# WEAK GRAVITATIONAL LENSING WITH SKA

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I shall outline the basic principles and some observational aspects of weak gravitational lensing, and discuss several applications of this powerful tool in observational cosmology. It will be explained why the applications have been restricted to optical observations up to now, and why SKA is going to change this. I conclude with a few general remarks on a comparison between SKA and the NGST, both being facilities which will provide a tremendous step forward in radio and near-IR astronomy, respectively, into completely unknown territory.

## 1 Introduction

Gravitational light deflection provides astronomers with a unique tool to study the mass distribution in the Universe. The resulting phenomena, called gravitational lensing, have attracted great interest in recent years, since gravitational light deflection is independent of the nature and the state of the matter which causes the deflection. Hence in particular, lensing is sensitive to the dark matter, and is therefore extensively employed for investigating the dark matter distribution, on scales ranging from stellar-mass objects (as done in microlensing studies), galaxy-mass scales (as in multiply-imaged AGNs), to clusters (e.g., through giant luminous arcs) and larger-scale matter distribution. Lensing is popular also because the underlying physics is very simple (it is just gravity, hence only General Relativity is needed, and in most cases, the correct answers are obtained from quasi-Newtonian theory, with all deflection angles multiplied by a factor 2), in marked contrast to nearly all other methods used for understanding the mass distribution in the Universe.

The phenomena mentioned above can be summarized as "strong gravitational lensing". For them, the lensing phenomenon is readily detected in individual sources, e.g., the characteristic brightness variation of one background star signifies the occurrence of microlensing, or the strongly distorted images (arcs) of background galaxies seen in a foreground galaxy cluster are sufficient to identify the lensing property of the deflector. Strong lensing therefore yields strong constraints on the deflecting mass distribution: for example, the mass of a lensing galaxy must be such that the observed multiple images are mapped back to the same source position; in particular, the total projected mass of the lensing galaxy in a circle tracing the multiple images is very well constrained then.

If a galaxy cluster is strong enough to distort the images of one or a few background galaxies very strongly, it appears obvious that for larger radial distances from the cluster center, the images of background galaxies are distorted as well, though weaker. If all galaxies were circular, this smaller distortion could be measured on individual images as well. However, galaxies have an intrinsic ellipticity which prevents the measurement of weak image distortions from individual images. But if one considers an ensemble of background galaxies whose intrinsic orientation is random, the weak distortion will impose a coherent alignment of the images which is measurable, provided the distortion is larger than $\sim \sigma_\epsilon/\sqrt{N}$, where $\sigma_\epsilon$ is the intrinsic rms ellipticity of the background galaxies, and $N$ is the number of images from which the shear shall be measured. The accuracy with which the average distortion over a small solid angle can be measured therefore depends on the number density of sources for which the shape measurement can be carried out. Whereas the optical sky is densely covered with faint galaxies (at the magnitude limit at which these *weak lensing* measurements are typically carried out, the number density of galaxies is $\sim 30/\mathrm{arcmin}^2$), the radio sky is literally empty (cf. the number of optical and radio sources in the HDF)! Therefore, weak lensing has up to now been restricted to optical astronomy (with the exception of the work by Kamionkowski et al. 1999). The SKA will change this situation drastically: the expected number density of radio sources routinely achieved with SKA is expected to be at least as large as that currently obtainable with deep ground-based optical observations.

In this contribution, I shall describe the basic theory of (weak) lensing in Sect. 2, and discuss a few technicalities in Sect. 3. Then, some applications of weak lensing are discussed in Sect. 4. I shall then try to put SKA into perspective with respect to weak lensing, and close with a few general remarks.

## 2 Theory of (weak) lensing

The basic theory of gravitational lensing can be found in our monograph [28], or in several more recent review articles (e.g., [4,13]). Here I shall just state a few general relations needed for the following discussion.

