On Metric Preheating

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We consider the generation of super-horizon metric fluctuations during an epoch of preheating in the presence of a scalar field χ quadratically coupled to the inflaton. We find that the requirement of efficient broad resonance is concomitant with a severe damping of super-horizon $\delta\chi$ quantum fluctuations during inflation. Employing perturbation theory with backreaction included as spatial averages to second order in the scalar fields and in the metric, we argue that the usual inflationary prediction for metric perturbations on scales relevant for structure formation is not strongly modified.

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Introduction. In the inflationary paradigm the universe went through an early and extended period of accelerated expansion $\ddot{a} > 0$ with rapid growth of the scale factor a. Inflation may provide solutions to a variety of cosmological "problems" as well as making generic predictions for a nearly scale-invariant spectrum of density fluctuations, possibly acting as seeds for structure formation [1,2]. After an inflationary period cosmic energy density is dominated by oscillations of the inflaton φ. Efficient conversion of this energy to other coupled bosons by parametric resonance phenomena may occur during a period of preheating immediately following the end of inflation [3,4]. Consider, for example, a chaotic inflationary scenario with Lagrange density $L = \frac{1}{2}\varphi_{;\mu}\varphi^{;\mu} - V(\varphi,\chi)$ and potential

$$V(\varphi,\chi) = \frac{m^2}{2}\varphi^2 + \frac{g^2}{2}\varphi^2\chi^2\,, \tag{1}$$

where χ is a boson coupled to the inflaton φ. After the end of slow roll inflation, i.e. for $\varphi \lesssim \varphi_0 \simeq M_{pl}/(12\pi)^{1/2}$, with M_{pl} the Planck mass, the linearized equation of motion for a comoving Fourier mode with wavevector $k = |\mathbf{k}|$ of the χ field is,

$$\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \left[\left(\frac{k}{a}\right)^2 + g^2\Phi^2(t)\sin^2(mt)\right]\delta\chi_k = 0\,. \tag{2}$$

At early times, before significant transfer of energy from the inflaton to the coupled boson has ocurred, the amplitude $\Phi(t)$ of the inflaton oscillations, $\varphi = \Phi(t)\sin(mt)$, decays as $\Phi(t) = \Phi(t_0)(a/a_0)^{-3/2}$, due to the Hubble expansion of the universe which is governed by the homogeneous Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\dot{\chi}^2 + V(\varphi,\chi)\right] \tag{3}$$

for the scale factor $a(t)$, where $\kappa^2 = 8\pi/M_{pl}^2$.

Eq. (2) exhibits parametric resonance, i.e. exponentially growing modes for $\delta\chi_k$, due to the time-dependence of the χ mass. One may associate a resonance-parameter $q(t) = g^2\Phi(t)^2/4m^2$ with the system. In an expanding universe, parametric resonance is efficient for $q > 1$ where resonance occurs for wavevectors k within broad bands, and is rendered inefficient for $q \ll 1$ [4]. In the broad resonance regime $q > 1$, exponential growth of bosonic fluctuations is possible for super-horizon modes even in the limit $k \to 0$. Recently, the important realization has been made that growth of coherent, bosonic matter fluctuations during preheating is accompanied by production of metric fluctuations possibly leading to abundant primordial black hole formation [5]. It has also been pointed out that a significant modification of the power spectrum of density fluctuations relevant for structure formation and cosmic microwave background anisotropies is conceivable [5,6].

