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The chemical pollution of the secondary of a cataclysmic variable by novae

R. Stehle[1], H. Ritter[2]

[1] *Astronomy Group, University of Leicester, Leicester LE1 7RH, U.K.*
[2] *Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, D–85740 Garching, Germany*

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ABSTRACT

We study the chemical evolution of population I and population II secondaries in cataclysmic variables (CVs) assuming that during nova explosions a part of the high metallicity nova ejecta is intercepted by the secondary and mixed into its convective envelope. We derive analytic expressions for the chemical composition of the secondary's envelope as a function of the chemical composition of the nova ejecta $X_{\rm i,ej}$ and the secondary's cross section σ. For Pop I CVs we find that the increase of the secondary's metallicity is comparable to its initial metallicity only if σ is larger by an order of magnitude than the geometrical cross section. A significant accumulation is therefore possible only in those species which are highly overabundant in the nova ejecta. Because the changes in the abundances of even those species depend strongly on the poorly known cross section σ, the predictive power of our model is weak for Pop I CVs as long as σ is not well determined. In case of Pop II CVs the accumulation of heavy elements by this process dominates over the initial metallicity of the secondary even for values of σ which are smaller by an order of magnitude than the geometrical cross section. Thus, within a short time after turn–on of mass transfer, the relative metal abundances in the secondary's envelope reflect those in the nova ejecta. This is nearly independent of the cross section σ.

Key words: novae, cataclysmic variables — binaries: close — stars: evolution

1 INTRODUCTION

The global picture of a cataclysmic variable (CV) as a mass transferring close binary consisting of a low mass star (i.e. the secondary) which fills its critical Roche volume, and a mass accreting white dwarf (WD) is now well established. The long–term (i.e. secular) evolution of the mass transfer in CVs has been successfully modelled in the past in numerous investigations, including Kolb (1993) and Stehle, Kolb & Ritter (1997). Most of these calculations are based on the model of disrupted magnetic braking (Spruit & Ritter 1983, Rappaport, Verbunt & Joss 1983). In order to account for the period gap of CVs, i.e. the observed dearth of systems in the period range $2h \lesssim P \lesssim 3h$ (see e.g. Ritter & Kolb 1998) this model postulates that orbital angular momentum loss by a magnetic wind from the secondary becomes rapidly less efficient as soon as the secondary becomes fully convective.

Whereas in the above–mentioned calculations the WD is treated as a point mass, more recent work has relaxed this simplification and focused on the questions to what extent accretion on to the WD and its reaction upon it can influence the secular evolution. Ritter, Zhang & Kolb (1995), Ritter, Zhang & Hameury (1996) and King, Frank, Kolb and Ritter (1997) show, e.g., that irradiation of the secondary by the high energy accretion luminosity coming from the WD can give rise to mass transfer cycles in long period CVs. Besides irradiation the secular evolution of CVs is likely to be affected by another consequence of accretion: namely nova explosions. Model calculations (e.g. Prialnik & Kovetz 1995) show that hydrogen burning begins at the base of the accreted hydrogen–rich envelope, resulting in a thermonuclear runaway, after the accumulation of $\sim 10^{-7}$–$10^{-4}\,M_\odot$. Given the typical mass transfer rates in CVs of $|\dot M_2| \approx 10^{-10} - 10^{-9}\,M_\odot{\rm yr}^{-1}$ the interoutburst times are of the order $\sim 10^3$–10^5yr. As a result of a nova outburst the binary system is engulfed in the ejected nova envelope for some time. During this common envelope phase, which can last anywhere from months to several years, angular momentum is transferred from the orbit to the expanding nova shell. This process thus results in what is known as frictional angular momentum loss (see e.g. Livio, Govarie & Ritter 1991, Schenker, Kolb & Ritter 1998). In addition to this, as has recently been pointed out by Stehle (1993), Stehle & Kolb (1994) and Stehle & Ritter (1994, poster presented at the Padova–Abano Conference on Cataclysmic Variables), part of the nova ejecta might also be intercepted by the secondary and mixed into its convective envelope. Because the chemical composition of the nova ejecta is often highly non–solar, the chemical composition of the secondary's convective envelope can be drastically changed. It is commonly accepted that the observed high metallicity in nova ejecta (see Livio & Truran 1994 for a summary) originates from mixing WD core material into the accreted hydrogen–rich envelope before or during the thermonuclear runaway. Depending on the internal structure, i.e. the mass of the WD, the envelope will either mainly be enriched with carbon (C) and oxygen (O) (in the case of a CO WD), or with O, neon (Ne) and magnesium (Mg) (in case of a higher mass O Ne Mg WD). During an outburst the already peculiar chemical composition will be further changed through nuclear burning. The main result of the latter is to transform most of the C and O into N (see e.g. Wiescher, et al. 1986; Weiss & Truran 1990 for details).

Recently Marks, Sarna & Prialnik (1997) have pursued the idea that the chemical composition of the secondary's envelope can change as a consequence of nova outbursts. Based on results obtained by Prialnik & Kovetz (1995) for a grid of nova simulations they computed the temporal evolution of the secondary's chemical composition in population I CVs (Pop I CVs), i.e. the class of CVs in which the initial metallicity of the secondary is typically $Z_{2,\mathrm{in}} \simeq 2\%$. Because they started from a value for $Z_{2,\mathrm{in}}$ which was already high and because the contribution from pollution to the secondary's chemical composition depends strongly on the cross section σ with which the nova ejecta are mixed into its convective envelope (see Sec. 3), some of their results, especially the absolute abundance of the secondary's convective envelope depend strongly on the assumed value for σ. Yet the precise value of σ is only poorly known, to say the least, as we shall show in Sec. 2. Only for relatively large values of σ does the metallicity of the envelope change significantly as a result of pollution through novae. With regard to the unknown value of σ it is the purpose of this paper to investigate how strongly the chemistry of CV secondaries can be affected by pollution due to novae and what cross sections are needed to leave significant chemical fingerprints in the secondary's convective envelope.

