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The distributed burning regime in type Ia supernova models

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Abstract. The deflagration mode of flame propagation in type Ia supernova (SN Ia) models requires a correct description of the interaction of the flame with turbulent motions. It is well-known that turbulent combustion proceeds in different regimes. For most parts of the deflagration in SNe Ia the flamelet regime applies. This has been modeled in previous multi-dimensional simulations. However, at late stages of the explosion, the flame will propagate in the so-called distributed regime. We investigate the effects of this regime on SN Ia models in a first and simplified approach and show that the trend of effects seems capable of curing some problems of current pure deflagration models.

Key words. stars: supernovae: general – hydrodynamics – methods: numerical

1. Introduction

Damköhler (1940) was the first to distinguish between different regimes of turbulent combustion. Since his pioneering work progress in the theoretical understanding of the turbulent combustion process (e.g. Peters 2000) as well as flame experiments in the laboratory (see Abdel-Gayed & Bradley 1981, for a collection of data) have confirmed his ideas. He described a regime of "large-scale turbulence" where the flame is deformed by interaction with turbulent motions and a "small-scale turbulence" regime where turbulent eddies actually penetrate the internal flame structure. In modern perception these regimes are identified with the *flamelet* and the *distributed* regimes, respectively.

As pointed out by Niemeyer & Woosley (1997), both regimes should be reached subsequently in the deflagration model of SN Ia explosions (for a review of models see Hillebrandt & Niemeyer 2000). In a statistical approach the distributed burning regime was addressed in the context of SNe Ia by Lisewski et al. (2000). Direct numerical simulations of flames in degenerate matter have recently been able to reach the distributed burning regime (Bell et al. 2004b). Nonetheless, the distributed burning regime has never been implemented in global deflagration models of thermonuclear supernovae (e.g. Reinecke et al. 2002; Gamezo et al. 2003). This was motivated by the fact that due to the low fuel densities to which the regime applies, no additional iron group nuclei are synthesized. Moreover, it was assumed that the flame propagation here would be too slow to significantly contributing to the energy generation. This may be a reasonable approach as long as the models aim at first order effects of the explosion characteristics. However, constant development in modeling and increasing computational resources facilitate systematic tests of initial parameters (see e.g. Röpke & Hillebrandt 2004b,a) and first synthetic light curves were derived by Sorokina & Blinnikov (2003). Even synthetic spectra may be calculated from multi-dimensional models soon.

Here the question arises whether the pure deflagration models (in which the flame propagates subsonically) in their current form are consistent with observational data. The physics input of these models may be incomplete. Possible problems arise from their low explosion energies which may only reproduce the low side of the average observed velocities of the ejecta in SNe Ia. Furthermore, they seem to underproduce intermediate mass elements and leave unburnt material at low velocities. Based on their success in one-dimensional parametrized models, delayed detonations have been put forward as a favorable scenario (Gamezo et al. 2004). In a detonation the flame is mediated by shock waves and travels with sound speed. The problem with the delayed detonation scenario is that no physical mechanism could be identified yet that would trigger the transition from a deflagration to a detonation under conditions of SN Ia explosions (Niemeyer 1999; Röpke et al. 2004a,b).

With reconsidering the distributed burning regime in the deflagration model of thermonuclear supernovae we propose an alternative mechanism that may help to overcome some of the difficulties of that model without artificially evoking a detonation. The advantage of this extended deflagration model is that it rests on a sound basis of known physics. If no other effects terminate the deflagration phase, burning inevitably enters the distributed regime.

Here, we study the effects of the distributed burning in very simplified two-dimensional models. As will be discussed below the employed description of this regime is no more than a coarse first-order estimate and can only point out the trends how it would change the model. Substantially more effort will be needed to implement details of the distributed burning regime in global SN Ia explosion simulations.

2. Modeling turbulent combustion in SNe Ia

The propagation speed of a laminar deflagration flame is determined by a balance between energy production in the reactions and diffusive energy transport. Based on this simple idea, Mikhel'son (1889) derived an expression for the laminar burning velocity. This expression was later corrected by Zel'dovich & Frank-Kamenetsky (1938) taking into account the activation energy necessary to induce the reaction. This results in a division of the flame structure into a preheat zone and a reaction zone. In both theories, the laminar flame speed s_l turns out to be proportional to the square root of the diffusivity D.