As light is deflected by a single mass distribution at distance $D_\mathrm{d}$, a source at distance $D_\mathrm{s}$ which, without an intervening mass distribution would be seen at an angular position $\beta$, will be seen at a slightly different position $\theta$, such that the lens equation $\beta = \theta - \alpha(\theta)$ is satisfied. Here, $\alpha(\theta)$ is the (scaled) deflection angle which depends on the impact vector $\theta$ of the light ray, and on the mass distribution. Drawing the analogy to Newtonian gravity, $\alpha$ can be considered as a 'force field'. Indeed, it can be obtained as the gradient of a potential, the so-called deflection potential $\psi(\theta)$. The source of this potential is the surface mass density of the deflector $\Sigma(\theta)$, or its dimensionless analogon,

$$\kappa(\theta) = \frac{4\pi D_\mathrm{d} D_\mathrm{ds}}{c^2 D_\mathrm{s}} \Sigma(\theta) \; . \tag{1}$$

Here, $D_\mathrm{ds}$ is the distance from the lens to the source. All these distances are to be interpreted as angular-diameter distances, and they depend, for given source and lens redshifts, on the cosmological model. The deflection potential obeys a Poisson-like equation, $\nabla^2\psi = 2\kappa$. Multiple images of a source occur if the lens equation has more than one solution $\theta$ for a given source position $\beta$.

Light is not only deflected as a whole, but undergoes differential deflection which distorts the size and shape of light bundles. If the size of the source is much smaller than the characteristic angular scale of the deflector, one can locally linearize the lens mapping, which is then described by the Jacobian of the lens equation

$$\mathcal{A}(\theta) = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} = (1-\kappa) \begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix} \; , \tag{2}$$

Here, $\gamma_i$ are the two components of the tidal gravitational field, called *shear*, which is described by the trace-free part of the Jacobian (as in Newtonian theory), and which are conveniently written as a complex number, $\gamma = \gamma_1 + \mathrm{i}\gamma_2 = (\psi_{,11} - \psi_{,22})/2 + \mathrm{i}\psi_{,12}$, and indices separated by a comma denote partial derivatives. The final version of $\mathcal{A}$ in (2) shows that the shape distortion of images is described solely by the *reduced shear* $g := \gamma/(1-\kappa)$. The change of the apparent flux of an image of a small source is given by the local area distortion of the lens mapping, $\mu = [(1-\kappa)^2 - |\gamma|^2]^{-1}$, where the magnification $\mu$ is the ratio of the observed flux of an image and that of the unlensed source. Giant arcs are obtained at places where one of the eigenvalues of $\mathcal{A}$ is close to zero, i.e., where $\mu \gg 1$.

In the next section we shall describe how one can obtain the (reduced) shear from observed image ellipticities. Suppose we have measured $\gamma$ over some field (say, around a cluster). Since

$\gamma$ and $\psi$ are linearly related, one can determine $\psi$ from $\gamma$, and from that, the surface mass density can be obtained. Hence, the measurement of the shear can be used directly to obtain the underlying surface mass distribution.

## 3 Technicalities

The theory of weak lensing is well understood, as is the application of this effect to several astrophysical situations, as will be described later. The main difficulty of applying these concepts to observational data lies in the measurement of what can be called "ellipticity". Since the SKA may be well superiour to optical observations in this regard, some of the technical issues shall be described here.

### 3.1 The principle of shear measurements

The isophotes of faint sources are not elliptical in general, and hence, one must define a quantity which characterizes the shape of an object and how this shape quantity is affected by shear. A convenient way to proceed, at least for theoreticians, is to define the tensor of second brightness moments,

$$Q_{ij} = \int \mathrm{d}^2\theta \; I(\theta) \, (\theta_i - \bar{\theta}_i) \, (\theta_j - \bar{\theta}_j) \; , \tag{3}$$

where $I(\theta)$ is the surface brightness of an image, and $\bar{\theta}$ is the center of the image, defined such that the dipole vector [which is defined in analogy to (3)] vanishes. Similarly, one can define the tensor $Q_{ij}^{(\mathrm{s})}$ of second brightness moments of the corresponding source. The lens equation then relates these two tensors by

$$Q^{(\mathrm{s})} = \det \mathcal{A} \, (\mathcal{A} \, Q \, \mathcal{A}) \; . \tag{4}$$

