Modeling Turbulent Nuclear Flames in Type Ia Supernovae

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Abstract. In this paper we present the application of the so-called *level set method* as a flame model in simulations of Type Ia supernovae. After giving a short introduction and motivation for our choice, we discuss two different implementations and preliminary results of test calculations and SN Ia explosions.

1 Introduction

Classical Type I supernovae, commonly named SN Ia today, are generally believed to be the result of the thermonuclear disruption of Chandrasekhar-mass carbon-oxygen white dwarfs (see, e.g., Wheeler & Harkness (1990) and Woosley (1990) for reviews). Several observational facts strongly support this view, including the absence of hydrogen lines in their spectra, their occurrence in old populations of stars, and the apparent homogeneity of this class. Moreover, thermonuclear explosions can, in principle, easily account for the production of more than 0.5 solar masses of ^{56}Ni which is needed to explain the light curves and the late-time spectra.

Thorough understanding of Type Ia supernovae is particularly important for the determination of cosmological parameters, such as the Hubble constant H_0 or the deceleration parameter q_0. If an expression for the maximum absolute brightness as a function of light curve shape and spectrum can be provided by theory or numerical computations, this would – in combination with SN observations at cosmological distances – put very tight constraints on the quantities mentioned above. An empirical correlation of this kind is given by Riess et al. (1995), but the underlying physical processes are not yet fully understood.

In the present work we concentrate on the physics and numerical simulations of thermonuclear burning in a degenerate mixture of ^{12}C and ^{16}O. In particular, we focus our attention on subsonic ("deflagration") fronts, i.e. we investigate fronts propagating by heat diffusion and convection rather than by compression.

2 Desired properties of the flame model

The thermonuclear reactions taking place during a Type Ia event can be summarized in relatively good approximation as the fusion of ^{12}C and ^{16}O to (finally) ^{56}Ni with an energy release of about $7 \cdot 10^{17}$erg/g. Compared to the typical length of a grid cell in Type Ia simulations ($\approx$ 10km) the width of the reaction zone is extremely small ($\ll$ 1cm). Consequently, the flame should be modeled as a sharp discontinuity.

Due to the lack of more appropriate numerical techniques, most of the Type Ia simulations performed up to now used the so-called *flame capturing method* (Khokhlov 1993) to describe the flame front. Unfortunately, this approach requires that the front is artificially smeared out over 8 to 10 zones, which stabilizes the flame against some hydrodynamical instabilities and suppresses the creation of small wrinkles in the front. This can lead to significant underestimation of the total flame surface and the energy generation rate.

3 The level set method

This technique was proposed first by Osher & Sethian (1988) as a natural way to describe moving interfaces in hydrodynamical simulations. Its central aspect is the association of the front $\Gamma(t)$ with the zero level set of a time dependent scalar function G:

$$\Gamma(t) := \{\mathbf{r} \mid G(\mathbf{r},t) = 0\} \tag{1}$$

Assuming that G is positive in the burnt regions and negative in the unburnt regions, the temporal evolution of this function (and thereby also the flame front) can be described by the following equation (Sethian 1996):

$$\frac{\partial G}{\partial t} = \underbrace{-\,\mathbf{v}_{\mathrm{F}}\boldsymbol{\nabla} G}_{\text{advection by the fluid}} + \underbrace{s|\boldsymbol{\nabla} G|}_{\text{propagation by combustion}} \tag{2}$$

This description is valid in one, two and three spatial dimensions and can handle complicated situations (like sharp corners in the front or changes of topology) without problems.

Minimal Implementation: The simplest way of modeling a flame front with a level set function is to propagate G according to equation (2) and, after every timestep, to burn all the fuel that is found behind the front. This technique can be implemented easily, but has the disadvantage that ashes can penetrate into the unburnt regions because of numerical diffusion; this results in a slightly smeared-out transition between burnt and unburnt states (the exact amount depends on the employed Riemann solver).

Reconstruction and Flux Splitting: According to Smiljanovski et al. (1997), the geometrical information provided by the G-function can be used to reconstruct the exact pre- and post-front states from the average values in the cells cut by the front. Having obtained these results, the flux of burnt and unburnt material over a cell interface can be calculated separately. This completely eliminates numerical diffusion of ashes into fuel and vice versa (see fig. 1). When using this extension, the fluid velocity and burning speed in equation (2) must both be taken from either the burnt or the unburnt material; traditionally, the latter is chosen.