The wrinkling of the thermonuclear flame in SN Ia explosions begins at large scales. An inverse stratification of light ashes below dense fuel in the gravitational field of the exploding white dwarf (WD) star makes burning from its center outward intrinsically unstable. Buoyancy (Rayleigh-Taylor) instabilities lead to the formation of burning bubbles that rise into the cold fuel. This effect is directly resolved in multidimensional SN Ia simulations. Secondary shear (Kelvin-Helmholtz) instabilities at the interfaces of the burning bubbles give rise to the generation of turbulent motions. The formed turbulent eddies decay to smaller scales in a turbulent cascade.

The eddies of the turbulent cascade interact with the flame further wrinkling and stretching it on smaller scales. This leads to a rapid increase in the flame surface and results in an enhanced burning rate. This effect acts down to the so-called Gibson scale (Peters 2000). Below that scale turbulent velocities are so small compared with the laminar burning speed of the flame that the flame burns through turbulent eddies before they can significantly alter its shape. If the Gibson length is large compared with the width of the flame, turbulent burning proceeds in the *flamelet regime*. Here the flame is deformed by turbulence, but its internal structure is not affected.

In the flamelet regime, the effect of surface enlargement on numerically unresolved small scales can be compensated by attributing an adequate effective turbulent burning velocity s_t to a (resolved) smoothed flame front. This effective turbulent burning velocity has to be determined in a way that the mass flux through the smoothed flame equals that through the unsmoothed flame propagating with its laminar speed. Damköhler (1940) found that in the flamelet regime the turbulent flame velocity completely decouples from the laminar speed and is proportional to the turbulent velocity fluctuations v':

$$s_t \propto v'. \tag{1}$$

Due to the expansion in the explosion process, the fuel density drops and this broadens the flame width (cf. Timmes & Woosley 1992). At some stage the turbulent motions become capable of penetrating the internal flame structure. The flame enters the *distributed burning regime* (see Niemeyer & Kerstein 1997, for an analysis of this transition in SNe Ia). Damköhler's idea to describe this regime was to assume that the turbulent eddies entering the preheat zone alter the diffusivity by mixing its thermal structure. Therefore, in analogy to the expression for the laminar burning velocity, the turbulent burning velocity in the distributed regime should scale like

$$s_t/s_l \sim (D_t/D)^{1/2},$$

with D_t denoting a *turbulent diffusivity*. The laminar diffusivity D is proportional to product of the flame thickness l_f and the laminar flame speed s_l. Analogously, D_t is set proportional to the turbulent velocity fluctuations v' at a certain length scale l. For the turbulent flame speed in the distributed regime this yields a square-root dependency on v',

$$s_t \sim (s_l\, v'\, l/l_f)^{1/2}, \tag{2}$$

which is confirmed by experimental data. In this regime s_t scales with the laminar flame speed. Turbulent mixing of the internal structure of the flame accelerates its propagation velocity.

To model both turbulent burning regimes in numerical simulations of SNe Ia we apply the scheme proposed by Reinecke et al. (1999, 2002) and Röpke (2004). The hydrodynamics is solved for by the piecewise parabolic method on a computational grid co-expanding with the WD star and the flame is modeled using the level-set approach. In the latter, the effective burning velocity of the flame has to be provided. To this end v' is derived from a sub-grid scale model (Niemeyer & Hillebrandt 1995). The maximum value of s_l and expression (1) then gives the flame propagation velocity in the first stages of the explosion process.

Previous models stopped burning in the flame once the fuel density fell below 10^7 g cm^{-3}. To take into account the distributed regime we extend this burning and and derive s_t form Eq. (2) at densities below 10^7 g cm^{-3} down to 5×10^5 g cm^{-3}. The necessary values for the laminar flame speeds s_l and the flame thickness l_f at low fuel densities are provided by fits to the data given by Bell et al. (2004a,b). The turbulent velocity fluctuations v' on the computational grid scale l are derived from the sub-grid scale model.

The feasibility of modeling the distributed burning regime via the level set approach – though in a more elaborate way of formulating the flame speed – has been shown and validated against experiments by Duchamp de Langeneste & Pitsch (2002).

3. Results of numerical models

We compare two very simple two-dimensional SN Ia models calculated on a 1024 × 1024 cells grid: one ignoring the distributed burning (referred to as "old" model) and one taking into account this regime ("new" model). In both cases the flame was ignited in the *c3* shape of Reinecke et al. (2002) – a central ignition superposed by sinusoidal perturbations. The initial resolution of both models was 1.975 km. Due to the different energy releases the final resolutions were 96 km for the old and 100 km for the new model.

