0.65$ at $\zeta_{RX} \sim 0.5$, compared to the value of $R = 0.4$ obtained for $\zeta_{RX} = 1$ (note that this correlation does not use a symmetric method, thus resulting in a different value than the $\zeta_{RX} \sim 0.6$ found in the regression analysis of MHD03). This difference is significant to the 99.99% level.

One may ask if in our heterogeneous sample the effect of distance may somehow induce such correlation, as the quantity $L_R/L_R^{\zeta_{RX}}$ is now a distance-dependent one in a flux limited sample, for $\zeta_{RX} \neq 1$. However, this is not the case, simply because black hole mass itself is only very slightly correlated with distance within the extragalactic SMBH sample, as can be seen from Fig. 4.

Yet another related, visually clear, illustration of the fact that the fundamental plane correlation is much stronger than any distance induced bias can be shown by plotting the data in the flux–flux–mass space. Fig. 5 shows in the upper left panel the data viewed across the fundamental plane relationship expressed in fluxes and with the distance as a fourth variable. The correlation found in MHD03, expressed this way, reads:

$$\text{Log} F_R = 0.6 \text{Log} F_X + 0.78 \text{Log} M - 0.8D + 7.33 \quad (2)$$

The other three panels of Fig. 5 show the data points after a randomization of radio fluxes (upper right panel), of X-ray fluxes (lower left panel) and of black hole mass (lower right panel). A visual inspection is sufficient to show that the correlation in the original data is much stronger than the residual correlation in the lower left panel (scrambled X-ray fluxes – note that a residual correlation should be expected in this case, as the radio luminosity should be related to black hole mass even for a random set of X-ray

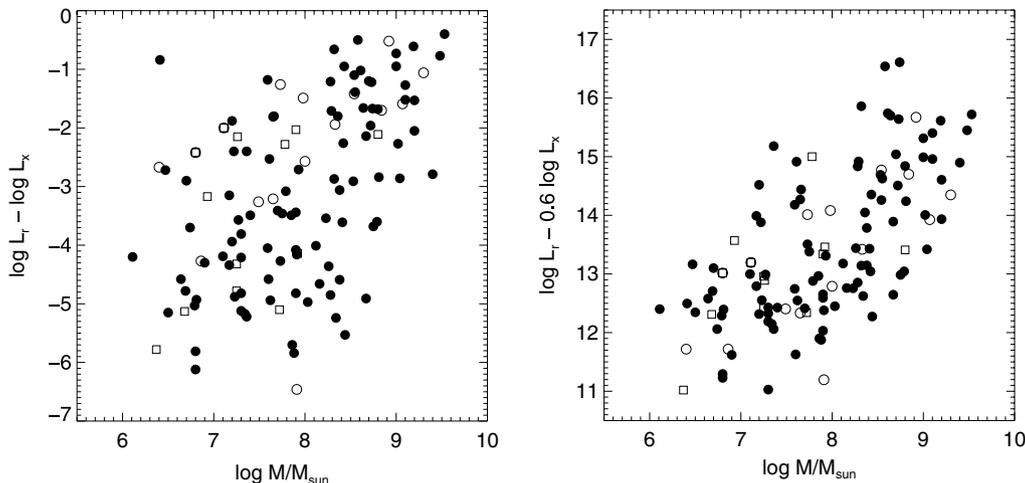


Fig. 3. The left panel shows the logarithm of the ratio of radio to X-ray luminosity, $\log L_R - \log L_X$ vs. the logarithm of black hole mass for the SMBH in the sample. The right panel shows instead the ratio $L_R/L_X^{0.6}$. The latter shows clearly a stronger correlation with black hole mass than the former. Note that this plot removes the distance bias up to the level that black hole mass is only very slightly correlated with distance within the extragalactic SMBH sample (this can be seen from Fig. 4). Open symbols are upper limits.

