ASYMMETRY OF COMPTON SCATTERING BY RELATIVISTIC ELECTRONS WITH AN ISOTROPIC VELOCITY DISTRIBUTION: ASTROPHYSICAL IMPLICATIONS

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Abstract. The angular distribution of low-frequency radiation after a single scattering by relativistic electrons with an isotropic velocity distribution differs markedly from the Rayleigh angular function. In particular, the scattering by an ensemble of ultrarelativistic electrons is described by the law $p = 1 - \cos\alpha$, where α is the scattering angle. Thus, photons are mostly scattered backward. We discuss some consequences of this fact for astrophysical problems. We show that a hot atmosphere of scattering electrons is more reflective than a cold one: the fraction of incident photons reflected after a single scattering can be larger than that in the former case by up to 50 per cent. This must affect the photon exchange between cold accretion disks and hot coronae (or advective flows) near relativistic compact objects, as well as the rate of cooling (through multiple inverse Compton scattering of photons supplied from outside) of optically thick clouds of relativistic electrons in compact radio sources. Scattering asymmetry also causes the spatial diffusion of photons to proceed slower in a hot plasma than in a cold one, which affects the shapes of Comptonization spectra and the time delay in the detection of soft and hard radiation from variable X-ray sources.

1 INTRODUCTION

In our recent paper (Sazonov and Sunyaev 2000a), we have called attention to the fact that the differential cross section for Thomson scattering of low-frequency photons averaged over an isotropic distribution of relativistic electrons differs markedly from the Rayleigh phase function, which corresponds to the scattering by cold electrons. The resulting angular function is backward-oriented, i.e., *photons tend to be scattered backward* rather than forward (see Fig. 1). It should be immediately emphasized that this angular function is a characteristic of the scattering by an ensemble of electrons. Similar terminology is used when treating the Compton scattering in dense plasmas, where collective effects are important (see, e.g., Bekefi 1966).

The above phenomenon results from the joint action of two effects. One is that a photon has a higher probability of undergoing a scattering by an electron that is moving toward it rather than away from it (the probability is proportional to the factor $1 - \cos\theta\, v/c$, where v is the electron velocity and θ is the angle at which the photon and the electron collide). The other effect is that photons emerge after the scattering preferentially in the direction of motion of the relativistic electron. The angular function in the case under consideration is virtually opposite to the forward-oriented Klein–Nishina angular function, which corresponds to the scattering of energetic photons by an electron at rest.

In this paper, we consider some astrophysical implications of this peculiar scattering behavior of a hot plasma. The most evident consequence of the backward scattering law is that a larger (than in the case of a cold plasma) fraction of the low-frequency photons incident on the surface of an optically thick cloud of relativistic electrons will be reflected from it after a single scattering by an electron. Therefore, fewer photons will be involved in the Comptonization (through multiple scatterings) inside the cloud, and the rate of energy loss by electrons will be lower. Using classical results of the theory of radiative transfer in scattering atmospheres, we derive (see Section 3) simple formulae for the albedos of atmospheres that consist of either mildly relativistic or ultrarelativistic thermal electrons. Our results can be of use, for example, in calculating the photon exchange between cold accretion disks and hot coronae (or mass outflows) above them, as well as in studying the illumination of advective accretion flows by external low-frequency radiation.

Another consequence of the backward-scattering asymmetry is that the coefficient of spatial diffusion for photons in a hot plasma must be different than that in the case of a nonrelativistic plasma. This affects the Comptonization spectra and time delays in the detection of soft and hard radiation from variable X-ray sources. We discuss this issue in Section 4.

It should be noted that the anisotropic (backward) scattering of low-frequency radiation in a hot plasma has been previously discussed in literature, and its various astrophysical manifestations have been studied in detail, mostly when considering the formation mechanisms of hard X-ray spectra for compact sources (Ghisellini et al. 1991; Titarchuk 1994; Stern et al. 1995; Poutanen and Svensson 1995; Gierlinski et al. 1997; Gierlinski et al. 1999). The scattering angular function proper for an ensemble of Maxwellian electrons was investigated by Haardt (1993), who derived approximate formulae for it from Monte Carlo simulations. The main goal of this study is to present simple analytic formulae for the scattering angular function and related quantities in the ultrarelativistic and mildly relativistic limits.

