The fate of a neutron star just below the minimum mass: does it explode?

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Abstract. First results of numerical simulations are presented which compute the dynamical evolution of a neutron star with a mass slightly below the minimum stable mass by means of a new implicit (general relativistic) hydrodynamic code. We show that such a star first undergoes a phase of quasi-static expansion, caused by slow nuclear β-decays, lasting for about 20 seconds, but then explodes violently. The kinetic energy of the explosion is around $10^{49} erg$, the peak luminosity in electron anti-neutrinos is of order $10^{52} erg/s$, and the thermodynamic conditions of the expanding matter are favorable for r-process nucleosynthesis. These results are obtained for the Harrison-Wheeler equation of state and a simple and, possibly, unrealistic treatment of β-decay rates and nuclear fission, which were adopted for comparison with previous works. However, we do not expect that the outcome will change qualitatively if more recent nuclear input physics is used.

Although our study does not rely on a specific scenario of how a neutron star starting from a bigger (and stable) mass can reach the dynamical phase, we implicitly assume that the final mass-loss event happens on a very short time scale, i.e., on a time scale shorter than a sound-crossing time, by removing a certain amount of mass as an initial perturbation. This assumption implies that the star has no time to adjust its nuclear composition to the new mass through a sequence of quasi-equilibria. In the latter case, however, there exists no stable configuration below the minimum mass, because the equation of state of fully catalyzed matter is too soft. Therefore, the dynamics of the explosion will not be too different from what we have obtained if different initial perturbations are assumed.

Key words: stars: neutron – physical data and processes: hydrodynamics – instabilities – nuclear reactions, nucleosynthesis, abundances

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1. Introduction

Ever since the discovery of binary neutron star systems their final fate has been subject of intensive work and also of many speculations. There are no doubts that because of orbital momentum loss by gravity waves the neutron stars will ultimately merge together, ending as a fast spinning black hole or massive neutron star, depending on the initial total mass and the amount of mass possibly lost from the system during the merger event. To the outside world such a merger will manifest itself by a burst of gravitational radiation, probably the best candidate for gravitational wave detectors, a burst of neutrinos and, likely, also a burst of γ-rays. It has also been speculated that during the merger a small amount of neutron star matter may be ejected which could, in principle, have nuclear abundances similar to those commonly attributed to the r-process, posing interesting limits on the rate of neutron star mergers over the history of the Galaxy. An alternative scenario which motivates in part our present work is the speculation that one of the neutron stars (the one with initially less mass) might lose most of its mass to the companion prior to the merging event. It may then end with a mass near the minimum equilibrium configuration. Such a scenario requires conservative mass-transfer, certain properties of the neutron star's equation of state, etc., and it may be difficult to find the right conditions. On the other hand, it cannot be excluded on simple grounds.

Most investigations so far start from the assumption that both neutron stars have nearly equal masses, an assumption which is in fact supported by the observed binary neutron star systems with well determined masses. Also, nearly all neutron stars seem to have masses at least consistent with a gravitating mass of about 1.4 $M_\odot$. The final coalescence phase of two neutron stars of initially 1.4 $M_\odot$ has recently been studied extensively by Ruffert et al.

(1996, 1997). They performed 2- and 3-dimensional simulations with state-of-the-art micro-physics, but Newtonian hydrodynamics. The outcome of their simulations was a final black hole, a strong burst of neutrinos, accompanied by γ-rays (with insufficient luminosity to explain the observed γ-ray bursts at cosmological distance), and ejection of some heavy and neutron-rich elements. So in case of (nearly) equal masses of both neutron stars it is very unlikely that one of them (i.e. the one with the slightly lower initial mass) will undergo slow mass-loss by Roche-lobe overflow and end up near the minimum mass prior to merging.

However, the situation may be different for neutron star binaries with very different initial masses. In that case the Roche-model may be more applicable and a scenario outlined by Imshennik and Popov (1996) may result, namely the smaller neutron star may, due to its considerably larger radius, lose some fraction of its mass before dynamic mass transfer sets in. Whether or not this can happen will depend on the true equation of state of neutron star matter, on the mass-ratio of the two stars, on whether or not conservative mass transfer can be obtained for certain times, etc. Investigating these questions is beyond the aim of this paper, and a definite answer is hard to get (see however, e.g., Lai et al. 1994). It might well happen that in a few rare cases a neutron star pair is born with largely different initial masses and one of them might reach the critical (minimum) mass. If it should explode at this stage this would have several interesting observable consequences (see, e.g., Eichler et al. (1989)). For example, the more massive component could obtain a kick-velocity of up to 2000 km/s (Imshennik and Popov, 1996), well in the range of observed pulsar proper motions (Lyne and Lorimer, 1994). They would be the source of considerable emission of γ-rays and neutrinos, and they should eject matter of rather unusual chemical composition. A second possibility to form neutron stars near the minimum mass, which we just mention in passing, could be by fragmentation of rapidly rotating cores of collapsing massive stars.