The (complex) ellipticity $\epsilon$ is now defined in terms of $Q$, as

$$\epsilon = \frac{Q_{11} - Q_{22} + 2\mathrm{i}Q_{12}}{Q_{11} + Q_{22} + 2\sqrt{Q_{11}Q_{22} - Q_{12}^2}} \quad , \tag{5}$$

and correspondingly the ellipticity $\epsilon^{(\mathrm{s})}$ of the intrinsic brightness profile of the galaxy in terms of $Q_{ij}^{(\mathrm{s})}$. For example, if an image has elliptical contours of axis ratio $r \leq 1$, then $|\epsilon| = (1-r)/(1+r)$. From (4) one then derives the transformation between intrinsic and observed ellipticity

$$\epsilon^{(\mathrm{s})} = \frac{\epsilon - g}{1 - g^* \epsilon} \; , \tag{6}$$

where the asterisk denotes complex conjugation. Finally, since the sources are assumed to be randomly oriented intrinsically (and this assumption is not seriously challenged – note that the faint galaxies used for weak lensing come from a large range of redshifts, and so most of them are not at all physically related), the expectation value of $\epsilon^{(\mathrm{s})}$ vanishes; this can be used to show that the expectation value of the image ellipticity is just the reduced shear $g$. Hence, the ellipticity of every galaxy image yields an unbiased (though very noisy) estimate of the reduced shear. So much for theory ...

### 3.2 Practical measurement of the shear

Unfortunately, the scheme presented above cannot be applied in real situations, for several reasons:

- Observations are noisy; e.g., CCD images have a (near Poisson) noise due to the finite number of photons per pixel. The brightness profile of an object falls off, and outside its characteristic size, $I(\theta)$ becomes dominated by noise. The large weighting given by (3) to large $|\theta - \bar{\theta}|$ then means that the integral is completely dominated by noise. In order to prevent this, one needs to limit the range of integration. This can be achieved in several ways: either by adding a weight function into the integrand which depends on the surface brightness $I$, or by a weight function depending explicitly on $|\theta - \bar{\theta}|$. The former choice renders the relation between $I$ and $Q$ non-linear, and so the effects of noise are more complicated to account for. Therefore, in the scheme described below, a weight function $W(|\theta - \bar{\theta}|)$ is chosen. In addition, since galaxies are not isolated, one needs a cut-off in the integration before the neighboring object enters the integration range. The size of the weight function should be chosen such as to match the size of the image. With a spatially dependent weight function in the definition of $Q$, the transformation between source and image plane is no longer given by the simple relation (4) or (6).

- Atmospheric turbulence causes seeing; i.e., the point-spread function (PSF) for ground-based optical astronomy is limited by the atmosphere. The PSF is further affected by the imaging properties of the telescope. In any case, since the size of faint galaxies is comparable to the size of the seeing disk, images are severely smeared, i.e., they appear rounder than they would be without seeing. In effect, seeing degrades the measured shear.

- Owing to imperfect imaging properties in the telescope, tracking errors, wind shake, or other reasons, the PSF is not necessarily circularly symmetric. Even a circular image would then appear elongated if observed through an anisotropic PSF. Hence, an anisotropic PSF mimics shear.

These effects have to be accounted for in order to obtain a reliable estimate of the shear. Several image analysis tools have been written specifically for weak lensing applications. We shall now indicate the result of the necessary correction steps, as used in the *imcat* method [15, 21, 23]. The observed image ellipticity $\epsilon^{(\mathrm{obs})}$ (defined as in (4) and (6), but with a spatial weight function added) can be written as

$$\epsilon^{(\mathrm{obs})} = \epsilon^{0} + P^{\mathrm{sm}} q + P^{\mathrm{g}} g \, , \tag{7}$$