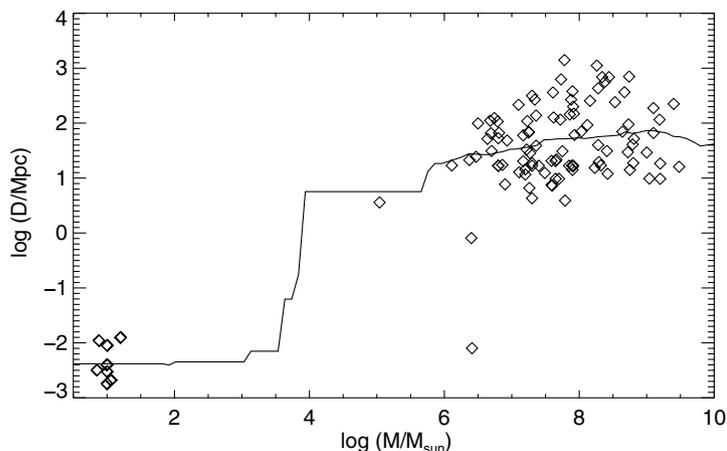


Fig. 4. Distance vs. black hole mass for the objects in the MHD03 sample, the solid line is the sliding mean. This shows that the AGN sample is homogeneous in distance with mass and therefore any L_R/L_X^{RX} vs. M relation in the AGN sample cannot be driven by distance.

luminosities) and that no correlation is present in the other two panels. By construction, this correlation cannot be a spurious distance effect.

2.5. On sample selection

Clearly, the plots in Fig. 5, as well as the confidence in the regression slopes, could be improved by a more carefully crafted, more complete sample than what we have currently available. We shall briefly address the question of whether a volume limited sample would, in fact, be the best way to treat this problem, as advocated, for example, in Bregman (2005).

In what follows, it is important to keep in mind that the original sample studied in MHD03 was *neither* a flux limited sample, *nor* a combination of flux limited samples, but rather a combination of flux and volume limited sam-

ples, observed in both X-ray and radio bands with different sensitivities (see Fig. 1). For example, MHD03 considered all known low-luminosity AGN within 19 Mpc observed by Nagar et al. (2002) with the VLA. Upper limits were recorded as far as possible, whenever the information regarding a source with reasonably well measured/estimated black hole mass was available from radio or X-ray surveys, but no effort was made to account for the incompleteness derived from the requirement of a source having a measured black hole mass itself. The heterogeneity of the resulting sample may well introduce biases which are hard to account for in a luminosity–luminosity correlation; however, it is also a safeguard against systematic effects that might arise from any one technique of estimating black hole masses.

Furthermore, the two populations have vastly different distances, masses, and luminosities. Clearly, these distinct