One can, of course, turn the argument around and ask: Would the impact of such explosions on galactic nucleosynthesis be so strange that we can exclude them immediately? Or can we impose so strong constraints on their event rate that they would be of little interest? It is this latter argument which motivated our work and which led us to reinvestigate the numerical studies of Colpi et al. (1989. 1991, 1993) with improved numerical techniques (this paper) and with more realistic micro-physics input data (forth-coming papers). Consequently, our final aim is to make firm predictions for the yields of neutron-rich isotopes ejected during the explosion of a neutron star just below the minimum mass, but the aim of the present paper less ambitious. We only want to demonstrate that even under unfavorable conditions, e.g. for an equation of state that predicts the transition to homogeneous nuclear matter at rather low density (the Harrison-Wheeler equation of state) neutron stars do explode if their mass drops below the minimum mass allowed by an equation of state of matter in nuclear statistical *and* β-equilibrium. This result is in qualitative agreement with that obtained by Colpi et al. (1993), but it is more rigorous for the following reason. The early evolution of such a neutron star and the transition from a slow expansion to the fast explosion proceeds on the time scale of (slow) β-decays, and its modeling requires a code that is able to follow the evolution over many dynamical times. Colpi et al. (1993) circumvented this problem by rescaling the β-decay rates in order to make both time scales comparable, an approach that is not fully satisfactory. Here, we did not have to use this trick because we could compute the evolution by means of an *implicit* hydro-code which is described in Section 2. In order to be able to make a detailed comparison with their work we left the micro-physics input essentially unchanged (see Section 2).

Given this assumption, e.g., the zero temperature Harrison-Wheeler equation of state and β-equilibrium, the minimum possible mass of a neutron star is $M_{min} = 0.189 M_\odot$ and it has a central density of $\rho_c = 2.67 \times 10^{13} g/cm^3$. It has a radius of $230 km$ and consists of a core region of homogeneous nuclear matter of about $0.13 M_\odot$ extending to $18 km$ and a crust that makes up for the rest. This initial configuration is identical to the one used by Colpi et al. (1993). We then take off some mass from the surface and follow the evolution by means of our implicit hydro-code. The results are given in Section 3, including first estimates of the nucleosynthesis yields that can be expected from the explosion, and a discussion and summary conclude the paper.

2. The Hydrodynamical Model

2.1. Hydrodynamics

We compute the evolution of a neutron star below the minimum mass by solving the equations of general relativistic hydrodynamics in spherical symmetry. The equations are solved numerically by an implicit method using Lagrange (baryon mass) coordinates. The code was developed and has been tested for a series of basic problems by Yamada (1997). In the present application we use a mesh of 100 equal mass-zones and micro-physics input, such as the equation of state, nuclear reactions and neutrino cooling, is implemented in a way described below into the original hydrodynamics code.

In addition, we have modified the original code to handle time-dependent nuclear reactions and neutrino cooling. In order to guarantee stability, an entropy equation was added to the implicit hydrodynamics scheme (Yamada 1997b), in addition to the energy equation. The temperature is calculated from the entropy, and the internal energies obtained from the energy equation and the equation of state are used to check for numerical errors only. The entropy equation reads (Meyer 1989),

$$T\frac{dS}{dt} = Q_{beta} + Q_{fission} - Q_{cooling}, \quad (1)$$

where S is the entropy per baryon, Q_{beta} and $Q_{fission}$ are the heating rate per baryon due to the energy released by β-decays and fission of nuclei, respectively. $Q_{cooling}$ is the cooling rate per baryon due to neutrino emission through β-decays. The equation for the evolution of the electron fraction, Eq. (3), is not solved implicitly but is followed separately by an explicit scheme. We first update the value of the electron fraction by the Runge-Kutta method and then solve the hydrodynamics with the new values of the electron fraction. The time step is constrained by limiting variations of all quantities to be less than 10 %.

We point out that in contrasts to earlier studies (e.g. Colpi et al (1993)) we adopted an implicit hydro code rather than an explicit method. Therefore, time steps are not constrained by the Courant condition as in explicit schemes. Because of this advantage we can study the evolution of a neutron star just below the minimum mass for time scales as long as several seconds or more, which is necessary because the early evolution is governed by the time scales of β-decays which are much longer than, e.g., the sound crossing time which would limit the time step in an explicit code.

2.2. The Equation of State

One of the main aims of the present paper is to demonstrate that neutron stars with a mass slightly less than the minimum mass encounter an instability and explode. Since we want to compare our numerical results, based on a newly developed hydrodynamics code, with the so far most elaborate study of Colpi et al. (1993) we decided to adopt the same equation of state as they did, namely the Harrison-Wheeler equation of state (Harrison et al. 1965; see also Shapiro & Teukolsky, 1983) for neutron star matter in β-equilibrium, and an extension of it for matter out of β-equilibrium and at finite temperatures (Colpi et al. 1989). Details of this treatment are given below. One has to keep in mind, however, that the Harrison-Wheeler equation of state is not very realistic at densities near nuclear matter density and that it predicts a transition to homogeneous nuclear matter far below the saturation density. Consequently, for more realistic equations of state, neutron stars near the minimum mass will have no central core of homogeneous nuclear matter but will consist mainly of crust material. This will lead to more violent explosions than we find in our study which thus, in a sense, is conservative if we want to answer the question whether or not these objects explode at all. In a subsequent paper we will discuss the results obtained for more realistic equations of state.

2.2.1. The Equilibrium Equation of State

According to the Harrison-Wheeler equation of state, cold neutron star matter has three different phases: homogeneous nuclear matter and electrons above the transition density $\rho_{tr} \equiv 4.3 \times 10^{12} g/cm^3$, a mixture of neutrons, nuclei, and electrons down to the neutron-drip density $\rho_{dr} \equiv 3.2 \times 10^{11} g/cm^3$, and nuclei and electrons below ρ_{dr}. The highest density regime will be referred to as the neutron star's *core*. The density range $\rho_{tr} \geq \rho \geq \rho_{dr}$ is called the *inner crust*, and the remaining part is the *outer crust*. We recall again that the composition of the central part of the neutron star depends on the equation of state, and that for neutron stars near the minimum mass nuclei exist even in the central part (Colpi et al. 1993) when microscopically better motivated equations of state such as the BPS equation of state (Baym et al. 1971) are adopted.