where $\epsilon^{0}$ is the image ellipticity one would observe if there were no shear, and if the PSF were isotropic. Although the relation between $\epsilon^{0}$ and the intrinsic source ellipticity is complicated, this is irrelevant in this scheme: if the expectation value of $\epsilon^{(\mathrm{s})}$ is zero, then necessarily that of $\epsilon^{0}$ also vanishes. For small values of the PSF anisotropy and the shear, the difference between $\epsilon^{(\mathrm{obs})}$ and $\epsilon^{0}$ can be linearized, which has been done in (7): $g$ is the reduced shear, and $q$ is the ellipticity of the PSF. The two factors (they are indeed tensors in general, but in practice turn out to be close to diagonal) $P^{\mathrm{sm}}$ and $P^{\mathrm{g}}$ describe the response of the image ellipticity to a PSF anisotropy and to a shear, respectively. These response factors depend on the brightness profile of the images and have to be calculated for each individual image. For example, a big image responds less to a PSF anisotropy than a small image, and for a small image, the response to a shear will be smaller than for a big one, owing to the smearing by the PSF. Appropriately averaging (7) over a set of galaxy images, using that the expectation value of $\epsilon^{0}$ vanishes, yields an estimate of the reduced shear.

The PSF anisotropy $q$ can be obtained by considering stellar images. Light from a star is unaffected by shear, and in addition, if smeared by an isotropic PSF, its image is round. Hence, for a star, $\epsilon^{0}_{*} = 0$ and $P^{\mathrm{g}}_{*} = 0$, yielding $q = \epsilon^{(\mathrm{obs})}_{*} / P^{\mathrm{sm}}_{*}$. Provided that the number of stars in the data field is sufficiently large, and that $q$ is a slowly varying function of position on the CCD image, one can use the $q$'s as measured from the stars to fit a (low-order) polynomial which then yields $q$ for every position on the frame. This determination of $q$ works quite well for some imaging instruments.

In addition, optical images show features which have to be dealt with carefully if shear measurements are to be carried out: cosmic rays have to be removed, diffraction spikes, trails and bleeding can lead to artificial object detections which characteristically have large moduli of the ellipticity etc. Whereas these artefacts can be recognized and thus corrected for or avoided, they complicate the weak lensing analysis.

## 4 Applications of weak lensing

Weak lensing is a young research area (the first arclets have been identified by Fort et al. in 1988 [12], and the first coherent shear around clusters was published by Tyson et al. in 1990 [36]), and the range of potential applications has grown at a substantial rate in the past few years. I shall outline a few of those in this section, without claiming that this list is even approximately complete; see [24] for an excellent review of weak lensing.

### 4.1 Mass and mass distribution of clusters of galaxies

Clusters of galaxies are the most massive bound structures in the Universe, and one might therefore expect that these systems produce the strongest weak lensing effect. Indeed, the vast majority of successful weak lensing observations have been targeted towards clusters. The results of these studies are two-dimensional maps of the projected mass density, and estimates of the bulk mass of the cluster. As mentioned at the end of Sect. 2, if the shear can be measured, the surface mass density $\kappa(\theta)$ can be reconstructed, due to the linear relations between $\kappa$, $\psi$ and $\gamma$. However, since a uniform surface mass density does not give rise to a shear contribution (as in Newtonian gravity!), $\kappa$ is determined only up to an additive constant. In addition, we have seen that not the shear itself is an observable, but only the reduced shear $g$ can be observed. This then leaves the invariance transformation $\kappa(\theta) \to \lambda\kappa(\theta) + (1-\lambda)$ [10,29], which for $\lambda \approx 1$ is similar to the addition of a homogeneous mass density. The scaling factor $\lambda$ cannot be determined from shear observations only, and is usually set such that, at least for large data fields, $\kappa \approx 0$ near the boundary of the field, or far from the cluster center. The mass reconstruction from the observable reduced shear is not more difficult than it would be from the shear itself (e.g., [31,33]).

Another way to quantify mass properties of clusters consists in defining the filtered mass density

$$M_{\rm ap}(\theta_0) := \int \mathrm{d}^2\theta \; \kappa(\theta)\, U(|\theta - \theta_0|) \;, \tag{8}$$

where $U(\theta)$ is a compensated filter, $\int \mathrm{d}\theta\, \theta\, U(\theta) = 0$ (this condition makes $M_{\rm ap}$ independent of the addition of a homogenous mass sheet) which vanishes beyond a filter radius $R$. One can then show [18,27] that the aperture mass can also be expressed as

$$M_{\rm ap}(\theta_0) = \int \mathrm{d}^2\theta \; \gamma_{\rm t}(\theta_0;\theta)\, Q(|\theta|) \;, \tag{9}$$