2 SCATTERING ANGULAR FUNCTION

The scattering angular function $p(\mu_{\rm s})$ allows the probability of scattering through a specified angle $\alpha = \arccos\mu_{\rm s}$ of a photon by an ensemble of electrons to be determined. We normalize $p(\mu_{\rm s})$ in such a way that the photon mean free path in a plasma is

$$\lambda = \frac{1}{N_{\rm e}\sigma_{\rm T}\int p(\mu_{\rm s})\,d\mu_{\rm s}/2}, \tag{1}$$

where $N_{\rm e}$ is the electron number density and $\sigma_{\rm T}$ is the Thomson scattering cross section. Note that, by the definition (1), $\int p(\mu_{\rm s})\,d\mu_{\rm s}/2$ is generally < 1 (because of the Klein-Nishina corrections).

Here, we restrict ourselves to the case where low-frequency radiation is scattered by electrons with a relativistic Maxwellian distribution. Two opposite limits can be considered in this situation: (A) the electrons are ultrarelativistic, $kT_{\rm e} \gg m_{\rm e}c^2$; and (B) the electrons are mildly relativistic, $kT_{\rm e} \ll m_{\rm e}c^2$ (where $T_{\rm e}$ is the electron temperature). In case (A), the following asymptotic formula is applicable (Sazonov and Sunyaev 2000a):

$$p(\mu_{\rm s}) = 1 - \mu_{\rm s}. \tag{2}$$

This expression closely corresponds to the ideal situation when $T_{\rm e} \to \infty$ and $\nu \to 0$ (ν is the initial photon energy). In this case, the angular function is backward-oriented (see Fig. 1). First-order correction terms, one of which corresponds to the Klein–Nishina correction and the other takes into account the electron finite energy, can be added to equation (2) (Sazonov and Sunyaev 2000a):

$$p(T_{\rm e},\nu,\mu_{\rm s}) = 1-\mu_{\rm s}-6\frac{h\nu}{m_{\rm e}c^2}\frac{kT_{\rm e}}{m_{\rm e}c^2}(1-\mu_{\rm s})^2+\frac{-1+3\ln 4+6\Gamma(0,m_{\rm e}c^2/kT_{\rm e})}{8}\left(\frac{m_{\rm e}c^2}{kT_{\rm e}}\right)^2\mu_{\rm s}, \tag{3}$$

where $\Gamma(\alpha,z) = \int_z^\infty x^{\alpha-1}e^{-x}dx$ is the incomplete Gamma function. Formula (3) can serve as a good approximation for $kT_{\rm e} \gtrsim 2m_{\rm e}c^2$ and $h\nu kT_{\rm e} \lesssim 0.01(m_{\rm e}c^2)^2$.

In case (B), the angular function can be described by the formula (Sazonov and Sunyaev 2000b)

$$\begin{aligned} p(T_{\rm e},\nu,\mu_{\rm s}) = \frac{3}{4}\Bigg[&1+\mu_{\rm s}^2-2(1-\mu_{\rm s})(1+\mu_{\rm s}^2)\frac{h\nu}{m_{\rm e}c^2}+2(1-2\mu_{\rm s}-3\mu_{\rm s}^2+2\mu_{\rm s}^3)\frac{kT_{\rm e}}{m_{\rm e}c^2} \\ &+(1-\mu_{\rm s})^2(4+3\mu_{\rm s}^2)\left(\frac{h\nu}{m_{\rm e}c^2}\right)^2+(1-\mu_{\rm s})(-7+14\mu_{\rm s}+9\mu_{\rm s}^2-10\mu_{\rm s}^3)\frac{h\nu}{m_{\rm e}c^2}\frac{kT_{\rm e}}{m_{\rm e}c^2} \\ &+(-7+22\mu_{\rm s}+9\mu_{\rm s}^2-38\mu_{\rm s}^3+20\mu_{\rm s}^4)\left(\frac{kT_{\rm e}}{m_{\rm e}c^2}\right)^2+...\Bigg], \end{aligned} \tag{4}$$

which is a good approximation when $kT_{\rm e} \lesssim 0.05m_{\rm e}c^2$ and $h\nu \lesssim 0.1m_{\rm e}c^2$. The main term in the above expansion series, $3(1+\mu_{\rm s}^2)/4$, describes the standard Rayleigh function, which corresponds to the nonrelativistic case.

