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Pacman (I): A new algorithm to calculate Faraday Rotation maps

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ABSTRACT
We propose a new method to calculate Faraday rotation measure maps from multi-frequency polarisation angle data. In order to solve the so called $n\pi$-ambiguity problem which arises from the observationally ambiguity of the polarisation angle which is only determined up to additions of $\pm n\pi$, where n is an integer, we suggest using a global scheme. Instead of solving the $n\pi$-ambiguity for each data point independently, our algorithm, which we chose to call *Pacman* (Polarisation Angle Correcting rotation Measure ANalysis), solves the $n\pi$-ambiguity for a high signal-to-noise region "democratically" and uses this information to assist computations in adjacent low signal-to-noise areas.

Key words: Intergalactic medium – Galaxies: cluster: general

1 INTRODUCTION

Magnetic fields are a common phenomenon on different astrophysical scales. They are observed around earth, other planets, the sun, in pulsars, AGNs, quasars, galaxies, in the interstellar and the intra-cluster medium. They influence the physics of all of these objects and media. The fields have different structures, ordering scales and strengths, for example the magnetic field of the earth is measured to be 300 – 600 mG whereas the fields in the intra-cluster gas are believed to be of the order of a few microgauss (Carilli & Taylor 2002).

One possibility to observe astrophysical magnetic fields and thus, gain insight into their structure, ordering scale and field strength is given by the Faraday rotation effect. It arises whenever linearly polarised radio emission passes through a magnetised medium. The polarisation angle φ of the radiation will be rotated due to the interaction between the magnetic field component parallel to the propagation direction of radiation and the radio emission itself. The resulting change in polarisation angle is proportional to the wavelength squared λ^2 which can be expressed by $\varphi = \mathrm{RM}\,\lambda^2 + \varphi^0$, where the proportionality constant RM in this relation is called Faraday rotation measure and φ^0 is the intrinsic polarisation angle of the emission at the radio source.

The latter two quantities, RM and φ^0, can be measured through multi-frequency polarisation observations. The RM can be described as the line of sight integral over the electron density and the magnetic field component parallel to the line of sight. Therefore, the RM contains valuable information on the magnetic field in the foreground of the polarised radio source. The intrinsic polarisation angle φ^0 and thus, the intrinsic magnetic field direction at the radio source gives insight into the magnetic field configuration at the source.

Faraday rotation studies are applied at various astrophysical scales. Polarisation observations concentrating on the intrinsic magnetic field configuration of galaxies (e.g., Beck et al. 1996; Beck 2000; Krause 2003) reveal large scale spiral structure of the fields in grand design spirals and irregular galaxies. Furthermore, observations of edge-on galaxies show that the magnetic fields are parallel to the disc. In contrary, RM maps of these galaxies reveal the coherency of the Faraday rotation producing magnetic fields which allows to distinguish between field generating mechanisms.

The large scale magnetic field of our own galaxy, the Milky Way, has been studied by compiling RM catalogues of extragalactic radio sources and pulsars located at low Galactic latitudes of the Galactic plane (e.g. by Simard-Normandin & Kronberg 1980; Broten et al. 1988; Brown et al. 2003). These catalogues are used to trace the RM distribution of the galactic plane. Various statistical methods have been applied to the RM distribution derived in order to understand the magnetic field structure of our own galaxy (e.g., Simard-Normandin & Kronberg 1980; Han et al. 1997; Frick et al. 2001).

Magnetic fields have been observed on even larger scales (in clusters of galaxies) using so called cluster radio *haloes* and radio *relics* suggesting that the intra-cluster medium is also magnetised. RM studies of point sources behind or embedded in galaxy clusters (Clarke et al. 2001) and of extended extragalactic radio sources (for a recent review, see Carilli & Taylor 2002) have been performed to gain insight into magnetic field structure and strengths (in the intra-cluster gas). For RM maps of extended extragalactic radio sources, Enßlin & Vogt (2003) and Vogt & Enßlin (2003) developed a statistical correlation analysis to determine the magnetic field power spectrum of cluster magnetic fields.

