Two- and Three-Dimensional Simulations of the Thermonuclear Runaway in an Accreted Atmosphere of a C+O White Dwarf

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Abstract. We present the results of two- and three-dimensional calculations of turbulent nuclear burning of hydrogen-rich material accreted onto a white dwarf of 1.0 $M_\odot$. The main aim of the present paper is to investigate the question as to whether and how the general properties of the burning are affected by dimensionality and numerical resolution effects. In particular, we want to see whether or not convective overshooting into the surface layers of the C+O white dwarf can lead to self-enrichment of the initially solar composition of the hydrogen-rich envelope with carbon and oxygen from the underlying white dwarf core.

1 Introduction

Today, we know that novae are white dwarfs in close binary systems with a main sequence star as their companion. The white dwarf collects hydrogen-rich material from the companion which first settles into an accretion disk and is then accreted onto the white dwarf's surface. Due to compressional heating this envelope can reach temperatures sufficiently high to burn hydrogen into helium, first by the proton-proton chain and later by the CNO-cycles, where C, N and O nuclei act as catalysts for the burning of hydrogen. Numerical simulations have shown that these reactions can fuse all H into He and eject the envelope, provided the matter is electron degenerate and the CNO-abundances are sufficiently high, several times their solar values [Starrfield et al. (72), Starrfield et al. (74)]. If these conditions are met, the outburst resembles the observed properties of classical novae very well.

It is an important feature that the presence of C, N, and O nuclei enhances the energy production during the violent stages of the nova outburst dramatically. Spherically symmetric hydro-dynamical simulations of the accretion process and the following outburst [Starrfield et al. (78), Fujimoto (82)] have shown that it is crucial to have large overabundance of CNO nuclei in the H-rich envelope already at the onset of the violent burning phase in order to get a thermonuclear runaway (TNR) at all. According to these calculations a mass fraction of CNO nuclei close to 30 % is required for a strong outburst and a fast nova. The crucial question is then: Can we explain the huge CNO abundances in the accreted envelope of the white dwarf? Here N is not a problem since it is directly produced from C and O during H-burning. The problem are C and O.

2 Calculations

Here we present the results of simulations we have carried out for a white dwarf of 1 $M_\odot$ accreting hydrogen-rich gas at a rate of $5\times10^{-9}M_\odot$ per year. The calculations were carried out in a carthesian system of coordinates. Only a fraction of the white dwarf's surface was covered by the computational grid, and periodic boundary conditions were implied horizontally. Two-dimensional and three-dimensional calculations were carried out. The dimensions of the calculated domain was 1000 x 1800 km (radial x lateral) and 1000 x 1800 x 1800 km (radial x lateral x lateral) coverd by a grid of the dimensions 100 x 220 and 100 x 220 x 220 for the two-dimensional and three-dimensional simulation, respectively.

The calculations were done with PROMETHEUS, an explicit multi-D Eulerian hydro code. The code contains a hydro solver which is based on a higher order Godunov method with an exact Riemann-solver, namely a PPM(piecwise parabolic method)-scheme according to Colella & Woodward (84). This hydro solver is one-dimensional and generalized to more than one spacial dimensions by applying operator splitting. A realistic equation of state is also part of PROMETHEUS including the Boltzmann gases of the nuclei under consideration, an arbitrarily degenerate non relativistic electron gas and a photon gas. The code also contains a nuclear reaction network including protons, helium and 10 CNO isotopes, solving stiff ordinary differential equations to model nuclear reactions by a Newton-Raphson technique.

The parallelization is done by dividing the computational grid into small blocks each corresponding to a processor element (PE). The most challenging part in terms of parallelization is the hydro solver since the hydrodynamical equations define a boundary value problem coupling the whole computational grid, in general. The PPM-sceme is an approximation which needs only 4 additional zones to uniquely define boundary conditions at every boundary of a computational grid. Therefore, each block has to be provided with four additional so-called ghost zones to uniquely define the boundary value problem for a single block. Before the integrations of the hydrodynamical equations on each block for a timestep are done the ghost zones are filled with information from the neighbouring blocks. This means that there has to be communication between neighbouring PEs preceding each hydrodynamical timestep. This is done by explicit message passing using the MPP- and, in a more recent version of PROMETHEUS, the MPI-library.

