A&A manuscript no.
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02.01.2; 08.14.2; 08.09.2; 13.25.5; 08.03.5

ASTRONOMY
AND
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WZ Sagittae - an old dwarf nova

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Received date; accepted date

Abstract. We model the evolution of the accretion disk of WZ Sagittae during the long quiescence. We find that the large amount of mass in the disk derived from the outburst luminosity is a severe constraint and demands values of $\alpha_c \approx 0.001$ in contradiction to some recent suggestions. We include in our computations the formation of an inner disk hole and the growth of the disk due to redistribution of angular momentum. We find a new mode of disk evolution. The disk is quasi-stationary. Only about half of the mass transfered from the companion star flows through the disk, the other half is needed to build up the steadily growing outer disk. When the 3:1 resonance radius is reached the disk growth ends. From then on all transferred matter flows inward, the surface density increases, leading to an outburst within a few years. We predict superhumps at low luminosity during this last phase. We discuss X-rays expected, the white dwarf mass and distance to WZ Sagittae.

Key words: accretion disks – cataclysmic variables – stars: individual: WZ Sge – X-rays: stars – stars: coronae

1. Introduction

WZ Sagittae is a dwarf nova with extremely long time intervals between outbursts. During the vvlast outburst superhumps were discovered, which led to the classification of WZ Sge as a SU UMa type dwarf nova (Patterson et al. 1981). A review of investigations of WZ Sge in the past as well as a redetermination of the system parameters is given by Smak (1993). The orbital period of 81 minutes is close to the period minimum of cataclysmic variables and by standard interpretation puts WZ Sge into a late state of evolution of dwarf nova systems. For these close binaries the distance of secondary star and white dwarf primary decreases continously due to loss of angular momentum from the system by gravitational radiation until a period minimum is reached and it grows again for later times. Osaki (1996, review) showed that, in the framework of the thermal-tidal instability model, the decreasing number of normal outbursts of SU UMa systems between superoutbursts can be explained as an evolutionary sequence with decreasing mass overflow rate from the secondary star. WZ Sge is an extreme case in only showing superoutbursts. Due to its outburst behaviour it was estimated that the viscosity parameter in the quiescent state should be as low as 10^{-3} or even 10^{-4} (Smak 1993). Recently two different suggestions were made, by Lasota et al. (1995), Hameury et al. (1997) and by Warner et al. (1996) how the behaviour of WZ Sge could be modeled using a viscosity parameter in the cool state α_c =0.01, which is only slightly smaller than what is usually chosen to describe ordinary dwarf nova outburst cycles. We show with our computations that the viscosity has to be low, if we take the large amount of mass in the disk, 1-2 10^{24}g, as a constraint. This mass was deduced in two ways, from the UV outburst luminosity and from the hot spot brightness during quiescence (Smak 1993). Also the optical outburst lightcurve shows the high mass flow rate in the beginning. SIn addition Smak's value for the distance to WZ Sge is in the lower range of values given in the literature. Thus a high amount of mass in the disk seems unavoidable.

The evaporation of the inner disk is an important feature for the evolution of the disk, because it prevents the early onset of the instability. We include the physics of the hole formation and find that the hole can either persist during the long quiescence for systems containing a white dwarf of large mass or the hole can be closed for small white dwarf mass. But evaporation still continues at the inner disk edge close to the white dwarf. Important for the evolution is also to follow in detail how the disk grows due to the redistribution of angular momentum until, for some cases, the 3:1 resonance radius is reached and finally the onset of the disk instability occurs. We found that during the long quiescence the disk can reach a quasi-stationary state. X-rays originate from an inner disk corona and the thermal boundary layer of the white dwarf.

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We give a short description of observational data and model parameters in Sect. 2. In Sect. 3 we describe the previous modelling of the long-term behaviour of WZ Sge. The computer code is modified to include the evaporation in the inner disk and the growth of the disk due to redistribution of angular momentum (Sect. 4-6). We show the results for the disk evolution and discuss the influence of parameters in Sect. 7. The accretion of matter on to the white dwarf releases X-rays. In Sect. 8 we evaluate how much one may expect to observe and compare with ROSAT and earlier observations. In Sect. 9 we discuss modelling with higher values of α_c in previous work. We point out the consequences from the fact that the 3:1 resonance radius may be reached several years before the onset of the outburst. We further present a new view of Smak's (1993) interpretation of white dwarf luminosity variation, arising from our results. In the conclusion we sumarize what follows from our computations: (1) on the low viscosity, (2) on the importance of evaporation and (3) on the late evolutionary state of cataclysmic variables.

2. Observational data and derived model parameters

WZ Sg is the dwarf nova with the longest observed outburst period. The three outbursts recorded in the past, November 22 1913, June 29 1946 and December 1 1978 occurred after remarkably similar time intervals of 32.6 and 32.4 years, respectively (Mattei 1980). WZ Sge is an SU UMa type dwarf nova, but unique in the way that no normal outbursts occur in between the superoutbursts. Smak (1993) argued, that the extremely long outburst cycle can only be explained by a very low viscosity in the disk during quiescence. From this work we take the following parameters: mass of white dwarf M_1=0.45M$_\odot$ and mass of secondary star M_2=0.058 M$_\odot$, mass overflow rate from the secondary star of about $\dot{M}= 2\,10^{15}$ g/s. The amount of matter in the disk M_d accumulated until the outburst should be 1-2$\,10^{24}$ g. We also consider other parameters to see the dependence of the results on the assumed values. The white dwarf mass is the most important one for the evolution. Sion et al. (1995) and Cheng et al. (1997) analysed HST observations of WZ Sge and from spectra derived values for rotational velocity and gravity. Together with the observed 28 s period they inferred a white dwarf mass of 0.3 M$_\odot$ (or higher, if the star is differentially rotating). We add an independent estimate of about 0.7 M$_\odot$ from the observed period and assuming criticical rotation. Spruit and Rutten (1998) found in their recent investigation 1.2±0.25$M_\odot$ from an analysis of the stream impact region.

3. Previous modelling of the long-time behaviour of WZ Sge

It is difficult to understand the long outburst recurrence time. In previous investigations this behaviour was modelled with either a very small value of the viscosity parameter in quiescence α_c=0.001 or a moderate value α_c=0.01, which is only slightly smaller than the values usually chosen to describe ordinary dwarf nova outbursts.

