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ON THE IMPORTANCE OF COHERENCE EFFECTS IN HIGH ORDER STATISTICS: A COMMENT ON "LOWERING INFERRED CLUSTER MAGNETIC FIELD STRENGTHS – THE RADIO GALAXY CONTRIBUTIONS"

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ABSTRACT

Rudnick & Blundell (2003, hereafter RB) claimed to have found a relationship between the intrinsic polarization angle (PA) of the radio source PKS 1246-410 and its Faraday rotation measure (RM) by testing the clustering strength of the PA-RM scatter plot. They argue that this would imply that most of the measured RM is due to the source itself and not due to magnetic fields in the foreground intra-cluster medium (ICM). This would imply that former ICM magnetic field estimates have to be strongly reduced. However, we show that the claimed relationship is an artifact of an improperly performed Null-experiment. We demonstrate that strong clustering in the PA-RM scatter plot naturally arises even if two completely independent PA and RM maps are compared, with the only requirement that these maps have patchy structures. The Null-experiment of RB was performed with a smooth, non-patchy RM map, whereas the observed RM map clearly shows distinct individual patches. This fundamental difference of the observed and simulated data explains why RB found a stronger clustering signal in the observed data than in their Null-experiment. Further, we argue, that an inspection of the PA-RM scatter plot of the observed data gives strong support for the PA and RM being independent in PKS 1246-410, indicating that the measured RM values are mostly from the foreground ICM magnetic field and are not source-intrinsic. Therefore the claim of RB that RM based ICM magnetic field estimates have to be strongly reduced is premature at best.

Subject headings: galaxies: clusters: general — magnetic fields — polarization — radio continuum: galaxies

1. PA-RM SCATTER PLOTS

A summary of the observational evidence for the presence of magnetic fields embedded within the thermal gas in clusters of galaxies is presented in two recent review papers (Carilli & Taylor 2002, and Widrow 2002). Rudnick & Blundell (2003, hereafter RB) use Faraday rotation measure observations of PKS 1246-410 by Taylor, Fabian & Allen (2002) to attempt to address an important question on ICM magnetic fields: What fraction of the measured RM signal of a radio galaxy embedded in the ICM is internal to the radio source or in a local mixing layer with the thermal ICM, as opposed to the foreground intracluster medium? If most of the RM signal is intrinsic to the radio source then the ICM magnetic field strength would be significantly lower than if all of the RM is extrinsic to the source.

RB's approach is to search for co-aligned structures in the polarization angle (PA) and rotation measure (RM) maps of the radio galaxy PKS 1246-410. Such a co-alignment would be a strong signature of a source intrinsic RM origin. Since no direct correlation between the PA and RM values can be expected even for co-aligned distributions, RB analyze the PA-RM scatter plot, which is generated by plotting for each map pixel the PA and the RM value in the same diagram. They argue that local co-alignment of PA and RM values should lead to a strongly clustered point distribution in the scatter plot. Since also statistically independent PA and RM maps may produce such clusters they perform a Null-experiment. They generate synthetic RM maps, which have the same power-spectrum as the observed RM map, but random phases. With these simulated maps they repeat their analysis and find much less clustering in the scatter plot. From this they conclude that there is a strong co-alignment in the PA-RM maps of PKS 1246-410, and therefore most of the RM should be local to the radio source. If true, this would imply that published ICM magnetic field strengths are overestimates.

In the following we explain why RB's Null-experiment is set up improperly, so that the strong conclusions drawn from it are at least premature. We do this first through a general argument, then by an illustrative numerical example. Finally we give independent reasons why the ICM magnetic field strength should be much higher than RB claim.

2. PATCHES AND HIGH-ORDER STATISTICS

The statistics of a map can be described by its N-point autocorrelation function, or equivalently by the power-spectrum and relation of the Fourier phases. Fixing the 2-point autocorrelation function is equivalent to specifying the power spectrum. RB designed their Null-experiment to be similar to the observational data on the 2-point level, but higher order correlations or phase coherence are neglected by their random phase realization.

Unfortunately, the chosen statistic is very sensitive to such higher order correlations. Any clustering in the PA-RM scatter plot is caused by having patches of nearly con-

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stant values in the PA and RM maps. The appearance of patches is a coherence effect, which requires special relations of the Fourier phases, or – equivalent – non-trivial higher order correlation functions to be specified.