On the other hand, Stehle, Kolb & Ritter (1997) have pointed out that the situation is completely different for population II CVs (Pop II CVs) which are born with an initial metallicity as low as $Z_{2,\mathrm{in}} = 10^{-4}$. There, as we shall show in a semi–analytical model in Sec.(3), a value of σ as low as one tenth of the geometrical cross section of the secondary as seen by the WD is already sufficient for the chemical composition of the secondary's envelope to be dominated by the composition of the intercepted nova ejecta. Furthermore, as we shall show analytically in Sec.(4), the chemical pollution of the envelope of the secondary in a Pop II CV can drive additional mass transfer which is of the same order as that driven by magnetic braking. In Sec.(5) we illustrate and confirm our analytic approach with numerical calculations using a full stellar evolution code. We finish our paper in Sec.(6) with a discussion of our results.

2 ACCRETION ON TO THE SECONDARY DURING NOVAE

Mass is transferred from the secondary through the nozzle at the inner Lagrangian point L_1 with a mass loss rate $|\dot{M}_{\mathrm{L}_1}|$ and eventually accreted by the WD. During a nova outburst the mass which has accumulated since the previous outburst and also a part of the WD core mass is ejected by the WD, most of it escaping from the binary. A small part of the ejecta is however intercepted by the secondary and eventually mixed into its convective envelope. Thus during a nova outburst the secondary accretes from the nova envelope. We denote the time–averaged accretion rate from this process by $\dot{M}_{\mathrm{int}}$. Regardless of the details of the accretion process we can relate the mass transfer rate $\dot{M}_{\mathrm{L}_1} < 0$ to the mass accretion rate $\dot{M}_{\mathrm{int}}$ by

$$\dot{M}_{\mathrm{int}} = -g\left(\dot{M}_{\mathrm{L}_1} + \dot{M}_{\mathrm{WD}}\right) = -\alpha \dot{M}_{\mathrm{L}_1} , \tag{1}$$

where $\dot{M}_{\mathrm{WD}}$ denotes the average rate of change of the WD mass during a nova cycle and

$$\alpha = g\left(1 + \frac{\dot{M}_{\mathrm{WD}}}{\dot{M}_{\mathrm{L}_1}}\right) . \tag{2}$$

For convenience we also introduce the cross section σ of the secondary, which is related to g by

$$\sigma = 4\pi a^2 g \tag{3}$$

where a is the orbital separation of the binary.

The values of g and α are at present unknown and have yet to be specified. As illustrated in Fig.(1), at least that fraction of the ejecta which corresponds to the solid angle of the secondary as seen from the WD, i.e.

$$g_{\mathrm{geo}} = \frac{1}{2}\left[1 - \cos\left(\arc\sin\frac{R_2}{a}\right)\right] , \tag{4}$$

where R_2 is the secondary's radius, hits the secondary and interacts with its atmosphere (assuming isotropic mass ejection). In addition, the secondary attracts the ejecta gravitationally, thus increasing the effective cross section.

The gravitational cross section σ_{BH} of a single star accreting from a steady and parallel flow of gas was first derived analytically under the assumption of vacuum boundary conditions by Bondi & Hoyle (1944) and Bondi (1952). Later time–dependent, adiabatic calculations (e.g. Ruffert 1994, Ruffert & Arnett 1994) have demonstrated that the Bondi–Hoyle cross section for a parallel flow on to a vaccum accretor is accurate to within $\sim 10\%$.

However, it is not at all clear whether application of the Bondi–Hoyle cross section is appropriate for the situation in question. After all, in a close binary system the flow of the ejecta is neither parallel nor steady and the accretor is not spherically symmetric. Instead the secondary, filling its critical Roche volume, is highly deformed, and the flow past it is divergent and highly time dependent. In addition, because we are dealing here with accretion on to a star, where the incoming matter has first to be stopped before it can be accreted (in contrast to a black hole) the vacuum boundary condition under which the Bondi–Hoyle cross section was derived does not apply.

The efficiency of accretion on to a star depends strongly on the Mach number M of the accretion flow (Kley, Shankar and Burkert 1995) and, in the case where $M > 1$, how efficiently the energy dissipated in the accretion shock can be

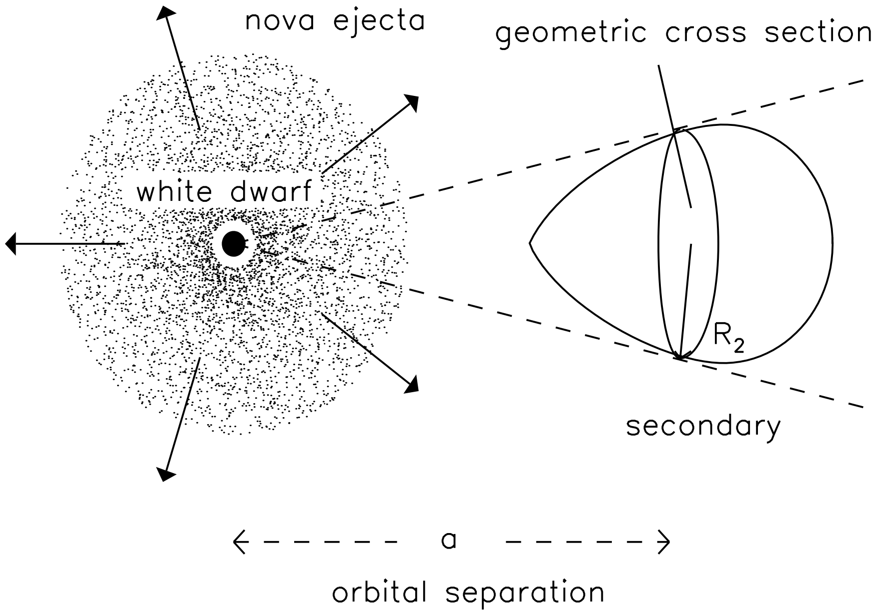


Figure 1. A cartoon illustrating the geometry of the binary during a nova explosion. Mass is ejected from the WD, part of which flows past the secondary, is intercepted by it and mixed into its convective envelope.

radiated away. Efficient cooling requires that close to the secondary the nova ejecta have optical depth of order unity. If the optical depth is too small they cannot radiate efficiently. On the other hand, if it is too high the thermal time scale of the gas is longer than the accretion time scale, and because of the high entropy of the inflowing gas an entropy shield is established around the secondary with the result that only a fraction of the nova ejecta within the solid angle subtended by the secondary as seen from the WD can be accreted (see e.g. Webbink 1988). Kley, Shankar and Burkert (1995) confirmed in terms of numerical calculations that for an optically thick supersonic wind the cross section is orders of magnitudes smaller than $\sigma_{\rm BH}$, if not zero. However, radiation pressure will eventually dominate over gas pressure if the temperature of the nova ejecta is high or its density low, resulting in a subsonic mass flow. The accretion then occurs in a more Bondi–Hoyle fashion with a cross section similar to $\sigma_{\rm BH}$ (Kley, Shankar & Burkert 1995). At present the situation is unclear and the cross section of the secondary can be anything between zero and $\sigma_{\rm BH}$.

