Investigating the Flame Microstructure in Type Ia Supernovae

F.K. Röpke, W. Hillebrandt, J.C. Niemeyer

Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, D-85740 Garching, Germany

Introduction

Type Ia Supernovae (SN Ia in the following) are the most accurate cosmological distance indicators at the moment. Through light curve shape corrections accuracies of better than 10% in distance are claimed. In addition, claims of an accelerated cosmic expansion and thus a new constituent of the universe are mainly based on supernova observations. It is, however, evident that an understanding of the explosion mechanism is needed in order to validate such interpretations of the data and to control possible evolutionary effects.

One way to perform such investigations is by means of numerical simulations. These simulations should be self-consistent, independent of any phenomenological parameters, and based on models that are in accord with observational constraints. However, the main problem with the numerical approach is the vast range of relevant length scales, 11 orders of magnitude from the flame width to the radius of the star, that remains unresolvable in multidimensional simulations. Following [1], attempts to overcome this difficulty can be classified into large scale calculations (LSCs) and small scale calculations (SSCs). The former try to simulate SN Ia on scales of the radius of the exploding white dwarf star relying on assumptions on the microscale physics, whereas the latter are used to study the flame dynamics through a small window in scale space. In this manner it is possible to isolate specific physical effects and eventually to include the information gained here into LSCs. Examples of LSCs are given in M. Reinecke's contribution to these proceedings. The present work is concerned with SSCs to investigate the dynamics of thermonuclear flames in SN Ia explosions.

The astrophysical model

The SN Ia model adopted in the present study describes them as thermonuclear explosions of Chandrasekhar-mass C+O white dwarf stars. In the standard model nuclear burning is ignited near or at the star's center. At the typical temperatures of around 10^{10} K the thermonuclear burning rate scales with the temperature as $\dot{S} \propto T^{12}$. Hence the burning takes place in a very thin region which is called a flame.

Flames can travel in two distinct modes: as a supersonic compression (shock) wave in the detonation mode and as a subsonic conductive wave in the deflagration mode. A pure detonation model for SN Ia can be ruled out because the outcome does not match the observed spectra. A more promising model (see M. Reinecke's contribution) is one in which the explosion starts out as a (slow) deflagration, is accelerated by turbulence, and possibly undergoes a transition to a detonation. The questions arising here concern the mechanism of the generation of turbulence as well as the likelihood of a deflagration-to-detonation transition. They are still not answered and are the motivation to study thermonuclear flames at microscopic scales.

In this work we will investigate length scales much larger than the diffusive flame width (less than one millimeter for C+O white dwarfs [2]) but much smaller than the stellar radius. On such scales the so-called discontinuity approximation holds which describes the flame as a discontinuity in temperature and density. This picture does not resolve the inner structure of the flame and it is therefore necessary to prescribe the laminar burning velocity as taken from direct numerical simulations, e.g. [2].

Some theoretical considerations

A laminar flame is subject to various instabilities. In the context of SN Ia explosions the most important ones are the Rayleigh-Taylor instability, the Kelvin-Helmholtz instability, and the Landau-Darrieus instability. While the first two are more important on large scales, the Landau-Darrieus (LD) instability acts unconditionally on all scales provided the flame can be described by the discontinuity approximation. This instability shall be investigated here, and its cause will be described briefly.

As was first discussed by Landau [3] and Darrieus [4] the cause of this instability is the refraction of the streamlines of the flow at the density jump of the flame. Consider a flame front that is perturbed from its originally planar shape. In the vicinity of a bulge of the perturbation the flow tubes are broadened. This leads to a decrease of the local fluid velocity with respect to the velocity far away from the front. Therefore the burning velocity s_l of the flame is higher than the corresponding local fluid velocity leading to an accrual of the bulge. The opposite holds for recesses of the perturbed front. In this way the perturbation keeps growing. By means of a linear stability analysis Landau found for the growth rate of the perturbation amplitude

$$\omega_{\mathrm{LD}} = k s_l \frac{\mu}{1+\mu} \left(\sqrt{1 + \mu - \frac{1}{\mu}} - 1 \right), \tag{1}$$

where k denotes the perturbation wavenumber and $\mu = \rho_u/\rho_b$ is the expansion across the flame front. In white dwarf matter at densities relevant for SNe Ia, the existence of the LD instability was first demonstrated numerically in [5].