Multiple scalar fields and metric perturbations. Consider perturbations around a Robertson-Walker metric in longitudinal gauge

$$ds^2 = (1+2\phi)dt^2 - a^2(t)(1-2\phi)d\mathbf{x}^2\,, \tag{4}$$

where ϕ is a gauge-invariant potential, quantifying the density perturbation $\delta\rho/\rho$ at horizon-crossing in unperturbed Hubble flow [2]. The Fourier-transformed, first-order Einstein equations for coupled bosons $\varphi_I(t)$ and their perturbations $\delta\varphi_I(t,\mathbf{x})$ give [7]

$$3H\dot{\phi}_k + \left[(k/a)^2 + 3H^2 - \frac{\kappa^2}{2}\sum_I \dot{\varphi}^2\right]\phi_k = -\frac{\kappa^2}{2}\sum_I\left(\dot{\varphi}_I\delta\dot{\varphi}_{Ik} + V_{\varphi I}\delta\varphi_{Ik}\right)\,, \tag{5}$$

$$\delta\ddot{\varphi}_{Ik} + 3H\delta\dot{\varphi}_{Ik} + \left[(k/a)^2 + V_{\varphi_I\varphi_I}\right]\delta\varphi_{Ik} =$$

$$4\dot{\phi}_k\dot{\varphi}_I + 2(\ddot{\varphi}_I + 3H\dot{\varphi}_I)\phi_k - \sum_{J\neq I} V_{\varphi_J\varphi_I}\delta\varphi_{Jk}\,, \qquad (6)$$

$$\dot{\phi}_k + H\phi_k = \frac{\kappa^2}{2}\sum_I \dot{\varphi}_I\delta\varphi_{Ik}\,, \qquad (7)$$

where a subscript φ_I denotes derivative with respect to φ_I. Eq. (5) and Eq. (7) may be combined to

$$\phi_k = -\frac{\kappa^2}{2}\frac{\sum_I\left(\dot{\varphi}_I\delta\dot{\varphi}_{Ik} + 3H\dot{\varphi}_I\delta\varphi_{Ik} + V_{\varphi_I}\delta\varphi_{Ik}\right)}{(k/a)^2 - (\kappa^2/2)\sum_I\dot{\varphi}_I^2}\,, \quad (8)$$

explicitly showing that ϕ is completely determined when the evolution of the matter fields is known. Eq. (8) has a sub- and super- horizon limit, $(k/a)^2 \gg$ and $\ll \kappa^2/2\sum_I\dot{\varphi}_I^2 \sim H^2$, with the assumption that the kinetic energy is a substantial fraction of the total energy. In the super-horizon limit metric perturbations ϕ_k are independent of k and only dependent on the perturbed energy (denominater) and the kinetic energy (numerator). For subhorizon scales one may easily show that the condition $\phi_k > 1$ implies that the perturbed (excess) energy of a region is sufficiently large to have a Schwarzschild radius exceeding the regions dimension.

Whether or not parametric resonance for $k \to 0$ induces significant secondary perturbations in ϕ is largely a question of initial conditions. The evolution equations for the unperturbed background fields are

$$\ddot{\varphi}_I + 3H\dot{\varphi}_I + V_{\varphi_I} = 0\,. \qquad (9)$$

Consider a chaotic inflationary period with inflaton potential as in Eq. (1). Assuming that inflation is driven by φ (i.e. $g^2\chi^2 \ll m^2$) we have $\dot{\varphi} = -(m^2/3H)\varphi$ by the slow-roll approximation for $\varphi \gtrsim \varphi_0$. The evolution of the scalefactor may be written as

$$\ln(a) \simeq -\int_t^{t_0} H dt \simeq \frac{2\pi}{M_{pl}^2}\left(\frac{M_{pl}^2}{12\pi} - \varphi^2\right)\,, \quad t \leq t_0\,, \quad (10)$$

