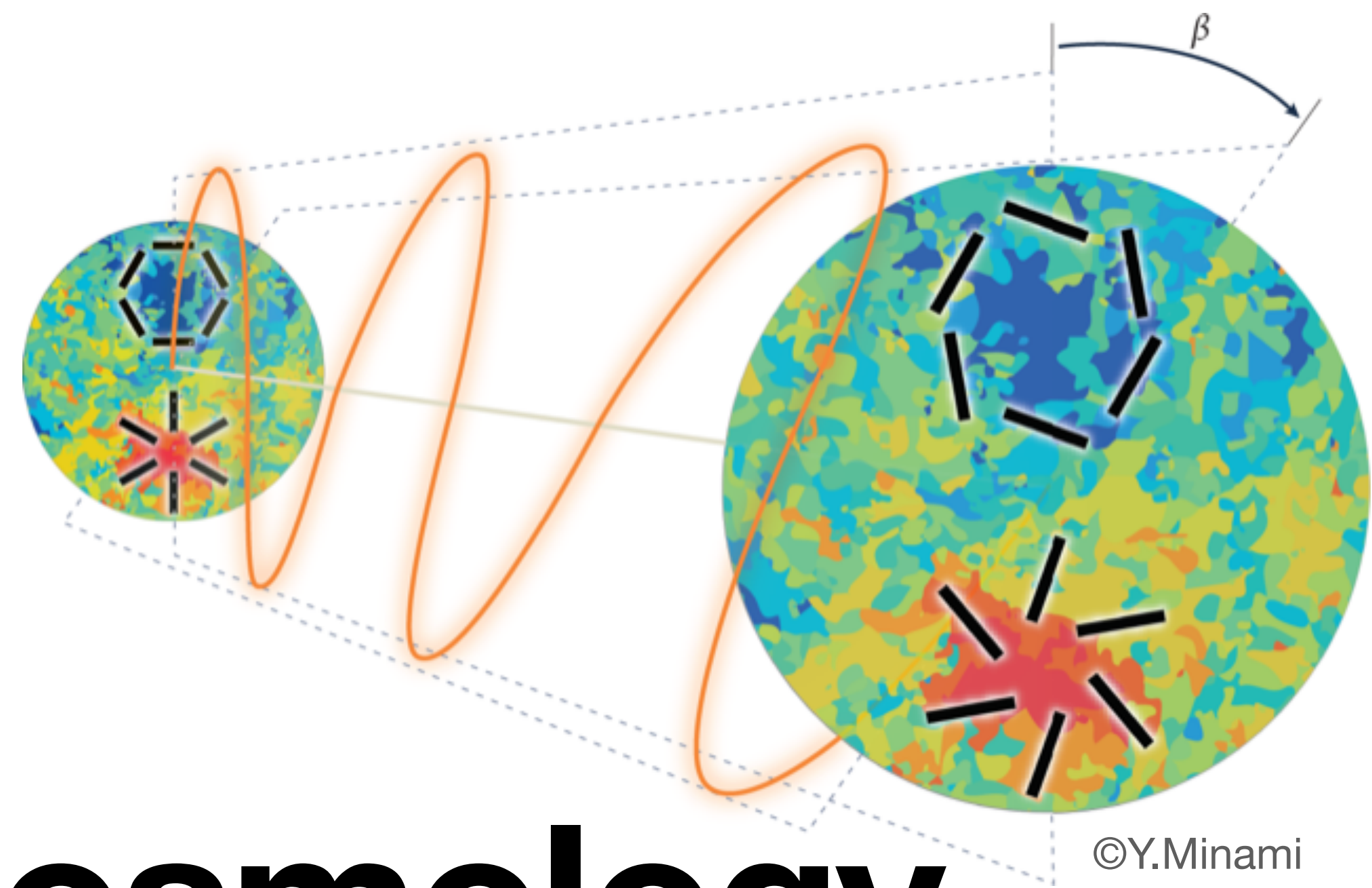


$$I_{CS} = \int d^4x \sqrt{-g} \left(-\frac{\alpha}{4f} \chi F \tilde{F} \right)$$



Parity Violation in Cosmology

In search of new physics for the Universe

Journal of **C**osmology and **A**stroparticle **P**hysics
an IOP and SISSA journal

Eiichiro Komatsu (Max Planck Institute for Astrophysics)
 JCAP Colloquium, September 4, 2023
 XXV SIGRAV Conference on General Relativity and Gravitation

Jcap **20th**
 2003-2023

Overarching Theme

Let's find new physics!

- The current cosmological model (*flat Λ CDM*) **requires** new physics beyond the standard model of elementary particles and fields.
 - What is dark matter (*CDM*)?
 - What is dark energy (Λ)?
 - Why is the spatial geometry of the Universe Euclidean (*flat*)?
 - What powered the Big Bang?

Overarching Theme

There are many ideas

- The current cosmological model (*flat Λ CDM*) **requires** new physics beyond the standard model of elementary particles and fields.
 - What is dark matter (*CDM*)? => CDM, WDM, FDM, ...
 - What is dark energy (Λ)? => Dynamical field, modified gravity, quantum gravity, ...
 - Why is the spatial geometry of the Universe Euclidean (*flat*)? => Inflation, contracting universe, ...
 - What powered the Big Bang? => Scalar field, gauge field, ...

Overarching Theme

There are many ideas

- The current cosmological model (*flat*) and the standard model of elementary particles
- What is dark matter (*CDM*)? => CDM, WDM, FDM, ...
- What is dark energy (Λ)? => Dynamical field, modified gravity, quantum gravity, ...
- Why is the spatial geometry of the Universe Euclidean (*flat*)? => Inflation, contracting universe, ...
- What powered the Big Bang? => Scalar field, gauge field, ...

New in cosmology!
Violation of parity symmetry may hold the answer to these fundamental questions.

Reference: nature reviews physics

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New physics from the polarized light of the cosmic microwave background

[Eiichiro Komatsu](#) 

[Nature Reviews Physics](#) **4**, 452–469 (2022) | [Cite this article](#)

Key Words:

1. Cosmic Microwave Background (CMB)
2. Polarization
3. Parity Symmetry

Lectures & Reviews

2023

- ▶ **Lecture Slides: "Parity Violation in Cosmology" [7 x 85 min]**
 - ▶ MC Specialized Course, Department of Physics, Nagoya University (June 6–30)
 - ▶ The syllabus is available [here](#).
 - ▶ Reference: "*New Physics from the Polarized Light of the Cosmic Microwave Background*"
 - ▶ **Nature Reviews Physics, 4, 452-469 (2022 May 18)**. You can have access to the full text via [this link](#). Supplementary information is available [here](#).
- ▶ **Lecture 1:** What is parity symmetry? (PDF 3.9 MB; last updated, June 5, 2023)
 - ▶ 1.1 Parity
 - ▶ 1.2 Vector and pseudovector
 - ▶ 1.3 Discovery of parity violation in β -decay
 - ▶ 1.4 Helicity
- ▶ **Lecture 2:** Chern-Simons interaction (PDF 1.6 MB; last updated, June 8, 2023)
 - ▶ 2.1 Parity symmetry in electromagnetism (EM)
 - ▶
 - ▶

Probing Parity Symmetry

Definition

- **Parity transformation = Inversion of all spatial coordinates**
 - $(x, y, z) \rightarrow (-x, -y, -z)$
- Parity symmetry in physics states:
 - *The laws of physics are invariant under inversion of all spatial coordinates.*
- Violation of parity symmetry = The laws of physics are **not** invariant under...
- Ask “***When we observe a certain phenomenon in nature, do we also observe its mirror image(*) with equal probability?***”
 - (*) “Mirror image” is an ambiguous word. A parity transformation is $(x, y, z) \rightarrow (-x, -y, -z)$, whereas a “mirror image” often refers to, e.g., $(x, y, z) \rightarrow (-x, y, z)$, where only one of (x,y,z) is flipped.



Do we also observe this with equal probability?



Note that this is not full parity transformation, as only one axis is flipped.

Parity and Rotation

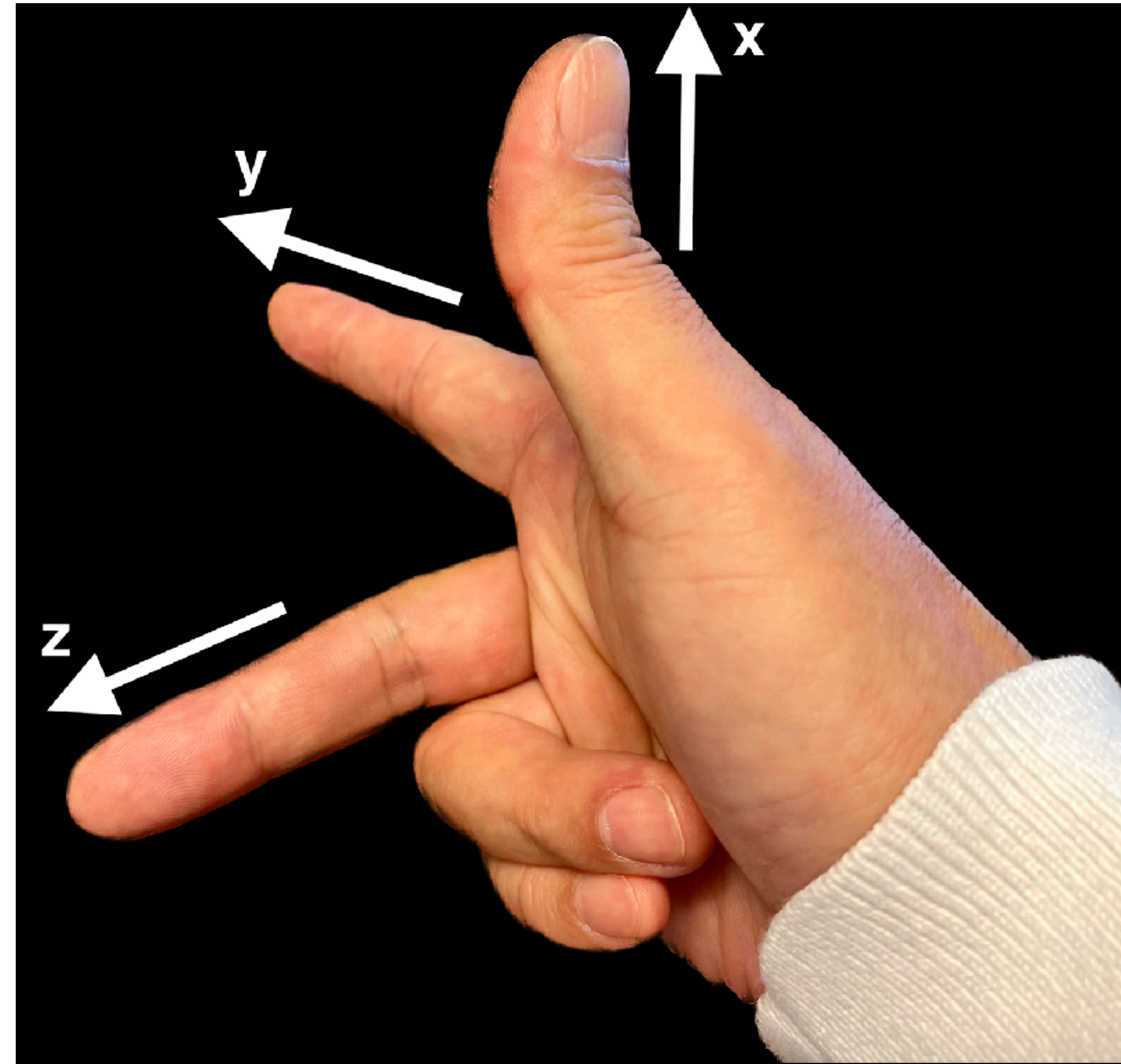
- Parity transformation ($\mathbf{x} \rightarrow -\mathbf{x}$) and 3d rotation ($\mathbf{x} \rightarrow R\mathbf{x}$) are different.
 - R is a continuous transformation and the determinant of R is $\det(R) = +1$.
 - Parity is a discrete transformation and the **determinant is -1**, as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

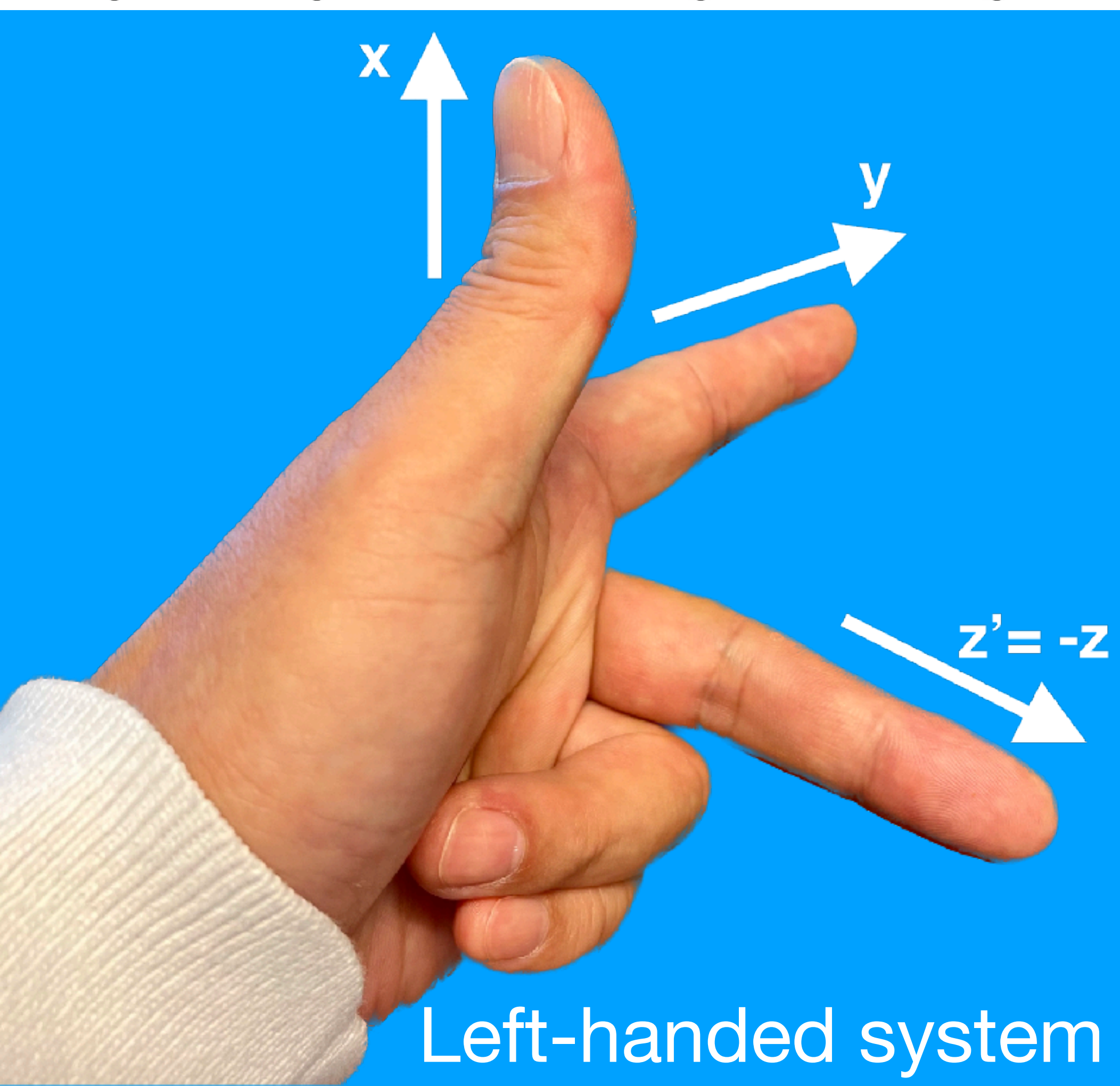
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Parity = Mirror + 2d Rotation

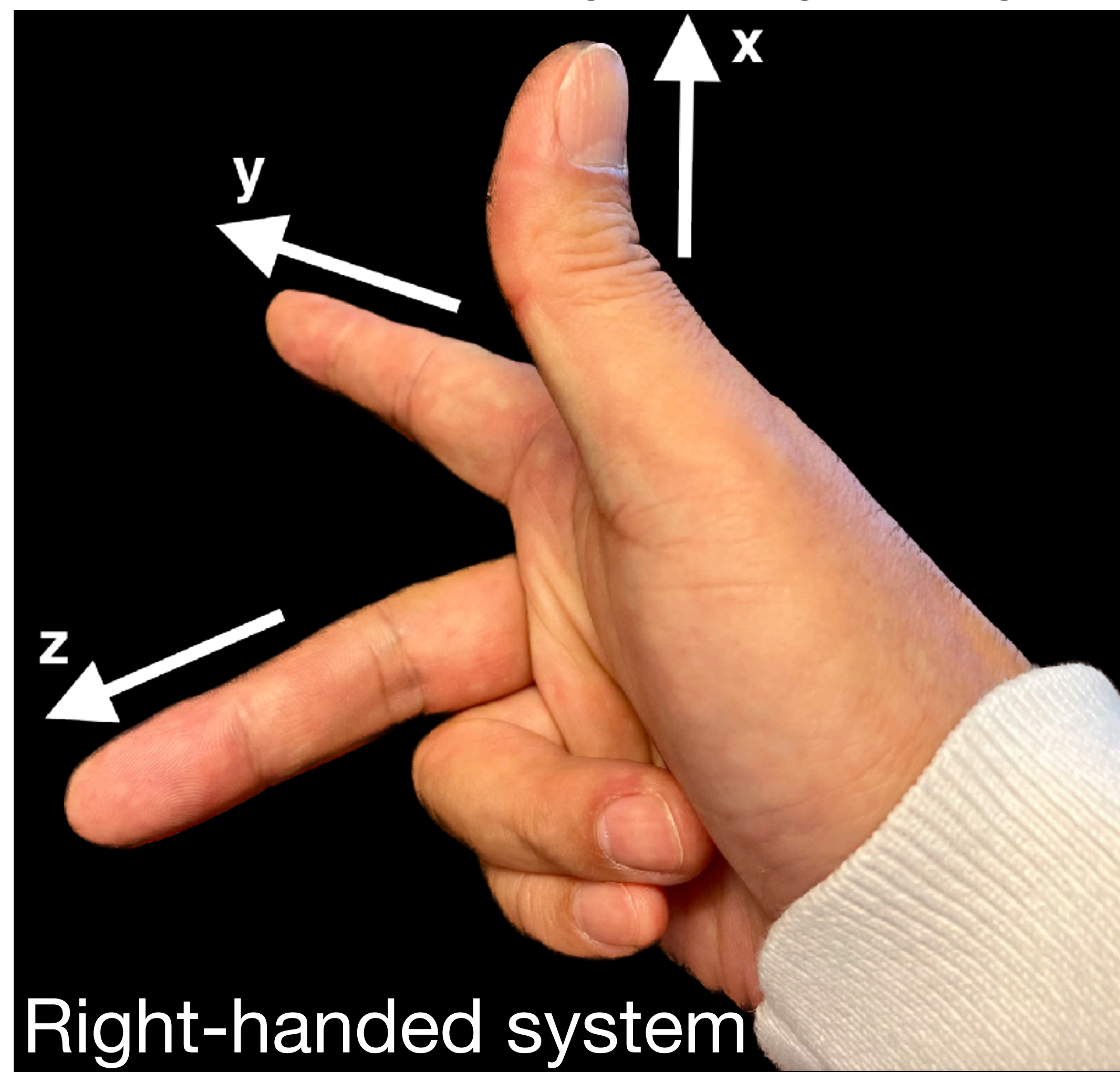
- One may think of parity transformation as a mirror in one of the coordinates (e.g., $z \rightarrow -z$) and **2d** rotation by π in the others.
- Dimostriamo!



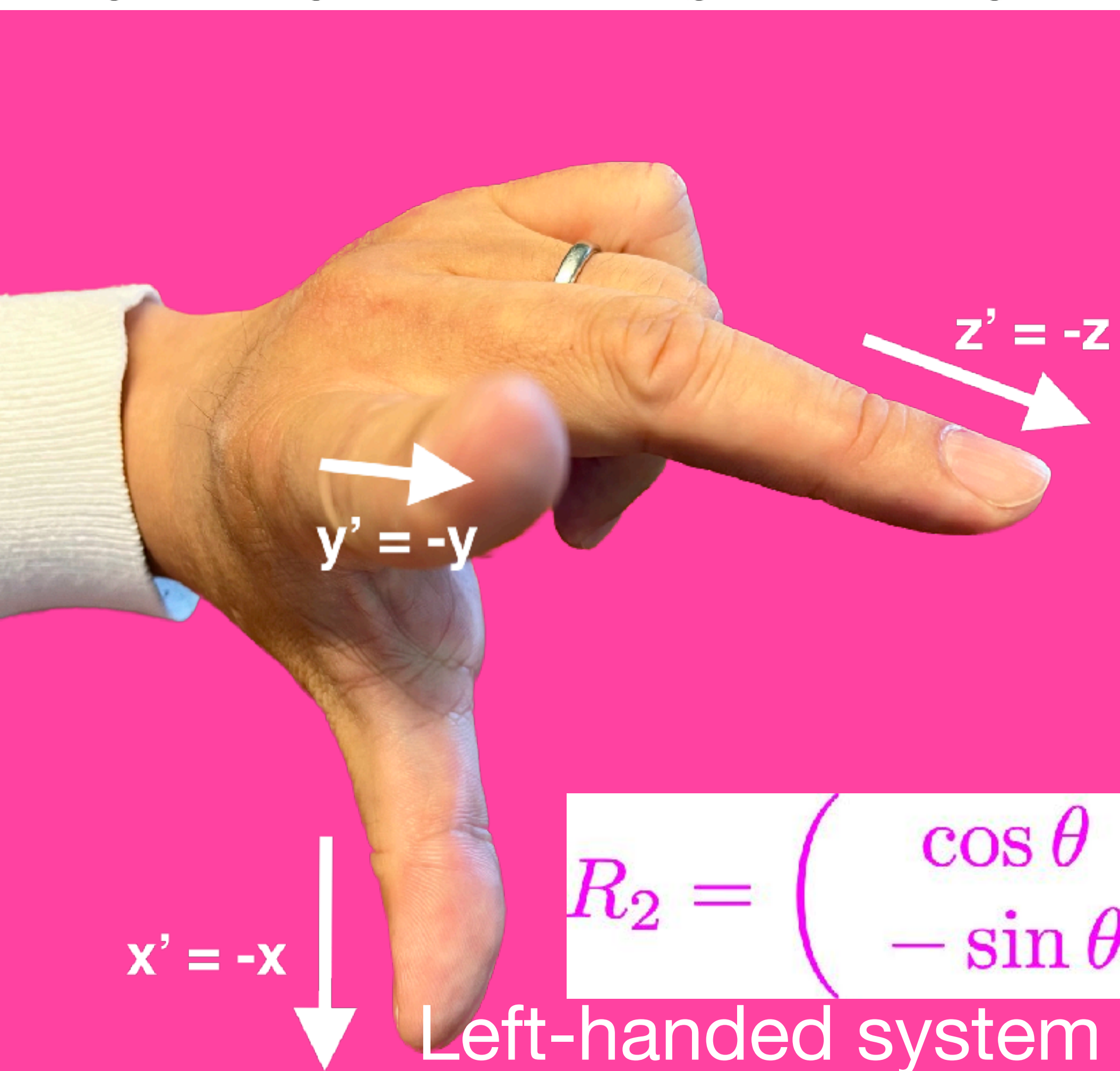
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$z \rightarrow z' = -z$$