The strength of the clustering in the scatter plot is indeed strongest, if the PA and RM structures are co-aligned. However, the clustering does not disappear if the RM and PA patches have independent distributions, since every PA patch is still overlaid by a small number of RM patches, so that the associated clustering in the scatter plot only gets split into a corresponding number of smaller clusters. These clusters happen to be co-aligned on a vertical PA = constant line in the plot, since they all belong to the same PA coherence patch. A corresponding effect splits the pixels of an RM cell into a horizontal line of clusters. Such horizontal and vertical structures are indeed visible in the scatter plot of PKS 1246-410 (Fig. 1, also Fig. 4 of RB).

We note that a proper Null-experiment, which would have the higher-order correlations, can also be constructed empirically by exchanging subregions of the observed RM image. This should keep the same RM correlation functions, but destroy any real PA-RM alignment since the different regions of the source are independent. The results of such an experiment are also presented in Fig. 1. Clustering of points in the PA-RM scatter plots are clearly seen in both plots in contrast to RB's claim that clustering results from RMs local to the source.

3. A NUMERICAL EXAMPLE

For illustration we construct a simple example, which captures the main effects but is not meant to be exact in all respects. We construct patchy RM and PA maps, which are statistically independent from each other. For both maps we use a similar recipe[4].

First a number N of random seed points $\vec{X}_i$ ($i \in \{1...N\}$) within a square area is drawn, and then the area is split into cells around the seed points by means of a Voronoi-tessellation: Each point $\vec{x}$ of the area belongs to the cell of its nearest seed $\vec{X}_i$. Then every seed is attributed with a random value ψ_i (ψ stands in the following for both, RM and PA) and a small two-dimensional random vector $\vec{\nabla}\psi_i$ (= RM or PA gradient). Each pixel $\vec{x}$ within the cell of seed i gets a value

$$\psi(\vec{x}) = \psi_i + \vec{\nabla}\psi_i \cdot (\vec{x} - \vec{X}_i) + \sigma\,, \qquad (1)$$

where sigma is a small random noise term. The resulting map consists of patches with nearly constant values, but which exhibit some internal trends and noise.

The PA and RM maps are slightly smoothed, a 20% border region is cut away in order to suppress edge effects, and a PA-RM scatter plot is generated. A typical realization of such a scatter plot is shown in Fig. 2. A strongly clustered distribution is visible even though the individual PA and RM maps were completely independent statistically. Further, nearly horizontal and vertical chains of clusters are visible for the reason given above. The map smoothing produces bridges between these clusters, since it gives intermediate values to pixels which are at boundaries of PA or RM cells.

The deviations from the strict horizontal and vertical directions visible in the scatter plot of PKS 1246-410 should be caused by trends within the coherence cells. We note that such structures are also visible in the simulated scatter plots of RB, although the smooth realizations of their RM maps has smeared them out (see their Fig. 2 for a comparison of the patchiness of the observed and their simulated RM maps).

We therefore state, that the data of PKS 1246-410 seems to favor statistically independent PA and RM maps, since the occurrence of vertical and horizontal lines in the scatter plot of the observational data demonstrates that the PA and RM patches are indeed not aligned.

In order to further investigate this, we construct a model in which the PA and RM patch positions are absolutely identical. We construct this by moving the Voronoi-tessellation seed points of the RM map to the locations of nearby seed points of the PA map, thereby assuring that there is a one-to-one mapping. All other variables (RM_i, $\vec{\nabla}\mathrm{RM}_i$, etc) were kept as before. The recomputed RM map has therefore exactly the same patch locations as the PA map (see Fig. 2). The horizontal and vertical cluster alignments and stripes are absent there. There are now several stripes with diagonal orientations due to pixels at the PA-RM cell boundaries which got simultaneously intermediate values in PA and RM by the smoothing.

