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J.M. Hameury and J.P. Lasota

Irradiation-driven Mass Transfer Cycles in Compact Binaries

Andreas Büning and Hans Ritter

Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, D-85741 Garching bei München

Abstract. We elaborate on the analytical model of Ritter, Zhang, & Kolb (2000) which describes the basic physics of irradiation-driven mass transfer cycles in semi-detached compact binary systems. In particular, we take into account a contribution to the thermal relaxation of the donor star which is unrelated to irradiation and which was neglected in previous studies. We present results of simulations of the evolution of compact binaries undergoing mass transfer cycles, in particular also of systems with a nuclear evolved donor star. These computations have been carried out with a stellar evolution code which computes mass transfer implicitly and models irradiation of the donor star in a point source approximation, thereby allowing for much more realistic simulations than were hitherto possible. We find that low-mass X-ray binaries (LMXBs) and cataclysmic variables (CVs) with orbital periods $\lesssim$ 6hr can undergo mass transfer cycles only for low angular momentum loss rates. CVs containing a giant donor or one near the terminal age main sequence are more stable than previously thought, but can possibly also undergo mass transfer cycles.

1. Introduction

The possible importance of irradiating the donor star of a semi-detached compact binary by accretion luminosity for its long-term evolution has first been pointed out by Podsiadlowski (1991). Subsequently, the stability of mass transfer with irradiation feedback has been studied in some detail by King et al. (1996, 1997, hereafter KFKR96 and KFKR97), and by Ritter, Zhang, & Kolb (2000, hereafter RZK00). In KFKR96 and KFKR97 it was also shown that mass transfer in cataclysmic variables (CVs) and low-mass X-ray binaries (LMXBs) can become unstable against irradiation feedback, and that in case of instability mass transfer proceeds in cycles in which episodes of irradiation-driven mass transfer alternate with low-states during which mass transfer is essentially shut off. Evolutionary calculations of mass transfer cycles which were based on a homology model for the donor and a simplified irradiation model, have been presented in KFKR97 and RZK00. Here we elaborate on the work done in previous studies in two respects: First, we have developed a binary evolution code which allows us to simulate the evolution of compact binaries with mass transfer cycles more realistically than was possible in previous studies, and second, we elaborate on the stability analysis of KFKR96, KFKR97 and RZK00, thereby taking into account a contribution to the thermal relaxation of the donor star which is unrelated to irradiation and which has been neglected in previous studies.

2. Input Physics and Model Assumptions for the Numerical Calculations

In the following we are briefly listing the main model assumptions and details of the input physics adopted for our numerical calculations. For details the reader is referred to Büning & Ritter (2004), hereafter BR04.

2.1. The Stellar Evolution Code

Basically, we use the 1D stellar evolution code described by Schlattl, Weiss, & Ludwig (1997) and Schlattl (1999). For calculating binary evolution, in particular for determining the mass transfer rate essentially free of numerical noise, considerable refinements in the calculation of the equation of state and the opacities were necessary. A detailed description of what has been done and how, is given in BR04.

2.2. Computing Mass Transfer

The mass loss rate from the donor star $-\dot{M}_2$ is computed following Ritter (1988), i.e. from an explicit relation of the form

$$-\dot{M}_2 = \dot{M}_0 \; e^{-\frac{R_{\mathrm{R},2}-R_2}{H_{\mathrm{P}}}} \tag{1}$$

where R_2 and $R_{\mathrm{R},2}$ are respectively the radius of the donor star and the corresponding critical Roche radius, H_{P} the effective photospheric pressure scale height of the donor, and $\dot{M}_0 \geq 0$ a slowly varying function of the donor's mass, radius, and photospheric parameters. For our numerical computations Eq. (1) is formulated as an outer boundary condition for the donor star and is thus solved implicitly with the stellar structure equations. The numerical setup is similar to what has been used by Benvenuto & de Vito (2003) and is described in detail in Büning (2003, hereafter B03).