The statistical analysis of RM measurements requires that the RMs are unambiguously determined. Thus, any ambiguous RM can

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lead to misinterpretation of the data investigated. For the calculation of RM and φ^0 using the relationship $\varphi = \mathrm{RM}\,\lambda^2 + \varphi^0$ mentioned above, a least squares fit is normally applied to the polarisation angle data. Since the measured polarisation angle φ is constrained only to values between 0 and π leaving the freedom of additions of $\pm\, n\pi$, where n is an integer, the determination of RM and φ^0 is ambiguous, causing the so called $n\pi$-ambiguity. Therefore, a least squares fit has to be applied to all possible $n\pi$-combinations of the polarisation angle data at each data point of the polarised radio source while searching for the $n\pi$-combination for which χ^2 is minimal.

In principle, χ^2 can be decreased to infinitely small numbers by increasing RM substantially. Vallée & Kronberg (1975) and Haves (1975) suggested to avoid this problem by introducing an artificial upper limit for $|\mathrm{RM}| \leqq \mathrm{RM}_{\max}$. Since this is a biased approach, Ruzmaikin & Sokoloff (1979) proposed to assume that no $n\pi$-ambiguity exists between the measurements of two closely spaced wavelengths taken from a whole wavelength data set. The standard error of the polarisation measurements is then used to constrain the possible $n\pi$-combinations for the least squares fit for the subsequent observed frequencies. We refer to these methods as the "standard fit" algorithms, as they are currently the most widely used methods. However, these methods might still give multiple acceptable solutions for data with low signal-to-noise, requiring the solution to be flagged and all the information carried by these data is lost. Furthermore, it still can happen that the algorithm chooses a wrong RM and imprints spurious artefacts on the RM and the φ^0 maps.

Recently, a completely different approach was proposed by Sarala & Jain (2001) which takes the circular nature of the polarisation angle into account. The authors apply a maximum likelihood method to spectral polarisation data. Although this approach is not biased towards any RM value, it is rather designed for a large number of observed wavelengths. However, typically the observations are only performed at three or four wavelengths especially for extended (diffuse) radio sources.

Here, we propose a new approach for the unambiguous determination of RM and φ^0. We assume that if regions exhibit small polarisation angle gradients between neighbouring pixels in all observed frequencies simultaneously, then these pixels can be considered as connected. Note, that we calculate the gradient modulo π, which implies that polarisation angles of 0 and π are regarded as having the same orientation and thus, the cyclic nature of polarisation angles is reflected. Information about one pixel can be used for neighbouring ones and especially the solution to the $n\pi$-ambiguity should be the same.

In cases of small gradients, assuming continuity in polarisation angles allows us to assign an absolute polarisation angle for each pixel with respect to a reference pixel within each observed frequency. This assignment process has to be done for each spatially independent patch of polarisation data separately, such as each side of a double-lobed radio source. The reference pixel is defined to have a unique absolute polarisation angle and the algorithm will start from this pixel to assign absolute polarisation angles with respect to the reference pixel while going from one pixel to its neighbours. Figuratively, the algorithm eats its way through the set of available data pixel. It might become clear now why we have chosen to call the algorithm *Pacman*[1] (Polarisation Angle Correcting rotation Measure ANalysis).

Pacman reduces the number of least squares fits in order to solve for the $n\pi$-ambiguity. Preferably, the reference point is chosen to have a high signal-to-noise ratio so that in many areas, it is sufficient to solve the $n\pi$-ambiguity only for a small number of neighbouring pixels simultaneously and to use this solution for all spatially connected pixels. Pixels with low signal-to-noise will profit from their neighbouring pixels allowing a reliable determination of the RM and φ^0.

In Sect. 2, we describe in detail the idea and the implementation of the *Pacman* algorithm. Application to observational polarisation data and intensive tests of the algorithm will be discussed in the second paper (Vogt et al.), to which we will refer as Paper II.