where $\gamma_{\rm t}(\theta_0;\theta)$ is the component of the shear at position $\theta_0 + \theta$ in the direction tangent to the direction of $\theta$, and $Q$ is a filter function which can be calculated directly in terms of $U$, and which also vanishes for $|\theta| > R$. Putting aside for a moment the difference between shear and reduced shear, the latter expression for $M_{\rm ap}$ can be estimated directly from observations by replacing the integral over the shear by a sum over the (tangential component) of image ellipticities. One example for the choice of $U$ is the so-called $\zeta$-statistics [19], where $U$ is a positive constant for $0 \le \theta \le \theta_1$, and a negative constant for $\theta_1 \le \theta \le R$, chosen such as to satisfy the compensation condition; then, $Q$ is different from zero only in the annulus $\theta_1 \le \theta \le R$, and if $\theta_1$ is chosen sufficiently large as to avoid the central part of the cluster where $\kappa$ takes appreciable values, the difference between $g$ and $\gamma$ in that annulus is small. For that choice, $M_{\rm ap}$ becomes the mean surface mass density within the circle $\theta_1$, minus the mean density in the annulus. Since the latter cannot be negative, this method yields a lower limit of the cluster mass within $\theta_1$.

Cluster mass profiles have been reconstructed for about 20 clusters so far (see [24] for a recent review), with data and methods which span quite in range in quality. The resolution of these mass maps depends on the number density of background galaxies used for the shear measurement, and is highest for observations taken with the HST. Several of the most detailed mass maps indicate that the bright cluster galaxies follow the underlying mass distribution quite well (e.g., [15, 20, 32]). The detection of strong weak lensing signals in high-redshift clusters ($z \sim 0.8$, see [8, 23]) shows that they are genuine massive structures which may pose very strong constraints on the cosmological parameters (e.g., [2]).

The above mentioned invariance transformation leaves the reduced shear invariant, but the magnification changes according to $\mu \to \mu/\lambda^2$, so that by measuring a magnification the invariance can be broken. Observable magnification effects include the change of the local number counts of faint galaxies (magnification bias) and the change in image size at fixed surface brightness. The former of these effects [7] has been observed in at least two clusters [11, 35], whereas the latter [3] may be difficult to observe from the ground (due to the PSF), but may be detectable in space-based observations.

### 4.2 Galaxy-galaxy lensing

Whereas clusters are massive enough to be studied individually with weak lensing techniques, galaxies are not. However, if one is not interested in the mass properties of an individual galaxy, but in the statistical mass properties of an ensemble of galaxies, the weak lensing signals of these galaxies can be statistically combined. If one considers two populations of galaxies, foreground and background galaxies, one would expect the ellipticities of the latter to be aligned preferentially in the direction tangent to the nearest foreground galaxies. Thus if one considers pairs of foreground-background galaxies, one should see a signature of weak lensing in the alignment statistics.

This idea can be formulated quantitatively, and the first galaxy-galaxy lensing signal has been detected in [6], using a single ground-based image. The foreground-background separation is made statistically, on the basis of apparent magnitude – fainter galaxies are on average further away – and the galaxy population is parametrized according to Tully-Fischer-type relations. The analysis found the characteristic rotational velocity of an $L_*$ galaxy to be about 220 km/s, in accordance to what is known from their rotation curves, and yielded a lower bound on the truncation radius of the dark halo; here, $L_*$ is the characteristic luminosity scale entering the Schechter luminosity function. Although this latter result is not very powerful yet, future observations using much larger data fields will be able to improve on this dramatically. Further galaxy-galaxy lensing results have come from HST observations (e.g., [14, 17]). With large enough samples, the request for a parametrization of the galaxy population may vanish, since then the galaxies can be binned according to 'similar types'. In this regard, the knowledge of even approximate redshifts will be very useful, and photometric redshift techniques seem to provide the necessary redshift accuracy.