Several examples of the scattering angular function for high-temperature electrons, as calculated both from Monte-Carlo simulations and from equations (3) and (4), are presented in Fig. 1.

3 REFLECTION FROM A HOT PLANE-PARALLEL ATMOSPHERE

The angular laws of scattering by an ensemble of Maxwellian electrons described above are readily applicable to any classical problem of radiative transfer in scattering atmospheres. Moreover, as long as the evolution of the photon energy in time and space is unimportant, i.e., the scattering angular function can be considered constant (such a situation takes place, for example, if the energy $h\nu$ remains so small that the Klein–Nishina corrections may be disregarded), we can use some classical results [presented, for example, in the textbooks by Chandrasekhar (1950) and Sobolev (1963)] for hot electron-scattering atmospheres by simply substituting the ensemble-averaged angular function for the classical phase function in the corresponding formulae. Below, we consider the reflection of light from such an atmosphere.

Since Compton scattering involves the transfer of energy between electrons and radiation, it is convenient to perform our analysis in terms of the number of photons rather than in terms of intensity. Our objective is to determine what fraction of the photons incident on the atmosphere is reflected from it after the first scattering. At the same time, this will immediately tell us what fraction of the incident photons can penetrate into the atmosphere while undergoing multiple scatterings, thus taking part in the Comptonization (here, we are talking about an atmosphere with an optical depth $\tau \gtrsim 1$; otherwise, a large fraction of the photons will pass through the atmosphere unscattered). During the subsequent discussion, we restrict ourselves to the case where the energy of the incident photons is low enough for the Klein–Nishina corrections to be unimportant. It should be particularly noted that $\int p(\mu_{\rm s})\, d\mu_{\rm s}/2 = 1$ in this (Thomson) limit.

In the case of a semi-infinite atmosphere with an optical depth $\tau \to \infty$, the number of photons that have undergone a single scattering and emerged in a specified direction $\mu = \cos\theta$, ϕ (the angles are measured relative to the normal to the atmosphere) is given by the relation (Chandrasekhar 1950)

$$N^{(1)}(\mu,\phi;\mu_0,\phi_0) = \frac{1}{4\pi(\mu+\mu_0)} p(\mu,\phi;-\mu_0,\phi_0) F_0, \tag{5}$$

where μ_0, ϕ_0 specify the direction from which the radiation is incident on the atmosphere, F_0 is the incident photon flux per unit surface area, and p is the scattering phase function.

We may introduce the concept of atmosphere albedo with respect to singly scattered photons:

$$A^{(1)}(\mu_0) = \frac{\int\int N^{(1)}(\mu,\phi;\mu_0,\phi_0)\mu\, d\mu d\phi}{F_0}, \tag{6}$$

which is a function of the angle of incidence μ_0.

The calculation using equation (6) presents no difficulty for the two scattering laws of interest: (A) $p = 1 - \mu_{\rm s}$ and (B) $p = 3(1+\mu_{\rm s}^2)/4$ (the Rayleigh law), for which the albedos with respect to singly scattered photons are, respectively,

$$\text{(A)} A^{(1)}(\mu_0) = \frac{1}{2}\left[1 + \frac{1}{2}\mu_0 - \mu_0^2 - \mu_0(1-\mu_0^2)\ln\frac{1+\mu_0}{\mu_0}\right], \tag{7}$$

$$\text{(B)} A^{(1)}(\mu_0) = \frac{3}{16}\left[\frac{8}{3} + \frac{1}{2}\mu_0 - \mu_0^2 - \frac{3}{2}\mu_0^3 + 3\mu_0^4 - \mu_0(3 - 2\mu_0^2 + 3\mu_0^4)\ln\frac{1+\mu_0}{\mu_0}\right]. \tag{8}$$