4. INDEPENDENT EVIDENCE OF STRONG ICM MAGNETIC FIELDS

A number of galaxy clusters show large regions of diffuse synchrotron emission which is not associated with individual galaxies in the cluster. Although initially considered to be relatively rare objects, recent surveys have significantly increased the number of radio halos and relics (Giovannini, Tordi, & Feretti 1999; Giovannini & Feretti 2000; Kempner & Sarazin 2001; Bacchi et al. 2003). The presence of this diffuse synchrotron emission reveals the widespread distribution of relativistic particles and magnetic fields within the ICM in these clusters. Equipartition assumptions provide minimum energy magnetic field estimates in radio halos of 0.1 - 1 μG (Feretti 1999; Bacchi et al. 2003) and 0.4 - 2.7 μG in radio relics (Enßlin et al. 1998). Note that inverse Compton constraints indicate that in the case of the Coma cluster, the magnetic field strength is expected to be larger than 0.3μG (e.g. Enßlin & Biermann 1998).

Faraday rotation measure studies of extended radio sources embedded in clusters generally reveal large RMs. Carilli & Taylor (2002) summarize the evidence that indicates that these embedded sources are indeed probing a foreground ICM with conservative estimates of magnetic field strengths between a few and 10 μG. Similar magnetic field strengths are determined from statistical RM studies of Kim, Tribble & Kronberg (1991) and Clarke, Kronberg & Böhringer (2001). We note that the RM sample of Clarke et al. (2001) contains 27 cluster probes, of which 15 are embedded and 12 are background (Clarke et al. in preperation).

5. SUMMARY

We have shown that the Null-experiment performed by RB was poorly designed to test the correlation between

[4] The computer code can be requested from TAE (`ensslin@mpa-garching.mpg.de`).

PA and RM in PKS 1246-410. The lack of phase coherence in the simulated data resulted in less clustering in the simulated PA-RM scatter plots than in the observational scatter plots. Using independent, patchy distributions of PA and RM we show that the correlations due to the overlap of the patches produce horizontal and vertical chains of clusters as seen in the PKS 1246-410 data, whereas co-aligned PA and RM patches produce diagonal stripes. The observed clustering therefore favors independent PA and RM maps resulting from foreground intracluster fields as opposed to local to the source as claimed by RB.

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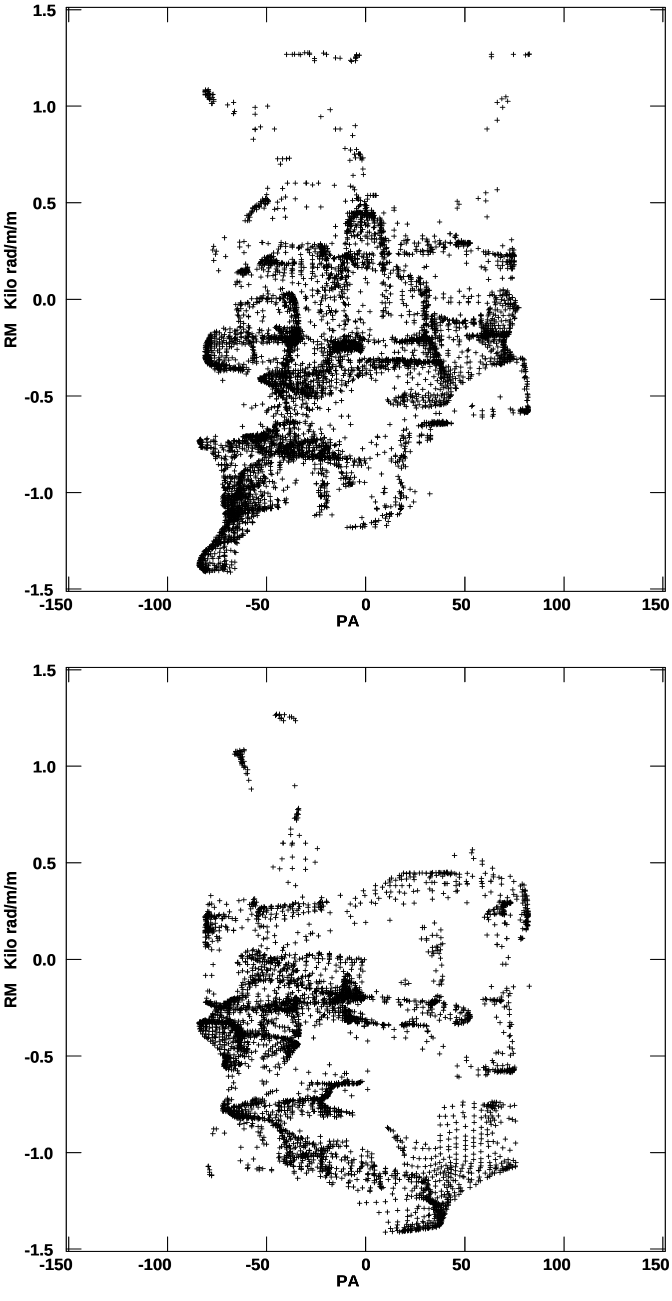