The hydrodynamical timestep is given by the CFL(Courant-Friedrich-Levy)-condition (Colella & Woodward (84)). The timestep is computed in every zone and the minimal timestep accounts for the following global hydrodynamical timestep applied to the whole grid.

All calculations were performed on the CRAY T3E of the Rechenzentrum Garching. The three-dimensional simulations consumed about 250 000 PE hours on the full 512 PE system. The parallel efficiency is over 90%.

3 Numerical Results

First simulations were done in two spatial dimensions, assuming that all physical quantities are independent of the third dimension. In this symmetry convective eddies are represented by infinitely long rolls. Some of our results are displayed in Fig.1 and 2. Fig.1 shows snapshots of the absolute value of the velocities obtained in this 2D simulation. A small temperature increase was imposed on one zone as an initial perturbation. Already 14 seconds later sound waves emerging from the ignition region ignited the whole bottom layer of the envelope (Fig.1a). As time progresses quasi-stationary axially symmetric flow fields appear (Fig.1b) which dominate the flow patterns after about 100s (Fig.1c). These vortices are very stable and carry a considerable amount of the turbulent kinetic energy, and they disappear only when the nuclear energy generation reaches its maximum after several hundred seconds.

The appearance of axi-symmetric quasi-stable structures in 2D flows is not new but has been found earlier in 2D simulations of convection in plane-parallel geometry, and has also been observed in the Earth's oceans (McWilliams (84)). It is for the first time however, that such deviations from standard inertially driven turbulence, where the energy is fed into the turbulent eddies on the largest scales, cascades down into the small scales, and is dissipated there by viscosity, are found in astrophysical applications and have important implications there.

As a direct consequence we cannot confirm the results of the simulations of Glasner & Livne (95) and Glasner et al. (97). We find considerably less dredge-up of C and O, mainly because on average less turbulent kinetic energy is available for the largest (and fastest) convective eddies. Although 500 seconds after ignition a considerable amount of ^{12}C can be seen in the envelope (40%) the mixing process has been too slow to yield a strong nova outburst, again in contrast to earlier calculations.

In order to make sure that all relevant scales have been resolved in our simulations we performed one additional run with five times higher spatial resolution, but did not find major differences, as far as the global properties are concerned. Mixing was slightly less efficient than in the model described in some detail here, leading to the conclusion that, at least in 2D, self-enrichment of a nova envelope by violent convective motions is a rather unlikely process. Finally, we also performed first 3D calculations, again with the same input physics and similar resolution as before, but found even less mixing, therefore less energy generation and a completely different flow structure (Fig.2) than in 2D as might have been suspected. The stable vortices do not appear in the 3D calculations since the flow is not restricted to two spacial dimensions anymore and therefore vorticity is not conserved along a fluid particle path. We rather observe convective and turbulent patterns as one might expect for a 3D convective flow.

It is obvious, according to the differences between the 2D and 3D calculations, that 3D simulations of the TNR on top of a white dwarf are indeed necessary to model the turbulent combustion process.

4 Conclusions

We have demonstrated that direct numerical simulations of nova outbursts, as far as the physics is concerned, require severe enrichment of the H-rich envelope *prior* to the TNR. If for some reason this does not happen the white dwarf will enter into a phase of quiet hydrostatic burning and will, if the envelope fulfills certain requirements, resemble a super-soft X-ray source more than a nova. We also have shown that direct numerical simulations of reaction hydrodynamics have become feasible, even in 3D, and should be done when fuel and ashes are well mixed and burning times are much longer than those of turbulent mixing.