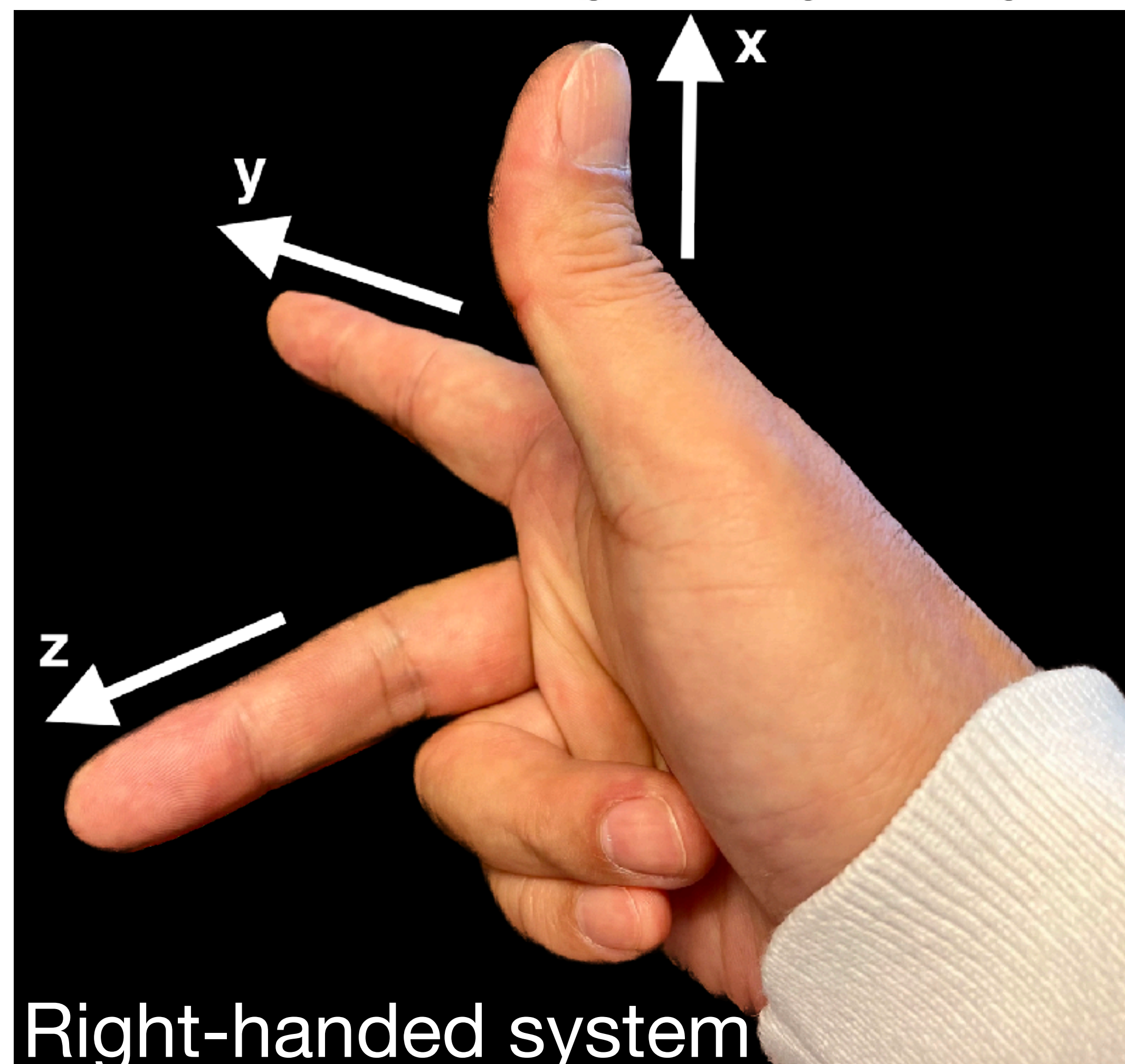


$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} \boxed{-1} & \boxed{0} & 0 \\ \boxed{0} & \boxed{-1} & 0 \\ 0 & 0 & \boxed{-1} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$R_2 = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

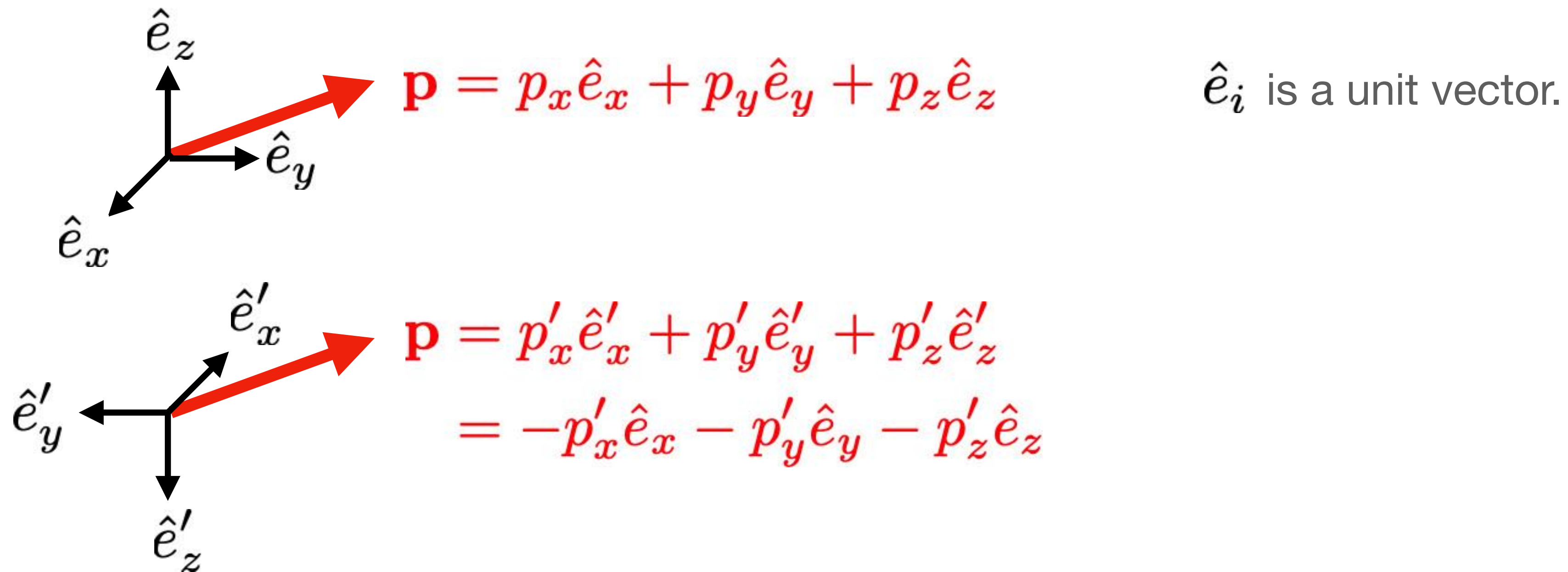
Left-handed system with $\theta \stackrel{13}{=} \pi$



Right-handed system

Parity Transformation: Vector

E.g., momentum, electric field



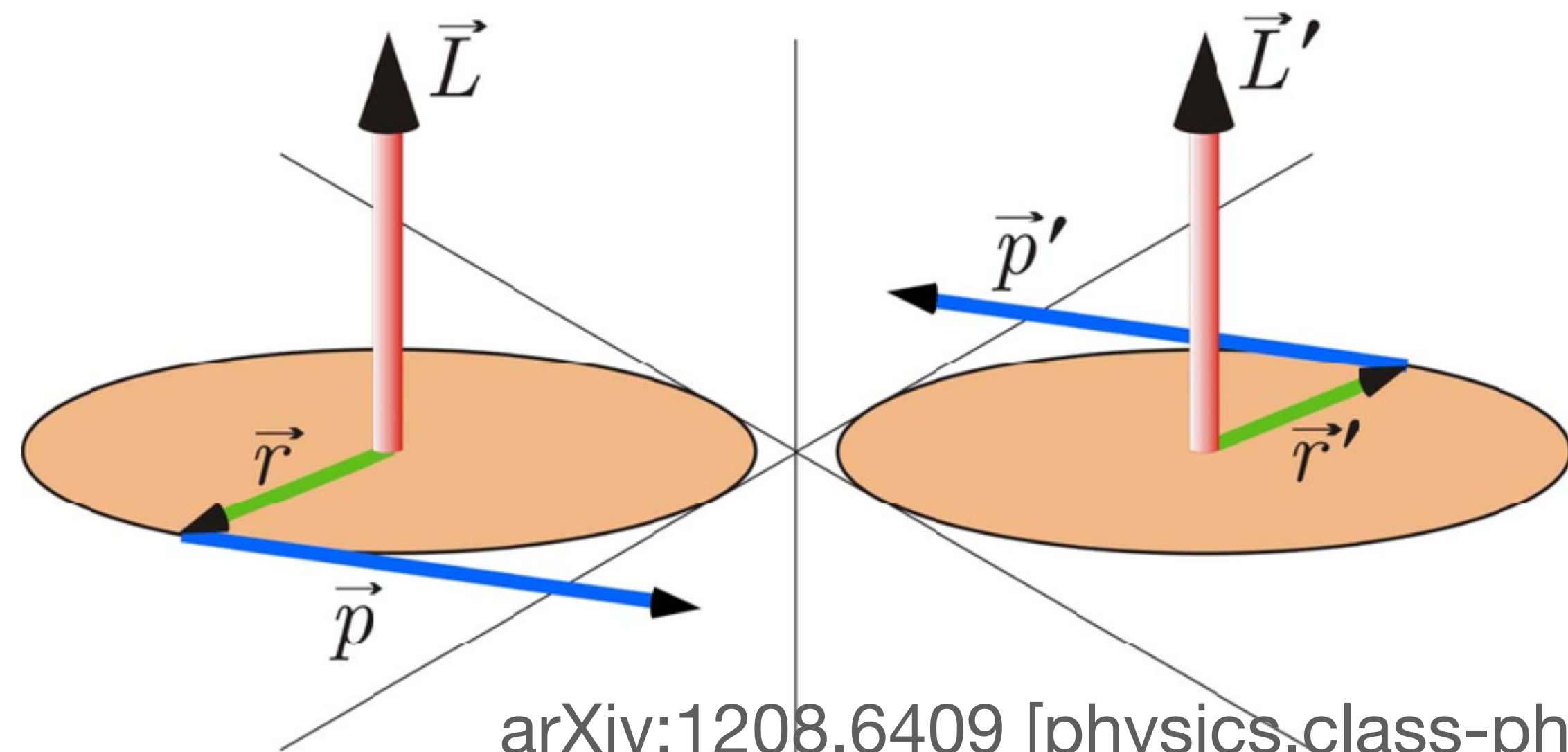
- \mathbf{p} is the same vector, written using two different basis vectors.
- Therefore, \mathbf{p} 's components are transformed as $(p'_x, p'_y, p'_z) = (-p_x, -p_y, -p_z)$

Parity Transformation: Pseudovector

E.g., angular momentum, magnetic field

- Orbital angular momentum, $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, is a *pseudovector*. Its *components* do **not** change under parity transformation: $(L'_x, L'_y, L'_z) = (L_x, L_y, L_z)$
- Both $\mathbf{r} = (X, Y, Z)$ and $\mathbf{p} = (p_x, p_y, p_z)$ are vectors whose components change sign. Thus, their products do not change, e.g.,

$$\begin{aligned} L'_x &= Y' p'_z - Z' p'_y \\ &= (-Y)(-p_z) - (-Z)(-p_y) \\ &= L_x \end{aligned}$$



Parity Transformation: Pseudoscalar

How to test parity symmetry?

- A dot product of a vector and a pseudovector is a **pseudoscalar**.
 - Like a scalar, a pseudoscalar is invariant under rotation.
 - But, a pseudoscalar changes sign under parity transformation.
- **Experimental test of parity symmetry**: Construct a pseudoscalar and see if the average value is zero. If not, the system violates parity symmetry!
 - *Example*: a dot product of particle A's momentum and particle B's angular momentum: $\mathbf{p}_A \cdot \mathbf{L}_B$. Measure this and average over many trials. Does the average vanish, $\langle \mathbf{p}_A \cdot \mathbf{L}_B \rangle = 0$?

Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

AND

E. AMBLER, R. W. HAYWARD, D. D. HOPPES, AND R. P. HUDSON,
National Bureau of Standards, Washington, D. C.

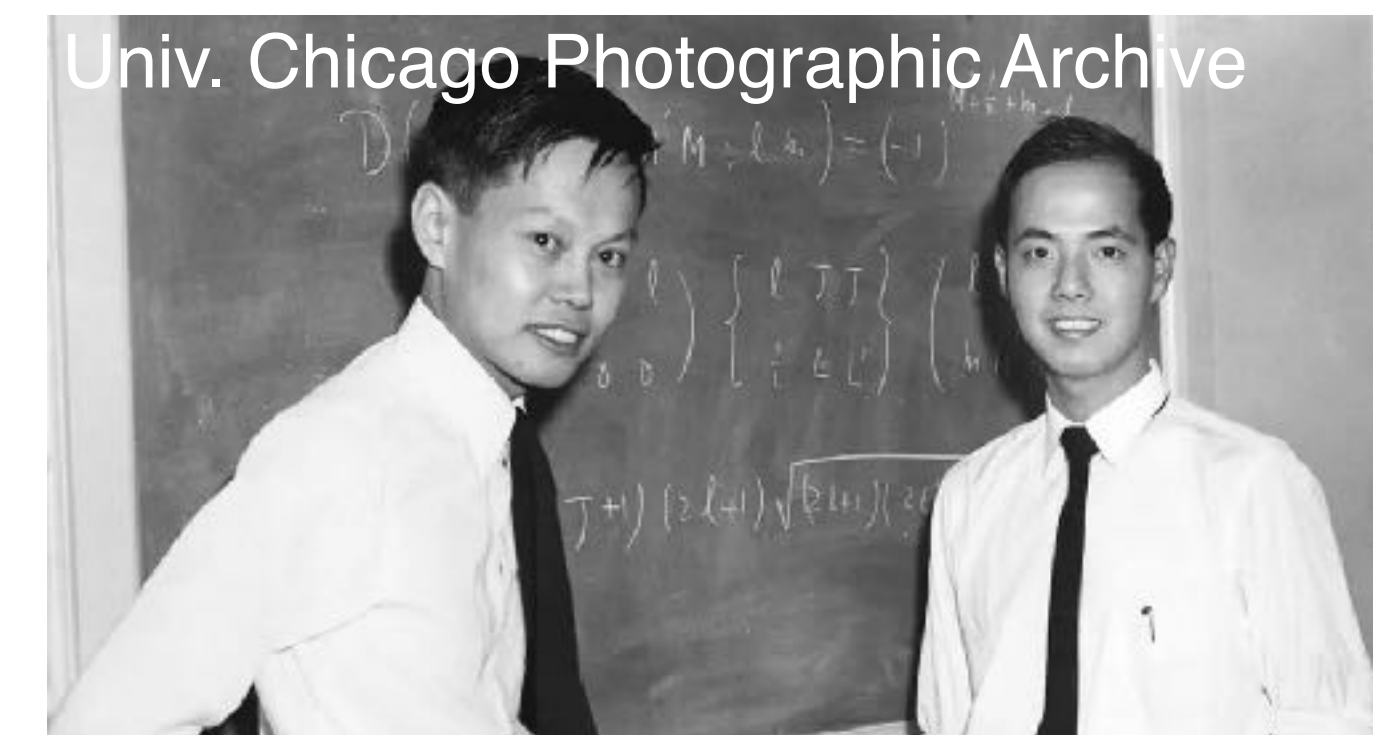
(Received January 15, 1957)

IN a recent paper¹ on the question of parity in weak interactions, Lee and Yang critically surveyed the experimental information concerning this question and reached the conclusion that there is no existing evidence either to support or to refute parity conservation in weak interactions. They proposed a number of experiments on beta decays and hyperon and meson decays which would provide the necessary evidence for parity conservation or nonconservation. In beta decay, one could measure the angular distribution of the electrons coming from beta decays of polarized nuclei. If an asymmetry in the



Smithsonian Institution Archives

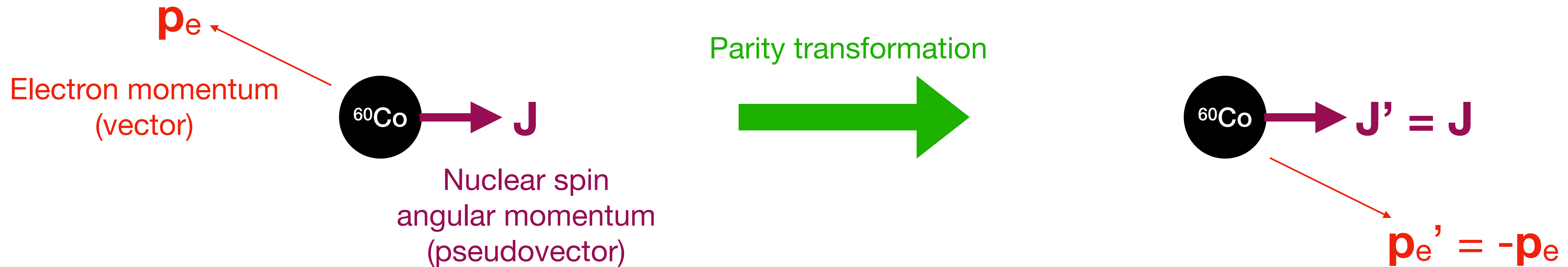
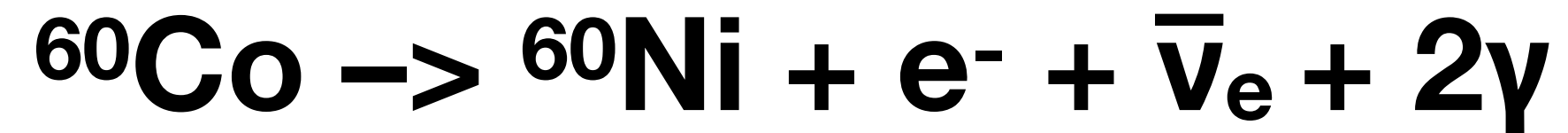
Chien-Shiung Wu



Chen-Ning Yang

Tsung-Dao Lee

The Wu Experiment of β -decay



- Electrons must be emitted with equal probability in all directions relative to \mathbf{J} , if parity symmetry is respected in β -decay.
- This was not observed: $\langle \mathbf{p}_e \cdot \mathbf{J} \rangle \neq 0$. **Parity symmetry is violated in β -decay!**

Initial reaction

Many physicists did not believe it initially.



Bildarchiv der ETH-Bibliothek

- To Lee and Yang’s theoretical paper on parity violation in β -decay:
 - Wolfgang Pauli said, “*Ich glaube aber nicht, daß der Herrgott ein schwacher Linkshänder ist*” (I do not believe that the Lord is a weak left-hander).
- To Wu’s discovery paper:
 - Wolfgang Pauli said, “*Sehr aufregend. Wie sicher ist die Nachricht?*” (Very exciting. How sure is this news?)
- **This was shocking news. The weak interaction distinguishes between left and right!**
- In this talk we ask, “*Does the Universe distinguish between left and right?*” Most scientists answer, “No, of course it doesn’t”. Only experiments will decide.

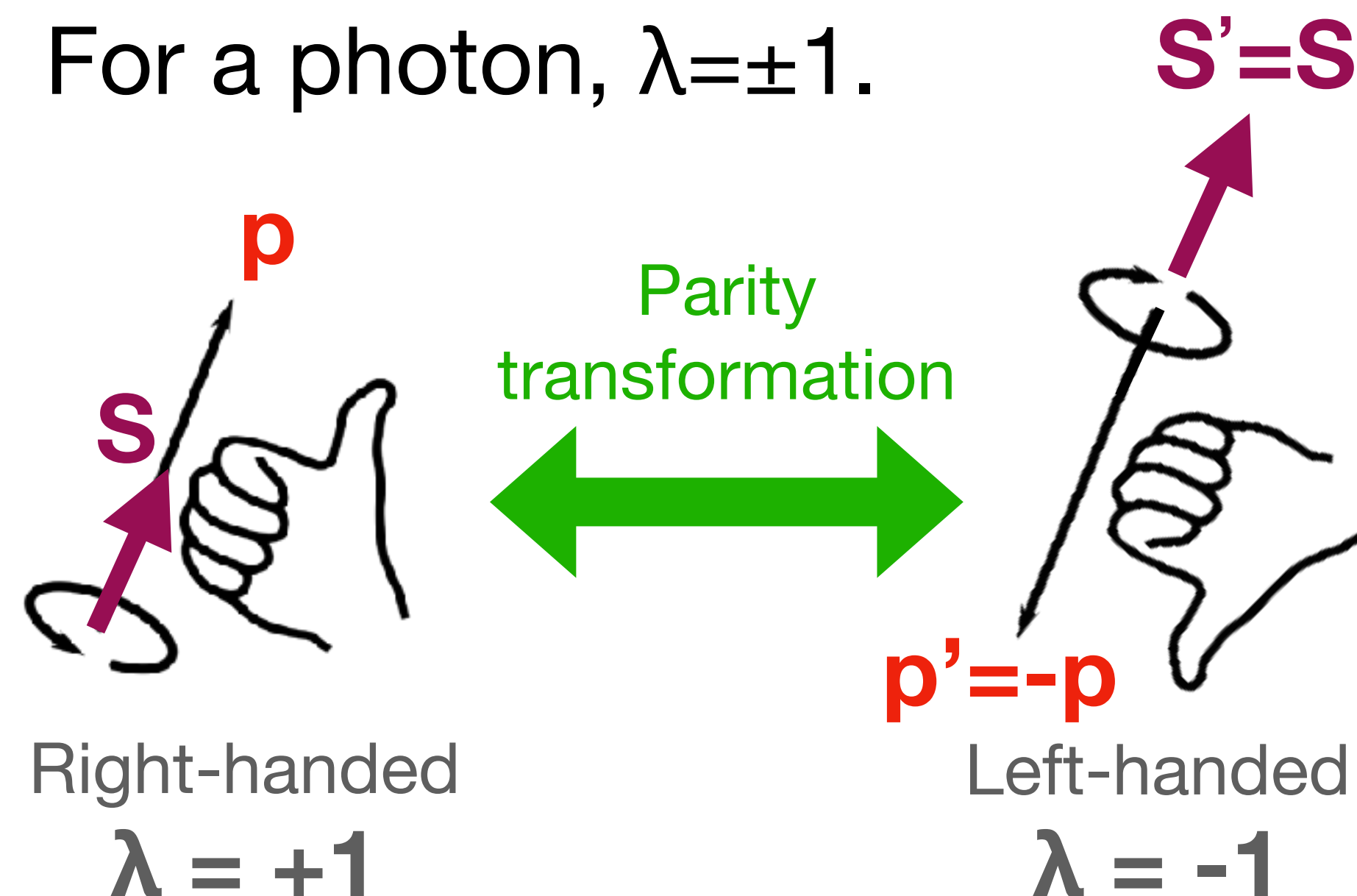
Helicity is a pseudoscalar

Parity transformation changes “right-handed” to “left-handed” and vice versa

- For massless particles, we define the “helicity”, λ , as

$$\mathbf{S} \cdot \frac{\mathbf{p}}{|\mathbf{p}|} = \lambda \hbar$$

- For a photon, $\lambda = \pm 1$.



- λ is a pseudoscalar because it is a product of a momentum vector (\mathbf{p}) and a spin pseudovector (\mathbf{S}).
- On the other hand, “scalar”, such as \mathbf{p}^2 and \mathbf{S}^2 , does not change sign.
- For a graviton, $\lambda = \pm 2$.
- Asymmetry between $\lambda = \pm 1$ and ± 2 is the sign of parity violation!

Maxwell's Equations

In Minkowski space, Heaviside units and $c=1$

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \rho, & -\dot{\mathbf{E}} + \nabla \times \mathbf{B} &= \mathbf{j} \\ \nabla \cdot \mathbf{B} &= 0, & \dot{\mathbf{B}} + \nabla \times \mathbf{E} &= 0\end{aligned}$$

- These equations are invariant under spatial translation and rotation and Lorentz transformation.

Parity-flipping Maxwell's Equations

In Minkowski space, Heaviside units and $c=1$

$$\begin{aligned}(-\nabla) \cdot (-\mathbf{E}) &= \rho, & -(-\dot{\mathbf{E}}) + (-\nabla) \times \mathbf{B} &= (-\mathbf{j}) \\ (-\nabla) \cdot \mathbf{B} &= 0, & \dot{\mathbf{B}} + (-\nabla) \times (-\mathbf{E}) &= 0\end{aligned}$$

- These equations are invariant under spatial translation and rotation and Lorentz transformation.
- They are also invariant under parity transformation, if \mathbf{E} and \mathbf{j} are vectors, ρ is a scalar, and \mathbf{B} is a pseudovector.