The total energy release of both models is plotted in Fig. 1, which also includes results of a resolution study of the new model showing convergence for more than $[512]^2$ cells. Contrary to more sophisticated three-dimensional models

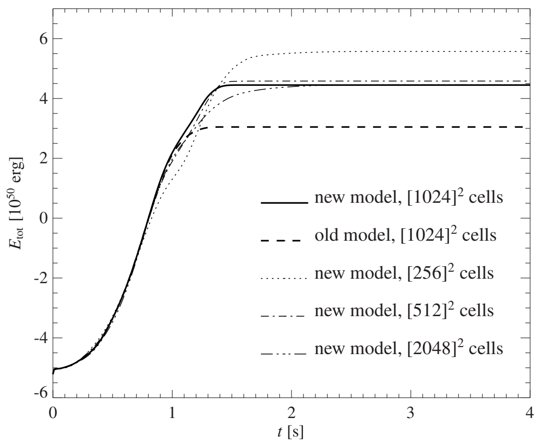

Fig. 1. Total energies released by the model ignoring distributed burning (dashed) and the model including the distributed burning phase (solid).

the simulations discussed here are based on very simple two-dimensional setups and they cannot be expected to reproduce the explosion strength of observed SNe Ia. Nevertheless, trends can be inferred even from these simple models. Clearly, the distributed regime contributes substantially to the energy generation. About 1.1 s after ignition the energy generation in the old model ceases because the fuel density reaches low values. In the new model, however, the burning proceeds for another 0.3 s, leading to a ~17% increase in the nuclear energy release. In this time range, the distributed burning produces large amounts of intermediate mass elements.

This is evident by comparing the snapshots of both simulations at t = 10 s shown in Fig. 2. They display the position of the zero level set as white curve (indicating the position of the flame front while burning is active and still providing an approximate separation between fuel and ashes later in the simulations; see Röpke 2004). The distribution of the species is color-coded. Our simplified description of the nuclear burning (for details see Reinecke et al. 2002) pools the intermediate mass nuclei in the representative element "Mg" and represents the iron group nuclei by "Ni". The evolution of the chemical composition in both models is compared in Fig. 3.

Implementing the distributed burning regime into our simulations we note various important aspects of the results. Due to the larger energy release the ejecta reach higher velocities and expand faster. This is obvious from Fig. 2. The transition from burning in the flamelet regime to the distributed regime takes place at low fuel densities (and is implemented in our models this way). Therefore the new and old models differ only in the production of intermediate mass nuclei. The amount of synthesized iron group elements is unaffected (cf. Fig. 3). Moreover, Fig. 2 shows that the distribution of "Ni"-rich regions in the new model resembles that in the old one. However, in the new model these regions are embedded in a layer of intermediate mass elements, which also shows a turbulent structure.

Taking into account the distributed regime, the flame naturally consumes more fuel. Therefore, the unburnt material in the central parts is strongly depleted. This is corroborated by Fig. 4 showing an angular average over the distribution of

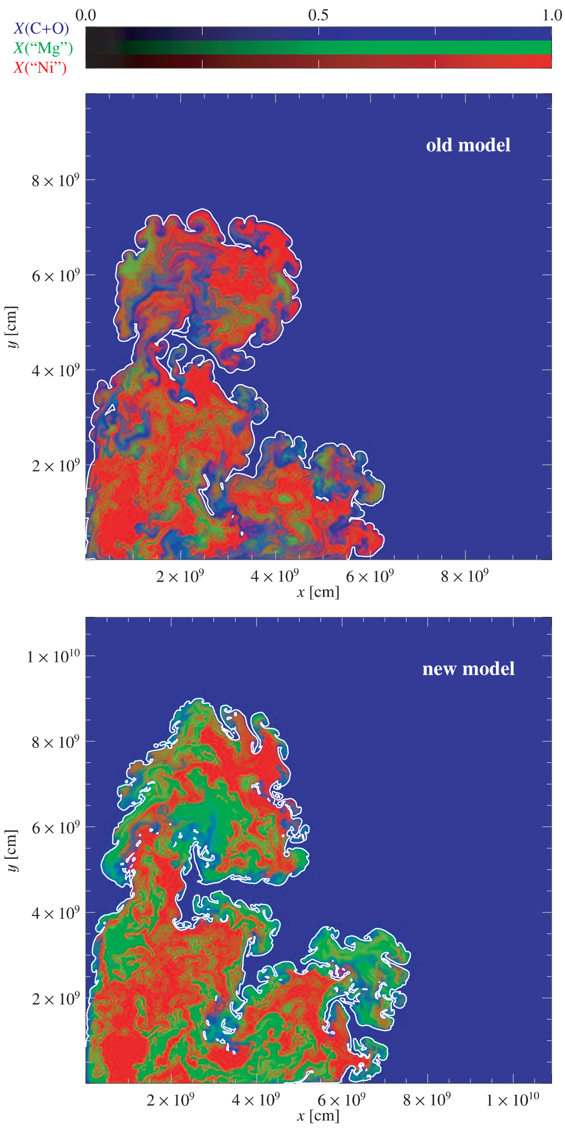

Fig. 2. Snapshots of the models with $[1024]^2$ cells at t = 10 s.