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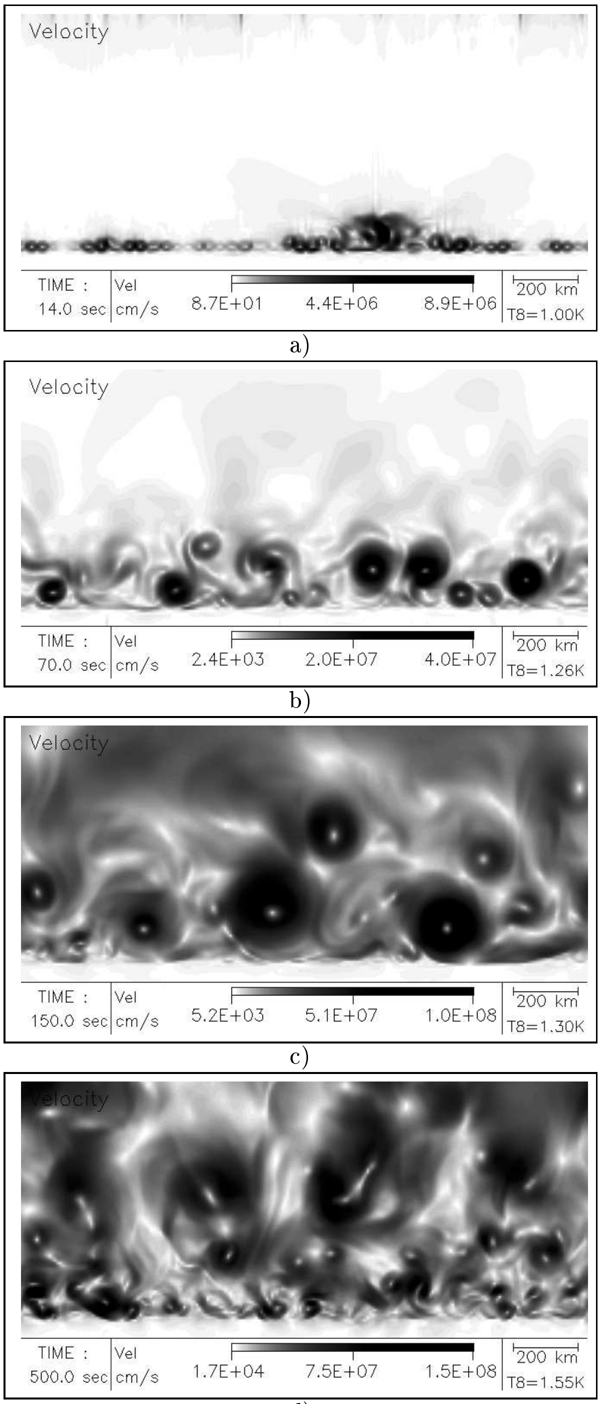

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Fig. 1. Velocity field at different stages of the evolution for the 2D model explained in the text. Given in color is the absolute value of the velocity at each point. T8 denotes the temperature of the hottest individual zone in units of 10^8K.

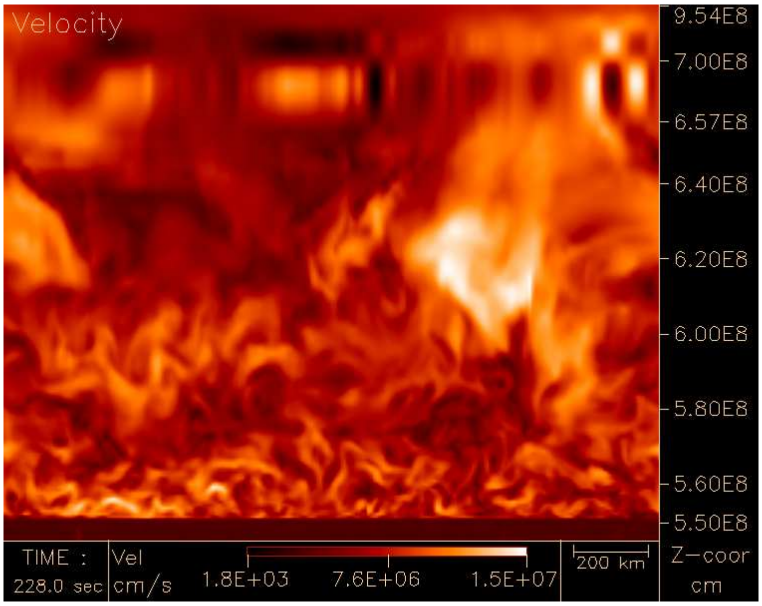

Fig. 2. 2D slice of the velocity field of the 3D model at 228 seconds.