where we normalized $a_0 = 1$ at the end of slow roll $\varphi_0 = M_{pl}/\sqrt{12\pi}$. One may search solutions to Eq. (9) for the χ boson in the adiabatic (WKB) approximation; i.e. $\chi \sim \exp i\int\omega_\chi dt$. For $q_0 \gtrsim 1$ one finds $\omega_\chi \simeq i(3/2)H \pm g\varphi$, yielding $\chi \propto (\omega_\chi/\omega_{\chi,i})^{-1/2}(a/a_i)^{-3/2}\cos\int g\varphi dt$ by using Eq. (10), where the subscript i denotes that quantities are to be taken at the beginning of inflation. (Note that one may add the factor $(\omega_\chi/\omega_{\chi,i})^{-1/2}$ witin the WKB approximation.) It is straightforward to show that the WKB approximation (i.e. $\dot{\omega}_\chi \ll \omega_\chi^2$) is fulfilled if $q_0^{-1/2}(\varphi/\varphi_0)^{-2} \ll 2$, quite generally when the resonance parameter at the beginning of preheating satisfies $q_0 \gtrsim 1$. Similarly, the amplitudes of superhorizon fluctuations in the χ field decay as $\delta\chi_k \propto (\omega_\chi/\omega_{\chi,i})^{-1/2}(a/a_i)^{-3/2}\cos\int g\varphi dt$. This may be seen by noting that the R.H.S. of Eq. (6) is negligible since it contains factors of χ and $\dot{\chi}$ whose values are strongly suppressed due to the inflationary expansion, such that Eq. (6) for $\delta\chi_k$ reduces to Eq. (9) for modes relevant for structure formation. When interpreted classically, the decay of the amplitude of the $\delta\chi_k$ fluctuations corresponds to a dilution of particle number densities associated with the $\delta\chi_k$ field by the expansion of the universe. In contrast, it is well known that super-horizon fluctuations in the inflaton field $\delta\varphi_k$ are not exponentially damped and lead to the usual prediction of a nearly scale-invariant Harrison-Zel'dovich spectrum of density fluctuations [2]. Hawking radiation in the de Sitter phase of an inflationary universe enhances the $\delta\varphi_k$ amplitude for superhorizon modes, which is possible since the mass of the inflaton m is somewhat smaller than the Hubble constant H. Demanding $M_\chi = g\varphi \lesssim H$, in order to have a similar effect for $\delta\chi_k$ fluctuations, we arrive at the condition $q_0 \lesssim 1/6^{1/2}$. In particular, damping seems inevitable for systems in the broad resonance regime.

Consider the amplitudes of vacuum quantum fluctuations in the bosonic fields on sub horizon scales $k/a \gtrsim H$. These may be estimated from quantum field theory in flat space where the $\delta\varphi_I$'s are replaced by operators $\delta\widehat{\varphi}_I(t,\mathbf{x}) = \int(d^3\mathbf{k})/(2\pi)^3[\widehat{a}_\mathbf{k}\delta\widetilde{\varphi}_{I\mathbf{k}}e^{-i\mathbf{kx}} + H.C.]$, with $\widehat{a}_\mathbf{k}$ the usual annihilation operators and $H.C.$ denoting the hermitian conjugate. A typical amplitude may be estimated by the square-root of the expectation value of $\widehat{\varphi}_I^2(t,\mathbf{x})$ in the vacuum. This gives

$$\langle 0|\delta\widehat{\varphi}_I^2(t,\mathbf{x})|0\rangle = \int\frac{d^3\mathbf{k}}{(2\pi)^3}|\delta\widetilde{\varphi}_{I\mathbf{k}}|^2 \simeq \int\frac{d^3(\mathbf{k}/a)}{(2\pi)^3\omega_{Ik}}\,, \quad (11)$$

where $\omega_{Ik}^2 = (k/a)^2 + M_{\varphi_I}^2$. Therefore, the vacuum amplitudes in a comoving wavevector range k are given by

$$|\delta\varphi_{Ik0}|^2 \equiv k^3\,|\delta\widetilde{\varphi}_{Ik}|^2 = \frac{(k/a)^3}{2\pi^2\omega_{Ik}}\,. \qquad (12)$$

At horizon exit of a mode, i.e. at $k/a = H$, perturbations in the inflaton therefore have $\delta\varphi_k|_{ex} \simeq H/(2\pi^2)^{1/2}$ since $m \lesssim H$. Similarly, for the massive field $\delta\chi_k$ we find $\delta\chi_k|_{ex} \simeq H^{3/2}/|g\varphi|^{1/2}/(2\pi^2)^{1/2}$. In what follows we will compute the evolution of the boson fields by the classical equations which is a good approximation in the limit of large occupation numbers, in particular, when the amplitudes are much larger than their vacuum expectation values.