3.1. Model based on α_c=0.001

Osaki (1995) simulated the outburst lightcurve with the the parameters α_c=0.001, α_h=0.03 (α_h viscosity in the hot state), $\dot{M}$=1.2 10^{15}g/s, M_1 = 1$M_\odot$ and M_2=0.1M$_\odot$ It was assumed that mass is accumulated in a torus at the circularisation radius ("Lubow-Shu" radius). This computation was based on the simplifying model of Osaki (1989). The observed outburst amplitude and duration are then well described. In the thermal-tidal instability model for SU UMa stars a decrease of the mass transfer rate leads to a longer superoutburst recurrence time and simultanously fewer normal outbursts in between (for a review see Osaki 1996). According to the standard scenario of CV evolution (Kolb 1993) mass transfer rates are expected to decrease with age. For the modelling of WZ Sge at the very end of this "activity sequence" not only $\dot{M}$ has to be low, also α_c must be extremely low.

3.2. Models based on α_c=0.01

To avoid the assumption of extremely small values α_c two different attempts were made to understand the behaviour of WZ Sge, both using a moderate value in combination with a hole assumed in the inner disk.

Lasota et al. (1995) and Hameury et al. (1997) pointed out that a hole in the inner disk due to evaporation into a coronal layer above the cool disk (Meyer & Meyer-Hofmeister 1994) or due to the presence of a magnetosphere of the white dwarf (Livio & Pringle 1992) prevents the onset of an outburst in the inner disk. They observe that the X-ray flux from WZ Sge soon after the outburst was roughly the same as several years later and argue that this points to a quasi-stationary disk structure during quiescence. Their analysis shows that with an inner hole reaching to 2.5 - 5 10^9cm, $\dot{M}$ =10^{15}g/s and α_c=0.01 no outburst occurs, the disk is stable. The mass contained in the disk is then much less than the value M_d derived by Smak. According to the estimate in Hameury et al. (1997) it is below 3 10^{23}g/s. To solve the two problems how to get the instability triggered in the stable disk and how to supply a high amount of mass the authors suggest that a fluctuation in the mass overflow rate from the companion star might lead to an increase of surface density in the disk beyond the critical value and trigger the instability. Then a very high mass overflow is expected due to irradiation of the secondary star. In their scenario this could happen any time after the disk is stationary.

Warner and al. (1996) try to model WZ Sge with the same value α_c= 0.01, a hole of 3 10^9cm , $\dot{M} = 6\,10^{14}$g/s, $M_1 = 0.6 M_\odot$ and an outer disk radius of 1.1 10^{10} cm. The assumed hole makes it possible to accumulate mass over a long time until after 36 years the outburst occurs. But due to the low accretion rate only 7 10^{23} g/s matter were transferred from the compagnion to the disk. The computed outburst lasts only 6 days.

4. Models with disk evaporation

Close to the white dwarf a hot corona above the cool disk leads to evaporation (Meyer & Meyer-Hofmeister 1994, F.K. Liu et al. 1995) of the inner disk. B.F. Liu et al.(1997) have shown in detail how the evaporation affects the disk for a typical dwarf nova. The evaporation rate scales as ${M_1}^{2.34}$ and $r^{-3.17}$ (F.K. Liu. et al. 1995). This allows to evaluate the coronal flow rate and the mass exchange between cool disk and hot corona (evaporation and condensation) as a function of radius. In a system like VW Hydri a hole is formed after the end of an outburst extending to about 2 10^9cm and it remains that size during the quiescence until the next outburst starts.

The position of the inner edge of the disk is determined by the balance between evaporation of gas from the cool disk into the corona and the supply of new matter from outer disk regions. The mass flow rate in the disk $\dot{M}(r)$ decreases with decreasing r. ($\dot{M}(r)$ is related to the surface density; for a description of disk structure see Ludwig et al. 1994). Because of this there is always a position where the rates are equal. The hole is larger the smaller $\dot{M}(r)$ is. The cool disk in WZ Sge relaxes to a quasi-stationary inward flow of mass due to the long time available in quiescence and the hole can be closed after a few years depending on the white dwarf mass. But evaporation is going on all the time and, at the inner edge, puts a significant fraction of this flow into the form of hot coronal gas.

5. The redistribution of angular momentum in the disk

The outer edge is determined by the redistribution of angular momentum. The gas in the incoming stream from the secondary star has a certain specific angular momentum. The radius at which the gas in the Keplerian orbit around the white dwarf has the same specific angular momentum is called "Lubow-Shu" radius.

Generally, in dwarf nova systems, there is no significant accretion on the white dwarf, all mass is kept in the disk during the comparatively short duration of quiescence. With the addition of low specific angular momentum gas from the secondary the outer disk radius shrinks towards the "Lubow-Shu" radius, only to expand again during outburst due to the flow of matter towards the white dwarf. The situation in WZ Sge and comparable systems is different. During the long quiescence quasi-stationary accretion on to the white dwarf is established. The specific angular momentum of this accreted matter is stored in the disk and as a consequence the disk grows. This goes on until it reaches the radius where the 3:1 resonance between Kepler binary period and the Kepler rotation occurs (which for these low mass ratio M_1/M_2 systems is inside the tidal truncation radius). Thus the tidal instability (Whitehurst 1988, Lubow 1991) sets in and the excentric disk produces superhumps in the lightcurve as discussed by Lubow (1994). To follow the evolution of systems with a long lasting quiescence like WZ Sge the radius change has to be taken into account in the computations. Ichikawa & Osaki (1992) had already included in their numerical code radius changes for the evolution of U Gem.

6. The numerical code

In our computations we solve the diffusion equation for mass and angular momentum flow and take the dependence of the critical values of surface density and the viscosity- surface density relation as described in Ludwig et al. (1994). In addition we take into account that the positions of both inner and outer edge of the disk change during the evolution.

We use the following formulae. $\dot{M}_0$ (taken positiv inward), Σ and Ω are mass flow rate, surface density and Kepler frequency in the disk, r_{out} is the outer disk radius and index "out" designates values at r_{out}, index "LS" at the Lubow-Shu radius r_{LS}. Conservation of mass and angular momentum at r_{out} give

$$\dot{M}_0 + \dot{M}_{out} = 2\pi\,(r\Sigma)_{out}\,\frac{\mathrm{d}r_{out}}{\mathrm{d}t} \tag{1}$$

$$\dot{M}_0 (r^2\Omega)_{LS} = \left((\dot{M} - 3\pi f) r^2\Omega\right)_{out} + \left(2\pi r\Sigma r^2\Omega\right)_{out}\frac{\mathrm{d}r_{out}}{\mathrm{d}t}, \tag{2}$$

where $\dot{M}_{out}$ is the mass flow in the disk at r_{out} (taken positive inward). Further $f = \int_{-\infty}^{\infty} \mu dz$ viscosity integral, z the height above the midplane, μ the effective viscosity, parametrized proportional to the value α (Shakura & Sunyaev 1973, compare Ludwig et al. 1994).