On top of all these uncertainties the amount of mass accreted will also depend on the anisotropy of the envelope ejection. If ejection occurs preferentially along an axis pointing away from the orbital plane of the secondary the accretion rate on to the secondary will be smaller than if ejection is isotropic or if it occurs preferentially in the orbital plane.

In view of the fact that neither g nor α is well constrained we shall subsequently investigate the chemical evolution of the secondary as a function of α in terms of analytic approximations. We will thus derive lower limits for the cross section of the secondary for which we expect pollution to become important.

3 THE CHEMICAL COMPOSITION OF THE SECONDARY'S CONVECTIVE ENVELOPE

3.1 An analytic model

In Fig. (2) we sketch the mixing of an arbitrary chemical element i into the convective envelope of a low–mass secondary. Denoting the mass of the secondary by M_2 and that of its radiative core by $M_{\rm rad}$ the mass of its convective envelope is given by $M_{\rm con} = M_2 - M_{\rm rad}$. Thermohaline mixing can set in at times where the total pollution since onset of mass transfer results in a significant discontinuity in the mean molecular weight across the boundary between the radiative core and the convective envelope (Ulrich 1972, Kippenhahn, Ruschenplatt & Thomas 1980). Then $M_{\rm con}$ is determined by the bottom of the thermohaline mixing zone.

By $X_{\rm i,con}$,$X_{\rm i,rad}$, and $X_{\rm i,ej}$ we denote respectively the mass fraction of the element i in the secondary's convective envelope, in its radiative core, and in the nova ejecta. If $\dot{M}_{\rm L_1}$ is the average mass loss rate through the L_1 point and $\dot{M}_{\rm int}$ the time averaged mass accretion rate due to novae, the average mass loss rate from the secondary star is $\dot{M}_2 = \dot{M}_{\rm L_1} + \dot{M}_{\rm int}$. In the following we shall assume that the matter accreted during a nova outburst is instantly mixed into the secondary's convective envelope, whereas the chemical composition of the core remains unaffected. The chemical composition of the core does change, however, because of nuclear evolution, but only on the rather long nuclear

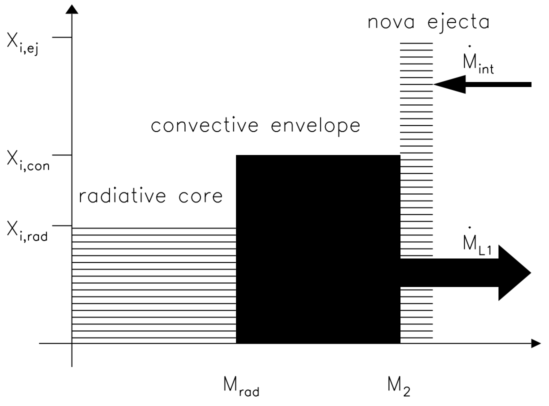


Figure 2. A sketch illustrating the change of the chemical composition in the convective envelope. Mass is lost from the L_1 point with a loss rate $\dot{M}_{\rm L_1}$ and partly re–accreted during a nova outburst at the time averaged rate ($\dot{M}_{\rm int}$). In addition, with decreasing M_2, core mass is mixed into the convective envelope.

time scale. Therefore we shall neglect nuclear evolution of the core in our simple analytical description.

With decreasing mass of the secondary the mass of the radiative core shrinks, and unpolluted but possibly nuclear–processed material from the core is mixed into the convective envelope. The total mass in the convective envelope of the i–th element, given by $(X_{\rm i,con}M_{\rm con})$, will thus change with time through mass transfer, mass accretion and mixing from the secondary's radiative core. The balance equation for $(X_{\rm i,con}M_{\rm con})$ reads

$$\frac{\partial(X_{\rm i,con}M_{\rm con})}{\partial t} = -\frac{\partial(X_{\rm i,rad}M_{\rm rad})}{\partial t} + \dot{M}_{\rm L_1}X_{\rm i,con} + \dot{M}_{\rm int}X_{\rm i,ej} \quad (5)$$

as long as $\partial M_{\rm rad}/\partial t < 0$, which is always fulfilled for low mass stars. Rewriting eq. (5) for $\alpha < 1$ yields

$$\begin{aligned}\frac{\partial X_{\rm i,con}}{\partial t} &= \frac{1}{M_{\rm con}}\left[\frac{\partial(M_{\rm rad})}{\partial t}\,(X_{\rm i,con}-X_{\rm i,rad})\right. \\ &+ \left.\frac{\alpha}{1-\alpha}\dot{M}_2\,(X_{\rm i,ej}-X_{\rm i,con})\right],\end{aligned} \quad (6)$$

where we have made use of $\dot{M}_{\rm ej} = -\alpha\dot{M}_{\rm L_1} = g(\dot{M}_{\rm L_1}+\dot{M}_{\rm WD})$ (eq.1). For $(X_{\rm i,con}-X_{\rm i,rad}) > 0$ the mixing of core material into the envelope results in a dilution of the i–th element in the convective zone and for $(X_{\rm i,ej}-X_{\rm i,con}) > 0$ the mass accretion during nova outbursts results in its enrichment.

For the case $\alpha = 1$ (i.e. $\dot{M}_{\rm int} = -\dot{M}_{\rm L_1}$ or $\dot{M}_2$=0) equation (5) has to be solved explicitly as a function of time and rewriting it in terms of $\dot{M}_2 = 0$ is impossible. As this is a rather unrealistic situation we do not proceed any further on this special case.