FIG. 1.— Top: PA-RM scatter plot of PKS 1246-410 (similar to Figure 4 of RB). The underlying maps were smoothed by a median weight filter in the same way as RB smoothed the data in order to amplify the clustering. Bottom: The same, but with having the RM maps of the Eastern and Western radio lobe exchanged.

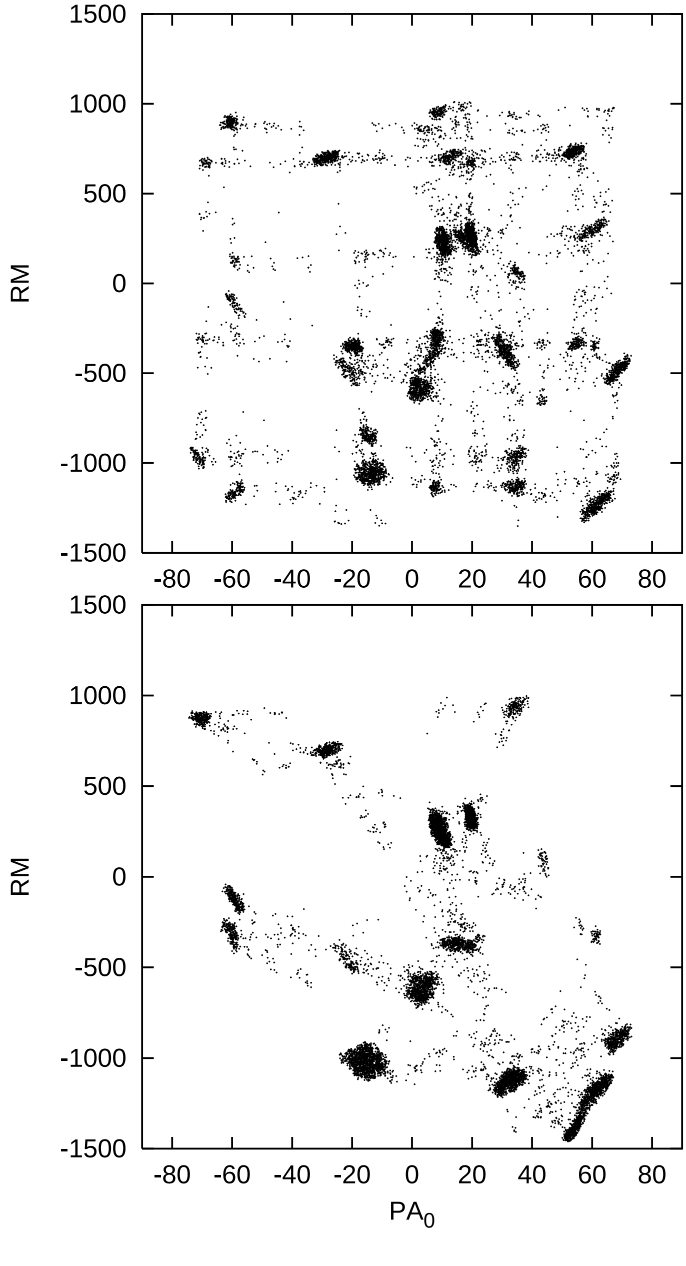

FIG. 2.— Simulated PA-RM scatter plot. Top: independent PA and RM coherence patches. Bottom: co-aligned PA and RM coherence patches. Note the horizontal and vertical patches in the plot of independent PA and RM. Similar structures are clearly visible in the scatter plots of PKS 1246-410 but are absent in the scatter plots of co-aligned PA and RM.