Let us evaluate the wavevectors for modes which contribute to the formation of large-scale structure. Let us define, $T_0 \equiv (m\varphi_0)^{1/2}$, which is the approximate reheat temperature, $T_{RH} \simeq T_0$, if reheating would be instantaneous. For what follows it is sufficient to approximate reheating to be instantaneous, such that a evolves as $a \simeq (t/t_0)^{1/2}$ for $t \gg t_0$ [setting $a(t_0) \equiv 1$]. Then the proper horizon distance evolves as $r_H^p(T) \simeq r_H^p(T_0)(T_0/T)^2$ with $r_H^p(T_0) \simeq M_{pl}/T_0^2$, the horizon distance immediately after inflation. A mode with comoving wave vector $k \simeq (T_{re}/T_0)/r_H^p(T_0) \simeq (m/3)(T_{re}/T_0)$ re-enters into the

horizon at $T_{re} \ll T_0$. This mode left the horizon during inflation at time t_{ex} when $k/a_{ex} \simeq H(\varphi_{ex})$. This allows one to derive the scale factor at horizon exit, $a_{ex} \simeq (T_{re}/T_0)(\varphi_{ex}/\varphi_0)^{-1}$. For $k/a \ll g\varphi$ the amplitude of the matter field at the beginning of preheating $\delta\chi_k(t_0) = \delta\chi(t_{ex})a_{ex}^{3/2}(\varphi_{ex}/\varphi_0)^{1/2}$ is given by

$$\delta\chi_k(t_0) \simeq 0.1 m q_0^{-1/4}(T_{re}/T_0)^{3/2}(\varphi_{ex}/\varphi_0)^{-1}. \qquad (13)$$

For small k and low $T_{re} \sim 1$eV this is very small compared to a typical inflaton perturbation $\delta\varphi_k \sim m$. Below we estimate the growth of $\delta\chi_k$ fluctuations during preheating on scales relevant for structure formation which, combined with their initial amplitudes Eq. (13) and Eq. (8), will serve to give a first-order estimate on the contribution of χ fluctuations to the total metric fluctuations.

Non-perturbative decay of the inflaton condensate into bosonic matter fields via parametric resonance occurs when the condition of adiabaticity for the χ field is violated, i.e. when $\dot{\omega}_{\chi_k} \gtrsim \omega_{\chi_k}^2$. In what follows, we follow closely the analytic treatment by Kofman, Linde, and Starobinsky [4] (KLS97) for the stochastic growth of bosonic particle number in the broad resonance parameter regime, $q \gg 1$, in an expanding universe. The largest bosonic wavevector k_{max} which receives parametric amplification is given by $k_{max}/a \simeq (gm\Phi)^{1/2}/2$. Due to phase space arguments, this is also the typical wavevector k_* in which most of the inflaton energy is "dumped". Let us define δX_k as the amplitude of the boson fields, i.e. $\delta X_k = \delta\chi_k$ at moments when $\delta\dot{\chi}_k = 0$. KLS97 were able to derive that the comoving occupation number n_k evolves as, $2n_k = \omega_{\delta\chi_k}|\delta\tilde{X}_k|^2 a^3 - 1 \simeq \exp(2m\int dt\mu_k(t))$ (δX_k and $\tilde{X}_k$ are related via Eq. [12]), where the integral may be replaced by $2m\mu t$, with an effective value $\mu \simeq 0.1 - 0.18$. Replacing the Floquet index by an effective index accounts for the stochasticity of the resonance, due to wavevectors shifting in and out of resonance bands with the expansion of proper wavelength and the time-dependency of the resonance parameter $q(t)$. Note that there is still some dependency of μ on k, which implies that the decay of the inflaton energy is dominated by decay into modes with k in a narrow neighborhood of the maximum unstable mode. These considerations allowed KLS97 to estimate the number of inflaton oscillations $N = mt/(2\pi)$ between $t_0 \lesssim t \lesssim t_1$, where t_1 is the time when backreaction effects of the produced matter particles on the inflaton become significant, i.e. when $m^2 \simeq g^2 X_{rms}^2(t_1)$. In the following for any function $f(\mathbf{x})$ we define $f_{rms} \equiv \langle f^2(\mathbf{x})\rangle^{1/2}$, where $\langle ...\rangle$ denotes spatial average. They found,