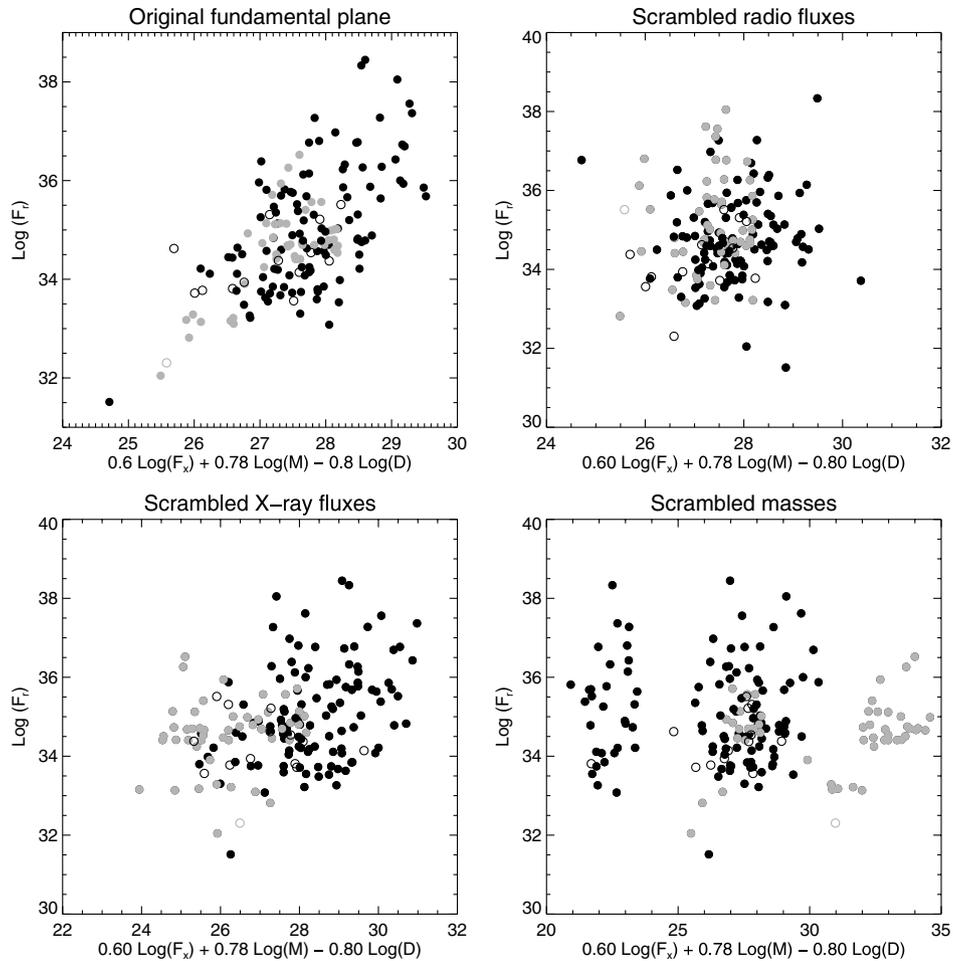


Fig. 5. The upper left panel shows the fundamental plane relation in a flux–flux–distance, rather than luminosity–luminosity plot (fluxes are calculated measuring distances in Mpc). The other three show the same dataset in which either radio flux, or X-ray flux or mass has been randomized. Filled black symbols are for SMBH, filled grey ones for XRB and open symbols for upper limits.

regions of parameter space are largely responsible for stretching out the original plot of the fundamental plane over 15 orders of magnitude on each axis. The question then arises whether a volume limited sample could address some of the concerns about spurious distance effects discussed above (after all, even the randomized data show a correlation coefficient of 0.94). Before addressing this question, however, it is important to note that it is not at all unreasonable to compare X-ray binaries and AGNs in the same flux range, and that a volume limited sample including both XRBs and SMBHs would, in fact, not make much sense. Physical intuition suggests that, when comparing black holes of vastly different masses, one should restrict the analysis to a similar range in dimensionless accretion rate, $\dot{m} \equiv \dot{M}/M$. By coincidence, the roughly seven orders of magnitude difference in M between XRBs and SMBHs are almost exactly canceled out by the roughly 3.5 orders of magnitude larger distance to the SMBH sample, making the flux ranges spanned by XRBs at least comparable. As it turns out, comparing the volume limited XRB sample with flux limited AGN sample puts both clas-

ses in roughly the same range of \dot{m} (individual sources like GX 339-4 and Sgr A* representing a small percentage of outliers).

In a volume limited sample that includes both AGNs and XRBs, one would be forced to compare objects at vastly different accretion rates, which would not be very meaningful from a physical point of view. In this sense, one could also argue that the distance bias that is invariably introduced when correlating XRBs and AGNs is in reality an accretion rate bias, which is warranted on physical grounds.

Furthermore, due to the cosmological evolution of the accreting black holes population, a volume limited sample would be strongly dominated by quiescent sources for AGNs. For fitting regression slopes, a sample crafted to have roughly equal density of points throughout the parameter space would presumably be much better suited for determining the regression slopes. While the MHD03 sample is certainly far from reaching that goal, it is another argument against a broad brush call for volume limited samples.

