Stochastic Gravitational Wave Background from Cosmological Supernovae

Alessandra Buonanno[a,b], Günter Sigl[a,b], Georg G. Raffelt[c], Hans-Thomas Janka[d], Ewald Müller[d]
[a] *GReCO, Institut d'Astrophysique de Paris, C.N.R.S., 98 bis boulevard Arago, F-75014 Paris, France*
[b] *Fédération de Recherche Astroparticule et Cosmologie, Université Paris 7, 2 place Jussieu, 75251 Paris Cedex 05, France*
[c] *Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany*
[d] *Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85741 Garching, Germany*

Based on new developments in the understanding of supernovae as gravitational-wave (GW) sources, we estimate the GW background from all cosmic supernovae. While our estimate is uncertain within a few orders of magnitude, this background may become detectable by second-generation space-based interferometers such as the proposed Big-Bang Observatory. At frequencies below about 1 Hz this GW background is stochastic. Down to about 0.01 Hz it could well be of comparable magnitude to the one expected from standard inflationary models, especially in the presence of an early population of massive stars.

PACS numbers: 04.30.Db, 04.80.Nn, 98.70.Sa, 98.70.Rz

Introduction.—Core-collapse supernova (SN) explosions are among the most violent astrophysical phenomena. The total energy of about 3×10^{53} erg is primarily released in a neutrino burst lasting for a few seconds. About 1% goes into the actual explosion while only a fraction of about 10^{-4} is emitted in visible light, yet a SN can outshine its host galaxy. In addition, SNe are expected to be strong gravitational-wave (GW) sources. About 1 SN per second takes place in the visible universe, but the neutrino burst has been observed only once from SN 1987A, the closest SN in modern history. GWs are even more difficult to detect and have never been measured from any source. On the other hand, the next galactic SN likely will be observed with high statistics in several large neutrino detectors as well as with the existing GW antennas. For the first time there is a realistic opportunity to observe such a cataclysmic event in all forms of radiation.

However, galactic SNe are rare so that the next SN neutrinos to be observed could well be the diffuse neutrino background from all cosmic SNe. The limit from Super-Kamiokande [1] already touches the upper range of theoretical predictions [2, 3]. The proposed gadolinium upgrade [4] or a corresponding future megatonne detector may well produce a positive detection. Therefore, it is natural to ask the analogous question whether the diffuse GW background from all past cosmic SNe will be important for current or future GW antennas that search for signals from individual astrophysical events or for the stochastic cosmic background that probably arises in the very early universe during the inflationary epoch.

The existing literature focuses on GWs from the SN bounce signal in the kHz range that is relevant for bar detectors or ground-based laser interferometers such as LIGO, VIRGO, GEO and TAMA [5]. However, recent studies of SNe as GW sources indicate that a much stronger signal may be expected from the large-scale convective overturn that develops in the delayed explosion scenario during the epoch of shock-wave stagnation that may last for several 100 ms before the actual explosion [6, 7]. Moreover, the relevant frequencies reach below 1 Hz where they may be relevant for space-based detectors, although we find that a first-generation instrument such as LISA is not sufficient. However, the GW background from SNe may become relevant for a second-generation detector such as the proposed Big-Bang Observatory (BBO) [18].

Estimating the Gravitational Wave Background.—We assume all SNe are identical GW sources defined by the Fourier transform $\tilde{h}(f) \equiv \int_{-\infty}^{+\infty} dt\, e^{-i2\pi f t} h(t)$ of the dimensionless strain amplitude $h(t)$ which is proportional to deviations from spherical symmetry of the energy-momentum tensor. (We always use natural units with $\hbar = c = 1$.) In general, $h(t)$ is obtained from numerical simulations. However, the contribution of asymmetric neutrino emission is explicitly [8, 9]

$$h(t) = \frac{2G_{\rm N}}{D} \int_{-\infty}^{t-D} dt'\, L_\nu(t') q(t')\,, \tag{1}$$

where $G_{\rm N}$ is Newton's constant, D the distance to our "standard SN," and $L_\nu(t)$ the neutrino luminosity. Further, $q(t) \leq 1$ is an asymmetry parameter defined as the angle-dependent neutrino luminosity folded over an angular function whose details are not of interest here.