Maxwell's Equations in a covariant form

$$\nabla \cdot \mathbf{E} = \rho, \quad -\dot{\mathbf{E}} + \nabla \times \mathbf{B} = \mathbf{j}$$

$$\nabla \cdot \mathbf{B} = 0, \quad \dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0$$

- These equations can be written in a covariant form as

$$\partial_\nu F^{\mu\nu} = j^\mu$$

$$\partial_\nu \tilde{F}^{\mu\nu} = 0$$

Dual tensor

$$\mu = 0, 1, 2, 3, \quad j^\mu = (\rho, \mathbf{j}), \quad \partial_\mu = \partial / \partial x^\mu, \quad x^\mu = (t, \mathbf{x})$$

Antisymmetric Field Strength Tensor, $F_{\mu\nu}$

$$F_{\mu\nu} = -F_{\nu\mu}$$

$$F_{\mu\nu} = \eta_{\mu\alpha}\eta_{\nu\beta}F^{\alpha\beta} \quad \text{where } \eta_{\mu\alpha} = \text{diag}(-1, 1, 1, 1)$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}$$

- Therefore,

$$F^2 \equiv F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B} \cdot \mathbf{B} - \mathbf{E} \cdot \mathbf{E})$$

This is a *scalar* and is invariant under parity transformation.

Dual Field Strength Tensor, $\tilde{F}^{\mu\nu}$

$$\tilde{F}^{\mu\nu} = -\tilde{F}^{\nu\mu}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \quad \text{where } \epsilon^{\mu\nu\alpha\beta} = \begin{cases} +1 & \text{if } (\mu, \nu, \alpha, \beta) \text{ is even perm. of } (0, 1, 2, 3) \\ -1 & \text{if } (\mu, \nu, \alpha, \beta) \text{ is odd perm. of } (0, 1, 2, 3) \\ 0 & \text{otherwise} \end{cases}$$

Levi-Civita symbol

$$\tilde{F}^{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{pmatrix}$$

- Equivalently,

$$\begin{aligned} \tilde{F}^{0i} &= B_i \\ \tilde{F}^{ij} &= -\epsilon^{ijk} E_k \end{aligned}$$

- Therefore,

$$F \tilde{F} \equiv F_{\mu\nu} \tilde{F}^{\mu\nu} = -4\mathbf{B} \cdot \mathbf{E}$$

This is a *pseudoscalar* and changes sign under parity transformation!

$F\tilde{F}$ in the action?

$$I = -\frac{1}{4} \int d^4x F^2 + \int d^4x A_\mu j^\mu$$

- This action is sufficient to produce all of Maxwell's equations.

- Can we add $\int d^4x F\tilde{F}$ to the action?

- The answer is yes. However, **this is only a surface term**, since $F\tilde{F}$ is a total derivative:

$$F_{\mu\nu}\tilde{F}^{\mu\nu} = 2\partial_\mu (A_\nu \tilde{F}^{\mu\nu}) \text{ where}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Ni (1977); Turner, Widrow (1987); Carroll, Field, Jackiw (1990)

$\tilde{F}\tilde{F}$ in the action

Chern-Simons term

• Consider $I_{\text{CS}} = -\frac{1}{4}\alpha \int d^4x \theta F \tilde{F}$ with $F \tilde{F} = 2\partial_\mu (A_\nu \tilde{F}^{\mu\nu})$

• α : a dimensionless constant

• θ : a dimensionless pseudoscalar field

• **This is not a surface term!** Integration by parts gives

$$I_{\text{CS}} = \frac{1}{2}\alpha \int d^4x (\partial_\mu \theta) A_\nu \tilde{F}^{\mu\nu}$$

• This is a special case of the so-called *Chern-Simons term*, $p_\mu A_\nu \tilde{F}^{\mu\nu}$

with $p_\mu = \partial_\mu \theta$



<https://einstein-chair.github.io/simons2023/>

Consistency with gauge invariance

p_μ cannot be arbitrary

$$I_{\text{CS}} = \frac{1}{2} \alpha \int d^4x p_\mu A_\nu \tilde{F}^{\mu\nu}$$

- This action is invariant under the gauge transformation, $A_\nu \rightarrow A_\nu + \partial_\nu f$
if $\partial_\nu p_\mu - \partial_\mu p_\nu = 0$ Hint: Use integration by parts and the identity $\partial_\nu \tilde{F}^{\mu\nu} = 0$
- For example: This implies the presence of a preferred direction in spacetime and violation of Lorentz invariance!
 - p_μ is a constant vector and not dynamical, or
 - p_μ is a gradient of a dynamical (pseudo)scalar field, such as $p_\mu = \partial_\mu \theta$.

The main goal of this talk

Let's find new physics!

- We study the cosmological consequence of

$$I_{\text{CS}} = -\frac{1}{4}\alpha \int d^4x \theta F \tilde{F}$$

- Specifically, we ask if θ is —
 - responsible for dark matter and dark energy, or
 - active during cosmic inflation.

The main goal of this talk

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- We study the cosmological consequence of

$$I_{\text{CS}} = -\frac{1}{4}\alpha \int d^4x \theta F \tilde{F}$$

- Specifically, we ask if θ is –
 - responsible for dark matter and dark energy, or
 - active during cosmic inflation.

- More examples:

- **Non-Abelian gauge fields**
[Maleknejad, Sheikh-Jabbari, Soda, Phys. Rept. **528**, 161 (2013)]

$$F \tilde{F} = F_{\mu\nu}^a F^{\mu\nu a}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_A \epsilon^{abc} A_\mu^b A_\nu^c$$

- **Gravitational CS**
[Alexander, Yunes, Phys. Rept. **480**, 1 (2009)]

$$R \tilde{R} = R^\beta_{\alpha\mu\nu} \tilde{R}^\alpha_{\beta\mu\nu}$$

You can have both!

Jcap Mirzaghali, EK, Lozanov,
Watanabe, JCAP **06** (2020) 024

Is there a known example of this term in particle physics?

Yes, a pion.



Credit: HiggsTan

- A pion is a composite meson composed of a quark and an antiquark.
 - A neutral pion, π^0 , is composed of either $u\bar{u}$ or $d\bar{d}$, and **is a pseudoscalar**.
(Chinowsky & Steinberger, 1954)
 - π^0 is coupled to photons via L_{CS} where
 - $\theta = \pi^0 / f_\pi$ with $f_\pi \sim 184$ MeV (pion decay constant)
 - $\alpha = 2\alpha_{EM}N_c / (3\pi)$ with $N_c = 3$ (the number of quark colors) and $\alpha_{EM} \sim 1/137$ (EM fine structure constant)
- **π^0 decays into 2 photons via this term, which has been observed.** So, this possibility is not completely crazy!

Correction to Maxwell's equations

In Minkowski space, Heaviside units and $c=1$

- We now derive the correction to Maxwell's equations from

$$d^4x = dt d^3\mathbf{x}$$

$$I = -\frac{1}{4} \int d^4x \left(F^2 + \alpha \theta F \tilde{F} \right) + \int d^4x A_\mu j^\mu$$

- Finding the path that gives a stationary point,

$$\partial_\nu F^{\mu\nu} + \alpha (\partial_\nu \theta) \tilde{F}^{\mu\nu} = j^\mu$$

As expected, only the space-time dependence of the θ field affects Maxwell's equation.

Correction to the EM wave equation

With the Chern-Simons term

$$\partial_\nu F^{\mu\nu} + \alpha(\partial_\nu \theta) \tilde{F}^{\mu\nu} = 0 \quad \text{where } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- With $A^0 = \phi = 0$ in the Lorenz gauge, we find

$$-\square A^i + \alpha(\partial_\nu \theta) \tilde{F}^{i\nu} = 0$$

$$\square = \eta^{\alpha\beta} \partial_\alpha \partial_\beta = -\frac{\partial^2}{\partial t^2} + \nabla^2$$
$$A^\mu = \eta^{\mu\alpha} A_\alpha = (\phi, \mathbf{A})$$

$$\rightarrow \ddot{\mathbf{A}} - \nabla^2 \mathbf{A} + \alpha \left[-\dot{\theta}(\nabla \times \mathbf{A}) + (\nabla \theta) \times \dot{\mathbf{A}} \right] = 0$$

Correction to the EM wave equation!

Note: \mathbf{A} is a vector and θ is a pseudoscalar.

Helicity basis to probe parity symmetry

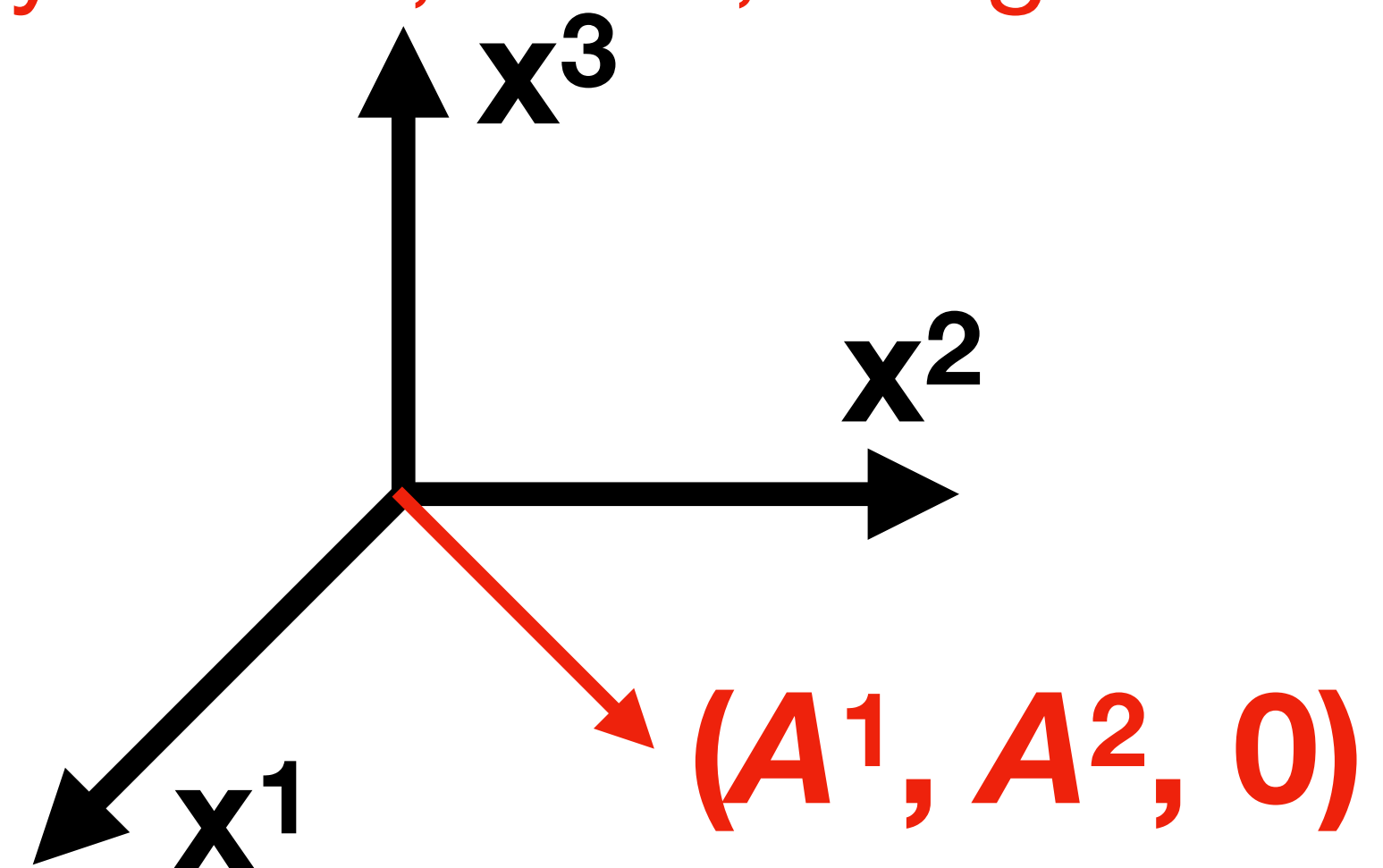
Going to Fourier space

- Fourier transform of $\mathbf{A}(t, \mathbf{x})$ is $\mathbf{A}(t, \mathbf{x}) = (2\pi)^{-3/2} \int d^3\mathbf{k} \mathbf{A}_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}$
- The EM wave propagates in the direction of \mathbf{k} . The change in $\mathbf{A}_{\mathbf{k}}$ is perpendicular to \mathbf{k} .

“Coulomb gauge” $\nabla \cdot \mathbf{A}(t, \mathbf{x}) = 0 \rightarrow \mathbf{k} \cdot \mathbf{A}_{\mathbf{k}}(t) = 0$

- Choose \mathbf{k} to be on the $z(=x^3)$ axis. The helicity states, $\lambda=\pm 1$, are given for each Fourier mode by

$$A_{\pm} = \frac{A_{\mathbf{k}}^1 \mp i A_{\mathbf{k}}^2}{\sqrt{2}}$$



Helicity basis to probe parity symmetry

Transformation property under rotation

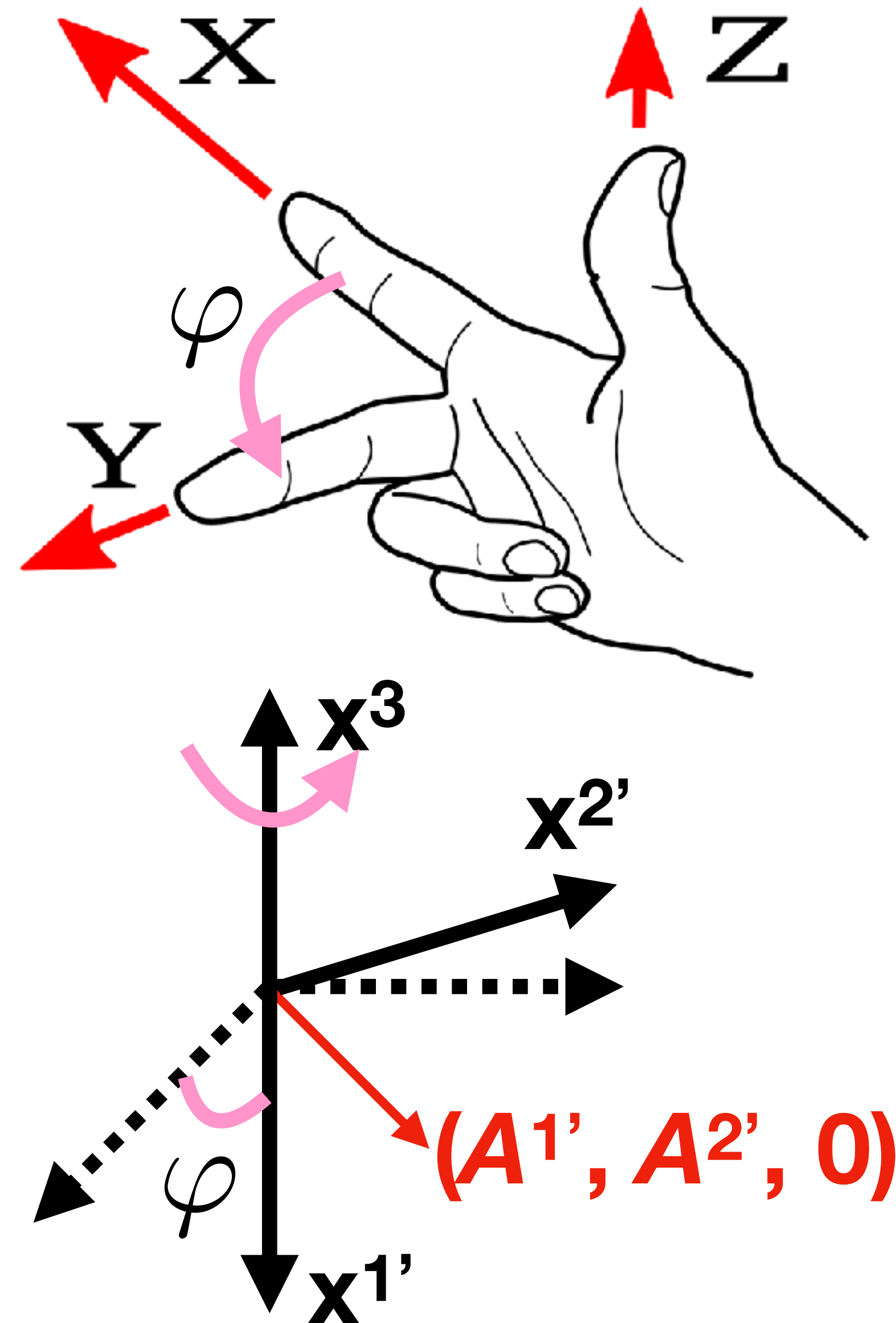
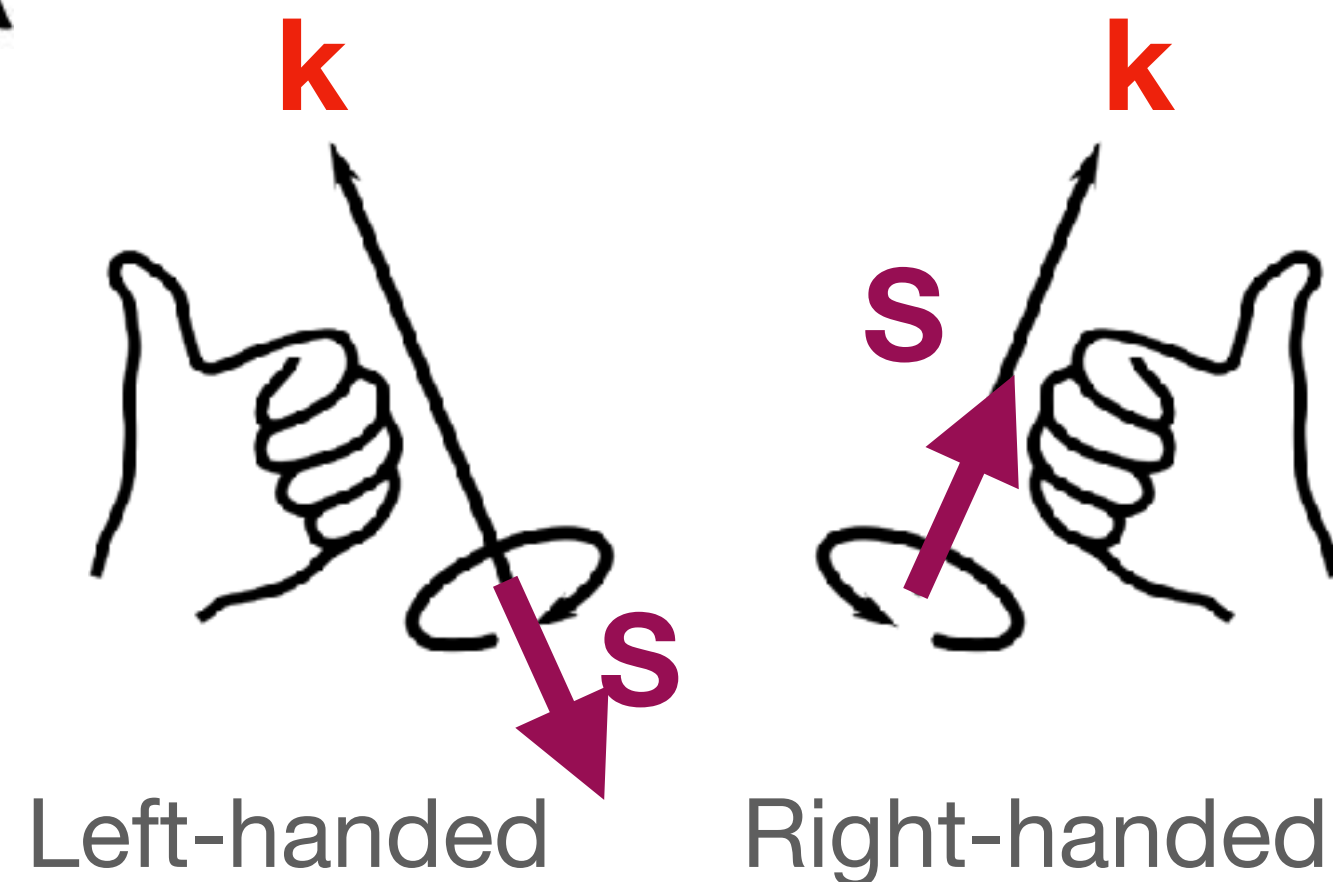
- To show that A_{\pm} represents the helicity states, rotate the spatial coordinates around the z axis in the right-handed system by an angle φ .
- The helicity states, $\lambda = \pm 1$, transform as

$$A_{\lambda} \rightarrow A'_{\lambda} = e^{i\lambda\varphi} A_{\lambda}$$

Helicity

A_+ : Right-handed state

A_- : Left-handed state



Correction to the EM wave equation

In the helicity basis

Note: \mathbf{A} is a vector and θ is a pseudoscalar.

$$\ddot{\mathbf{A}} - \nabla^2 \mathbf{A} + \alpha \left[-\dot{\theta}(\nabla \times \mathbf{A}) + (\nabla \theta) \times \dot{\mathbf{A}} \right] = 0$$

Correction to the EM wave equation!

- If θ has a time-dependent vacuum expectation value, $\theta(t, \mathbf{x}) \rightarrow \bar{\theta}(t)$, we find in Fourier space

$$\ddot{\mathbf{A}}_{\mathbf{k}} + k^2 \mathbf{A}_{\mathbf{k}} - i\alpha \dot{\bar{\theta}}(\mathbf{k} \times \mathbf{A}) = 0$$

$$\rightarrow \ddot{A}_{\pm} + \left(k^2 \boxed{\mp} k\alpha \dot{\bar{\theta}} \right) A_{\pm} = 0$$

Parity violation

The equation of motion depends on handedness!

Parity Violation in EM Waves due to Dark Matter and Dark Energy

Scalar field DM/DE coupled to the CS term

DM = Dark Matter; DE = Dark Energy

$$I = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\partial\chi)^2 - V(\chi) - \frac{1}{4} F^2 - \frac{\alpha}{4f} \chi F \tilde{F} \right]$$

- χ is a neutral pseudoscalar field (spin 0).
- Why consider χ as a good DM/DE candidate?

We wrote

$$\theta = \frac{\chi}{f}$$

- *Why not?* We have an example in the Standard Model: a neutral pion.
- We expect $\alpha \simeq \alpha_{\text{EM}} \simeq 10^{-2}$ and $f < M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV.
- χ can be composed of fermions like a pion, or a fundamental pseudoscalar like an “axion” field.

Distinction between DE and DM

How small is its mass? Example of $V(\chi) = m^2\chi^2/2$

- The useful criterion is the equation of state parameter, w .