2 THE NEW *PACMAN* ALGORITHM

2.1 The Idea

As described in the introduction, the Faraday rotation measure RM_{ij} at each point with map pixel coordinate (ij) of the source, is usually calculated by applying a least squares fit to measured polarisation angles $\varphi_{ij}(k)$ observed at frequency $k \in 1...f$ such that

$$\varphi_{ij}(k) = \mathrm{RM}_{ij}\,\lambda_k^2 + \varphi_{ij}^0, \tag{1}$$

where φ_{ij}^0 is the intrinsic polarisation angle at the polarised source.

Since every measured polarisation angle is observationally constrained only to a value between 0 and π, one has to replace $\varphi_{ij}(k)$ in the equation above by $\tilde{\varphi}_{ij}(k) = \varphi_{ij}(k) \pm n_{ij}(k)\,\pi$, where $n_{ij}(k)$ is an integer, leading to the so called $n\pi$-ambiguity. Taking this into account, a least squares fit to calculate RM_{ij} and φ_{ij}^0 at each pixel has to be applied by allowing all possible combinations of $n_{ij}(k)\pi$ while determining the $n_{ij}(k)$ for which the χ_{ij}^2 is minimal. The presence of observational noise might cause a standard least squares fit, as suggested by Vallée & Kronberg (1975), Haves (1975) or Ruzmaikin & Sokoloff (1979), to choose a spurious RM value especially for areas of low signal-to-noise.

The idea of *Pacman* is to reduce the number of pixels for which the $n\pi$-ambiguity has to be individually solved. This is done by splitting the solution of the $n\pi$-problem into two problems, a local and a global one,

$$n_{ij}(k) = \tilde{n}_{ij}(k) + n(k), \tag{2}$$

where $\tilde{n}_{ij}(k)$ is the local solution, linking polarisation angles of neighbouring pixels within a frequency map, and $n(k)$ is the global solution to the problem, linking polarisation angles of the different frequencies. The local part $\tilde{n}_{ij}(k)$ is determined by construction of absolute polarisation angle maps for each frequency with respect to a high signal-to-noise reference pixel being defined to possess a unique polarisation angle. The term absolute polarisation angle is to be understood as a value determined relative to the reference pixel by adding $\pm n\pi$ to the measured polarisation angles in order to remove jumps of the order of π in the measured polarisation angle map of each observed frequency. The global $n\pi$ ambiguity is solved for a high signal-to-noise area surrounding the reference pixel resulting in $n(k)$. This is then also the solution of the global

[1] The computer code for *Pacman* will be publicly available at `http://dipastro.pd.astro.it/~cosmo/Pacman`

$n\pi$-problem for all spatially connected points which are assigned absolute polarisation angles with respect to this reference area.

The splitting of the problem in a local and a global one is possible if the real polarisation angle $\bar{\varphi}_{ij}(k)$ ($\bar{\varphi}_{ij}(k) = \mathrm{RM}_{ij}\,\lambda_k^2 + \varphi_{ij}^0$) is a smooth quantity which does not change more than $\pm\pi/2$ between neighbouring pixels. In Sect. 2.2, it is described how *Pacman* ensures that only pixels fulfilling this condition are used.

The source might consists of several spatially independent areas of polarisation, to which we will refer as patches in the following. The $n\pi$-ambiguity has to be solved for each of these patches separately, requiring separate reference pixels to be defined for the construction of absolute polarisation angles.

The advantage of *Pacman* is that the global $n\pi$-ambiguity is solved only for pixels having the highest signal-to-noise ratios. Therefore, noisier pixels which are situated at the margin of the source profit from an already defined global solution $n(k)$ to the $n\pi$-ambiguity, making a reliable determination of RM_{ij} and the intrinsic polarisation angle φ_{ij}^0 for these pixels possible.