2.3. Irradiation Physics

If the unperturbed donor star has a deep outer convective envelope, i.e. if it is a cool star, irradiation, if not too strong, can be treated as a local problem (for a detailed justification see RZK00). Therefore, the problem reduces to specifying the intrinsic flux F_{int}, i.e. the true energy loss of the donor per unit surface area and unit time as a function of the component of the irradiating flux perpendicular to the stellar surface F_{irr}. For our numerical computations we use for $F_{\mathrm{int}}(F_{\mathrm{irr}})$ results tabulated by Hameury & Ritter (1997) and additional data kindly computed by Hameury (private communication) at our request.

2.4. Irradiation Model

The irradiation model links the momentary accretion luminosity L_{accr} (liberated by accretion onto the compact star) to the irradiating flux seen by a surface element of the irradiated donor star. For our numerical computations we adopt the so-called point source model which assumes that the spherical donor (of radius $R_2 \approx R_{\mathrm{R},2}$) is illuminated by a point source with luminosity L_{accr} at the position of the accretor, i.e. at the orbital distance a. Because the accretion luminosity is not necessarily radiated isotropically or staedily from the accretor, and because only a part of the irradiating flux is absorbed below the donor's photosphere, the link between $F_{\rm irr}$ and $L_{\rm accr}$

$$F_{\rm irr} = \alpha \frac{L_{\rm accr}}{4\pi a^2} h(\theta) =: \langle F_{\rm irr} \rangle \; h(\theta) \tag{2}$$

involves a dimensionless and a priori unknown efficiency parameter $\alpha < 1$ which is the main unknown quantity in the irradiation problem. In the framework of the point source model the flux $F_{\rm irr}$ depends also on the substellar latitude θ of the irradiated surface element. This is taken into account by the function $h(\theta)$ in Eq. (2) (for details see e.g. RZK00 or BR04).

3. Stability Analysis

The stability of mass transfer in the presence of irradiation has been studied previously in some detail by KFKR96, KFKR97 and RZK00. Because some of the numerical results which we have obtained with the above-described stellar evolution program were at variance with results of these stability analyses, we have carried out a more refined stability analysis and found that an additional term in the stability criterion which becomes particularly important for giant donors had been ignored in ealier considerations. Space limitations do not allow us to repeat our stability analysis in detail. For this we refer the reader to BR04. Rather we wish to give here only the main result. The criterion for the stability of mass transfer can be written as follows:

$$\frac{ds}{d\ln -\dot{M}_2} < \frac{\tau_{\rm ce}}{\tau'_{\rm d}} + \frac{H_{\rm P}}{R_2}\,\delta \tag{3}$$

Here s is the effective fraction of the donor surface through which energy outflow from its interior is totally blocked by irradiation (for a detailed definition of s see BR04), $\tau_{\rm ce}$ is the thermal time scale of the donor's convective envelope, $\tau'_{\rm d}$ the time scale on which mass transfer is driven, i.e.

$$\frac{1}{\tau'_{\rm d}} = \left(\frac{\partial \ln R_2}{\partial t}\right)_{\rm nuc} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{\rm th} - 2\,\frac{\partial \ln J}{\partial t} \tag{4}$$

is the sum of the driving terms resulting from nuclear evolution and thermal relaxation of the secondary, and from systemic loss of orbital angular momentum J. In addition, if homology is used for describing the structure of the secondary, δ in (3) can be approximated as

$$\delta = 4\,(1-s)\left(\frac{R_2}{R_{2,\rm e}}\right)^3 + (n+1)\left(\frac{R_2}{R_{2,\rm e}}\right)^{-(n+2)} \sim (n+5), \tag{5}$$

where $R_{2,\rm e}$ is the thermal equilibrium radius of the irradiated donor (under stationary irradiation) and $n = (\partial \ln \varepsilon_{\rm nuc} / \partial \ln T)$ is the temperature exponent of the nuclear energy generation rate $\varepsilon_{\rm nuc}$. In the case of low-mass main sequence donors which burn hydrogen via the pp-chain $n \approx 5$. In the case of giant donors one has to set $n = -3$ for self-consistency.