The properties of cold neutron star matter are determined by several equilibrium conditions at zero temperature. β-equilibrium with vanishing neutrino chemical potential is maintained in the whole star and determines the electron fraction $Y_e \equiv \frac{n_e}{n_B}$ together with the condition of charge neutrality. Nuclear species in the inner and outer crust are determined so as to minimize the total energy per nucleon of the matter in β-equilibrium and charge neutrality. The equilibrium between neutrons inside nuclei and dripped neutrons is maintained additionally in the inner crust. These conditions determine the composition of the matter, i.e. the electron fraction Y_e, the number fraction of nuclei $Y_A \equiv \frac{n_A}{n_B}$, the mass number A and proton number Z of the nuclei. Other quantities such as the pressure are calculated accordingly with the composition. We treat neutrons, protons and electrons in the core and dripped neutrons and electrons in the crust as ideal Fermi gases. Properties of nuclei in the crust are calculated by means of the mass formula of the Harrison-Wheeler model. We use this equilibrium equation of state of cold neutron star matter to construct the initial model (see section 2.6).

2.2.2. The Non-Equilibrium Equation of State

The equilibrium conditions described above are usually assumed if cold neutron stars in hydrostatic equilibrium are studied. In order to investigate the instability of neutron stars below the minimum mass, however, one has to take into account departures from equilibrium because nuclear reaction times can be considerably longer than the hydrodynamical time scale which is of order milliseconds.

At low temperatures β-decays of free neutrons in the neutron star's core are suppressed due to the limited phase space available in the highly degenerate environment there (Colpi et al. 1989). Therefore, the composition of the core is frozen and the electron fraction, Y_e, is fixed to its initial value during the evolution.

The β-decay times of nuclei in the crust of a neutron star range over many orders of magnitude and play an essential role in initiating the dynamical instability. These time scales are determined by the properties of the matter out of β-equilibrium and will be described in more detail in section 2.3. Due to the decay of nuclei the electron fraction in the crust changes from its initial value once the matter moves out of β-equilibrium. The change of the electron fraction leads to a change of the pressure and affects the hydrodynamics, which in turn changes the thermodynamic conditions which determine the β-decay rates.

Neutrons inside and outside the nuclei in the inner crust remain in equilibrium during the evolution since the density of free neutrons and the temperature are high enough for the time scales of neutron captures (and their inverse process) to be much shorter than the β-decays in the expanding neutron star matter (Meyer 1989). Therefore, we can assume that the equilibrium between neutrons inside and outside the nuclei is maintained during the evolution. We stress that the combination of rapid neutron captures with slow β- decays resembles the classical conditions for r-process nucleosynthesis.

In a neutron stars crust a complete statistical equilibrium is not achieved unless the temperature becomes very high (above $10^{10}K$; see, e,g., Lattimer et al. (1985)). The neutron star matter is heated by the energy released by the β-decay and fission of nuclei during the expansion of the neutron star. The amount of heating depends on details of the hydrodynamical evolution and on micro-physics data such as the equation of state and the β-decay rates, and only if the temperature becomes sufficiently high one has to compute the equation of state in nuclear statistical equilibrium including weak, electro- magnetic and strong interactions. In the present study, we assume one species of nuclei instead of computing the equilibrium composition for a network of nuclei. Moreover, we assume that the number fraction of nuclei, Y_A, is fixed during the evolution because charged particle reactions are essentially frozen. For calculating the equation of state, this latter assumption can be justified by the fact that the temperatures of neutron star matter we find in the current study never exceed a few times 10^9K.

2.2.3. Properties of the Expanding Matter

Since we calculate the hydrodynamics in a Lagrangian mesh and can ignore transport effects, we follow the evolution of each fluid mass element independently and evaluate the equation of state and the composition at every time step accordingly. To be more precise, the properties of expanding neutron star matter are obtained in the following way.

The equation of state of the matter at high densities, $\rho \geq \rho_{tr}$, is calculated for a mixture of ideal Fermi gases of neutrons, protons and electrons. The composition in each mass element is determined by the electron fraction and charge neutrality. The electron fraction, $Y_e(j,t)$, of the Lagrangian mass element indexed by j at time t is fixed to its initial value

$$Y_e(j,t) = Y_e(j,0) = Y_e^0(j). \tag{2}$$

If the mass element originally belonged to the core of the initial model and its density reaches ρ_{tr} due to expansion, we assume that the matter is converted into a mixture of nuclei and ideal Fermi gases of neutrons and electrons. The composition (Y_e and Y_A) is assumed to be that of cold neutron star matter at the transition density, ρ_{tr}. The entropy is assumed to be continuous over the phase transition. Afterwards, such a mass element evolves like an element composed of crust material as will be described below.

The equation of state at densities $\rho \leq \rho_{tr}$ is calculated as a mixture of nuclei and ideal Fermi gases of neutrons and electrons. The composition is determined by the electron fraction, the number fraction of nuclei, the equilibrium condition for the neutrons and baryon number conservation. The electron fraction evolves because of β-decays according to

$$\dot{Y}_e(j,t) = \lambda_\beta(j,t) Y_A(j,t), \tag{3}$$

where $\lambda_\beta(j,t)$ is the β-decay rate of the nuclei in the mass-zone j at time t. The initial condition for Eq. (3) is the value of the electron fraction in the initial model. For mass elements originally belonging to the core equation (3) is solved only for times after the density reaches ρ_{tr}, and the initial value of Y_e is taken to be the equilibrium value at ρ_{tr}. The number fraction of nuclei, $Y_A(j,t)$, is kept constant,

$$Y_A(j,t) = Y_A(j,0) = Y_A^0(j), \tag{4}$$

unless after β-decay the spontaneous fission line is reached. For mass elements originally in the core, the equilibrium value of Y_A at ρ_{tr} is adopted as the value of Y_A^0 in Eq. (4). The proton number of nuclei is calculated from the relation