2.2 The Basic Algorithm

The algorithm starts with reading the maps of polarisation angles and of its errors, processes them, and when finished, it saves the different calculated maps. A flow chart of our *Pacman* algorithm is exhibited in Fig. 1, which shows schematically the procedure the algorithm follows in order to determine the solution to the $n\pi$-ambiguity and to calculate the various maps.

After loading of the various polarisation data for the different frequencies, the algorithm calculates a quality measure map. Different actions are involved in the processing of the data. One of the processes is building a patch which involves the construction of absolute polarisation angle maps. First, the reference pixel for a patch of polarisation is determined from the quality measure map. This step is followed by solving the local $n\pi$-ambiguity in each frequency map for pixels being spatially connected to the defined patch reference pixel and having small gradients in polarisation angle to the neighbouring pixels. This process is mainly governed and stopped by quality requirements.

When the patch building process is stopped then the algorithm solves the global $n\pi$-ambiguity across the frequency maps individually for a selected set of superior quality pixels (voters) within the patch. The majority of votes is used as the global patch $n\pi$-solution and the resulting RM and φ^0 values are stored. If there are still unprocessed high quality pixel left, the algorithm starts again to build a new patch. Otherwise *Pacman* finishes and saves the resulting complete maps.

The values for RM and φ^0 are calculated following a weighted least squares fit expressed by

$$\mathrm{RM}_{ij} = \frac{\overline{S_{ij}}\,\overline{\lambda^2\varphi_{ij}} - \overline{\lambda^2}\,\overline{\varphi_{ij}}}{\overline{S_{ij}}\,\overline{\lambda^4} - \overline{\lambda^2}^2} \quad (3)$$

$$\varphi_{ij}^0 = \frac{\overline{\varphi_{ij}}\,\overline{\lambda^4} - \overline{\lambda^2}\,\overline{\lambda^2\varphi_{ij}}}{\overline{S_{ij}}\,\overline{\lambda^4} - \overline{\lambda^2}^2} \quad (4)$$

where $\overline{S_{ij}} = \sum_{k=1}^{f} 1/\sigma_{k_{ij}}^2$, $\overline{\varphi_{ij}} = \sum_{k=1}^{f} \varphi_{ij}(k)/\sigma_{k_{ij}}^2$, $\overline{\lambda^2} = \sum_{k=1}^{f} \lambda_k^2/\sigma_{k_{ij}}^2$, $\overline{\lambda^2\varphi_{ij}} = \sum_{k=1}^{f} \lambda_k^2\,\varphi_{ij}(k)/\sigma_{k_{ij}}^2$ and $\overline{\lambda^4} = \sum_{k=1}^{f} \lambda_k^4/\sigma_{k_{ij}}^2$. In these relations, $\sigma_{k_{ij}}$ is the standard error of the polarisation angle $\varphi_{ij}(k)$ at the pixel coordinate (ij) at the kth wavelength λ_k.

Since the performance of the algorithm is mainly governed by quality requirements, we need some quality measure in order to rank the map pixels. One good candidate is the expected uncertainty $\sigma_{ij}^{\mathrm{RM}}$ of any RM value obtained in an error weighted least squares fit which is calculated by

$$\sigma_{ij}^{\mathrm{RM}} = \sqrt{\frac{\overline{S_{ij}}}{\overline{S_{ij}}\,\overline{\lambda^4} - \overline{\lambda^2}^2}}, \quad (5)$$

where the terms are defined as above.

The uncertainty $\sigma_{ij}^{\mathrm{RM}}$ of the RM value is used to assign each pixel an initial quality $q_{ij} = \sigma_{ij}^{\mathrm{RM}}$ since $\sigma_{ij}^{\mathrm{RM}}$ accounts for the