In the computer code we have an equidistant grid in $x = 2\sqrt{r}$ and use $b = \sqrt{r} f$ instead of f. Conservation of mass and angular momentum in the disk are guaranted with the proper determination of the position of the outer edge r_{out} and the appropriate value b_{out} there. b_{out} and $\frac{\mathrm{d}x_{out}}{\mathrm{d}t}$ are then determined by Eqs. (1) and (2).

$$b_{out} = x_{out}\frac{\dot{M}_0}{6\pi}(1 - x_{LS}/x_{out}) \tag{3}$$

$$\frac{\mathrm{d}x_{out}}{\mathrm{dt}} = \frac{4}{\pi x_{out}^3 \Sigma_{out}} \left[\dot{M}_0 - 6\pi (\frac{\mathrm{d}b}{\mathrm{d}x})_{out} \right] \quad (4)$$

x_{LS} is related to r_{LS} (we took the values of r_{LS} from Table 2 in the investigation of Lubow & Shu (1975)).

If the 3:1 resonance radius is reached a further growth of the disk is effectively cut off since the tidal instability quickly increases to such a strength, that all surplus angular momentum is transferred back to the binary orbit. We implement this by replacing Eqs. (3) and (4) by

$$\frac{\mathrm{d}b_{out}}{\mathrm{d}x} = \frac{\dot{M}_0}{6\pi} \quad (5)$$

and

$$\frac{\mathrm{d}r_{out}}{\mathrm{d}x} = 0 \quad (6)$$

7. New computation of the disk evolution during quiescence

We follow the evolution of the cool disk from the early quiescence to the onset of the next outburst. The orbital period is known. The geometry of the binary system containing white dwarf and Roche lobe filling secondary star is then determined, if we assume white dwarf mass. Further parameters are the mass overflow rate from the secondary star and α_c. We start our investigation with the parameters for M_1 and M_2 suggested by Smak (1993). We want to model the evolution that means find the onset of an outburst after 32 years. We take as constraints the amount of matter in the disk at this time, 1-2 10^{24}g, and the mass accretion rate of 2 10^{15}g/s , also derived by Smak. Note that this accretion rate will lead to the accumulation of the right amount of matter during the 32 years, but whether an outburst occurs at this time depends on the disk evolution, which is governed by the viscosity. Evaporation is an additional important feature, which influences the location where the instability is triggered. In the following we show the results, (1) the time until an outburst arises and (2) the amount of matter accumulated in the disk until then. We take α_c=0.001 fixed and vary the mass accretion rate. Then we discuss the variation of these quantities with α_c and the influence of evaporation. Finally we show results for a larger white dwarf mass.

7.1. Initial distribution

We take an initial distribution of surface density in the disk according to the results for ordinary dwarf novae of Ludwig & Meyer (1998), but scaled to the lower values of α_c used here. To start the computation of the long-time evolution we have to assume the disk size in the beginning. To test the effect of this we study two cases. In case (a) we take a relatively small disk. Such a disk then contains not much mass ($\approx$ 0.1 10^{24}g) and angular momentum. As an alternative, case (b), we start with a disk reaching almost to the resonance radius. Then more mass is in the disk ($\approx$ 0.28 10^{24}g). Consequently more matter should be in the disk at the onset of the outburst, so that the difference in mass ΔM_d corresponds to the amount 1-2 10^{24}g derived from the outburst luminosity. The total amount accumulated should be higher in case (b) and the matter has to be packed more densily. This demands a smaller viscosity α_c. All computations if not stated otherwise are case (a). The results for the computations case (b) ($\dot{M}$=2 10^{15}g/s, α_c= 0.001 and 0.0008) are marked by the two asterisks in Fig. 3. Comparing the evolution in case (a) and (b) we find, that the onset of the outburst happens for the same total amount of matter in the disk and at nearly the same time. An exact initial distribution could only be determined by a detailed computation of the outburst. Due to the long quiescence the evolution is only slightly influenced by this uncertainty.

7.2. Results for $M_1 = 0.45 M_\odot$ and α_c=0.001 ("standard parameters")

In Fig. 1 we show the evolution of the disk for our "standard" parameters. After 30.3 years the critical surface density Σ_B is reached at the distance r=1.86 10^9 cm from the white dwarf and the outburst starts. In the disk 1.44 10^{24}g matter are accumulated. In the left panel we show the surface density $\Sigma(r)$ in the disk, in the right panel the corresponding rates $\dot{M}(r)$ every 3 years during the evolution. The run of $\dot{M}$(r) during the first 3 years is due to the assumed initial distribution of surface density. In the early evolution the mass flow rate in the inner disk is small and evaporation creates a hole. The maximal extent of the hole r=3 10^9 cm, is already reached after less than 3 years. The position of the inner edge depends on the balance between the rate of mass flow inward and evaporation. The surface density increases continously during quiescence and with it $\dot{M}(r)$. The hole is closed after 9 years of disk evolution. Evaporation is still going on, then with the high rate of $\approx$ 10^{15}g/s. About 20% of this flow goes into wind loss, the remaining part accretes via the hot corona on to the white dwarf.

After the first decade we find an almost constant mass flow rate of 7-8 10^{14} g/s over a large part of the disk. The rate of evaporation, efficient in a ringlike region around the hole, can also be seen from the Fig. 1, left panel. It is the difference between the nearly constant value and the lower values at distances r less than about 2.5 10^9 cm. The values at the inner disk edge indicate how much mass flows via the cool disk on to the white dwarf. Differences resulting from accretion of hot versus cool gas are discussed in Sect. 6. The outer radius of the disk increases as can be seen from the extent of the lines at different evolutionary times. In Fig. 2 we show the changes of the outer and inner disk radius for "standard" parameters, but an