$$Z(j,t) = \frac{Y_e(j,t)}{Y_A(j,t)}. \tag{5}$$

The chemical potentials of neutrons inside and outside nuclei are balanced,

$$\mu_n(j,t) \equiv \mu_n^{in}(j,t) = \mu_n^{out}(j,t). \tag{6}$$

Here, $\mu_n^{in}(j,t)$ is the neutron chemical potential inside the nuclei and is calculated as a function of proton number, Z, and mass number, A, of nuclei obtained from the mass formula used in the Harrison-Wheeler equation of state. The chemical potential of dripped neutrons, $\mu_n^{out}(j,t)$, is given by the ideal Fermi gas expression. The condition of the baryon number conservation is expressed as

$$A(j,t) = \frac{1}{Y_A(j,t)}(1 - Y_n(j,t)), \tag{7}$$

where Y_n is the number fraction of dripped neutrons defined by $Y_n \equiv \frac{n_n}{n_B}$. The density of dripped neutrons, n_n, is calculated in terms of the chemical potential, μ_n^{out}, and the temperature, T.

The mass number of nuclei, A, and the neutron chemical potential, μ_n, are determined by solving the equations for the chemical equilibrium of the neutrons, Eq. (6), and baryon number conservation, Eq. (7). Numerically, we search for a solution Y_n in the range between 0 and 1 by comparing the values of μ_n^{out} and μ_n^{in}. μ_n^{out} is determined through Y_n and μ_n^{in} is determined through A and Z, which are given by Eqs. (5) and (7), respectively.

When the matter is not in equilibrium and/or at finite temperature, the transition density from inner crust to outer crust is not necessarily in accord with the neutron-drip density ρ_{dr} for the cold neutron star matter. Whether the matter belongs to the inner or outer crust is solely determined by the free neutron fraction, Y_n. If $\mu_n^{in} \leq \mu_n^{out}$ at $Y_n = 0$, nuclei are bound and there are no dripped neutrons. Therefore, the matter belongs to the outer crust. In this case, the equation of state is calculated as a mixture of nuclei and an ideal Fermi gas of electrons. The composition is determined by the electron fraction and the number fraction of nuclei as in Eqs. (3) and (4). The nuclear species are determined from Eqs. (5) and (7) with $Y_n = 0$.

In order to take into account temperature effects, we evaluate all quantities of the Fermi gases of neutrons, protons, and electrons at finite temperature. We use the mass formula in the Harrison-Wheeler equation of state even at finite temperature. This should be a good approximation for temperatures well below $10^{10}K$. The entropy of the nuclei is approximated by that of a non-degenerate ideal Fermi gas.

2.3. β-decay Rates

The β-decay rates appropriate for neutron star matter are evaluated in the same manner as was done by Colpi et al. (1989). The β-decay of free neutrons is completely suppressed in the core and in the inner crust due to the high electron degeneracy there. Therefore, only nuclei in the crust can decay.

The β-decay rate of nuclei, $\lambda_\beta(j,t)$, can be estimated (Lattimer et al. 1977)

$$\lambda_\beta = \langle \frac{\rho}{ft} \rangle (\frac{\Delta^6}{6} + \Delta^5 \mu_e + \frac{5}{2}\Delta^4 \mu_e^2), \tag{8}$$

where μ_e is the chemical potential of electrons and Δ is defined by

$$\Delta \equiv \mu_n - \mu_p - \mu_e. \tag{9}$$

This quantity measures the deviation from β-equilibrium and is equal to zero when β-equilibrium is achieved. It is equal to the energy available for the decay, given by the Q_β value of the parent nucleus minus the electron Fermi energy, μ_e. The coefficient $\langle \frac{\rho}{ft} \rangle$ contains nuclear structure information (matrix elements and level density in the daughter nuclei) and is rather uncertain. Here we take this parameter to be $10^{-2.8} MeV^{-6} s^{-1}$ following the work of Colpi et al. (1993). We note that the value of this coefficient has an ambiguity ranging from $10^{-5.5}$ to $10^{-2.8} MeV^{-6} s^{-1}$, when Δ and μ_e are measured in units of MeV (Lattimer et al. 1977). The energy release by β-decays enters the entropy equation, Eq. (1), and is expressed as (Meyer 1989),

$$Q_{beta} = -\sum_i \mu_i \frac{dY_i}{dt} = \Delta \frac{dY_e}{dt}, \tag{10}$$

where the sum is over neutrons, protons and electrons, and we have used the charge neutrality condition.

2.4. Neutrino Cooling

In the β-decays of nuclei anti-neutrinos are emitted. It is assumed that they escape freely from the star and contribute only by cooling the matter inside the star (Meyer 1989). This is a fair approximation since the time scale for the expansion of the star is much longer than neutrino diffusion times. The average energy of anti-neutrinos is evaluated by

$$\varepsilon_{\bar{\nu}_e} = \frac{3}{7}\Delta \frac{\Delta^2 + 7\Delta\mu_e + 21\mu_e^2}{\Delta^2 + 6\Delta\mu_e + 15\mu_e^2}, \tag{11}$$

in accord with the β-decay rates adopted here (Lattimer et al. 1977). The local luminosity of anti-neutrinos per baryon is then calculated from

$$l_{\bar{\nu}_e} = \varepsilon_{\bar{\nu}_e} Y_A \lambda_\beta. \tag{12}$$

Finally, the total neutrino luminosity is obtained by integrating over the whole star,