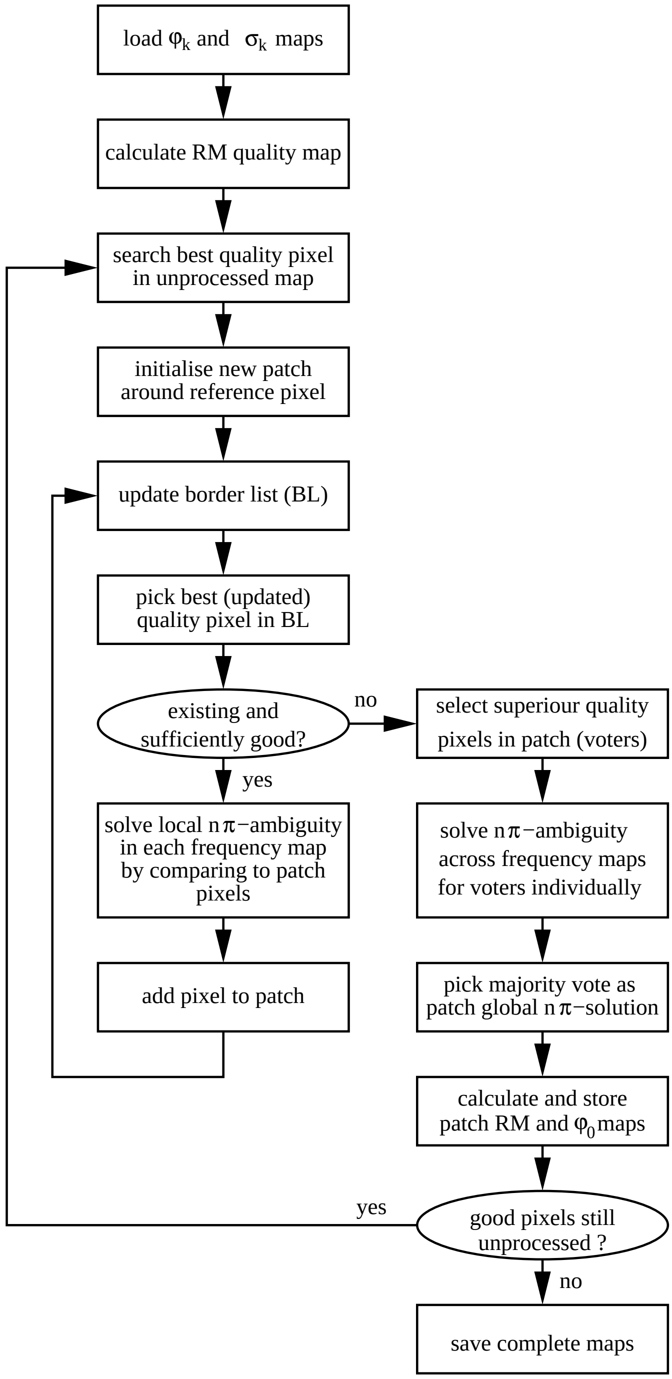

Figure 1. A flow chart representing the individual steps involved in the calculation of RM and φ^0 maps as performed by our algorithm *Pacman*.

statistic used to determine the RM maps. Hence, small values of q_{ij} indicate high quality pixels. The quality is then used to determine the way *Pacman* goes through the data which will be preferably from high to low quality data pixels.

For the construction of absolute polarisation angle maps for each frequency, *Pacman* starts at the best quality pixel having the smallest value q_{ij}, which is defined to be the reference pixel. For this first point, the measured polarisation angle $\varphi_{ij}(k)$ is defined to possess a unique polarisation angle value $\bar{\varphi}_{ij}(k) = \varphi_{ij}(k)$ for each observed frequency. It is important to note that the reference pixel is the same for all frequencies.

Then, *Pacman* compiles what we call the border list (BL). It contains pixels being direct neighbours to points which have been already assigned an absolute polarisation angle. In the following, the set of adjacent pixels ($i \pm 1\ j \pm 1$) to the pixel (ij) will be referred to as direct neighbours $(i'j')$. Beginning with the direct neighbours of the reference pixels the border list is continuously updated during the progression of the algorithm. A pixel can be rejected from the border list if the standard error $\sigma_{k_{ij}}$ of the polarisation angle for any frequency at this pixel exceeds a certain limit $\sigma_k^{\max}$ which is set at the beginning of the calculation.