unburnt material in velocity space. No unburnt material is left at low velocities.

4. Conclusions

By reconsidering the distributed burning regime in global SN Ia models we have proposed a way to alleviate some of the problems that might arise when comparing the deflagration model with observations. By extending the burning of material to intermediate mass elements it acts against a possible underproduction of those in previous deflagration models. The additional energy released here helps to accelerate the ejecta to higher velocities. Near the center the burning consumes now most of the fuel leaving no low-velocity carbon and oxygen behind.

The newly implemented burning regime does, however, not affect the production of iron group elements since it applies to low fuel densities. It was claimed that to reproduce the observed spectra a layered composition of the ejecta would be needed (Gamezo et al. 2004). This is, however, not yet

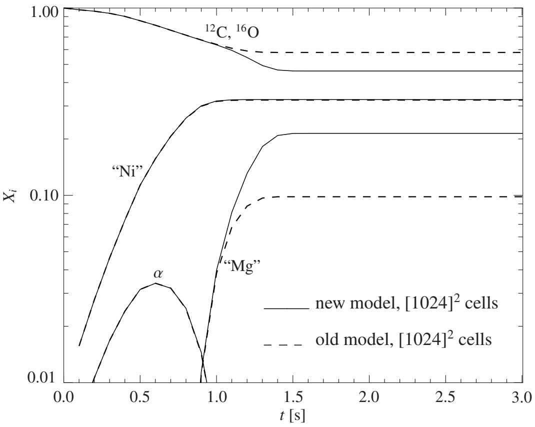

Fig. 3. Temporal evolution of the chemical composition in the models.

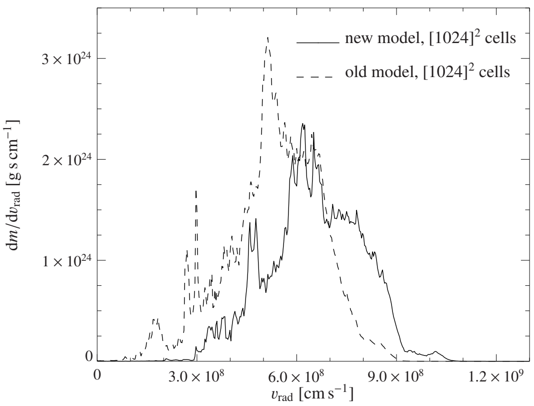

Fig. 4. Distribution of unburnt material in velocity space after 10 s.

confirmed since synthetic spectra derived with multi-dimensional radiation transport schemes are still lacking. Recent spectrapolarimetry data (Wang et al. 2004) seem to indicate that a turbulent structure of the intermediate mass elements is required. In any case, the deflagration phase does not remove the mixed distribution of species in the ejecta that was present already in the old models.

The results presented here have to be considered preliminary for several reasons. Our simulations are performed only in two spatial dimensions. The turbulent structure develops differently in three-dimensional models resulting in an overall more vigorous explosion (e.g. Reinecke et al. 2002; Röpke 2004). An even more important issue is that the modeling of the distributed burning regime applied here is very crude and no more than a first-order approach. The value of transition density is an ad hoc choice. Moreover, Eq. (2) is in a strict sense a limiting case for $l_{\rm f} \gg l$, which is certainly not yet met at the presumed transition density. Therefore our description by simply switching from the flamelet regime to the distributed regime will overestimate the effects.

However, the presented study points to the trends that can be expected. The distributed burning regime has to be implemented into global SN Ia simulations to complete the deflagration model. Important consequences for the energetics and the derived synthetic spectra may result from this extension, although the effects may not be as large as in the crude first models presented here. Whether they suffice to remove the current problems of the deflagration scenario will be investigated in forthcoming studies. For instance, a more general description of the turbulent flame propagation velocity (featuring a smooth transition between both burning regimes; see Peters 2000) will be tested and also applied to three-dimensional models.

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