3.2 Fully convective stars

Let us start investigating eq. (6) by considering fully convective stars. In that case $M_{\rm 2,rad} = \dot{M}_{\rm 2,rad} = 0$ and eq. (6) reads

$$\frac{\partial X_{\rm i,con}}{\partial t} = \frac{\alpha}{1-\alpha}\frac{|\dot{M}_2|}{M_2}(X_{\rm i,ej}-X_{\rm i,con})\,. \quad (7)$$

Integrating (7) yields

$$X_{\rm i,con}(M_2) = X_{\rm i,ej} + \left(X_{\rm i,con}^{\rm in}-X_{\rm i,ej}\right)\left(\frac{M_2}{M_2^{\rm in}}\right)^{\alpha/(1-\alpha)}, \quad (8)$$

where $X_{\rm i,con}^{\rm in}$ and $M_2^{\rm in}$ are respectively the initial mass fraction of the i–th element and the initial mass of the secondary at the onset of mass transfer.

As an example let us consider the evolution of the total metallicity Z of a Pop I ($Z_{\rm con}^{\rm in} = 0.02$) and a Pop II ($Z_{\rm con}^{\rm in} = 10^{-4}$) CV with a fully convective secondary of initially $M_2^{\rm in} = 0.3M_\odot$. Eq. (8) then yields for the secondary's metallicity $Z^{\rm I}(M_2 = 0.1M_\odot) = 0.0260$ (Pop I CV) and $Z^{\rm II}(M_2 = 0.1M_\odot) = 0.0068$ (Pop II CV), if we assume $Z_{\rm ej} = 0.2$, $g = g_{\rm geo} \simeq 0.03$ for the cross section and $\dot{M}_{\rm WD} = 0$. Thus in both cases the metallicity is increased by about 1/2%. This value, however, can be higher if $Z_{\rm ej}$ is larger. Thus a significant increase compared to the initial metallicity is expected for both populations. As discussed in Sec. 2, however, the secondary's cross section can be at least one order of magnitude smaller than the geometrical cross section. For $g = 0.1\,g_{\rm geo} \simeq 3\,10^{-3}$ we find $\Delta Z_{\rm poll} \simeq 6\,10^{-4}$. A value 10 times smaller than the geometrical cross section will therefore still alter the chemistry of Pop II secondaries significantly, whereas Pop I secondaries are left fairly unaffected by pollution.

3.3 Low–mass stars with a radiative core

In the case of a low–mass star with a radiative core eq. (6) can be solved only if the change of the mass of the radiative core with total mass is known. This, however, cannot be derived in closed form from first principles. Therefore we are forced to take this information from full stellar evolution calculations. We will show in Sec. 5 that the mass of the radiative core decreases approximately linearly with the total mass of the secondary. Therefore we can approximate the evolution of the mass of the radiative core and of the convective envelope by

$$M_{\rm con} = m_1 + m_2M_2 \quad (9)$$

$$M_{\rm rad} = -m_1 + (1-m_2)M_2\,, \quad (10)$$

where m_1 and m_2 have to be determined in full stellar evolution calculations from the bottom of the thermohaline mixing zone.

Although eqs. (9) and (10) are not particularly accurate over the whole mass range we are interested in, the approximation will always be correct locally (see Fig.5). Using eqs. (9) and (10) we find as a solution for eq. (6)

$$X_{\rm i,con} = -\frac{a_1}{a_2} + fM_{\rm con}^{-(a_2/m_2)}\,, \quad (11)$$

where

$$a_1 = (1-m_2)X_{\rm i,rad} + \frac{\alpha}{1-\alpha}X_{\rm i,ej} \quad (12)$$

and

$$a_2 = m_2 - 1 - \frac{\alpha}{1-\alpha}\,. \quad (13)$$

f depends on the initial values $M_2^{\rm in}$ and $X_{\rm i,con}^{\rm in}$ and is given by

$$f = \frac{1}{a_2} \frac{a_1 + a_2 X_{\rm i,con}^{\rm in}}{(m_1 + m_2 M_2^{\rm in})^{-a_2/m_2}} \,. \qquad (14)$$

Later (Sect. 5) we shall find from numerical calculations that m_2 is of the order $m_2 \simeq -0.5$ ... -1.0. Assuming for the moment $m_2 = -0.75$ and $g = g_{\rm geo}$ we find $a_2 \simeq -2$ and $-a_2/m_2 \simeq -2.5$. f turns out to be rather small for secondaries with an initially small convective envelope. Therefore the second term on the right hand side of eq. (11) is important only if we are interested in the chemical composition shortly after the onset of mass transfer. Asymptotically, i.e. for $M_2 \ll M_2^{\rm in}$ we find that $X_{\rm i,con}$ is given by

$$\begin{aligned} X_{\rm i,con}^{\rm a} &= -\frac{a_1}{a_2} = \\ &= \frac{(1-m_2)X_{\rm i,rad} + \alpha/(1-\alpha)X_{\rm i,ej}}{1 - m_2 + \alpha/(1-\alpha)} \,. \end{aligned} \qquad (15)$$

This result can also be obtained directly from the balance equation (6) by setting $\partial_t X_{\rm i,con} = 0$. This also shows that $X_{\rm i,con}^{\rm a}$ is an extremum and thus a limiting value. Esp. for the initial condition where $X_{\rm i,con} < X_{\rm i,ej}$, as this is true cf. for the total metallicity, the value given by $X_{\rm i,con}^{\rm a}$ is a maximum and thus an upper limit. The total time needed to reach the limiting value $X_{\rm i,con}^{\rm a}$ will be shorter for smaller $M_{\rm con}^{\rm in}$, and the mass transfered lower.

$X_{\rm i,con}^{\rm a}$ is dominated by the chemical composition of the nova ejecta if

$$\frac{\alpha}{1-\alpha} X_{\rm i,ej} > (1-m_2) X_{\rm i,rad}. \qquad (16)$$

This can be written as a condition for α which is sufficient to leave recognizable fingerprints on the chemical composition of the secondary by nova pollution. This condition reads

$$\alpha \gtrsim (1-m_2) X_{\rm i,rad}/X_{\rm i,ej}. \qquad (17)$$

For Pop II CVs with $Z_{\rm rad} \simeq 10^{-4}$ a value $\alpha \sim 10^{-3}$ is already sufficient. This is a factor ~ 10 smaller than the value obtained by assuming the geometrical cross section for σ. It is therefore likely that chemical pollution of the secondary's convective envelope is important in Pop II CVs. In Pop I CVs, however, we require $\alpha \geq 0.1$ for the composition of the nova ejecta to dominate that of the secondary. This is a factor ~ 3 larger than obtained with the geometrical cross section and thus larger than estimated realistic. However, because nova ejecta can be drastically overabundant in some particular constituents, e.g. in Ne in the case of Ne–novae, the chemical composition of the secondary can be significantly changed even in Pop I CVs as far as these constituents are concerned.