Having defined the reference pixel, *Pacman* assigns absolute polarisation angles to pixels within the border list always starting with the pixel having the best quality, i.e. the lowest q_{ij} value. For this pixel, the algorithm solves the local $n\pi$-ambiguity with respect to $\tilde{n}_{ij}(k) \in \boldsymbol{Z}$ by minimising the expression

$$\sigma^{\Delta}_{ij} = \sum_{\{i'j'\} \in D_{ij}} \left[(\varphi_{ij}(k) \pm \tilde{n}_{ij}(k)\pi) - \bar{\varphi}_{i'j'}(k) \right]^2 , \qquad (6)$$

where D_{ij} is the set of all direct neighbours $(i'j')$ to the pixel (ij) which have already been assigned an absolute polarisation angle. The resulting $\tilde{n}_{ij}(k)$ value determines the absolute polarisation angle $\bar{\varphi}_{ij}(k) = \varphi_{ij}(k) \pm \tilde{n}_{ij}(k)\pi$ which has the smallest difference to the already defined absolute polarisation angles of adjacent pixels $\bar{\varphi}_{i'j'}(k)$. For each pixel, this is done at each frequency $k \in 1....f$ independently but simultaneously. Thus, *Pacman* goes the same way through the data in each frequency. Note that one can introduce a value $\sigma^{\Delta}_{\max}$ which causes *Pacman* to reject the pixel if σ^{Δ}_{ij} exceeds $\sigma^{\Delta}_{\max}$ (see Sect. 2.6). Additionally the border list is updated to include direct neighbours to the recently processed pixel (ij) which have not yet been assigned an absolute polarisation angle.

Pacman repeats this process for the best remaining pixel in the border list which has the lowest q_{ij} and so on, until the whole patch consists of spatially connected pixels with assigned absolute polarisation angles and no acceptable neighbouring pixels remain in the border list.

At this stage, *Pacman* solves the global $n\pi$-ambiguity $n(k)$ by applying a standard least squares fit to a set of best constrained pixels within the patch (i.e. the surrounding area of the reference pixel). *Pacman* solves for each of these best constrained pixels the global $n\pi$-ambiguity independently by either using the method of Haves (1975); Vallée & Kronberg (1975) or of Ruzmaikin & Sokoloff (1979) minimising the χ^2 for all possible combinations of $n\pi$. It searches then for the solution $n(k)$, which has been derived for the majority of pixels. This democratically determined solution $n(k)$ is used to perform a least squares fit of the other pixels defined within this patch. Note that in an extreme case, only the reference pixel could be used to solve the global $n\pi$-ambiguity.

If there are still good quality pixels left in the map which were not connected to the first patch of good quality data, the algorithm begins a new patch of absolute polarisation angles, starting again with the best remaining pixel with smallest q_{ij}. A new patch of absolute polarisation angles is constructed. As a result, the source will be divided in a set of spatially independent polarisation patches, and the global $n\pi$-ambiguity is solved once for each patch separately. A q_{ij}^{thresh} can be introduced to prevent *Pacman* from starting new patches if the pixels remaining have quality values above that threshold value whose value is chosen in the beginning.

2.3 Improving Quality

The solution of the local $n\pi$-ambiguity for pixel (ij) becomes more reliable as more direct neighbours of these pixels have their absolute polarisation angle $\bar{\varphi}_{i'j'}$ defined. This is taken into account by modifying the quality q_{ij} of the pixels added to the border list to

$$1/q_{ij} = 1/\sigma_{ij}^{RM} + \frac{\alpha}{n_{D_{ij}}^{\beta}} \left(\sum_{i'j' \in D_{ij}} 1/\sigma_{i'j'}^{RM} \right) , \qquad (7)$$

where $n_{D_{ij}}$ is the number of already defined neighbours $(i'j')$, and α and β are free parameters. This ensures that pixels having more already defined neighbours are considered before others although the original data point might have a smaller signal quality, i.e a higher q_{ij}. Values between 0 and 1 for the free parameter β are reasonable and yield good results. We used $\alpha = \beta = 1$ and observed that *Pacman* goes through the acceptable data points of the polarisation patches in a uniform manner.