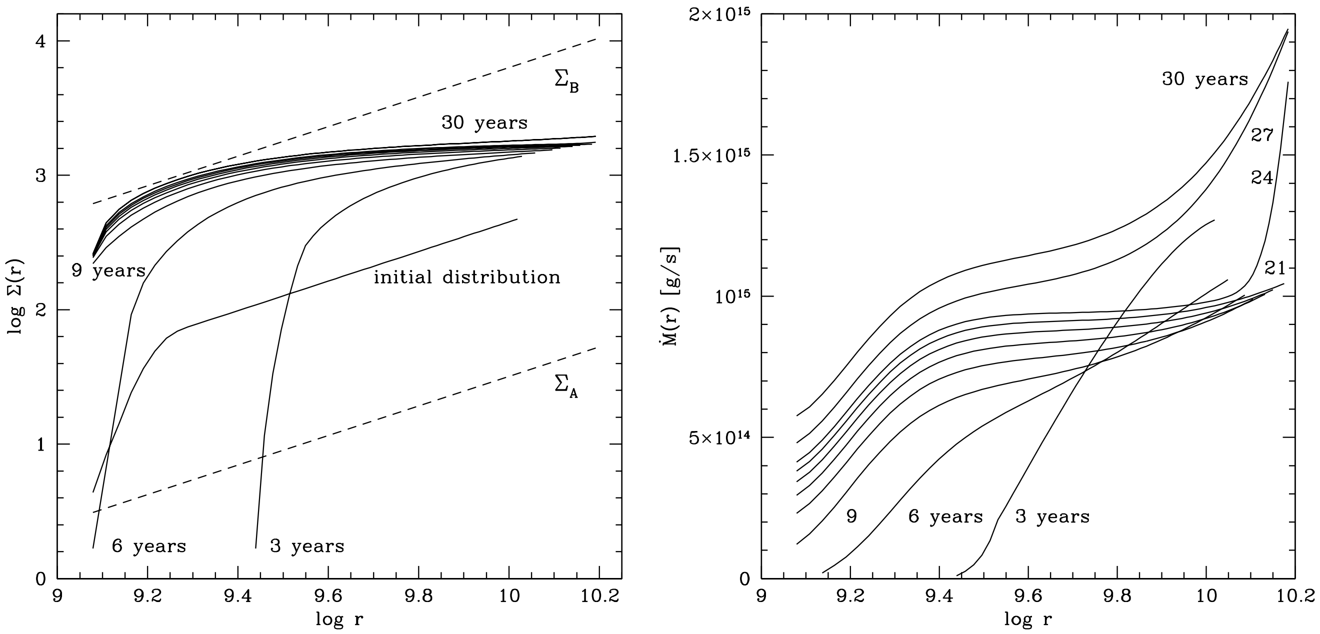

Fig. 1. Evolution of the disk during quiescence. Left panel: Surface density Σ given at time intervals of 3 years. The radial growth of the disk can be seen from the extent of the surface density lines. Σ_A and Σ_B are critical surface densities; above Σ_B only hot state possible, which means the outburst starts, if it is reached; r distance to white dwarf. Right panel: Corresponding mass flow rates in the disk $\dot{M}(r)$

initially larger disk (initial distribution case b),where the shrinking of the outer edge is more prominent. The disk grows until the resonance radius r=1.6 10^{10}cm ("standard" parameters) is reached.

The size of the disk is an interesting feature. Before the 3:1 resonance radius is reached roughly half of the newly accreted matter is needed to fill the larger and larger outermost disk areas. The remaining part is too low to trigger an outburst. After the resonance radius is reached the disk does not grow anymore, the mass flow in the disk suddenly is nearly doubled, and the surface density increases to the critical value. In Fig. 1, right panel, we see the higher mass flow rate after the resonance radius is reached. We point out that evaporation prevents instability in the early evolution. Without evaporation the critical surface density would have been reached at r= 1.5 10^9 after 9.6 years (compare Fig. 3). The fact that the hole is closed does not change this situation.

If we follow the disk evolution for given parameters M_1, M_2 and α_c the accumulation of mass depends further on $\dot{M}$. To get a model appropriate for WZ Sge the critical surface density has to be reached after $\approx$ 32 years and the amount of matter accumulated then has to be 1-2 10^{24} g. Fig. 3 shows this accumulation process for different rates of $\dot{M}$. Higher rates make the outburst happen too early. The opposite happens for low rates. With $\dot{M}$=10^{15} g/s, which is half of what was suggested for WZ Sge (Smak

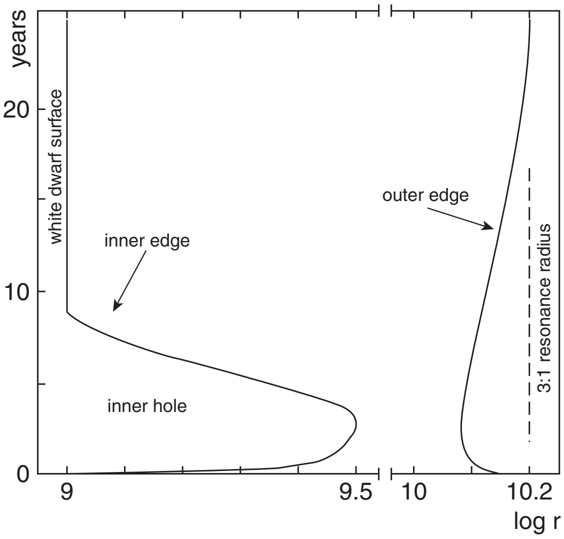

Fig. 2. Changes of the position of outer and inner disk edge during the evolution for "standard case " parameters, but a large disk initially.

1993) the surface density in the disk never reaches the critical value, no outburst occurs. The disk is stationary. All matter accreted from the secondary star flows through the disk on to the white dwarf (except the part lost by the wind). This means, that mass is accreted continously via

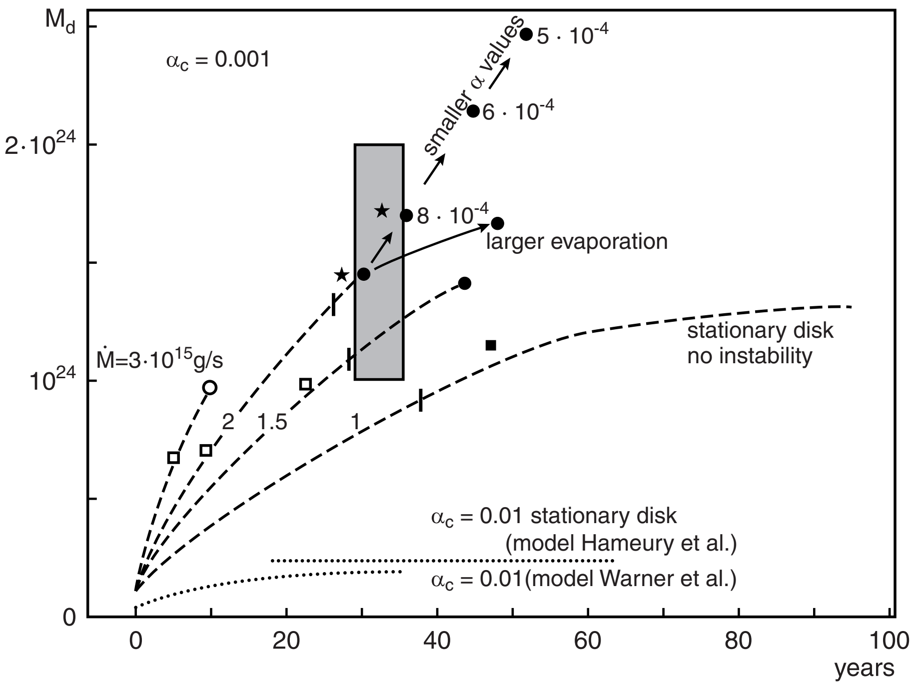

Fig. 3. Amount of matter accumulated in the disk after time t for mass overflow rates from the companion star $\dot{M}$=1, 1.5, 2 and 3 10^{15}g/s. Broken lines describe the accumulation of matter during the evolution. The shaded box marks the observed recurrence time and the amount of matter derived by Smak (1993), which we want to model. Circles and squares indicate the onset of an outburst, either including evaporation or not; bars at the evolutionary lines mark the time, when the 3:1 resonance radius is reached; circles/squares are filled if this is the case, otherwise open. The 2 asterisks show the results based on initial distribution case (b), to be compared with the filled neighbouring circles (see Sect. 7.1); arrows indicate the effect of variation of α_c and evaporation. M_1=0.45$M_\odot$, M_2=0.058$M_\odot$. Models by Hameury et al. and Warner et al. (see text) indicated by dotted lines.

the corona and the disk on to the white dwarf. In Fig. 4 we show how the disk reaches such a stationary state.