Let us now estimate typical values for $Z_{\rm con}^{\rm a}$. For a PopI CV with $Z_{\rm rad}^{\rm I} = 0.02$, $m_2 \simeq -0.5$ (taken from full stellar evolution calculations, see Fig.5) we find $Z_{\rm con}^{\rm I,a} = 0.0235$. For a Pop II CV with $Z_{\rm rad}^{\rm II} = 10^{-4}$ and $m_2 \simeq -1$ (see Fig.5) the balance of mixing core material and nova ejecta into the envelope yields $Z_{\rm con}^{\rm II,a} = 0.0031$. In both cases we have assumed $Z_{\rm ej} = 0.2$, $g = g_{\rm geo} = 0.03$ and $\dot{M}_{\rm WD} = 0$. These examples show that if the geometrical cross section is adopted a significant change in the envelope's metallicity is expected only for the secondaries of Pop II CVs. For cross sections smaller than the geometrical cross section, say for $g = 0.1\, g_{\rm geo} \simeq 3\,10^{-3}$, we find $Z_{\rm con}^{\rm I,a} = 0.02036$ and $Z_{\rm con}^{\rm II,a} = 4\,10^{-4}$, i.e. a negligible increase in metallicity for Pop I CVs, but still noticeable in Pop II CVs. Uncertainties in the physics of the secondary's convective envelope, i.e. errors in the value of m_2 which we assumed to be -0.5 and -1.0 respectively, will add to the above error by about 1% for Pop I CVs and about 25% for Pop II CVs. Both of these errors are negligible compared to the order of magnitude approximation of the secondary's cross section.

4 POLLUTION DRIVEN MASS TRANSFER

We have shown in the previous section that in a Pop II CV pollution, as a long–term consequence of nova outbursts, can significantly change the chemical composition of the convective envelope of the secondary. As a result of this process the secondary attains a rather peculiar chemical structure. Whereas the metallicity in the core remains that of a Pop II star, that in its envelope approaches gradually that of a Pop I star. With the metallicity in the convective envelope, also the opacity in the photosphere and the sub-photospheric layers (where H^- is the dominant source of opacity) and the mean molecular weight increases. This in turn has a direct influence on the stellar radius. Because the equilibrium radius of a main sequence star of a given mass is larger for higher metallicity, a chemically polluted Pop II star in thermal equilibrium is larger than an unpolluted one (i.e., $R_{\rm e}^{\rm II,pol} > R_{\rm e}^{\rm II}$, where $R_{\rm e}^{\rm II,pol}$ is the thermal equilibrium radius of the polluted star and $R_{\rm e}^{\rm II}$ that of the unpolluted one). In the context of CV evolution this means that pollution of the secondary through novae can give rise to enhanced mass transfer. This effect is particularly important if the relative increase of the metallicity occurs on a short time scale (which is the case if the secondary initially has only a shallow convective envelope) such that the star is driven significantly out of thermal equilibrium. In that case the secondary tries to adjust its radius as fast as it can, i.e. on the thermal time scale of the convective envelope, to the radius given by the uniform evolution corresponding to its actual chemical composition (Stehle, Ritter Kolb 1996). The relaxation time scale can be written as

$$\tau_{\rm relax} = c\, \tau_{\rm KH} \,, \qquad (18)$$

where $\tau_{\rm KH}$ is the Kelvin–Helmholtz time of the secondary and the correction factor c can be estimated analytically using homology. Following Stehle, Ritter & Kolb (1996) we find $c \simeq 0.02$.

The contribution to thermal relaxation of the secondary from chemical pollution of its envelope drives an additional mass transfer at a rate $\dot{M}_{\dot{Z}}$. To estimate $\dot{M}_{\dot{Z}}$ we use (e.g. Ritter 1996)

$$\begin{aligned} -\dot{M}_2 &= -(\dot{M}_{\dot{Z}} + \dot{M}_{\tau_{\rm J}}) \\ &= \frac{M_2}{\zeta_{\rm s} - \zeta_{\rm R}} \left[\left(\frac{\partial \ln R_2}{\partial t} \right)_{\dot{Z}} + \left(\frac{\partial \ln R_2}{\partial t} \right)_{\rm th} - \frac{2}{\tau_{\rm J}} \right] , \end{aligned} \qquad (19)$$

where the first term in the brackets on the right hand side has been added to take into account thermal relaxation because of pollution. The contribution to thermal relaxation due to mass loss is denoted by $(\partial \ln R_2/\partial t)_{\rm th}$. $\zeta_{\rm s}$ is the adiabatic mass–radius exponent of the secondary, $\zeta_{\rm R}$ the mass radius exponent of the corresponding Roche radius, and $\tau_{\rm J}$ the time scale of angular momentum loss. For a rough estimate of $\dot{M}_{\dot{Z}}$ in a Pop II CV we need to know the time scale on which the radius of the secondary would change in the absence of mass loss from R_e^II to $R_\mathrm{e}^\mathrm{II,pol}(M_2)$. If the time scale τ_pol on which the secondary is polluted is long compared to τ_relax, the stellar radius can closely follow its changing equilibrium radius and the relevant time scale is thus τ_pol. In the opposite case, i.e. if $\tau_\mathrm{pol} < \tau_\mathrm{relax}$ the radius increases on the relaxation time scale. Thus the change of the radius will occur on the longer of the two time scales τ_relax or τ_pol.