2.4 Restricting Gradients

The algorithm might be faced with a situation where a high quality pixel can be influenced by a very poor quality pixel, for example when the polarised radio source consists of two radio lobes each having a good signal-to-noise ratio which are connected by a bridge containing only low quality pixels. The *Pacman* algorithm would start by defining absolute polarisation angles from one of the two lobes, eventually reaching the bridge of low signal-to-noise and entering the second lobe from there. In such cases, it might happen that within the area of low quality data pixels a distinct determination of absolute polarisation angles by solving the local $n\pi$-ambiguity using Eq. (5) is no longer possible. The entire second lobe would then suffer, and wrong solutions would be introduced.

In order to avoid such situations, the algorithm is restricted to accept in the border list only neighbours which have a lower quality than the one under consideration. This forces the algorithm to go always from high to low quality pixels, leading to an artificial splitting of connected regions in the map into different patches.

However, such a strict rule would lead to heavy fragmentation and is not preferable. Therefore, we introduced a parameter g to relax this strict limitation such that a new pixel is only accepted in the border list when the relation

$$q_{i'j'} > g * q_{ij} \qquad (8)$$

is fulfilled. This relation is always accounted for when adding a new pixel to the border list. We found that g between 1.1 and 1.5 is a good choice.

2.5 Topological Defects

The algorithm can also be faced with situations which we call topological defects. These defects can be understood by supposing a ring-like polarised structure for which all polarisation angles point towards the centre of the ring. Starting at any point of the ring to define absolute polarisation angles and following the ring structure, a jump will appear on the border of the first to the last defined absolute polarisation angles after having performed a full circle.

In observational data, topological defects are often more complex structured. The artificial jump introduced will cause difficulties in the solution of the local $n\pi$-ambiguity because this procedure relies on already defined absolute polarisation angles of neighbouring pixels. When confronted with this situation, our algorithm divides the list of direct neighbours into sublists possessing similar polarisation angles. The sublist containing the most pixels is then used to assign the absolute polarisation angle by solving the local $n\pi$-ambiguity for the pixel under consideration.

The locations of the polarisation angle steps of topological defects are somewhat artificial since they depend on the actual path of the algorithm through the data. Since the algorithm processes all frequencies simultaneously, the steps of topological defects are at the same positions within all frequency maps, and therefore do not cause any further problems. In our experience, this situation is a rare event, but it is necessary to take this effect into account for the solution of the local $n\pi$-ambiguity.

2.6 Spurious Points

The intrinsic polarisation angle distribution might show strong gradients extending over a few pixels. This could lead to situations, where our algorithm can not solve the local $n\pi$-ambiguity at all frequencies simultaneously. Therefore, *Pacman* refuses to assign an absolute value $\bar{\varphi}_{ij}$ to pixels when Eq. (4) yields a value $\sigma_{ij}^{\triangle}$ above a certain threshold $\sigma_{\max}^{\triangle}$, which can be set at the beginning of the calculation.

The pixels in the regions where this might occur most often have a low signal-to-noise ratio. In our experience, such situations always occur at strongly depolarised areas, leading to blanked regions in the RM distribution.

2.7 Multi-Frequency Fits

The aim of any RM derivation algorithm should be to calculate RMs for an area as large as possible using as much information as is available. On the other hand, for radio observations the total radio intensity decreases with increasing frequency. This can lead to the problem that the area of acceptable polarisation data at a high frequency is much smaller than at a lower frequency. This is especially true for (diffuse) extended radio sources. Furthermore, the limit of allowed standard errors $\sigma_k^{\max}$ of the polarisation angle might be exceeded for only one frequency leaving the values for the other frequencies still in the acceptable range.