The new result is the last term in Eq. (3). It derives from the previously neglected fact that the thermal relaxation term, i.e. $(\partial R_2/\partial t)_{\rm th}$, not only has a non-vanishing derivative with respect to $\Delta R = R_2 - R_{\rm R,2}$ but also with respect to $\Delta R_{\rm e} = R_2 - R_{2,\rm e}$. Since the last term of (3) is always positive it stabilizes mass transfer. Its consequences are first that mass transfer with irradiation feedback is always stable for very small driving rates, i.e. very small mass transfer rates, and second that it is important for giant donors where $H_{\rm P}/R_2$ is much larger than for main sequence stars. Therefore, contrary to earlier results obtained by KFKR97, mass transfer from giant donors in CVs is much more stable than previously thought.

4. Results

Here we are mainly interested in answering the question whether irradiation-driven mass transfer cycles can occur in CVs or LMXBs. A necessary condition for such mass transfer cycles to be possible is violation of the stability criterion (3). As can be seen from (3), this is possible only if both terms on the right-hand side of (3) are sufficiently small. It follows then immediately that mass transfer cycles are most likely to occur if the donor star has a relatively shallow convective envelope (i.e. a small $\tau_{\rm ce}$), and/or mass transfer is driven on a long timescale $\tau'_{\rm d}$, and if the relative photospheric scale height, i.e. $H_{\rm P}/R_2$, is small. On the other hand, the left-hand side of (3) is itself also a function of the mass transfer rate (via the irradiating flux) which vanishes for very small and very large mass transfer rates and which attains a maximum value of ~ 0.1 for irradiating fluxes $1 \lesssim \langle F_{\rm irr}\rangle / F_0 \lesssim 10$, where F_0 is the unperturbed flux of the donor. Therefore, for a given donor star (i.e. given values of $\tau_{\rm ce}$ and $H_{\rm P}/R_2$) there is always only a restricted range of mass transfer rates, which also depends on the efficiency factor α defined in (2), for which mass transfer cycles can occur. Because of (2) the left-hand side of (3) vanishes for both very small and very large values of α. For physical reasons we can exclude values $\alpha \gtrsim 1$. But apart from this upper limit α has to be treated as a free parameter.

From what has just been explained it has probably become clear that irradiation–driven mass transfer cycles can occur only under special conditions. Given the binary parameters, the rate of systemic angular momentum loss $-\dot{J}$, and an adopted value of α one can explore the range of instability by using (3) and an analytical approximation for δ, e.g. the one given in (5). In this way one can narrow down the parameter space of interest before going to extensive (and expensive) numerical calculations. For more details on this point we refer the reader to BR04.

As an example of such a numerical calculation we show in Fig. 1 the variation of the mass transfer rate (in $M_\odot {\rm yr}^{-1}$) as a function of orbital period P (in hours) of a CV undergoing mass transfer cycles (full line). For this calculation we have assumed the following parameters: The systemic loss of orbital angular momentum is due to gravitational radiation only, i.e. $\dot{J} = \dot{J}_{\rm GR}$. The donor star is a standard Pop. I low-mass main sequence star with an initial mass $M_2 = 0.5 M_\odot$, an age of 10^{10}yr and a central hydrogen mass fraction of $X_{\rm c} \approx 0.62$. In this case $H_{\rm P}/R_2 \approx 10^{-4}$. The primary is a white dwarf with a mass of $M_1 = 0.8 M_\odot$ and a radius of $R_1 = 0.010 R_\odot$. The transferred mass is