For comparison we have also performed computations neglecting evaporation. As can be seen from Fig. 3 the outburst would happen much earlier, after 9.6 years instead of 32 years for the same parameters. The amount of accumulated matter would be lower than what corresponds to the observed increase of 7 magnitudes in the outburst lightcurve and the duration longer than 1 month (Mattei 1980). Such an outburst is prevented by the formation of a hole.

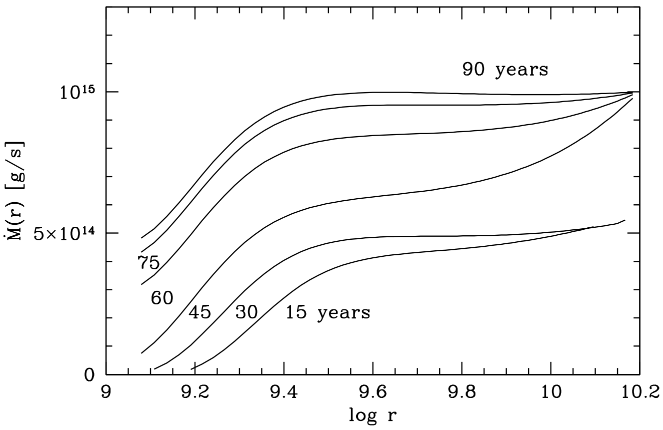

Fig. 4. Mass flow rates $\dot{M}(r)$ in the disk for a mass overflow rate from secondary $\dot{M}$=10^{15}g/s, otherwise "standard" parameters.

7.3. *Results for smaller α_c*

A smaller viscosity means that the mass flow in the disk is reduced and the matter is packed more densily. For the evolution this results in a higher M_d and a longer recurrence time. Figures 3 and 5 show the results for viscosity values α_c = 5-8 10^{-4} instead of 0.001.

7.4. *Results for different strength of evaporation*

We have computed one example of evolution for our "standard" parameters, but with an evaporation rate three times as high as the regular rate. This means a larger hole, a later onset of the outburst and more matter accumulated (see Fig. 3).

The opposite case is neglecting evaporation. We indicated already in Fig. 3 (squares) at which time the instability would be triggered without a hole in the inner disk.

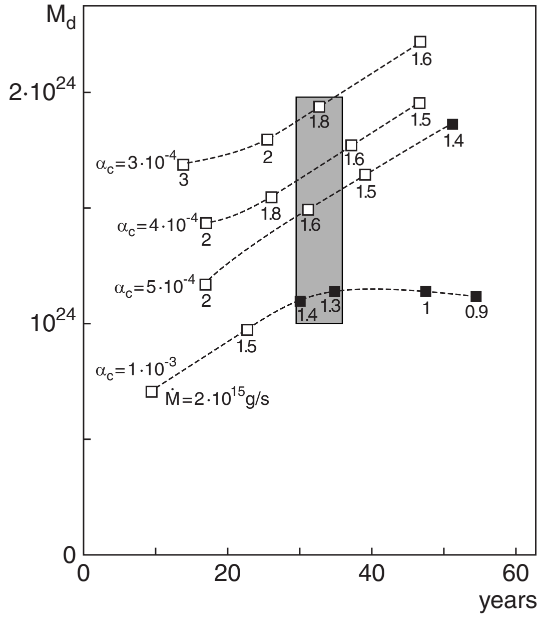

Fig. 5. Observed recurrence time and amount of matter in the disk at the onset of the outburst (shaded box). Squares are the computed accumulated mass and recurrence time at the onset of the outburst, without evaporation, as functions of cool state viscosity α_c and mass transfer rate $\dot{M}$.

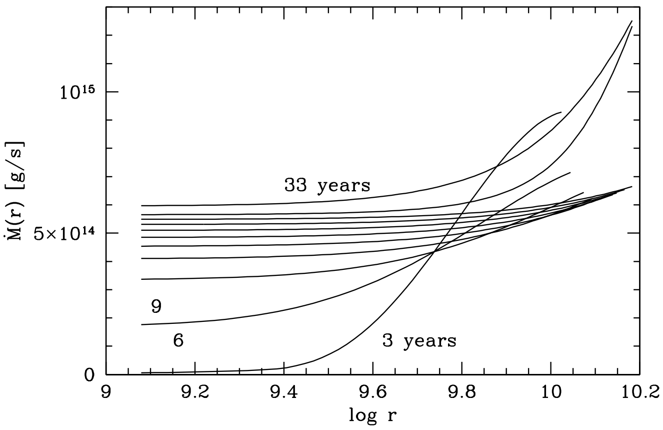

Fig. 6. Mass flow rates $\dot{M}(r)$ in the disk for a mass accretion rate from the secondary $\dot{M}$=1.3 10^{15}g/s otherwise "standard" parameters, but without evaporation

To fullfill the constraints, long recurrence time and large amount of matter in the disk, other parameters have to be taken.

We have performed a series of computations of disk evolution varying α_c and $\dot{M}$. The critical surface density is reached the earlier the higher the mass accretion rate. One needs small values for α_c= 3 10^{-4} to 1 10^{-3} in combination with low rates $\dot{M}$= 1.8 to 1.3 10^{15} g/s to get the right recurrence time and amount of matter in the disk. Figure 5 shows the results in terms of the mass accumulated at the onset of outburst. Only for the lowest rates $\dot{M}$ is the resonance radius reached before the outburst starts. In the other cases the disk expands to the 3:1 resonance radius during the outburst. Then superhumps would only appear in the outburst. Figure 6 shows the mass flow rates in the disk for various times during the evolution for a low mass transfer rate, 1.3 10^{15}g/s, from the seccondary star. The disk is quasi-stationary. Without evaporation the matter is accreted via the cool disk on to the white dwarf. For a rate a bit lower the critical surface density would never be reached. So this case is similar to a quasi-stationary disk with evaporation (see Sect. 7.2, evolution for $\dot{M}$=1 10^{15}g/s in Fig. 3).