Equation (1) shows that $h(t)$ converges to a constant value h_∞ for $t \gtrsim t_e$, with t_e the neutrino emission time scale of a few seconds. Therefore, the neutrino burst causes $h(t)$ to jump from zero to a non-vanishing constant value, an effect called the "burst with memory" [8, 10–13]. We can explicitly deduce the GW signal at frequencies $f \ll (2\pi t_e)^{-1}$ by applying the so-called zero-frequency limit [8, 10, 11, 14]. We write $2\pi f \tilde{h}(f) = -i \int_{-\infty}^{+\infty} dt\, e^{-i2\pi f t} \dot{h}(t) \;\approx\; -i(2G_{\rm N}/D) \int_{-\infty}^{+\infty} dt\, L_\nu(t) q(t)$, where in the second step we used Eq. (1). Assuming

$f \ll (2\pi t_e)^{-1}$ this implies

$$f|\tilde{h}(f)| \approx \frac{|h_\infty|}{2\pi} = \frac{G_\mathrm{N}}{\pi D}\langle q\rangle E_\nu \approx 2.6\times 10^{-19}\langle q\rangle \left(\frac{10\,\mathrm{kpc}}{D}\right)\left(\frac{E_\nu}{3\times 10^{53}\,\mathrm{erg}}\right), \quad (2)$$

where $E_\nu = \int_{-\infty}^{+\infty} dt'\, L_\nu(t')$ is the total emitted neutrino energy and $\langle q\rangle$ is a time average. Although the zero-frequency limit Eq. (2) is valid for $f \ll (2\pi t_e)^{-1}$, we will use it to evaluate the GW signal at $f \lesssim (2\pi t_\mathrm{sim})^{-1}$, where $t_\mathrm{sim} < t_e$ is the maximal simulation time (less than a second), and then we continuously extend $|\tilde{h}(f)|$ to lower frequencies. Current simulations do not cover the strain spectrum below fractions of a Hertz.

The cosmic star-formation rate and, as a consequence, the core-collapse SN rate is reasonably well known at redshifts $z \lesssim 5$ [15], but becomes rather uncertain at larger z. The SN rate can be parametrized by

$$R_\mathrm{SN}(z) = R^0_\mathrm{SN} \times \begin{cases} (1+z)^\beta & \text{for } z < 1 \\ 2^{\beta-\alpha}(1+z)^\alpha & \text{for } 1 \le z \le 20 \end{cases}. \quad (3)$$

If not otherwise stated, we will use $\beta = 2.7$, $\alpha = 0$, and the present-day rate $R^0_\mathrm{SN} = 2\times 10^{-4}\,\mathrm{Mpc}^{-3}\,\mathrm{yr}^{-1}$. These values are consistent with the Super-K limits on the diffuse neutrino background [1]. The parameter α is much less constrained than β and R^0_SN because α influences the rate only for $z > 1$ where the neutrinos are redshifted below the Super-K threshold. This situation may change if low-threshold detectors such as the gadolinium upgrade of Super-K [4] become operative.

The energy density in GWs at frequency f per logarithmic frequency interval in units of the cosmic critical density $\rho_\mathrm{c} = 3H_0^2/(8\pi G_\mathrm{N})$ can be written as [16]

$$\Omega_\mathrm{gw}(f) = \frac{16\pi^2 D^2}{15 G_\mathrm{N}\rho_\mathrm{c}} \int_0^\infty dz\, \frac{R_\mathrm{SN}(z)}{1+z}\left|\frac{dt}{dz}\right| f_z^3 \left|\tilde{h}(f_z)\right|^2, \quad (4)$$

where $f_z \equiv (1+z)f$ and $\tilde{h}(f)$ is the strain spectrum of our standard SN at distance D. Cosmology enters with $|dt/dz| = [(1+z)H(z)]^{-1}$ and, for a flat geometry,

$$H(z) = H_0\left[\Omega_\mathrm{M}(1+z)^3 + \Omega_\Lambda\right]^{1/2}. \quad (5)$$

We will use the parameters $\Omega_\mathrm{M} = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = h_0\, 100\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ with $h_0 = 0.72$.