4 Turbulent flame propagation speed

During the early stages of a SN Ia, combustion takes place in the flamelet regime, because the flame width lies below the Kolmogorov length. Since the amplitude of the turbulent velocity fluctuations on the grid scale is considerably higher than the laminar burning velocity in the degenerate C/O-material, the propagation speed of the front only depends on these fluctuations. This quantity is obtained from a model for the turbulent kinetic subgrid energy (Niemeyer & Hillebrandt 1995).

5 Results

5.1 Numerical tests

Both of the implementations mentioned above were incorporated into the hydrodynamics code PROMETHEUS (Fryxell et al. 1989). Several test cases were run to determine the correctness of the new algorithms. Thereby it turned out that the minimal implementation does not reproduce a flame propagating with a fixed, predefined speed very well, whereas the complete implementation produces perfect results (see fig. 2). The observed error is most probably caused by the fact that, using the minimal implementation, we do not know the velocity $\mathbf{v}_{\mathrm{F}}$ of the unburnt state needed in equation (2), but take the average fluid speed instead. On the positive side, the measured flame speed is in both versions nearly independent from the angle between the front and the grid directions.

5.2 Simulations of SNe Ia

Fig. 4 shows some snapshots of the front geometry and fluid velocity in a SN Ia simulated with the minimal implementation. The light, hot bubble of burnt material starts to rise immediately and soon develops the typical mushroom shape. Near the coordinate axes, cold fuel sinks down towards the center, causing strong shear flows at the sides of the bubble. Near the end of the simulation, this leads to clearly visible small-scale distortions and a highly complex front geometry, which could not be observed in earlier simulations.

In spite of the larger flame surface, the total energy release is still too low to explain a typical Type Ia event. In some of our simulations, the star becomes unbound (see fig. 3), but to accelerate the ejecta of the explosion to the observed speeds of about 10^4km/s, another $\approx 3 \cdot 10^{50}$ergs would be needed.

6 Conclusion

Using a level set function to describe the thermonuclear flame in Type Ia Supernovae greatly reduces the numerical flame thickness in comparison to the other models used so far; this represents a step towards the real situation, where the front is practically infinitely thin. As a consequence, deformations of the flame caused by hydrodynamical instabilities can be observed on smaller scales than before. The obtained results are still not consistent with real, observed events, but further improvement of the turbulence models and three-dimensional simulations might bring the desired breakthrough.

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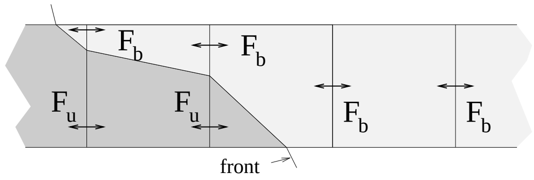

Fig. 1. Using the level set to determine the partial fluxes of burnt and unburnt matter over a cell interface

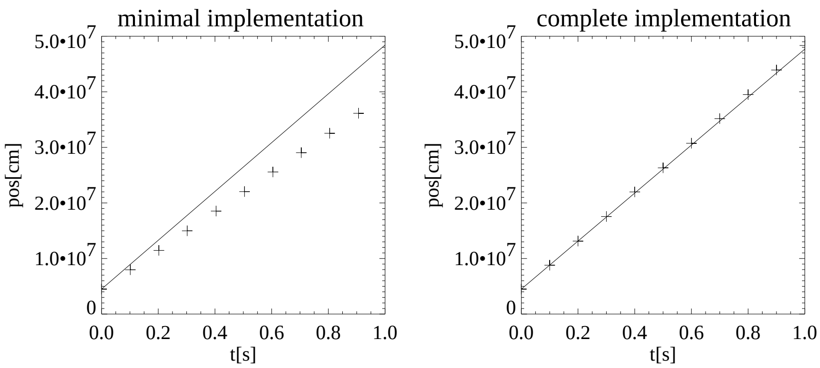

Fig. 2. Measured (dots) and expected (line) position of a 1D flame propagating with a fixed speed