However, the LD instability does not lead to unlimited growth of the amplitudes but is limited in the nonlinear regime. Fig. 1 illustrates this effect which can be described in a geometrical way (following [7]): The propagation of an initially sinusoidally perturbed flame is first determined by Huygens' principle. But once a cusp forms at point A Huygens' principle breaks down and the flame propagation enters the nonlinear regime. The propagation velocity at the cusps exceeds s_l. This leads to a stabilization of the flame in form of cells which propagate with a velocity not much higher than s_l. Therefore the LD instability is usually ignored in SN Ia simulations. An analytical investigation of the stabilizing effect is given in [8] and is applied to the context of SN Ia in [9]. But there exist the possibility that nonlinear stabilization is destroyed under certain conditions (e.g. certain densities; interaction with turbulent velocity fields). In the following section we present a numerical method that will allow us to study this question.

Numerical methods

The fluid dynamics is described by the reactive Euler equations that are discretized on an equidistant Cartesian grid. To solve these equations a Godunov scheme is applied using the

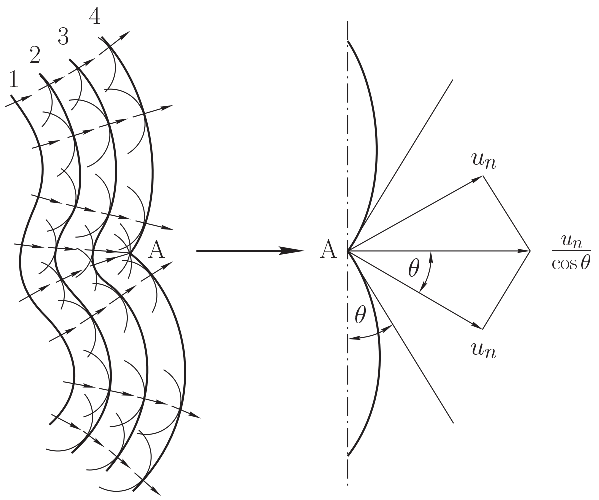

Figure 1: Nonlinear effect stabilizing the flame front (adopted from [6]).

piecewise parabolic method (PPM) [10]. For this we employ the PROMETHEUS implementation [11] with directional splitting in two dimensions.

The equation of state is that of white dwarf matter, including an arbitrarily degenerate and relativistic electron gas, local black body radiation, an ideal gas of nuclei and electron-positron pair creation/annihilation.

Thermonuclear burning as considered here takes place at fuel densities above $10^7\,\mathrm{g\,cm^{-1}}$ and terminates into nuclear statistical equilibrium, consisting mainly of ^{56}Ni. Because of the high computational costs of a full reaction network and because the objective of our SSC is to study flame dynamics rather than to provide an exact nucleosynthetic description we simplified the burning to an net reaction, $14\ ^{12}\mathrm{C} \longrightarrow 3\ ^{56}\mathrm{Ni}$, which gives an energy release of $7 \cdot 10^{17}\,\mathrm{erg\,g^{-1}}$.