For $\tilde{h}(f)$ we first use the recent 3-dimensional asymmetric simulation shown in Fig. 9 of Ref. [7]. We show the resulting GW background in Fig. 1. Shifting these curves downward by a factor of 100 is probably more realistic [7], in which case the total energy released in GWs is about $8\times 10^{-11}\,M_\odot$. We also show sensitivities of LISA, BBO [18], LIGO (or EGO) correlated third generation, and the ultimate DECIGO [19], which would be the quantum limited space-based interferometer with mirror masses of 100 kg. For some frequencies, even our lower estimate for the GW background from SNe is comparable to the most optimistic GW background from slow-roll inflation (horizontal lines).

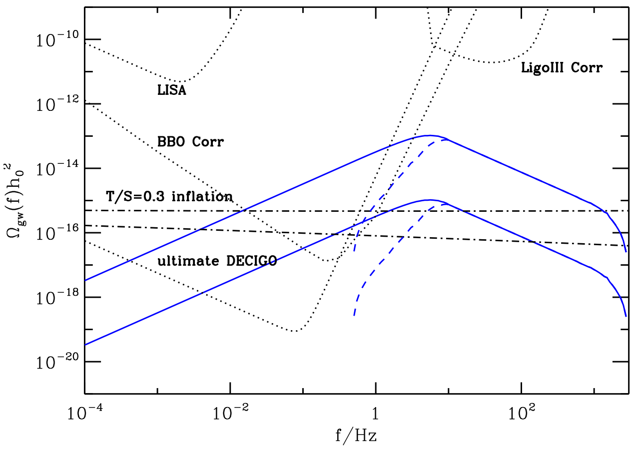

FIG. 1: Spectrum of GW background from Eq. (4) for a simulation from Ref. [7]. Dashed line: the source spectrum $\tilde{h}(f)$ is taken to vanish for frequencies not covered by the simulation (below 10 Hz in the rest frame). Solid line: the spectrum is continuously extended with the low-f tail Eq. (2). The lower set of curves is shifted downwards by a factor 100. Horizontal lines: GW stochastic spectrum produced during slow-roll inflation and evaluated from Eq. (6) of Ref. [17]. We assume $T/S = 0.3$ for the ratio of the tensorial and scalar contribution to the CMBR anisotropy and consider two values $\pm 10^{-3}$ of the running of the tensorial power-law index. Dotted lines: sensitivities of the indicated detectors.

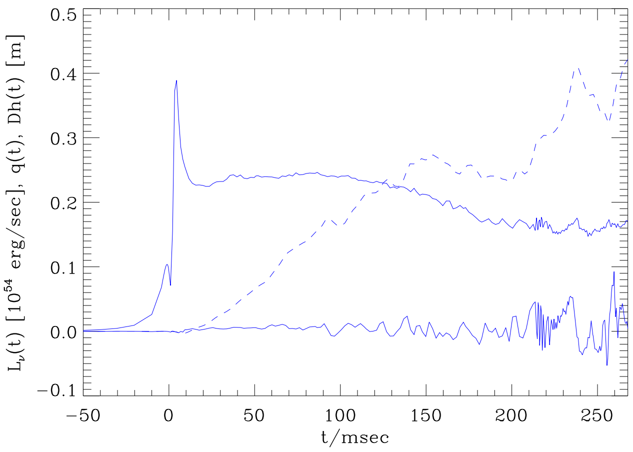

FIG. 2: Neutrino luminosity $L_\nu(t)$ (upper solid line), anisotropy $q(t)$ (lower solid line), and GW strain $h(t)$ times distance (dashed line), see Eq. (1), for model s15r of Ref. [6], as functions of time after bounce.

Next, we use the three models s15r, s11nr180, and pns180 from recent 2-dimensional core-collapse simulations based on a more detailed implementation of neutrino transport [6]. We focus here specifically on model s15r which gives more optimistic GW signals. Figures 2