$$N_1 \simeq \frac{1}{8\pi\mu}\ln\left[\frac{10^6 m(2\pi N_1)^3}{g^5 M_{pl}}\right]. \qquad (14)$$

During the time $t_0 \lesssim t \lesssim t_1$ the scale-factor evolves as $a(t) = (t/t_0)^{2/3} \simeq (\pi N)^{2/3}$, such that at t_1 the resonance parameter is $q_1 = g^2\Phi^2(t_1)/(4m^2) \simeq q_0/(\pi N_1)^2$. Subsequently, if $q_1 > 1/4$, the inflaton oscillation frequency increases to $m_{\varphi,\mathrm{eff}}^2 \simeq g^2 X_{rms}^2$, and Hubble expansion effects are negligible, i.e. $m_{\varphi,\mathrm{eff}} \gg H$. The second stage of preheating, $t > t_1$, is characterized by the resonance parameter $q_{\mathrm{eff}} \simeq g^2\Phi^2/(4g^2X_{rms}^2)$. Broad resonance is terminated at some time t_2, when q_{eff} decreased to $\simeq 1/4$. At this time $\Phi^2(t_2) \simeq X_{rms}^2(t_2)$. During the second stage of preheating, the inflaton typically makes only a few oscillations, $N_2 \simeq (1/4\pi\mu)\ln 4q_1^{1/4}$. The total amplification of the amplitudes of the bosonic matter fluctuations between t_0 and t_2 yields [4]

$$\frac{\delta X_k(t_2)}{\delta X_k(t_0)} \simeq \left[\frac{\Phi(t_0)}{\Phi(t_2)}\right]^{1/2} \frac{\exp\left[2\pi\mu(N_1+N_2)\right]}{a^{3/2}(t_2)}. \qquad (15)$$

The contribution of χ fluctuations to the large-scale metric fluctuations at t_2, if estimated via Eqs. (8) and (15) and employing the severely damped background χ, would be utterly negligible. Nevertheless, the problem is intrinsically non-linear since $\delta\chi_k \gg \chi$. This is because the homogeneous background χ is damped from the very beginning of inflation, whereas quantum fluctuations are only damped after horizon crossing at later stages during inflation. If instead, we were to include higher order contribution such as $\sim (g\varphi)^2\int d^3\mathbf{k}'\delta\widetilde{\varphi}_{I\mathbf{k}'}\delta\widetilde{\varphi}_{I(\mathbf{k}-\mathbf{k}')}$, into Eq. (8) we would find significant metric perturbations when backreaction becomes important, even in the limit $k \to 0$, sourced by the sub-horizon fluctuations in $\delta\chi$. This is not surprising though, at onset of backreaction there is approximate equipartition of energy between the inflaton and the coupled boson. Since the energy in the $\delta\chi$ fluctuations is not taken into account in Eq. (3), one expands effectively around an unphysical Hubble constant. We may rather attempt to estimate $\phi_{k\to 0}(t_2)$ by associating an effectively homogeneous background of the χ field on large scales by taking the root mean square X_{rms} over small-scale fluctuations. This would yield $\phi_{k\to 0}|_\chi(t_2) \sim g^2\Phi^2 X_{rms}\delta X_k/g^2\Phi^2 X_{rms}^2 \sim \delta X_k/X_{rms}\,(t_2)$, for the contribution of χ fluctuations to the metric perturbations via Eq. (8). Here we have used that at t_2 there is approximate equipartition between the kinetic energy densities and the interaction energy density in Eq. (1), i.e. $\dot{\varphi}^2(t_2) \simeq \dot{\chi}^2(t_2) \simeq g^2 X_{rms}^2(t_2)\Phi^2(t_2)$. Using Eq. (13) with $T_{re} \simeq 1$eV, $T_0 \simeq 4\times 10^{-4}M_{pl}$, and $\varphi_{ex} \simeq 3M_{pl}$ we find $\delta X_k(t_0) \sim 10^{-47}M_{pl}$. Such a small initial amplitude is only amplified by modest factors $\sim 10^4 - 10^5$, estimated via Eq. (15) for $\mu \simeq 0.13$ and $g \sim 3\times 10^{-4} - 10^{-2}$. Nevertheless, we will see below that this result represents a gross underestimate.