7.5. Results for larger white dwarf mass

For larger white dwarf mass the evaporation process is more important. The higher rate keeps the hole open longer. We computed the evolution for $M_1 = 1 M_\odot$, $M_1 = 1.2 M_\odot$ and the mass ratio M_2/M_1=0.075. For the larger white dwarf mass a larger fraction of matter passes continously through the disk and in the inner part through the corona on to the white dwarf. Less matter can be accumulated in the disk. We found the following results. (1) $M_1 = 1.2\,M_\odot$, α_c=0.001, $\dot{M}$=2 10^{15}g/s (rate "standard parameters") no outburst. (2) $M_1 = 1 M_\odot$, α_c=0.001, $\dot{M}$=2 10^{15}g/s, outburst after ≈100 years, this means this rate corresponds to an almost stationary disk. If we increase $\dot{M}$ to 3 10^{15}g/s an outburst arises after 50 years. In all three examples ≈3 10^{24}g of matter were accumulated. In agreement with our findings from parameter variations described earlier we get for the $1 M_\odot$ white dwarf about the right recurrence time 29.2 years and amount of matter 1.36 10^{24}g with α_c=0.003 and $\dot{M}$=2.5 10^{15}g/s. The mass flow rate on to the white dwarf increases to values around 2 10^{15} during the quiescence. This and the deeper gravitational potential of the more massive white dwarf implies a higher X-ray flux from the system.

8. X-rays

8.1. Observations for WZ Sge

Observations of X-rays from cataclysmic variables (CVs) with the Einstein Observatory also included WZ Sge (for a review see Cordova & Mason 1983, for the determination of temperatures Eracleous et al. 1991). WZ Sge was observed in April 1979, April and May 1980. It would be interesting to learn about changes shortly after the outburst. The low number of counts had been at about the same level for the three times of observations, but the derived best fit temperatures variied between 2.9 and 5.8 keV. The X-ray flux (0.1-3.25 keV), unabsorbed, was 0.46 to 0.62 10^{-11} ergcm^{-2}s^{-1}. Mukai & Shiokawa (1994) analysed the EXOSAT ME archival data on dwarf novae. From the brightest observation of WZ Sge in October

1985 they found the X-ray flux (2-10 keV) of 0.39 10^{-11} ergcm^{-2}s^{-1}. Richman (1996) analysed the X-ray flux from CVs observed with the ROSAT PSPC. From the observations of WZ Sge taken in April 1991 she found the best fit temperature 3.4 keV and the flux (0.1-2.4 keV) 0.13 10^{-11}ergcm^{-2}s^{-1}.

Using a correlation between mass accretion rate and the ratio of X-ray to visual flux (Patterson & Raymond 1985) the ROSAT observations indicate a mass accretion rate of $10^{14.6}$g/s (Richman 1996). This distant-independent estimate from the observations supports our theoretical picture, that accretion goes on all the time. The amount of mass flow in quiescence of WZ Sge as suggested by our model agrees with this estimate.

8.2. Expected X-ray flux

Accretion on to a white dwarf at a rate of $10^{-14.6}$g/s releases energy, which, if mainly in X-rays, would predict significantly higher fluxes at earth than observed. There are however a number of reduction factors, which together bring prediction and observation into approximate agreement.

8.2.1. Accretion on to a slowly rotating white dwarf

We first discuss the release of accretion energy in the surroundings of the white dwarf. During the quasi-stationary evolution the mass flow rate is around $10^{-11} M_{\odot}$/y in the middle disk (at distances larger than about $10^{9.5}$cm , compare Fig. 1, right panel). Farther in, roughly $\frac{1}{3}$ of the mass flow remains in the optically thick disk, $\frac{2}{3}$ evaporate into the corona and of these, $\approx$ 20% are lost in a wind.

If the white dwarf is not close to critically rotating, both flows, optically thick and thin, dissipate their rotational energy, about half of the accretion energy, in a frictional boundary layer close to the surface of the white dwarf (Pringle 1977, Pringle & Savonije 1979). The boundary layer related to the coronal flow is always optically thin. At these low accretion rates also the boundary layer related to the flow via the disk is probably optically thin (Narayan & Popham 1993, Popham & Narayan 1995). The other half of the accretion energy of the coronal flow is dissipated in the corona above the optically thick disk. What is not used up by the escaping wind is conducted down and radiated away in the thermal boundary layer between corona and disk. The energy dissipated in both, white dwarf and coronal boundary layer adds up to an efficiency of about 0.5 of the total gravitational energy release. With a hole the situation is roughly the same.

Another reduction is expected from the spectral distribution of X-rays. F.K Liu et al. (1995) investigated how much X-rays and UV radiation are caused by the accretion of matter evaporating from the disk and flowing via the hot corona on to the white dwarf. A theoretical spectrum was compared with ROSAT observations for VW Hydri (Meyer et al. 1996). The form of this spectrum is very general due to the scaling properties of such transition layers. Fig. 7 shows the luminosity contributions of the various temperature layers in the thermal boundary layer of an accreting white dwarf. The multi-temperature character clearly cannot be satisfacturely be described by a one-temperature spectrum (see also Richman 1996). Fig. 7 shows that a fraction of about 0.4 of the radiation is below the sensitivity limit of 0.1 keV of the various X-ray observatories. For ROSAT also an upper cut-off might matter.

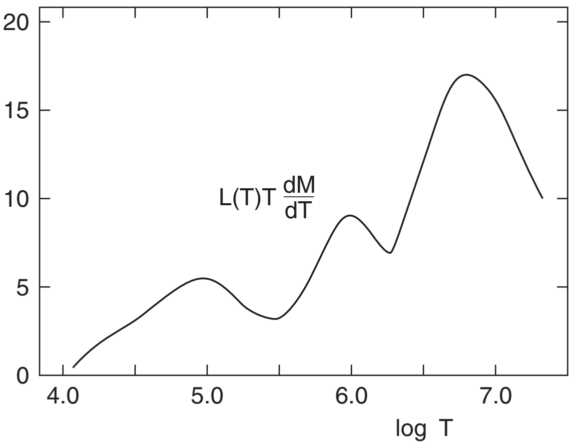

Fig. 7. Luminosity contributions of the various temperature layers in the thermal boundary of an accreting white dwarf, T temperature in Kelvin, M emission measure and L(T) luminosity per andunit emission measure of the optically thin plasma in non-dimensional units (according to F.K Liu et al. 1995). The area underneath the curve gives the contribution of the corresponding temperature interval. Note the multi-temperature character and the significant contributions below 0.1 keV ($\approx$ 1.2 10^6K).

A further reduction of the observable flux is due to the fact that half of the optically thin emission layers are hidden by the white dwarf and accretion disk surfaces, respectively. The reduction is 0.5 for a disk with an extended hole. If the disk reaches near to the surface of the white dwarf a further reduction results, because the white dwarf hemisphere towards the observer is partially covered by the disk and vice versa, depending on inclination. The total observable fractions are then 0.32 for the white dwarf and 0.4 for the disk (i=75° for WZ Sge, Smak 1993).