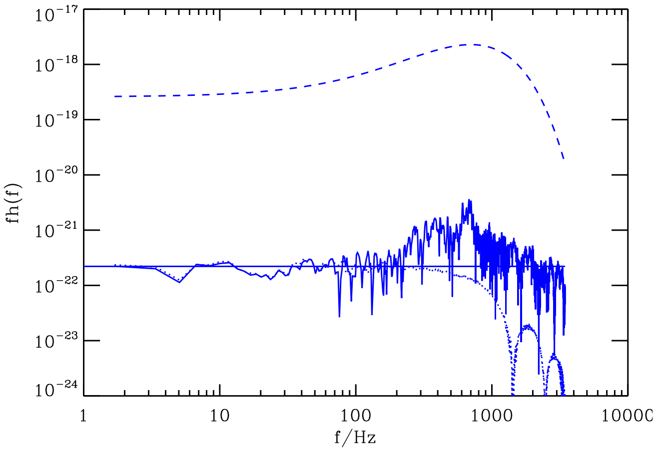

FIG. 3: GW source spectra: Solid and dotted ragged lines are total and neutrino contribution, respectively, for model s15r of Ref. [6] at distance $D = 10$ kpc. The straight solid line is the low-f tail Eq. (2) with h_∞ from Eq. (1) using $q(t)$ and $L_\nu(t)$ of Fig. 2, leading to $f|\tilde{h}(f)| \simeq 2.21\times10^{-22}$. The dashed line is the schematic spectrum for PopIII stars of Eq. (6).

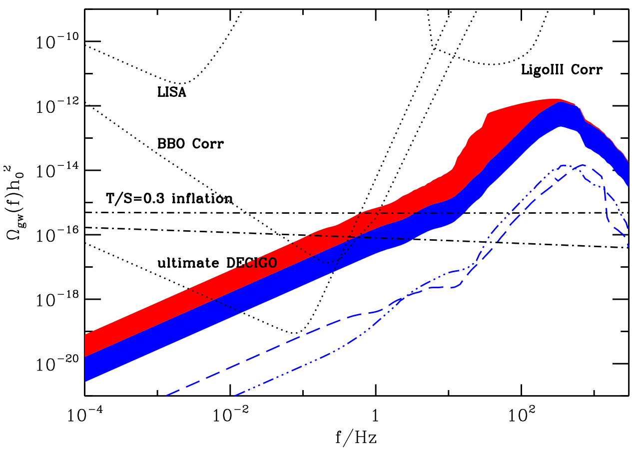

FIG. 4: GW background for the models s15r (colored bands), s11nr180 (dash-dot-dot), and pns180 (long-dash) from Ref. [6]. The source spectra have been continuously extended using the zero-f tail Eq. (2) for $f \lesssim 1$ Hz. The blue band for model s15r reflects the plausible range 6–36 of the enhancement factor to correct for the limited time of the simulation. For the other models the lower enhancement factor of 6 was assumed. We always use $\alpha = 0$ in Eq. (3), except for the red band which shows the difference between $\alpha = 0$ and $\alpha = 2$ for model s15r.

and 3 show neutrino luminosity $L_\nu(t)$, anisotropy $q(t)$, and GW strain $h(t)$ and GW source spectra $|\tilde{h}(f)|$, respectively. Figure 3 shows that anisotropic neutrino emission dominates at low frequencies. Note that the simulation stops at about 250 msec after the bounce when only about 0.5×10^{53} erg has been emitted in neutrinos, i.e. about 1/6 of the total, and when $\int dt\, L_\nu(t)q(t) = 2.55\times10^{50}$ erg. The average anisotropy over this time period is $\langle q\rangle \approx 0.45\%$, but $|q(t)|$ tends to increase (Fig. 2). If $q(t)$ were constant, according to Eq. (1), $h(t)$ would increase proportional to the emitted neutrino energy, whereas for a fluctuating $q(t)$, $h(t)$ increases roughly as the square root of that energy. Therefore, we expect enhancement factors in the strain between $\sim\sqrt{6}$ and ~ 6. In reality, $q(t)$ will first increase beyond the value at the end of the simulation, and then decrease, leading to a similar plausible range of enhancement factors.

The resulting GW backgrounds are shown in Fig. 4. For model s15r the blue band corresponds to the uncertainty of the enhancement factor discussed above, and the red band is a rough estimate of the uncertainty due to redshift evolution. For 100 Hz $\lesssim f \lesssim$ 1 kHz the signal is dominated by the convective motion in the neutrino-heated postshock layer, whereas at lower frequencies it is mostly due to asymmetric neutrino emission. This scenario corresponds to a total energy release in GWs of $\simeq 1.8\times10^{-8}\,M_\odot$ for the most conservative enhancement factor. Note that, although the zero-frequency signal is lower, the gravitational energy release is higher than in the case of Fig. 1 because it is dominated by the high-frequency part.

Equations (2) and (4) show that for our standard redshift evolution and $E_\nu \approx 3\times10^{53}$ erg liberated in neutrinos, an average anisotropy $\langle q\rangle \simeq 6\%$ would completely mask the inflation background down to 10 mHz.