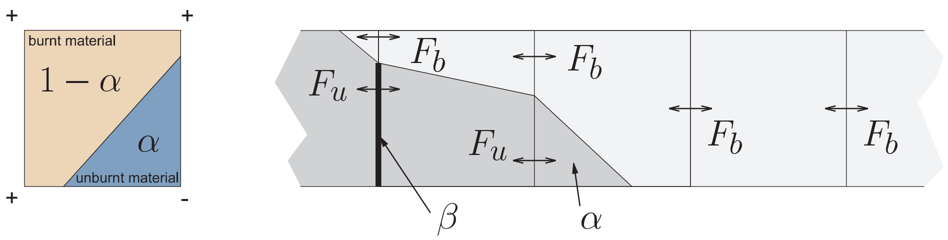

Figure 2: Determination of the unburnt volume fraction α (l.h.s.); flux-splitting method, adapted from [12] (r.h.s.)

The flame is described using the *level set technique*, associating the flame front with the zero level set of a signed distance function G. Details of the implementation of the level set method can be found in [12]. Without additional measures, however, the flame would smear out over 3-4 cells of the computational grid. This is not acceptable for our purposes and therefore we apply the *in-cell reconstruction/flux-splitting scheme* developed by Smiljanovski et al. [13]. This algorithm makes use of the fact that geometrical information on cells containing burnt and unburnt material ("mixed cells") can be obtained from the intersection of the zero level set of the G-function (representing the flame) with the interfaces of the mixed computational cells. In this way one can designate the volume fraction α of the unburnt cell part (Fig. 2, l.h.s.). The conserved quantities U are now interpreted as a linear combination of unburnt and burnt parts of each cell

$$\overline{U} = \alpha U_u + (1 - \alpha) U_b.$$

Together with the Rankine-Hugoniot jump conditions and the Rayleigh criterion for the flame front, the equation of state, the jump condition for the velocity component normal to the front, and the prescribed burning velocity one obtains a system of equations which allows to compute state variables of unburnt and burnt matter in each mixed cell from the mean values. To describe the energy generation and species conversion consistently it is necessary to treat the source terms in this reconstruction implicitly. The reconstruction equation system is solved with Broyden's method.

The knowledge of the values of the burnt and unburnt quantities makes it possible to treat the hydrodynamic fluxes as linear combinations of "unburnt" and "burnt" fluxes weighted with corresponding parts β of the cell interfaces (see Fig. 2, r.h.s.). These partial fluxes are calculated separately. Therefore it is guaranteed that for instance no unburnt material can flow through the third cell interface from the left in Fig. 2, r.h.s. This feature prevents the flame from smearing out and enables us to describe it as a sharp discontinuity.

Some preliminary results

In order to study the LD instability the flame evolution has been simulated for two values of the fuel density, namely for $\rho_u = 5 \cdot 10^7\,\mathrm{g\,cm^{-1}}$ and for $\rho_u = 5 \cdot 10^8\,\mathrm{g\,cm^{-1}}$. The corresponding expansion coefficients across the front are $\mu = 2.410$ and $\mu = 1.619$, respectively. The simulation runs were performed on a grid of 100×100 cells with a cell length of 500 cm. The physical domain was periodic in the y-direction. On the left boundary of the domain an outflow condition was enforced and on the right boundary we imposed an inflow condition with the unburnt material entering the grid with the laminar burning velocity s_l.

Fig. 3 shows the temporal evolution of the flame fronts. The flame was initially perturbed in a sinusoidal way with a wavelength of $\lambda_{\mathrm{pert}} = 5 \cdot 10^4\,\mathrm{cm}$ and an amplitude of 200 cm (0.4 grid cells). Each contour is shifted artificially in x-direction for better visibility. As expected, in the beginning the perturbation grows exponentially, and the growth rate of the perturbation amplitude meets the theoretical prediction (1). As the flame evolution enters the nonlinear regime the formation of a cusp sets in. The front develops a cellular structure. At the troughs the cells split forming new cusps. The new cells move downwards the initial cusp and disappear. Finally this merging process results in a single domain-filling cusp/trough structure which then propagates steadily forward.

We do not reproduce the result of [5] that nonlinear stabilization fails at $\rho_u = 5 \cdot 10^7\,\mathrm{g\,cm^{-1}}$. A detailed discussion of this discrepancy will be presented elsewhere.