$$L_{\bar{\nu}_e} = \int \frac{dm_B}{m_u} l_{\bar{\nu}_e}, \tag{13}$$

where m_B is the baryonic mass coordinate and m_u is the atomic mass unit. Cooling due to neutrino emission enters in the energy equation and entropy equation. The cooling term $Q_{cooling}$ is expressed as,

$$Q_{cooling} = l_{\bar{\nu}_e}. \tag{14}$$

2.5. Nuclear Fission

During the expansion, because of neutron captures followed by β-decays, the proton and neutron numbers of nuclei increase and they may become unstable to spontaneous fission. Fission of nuclei is treated in a very simple manner in this current study. We assume that a nucleus fissions spontaneously once its proton number Z satisfies the condition

$$Z \geq Z_0, \tag{15}$$

where Z_0 is the proton number which defines the spontaneous fission line. We take $Z_0 = 90$ in the current study. This criterion for fission is rather simple but it allows us to study the essential effects caused by this extra heating on the hydrodynamics. In a forthcoming paper we will incorporate a more realistic treatment of fission such as the one used in the study by Colpi et al. (1993) (see also Lattimer et al. 1977). We assume further that fission is symmetric and set the proton and neutron numbers to be one half of those of the parent nuclei. We set also the number fraction of nuclei to twice the original value. We evaluate the liberated energy from the difference of the total energy of the matter before and after fission. This energy contributes to the heating and enters into the entropy equation. The heating term due to fission is expressed as

$$Q_{fission} = \frac{\epsilon(\rho, Y_e, 2Y_A, A/2, Z/2) - \epsilon(\rho, Y_e, Y_A, A, Z)}{n_B}, \quad (16)$$

where $\epsilon(\rho, Y_e, Y_A, A, Z)$ is the total energy density of the matter at the density ρ with the composition Y_e, Y_A, A, and Z.

2.6. The Initial Model

An initial model is constructed as follows. We solve the Tolman-Oppenheimer-Volkoff equation with the Harrison-Wheeler equation of state to obtain a series of neutron star models in hydrostatic equilibrium and at zero temperature. For the configuration with the lowest mass we find $0.189 M_\odot$. The central density of this neutron star is $2.67 \times 10^{13} g/cm^3$ and its radius is $230 km$. As an initial perturbation we remove a certain amount of mass from the surface of the hydrostatic configuration, given by the mass ΔM removed in units of the solar mass:

$$\delta \equiv \frac{\Delta M}{M_\odot}. \quad (17)$$

Removing a surface layer from the star physically means that mass is lost on the dynamical time scale. This implies a certain scenario for the onset of the instability and may not be realistic. But in order to be able to compare with previous works we use instantaneous mass loss as an initial perturbation here.

We checked that the initial model remained hydrostatic if no perturbation (i.e. $\delta = 0$) was imposed. For practical reasons, we set the initial temperature in the whole star to $10^8 K$ when we map the hydrostatic configuration onto the hydrodynamics code. This temperature is sufficiently low such that it does not change the hydrostatic equilibrium because also the equation of state is essentially unchanged.

3. Numerical Results

We have carried out numerical simulations of the evolution of neutron stars below the minimum mass by changing the value of the mass-loss parameter δ. In the present paper, we report the results obtained for one particular choice of δ and concentrate on the question of the star's final fate, namely whether it explodes or not. The details of the results for various values of δ will be given elsewhere.

A similar study of the neutron stars below the minimum mass has been carried out by Colpi et al. (1993) and, in fact, we have chosen micro-physics input and initial model in such a way to be able to compare our results with their's directly. They found that a large amount of mass, more than $\delta = 0.20$, had to be removed to initiate a dynamical instability, which then lead to the disruption of the whole star. However, they had to assume either β-equilibrium or had to artificially increase the β-decay rates in order to enter into the dynamical phase. Moreover, they could follow the evolution only over about 0.1 second, and the final fate could not be clarified for more realistic β-decay rates. In particular, in their paper Colpi et al. (1993) calculated the fate of a neutron star with normal β-decay rates and $\delta = 0.22$ up to 0.1 second and found that the outer layers of the star were expanding, but the inner parts were still near to hydrostatic equilibrium. They inferred that an explosion should result on a time scale of about 100 seconds by extrapolating the results they had obtained for unrealistically fast β-decay rates. Here, we report the results of numerical simulations which follow the evolution over sufficiently long times avoiding artificial speed-ups of β-decays, and we chose also $\delta = 0.22$ as in Colpi et al. (1993) in order to be consistent with their work.

3.1. A Delayed Explosion

We show in Fig. 1 the results of our numerical simulations for $\delta = 0.22$. The position of selected mass-zones is plotted as a function of time. Shown are the trajectories of every 5th zone. The outer parts start expanding after the mass has been removed because they are no longer in pressure equilibrium. In addition, these shells move out of β-equilibrium because the density decreases quickly. Subsequent β-decays heat the matter and increase the electron fraction, leading to an increase of the pressure. This feedback accelerates the matter further and more shells begin to expand. The net result is a leptonization wave progressing inwards and affecting larger and larger fractions of the star. However, although the expansion of the outer parts continues due to this coupling of hydrodynamics and β-decays and the expansion becomes even faster as time proceeds due to shorter β-decays times, the central core stays almost hydrostatic for about 20 seconds. The region of expansion grows slowly inwards, nuclei form in the core region because of the decreasing density and undergo β-decays, and finally after 21 seconds also the core of the star becomes unstable and explodes.