For $\alpha \lesssim 2$ in Eq. (3), the integral Eq. (4) converges rather quickly. Cutting the integral at $z = 5$ instead of at $z = 20$, for example, lowers the predictions by less than a factor 2. In contrast, the poorly constrained redshift evolution for $z > 1$ causes more significant uncertainties (see, e.g. Fig. 4).

The core collapse of the hypothetical population III (PopIII) generation of first stars could be much more efficient emitters of GWs than today's SN populations [20]. Lacking more detailed numerical models, we assume a schematic standard case of mass $300\,M_\odot$, with a GW spectrum at $D = 10$ kpc of

$$f|\tilde{h}(f)| = 2.6\times10^{-19}\left(1+\frac{f}{0.2\ \mathrm{kHz}}\right)^3 e^{-f/0.3\ \mathrm{kHz}}\,, \quad (6)$$

shown in Fig. 3. The normalization implies $E_{\rm gw} = [16\pi^2D^2/(15G_{\rm N})]\int df\,|f\tilde{h}(f)|^2 \approx 2\times10^{-3}M_\odot$ for the total energy emitted in GWs as in Ref. [20]. Furthermore, according to Eq. (2), for $E_\nu \approx 10^{55}$ erg emitted in neutrinos [20], the average anisotropy corresponds to $\langle q\rangle \approx 3\%$. Finally, we assume that the PopIII rate is concentrated around $z = 15$. The resulting GW background is shown in Fig. 5 as a band delimited by two extreme assumptions about the total cosmic PopIII core-collapse rate observed from Earth, i.e. $R_{\rm III} = 0.2\ {\rm s}^{-1}$ [15] (upper edge of band) and $4.4\times10^{-4}\ {\rm s}^{-1}$ [22] (lower edge). Note that a PopIII core-collapse rate corresponds to an approximate

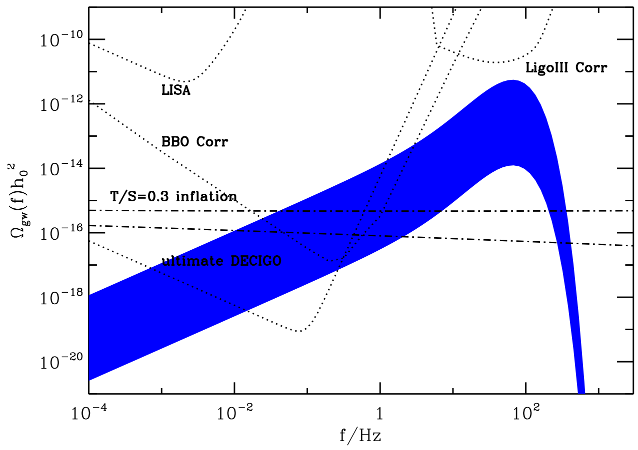

FIG. 5: GW background for our standard PopIII core collapse case and event rates $4.4 \times 10^{-4}\ \mathrm{s}^{-1} < R_{\rm III} < 0.2\ \mathrm{s}^{-1}$.

fraction $10^{-3}(R_{\rm III}/0.2\,\mathrm{s}^{-1})$ of baryons going into PopIII stars. Whereas neutrinos from PopIII stars are redshifted to energies probably too low to be detected [21], the GW background for 10 mHz $\lesssim f \lesssim$ 1 kHz could be dominated by these objects.

Stochasticity.—Finally, we estimate the duty cycle of the background at a given frequency f. It is defined as the product of the rate of events and the time scale of interest, f^{-1}. The SN rate as seen from Earth is

$$\int_0^\infty dz\,\frac{R_{\rm SN}(z)}{1+z}\frac{dV}{dz} = \int_0^\infty dz\,R_{\rm SN}(z)\frac{4\pi r^2(z)}{(1+z)H(z)}\,, \quad (7)$$

where dV/dz is the fractional volume element, the Hubble rate at redshift z is given by Eq. (5), and $r(z)$ is the comoving coordinate, $dr = (1+z)dt$.