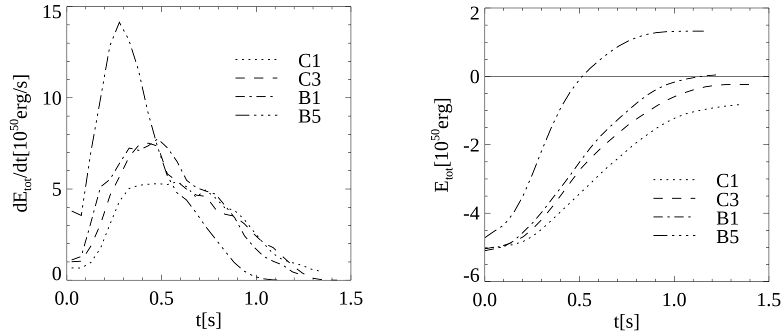

Fig. 3. Energy production rate and total energy of the White Dwarf for different flame geometries at $t = 0$ (B1: one burnt bubble near the center; B5: five burnt bubbles; C1 and C3: different kinds of central ignition)

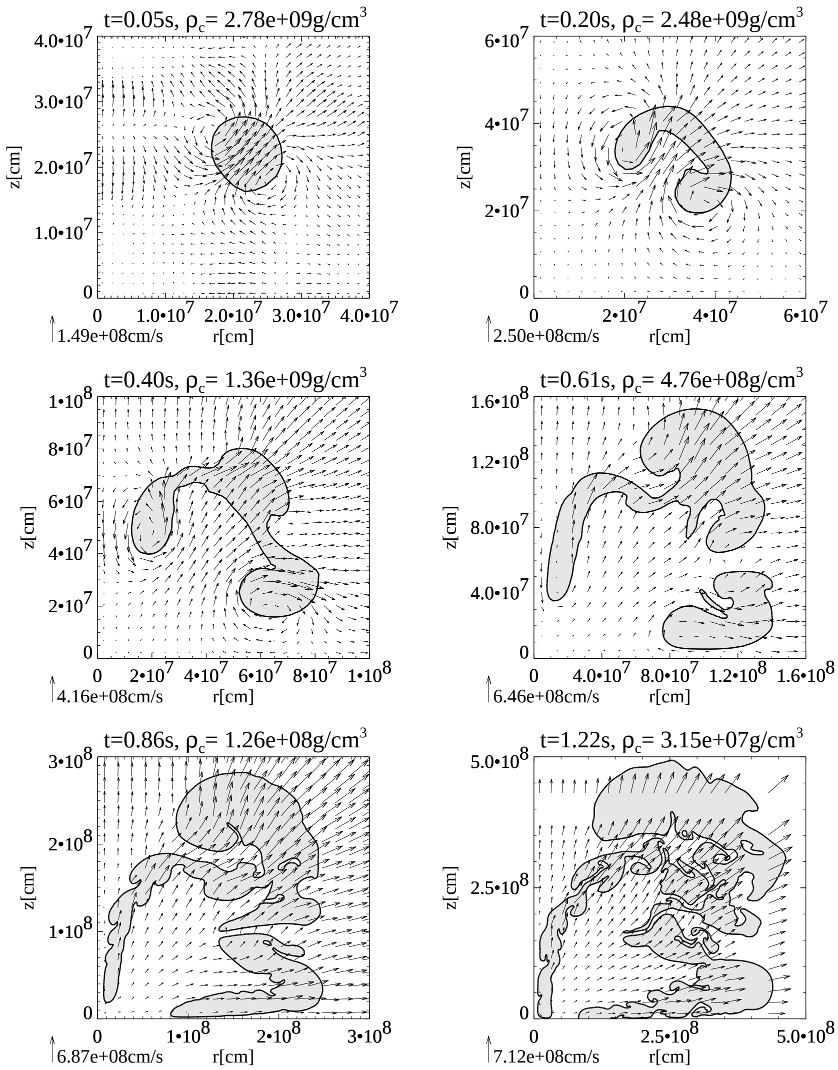

Fig. 4. Evolution of front and velocity field for a single, circular bubble ignited near the center of the White Dwarf (Model B1)