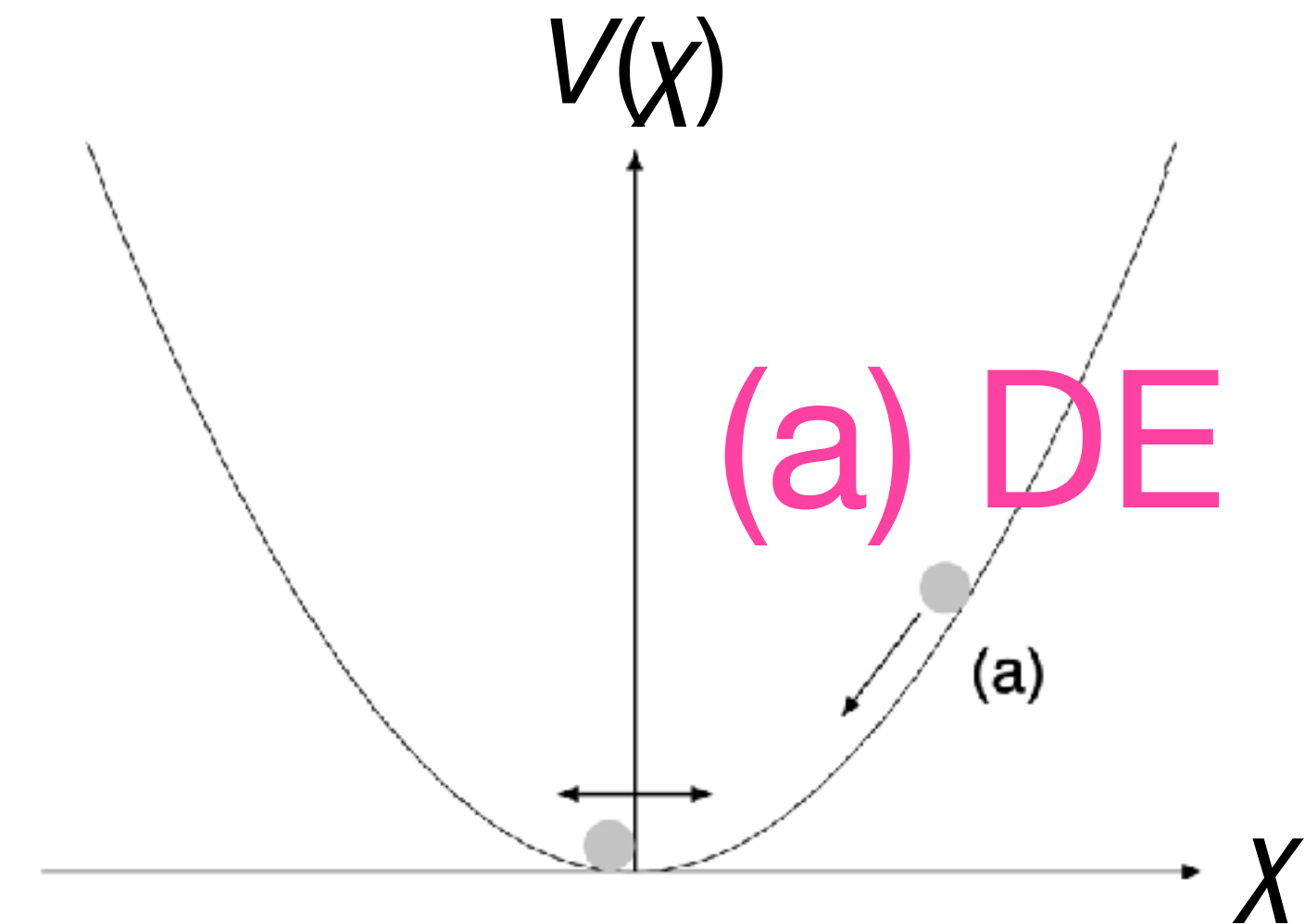
$$w = \frac{P}{\rho} = \frac{\langle \dot{\chi}^2 \rangle - m^2 \langle \chi^2 \rangle}{\langle \dot{\chi}^2 \rangle + m^2 \langle \chi^2 \rangle}$$

- $w \simeq -1$: Dark Energy (DE)

- $m \lesssim H_0 \simeq 10^{-33}$ eV

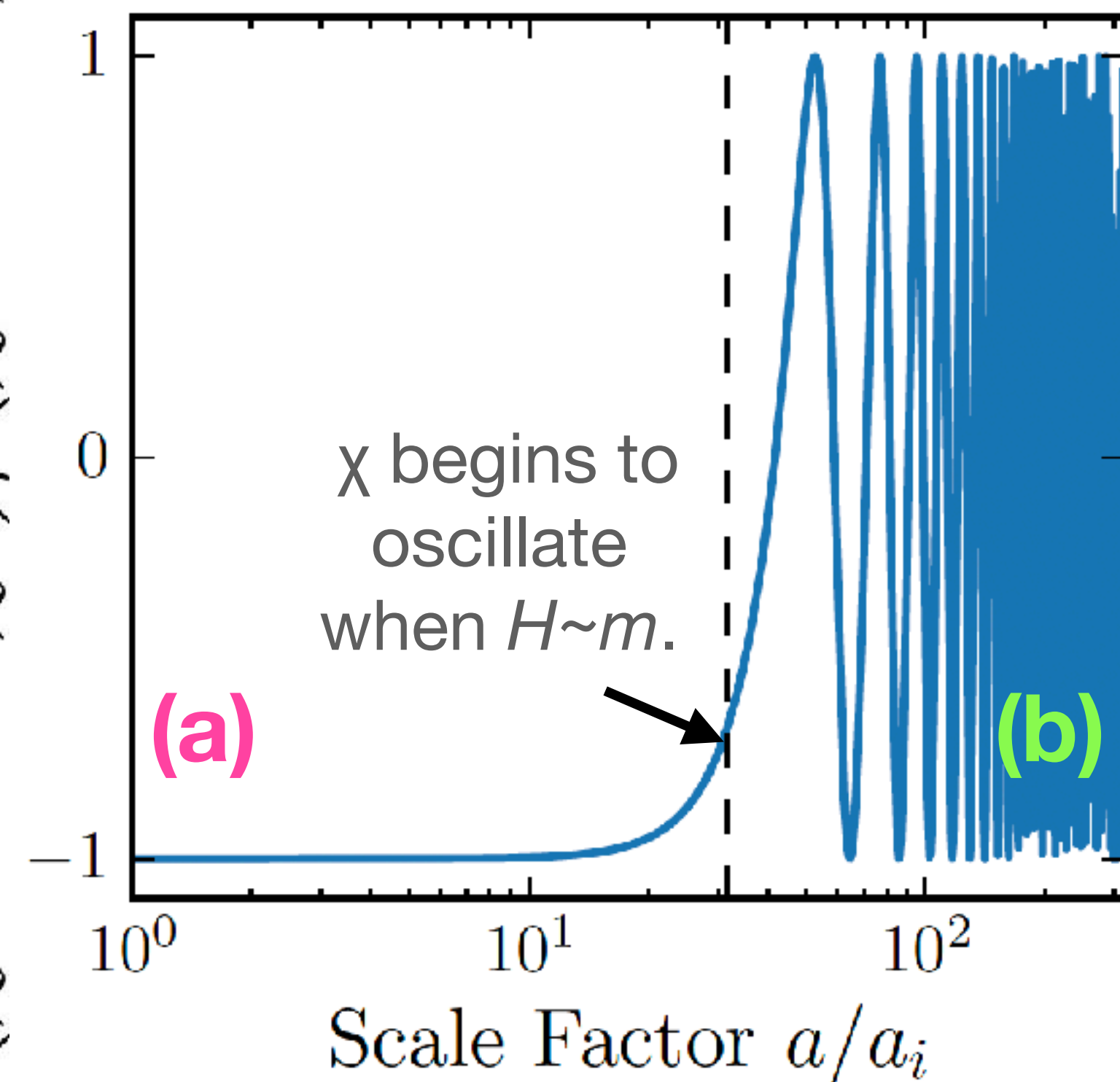
- $w \simeq 0$: Dark Matter (DM)

- $m \gtrsim H_0$



(a) DE

$$(\dot{\chi}^2 - m^2\chi^2) / (\dot{\chi}^2 + m^2\chi^2)$$



Phase velocity of circular polarization states

Expanding space, $c=1$

- We write

where $' = \frac{\partial}{\partial \tau} = a \frac{\partial}{\partial t}$

τ : conformal time

$$A''_{\pm} + \omega_{\pm}^2 A_{\pm} = 0, \quad \omega_{\pm}^2 = k^2 \mp \frac{k\alpha\chi'}{f}$$

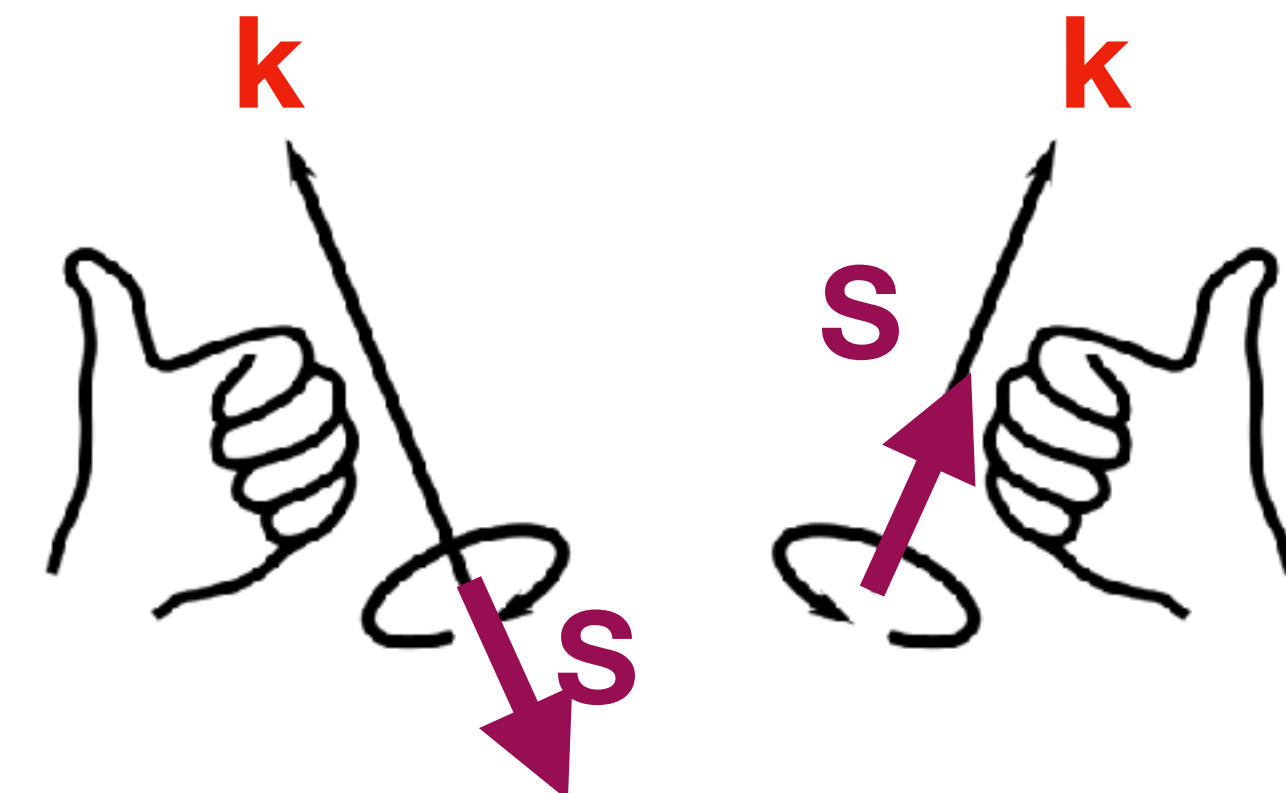
- We work in the limit of $k^2 \gg k\alpha\chi'/f$. This approximation is accurate for the photons we observe today. (However, ω_{\pm}^2 can become negative during inflation!)

- The phase velocity of circular polarization states, ω_{\pm}/k , is

$$\frac{\omega_{\pm}}{k} \simeq 1 \mp \frac{\alpha\chi'}{2kf}$$

+: Right-handed state

-: Left-handed state



Left-handed

Right-handed

Plane-wave (WKB) Solution

Expanding space, $c=1$

$$A''_{\pm} + \omega_{\pm}^2 A_{\pm} = 0, \quad \omega_{\pm} \simeq k \mp \frac{\alpha \chi'}{2f}$$

- For $|\omega'_{\pm}| \ll \omega_{\pm}^2$, which is satisfied here, an accurate solution is given by

$$A_{\pm} \simeq C_{\pm} \frac{\exp\left(-i \int d\tau \omega_{\pm} + i\delta_{\pm}\right)}{\sqrt{2\omega_{\pm}} \simeq \sqrt{2k}}$$

We can replace ω_{\pm} in amplitude (but not in phase) with k .

where C_{\pm} is the initial amplitude and δ_{\pm} is the initial phase.

Carroll, Field, Jackiw (1990); Carroll, Field (1991); Harari, Sikivie (1992)

Cosmic Birefringence

Rotation of the plane of linear polarization

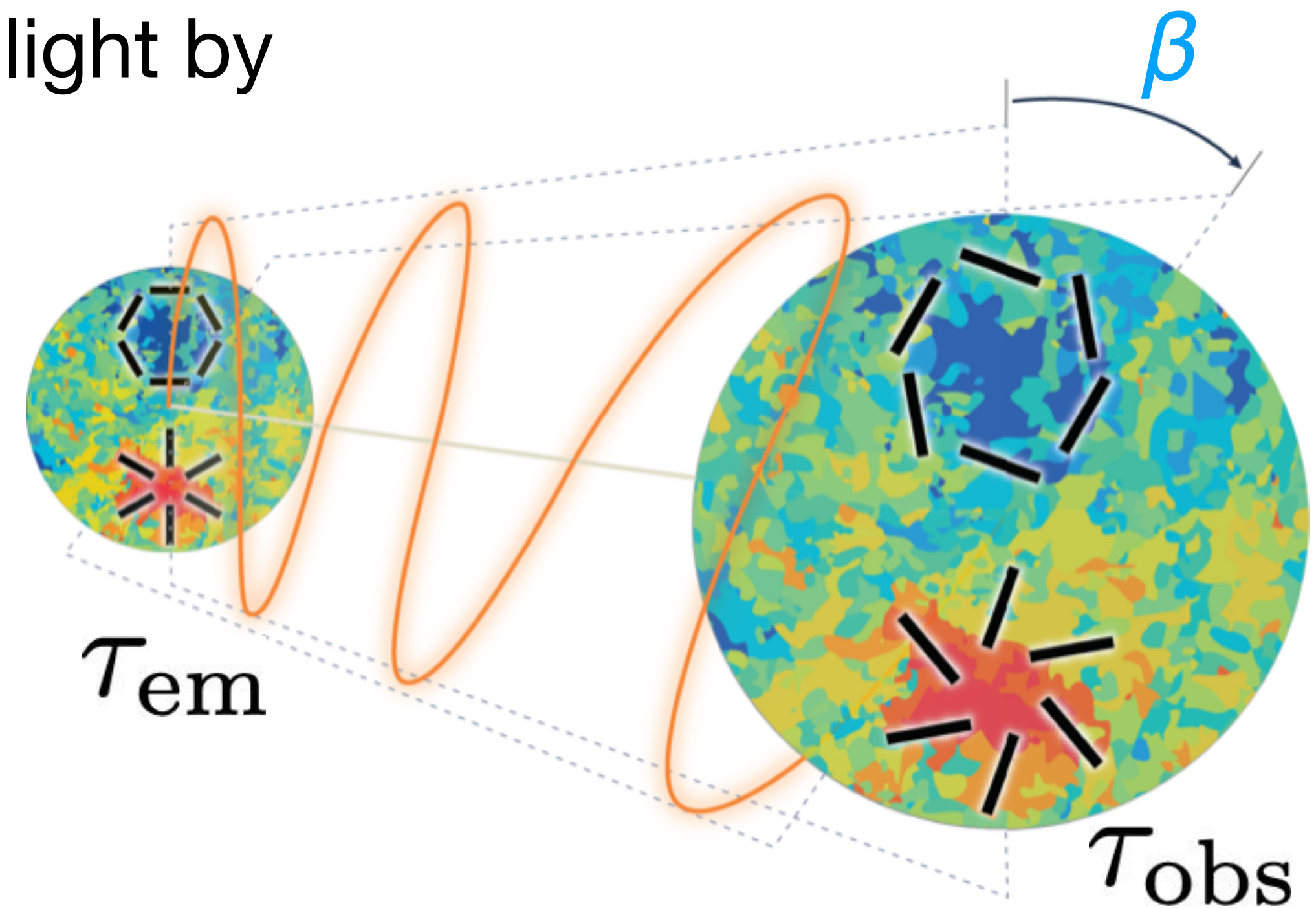
$$A_{\pm} \simeq C_{\pm} \frac{\exp\left(-i \int d\tau \omega_{\pm} + i\delta_{\pm}\right)}{\sqrt{2\omega_{\pm}} \simeq \sqrt{2k}}$$

with

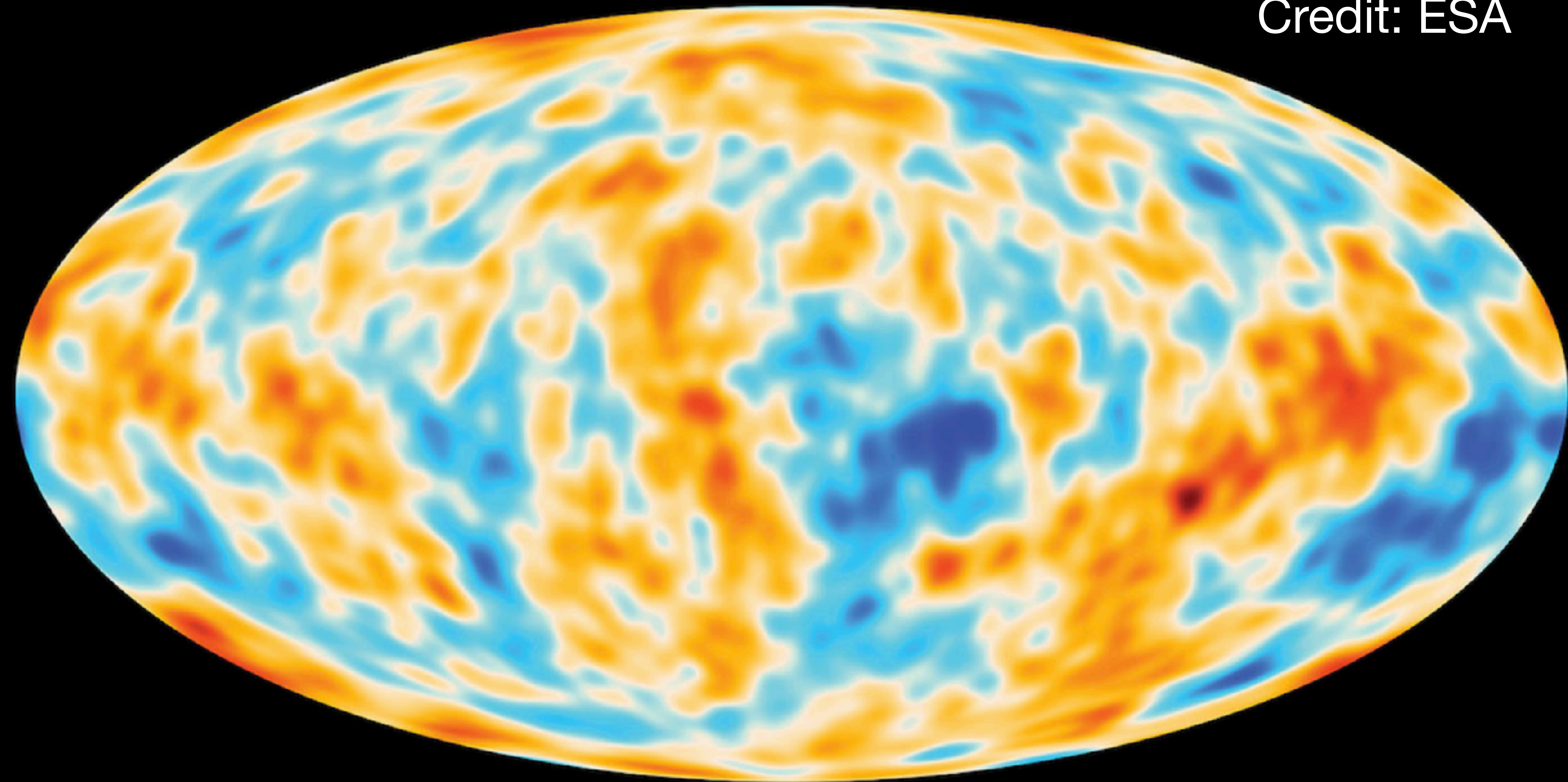
$$\frac{\omega_{\pm}}{k} \simeq 1 \mp \frac{\alpha\chi'}{2kf}$$

- This **rotates** the plane of linear polarization of light by

$$\begin{aligned} \beta &= - \int_{\tau_{\text{em}}}^{\tau_{\text{obs}}} d\tau (\omega_{+} - \omega_{-}) \\ &= \frac{\alpha}{2f} [\chi(\tau_{\text{obs}}) - \chi(\tau_{\text{em}})] \end{aligned}$$

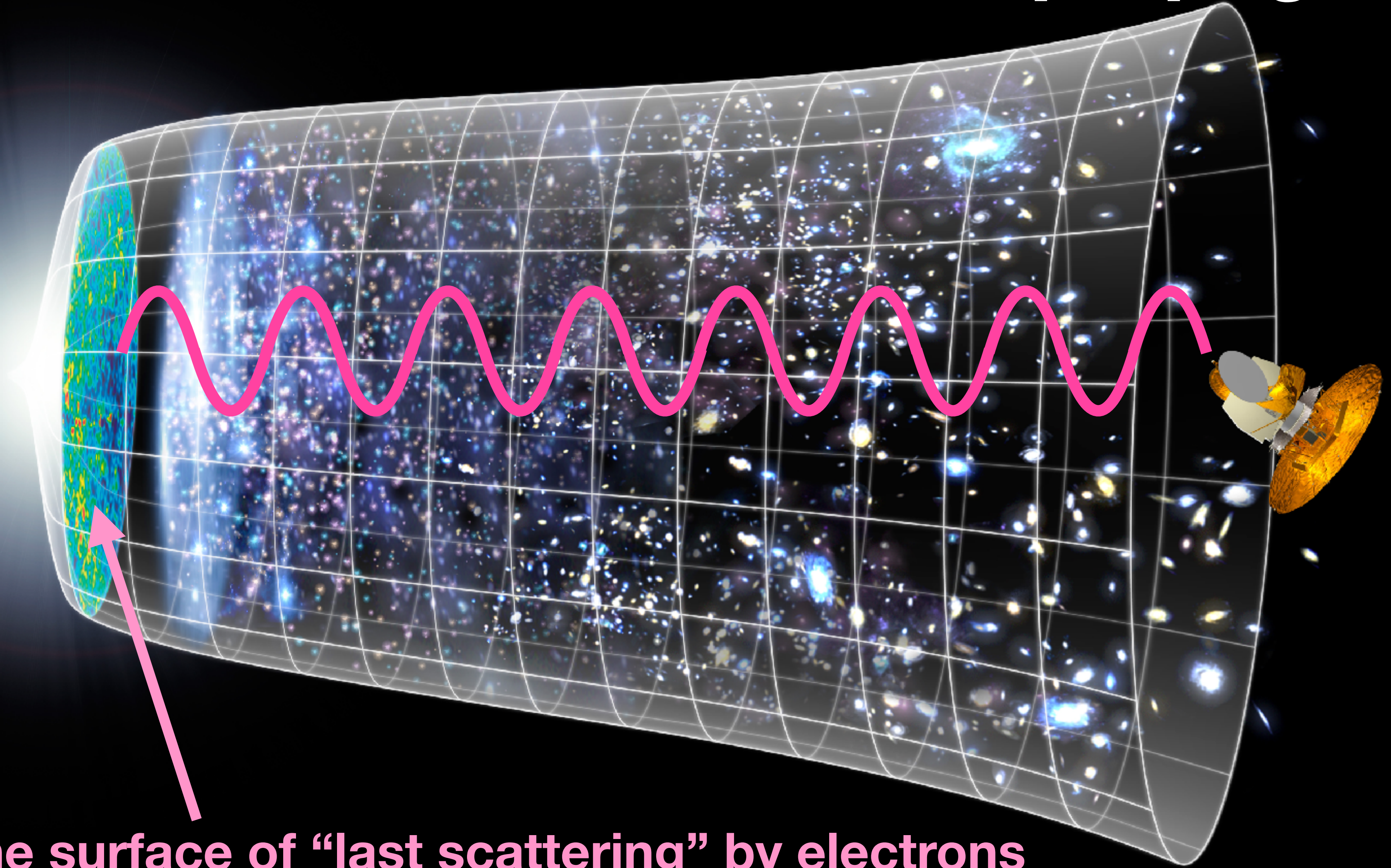


Credit: ESA



Emitted 13.8 billions years ago

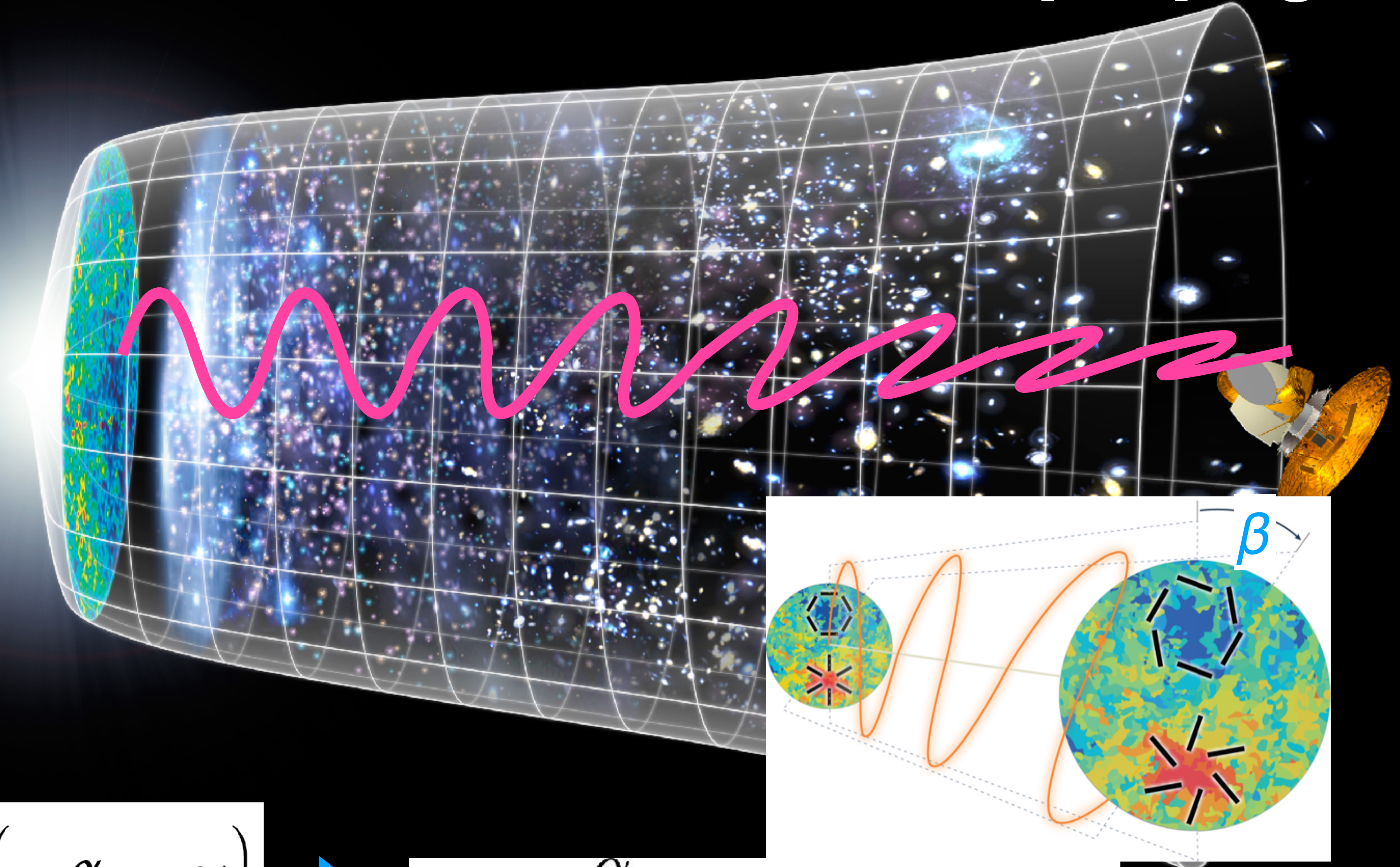
How does the EM wave of the CMB propagates?