An estimate of the upper limit for $\dot{M}_{\dot{Z}}$ is easily obtained as follows: If $\tau_\mathrm{pol} \ll \tau_\mathrm{relax}$, then

$$\left(\frac{\partial \ln R_2}{\partial t}\right)_{\dot{Z}} \simeq \frac{1}{\tau_\mathrm{relax}} \frac{R_\mathrm{e}^\mathrm{II,pol} - R_\mathrm{e}^\mathrm{II}}{R_\mathrm{e}^\mathrm{II}}. \tag{20}$$

For the ratio of the mass transfer rate driven by chemical pollution $\dot{M}_{\dot{Z}}$ to that caused by angular momentum loss $\dot{M}_{\tau_\mathrm{J}}$ we find

$$\rho = \frac{\dot{M}_{\dot{Z}}}{\dot{M}_{\tau_\mathrm{J}}} \simeq 0.5 \frac{R_\mathrm{e}^\mathrm{II,pol} - R_\mathrm{e}^\mathrm{II}}{R_\mathrm{e}^\mathrm{II}} \frac{\tau_\mathrm{J}}{c\tau_\mathrm{KH}}\,. \tag{21}$$

Assuming a relative radius change due to pollution of 5% and $\tau_\mathrm{J}/\tau_\mathrm{KH} \simeq 1$, a value typical for systems driven by magnetic braking, eq. (21) yields $\rho \simeq 1$. This shows that mass transfer driven by chemical pollution can be as important as mass transfer driven by angular momentum loss. However, this is the case only for about one relaxation time scale τ_relax after the onset of mass transfer, and only if the initial mass of the convective envelope is small enough that $\tau_\mathrm{pol} \ll \tau_\mathrm{relax}$. Detailed numerical calculations which we shall describe in the next section confirm our simple estimate.

5 NUMERICAL CALCULATIONS

We now perform numerical calculations to illustrate our analytic approach (see also Stehle 1993, Stehle & Kolb 1994, and Stehle & Ritter 1994, poster presented at the Padova–Abano Conference on Cataclysmic Variables). Similar calculations were done by Marks, Sarna & Prialnik (1997) including thermohaline mixing due to the effect of an inversion in mean molecular weight in the surface layers of the secondary (Ulrich 1972, Kippenhahn, Ruschenplatt & Thomas 1980). They also made use of the grid of nova simulations by Prialnik & Kovetz (1995) to derive the change in the composition of the ejecta on a mass transfer time scale. However, as we regard our calculations mainly as an illustration for our analytic analysis and as the secondary's cross section is determined only poorly to date we neglect thermohaline mixing and assume a sensible but constant chemistry for the nova ejecta (Livio, Truran 1994). The predictive power of these models will be poor in any case as long as the cross section with which the nova material is intercepted by the secondary is not determined with better accuracy. For our numerical calculations we use the stellar evolution code of Mazzitelli (1989) with OPAL opacities (Iglesias & Rogers 1996). Mass transfer from the secondary star was calculated using the method described in Kolb & Ritter (1992). Our calculations were done in the framework of the model of disrupted magnetic braking (Spruit & Ritter 1983, Rappaport, Verbunt & Joss 1983). In this model it is assumed that angular momentum loss via gravitational radiation operates for any mass and internal structure of the secondary, whereas magnetic braking does so only as long as the secondary is not fully convective. In our calculations we compute the rate of loss of orbital angular momentum using the parametrisation of Verbunt & Zwaan (1981) with the free parameter $f_\mathrm{VZ} = 1$. For a more detailed description of the standard secular evolution of a CV we refer to Kolb & Ritter (1992) or Stehle, Kolb & Ritter (1997).

In the simulations described below we have taken into account accretion on the secondary from nova ejecta as follows: For simplicity we assume that the nova envelope is ejected isotropically and that $g = g_\mathrm{geo}$, i.e. that the geometrical solid angle of the secondary is appropriate, although there is no a priori justification for this particular choice (see our discussion in Sec.2). Furthermore, the chemical composition of the ejecta is assumed to be $Y_\mathrm{ej} = 0.25$ and $Z_\mathrm{ej} = 0.2$ (see Tab. 1). Although the high fraction of metals in the nova ejecta requires the mass of the WD to decrease with time, we neglect this in our calculations, i.e. we assume for simplicity $\dot{M}_\mathrm{WD} = 0$. This assumption also allows to compare the results of our calculations directly with those of standard CV evolution (e.g. Stehle, Kolb & Ritter 1997). Given the poorly–determined cross section (see Sec. 2) on which any result for the chemical evolution of the secondary will depend sensitively, the above simplifications will not contribute significantly to the errors made at present.

Using these assumptions we have computed the evolution of a Pop I and Pop II CV with constant WD mass $M_\mathrm{WD} = 1M_\odot$ and an initial secondary mass $M_{2,\mathrm{i}} = 0.6M_\odot$. The chemical composition is specified in Tab. (1).

In Fig. (3) we show the mass transfer rate for the Pop I and Pop II model CVs, both in the $\dot{M}_2$–M_2 and the $\dot{M}_2$–P diagram. The mass transfer rate $\dot{M}_2$ for the Pop II model CV, taking into account the effects of chemical pollution, is shown again in Fig. (4), together with the results of an evolution of the same model system where the effects of chemical pollution have been ignored. As we have already pointed out in Sec. 4 the mass transfer rate of a Pop I CV is hardly changed by chemical pollution. However, for a Pop II CV the situation is different because the secondary can react to chemical pollution by inflating on a rather short time scale. Because in our model system the relative increase of the metallicity in the secondary's envelope occurs on a time scale shorter than the thermal time scale of the convective envelope, its radius increases on the thermal time scale. This, in turn, results in a phase of pollution–enhanced mass transfer. This phase of mass transfer, at the beginning of which the transfer rate is by about a factor of 4 larger than in the standard evolution, lasts for about 10^7yr, i.e. for about the secondary's thermal relaxation time. These numbers are entirely consistent with what we estimated in Sect. 4. Because the long–time average of the mass transfer rate reflects mainly the loss of orbital angular momentum (and where appropriate that of nuclear evolution), the excess mass loss during the initial phase of pollution–enhanced mass transfer has later to be compensated by a phase during which $\dot{M}_2$ is below the average for standard CV evolution.

In Fig. (5) we show how the mass in the secondary's convective envelope changes along the evolution of the Pop I and Pop II model CVs. In the mass range $0.25M_\odot \lesssim M_2 \lesssim 0.58M_\odot$ for Pop I CVs and $0.27M_\odot \lesssim M_2 \lesssim 0.45M_\odot$ for Pop II CVs $M_{2,\mathrm{con}}$ is approximately a linear function of M_2 with a slope as given in Sec. (4). This is the a posteriori justification for our ansatz in Sec. (4), eqs. (9) and (10).