Pacman can take this into account and performs the RM fit by omitting the polarisation angles at frequencies which do not meet the quality requirements. In order to do that, *Pacman* uses the standard errors $\sigma_{k_{ij}}$ of polarisation angles to define independently for each frequency, areas in which the RM map is permitted to be produced. An additional parameter $k_{\min} \leqq f$ is introduced which describes the minimum number of frequencies allowing the freedom to use any combination of the minimum of frequencies.

In cases of $k_{\min} < f$, *Pacman* will start to determine the solution to global and local $n\pi$-ambiguity only for the pixels fulfilling the quality requirements at all f frequencies. After finishing that, the algorithm proceeds to include pixels satisfying the quality criteria in less than f frequencies. For these pixels, the same patch building procedure as described in Sect. 2.2 applies with some modification in order to prevent the final maps from heavy fragmentation. The best quality pixel among the remaining pixel is picked but before starting a new patch, *Pacman* tests if the pixel under consideration adjoins a patch which has already been processed before. If the pixel adjoins such a patch, *Pacman* tries to solve the local $n\pi$-ambiguity following Eq.(6) and applies the patch solution of the global $n\pi$-ambiguity to the pixel. If the pixel is neither adjoined to an already processed patch nor the local $n\pi$-ambiguity solvable (i.e. $\sigma_{ij}^{\triangle}$ exceeds $\sigma_{\max}^{\triangle}$) then a new patch is initialised by this pixel.

Another possibility is to force *Pacman* to use certain frequencies in all circumstances. Thus, if the quality requirements are not fulfilled for these particular frequencies, there will be no RM value determined for the pixel under consideration. This has the advantage that one can use relatively close frequencies as a basis and then include other frequencies at points when a reliable polarisation signal is detected. The advantage of this technique is discussed in Paper II.

2.8 Additional Information

Apart from the resulting RM and φ^0 maps, *Pacman* provides sets of additional information about the data in order to estimate the reliability of the results. A patch map which contains all patches used is one example. Such a map is very useful, especially if one requires a minimal number of pixels in a patch in order to accept any calculated RM values from a particular patch. A map which includes rejected and thus flagged pixels, can also be obtained. Probably more important are the final χ^2- and $\sigma^{\rm RM}$- maps which are also provided by *Pacman*. This information allows one to understand the reliability of the RM maps obtained and can be used for further evaluation and analysis of the RM maps.

3 CONCLUSIONS

We have presented a new algorithm for the calculation of Faraday rotation maps from multi-frequency polarisation data sets. Unlike other methods, our algorithm uses global information and connects information about individual neighbouring pixels with one another. It assumes that if regions exhibit small polarisation angle gradients between neighbouring pixels in all observed frequencies simultaneously, then these pixels can be considered as connected, and information about one pixel can be used for neighbouring ones, and especially, the solution to the $n\pi$-ambiguity should be the same. We like to stress, that this is a very weak assumption, and – like all other criteria used within the *Pacman* algorithm – only depends on the polarisation data at hand and the signal-to-noise ratio of the observations.

Our *Pacman* algorithm is especially useful for the calculation of RM and φ^0 maps of extended radio sources. Global algorithms as implemented in *Pacman* are preferable for the calculation of RM maps and needed if reliable RM values are desired from low signal-to-noise regions of the source. Furthermore, since the resulting φ^0 maps are calculated from the whole set of polarisation angle maps, they do not represent a simple Faraday derotation of one frequency map using the RM maps. Thus, the φ^0 maps are less noisy than maps which would have been determined by Faraday derotation of a single frequency map.

Our *Pacman* algorithm reduces $n\pi$-artefacts in noisy regions and makes the unambiguous determination of RM and φ^0 in these regions possible. *Pacman* allows to use all available information obtained by the observation. Any statistical analysis of the RM maps will profit from these improvements.

A more detailed description of the application of *Pacman* to observations and various statistical tests performed on resulting RM maps in comparison to the standard fit algorithms are given in Paper II.

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