Case (B) corresponds to the situation where the electrons are cold ($kT_\mathrm{e} \ll m_\mathrm{e}c^2$), whereas case (A) is realized, as follows from the results of Section 2, if the plasma is very hot ($kT_\mathrm{e} \gg m_\mathrm{e}c^2$). The reflective properties of an atmosphere with the scattering law $p = 1 - \mu_\mathrm{s}$ differ markedly from those of a cold atmosphere, as shown by Fig. 2(a). In particular, the single-scattering albedo for normally incident photons ($\mu_0 = 1$) is 0.25 for an atmosphere consisting of ultrarelativistic electrons, whereas the corresponding value for case (B) is 0.168. This implies that 50 per cent more photons can be reflected from a hot atmosphere (after a single scattering) than from a cold one. It is evident from Fig. 2(a) that, even for $kT_\mathrm{e} \gtrsim 2m_\mathrm{e}c^2$, the albedo does not differ much from the value corresponding to the limiting, ultrarelativistic case (A).

If the electrons are only mildly relativistic, the appropriate expression for the albedo can be derived by integrating the angular function (4):

$$A^{(1)}(\mu_0) = \frac{3}{16}\left\{\frac{8}{3} + \frac{1}{2}\mu_0 - \mu_0^2 - \frac{3}{2}\mu_0^3 + 3\mu_0^4 - \mu_0(3 - 2\mu_0^2 + 3\mu_0^4)\ln\frac{1+\mu_0}{\mu_0} + \left[-2\mu_0 + 6\mu_0^2 \right.\right.$$
$$\left.\left. + 16\mu_0^3 - \frac{106}{3}\mu_0^4 - 10\mu_0^5 + 20\mu_0^6 - \mu_0(-2 + 16\mu_0^2 - 42\mu_0^4 + 20\mu_0^6)\ln\frac{1+\mu_0}{\mu_0}\right]\frac{kT_\mathrm{e}}{m_\mathrm{e}c^2}\right\}, \quad (9)$$

where we gave only the correction term proportional to $kT_\mathrm{e}/m_\mathrm{e}c^2$. This formula is accurate enough for $kT_\mathrm{e} \lesssim 0.2m_\mathrm{e}c^2$. As follows from equation (9) and Fig. 2(a), the albedo with respect to singly scattered photons of an atmosphere consisting of mildly relativistic electrons is only slightly larger than that of an atmosphere with the Rayleigh scattering law. The relative difference is less than 5 per cent if $kT_\mathrm{e} \lesssim 0.1m_\mathrm{e}c^2$. We may conclude that, in most practical applications when the reflection of radiation from a plasma with $kT_\mathrm{e} \lesssim 0.1m_\mathrm{e}c^2$ is studied, the required accuracy will be achieved by using the albedo corresponding to a cold atmosphere.

Although formulae (8) and (9) are, strictly speaking, applicable only to an atmosphere whose optical depth is infinite, a similar increase in the albedo when passing from cold to hot electrons will occur for a transparent atmosphere as well. As follows from Monte Carlo simulations, the single-scattering albedo for normally incident photons increases by approximately 50 per cent when passing from cold to ultrarelativistic electrons for arbitrary optical depths of the atmosphere. An example for which we took $\tau = 0.5$, a value typical of the advection flows near accreting compact objects, is shown in Fig. 2(b).

We can also calculate the single-scattering albedo for isotropic incident radiation (when $I_0(\mu_0, \phi_0) = const$):

$$\text{(A)} \qquad A^{(1)}_\mathrm{iso} = \frac{7 - 4\ln 2}{15} \approx 0.282, \quad (10)$$

$$\text{(B)} \qquad A^{(1)}_\mathrm{iso} = \frac{26 - 27\ln 2}{35} \approx 0.208. \quad (11)$$

Thus, we obtained the following result: 36 per cent more photons are reflected (after the first scattering) from a very hot atmosphere than from a cold one.

4 PHOTON DISTRIBUTION IN THE TIME OF ESCAPE FROM A CLOUD OF ELECTRONS

In the previous section, we showed that the fraction of the low-frequency photons incident on a cloud of electrons which become the seed photons for the Comptonization inside the cloud, i.e., the normalization constant for the distribution of multiply (more than once) scattered photons in the number of scatterings, depends on the plasma temperature. In this section, we turn to the shape of this distribution, which is also affected (although to a lesser degree) by the scattering angular function.