3. The slope of the fundamental plane

Fitting a regression through the data requires the *assumption* that one single underlying relation drives the data. Within that context, the regression will produce the correct slopes no matter what the sample is. The same is true for including XRBs: although they may have comparable slope to the AGNs and although the AGNs lie on the extrapolation of the XRB slope with the mass correction, the statement that these facts are truly an expression of the same accretion physics at work must be posited as an ansatz (see MHD03).

The fact that radio/X-ray correlations can be found in samples of XRBs and AGN, either in luminosity–luminosity, or flux–flux (with slaved distance) space that are consistent with each other within the uncertainties then supports the ansatz, and the correctness of the idea of jointly fitting *one* correlation. Within those limits, the slopes we derived are an accurate representation of the putative relation. This is in fact the customary and correct way to proceed. First one should test that the available data are indeed correlated, taking all possible biases (as those induced, for example, by distance, sample selection, etc.) into account. If, and only if, any such correlation is found to be statistically significant, then a linear (possible multivariate) regression fit to the data can be looked for.

Within the present context, the clear non-linearity of the correlation between radio and X-ray luminosity for XRB and the apparent non-linearity of the correlation for the SMBH-only sample (with the slope consistent with being the same in the two separate samples), not only reinforce strongly the validity of our approach, but also suggests that only by working in the luminosity–luminosity space can one recover the intrinsic properties of the objects under scrutiny (Feigelson and Berg, 1983; Kembhavi et al., 1986).

3.1. Using simultaneous radio/X-ray observations

As a final note, it is useful to point out one additional factor which should be considered when searching for the true nature (i.e. slope) of the fundamental plane. In XRBs, discovering and properly measuring the non-linear radio/X-ray correlation depended on the existence of good quality, quasi-simultaneous radio and X-ray band observations of a single source. The luminosity changes during outburst cycles which trace out this correlation occur on timescales of days to weeks. Radio and X-ray flux measurements separated in time by more than this would result in an altogether different correlation reflecting the lag in observation time. Our techniques of testing whether AGN follow a similar correlation as XRBs by using samples are viable only with the inherent assumption that non-simultaneous radio/X-ray observations are comparable for these sources. In general, this should be true because AGN are expected to vary on longer timescales than XRBs, by a factor that scales roughly linearly with their mass ratio. In other words, if a single AGN observa-

tion is equivalent to a single data point on the XRB radio/X-ray correlation, then we assume that we would in fact see the same type of correlation if we could study an AGN for millions of years.

Nearby low-luminosity AGN (LLAGN), however, often have smaller central masses ($\sim 10^6 - 10^7 M_\odot$) and can show variations in both radio and X-ray fluxes of tens of percent over month-long timescales. As an alternative test of the reality of the fundamental plane, one can study the fit to the plane and its scatter using just a few sources with very well measured mass and distance, so that the scatter is in fact dominated by intrinsic variability. An initial test was performed by Markoff (2005), using data from the best XRB displaying the radio/X-ray correlation in its hard state, GX 339-4, as well as our Galactic, underluminous SMBH Sgr A*, and two nearby LLAGN, M81 and NGC 4258. All of these sources have well-determined physical mass and distance, and are not highly beamed, allowing a detailed assessment of the fundamental plane coefficients, as well as the contribution to its scatter from intrinsic variability. A linear regression fit was performed on 10^4 “samples” of data, simulated using a Monte Carlo technique from decades of observations of the sources, and representing all possible configurations of their respective fundamental plane during different phases of variability. The best fit plane is shown in Fig. 6, with contours in average scatter indicated, which was used to estimate the relationship of Sgr A*’s flares to the fundamental plane relation. Interestingly, the resulting fundamental plane coefficients