These reductions add up to 0.15 and 0.10 with and without an extended hole. This gives an observable X-ray luminosity of

$$L_{\rm x,obs} = 0.10 \frac{G M_1 \dot{M}}{R} \tag{7}$$

R radius of white dwarf. For $M_1 = 0.5 M_\odot$ the flux density at earth (unabsorbed) is

$$f_x = L_{\rm x,obs}/4\pi d^2 = 9.7 \times 10^{-12} \dot{M}_{-11} \left(\frac{d}{60{\rm pc}}\right)^{-2} \frac{\rm erg}{\rm cm^2 s} \quad (8)$$

$\dot{M}_{-11}$ is the mass accretion rate in units of $10^{-11} M_\odot$/y. This is about double the observed flux.

8.2.2. WZ Sagittae, a critically rotating white dwarf

Now we discuss the consequences of critical rotation on the observable X-ray flux and the white dwarf mass.

The photometric period of 28s discovered in UV light from WZ Sge by Welsh et. al. (1997 and references to earlier work there) appears to originate at the surface of the white dwarf and indicates a very rapid rotation of this star (see also Cheng et al., 1997). Using the mass-radius relation and the variation of equatorial radius with rotation interpolated from Hachisu (1986) by Popham & Narayan (1995) we can determine which white dwarf rotates critically at its equator with this period. We find

$$M = 0.69 M_\odot,\ R_{\rm equ} = 1.2 \times 10^9 {\rm cm} \quad (9)$$

where R_{equ} is the equatorial radius of the critically rotating star, 1.6 times that of a non-rotating white dwarf with the same mass. This mass is a lower limit since with this period any less massive white dwarf with its larger radius and smaller gravity would rotate supercritically. Though the mass of the white dwarf in WZ Sge might be larger and the star then rotate subcritically it is fairly probable that WZ Sge has already reached critical rotation during its long evolution. One can evaluate how much mass transfer to the white dwarf is required to speed it up to critical rotation. For example, if one assumes that mass is accreted with Kepler angular momentum at the equator and that all accreted matter is lost again in consecutive nova explosions but with the mean angular momentum of the white dwarf surface then one obtains critical rotation after mass transfer of

$$\Delta M = \frac{3}{2} {r_g}^2 \ln 3\, M_1 = 0.21 M_\odot \quad (10)$$

where the radius of gyration r_g for a $0.69 M_\odot$ white dwarf, $r_g^2 = 0.184$ (H. Ritter, private communication) was used. If WZ Sge entered the cataclysmic variable evolution with an orbital period above 3 hours this amont of mass has already been transferred and the white dwarf will now be critically rotating.

The X-rays from accretion on to a critically rotating white dwarf are even more reduced since no frictional boundary layer forms. Material from an optically thick accretion disk then settles on the white dwarf surface without becoming optically thin and producing X-rays. The only remaining contribution comes from the coronal gas that releases 1/2 of its gravitational energy as it spirals towards the white dwarf surface. Again roughly 20% will be lost in a wind together with about twice its escape energy. The remaining energy release is then $\frac{1}{8} G M_1 \dot{M}_c / R$.

The coronal accretion rate $\dot{M}_c$ is 0.8 of the mass flow rate that has disappeared from the flow of the optically thick disk on approaching the white dwarf, in our example of Fig. 1, right panel, typically $\dot{M}_c \approx 0.5 \dot{M}$. Of the energy released in X-rays 1/2 is covered by the disk surface (or by the white dwarf if part is conducted to the white dwarf surface), about 0.2 is further hidden behind the white dwarf for WZ Sge's inclination, and 0.4 is out of spectral range (compare Sect. 8.2.1). Alltogether this gives an observable X-ray luminosity of

$$L_{\rm x,obs} = 0.03 \frac{G M_1 \dot{M}_c}{R} \approx 0.015 \frac{G M_1 \dot{M}}{R} \quad (11)$$

and a flux density at earth (unabsorbed) of

$$f_{\rm x} = L_{\rm x,obs}/4\pi d^2 = 1.6 \times 10^{-12} \dot{M}_{-11} \left(\frac{d}{60{\rm pc}}\right)^{-2} \frac{\rm erg}{\rm cm^2 s} \quad (12)$$

for $M_1 = 0.69 M_\odot$ and R the equatorial radius.
These estimates show that the accretion rates in quiescence determined by our models lead to X-ray fluxes in good agreement with what has been observed, 1.3 to 6.2 $10^{-12} {\rm erg cm^{-2} s^{-1}}$, resolving an apparent puzzling discrepancy. A slight preference for a critically rotating white dwarf is indicated.

8.2.3. Magnetic accretion spots on the white dwarf?

The pulsations observed in various wavelength ranges might suggest accretion on the rotating poles of a magnetic white dwarf. Such a field would have to be weaker than 10^4 gauss. Otherwise the disk/magnetosphere interface reaches beyond the co-rotation radius and matter entering the magnetosphere would be flung out instead of accreted.

However, the only intermediate quality of the optical pulsations, $Q \simeq 10^5$, the fact that the mean period varied by 1.3‰ between 1995 and 1996 (Patterson et al. 1998), and the difference between the UV-period (28.10s, Welsh et al. 1997) and the X-ray/ optical period (27.86s, Patterson et al. 1998) argues against an accretion spot frozen into the rotating white dwarf. It might rather suggest accretion spots formed by magnetic fields brought in with the accreted matter and circulating around the nearly critically rotating white dwarf, similar to what has been suggested for dwarf nova oscillations (Meyer 1997). Optically thin accretion at higher latitudes (X-rays, also reprocessed into optical light) could have a slightly shorter period than equatorial, optically thick disk accretion (UV light).

The situation remains unclear and needs further study. The magnetic pressure of a dipolar white dwarf field of surface strength 10^4 gauss could just interfere with coronal evaporation pressures at $r = 10^{9.4}$cm and affect the coronal flow, possibly reducing the X-rays even below the observed level. Accretion along the magnetic funnel of accretion formed accretion spots would concentrate the thermal boundary layer to a smaller surface area and result in an increase of the maximum temperature $\sim$ (area)$^{-2/7}$. The above estimates of X-rays observable will not be much affected by this.

9. Discussion

9.1. Comparison with previous modelling

We shortly discuss here the earlier modelling described in Sect. 3 in the context of our results.

9.1.1. Model of Osaki (1995)

The simplifying model made already clear that low viscosity values are needed. Our more detailed computations allow to study two effects, which are important for the evolution, (1) the amount of matter not accumulated in the disk, because it is accreted on to the white dwarf or lost in the wind from the corona during quiescence; (2) the growth of the disk with respect to the 3:1 resonance radius.