Table 1. Initial parameters for the numerical calculations. $M_{\rm WD}$ is the WD mass which is kept constant in time, $M_{2,\rm in}$ the initial mass of the secondary. The heavy element abundances in the secondary's convective envelope are denoted by ($\rm Z_{in}$, $\rm Y_{in}$, $\rm C_{in}$, $\rm N_{in}$, $\rm O_{in}$), and those in the nova ejecta by ($\rm Z_{ej}$, $\rm Y_{ej}$, $\rm C_{ej}$, $\rm N_{ej}$, $\rm O_{ej}$).

initial model	$M_{\rm WD}$	$M_{2,\rm in}$	Z	Y	C	N	O
Pop I CV	$1\,M_\odot$	$0.6\,M_\odot$	$\rm Z_{in} = 0.02$	$\rm Y_{in} = 0.25$	$\rm C_{in} = 2.6\,10^{-3}$	$\rm N_{in} = 8.6\,10^{-4}$	$\rm O_{in} = 7.8\,10^{-3}$
Pop II CV	$1\,M_\odot$	$0.6\,M_\odot$	$\rm Z_{in} = 10^{-4}$	$\rm Y_{in} = 0.25$	$\rm C_{in} = 1.3\,10^{-5}$	$\rm N_{in} = 4.3\,10^{-6}$	$\rm O_{in} = 3.9\,10^{-5}$
nova ejecta			$\rm Z_{ej} = 0.2$	$\rm Y_{ej}$=0.25	$\rm C_{ej} = 0.02$	$\rm N_{ej} = 0.09$	$\rm O_{ej} = 0.09$

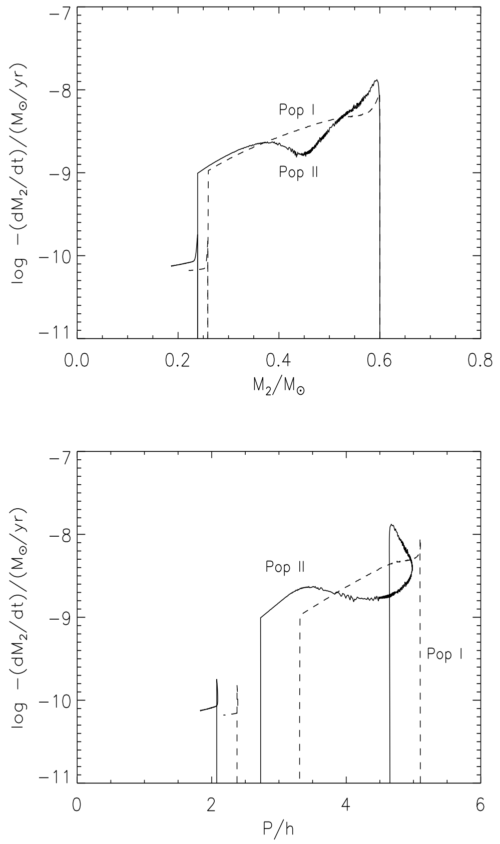


Figure 3. Evolution of the mass transfer rate in the $\dot M_2$–M_2 diagram (upper frame) and the $\dot M_2$–P diagram (lower frame) for the Pop I and the PoP II model CVs discussed in the text

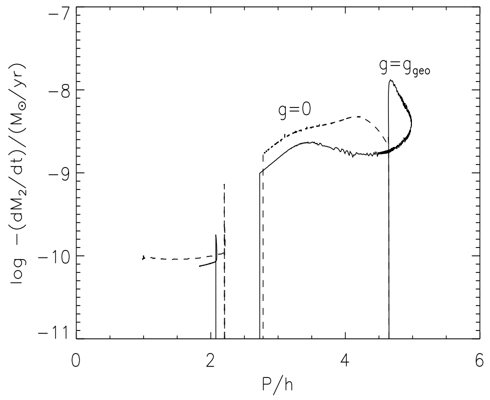


Figure 4. The mass transfer rate as a function of orbital period during the evolution of a Pop II model CV for which chemical pollution of the secondary is taken into account with $g = g_{\rm geo}$ and, for comparison, an evolution where chemical pollution is ignored ($g = 0$).

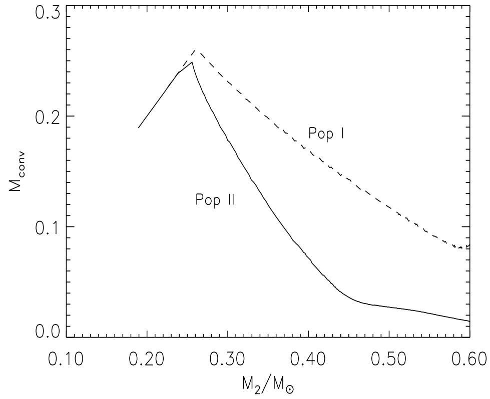
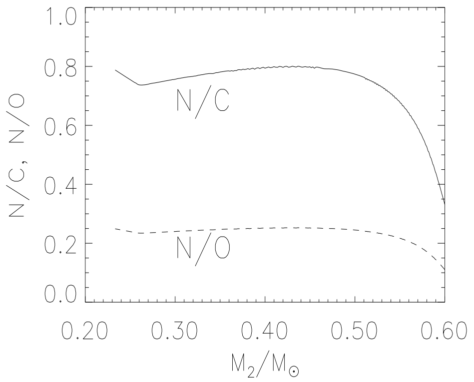


Figure 5. The mass in the convective envelope as a function of the secondary's total mass in the Pop I and Pop II CV evolutions (including mass loss and chemical pollution with $g = g_{\rm geo}$.)