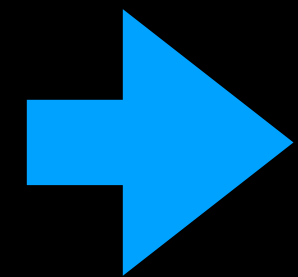
The surface of “last scattering” by electrons
(Scattering generates *polarization*!)

Credit: WMAP Science Team

How does the EM wave of the CMB propagagate?

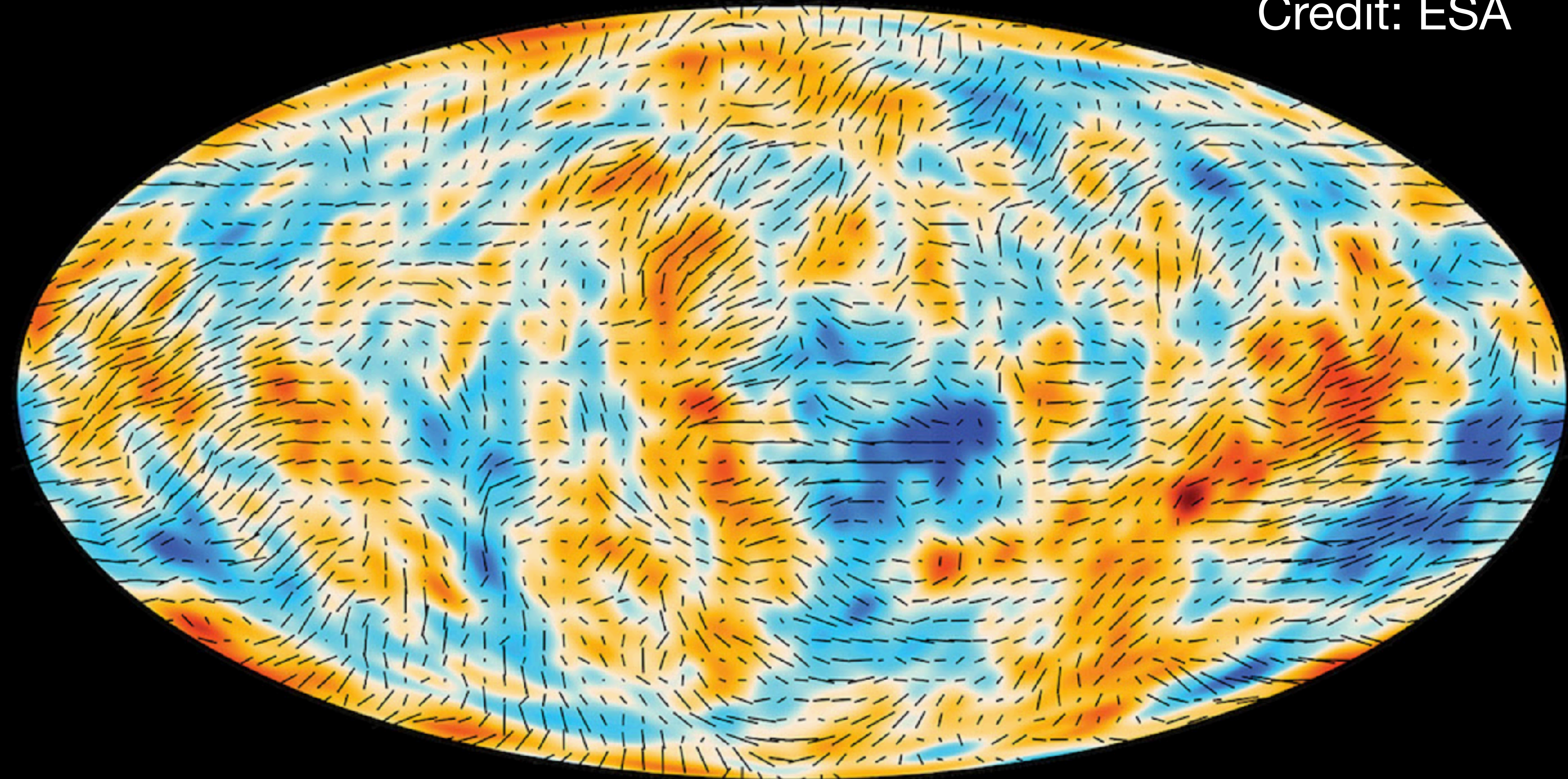


$$I_{\text{CS}} = \int d^4x \sqrt{-g} \left(-\frac{\alpha}{4f} \chi F \tilde{F} \right)$$



$$\beta = +\frac{\alpha}{2f} [\chi(\tau_{\text{obs}}) - \chi(\tau_{\text{em}})]$$

Credit: ESA



Temperature (smoothed) + Polarisation

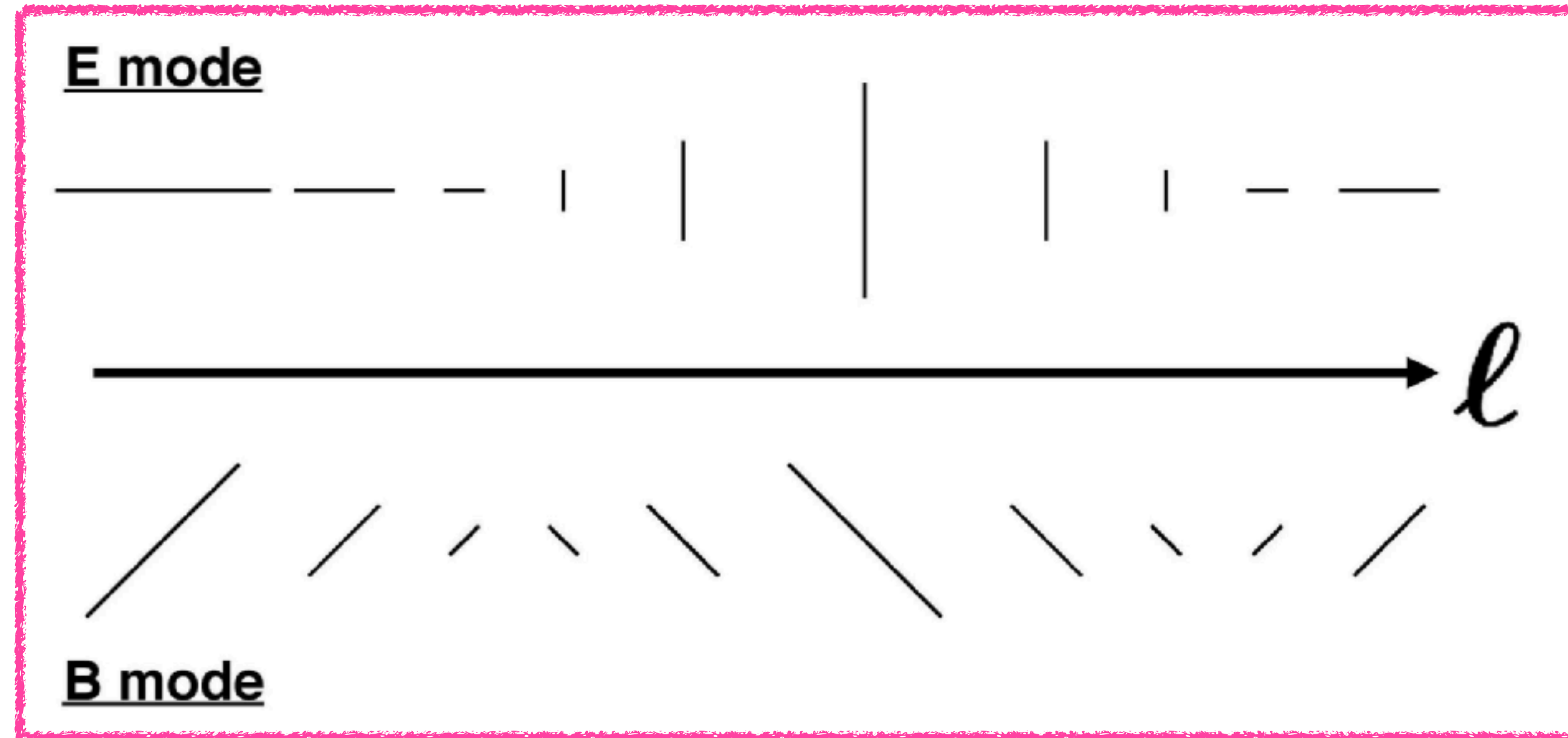
If the plane of linear polarization of the CMB is rotated uniformly by β , it is the sign of parity violation!

Pseudoscalar: EB correlation

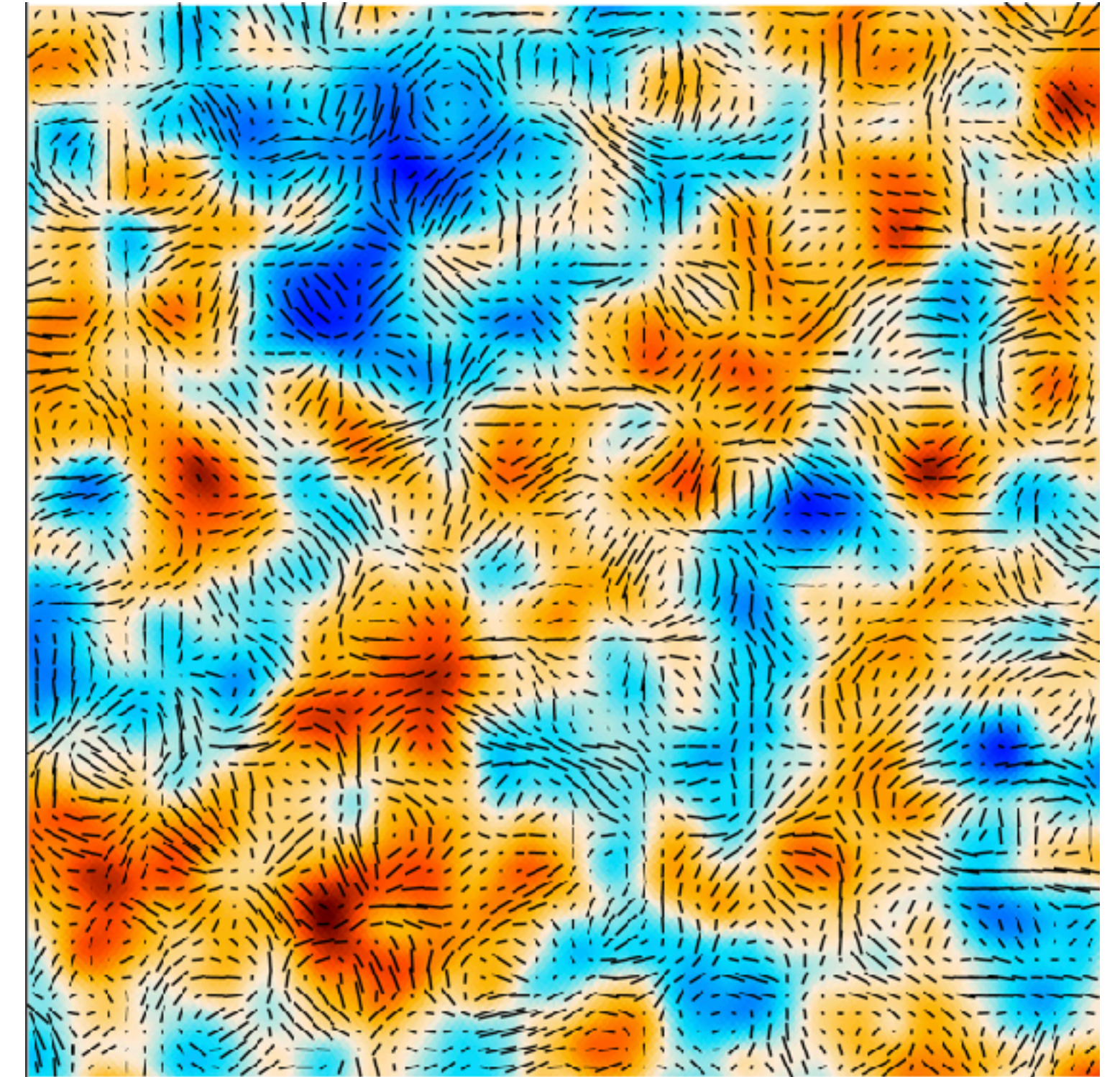
- The observed pattern of the CMB polarization can be decomposed into eigenstates of parity, called “E modes” and “B modes”.
 - Note that these are jargon in the CMB community and have nothing to do with electric and magnetic fields!
- E and B modes are transformed differently under the parity transformation. Therefore, the product of the two, **the “EB correlation”, is a pseudoscalar.**
- **The full-sky average of the EB correlation must vanish (to within the measurement uncertainty), if there is no parity violation!**

Parity eigenstates: E and B modes

Concept defined in Fourier space



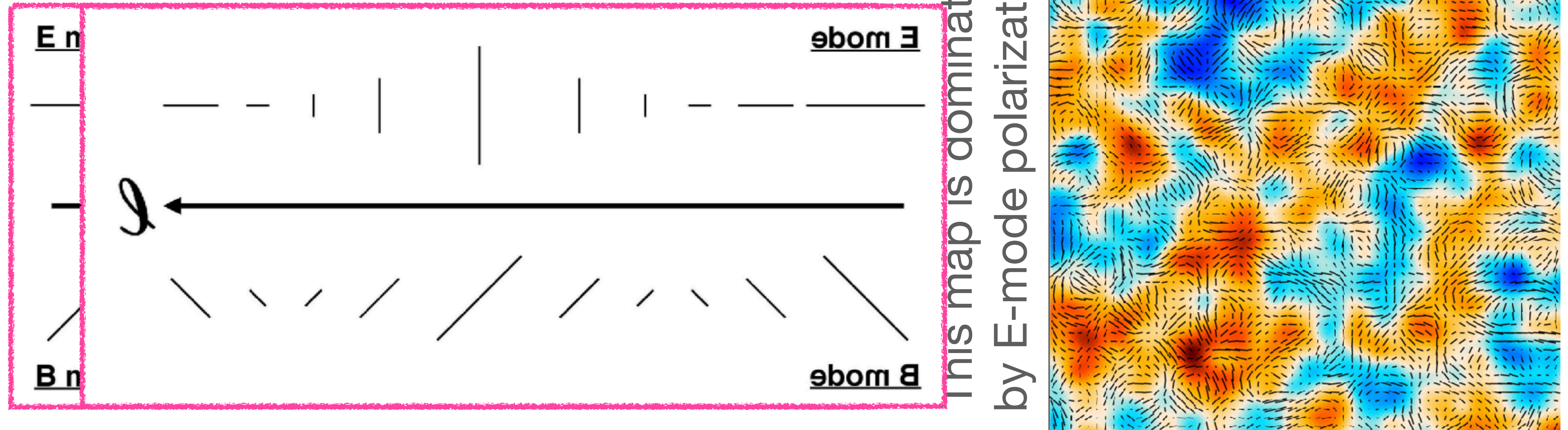
This map is dominated
by E-mode polarization



- **E-mode** : Polarisation directions are **parallel or perpendicular** to the wavenumber direction
- **B-mode** : Polarisation directions are **45 degrees tilted** w.r.t the wavenumber direction

Parity eigenstates: E and B modes

Concept defined in Fourier space

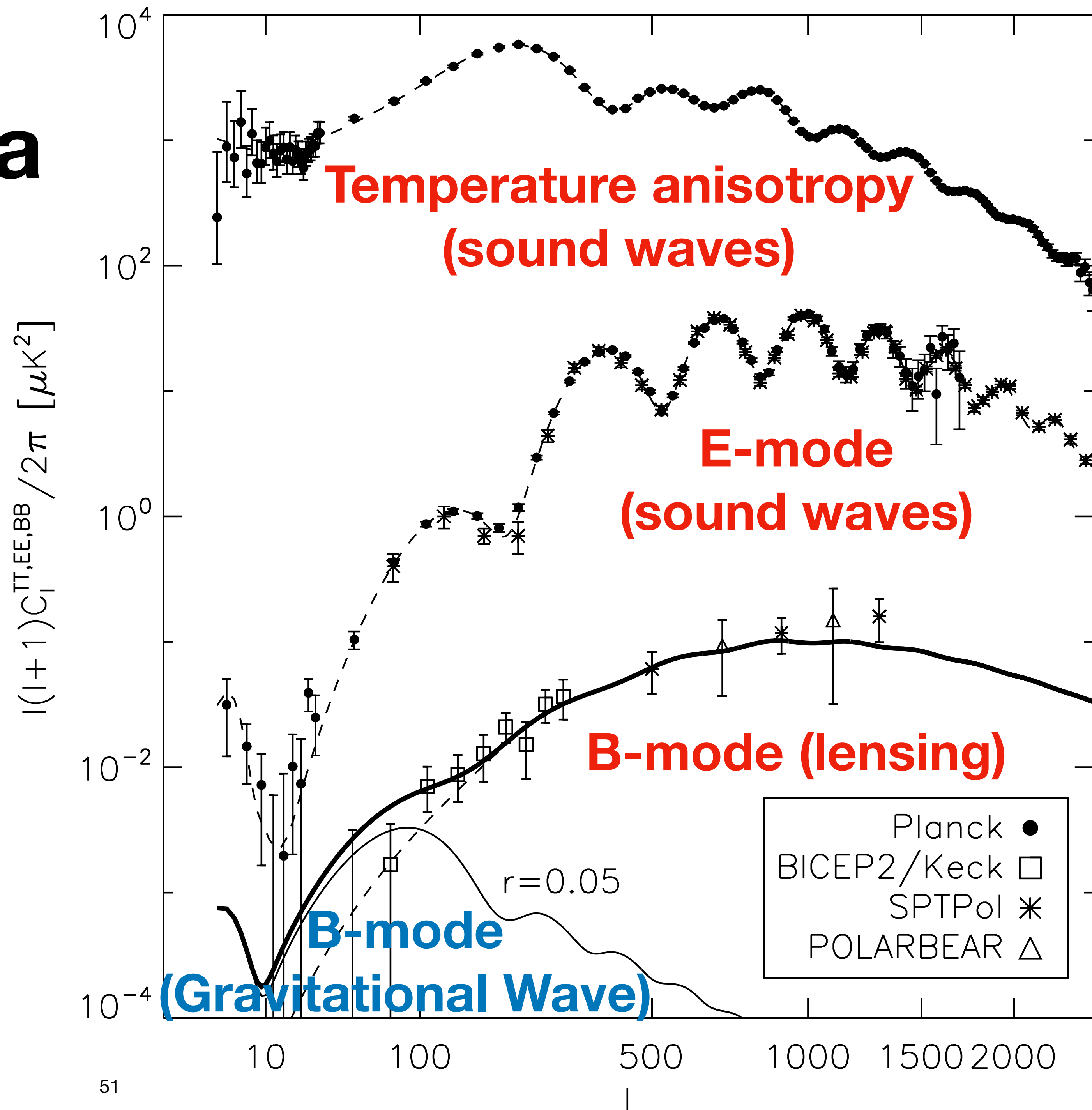


- **E-mode** : Polarisation directions are **parallel or perpendicular** to the wavenumber direction
- **B-mode** : Polarisation directions are **45 degrees tilted** w.r.t the wavenumber direction