9.1.2. Model of Lasota et al. (1995) and Hameury et al. (1997)

In this model the disk, with an assumed inner hole of specified radius, is stationary and stable. A fluctuation in the mass overflow rate is then thought to bring the disk to the point of instability and trigger the outburst. One would then expect quite a fluctuation in repetition times between consecutive outbursts. This contrasts with the nearly exactly (only 2% variation) regular repetition observed and the very regular build-up to outburst in the model presented in this paper. A further problem might lie in the (presumed irradiation caused) large mass transfer required during outburst. It becomes necessary since the assumed stationary disk contains too little mass to explain the observed outburst energy. It would require an increase in the overflow rate by a factor of about 300 while observed increases in some dwarf novae are limited to only a factor 2 (Smak 1995, Hameury et al. 1997).

9.1.3. Model of Warner et al. (1996)

Also in this model a hole is assumed. We note that here as in the preceding model, a more detailed physical picture of the hole formation and change of its size can have a significant effect on the duration of the quiescence. The chosen low mass overflow rate from the secondary makes it possible to have the outburst occur after only 36 years. But the low amount of matter provided for the outburst then shows up in the computed short duration. The authors suggest that in SU UMa systems small, regular outbursts are followed by larger superoutbursts. But for WZ Sge such regular outbursts were not observed. It seems that the claim “down with low α” cannot be realized as suggested.

9.2. Re-interpretation of the observed UV flux of WZ Sge after the outburst in the context of our results

Smak (1993) used IUE observations of WZ Sge, taken 1979 July 11 and 1981 November 22, 0.6 and 2.4 years after the December 1 1978 outburst to determine temperatures. Together with observed fluxes he derived values for the product $x(R/d)^2$, where x is the fraction of the full “stellar disk” πR^2 that is not occulted by the accretion disk and is radiating with the effective temperature determined. His values are $2.72\,10^{-23}$ and $1.56\,10^{-23}$, respectively. Smak assumed x_{79}=0.63 (x_{79} value in 1979, x_{81} in 1981) corresponding to a full disk whose lower half is partially occulted by the accretion disk (i=75°). From the ratio of the derived values $x(R/d)^2$ one gets x_{81}=0.35. This was interpreted as shrinking of the white dwarf radiating area to an equatorial belt. The long-term IUE observations indicate a cooling of the white dwarf (la Dous 1994).

Our model allows another interpretation. If in the 1979 observation an inner disk hole was present as expected theoretically in the early phase of quiescence, flux from one full hemisphere of the white is observable and, in Smak's terminology, the fraction x_{79} is equal to 1. From the ratio of the values determined by Smak then follows x_{81}=0.59. This agrees, within the errors with 0.63 corresponding to the fraction visible if the hole is closed. This changes the estimate for the distance to WZ Sge from Smak's value and error $d = 48 \pm 10$ pc to $d = 60.5 \pm 13$ pc for the $0.45 M_\odot$ white dwarf. For the critically rotating $0.69 M_\odot$ white dwarf one may estimate (see Hachisu 1986) a radius $\approx 0.77\ R_{\rm equ}$ of a “stellar disk” of the same area as the rotationally flattened cross section of the real star. With the equatorial radius of about $1.2\,10^9$cm this yields the distance estimate $d = 50.6 \pm 11$ pc. A not critically rotating star of larger mass would be correspondingly closer.

The observed change in the UV flux between 1979 and 1981 supports the prediction of formation and disappearance of an inner hole of the quiecent accretion disk.

9.3. Consequences from reaching the 3:1 resonance radius

From our computations we found that the 3:1 radius can be reached already several years before the outburst (in Fig. 3 the bars at the evolutionary lines mark the time when the resonance radius is reached). This means that the outer disk from this time on has an excentric shape and superhumps should be seen. They dissipate the work done in transferring accreted angular momentum back to the orbit, $\approx (\Omega_{out} - \Omega_{orb})(r^2\Omega)_{LS}\dot{M}$, which is about twice the hot spot luminosity (Ω_{orb} orbital frequency). We note that the weak early superhumps in outburst do not differentially precess (Whitehurst 1988, DeYoung 1995, Patterson 1995, Kato et al. 1996) and we might expect the same stationarity in the lightcurve for such quiescent superhumps.

10. Conclusions

Our detailed computations show that values α_c around 0.001 are necessary to accumulate enough matter in the disk to explain outburst duration and amplitude. Our investigation shows new features which are essential for the understanding of WZ Sge. The evaporation plays an important role in the way that it prevents a premature outburst. As long as the disk size grows, more than half the matter transferred from the secondary star is used up to fill the newly added outer disk areas. Only after the disk has reached the 3:1 resonance radius no further growth occurs and all accreted matter can flow inward. This results in a higher mass flow rate and triggers the outburst. Evaporation goes on all the quiescence with the rate corresponding to the inner edge of the disk close to the white dwarf. This means accretion of matter via the corona on to the white dwarf at a constant rate of about 10^{15}g/s. Without evaporation the outburst behaviour of WZ Sge can also be modeled, but then requires small values α_c for the cool state, $\leq$0.001, and low mass overflow rate from the companion sta,r of order $\dot{M}$=1.4 10^{15}g/s.

The finding that the value of α_c has to be smaller than usually assumed for dwarf novae raises again the question of the cause. A new interesting suggestion of Gammie & Menou (1998) relating low viscosity to low magnetic Reynolds numbers does not seem to explain why the dwarf novae in late evolutionary phase should have low values with otherwise very similar quiescent disks.

Our estimate of the observable X-rays from WZ Sge yields agreement with the observations and slightly favours a critically rotating white dwarf of $0.7\,M_\odot$ at a distance of 50 pc.

Most of our models predict superhumps before the outburst. Their appearance in late quiescence, observable as a change in the orbital lightcurve, would signal that the disk has reached its final radius at 3:1 resonance and that after the disk relaxation time the next outburst will occur.

Models for WZ Sge are relevant for dwarf novae in a late stage of secular evolution. WZ Sge is already a marginal system in that an outburst occurs only after a very long accumulation time. If the accretion rate would be only slightly smaller, no outbursts would arise. Instead the disk becomes stationary. Since the mass transfer rate from secondary star to white dwarf decreases during the secular evolution of dwarf novae (Kolb 1993) we expect a large number of systems "beyond WZ Sge". Such systems are not conspicuous optically, but the X-rays from all these binaries add up to quite an amount of radiation. Consequences arising from this will be investigated separately (Kolb et al., in preparation)

The features described for WZ Sge have relevance also for other systems including X-ray transients (Mineshige et al., to be published in PASJ).

Acknowledgements. We thank Ulrich Kolb, Hans Ritter and Henk Spruit for valuable discussions.

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