Second order contributions to the energy density from scalar field and metric fluctuations on scales exceeding the typical fluctuation scale may be accounted for by the effective energy-momentum tensor formalism of Ref. [8]. It is important to realize that for the potential Eq. (1) working to second order amounts to a com-

plete solution of the problem. Defining the potential $\tilde{V} \equiv V + \frac{1}{2}\sum_{IJ} V_{\varphi_I\varphi_J}\delta\varphi_I\delta\varphi_J$, the formalism corresponds to substituting Eq. (3) by

$$H^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}\sum_I \left(\dot{\varphi}_I^2 + \delta\dot{\varphi}_I^2 + a^{-2}(\nabla\delta\varphi_I)^2\right) + \tilde{V}(\varphi_I) + 2V_{\varphi_I}\phi\delta\varphi_I\right] \quad (16)$$

and using the modified equation

$$(\ddot{\varphi}_I + 3H\dot{\varphi}_I)(1+4\phi^2) + \tilde{V}_{\varphi_I} - 2\phi\delta\ddot{\varphi}_I - 4\dot{\phi}\delta\dot{\varphi}_I - 6H\phi\delta\dot{\varphi}_I + 4\dot{\varphi}_I\dot{\phi}\phi - 2a^{-2}\phi\nabla^2\delta\varphi_I = 0 \quad (17)$$

for the homogeneous fields. In these expressions second order terms are to be understood as spatial averages. Whereas for small scales $\sim k_{max}^{-1}$ such a scheme may be questionable, it should be appropriate for the super-horizon scales relevant for structure formation.