CMB Power Spectra

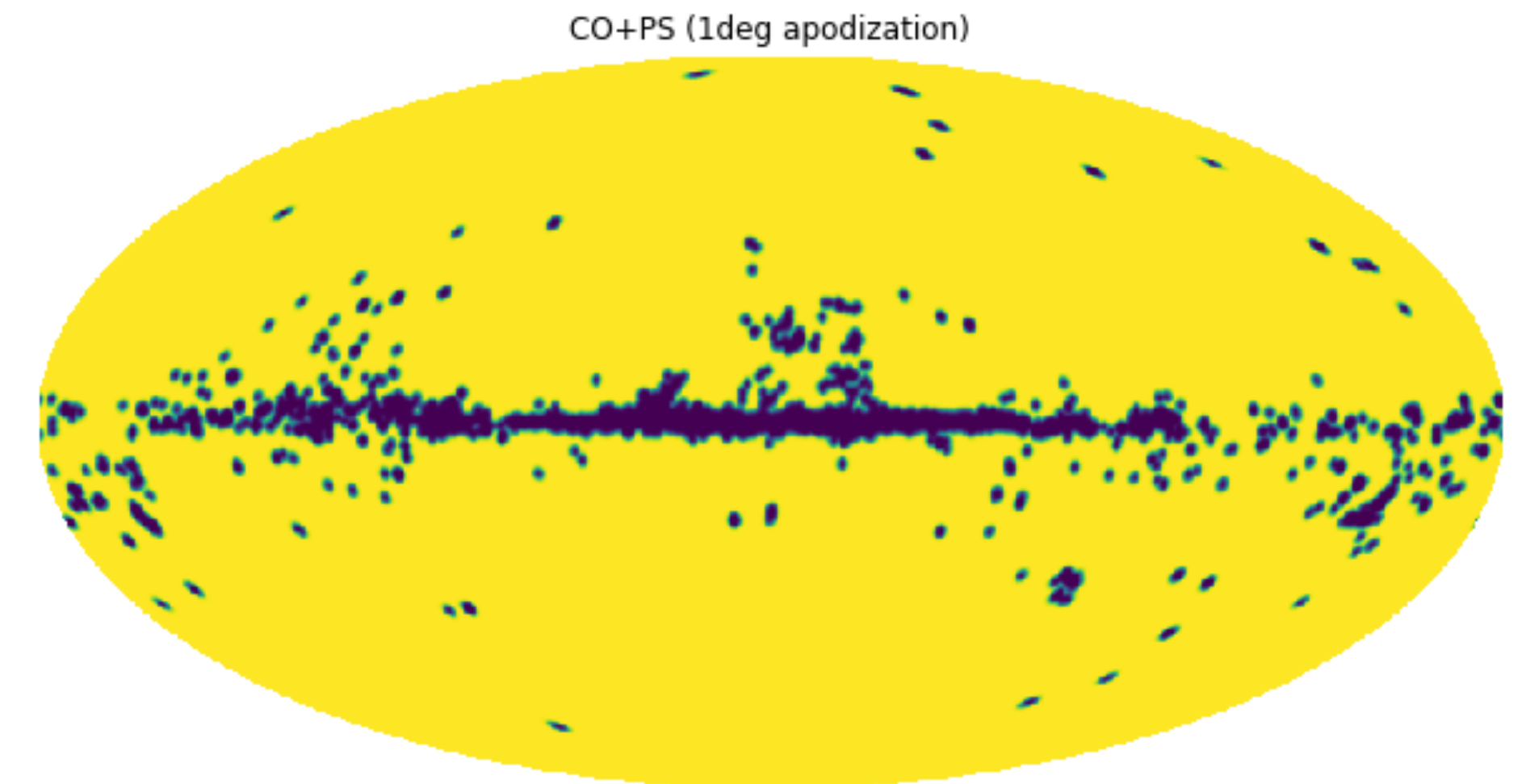
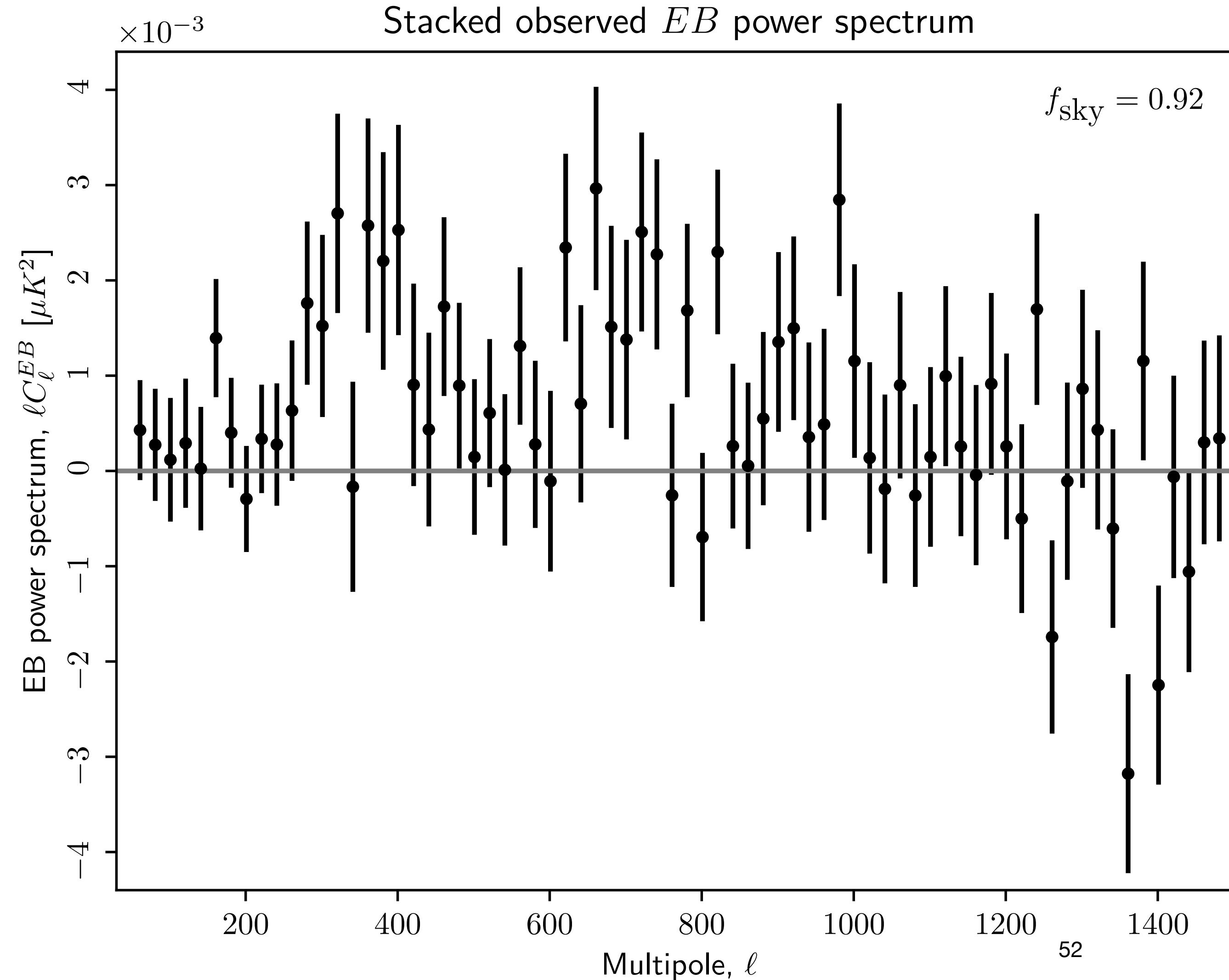
Progress over 30 years

- This is the typical figure seen in talks and lectures on the CMB.
- The temperature and the E- and B-mode polarization power spectra are well measured.
- **Parity violation appears in the TB and EB power spectra, not shown here.**



This is the EB power spectrum (WMAP+Planck)

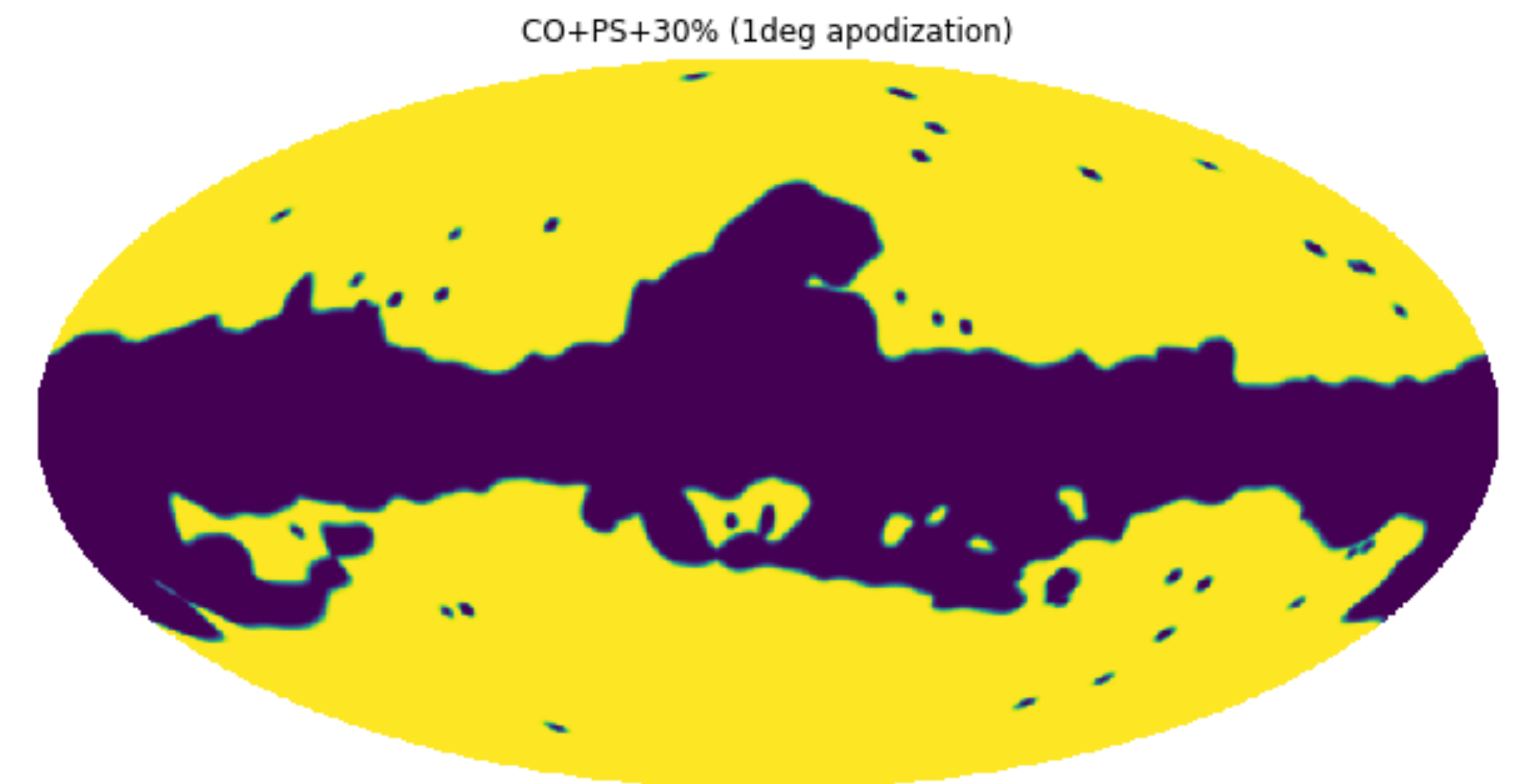
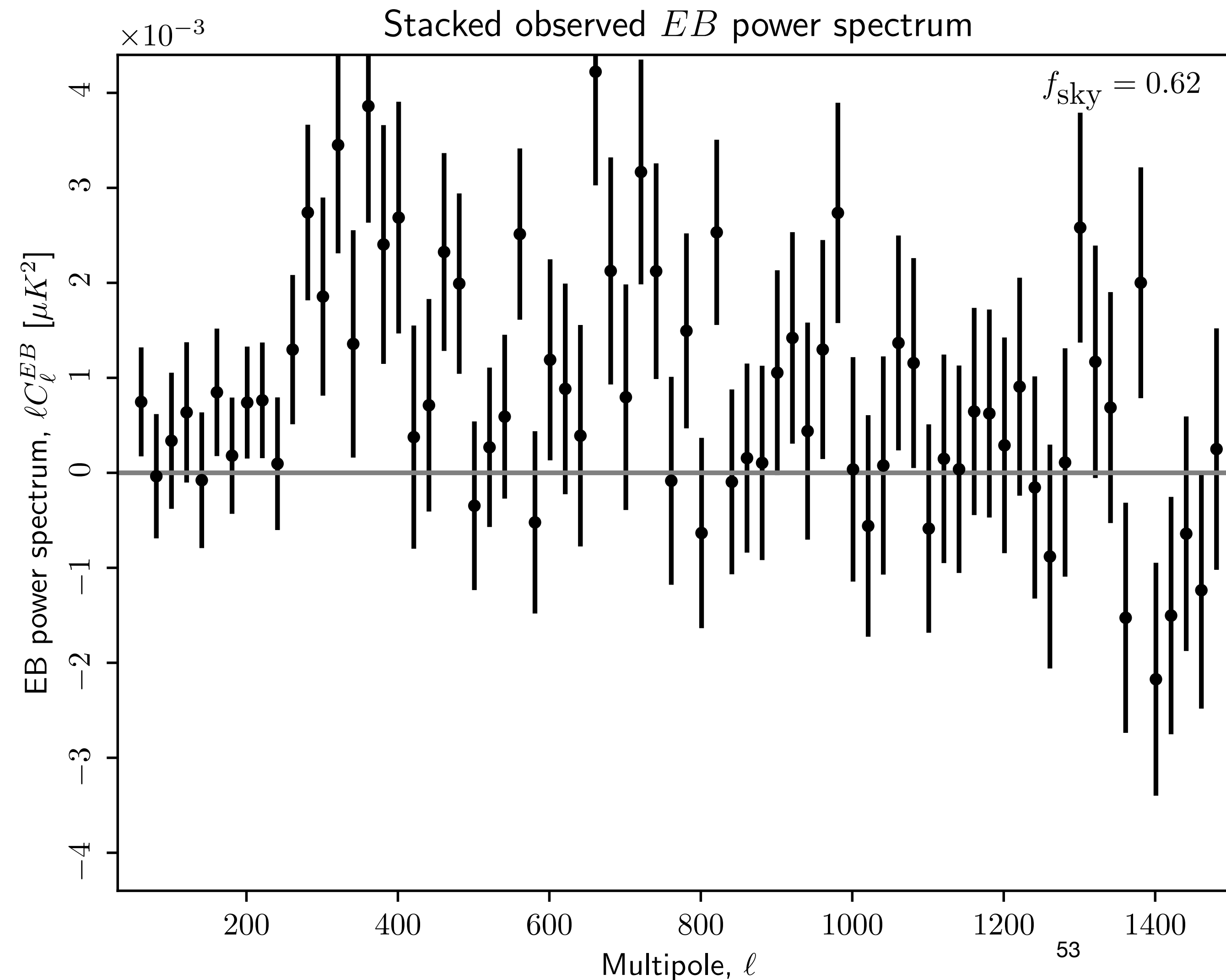
Nearly full-sky data (92% of the sky)



- $\chi^2 = 125.5$ for DOF=72
- Unambiguous signal of something!

This is the EB power spectrum (WMAP+Planck)

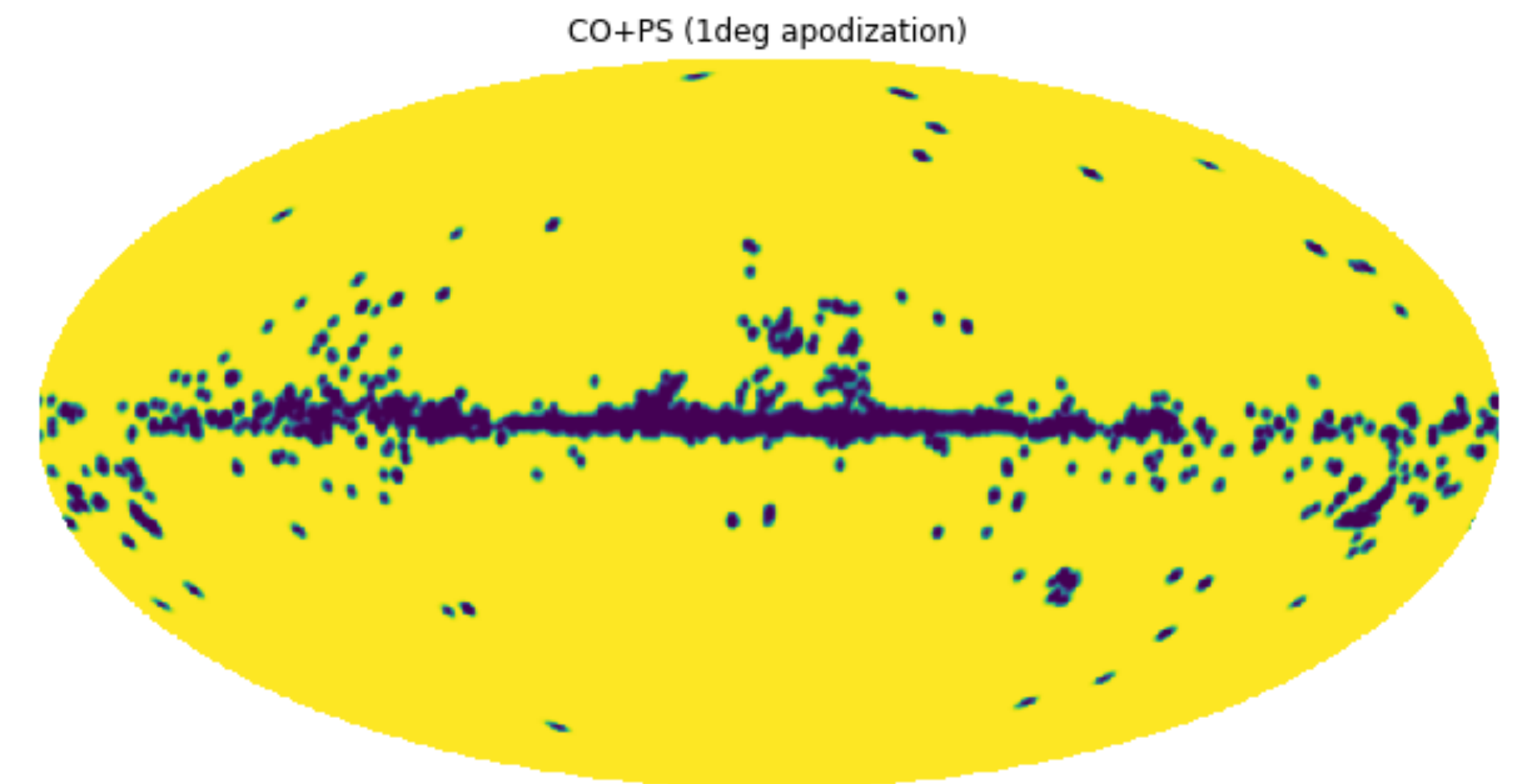
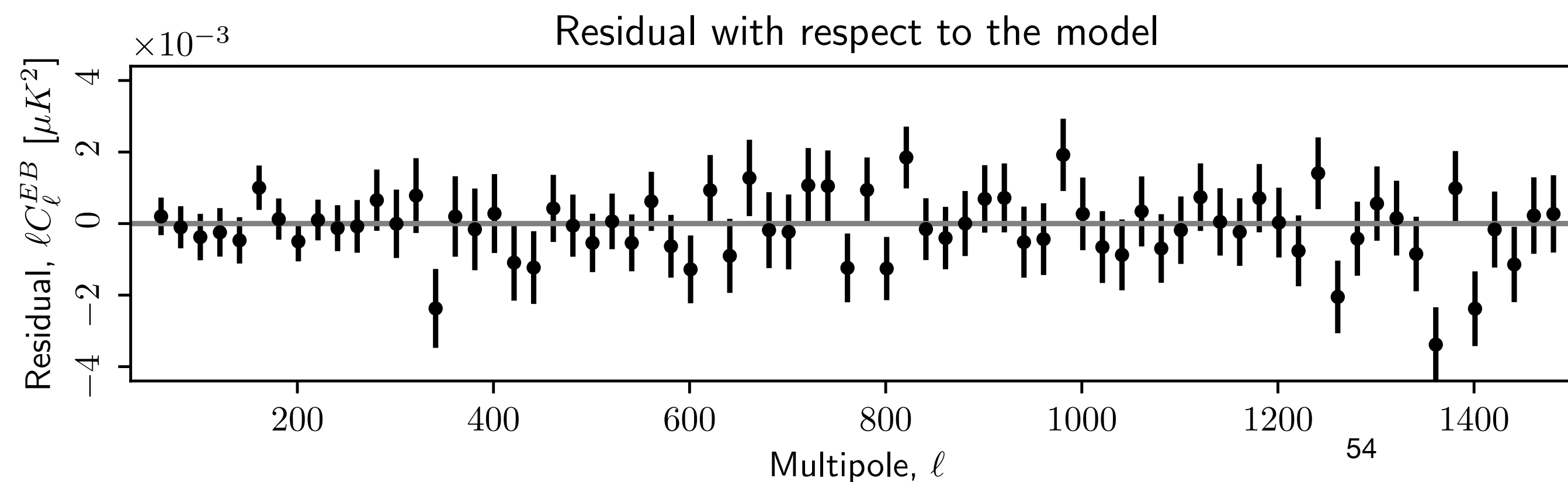
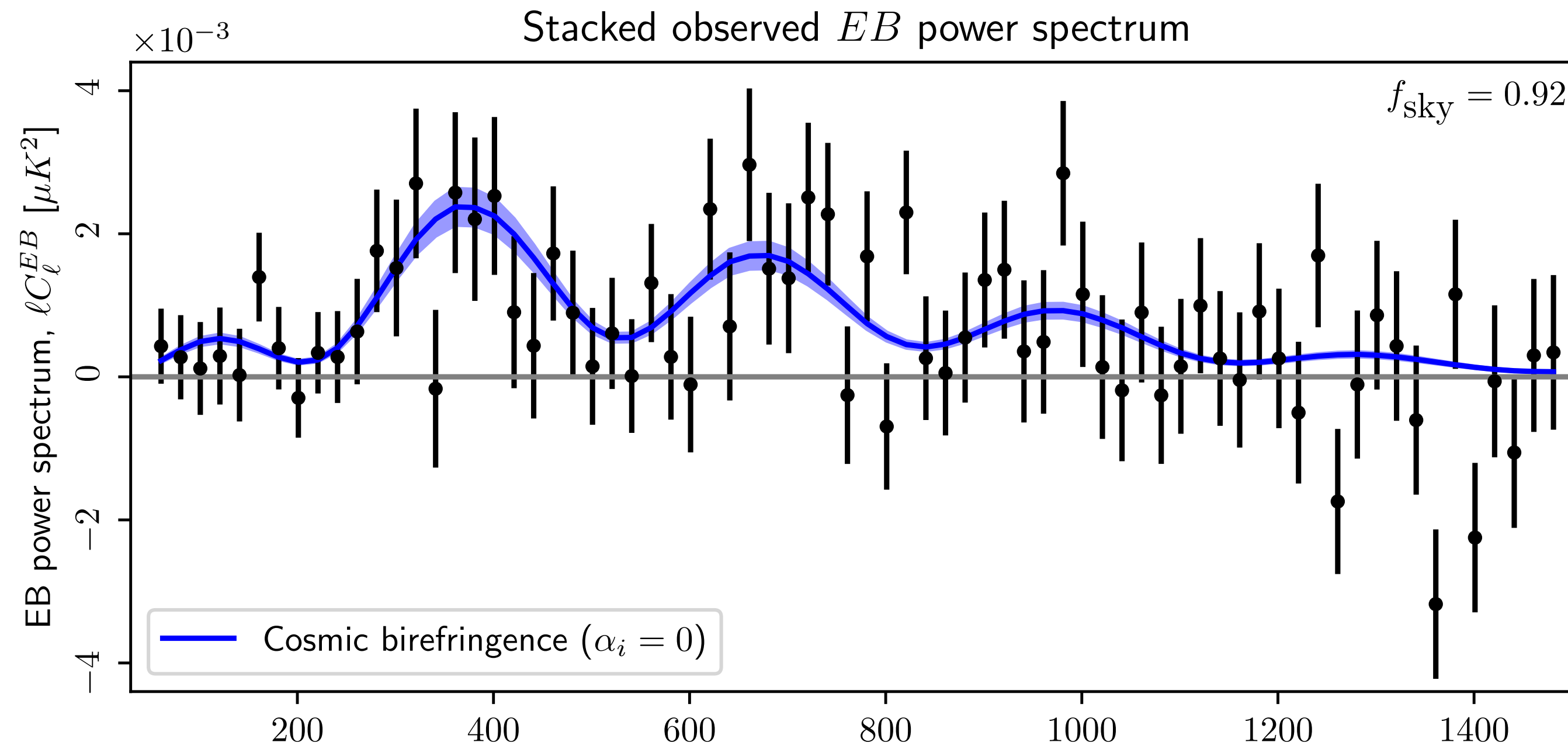
Galactic plane removed (62% of the sky)



- $\chi^2 = 138.4$ for DOF=72
- The signal exists regardless of the Galactic mask. This rules out the Galactic foreground.

Cosmic Birefringence fits well(?)