### 4.3 Detection of (dark) halos

A weak lensing observation around a massive cluster leads to the detection of a coherent shear signal. This signal is then used to determine the mass of the cluster, in the way described above. As a first step, though, one can ask whether the cluster is detected at all in the weak lensing map, and this question can be quantified well (is the tangential shear several $\sigma$ above the noise expected from randomly oriented galaxy images?). Of course, one can also set up the experiment in a slightly different way: by taking a wide-field image in an arbitrary direction, one can look for points in that image around which the tangential alignment of galaxy images is several $\sigma$ above the noise level expected for randomly oriented images. If that is the case (say in a 6-$\sigma$ situation), one would conclude that this point corresponds to the 'center' of a massive halo. If at the same point an overdensity of galaxies were detected, one would conclude to have detected a cluster – selected by its mass properties. If, on the other hand, this overdensity of galaxies is not seen, then what? Would one conclude that a massive halo has been detected which does not contain luminous galaxies, a cluster failed to produce light?

Before turning to this question, it should be pointed out that a mass-selected sample of clusters would be of great value for cosmology. Whereas usually clusters (like other astronomical objects) are selected by the (optical or X-ray) light they emit, the comparison with cosmological predictions (either semi-analytical or numerical) is hampered by the fact that these concern the mass properties of objects (halos) rather than the light. In order to make predictions of the luminous properties, additional assumptions have to be made, such as a mass-temperature relation for clusters, and its evolution with redshift. The uncertainties associated with that have hampered the use of the cluster abundance as a function of mass and redshift as a most sensitive tool for testing cosmological models.[a] The selection of clusters by weak lensing techniques (that is, by mass) would circumvent these difficulties, and the observational results could be compared directly with numerical N-body simulations of the evolution of the dark matter in the Universe.

A quantitative way to proceed is to use the aperture mass, defined in (8) and (9) above. Positive values of $M_{\rm ap}$ signify the presence of a mass peak, and the noise of $M_{\rm ap}$ can be determined either from the data themselves (e.g., by repeated randomizations of the orientation of images), or be estimated analytically. As an illustrative example, consider a singular isothermal sphere with velocity dispersion $\sigma_v$. The signal-to-noise ratio for its detection is

$$\frac{\rm S}{\rm N} = 12.7 \left(\frac{n}{30{\rm arcmin}^{-2}}\right)^{1/2} \left(\frac{\sigma_\epsilon}{0.2}\right)^{-1} \left(\frac{\sigma_v}{600{\rm km/s}}\right)^{2} \left(\frac{\ln(\theta_2/\theta_1)}{\ln(10)}\right)^{1/2} \left\langle \frac{D_{\rm ds}}{D_{\rm s}} \right\rangle \tag{10}$$

where the angular bracket denotes an average over the source population (the ratio $D_{\rm ds}/D_{\rm s}$ is set to zero for galaxies with smaller redshift than the lens), $\sigma_\epsilon$ is the dispersion of the intrinsic ellipticity distribution, and $\theta_1$ and $\theta_2$ denote the inner and outer radius of an annulus in which the shear is measured. This S/N is obtained if the filter function $Q(\theta)$ is optimized for the mass profile of a singular isothermal sphere; using more generic weight function, the S/N is reduced by factors of order 1.5. Nevertheless, the foregoing estimate shows that dark matter halos at intermediate redshift with velocity dispersion in excess of $\sim 600$km/s can be detected from their shear field, a prediction [25, 27] that was impressively verified in the case of the cluster MS1512 [34]. The expected abundance of such halos which can be detected with that technique with S/N $\geq 5$ depends on the cosmology and the normalization of the power spectrum, but is in excess of $\sim 10$ per square degree [22]. This abundance, and the corresponding $M_{\rm ap}$-spectrum, can be directly compared with predictions from N-body simulations [26]. Thus, even a modest wide-field survey of $\sim 10$ square degrees will lead to an extremely useful sample of mass-selected halos.

Probably, most of them will be luminous, so that one detects ordinary clusters. Then the redshift of these halos can be determined spectroscopically, and one could investigate the range of mass-to-light ratios for these clusters. If this range extends toward very high M/L values then it may appear plausible that there may be some halos for which this value even exceeds those for which it can be measured, because there is so little light, and one therefore might detect 'dark' clusters (against which the normal selection procedure clearly would bias).