Sunyaev and Titarchuk (1980) and Payne (1980) derived analytic formulae for the distribution of photons in the time of their escape from a spherical, optically thick plasma cloud. In their derivation carried out in the diffusion approximation, these authors implicitly assumed that the probabilities of forward ($\mu_{\rm s} > 0$) and backward ($\mu_{\rm s} < 0$) scattering were equal. This condition is actually satisfied, for example, if the scattering angular function is spherical ($p = 1$) or the Rayleigh one [$p = 3(1 + \mu_{\rm s}^2)/4$]. The latter is possible when both the photons and the electrons are nonrelativistic ($h\nu, kT_{\rm e} \ll m_{\rm e}c^2$). However, as we showed in Section 2, the forward–backward symmetry breaks down if the electrons are essentially relativistic.

It is easy to adjust the solution of Sunyaev and Titarchuk (1980) in such a way that it will take into account the dependence of the scattering law on the plasma temperature. To this end, we must use the correct diffusion coefficient, which is given by (see, e.g., Weinberg and Wigner, 1958)

$$D = \frac{1}{3(\sigma/\sigma_{\rm T})(1 - \langle\mu_{\rm s}\rangle)}, \tag{12}$$

where σ is the total scattering cross section, and

$$\langle\mu_{\rm s}\rangle = \frac{\int \mu_{\rm s} p(\mu_{\rm s})\, d\mu_{\rm s}/2}{\int p(\mu_{\rm s})\, d\mu_{\rm s}/2}. \tag{13}$$

In the nonrelativistic limit, $\sigma = \sigma_{\rm T}$, $\langle\mu_{\rm s}\rangle = 0$ and $D = 1/3$; it is the latter value that was used by Sunyaev and Titarchuk (1980).

Let us now assume (as we did in Section 3) that the energy of a photon remains low as it diffuses outward from the cloud while undergoing multiple scatterings. Hence, the Klein–Nishina corrections are unimportant. In this case, $\sigma = \sigma_{\rm T}$, and [see equation (12)] the diffusion coefficient is determined solely by the dependence of $\langle\mu_{\rm s}\rangle$ on the scattering law.

In the ultrarelativistic case ($kT_{\rm e} \gg m_{\rm e}c^2$), it can readily be found, using equation (3), that $\langle\mu_{\rm s}\rangle = -1/3$, and, consequently,

$$D = \frac{1}{4}. \tag{14}$$

Thus, the diffusion of low-frequency photons in an ultrarelativistic plasma proceeds a factor of 4/3 more slowly than it does in a cold one. However, it is unlikely that (14) is of practical value. Indeed, a photon acquires a huge amount of energy when scattering from an ultrarelativistic electron of energy $\gamma m_{\rm e}c^2$ ($\nu'/\nu \sim \gamma^2$); hence, it typically becomes relativistic (being initially of low frequency) after few scatterings, and our statement of the problem in the Thomson limit becomes improper.

In the mildly relativistic limit, using equation (4), we find that $\langle\mu_{\rm s}\rangle \approx -2kT_{\rm e}/5m_{\rm e}c^2$ and

$$D(T_{\rm e}) = \frac{1}{3(1 + 2kT_{\rm e}/5m_{\rm e}c^2)}. \tag{15}$$

We would like to point out that our analysis is similar to the previous studies (Illarionov et al. 1979; Grebenev and Sunyaev 1987), in which the diffusion coefficient for the scattering of relativistic photons by cold electrons was derived. There have also been studies (Cooper 1974; Shestakov et al. 1988) of the effect of relativistic corrections on the so-called transport cross section, which is used to describe the spatial diffusion of radiation in terms of the intensity rather than in terms of number of photons, as is done here.

Using the diffusion coefficient in the form (15), it is easy to recalculate the distributions of photons in escape time that were deduced by Sunyaev and Titarchuk (1980) for various distributions of photon sources over a spherical cloud. For example, if the source lies at the cloud center and if we are interested in the fate of the photons that underwent more scatterings than the average number, i.e., the dimensionless time $u \equiv c\sigma_{\rm T}N_{\rm e}t \gg u_0 = 0.5\tau^2$ (where $\tau \gg 1$ is the optical depth of the cloud along its radius), then

$$P(u) = \frac{2\pi^2 D(T_{\rm e})}{(\tau+2D(T_{\rm e}))^2}\exp\left[-\frac{u\pi^2 D(T_{\rm e})}{(\tau+2D(T_{\rm e}))^2}\right], \tag{16}$$

where $D(T_{\rm e})$ is given by equation (15).