This final explosion of the core is very violent and takes place on a short time scale of 0.1 seconds only. We show

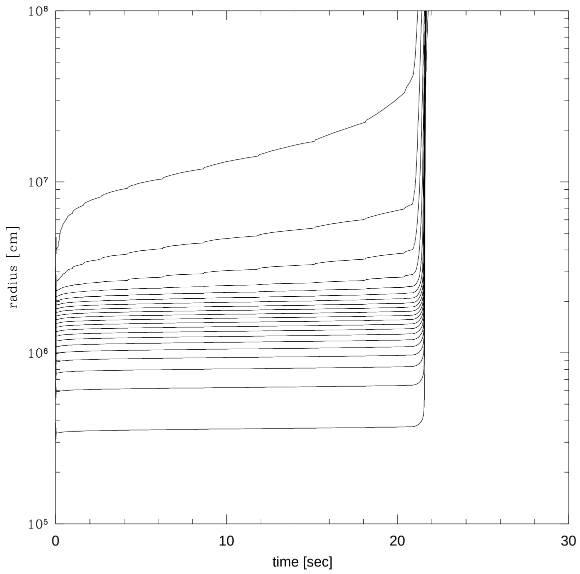

Fig. 1. The position of selected mass-zones of the neutron star is plotted as a function of time. The trajectories of every 5th mass-zone are shown.

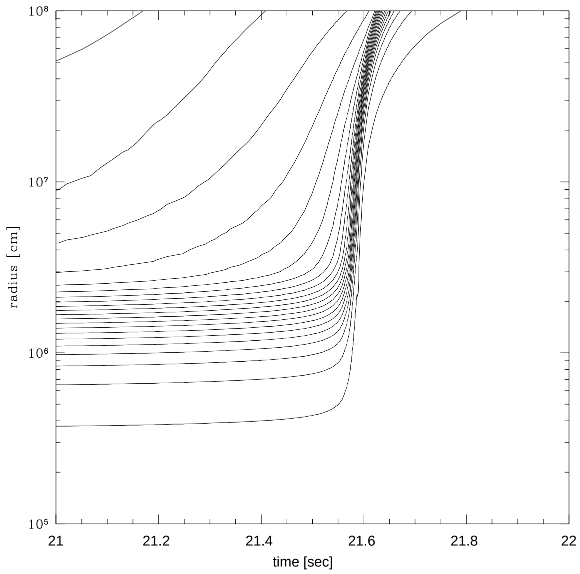

Fig. 2. The position of selected mass-zones is plotted as in Fig. 1, but expanded for the time between 21 and 22 second.

in Fig. 2 the trajectories of mass elements as in Fig. 1, but expanded for the time between 21 and 22 second after the onset of the calculations. This part of the explosion looks similar to the result obtained by Colpi et al. (1993) for much faster (and unrealistic) β-decay rates. So we conclude that the neutron star explodes in any case, and that β-decays are crucial for the onset of the explosion but matter less at later times.

In summary, we arrive at a first and important conclusion: Neutron stars near the minimum mass do explode if a moderate amount of mass is peeled off from their surface on a dynamical time scale. The rather poorly known β-decay rates of neutron-rich nuclei strongly affect the duration of the first quasi-static expansion phase but not so much the final outcome. The energy of the explosion comes mainly from nuclear decay energy, both from β-decays and from fission, and neutrino losses cannot be ignored but are not crucial. Thermodynamically, the matter of the expanding neutron star stays sufficiently close to a local β-equilibrium during the early phase and, therefore, the effective adiabatic index stays low, below 4/3, and the star does not find a new equilibrium configuration. Moreover, once the core region starts to expand, because of the low initial lepton fraction the nuclear energy release is big and fast enough to give rise to a violent explosion.

3.2. The Thermal History of the Expanding Neutron Star Matter and Nucleosynthesis

Next we discuss the thermal history of the expanding neutron star matter during the explosion. This is primarily interesting because it determines the nucleosynthesis predictions. We display the main results in Fig. 3 as trajectories of mass shells in the temperature - density plane. We show trajectories only of mass elements which are part of the central regions of the star. They correspond to one half of the total mass and every 5th mass-zone is given. It is remarkable that all trajectories are bunched together and that the thermal history is quite similar for all of them. We note that the thermal history of the rest of the star is also similar, but with a somewhat broader range around the ones shown in Fig. 3.

The trajectories all start from a density above $10^{13} g/cm^3$ at the initial temperature of $10^8 K$, and the density decreases due to the expansion. The temperature stays constant down to the transition density ρ_{tr} since no nuclear reaction take place in the core and, for practical reasons, we do not allow the temperature to drop below $10^8 K$. (This would be different for a more realistic equation of state.) The matter is converted into nuclei with dripped neutrons at ρ_{tr} and, from then on, is heated up due to β-decays despite the fact that it is expanding. It can clearly be seen in Fig. 3 that β-decay heating dominates over adiabatic losses. A large jump of the temperature at a density near $10^{11} g/cm^3$ indicates the onset of fission of nuclei and the temperature reaches peak values close to $5 \times 10^9 K$. From then on rapid expansion leads to some cooling (β-decay heating can no longer compete with adiabatic losses) until once again fission sets in and heats the matter once more to temperatures near $5 \times 10^9 K$, and finally the expansion cools down the matter continuously.