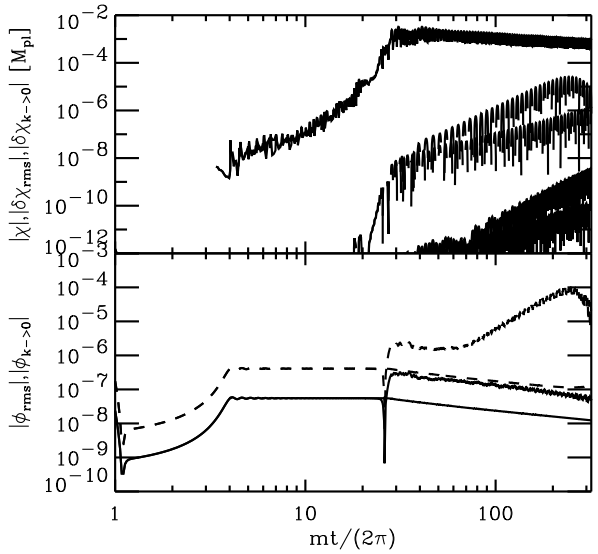

FIG. 1. Results of a numerical simulation of metric preheating with two scalar fields and parameters as discussed in the text. The top panel shows, from top to bottom, the total $|\delta\chi_{rms}|$, the homogeneous field $|\chi|$, and the large scale fluctuations $|\delta\chi_{k\to 0}|$. The curve for $|\delta\chi_{rms}|$ starts at its vacuum expectation value at the beginning of preheating. The bottom panel shows the total $|\phi_{rms}|$ (dashed lines) and the large scale metric potential $|\phi_{k\to 0}|$ (solid lines) as functions of time. The results obtained from integrating Eq. (5) (upper set of curves) and Eq. (7) (lower set of curves) diverge once backreaction sets in, reflecting the uncertainties within the current numerical approach.

Numerical Model. We have numerically integrated Eqs. (6) and (7) along with Eq. (17) for the homogeneous fields and the Friedman equation (3) with backreaction terms for the two field case and the potential Eq. (1). We started integration deep in the inflationary regime where the large scale structure modes relevant today were still within the Hubble horizon. We used the initial conditions $\varphi = 4M_{pl}$, $\chi = m/g$ and Eq. (12) for the matter fields, as well as $\delta\dot{\varphi}_{Ik} \sim \omega_{Ik}\delta\varphi_{Ik}$ for fluctuation derivatives (the results are qualitatively insensitive to the exact values). The initial value of ϕ_k was evaluated from Eq. (8). A grid of 100 equidistant logarithmic momentum modes was implemented between $k = m/2^{1/2}$ and $k = 1.5[gm\Phi(t_0)]^{1/2} = 3m(q_0/4)^{1/4}$ which covers the dominant resonant modes around the Hubble horizon. In addition, one mode with comoving scale of $\sim 100\,\mathrm{Mpc}$ today, corresponding to $T_{re} \simeq 0.5\,\mathrm{eV}$ in Eq. (13), was followed.

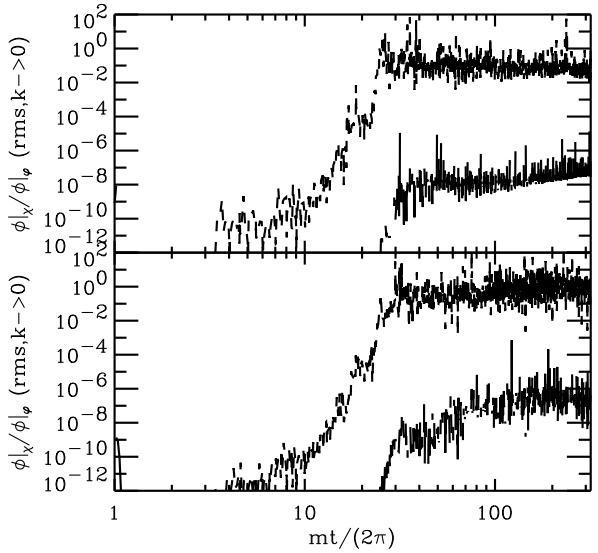

FIG. 2. The ratio of the $\chi-$dependent and the inflaton dependent source terms of the total (dashed lines, upper curves) and the large scale (solid lines, lower curves) metric perturbations, within the scenario from Fig. 1. Results from the simulations involving Eqs. (5) (upper panel) and (7) (lower panel) agree reasonably well.

In Fig. 1 we show results of simulations with $m = 10^{-6}M_{pl}$ and $q_0 = 10^4$. Note that Fig. 1 includes results from two simulations (a) as outlined above and (b) as above but by integrating Eq. (5) instead of Eq. (7). In the linear limit (i.e. $mt/(2\pi) \lesssim 25$) the results of both simulations coincide, providing a consistency check of our numerical routine. During the inflationary phase one may observe the standard growth of metric fluctuations ϕ due to the increase of $(1+w)$, where w is the ratio of pressure and density. For $mt/(2\pi) \gtrsim 5$ parametric resonance develops and for $mt/(2\pi) \gtrsim 25$ backreaction becomes important. The saturation of X_{rms} at $\simeq m/g = q_0^{-1/2}\Phi(t_0)/2 \simeq 3\times 10^{-3}M_{pl}$ due to backreaction is clearly visible in Fig. 1. Note that due to the choice of q_0 and m, $X_{rms}(t_2) \simeq X_{rms}(t_1)$ since $q_1 \simeq 1/4$, thus the second stage of preheating is very short. When backreaction becomes important, results obtained from