A first example of a shear-detected matter concentration has been found recently [9]. With two independent data sets of deep wide-field optical imaging, a tangential alignment was found

[a] An additional uncertainty of the association of X-ray selected clusters with prediction from, e.g., Press-Schechter theory is that many of the high-redshift clusters, which yield the most sensitive probe of the cosmological model, do show very substantial substructure which might indicate that the cluster formation is not yet completed for them – are those objects then to be identified with collapsed halos as counted by Press-Schechter?

around a point which, by random orientation of the galaxy images, would be as unlikely as $10^{-6}$. No obvious concentration of bright galaxies centered on the mass peak is found; however, faint X-ray emission from near the matter concentration has been identified from archival X-ray data. The interpretation of this result is unclear at present, and follow-up observations are needed to test whether the mass concentration is associated with a high-redshift cluster, or is an example of a 'dark clump'.

### 4.4 Cosmic shear and cosmology

If, as considered in the previous section, a blank field is targeted and investigated with respect to weak lensing, the strong tangential alignments show up as the most prominent features which can be identified with mass peaks. But of course, they cover only a small fraction of the total area. In the rest of the field, the image ellipticities are also affected by the tidal gravitational field between us and the source galaxies and therefore carry information about the larger-scale mass distribution in the Universe. Indeed, the two-point statistics of the shear (such as 2-pt. correlation function, or rms shear within circular apertures) is related directly to the projected power spectrum of the mass distribution in the Universe. Higher-order statistics correspondingly yields higher-order statistical properties of the Large Scale Structure; e.g., the skewness of the shear appears to be a sensitive probe of the density parameter $\Omega$ [5, 30, 37]. It should be noted that the weak lensing investigation of the LSS is the only method currently known which does not depend on assumptions about how the luminous matter traces the underlying dark matter distribution, with the exception of Cosmic Microwave Background experiments which study the LSS at much higher redshifts ($z \sim 1000$) and at large comoving length scale ($\gtrsim$ 10Mpc). Comparison between CMB and weak lensing results can be turned into a sensitive test of the gravitational instability picture of structure growth. In addition, even modestly large weak lensing surveys can break the degeneracies of cosmological model parameters left with the next generation of CMB experiments [16].

It must be mentioned, however, that the so-called cosmic shear is weak; depending on cosmology, the rms shear on a scale of a few arcminutes is about 1%. This implies that the systematic effects mentioned before (e.g., PSF anisotropy) must be understood to levels well below 1% to make a quantitative measurement of cosmic shear. Whereas no sure-stopper has been identified up to now for ground-based optical imaging, this field may especially profit from the control one may expect over the PSF of SKA, in particular on large angular scales.

## 5 Weak lensing and SKA

Let us summarize the observational requirements for an efficient weak lensing study:

- The number density $n$ of objects for which a shape can be measured reliably should be as high as possible;
- the mean redshift $\langle z \rangle$ of this source population should be high, to put a large fraction of them into the background of the lenses to be investigated and to maximize the average of $D_{\rm ds}/D_{\rm s}$;
- the source population should be as round as possible, i.e., $\sigma_\epsilon$ should be minimized;
- the ratio of the size of the PSF and that of the source should be as small as possible, to minimize the corrections that have to be applied to the observed ellipticities;
- for the same reason, the PSF anisotropy should be small;
- the PSF must be controllable, to allow the corrections for it;

- for most of the applications listed in Sect. 4, the field-of-view should be large to enhance the statistical significance of the results.

SKA will push radio astronomy in a position where the radio sky is as much filled with sources as in current optical astronomy. Since the limiting flux of SKA will be $\sim 100$ times fainter than currently achievable, predictions about $n$ and $\langle z \rangle$ are quite uncertain. If the dominant source population correspond to normal or star-forming galaxies, then $\langle z \rangle \sim 1$. If an additional source population turns up, potentially increasing $n$, their characteristic redshift will be most important. The value of $\sigma_\epsilon$ of the faint radio sources is also unknown; if they are dominated by core-jet type of sources, then $\sigma_\epsilon$ can be quite high, whereas hydrogen emission from normal galaxies yield probably much rounder sources. The FOV of SKA is large, and its PSF will be well controllable, perhaps better than it can ever be hoped for optical imaging. Therefore, SKA may be able to measure shears smaller than is possible with optical images.