The fact that rather high temperatures are reached during the explosion is important for the nucleosynthe-

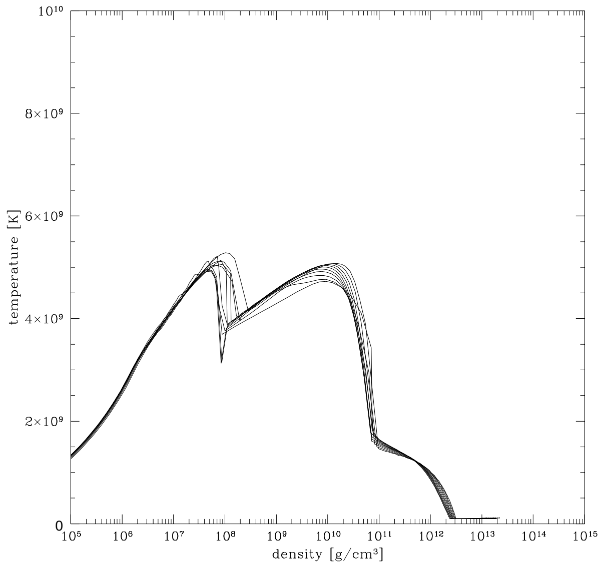

Fig. 3. Thermal history of neutron star matter during the explosion is displayed in the temperature - density plane. Shown are the trajectories of mass-zones (one half of the total mass and every 5th zone) in the central region of the star.

sis yields expected from such an event. On the one hand side, the peak temperatures are considerably higher than $10^9 K$, so significant processing by charged particle nuclear reactions can proceed. On the other hand side, the temperature is not too high, e.g. lower than about $5\times10^9 K$, so that a nuclear statistical equilibrium, which would erases the memory of the original composition, is never fully achieved. Therefore, nucleosynthesis proceeds with various nuclear reactions, but keeping some memory of neutron-rich cold neutron star matter.

Cold expansion of neutron star matter has been discussed as a possible site for r-process nucleosynthesis, and the abundances of r-elements produced in the expansion have been calculated by parameterizing plausible scenarios for the dynamics of the star (Sato 1974; Lattimer et al. 1977; Meyer 1989). The crucial questions concerning r-process-like nucleosynthesis in exploding low-mass neutron stars are whether or not adiabatic expansion limits β-decay heating (a question which has been answered in a positive way in this section) and whether or not a sufficient number of neutron captures and β-decays can produce also nuclei in, say, the Uranium region. Again, the answer to this question depends on the details of the dynamical evolution and, therefore, requires the use of numerical simulations. As was stated earlier, fission heating is an important ingredient in this respect, indicating that indeed in our computations nucleosynthesis proceeds all the way up to the spontaneous fission line, and we found by coupling the hydrodynamics to the various nuclear processes that r-process nucleosynthesis can in fact take place during the explosion of a neutron star at the minimum mass.

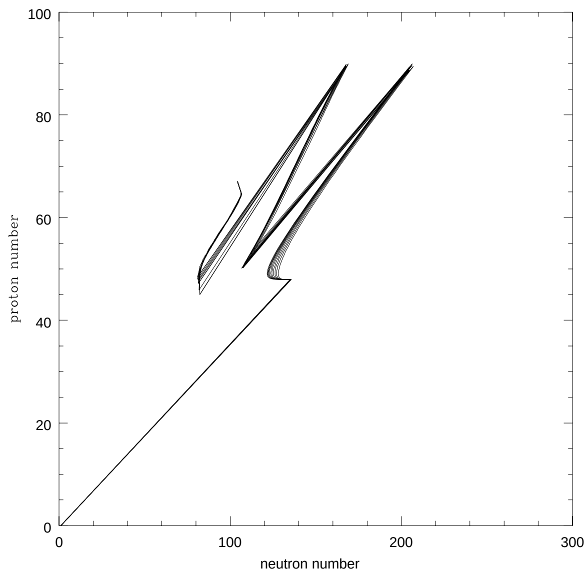

Fig. 4. The history of nuclei in neutron star matter during the explosion is displayed in the nuclear chart. Shown are the trajectories of mass-zones in the central region of the star as in Fig. 3.

To be a bit more quantitative, Figure 4 shows again the history of the matter during the explosion, but now in the chart of nuclides. We display the trajectories of the mass elements of the core as in Fig. 3. It is obvious that the mean nucleus present in the expanding neutron star matter is very neutron-rich, neutron-to-proton ratios of up to 2 are realized in all mass-shells originally belonging to the stellar core, conditions that are favorable for r-process nucleosynthesis. At the beginning of the evolution, this matter is composed of neutrons with a small fraction of protons and electrons and is converted into nuclei with many dripped neutrons and some electrons once the density drops below ρ_{tr}. During the expansion nuclei absorb the dripped neutrons rapidly, followed by slow β-decays, and the mass-number of the mean nucleus grows towards heavier elements. When the proton number of nuclei reaches $Z_0 = 90$ (where we have put the spontaneous fission line) fission takes place and the nuclei split in half, according to our simplified treatment. The light fission products again absorb neutrons rapidly, with subsequent β- decays, until they fission again. After two cycles the matter density has become so low and the expansion rate is so fast that β-decays do no longer happen and dripped neutrons are no longer around and, as the final product after freeze-out, we find nuclei in the rare earth region.

This result, although very promising, is not fully conclusive. Firstly, one has to bear in mind that some of the micro-physics input used here needs refinements, such as the equation of state, the treatment of fission, etc. Moreover, we have always assumed that the matter composition is well characterized by one representative nucleus. While for calculating the equation of state this is a rather good approximation, it is less obvious that it also remains true if we are concerned with β-decay rates and fission properties. It is interesting to note, however, that the various mass-shells follow almost the same trajectories in the (Z,N)-plane which means that even with more elaborate input physics the final composition will be characterized by a typical set of thermodynamic parameters. This will allow us to post-process typical shells with an extended r-process network, to compute detailed abundances, and to compare them with observations. These aspects of our work are now under investigation. Finally, the peak temperatures found in our computations (close to $5 \times 10^9 K$) may be too high to ignore charged particle reactions altogether. If it should turn out that with more realistic equations of state we still get peak temperatures of that order we would have to calculate the nuclear abundances from a full reaction network.