Finally we show in Fig. (6) the abundance ratios ($\rm N_{con}/C_{con}$) and ($\rm N_{con}/O_{con}$) in the secondary's convective envelope as a function of the secondary's total mass. As can be seen, already a very short time after the onset of mass transfer, i.e. after an initial mass loss of less than $\Delta M_2 = 0.01 M_\odot$, the relative abundances of C, N and O in the envelope of the Pop II secondary are practically those in the nova ejecta, and in the subsequent evolution the diluting effect of the shrinking radiative core is too small to change this. Thus ($\rm N^{II}_{con}/C^{II}_{con}$) $\simeq$ ($\rm N_{ej}/C_{ej}$) and ($\rm N^{II}_{con}/O^{II}_{con}$) $\simeq$ ($\rm N_{ej}/O_{ej}$). This means that the chemical composition of the secondary's envelope, as far as the heavy elements are concerned, is completely dominated by that of the nova ejecta. In the case of Pop II secondaries, this remains true even for an effective solid angle smaller than $g_{\rm geo}$. The solid angle g is important only for the absolute level of the metal enrichment, whereas the relative metal abundances are fairly independent of g.

For the Pop I secondary the chemical evolution is different because the total initial abundance of the CNO elements is much larger than in the Pop II star. However, because



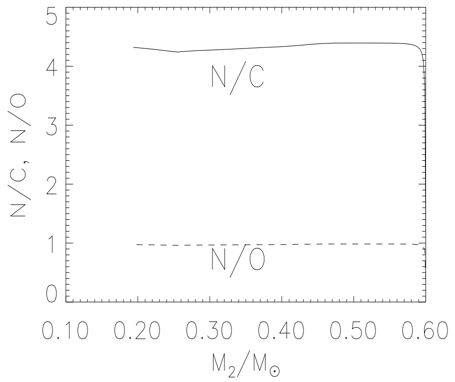


Figure 6. The N/C and N/O abundance ratios in the secondary's convective envelope as a function of M_2 during the evolution the Pop I (upper frame) and the Pop II (lower frame) model CV.

we have assumed that all the heavy elements in the nova ejecta are either C, N or O (in fact mainly N and O), their abundances in the secondary still change appreciably. Correspondingly the abundance ratios (N/O) and (N/C) increase by about a factor of two during the evolution. This increase is, however, much smaller than that obtained in the Pop II CV evolution, where the initial metal abundances were much smaller. For Pop I CVs the evolution of the secondary's chemical composition depends not only on the composition of the nova ejecta, but also very strongly on the solid angle g.

We compare the evolution of the secondary's metallicity in our numerical calculations with the results from our analytic investigation in Sec.(3). In the Pop I case the metallicity $Z^{\rm I}_{\rm con}$ rises slowly between $M_2 = 0.6 \rm M_\odot$ and $0.5 \rm M_\odot$ and levels at $Z^{\rm I}_{\rm con} \simeq 0.023$. For the calculation of the Pop II CV we find a steep increase in $Z^{\rm II}_{\rm con}$ between $M_2 = 0.6 \rm M_\odot$ and $0.58 \rm M_\odot$ towards a plateau phase at $Z^{\rm II}_{\rm con} \simeq 0.005$. This coincides with the initial phase where the secondary's convective envelope increases only slightly. Later, (i.e. for $M_2 \lesssim 0.45$) when $|{\rm d}_{\rm M_2} \rm M^{\rm II}_{\rm con}|$ becomes larger (see Fig. 5), $Z^{\rm II}_{\rm con}$ settles around 0.003. Both calculations are thus in perfect agreement with our analytic results of Sec.(3) where we found $Z^{\rm I,a}_{\rm con} = 0.0235$ and $Z^{\rm II,a}_{\rm con} = 0.0031$ respectively.

6 DISCUSSION

We have examined the effects of chemical pollution of the secondary of a CV resulting from accretion from the ejected envelope during nova outbursts. For this we have assumed that a fraction g of the high–metallicity nova ejecta is intercepted by the secondary and instantly mixed into its convective envelope. Because the chemical composition of nova ejecta is in general very different from that of the accreting secondary, this changes its chemical composition. We have shown that if $g = g_{\rm geo}$, i.e. that the geometrical cross section is relevant for capturing nova ejecta by the secondary, and if the metallicity of the ejecta is $Z_{\rm ej} \simeq 0.2$ the total enrichment in metals in the convective envelope is of the order of several 10^{-3}.

The effect of chemical pollution of high metallicity ($Z \simeq 0.02$) secondaries in Pop I CVs is thus always much smaller than their initial metallicity, unless the relevant cross section exceeds the geometrical one by at least an order of magnitude. In addition, the evolution of the relative metal abundances depends strongly not only on g (a factor which is poorly known) but also on the detailed composition of the ejecta. Therefore, for Pop I CVs no precise predictions concerning the secondary's chemical composition can be made, not even for the relative abundances. Because of the poorly determined value of g we can only conclude that pollution will be important with respect to species which are highly overabundant in the nova ejecta and which are rare otherwise. A case in point could be a secondary exposed to the ejecta of Ne novae.

For Pop II CVs the situation is different because of the much smaller initial metallicity ($Z = 10^{-4}$). For these systems we find that the chemical composition of the secondary is dominated by the composition of the nova ejecta even if the relevant capture cross section is smaller by an order of magnitude than the geometrical one. For Pop II CVs we expect that the relative metal abundances in the secondary's convective envelope are thus comparable to those of the nova ejecta. However, the evolution of the total metallicity still depends critically on the precise value of the cross section.

In Pop II CVs the relative increase of the metallicity in the convective envelope of the secondary can be rather large. If it occurs also on a sufficiently short time scale, i.e. shorter than the thermal relaxation time $\tau_{\rm relax}$, the secondary's radius increases on that time scale. In this way chemical pollution can drive additional mass transfer. This effect is particularly important shortly after the onset of mass transfer from a secondary with an initially shallow convective envelope and lasts for $\sim \tau_{\rm relax}$, i.e. a time scale comparable to that of magnetic braking in standard CV evolution.

Whether or not chemical pollution is important depends rather critically on the secondary's cross section, as discussed above in detail. Observations of abnormal C/O or C/N ratios in the secondary's convective envelope for systems with a CO white dwarf (see also Marks et al. 1997) or abnormal values of oxygen, neon or magnesium in systems with a neon white dwarf and which are rare otherwise might help to clarify the picture. The best candidates for observing abnormal chemical abundances are those CVs with a low premordial metallicity, i.e. Pop II CVs. There, the secondary's chemistry will be dominated by that the nova ejecta already shortly after the onset of mass transfer even for small cross sections. The galactic position of a CV or its space velocity might reveal its population II affiliation (Stehle, Kolb & Ritter 1997).

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