Figure 3 shows plots of $P(u)$ for several values of $kT_{\rm e}$. We see that the agreement between the results of Monte Carlo simulations and the result of equation (16) is satisfactory up to $kT_{\rm e} \sim 0.4m_{\rm e}c^2$. Because of the temperature correction to the diffusion coefficient, photons spend more time (and undergo more inverse Compton scatterings) in the cloud.

The knowledge of how photons are distributed in the time they stay in a scattering cloud is required virtually in all problems related to Comptonization in astrophysical plasmas. In particular, the shapes of Comptonization spectra and the time delays in the arrival of soft and hard radiation from variable X-ray sources strongly depend on this quantity. To solve such problems, it is necessary to additionally study the evolution of the photon energy with time, which is beyond the scope of this paper. Note, however, that much work along these lines (as applied to relativistic thermal plasmas) has already been done (Titarchuk 1994; Hua and Titarchuk 1996; see also references therein).

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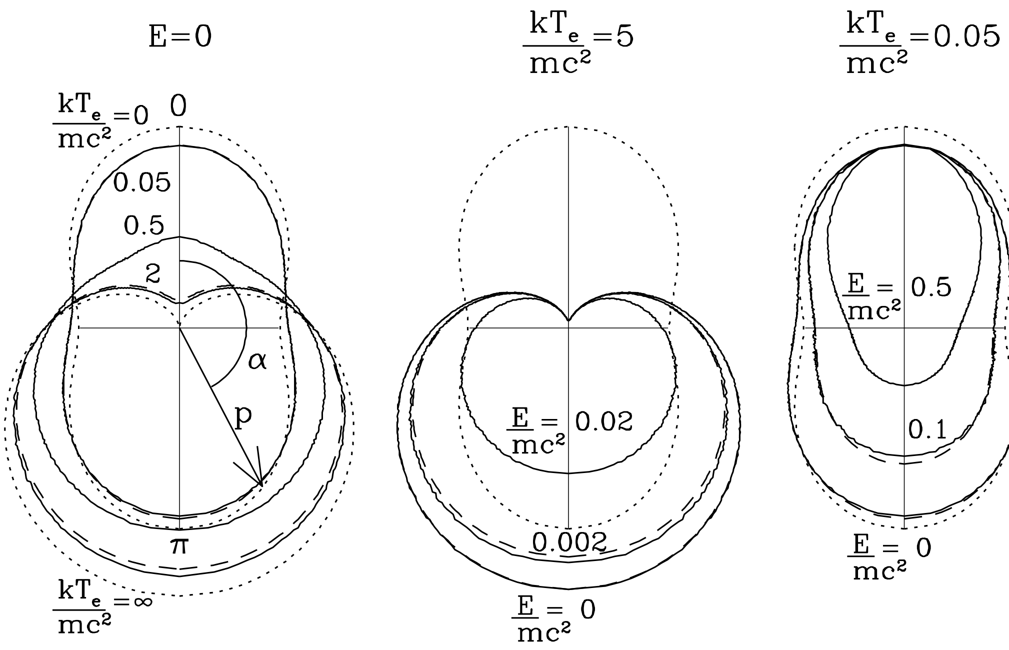

Figure 1: Angular function (in polar coordinates) for the Compton scattering of photons with energy E by an ensemble of relativistic Maxwellian electrons with temperature $T_{\rm e}$. The solid lines indicate the results of our Monte Carlo simulations. The dashed lines represent the fits (when available) given either by equation (3) or by equation (4). The dotted lines refer to the two extreme cases: an infinitesimal temperature (which corresponds to the Rayleigh angular function) and ultrarelativistic electrons ($kT_{\rm e} \gg m_{\rm e}c^2$). In the latter case (presented only in the left panel), the scattering obeys the law $p = 1 - \cos\alpha$.