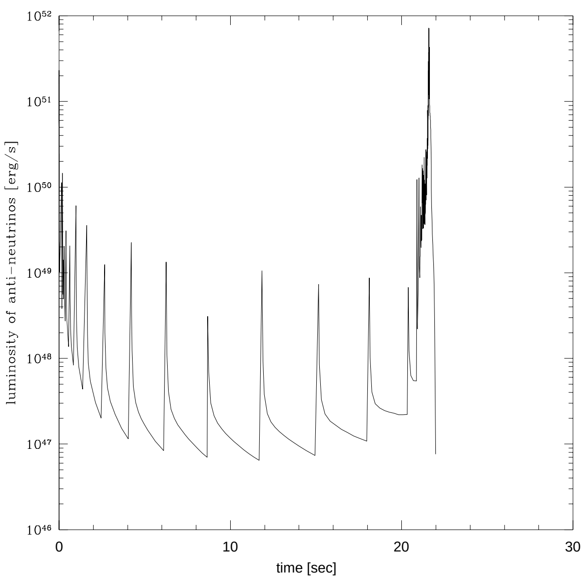

Fig. 5. Predicted luminosity of electron anti-neutrinos during the evolution of the neutron star is plotted as a function of time.

3.3. Neutrino Emission from an Exploding Neutron Star

We conclude this section with a brief discussion of the neutrino signal that is to be expected from an exploding neutron star. In fact, due to the β-decays of nuclei copious neutrinos are emitted, both during the quasi-static expansion of the neutron star and in its final explosion. Figure 5 shows the luminosity in electron anti-neutrinos as a function of time predicted from our computations. The red-shift factor is taken into account, but its effect is quite small. Typically, we find neutrino luminosities below $10^{50} erg/s$ during the slow expansion phase of 20 seconds. The spiky nature of the luminosity reflects always the onset of fission in some mass-zones, because fission not only provides heating of the matter but also shorter β-decay lifetimes than of the parent nuclei. Therefore, the neutrino emission suddenly increases whenever fission occurs.

The final explosion is accompanied by a short outburst of neutrinos lasting for about 0.2 seconds with a peak luminosity of nearly $10^{52} erg/s$. This is still only of the order of 10% of the electron (anti-) neutrino luminosity of a core-collapse supernova, but it would be observable with existing neutrino detectors if such an event occurred in our own Milky Way Galaxy.

4. Summary and Conclusions

We have demonstrated that a neutron star which happens to be close to the minimum stable mass will explode if of the order of 20% of its mass are removed from its surface. We found that because of the reduced gravity the outer layers expand first on time scales of about $20s$, leaving the core essentially untouched. However, due to slow β-decays also the core is heated somewhat and expands. Nuclear energy release, both from β-decays and from fission, finally unbinds the core and causes it (and the entire star) to explode on a time scale of a fraction of a second.

The peak temperatures we find never exceed $5 \times 10^9 K$. This is an interesting and important result because it indicates that a kind of r-process will take place in exploding neutron stars. The abundance yields still have to be calculated, but from this preliminary study it is already apparent that the temperatures necessary for a complete nuclear statistical equilibrium are not reached during the explosion. Therefore, some information from the pre-explosion phase is preserved and we expect that a significant fraction of the neutron star's mass is ejected in form of neutron-rich isotopes of heavy nuclei. This will allow us to place strong constraints on the event rate of such explosions, no matter whether they result from merging binary neutron star systems or from fragmenting cores of rapidly spinning collapsing massive stars. For example, if a total of 0.1 $M_\odot$ were ejected with typical solar system r-process composition (which seems possible on the basis of our present study) this would imply that the event rate in our Galaxy should not exceed about 1 per 10^4 years to avoid over-production of r-isotopes. Of course, this constraint will become more solid once we have performed full nuclear network calculations on the basis of our hydrodynamical models.

As we have discussed earlier a major short-coming of our study as well as of all previous ones is the rather unrealistic equation of state that has been applied. With a more realistic equation of state one expects (see, e.g., Baym et al. 1971; Colpi et al. 1993) that at the minimum mass almost the entire star will consist of crust material, maybe with a small central core of a few percent of a solar mass. According to what we discussed in Section 2 this implies that even if only a very small amount of mass is removed from the surface β-decays will set in everywhere in the star, leading to faster heating and, likely, to a more violent explosion. Whether or not the temperatures remain as low as was found here has to be seen.

A second question which has to be addressed in a forthcoming work is the sensitivity of the predictions to the rather uncertain β-decay rates. Here, we have used rates which are probably too fast (see, e.g., Takahashi et al. 1973; Kodama and Takahashi 1975). Slower ones will postpone the explosion and counteract the effect of an equation of state with nuclei up to nuclear saturation density. But in general terms we expect an effect on the explosion time scale only but not so much a change of the thermodynamical properties of the exploding matter. So the net-effect of changing the β-decay rates on the nucleosynthesis yields is expected to be small.

Finally, we have shown that exploding neutron stars are sources of electron anti-neutrinos and can, in principle, be detected by already existing neutrino detectors if such explosions would occur in our own Milky Way Galaxy. However, because of the very low event rate discussed above, this is very unlikely. More distant events, on the other hand, would not lead to a measurable signal in present neutrino experiments. Because of the compactness of the star the explosion will, however, give rise to emission of short wave-length electromagnetic radiation. A rough estimate shows that this emission will peak in the keV range with luminosities of the order of up to $10^{47} erg/s$ and a typical duration of around tens of seconds. Of course, despite of this enormous X-ray luminosity which in principle would allow to observe neutron star explosions at large distances, in practice, however, because of the low event rate and the short duration of the burst, real detections seem not to be very probable.

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