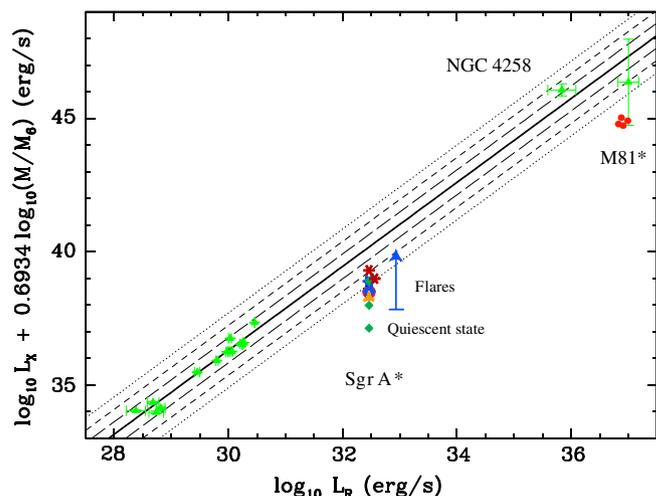


Fig. 6. The fundamental plane correlation for the three well-measured sources bracketing Sgr A*: the X-ray binary GX 339-4, and the LLAGN NGC 4258 and M81. For this latter source the red points represent last year’s simultaneous observations (Markoff et al., in preparation), while the green point with large error bars is the average and rms variation from all prior non-simultaneous observations. The solid line indicates the best fit correlation from Monte Carlo simulations (see Markoff, 2005 for details), with contours in the average scatter (σ) from the correlation represented as increasingly finer dashed lines. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

are similar to those derived by MHD03 and FKM04, although the mass-scaling factor ζ_{RM} is somewhat smaller.

In the past year, truly simultaneous observations of M81* have been carried out (Markoff et al., in preparation) and four actual data points like those for the XRBs can be added to the plane projection. These have been placed in Fig. 6 for comparison to the data point representing the average and rms variation from all prior non-simultaneous observations. If these points had been used in a single determination of the fundamental plane, along with the GX 339-4 data and the same average/rms variation from NGC 4258, linear regression would give the relation:

$$\text{Log}L_{\text{R}} = 0.586\text{Log}L_{\text{X}} + 0.656\text{Log}M + 8.211 \quad (3)$$

Again, the radio/X-ray correlation coefficient is very similar to the results derived by MHD03. Because this directly tackles the question of simultaneity, it is in fact a very strong test of the predicted mass scaling. It is compelling that the correlation derived from such well-measured sources is consistent with the relations derived from both XRB and the AGN samples.

4. Conclusions

We have presented further statistical evidence that the fundamental plane of black hole activity (i.e. the non-linear correlation between radio core luminosity, X-ray luminosity and mass of accreting black holes) is not an artifact due to an overlooked bias introduced by the range of distances spanned by our samples.

Partial correlation analysis techniques capable of handling censored data were already used in the original work of MHD03, following a decades long tradition in the multiwavelength study of AGN and QSOs. Here we have extended this analysis performing Monte Carlo simulations of randomized radio fluxes and found results entirely consistent with the partial correlation analysis. Additional Monte Carlo simulations of combined flux limited samples of XRBs and AGN have also demonstrated that the orientation of the fundamental plane (i.e. its slope) cannot be reproduced by spurious distance driven effects. Moreover, also distance-independent tests demonstrate that the fun-

damental plane correlation is real and has a non-linear slope, which further suggests that studying flux–flux relations only is *not* appropriate when dealing with the data.

With respect to the traditional studies of correlations between luminosities of AGN in different bands, the inclusion of a mass term in the analysis imposes a very complex selection criterion on any sample: mass can be estimated in a number of different ways, with different degrees of uncertainties, and different degrees of observational difficulty, so that it is almost impossible, at least with the current data, to estimate the degree of incompleteness of any black hole mass sample. With respect to this crucial aspect, we argue that volume limited samples are not necessarily the best tools to study and understand the physical origin of such a correlation, as the cosmological evolution of the population of accreting black holes introduces severe biases in the (M, \dot{m}) parameter space, which also have to be taken into account. Simultaneous observations in the radio and X-ray bands of accreting SMBH at the low-mass end of their distribution will be extremely useful in better determining the true correlation coefficients of the fundamental plane and thus place better constraints on its physical origin.

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