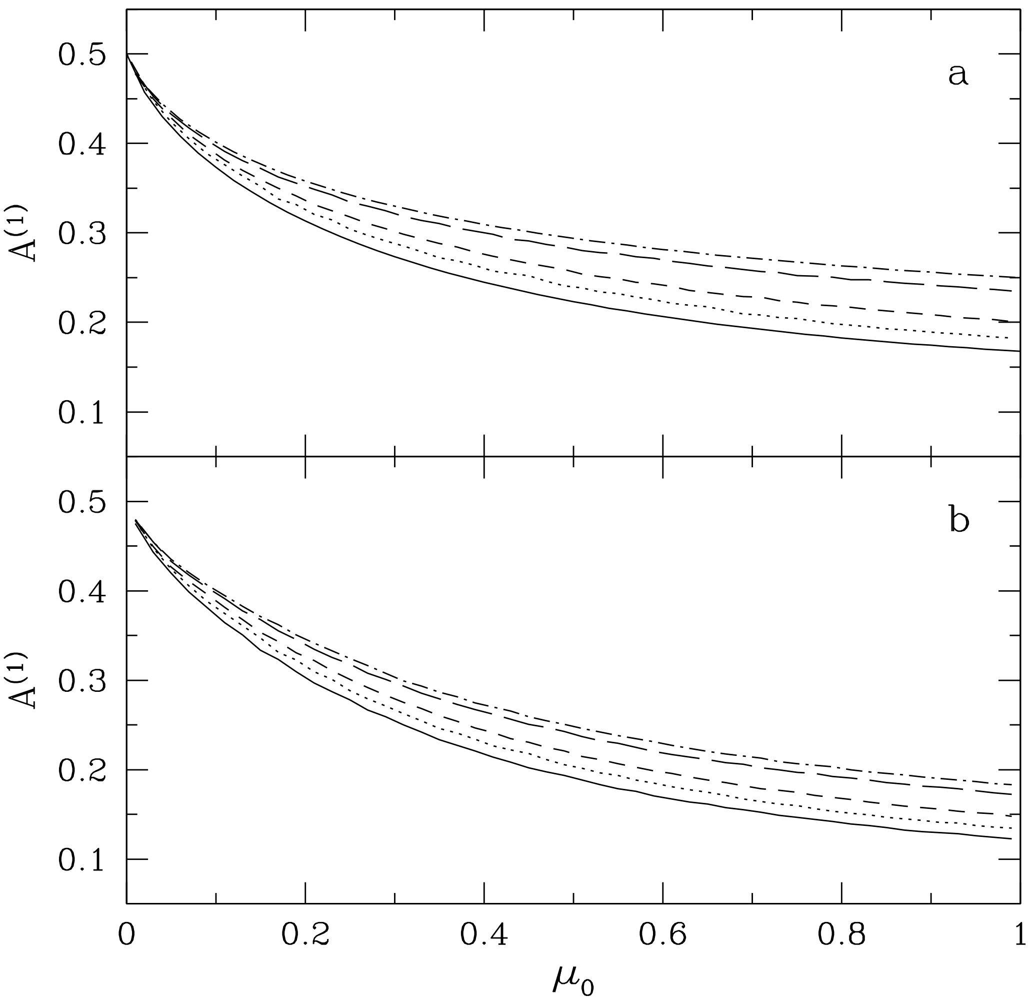

Figure 2: Albedo of an electron-scattering atmosphere with respect to singly scattered photons versus angle of incidence $\alpha = \arccos \mu_0$. The radiation is of low frequency, $h\nu kT_{\rm e} \ll (m_{\rm e}c^2)^2$. The results of our Monte Carlo simulations are shown. (a) The optical depth of the atmosphere is infinite ($\tau \to \infty$). Different curves correspond to different electron temperatures: $kT_{\rm e} \ll m_{\rm e}c^2$ (solid line), $0.2m_{\rm e}c^2$ (dotted line), $0.5m_{\rm e}c^2$ (short dashes), $2m_{\rm e}c^2$ (long dashes) and $\gg m_{\rm e}c^2$ (dash-dotted line). (b) The same as (a), but for a transparent atmosphere with $\tau = 0.5$.