The duty cycle is roughly given by dividing the total cosmic SN rate Eq. (7) by the frequency. A probably more realistic estimate is obtained by folding the rate integral over redshift with the redshift dependent contribution to the GW signal given by the integrand in Eq. (4). For model s15r of Ref. [6] and the evolution discussed around Eq. (3) these two estimates give approximately $4\,(f/\mathrm{Hz})^{-1}$ and $33\,(f/\mathrm{Hz})^{-1}$, respectively.

The signal from ordinary SNe becomes stochastic for $f \lesssim 1$ Hz, in agreement with a cosmic SN rate of about 1 s^{-1}. In contrast, the backgrounds in the 10^2–10^3 Hz range are not stochastic, consistent with Ref. [5] where only the bounce signal was considered that lasts about 10^{-3} s. At these frequencies the duty factor is $\lesssim 1\%$. The putative PopIII background becomes stochastic only for $f \lesssim R_{\rm III}$, i.e. probably for $f \lesssim 0.1$ Hz.

Conclusions.—The GW background from cosmological core-collapse SNe at frequencies around 1 Hz is stochastic. Its power is uncertain by a few orders of magnitude, mostly due to uncertainties of SNe as GW sources. However, this background can be detected by second-generation space-based detectors. It can be comparable to the most optimistic signal expected from standard inflationary models from which it is difficult to disentangle. A PopIII component could give a particularly strong contribution for the upper range of the very uncertain rate predictions in the literature. Our results can have important consequences for designing space-based detectors of second generation with sensitivity below 1 Hz. Numerical simulations and stronger constraints or detections of the corresponding neutrino background will lead to an improved prediction of the GW background.

Acknowledgements.—We thank Tom Abel, Luc Blanchet, Frédéric Daigne, Daniel Holz, and Scott Hughes for informative discussions. In Garching and Munich, this work was partly supported by the Deutsche Forschungsgemeinschaft under grants SFB-375 and SFB-Transregio 7.

[1] M. Malek *et al.* [Super-Kamiokande Collab.], Phys. Rev. Lett. **90**, 061101 (2003) [hep-ex/0209028].
[2] L. E. Strigari, M. Kaplinghat, G. Steigman and T. P. Walker, JCAP **0403**, 007 (2004) [astro-ph/0312346].
[3] S. Ando and K. Sato, New J. Phys., in press (2004) [astro-ph/0410061].
[4] J. F. Beacom and M. R. Vagins, Phys. Rev. Lett. **93**, 171101 (2004) [hep-ph/0309300].
[5] R. Schneider, A. Ferrara, B. Ciardi, V. Ferrari and S. Matarrese, Mon. Not. Roy. Astron. Soc. **317**, 385 (2000) [astro-ph/9909419].
[6] E. Müller, M. Rampp, R. Buras, H. T. Janka and D. H. Shoemaker, Astrophys. J. **603**, 221 (2004) [astro-ph/0309833].
[7] C. L. Fryer, D. E. Holz and S. A. Hughes, [astro-ph/0403188].
[8] R. Epstein, Astrophys. J. **223**, 1037 (1978).
[9] E. Müller and H. T. Janka, Astron. Astrophys. **317**, 140 (1997).
[10] M. S. Turner, Nature **274**, 565 (1978).
[11] L. Smarr, Phys. Rev. D **15**, 2069 (1977).
[12] V. B. Braginsky and K. T. Thorne, Nature **327**, 123 (1987).
[13] A. Burrows and J. Hayes, Phys. Rev. Lett. **76** (1996) 352 [astro-ph/9511106].
[14] R.J. Bontz and R. H. Price, Astrophys. J. **228**, 560 (1979).
[15] F. Daigne, K. A. Olive, E. Vangioni-Flam, J. Silk and J. Audouze, [astro-ph/0405355].
[16] E. S. Phinney, [astro-ph/0108028].
[17] M. S. Turner, Phys. Rev. D **55**, 435 (1997) [astro-ph/9607066].
[18] E. S. Phinney, private communication.
[19] N. Seto, S. Kawamura and T. Nakamura, Phys. Rev. Lett. **87**, 221103 (2001) [astro-ph/0108011].
[20] C. L. Fryer, S. E. Woosley and A. Heger, Astrophys. J. **550**, 372 (2001) [astro-ph/0007176].
[21] F. Iocco, G. Mangano, G. Miele, G. G. Raffelt and P. D. Serpico, astro-ph/0411545.
[22] J. H. Wise and T. Abel, astro-ph/0411558.