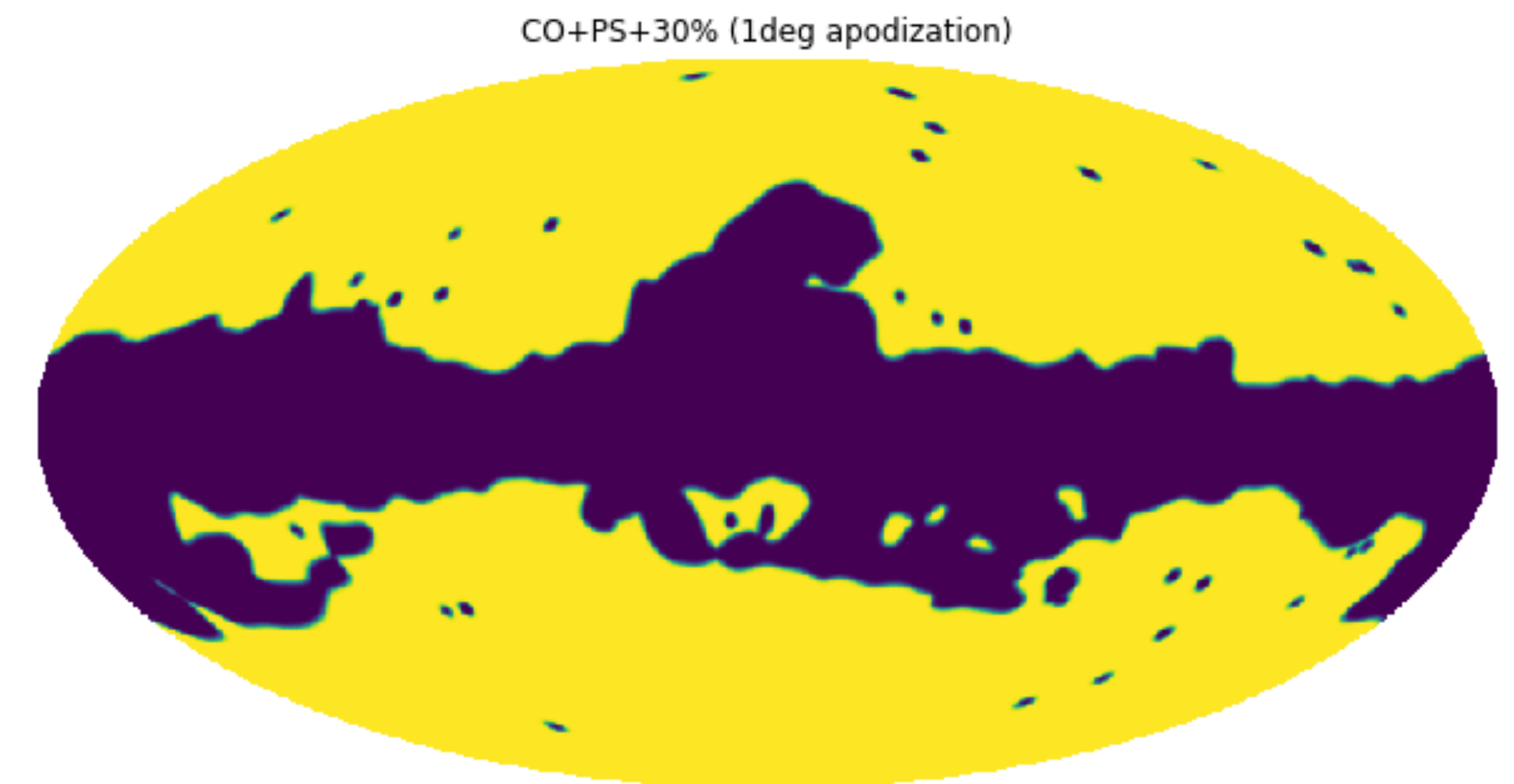
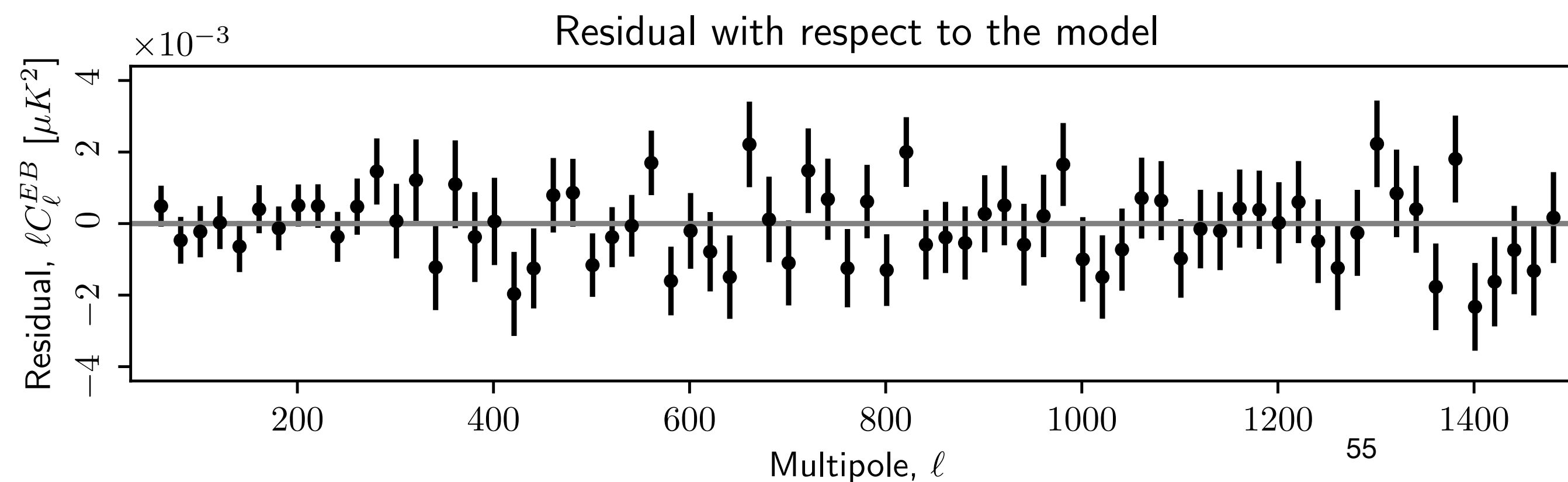
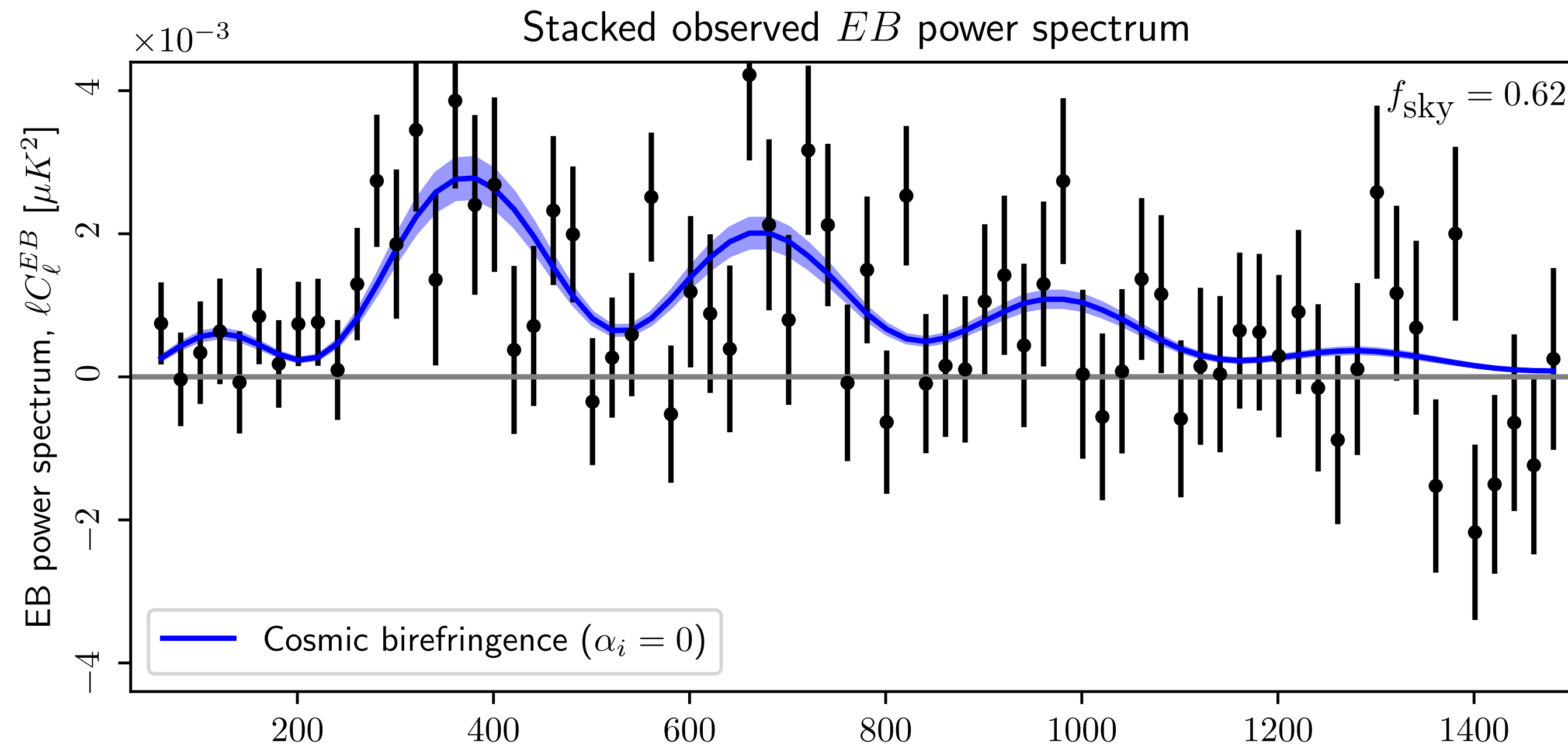
Nearly full-sky data (92% of the sky)



- $\beta = 0.288 \pm 0.032$ deg
- $\chi^2 = 66.1$
- Good fit! 9σ detection?

Cosmic Birefringence fits well(?)

Galactic plane removed (62% of the sky)



- $\beta = 0.330 \pm 0.035$ deg
- $\chi^2 = 64.5$
- Signal is robust with respect to the Galactic mask.

The Biggest Problem: Miscalibration of detectors

Impact of miscalibration of polarization angles

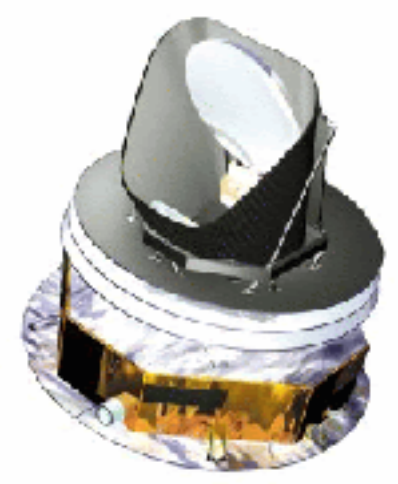
Cosmic or Instrumental?



- Is the plane of linear polarization rotated by the genuine cosmic birefringence effect, or simply because the polarization-sensitive directions of the detectors are rotated with respect to the sky coordinates (and we did not know it)?

- If the detectors are rotated by α , it seems that we can measure only the **sum $\alpha + \beta$** .

The Key Idea: The polarized Galactic foreground emission as a calibrator



ESA's Planck

Polarized dust emission within our Milky Way!

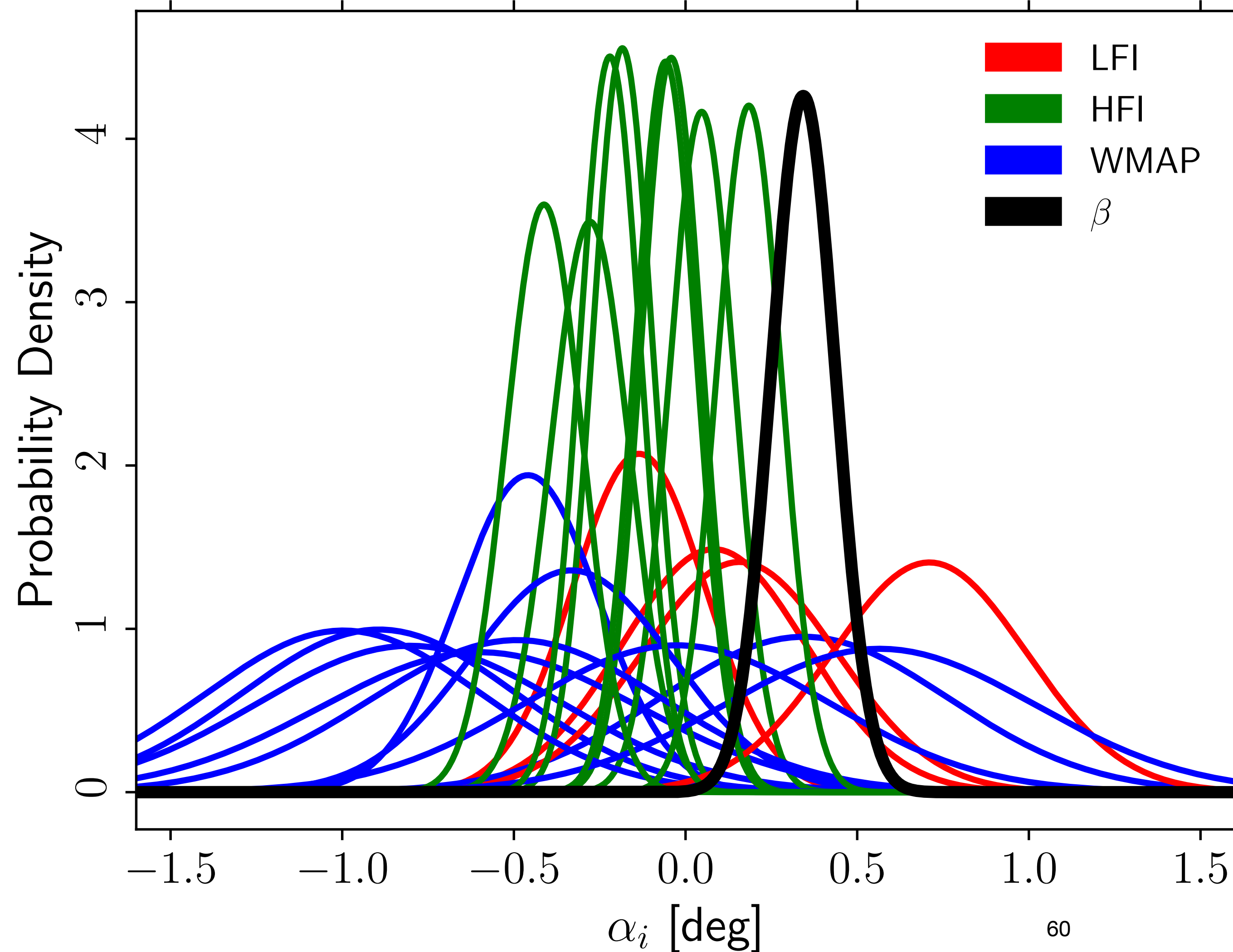
$$\beta = +\frac{\alpha}{2f} [\chi(\tau_{\text{obs}}) - \chi(\tau_{\text{em}})]$$

Emitted “right there” - it would not be affected by the cosmic birefringence.

Directions of the magnetic field inferred from polarization of the thermal dust emission in the Milky Way

Miscalibration angles (WMAP and Planck)

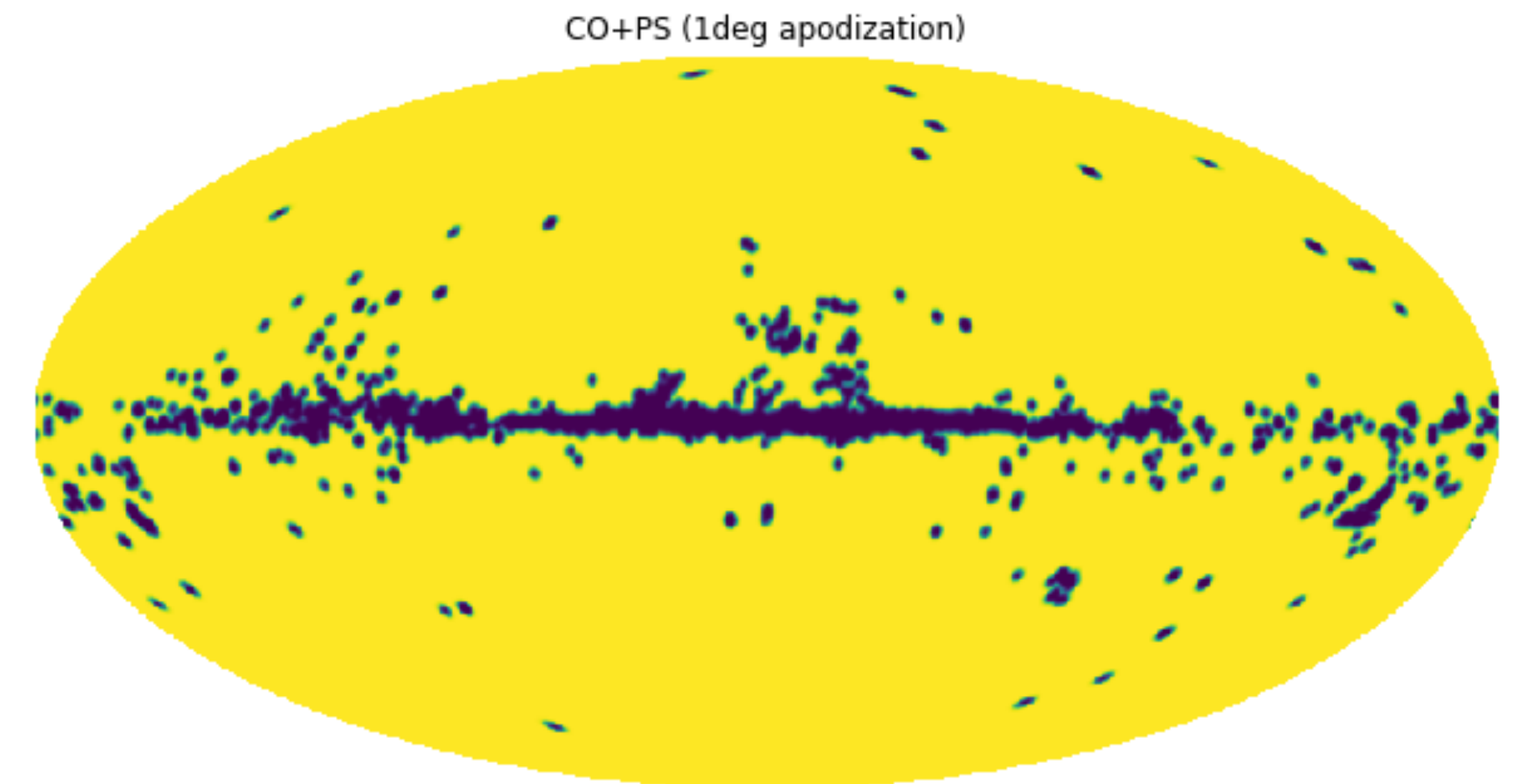
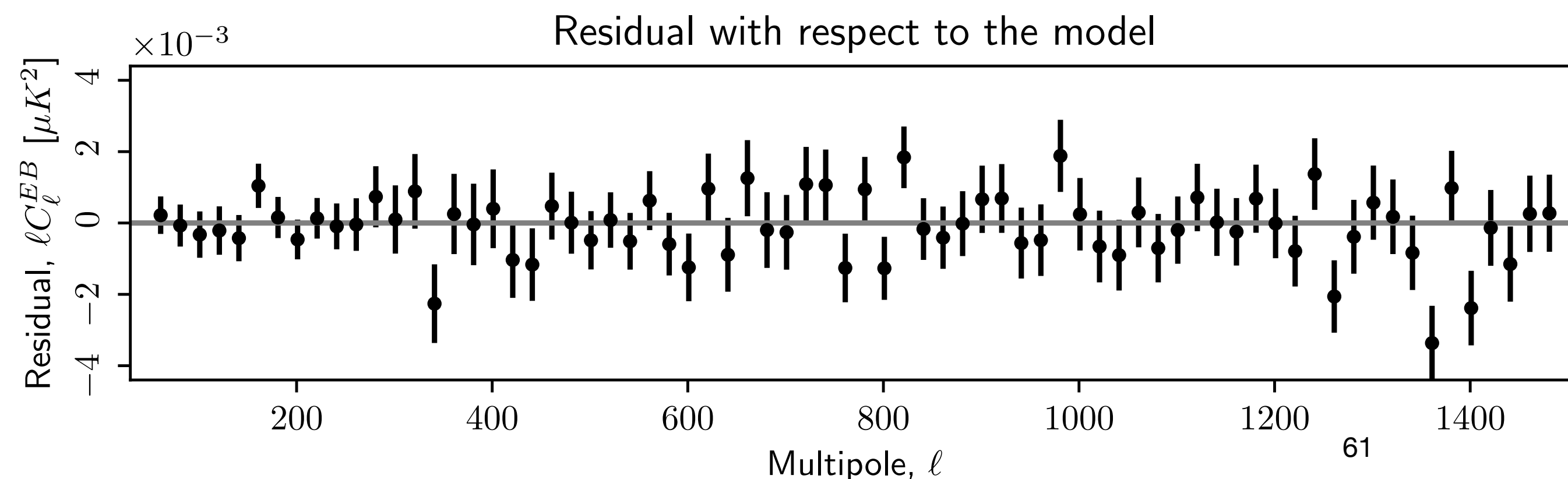
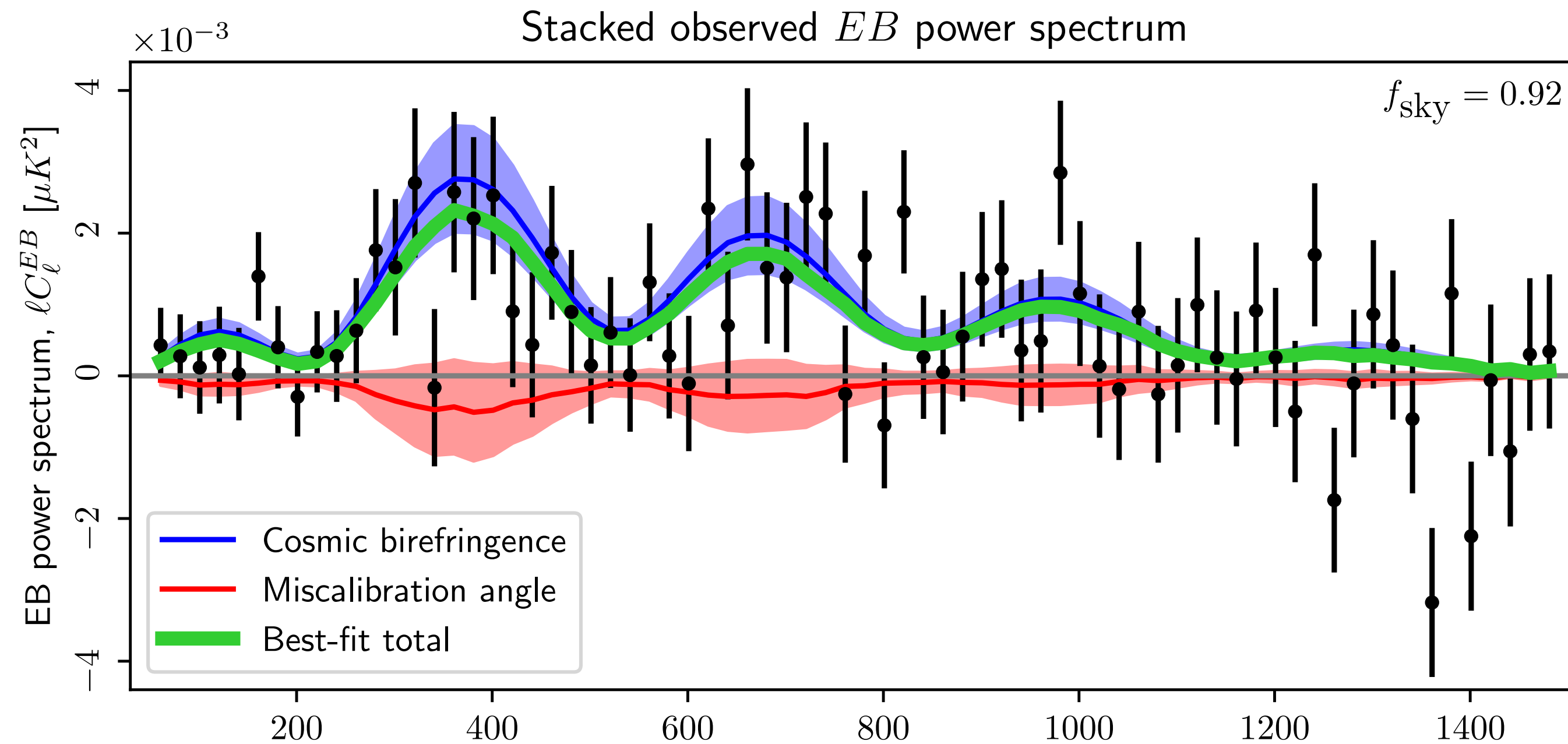
Nearly full-sky data (92% of the sky)



- The angles are all over the place, and are well within the quoted calibration uncertainty of instruments.
- 1.5 deg for WMAP
- 1 deg for Planck
- They cancel!
- The power of adding independent datasets.