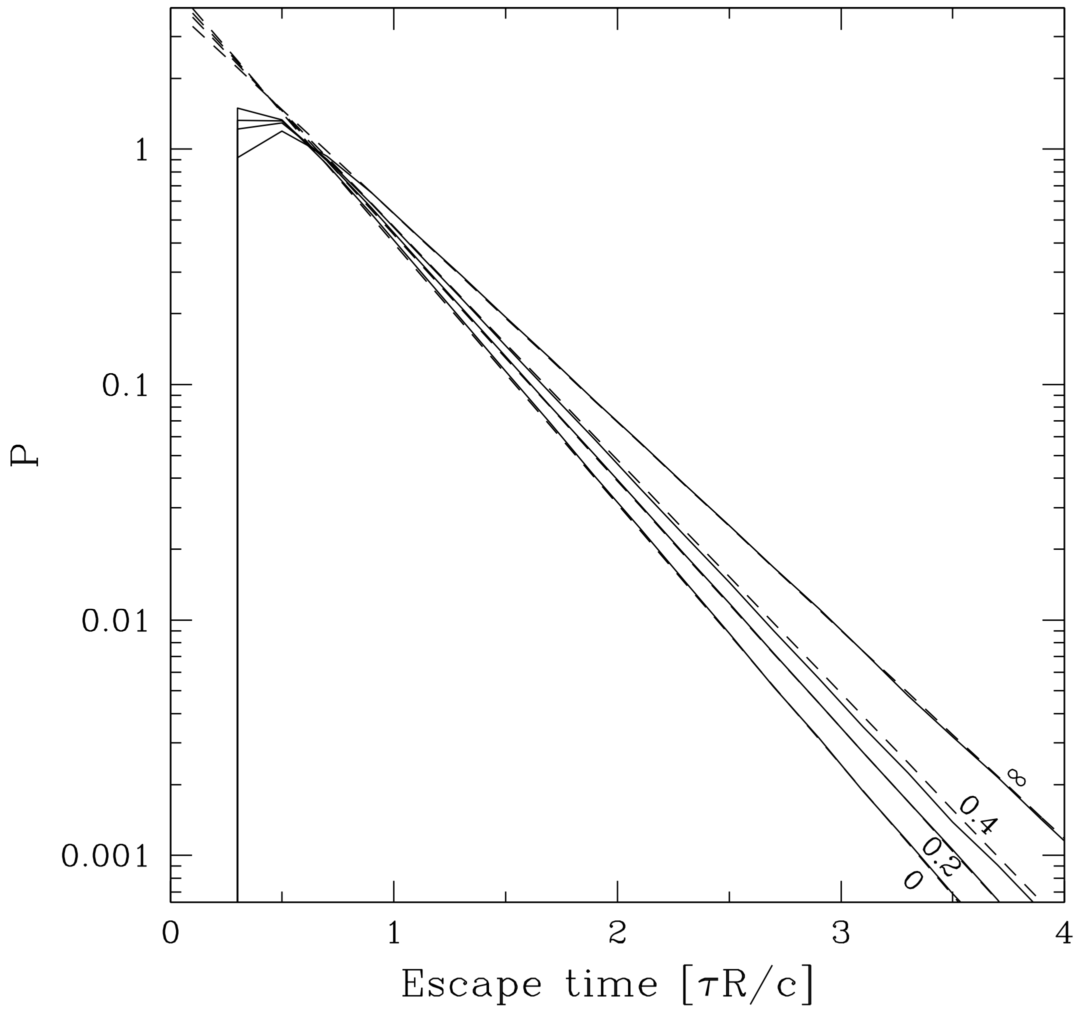

Figure 3: Distribution of low-frequency photons in the time of their escape from a cloud of electrons with the optical depth $\tau = 5$ (R is the cloud radius) for various plasma temperatures (the values of $kT_{\rm e}/m_{\rm e}c^2$ are shown alongside the corresponding curves). The solid lines represent the results of our Monte Carlo simulations. The dashed lines indicate the results of our calculations by means of the approximate formula (16), which can be used to describe the exponential (power-law in the linear-logarithmic coordinates used here) part of the distribution function. The diffusion coefficient D appearing in equation (16) is given by equation (14) in the case $kT_{\rm e} \gg m_{\rm e}c^2$ and by equation (15) in the case $kT_{\rm e} \lesssim 0.4 m_{\rm e}c^2$.