Summary and outlook

The presented model was proven to be applicable for studies of the microscale behavior of thermonuclear flames in the discontinuity approximation. The response of the flame to perturbations of its planar shape meets the theoretical expectations. Early stages of the flame evolution are consistent with Landau's stability analysis. The later evolution of the flame is nonlinear, it stabilizes in a cellular structure. Whether the final outcome is always

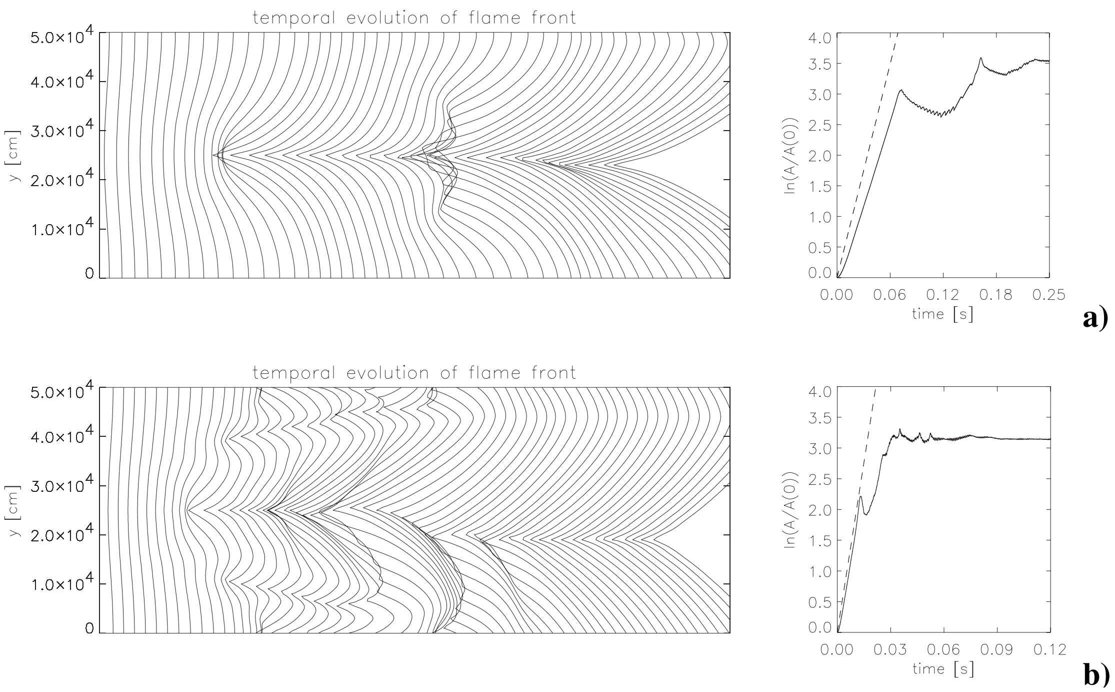

Figure 3: Flame front evolution. a) $\rho_u = 5 \cdot 10^7 \mathrm{g\,cm^{-3}}$, each contour corresponds to a time step of $\Delta t = 2.46 \cdot 10^{-3}\,\mathrm{s}$. b) $\rho_u = 5 \cdot 10^8 \mathrm{g\,cm^{-3}}$, $\Delta t = 1.23 \cdot 10^{-3}\,\mathrm{s}$. The plots on the r.h.s. show the growth of the perturbation amplitude over time (solid) and the prediction of Landau's linear stability analysis (dashed).

a single domain-filling cell has to be studied in larger simulations. With this new code we are now in a position to investigate the interaction of thermonuclear flames with imprinted velocity fields such as turbulence and to study the question of nonlinear stability. Moreover, if possible, we want to apply the in-cell reconstruction/flux-splitting technique to full two-dimensional LSCs of SN Ia explosions.

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