integrating Eq. (5) and Eq. (7), respectively, which are equal at early times, start to differ. In particular, whereas Eq. (5) predicts a second rise in small scale metric fluctuations at t_2, and oscillatory behavior, integration of Eq. (7) shows a smooth transition. These differences are likely due to the replacement of small scale fluctuations by numerical phase space sums which may introduce some coherence between the perturbed quantities which in reality, in particular for large scales, does not exist. In Fig. 2 we plot the time dependence of the ratio of the $\chi-$ dependent to the inflaton dependent source terms of the metric perturbations both on small and large scales within this approximative scheme. This is accomplished by comparison of the respective contributions of the χ boson and the inflaton to the R.H.S. of Eq. (5) and Eq. (7), depending if Eq. (5) or Eq. (7) is used for the numerical integration. We see that for the large scales, $\phi_{k\to 0}|_\chi/\phi_{k\to 0}|_\varphi(t_2) \sim 10^{-8}$, with a slight subsequent growth in the non-linear regime. This is still much smaller than 1, but much larger than our "naive" analytical estimate. In contrast, for the small scales we find $\phi_{rms}|_\chi/\phi_{rms}|_\varphi(t_2) \sim 1$.

The origin of the increased contribution of the χ field to metric perturbations in the presence of backreaction is the following: The coupling to small-scale metric ϕ and boson perturbations $\delta\chi$ in Eq. (17) tends to lift the homogeneous χ field to $\sim \phi\delta X$ which in turn induces a source term for $\delta\chi_{k\to 0}$ in Eq. (6). These source terms induce a growth of χ and $\delta\chi_{k\to 0}$, with growth due to parametric resonance subdominant. In contrast, the analytic estimate approximates growth of $\delta\chi_{k\simeq k_{max}}$, as well as backreaction on φ, by the usual parametric resonance theory [4], i.e. essentially unmodified by the existence of metric fluctuations. Our conclusions, i.e. $\phi_{k\to 0}|_\chi \lesssim \phi_{k\to 0}|_\varphi$ and $\phi_{k\to 0} \ll 1$, are not likely modified by a subsequent epoch of rescattering and thermalization, but may be if ϕ grows further by gravitational instability. Nevertheless, for $\phi \sim 1$ efficient formation of primordial black holes seems inevitable and the notion of an effectively homogeneous and isotropic universe on large scales, described by a FRW metric, is called into question.

Summary. We have investigated preheating after an inflationary epoch for a simple chaotic inflationary model with the inflaton coupled to another bosonic particle. Our study focuses on the possibility of generation of secondary metric perturbations on scales relevant for structure formation. Within first-order perturbation theory, but with backreaction included to second order in the scalar fields and in the metric in an effective way, we found that metric fluctuations on scales relevant for structure formation stay well within the linear regime during preheating. This is mainly due to the fact that at the beginning of preheating the homogeneous component of the scalar field χ, and its super-horizon quantum fluctuations $\delta\chi$, which couple to the inflaton, are severely suppressed compared to the inflaton [9]. Our conclusions are similar to those of Refs. [6,10] who investigated the case of general-relativistic parametric resonance of perturbations around the coherent oscillations of a massive, but otherwise uncoupled inflaton. However, we caution that our study does not fully address the possibility of efficient mode-mode coupling between sub-horizon and super-horizon metric fluctuations. This issue may, in principle, be resolved by numerical simulation as in Ref. [10].

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