Optical astronomy is also developing quickly, and to put SKA in perspective, one has to consider the expected evolution of optical imaging instruments in the next decade. In ground-based astronomy, two developments are of particular interest for weak lensing: the installment of 8-meter class telescopes, and the coming-on-line of wide-field imaging cameras, both at excellent observing sites (see [1]). These developments will allow to tremendously improve the weak lensing capabilities of optical astronomy, in particular in wide-field applications. However, seeing provides a fundamental limitation: as galaxies tend to become smaller when they are fainter, the number density of objects for which a shape can be measured reliably (i.e., corrected for seeing, which is only possible if it is not much smaller than the seeing disk) is limited, to about $60/\mathrm{arcmin}^2$, as shown by deep Keck images [8]. Space-based observations can achieve much higher number densities, as shown in the HDF, and the Advanced Camera on-board HST will yield substantial progress to the field. But it, as well as imaging instruments currently under discussion for the Next generation Space Telescope, will have a fairly limited FOV, of order $4' \times 4'$, and at least for wide-field application cannot easily compete with ground-based wide-field imaging cameras. In this respect, SKA will play a very important role for weak lensing, as it combines a high number density of objects for which ellipticities can be measured reliably (due to the knowledge of its PSF), with a large FOV – again, provided $\sigma_\epsilon$ is not excessively large and $\langle z \rangle$ not small for these faint radio sources.

## 6 General remarks

SKA will be superiour to current radio observatories by orders of magnitude, in several respects, in particular in sensitivity and FOV. Coupled with the angular resolution, it will allow observations one cannot even nearly approach today: Specifying the properties of SKA in order to carry out specific observations is, at least in part, wild guesswork. This specification is partly based on a fainter-N'-further philosophy, i.e., one can study objects of the same class at much lower luminosity (like faint AGNs) or similar objects at larger distance (e.g., SNRs in more distant galaxies). Although these may be valuable science drivers, this projection is very conservative. I think there can be no doubt that right in the first weeks of operation of SKA many new discoveries will be made (though the meaning of them may become clear only much later, e.g., after optical/IR spectroscopy and/or X-ray observations), since so much more volume of the observational phase space will open up. Also, even in the time span between now and the finishing of SKA, new science drivers will be identified.

Let me compare SKA with NGST which will also yield a jump in sensitivity by orders of magnitude relative to current observations, at least in the near-IR. Deepest K-band images today reach 24th magnitude, whereas NGST routinely will go to $\sim$29th magnitude. Predictions into that regime are of course very uncertain extrapolations, but the specification of instrumental capabilities needs to be taken from here. If one considers a possible distribution of observations for NGST, as obtained from the current version of the Design Reference Mission, then a large fraction goes into science programs which would not have been proposed only 5 years ago: for example, weak-lensing related imaging (5 years ago, the very first weak lensing results were published), the detailed investigation of the galaxy distribution by photometric redshifts (the U-dropout technique provided the first sizeable sample some 3 years ago), or the search for extra-solar planetary systems (with the first ones discovered only a few years ago). No question that the DRM will change in the course of time and may look very different at the time of launch of NGST. One can even argue that NGST will provide answers to question we have not yet dared to ask. One example would be the mass distribution in clusters: each massive cluster will contain tens of observable strong lensing features (arcs, multiple images) and a very cleanly outlined weak lensing structure, so that the projected mass distribution in these clusters can be obtained with very high angular resolution. What do we actually learn from these mass maps? Which cluster properties, or properties of cluster galaxies, can be probed, which models for the mass distribution in clusters can be critically checked (and eventually rejected) with these data?

The lesson to learn from that is that the current predictions of what one wants to do with NGST and SKA *are very conservative indeed.* This is to some degree unavoidable and can only partly be overcome with model calculations and simulations. What is important, though, is to construct these observatories such that they are flexible enough to allow a major modification of scientific goals and the corresponding observational programs. Since one does not know which direction these future requirements will take, the best guess may be to cover as much observational phase space as politically wise, financially affortable and compatible with the key science issues, to have a versatile instrument at hand. This aspect may be particularly relevant for SKA which is likely to render most other radio telescopes working in the same wavelength regime obsolete.

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