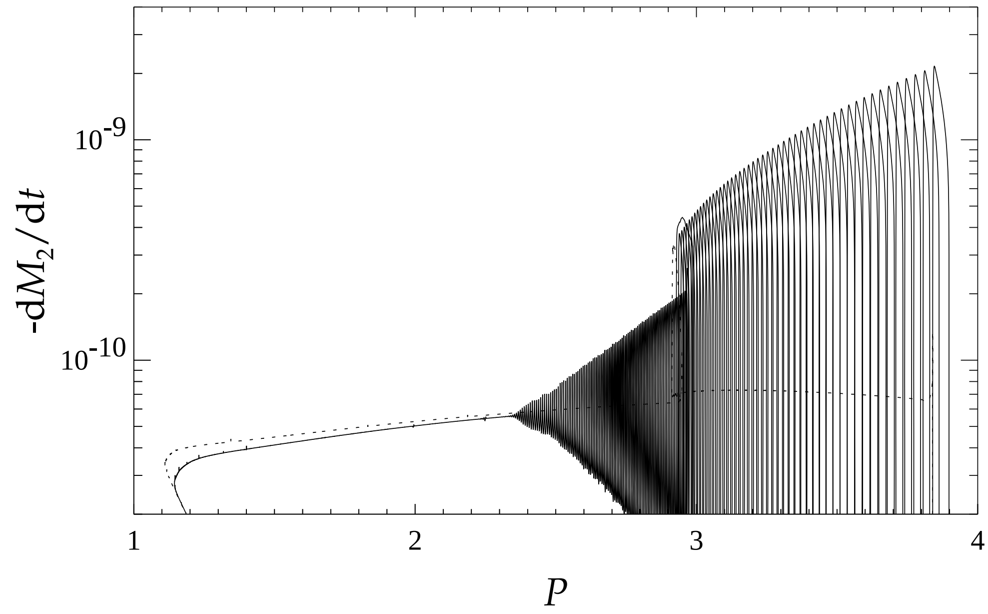

Figure 1. Mass transfer rate (in $M_\odot \mathrm{yr}^{-1}$) of a CV undergoing irradiation-driven mass transfer cycles (full line) as a function of its orbital period P (in hours). The corresponding evolution without irradiation feedback is shown as a dashed line. For the parameters underlying these calculations see text.

assumed to eventually leave the binary system (e.g. via nova explosions) with a dimensionless orbital angular momentum $\partial \ln J_{\mathrm{orb}} / \partial \ln(M_1 + M_2) = M_2/M_1$. The irradiation efficiency parameter is $\alpha = 0.3$. The dotted line shows the corresponding evolution without irradiation feedback.

What this example shows, is first how numerically accurate the stellar evolution code described in Sect. 2 can follow such an evolution. Second, we see that irradiation-driven mass transfer cycles can occur in CVs for not unrealistic values of α provided that the braking rate is small. In a corresponding evolution in which the absolute value of the angular momentum loss rate is much higher, e.g. according to the Verbunt & Zwaan (1981) prescription, no mass transfer cycles would occur. We note that the step in the amplitude of the mass transfer cycles at an orbital period of ~ 3 hr is due to the secondary becoming fully convective. Third, because the thermal time scale of the fully convective donor star increases with decreasing mass, mass transfer eventually becomes stable at $P \approx 2.5$ hr.

Based on numerous numerical simulations (detailed in B03 or BR04) and on the more general considerations outlined above we can summarize our results as follows:

1. In agreement with earlier results we find that CVs which contain a main sequence donor and in which the driving rate above the period gap is as high as required for explaining the period gap in the framework of the

model of disrupted magnetic braking (see e.g. Spruit & Ritter 1983) are stable against irradiation feedback except for the most massive donor stars $M_2 \sim 1 M_\odot$. On the other hand, mass transfer cycles can occur in short-period CVs if the driving rate is small, i.e. no larger than a few times the gravitational braking rate.

2. CVs containing a donor star which is near the terminal age main sequence turn out to be more stable than has been anticipated based on simple homology arguments.

3. CVs containing an extended giant donor with nuclear timescale–driven mass transfer are less likely to undergo mass transfer cycles than has been anticipated based on the results of KFKR97. The main reason for this discrepancy is the second term on the right-hand side of the stability criterion (3), i.e. the relatively large value of H_P/R_2 associated with such stars. If mass transfer cycles do occur they are characterized by comparatively very short phases with high, irradiation-driven mass trasnsfer rates which are followed by extended periods without mass transfer during which the system is essentially detached and reattachment is reached only on the nuclear time scale of the giant.

4. Adopting the point source irradiation model which takes into account irradiation of surface elements near the terminator of the donor we find that possibly also LMXBs can undergo mass transfer cycles. Regarding the braking rate which is necessary to drive cycles, basically the same restrictions apply as for short–period CVs. We confirm also that LMXBs containing a giant donor can undergo cyles.

5. Mass transfer cycles in CVs do occur only if $0.1 \lesssim \alpha \lesssim 1$ whereas in the case of LMXBs cycles do not occur if $\alpha \gtrsim 0.1$.

6. For systems containing an unevolved main sequence or a giant donor the results of our numerical computations and the predictions from the analytic model for the stability boundaries are in reasonable agreement.

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