Cosmic Birefringence fits well (WMAP+Planck)

Nearly full-sky data (92% of the sky)



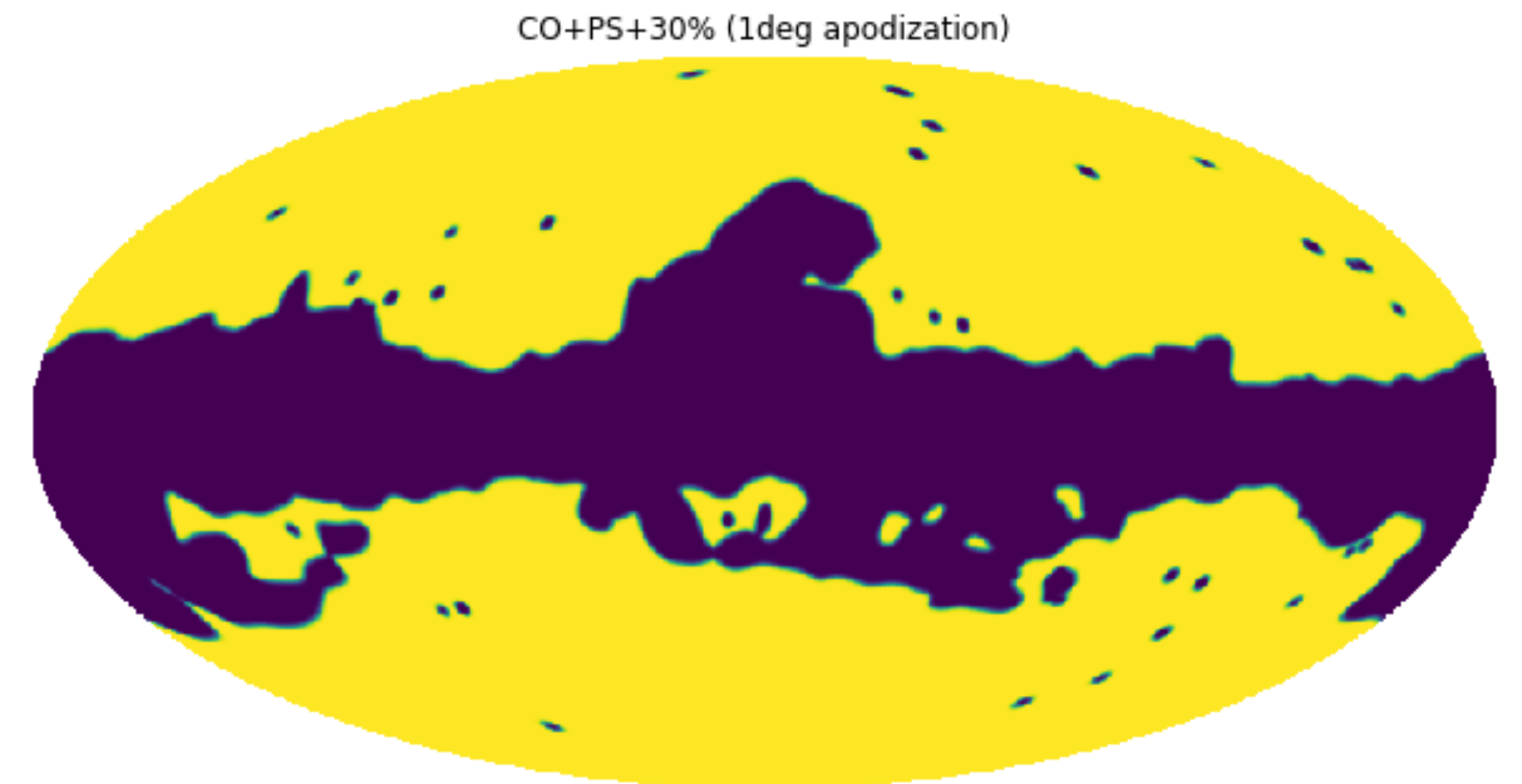
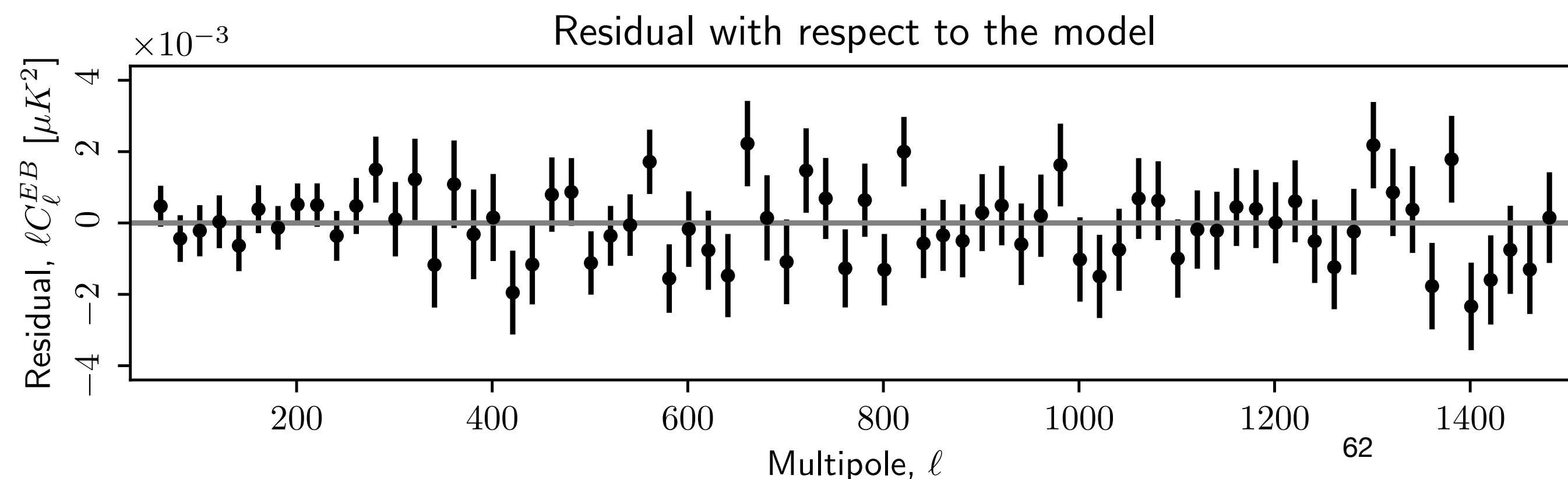
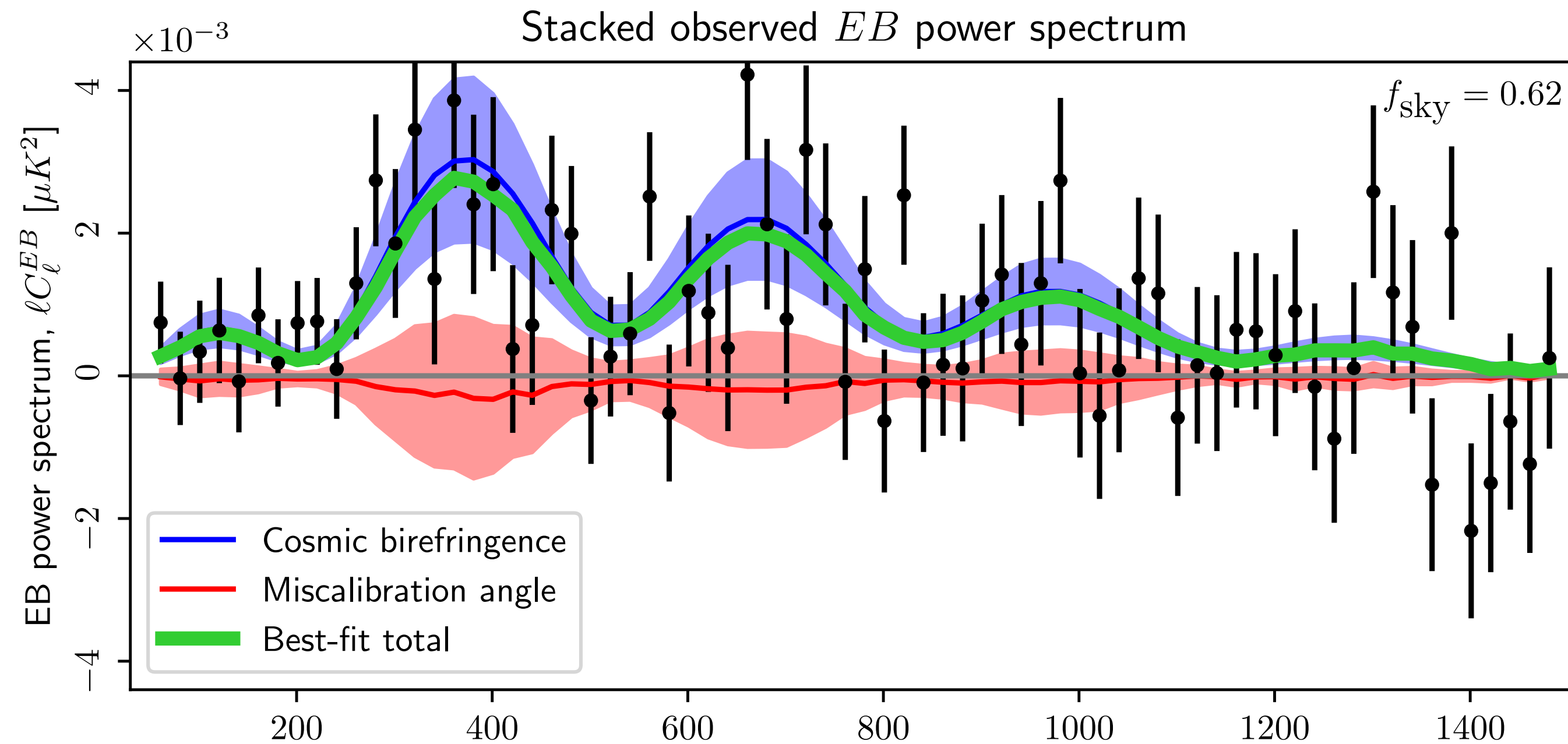
- **Miscalibration angles** make only small contributions thanks to the cancellation.

- $\beta = 0.34 \pm 0.09$ deg

- $\chi^2 = 65.3$ for DOF=72

Cosmic Birefringence fits well (WMAP+Planck)

Robust against the Galactic mask (62% of the sky)



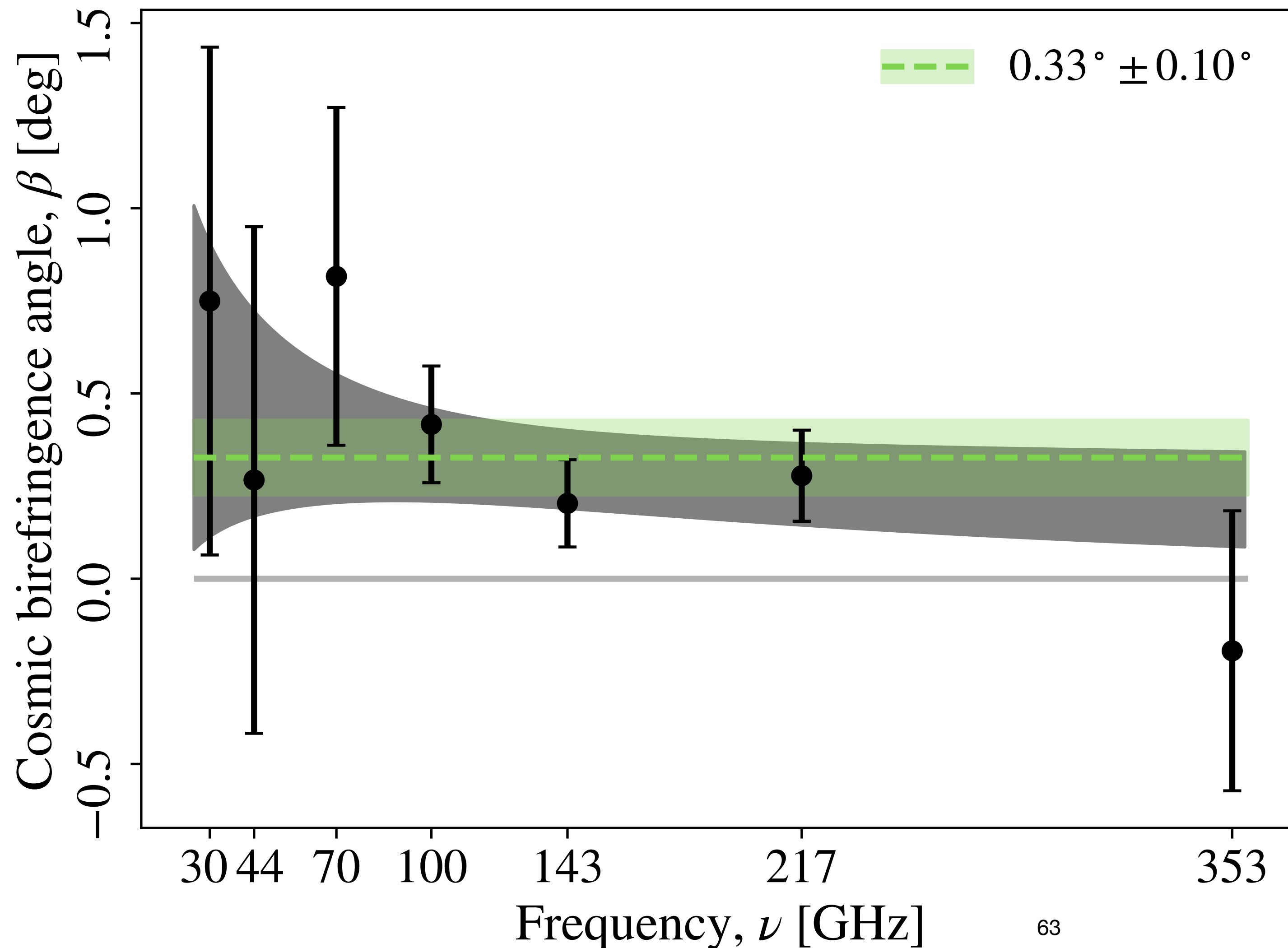
- **Miscalibration angles** make only small contributions thanks to the cancellation.

- $\beta = 0.37 \pm 0.14$ deg

- $\chi^2 = 65.8$ for DOF=72

No frequency dependence is found

Consistent with the expectation from cosmic birefringence



- Light traveling in a uniform magnetic field also experiences a rotation of the plane of linear polarization, called “**Faraday rotation**”. However, the rotation angle depends on the frequency, as $\beta(\nu) \propto \nu^{-2}$.
- No evidence for frequency dependence is found!
 - For $\beta \propto \nu^n$, $n = -0.20^{+0.41}_{-0.39}$ (68% CL)
 - **Faraday rotation ($n = -2$) is disfavoured.**

Is β caused by non-cosmological effects?

We need to measure it in independent experiments.

- The **known** instrumental effects of the WMAP and Planck missions are shown to have negligible effects on β .
 - However, we can never rule out **unknown** instrumental effects... We need to measure β in independent experiments.
- The polarized Galactic foreground emission was used to calibrate the instrumental polarization angles, α . The intrinsic EB correlations of the Galactic foreground emission (**polarized dust and synchrotron emission**) could affect the results.
 - We need to measure β without relying on the foreground by calibrating α well, e.g., Cornelison et al. (BICEP3 Collaboration), arXiv:2207.14796.

Implications

DM = Dark Matter; DE = Dark Energy

$$I = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\partial\chi)^2 - V(\chi) - \frac{1}{4} F^2 - \frac{\alpha}{4f} \chi F \tilde{F} \right]$$

- The measured angle, β , implies that the field has evolved by

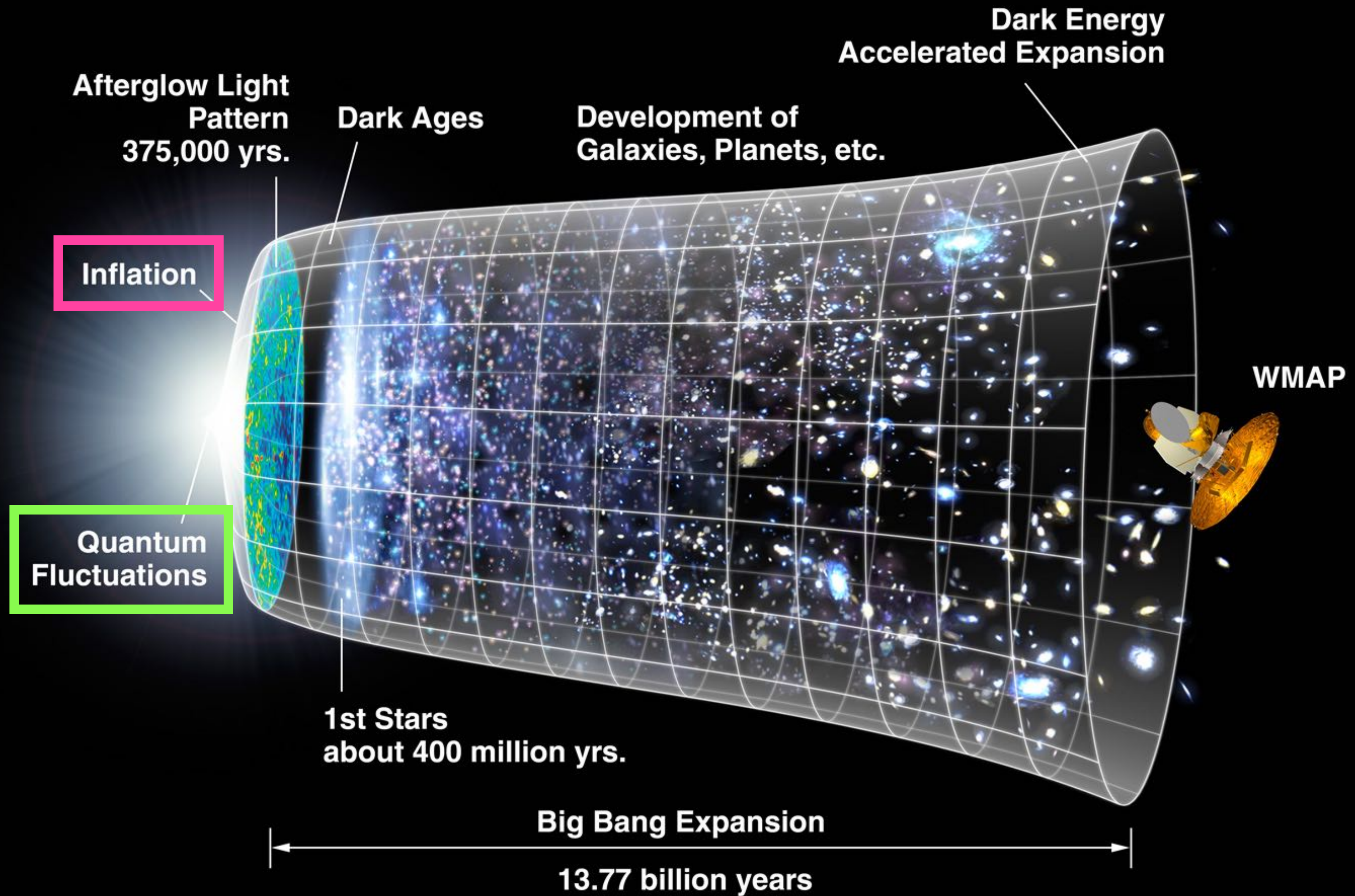
$$\Delta\chi = \chi(\tau_{\text{obs}}) - \chi(\tau_{\text{em}}) \simeq \frac{10^{-2}}{\alpha} f$$

We wrote

$$\theta = \frac{\chi}{f}$$

- If it is due to DE: this measurement rules out DE being a cosmological constant.
- If it is due to DM: at least a fraction of DM violates parity symmetry.

Parity Violation during Cosmic Inflation



Cosmic Inflation: Key Features

More than 40 years of research in a single slide

- Inflation is the period of **accelerated** expansion in the very early Universe.

- If the distance between two points increases as $a(t)$, **$d^2a/dt^2 > 0$** .

This is the definition of inflation.

- *Primordial fluctuations* are generated **quantum mechanically**.

- Scalar modes: Density fluctuations → The origin of all cosmic structure.

- Tensor modes: Gravitational waves → Yet to be discovered.

- Vector modes: ?

- **A New Paradigm**: Sourced contributions (this talk)

Anber, Sorbo (2010); Barnaby, Peloso (2011);
 Sorbo (2011); Barnaby, Namba, Peloso (2011)

The full action

Observational consequences

$$I = I_{\text{inflation}} \quad [\text{no one understands this}]$$

$$+ \int d\tau d^3\mathbf{x} \sqrt{-g} \left[\frac{R}{16\pi G} \right] \Rightarrow \text{Gravitational waves}$$

$$\square h_{ij} = 16\pi G (E_i E_j + B_i B_j)^{\text{TT}}$$

$$- \frac{1}{2} (\partial\chi)^2 - V(\chi) \Rightarrow \text{Scalar fluctuations}$$

$$\square\chi - \frac{\partial V}{\partial\chi} = -\frac{\alpha}{f} \mathbf{E} \cdot \mathbf{B}$$

$$- \frac{1}{4} F^2 - \frac{\alpha}{4f} \chi F \tilde{F} \Rightarrow \text{Parity violation in } A_\mu$$

$$A''_{\pm} + \omega_{\pm}^2 A_{\pm} = 0, \quad \omega_{\pm}^2 = k^2 \mp \frac{k\alpha\chi'}{f}$$

A note on terminology

“Photons” = Massless spin-1 particles

- Since inflation occurred long before the electroweak symmetry breaking, “photons” as we know them did not exist during inflation.
- We should think of them more generally as “**massless spin-1 particles**”.

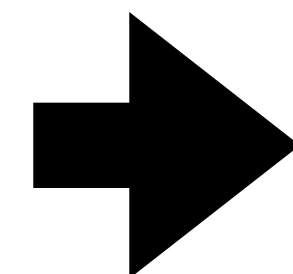
Gravitational waves

$$\square h_{ij} = 16\pi G (\underline{E_i E_j + B_i B_j})^{\text{TT}}$$

Scalar fluctuations

$$\square \chi - \frac{\partial V}{\partial \chi} = -\frac{\alpha}{f} \underline{\mathbf{E} \cdot \mathbf{B}}$$

Spin-1 sources, which violate parity symmetry due to the Chern-Simons term.



Non-Gaussian and parity-violating gravitational waves and scalar fluctuations!

Particle production due to $\chi F\tilde{F}$ during inflation

Kinetic energy of χ is used to produce massless spin-1 particles

$$A''_{\pm} + \omega_{\pm}^2 A_{\pm} = 0 \quad \text{where} \quad \begin{cases} \omega_{\pm}^2 = k^2 \mp \frac{2k\xi}{-\tau} \\ \xi = \frac{\alpha \dot{\chi}}{2fH} \quad (-\infty < \tau < 0) \end{cases}$$

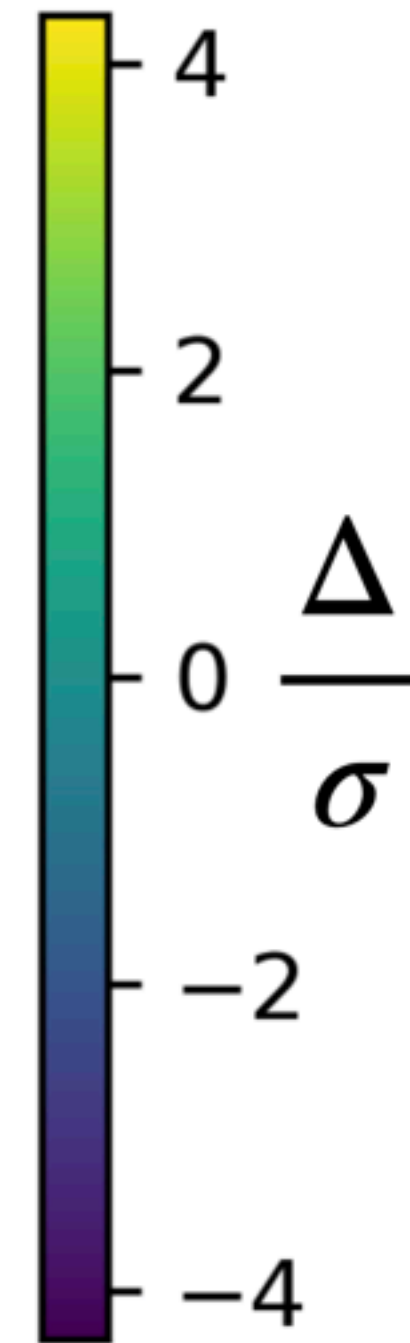
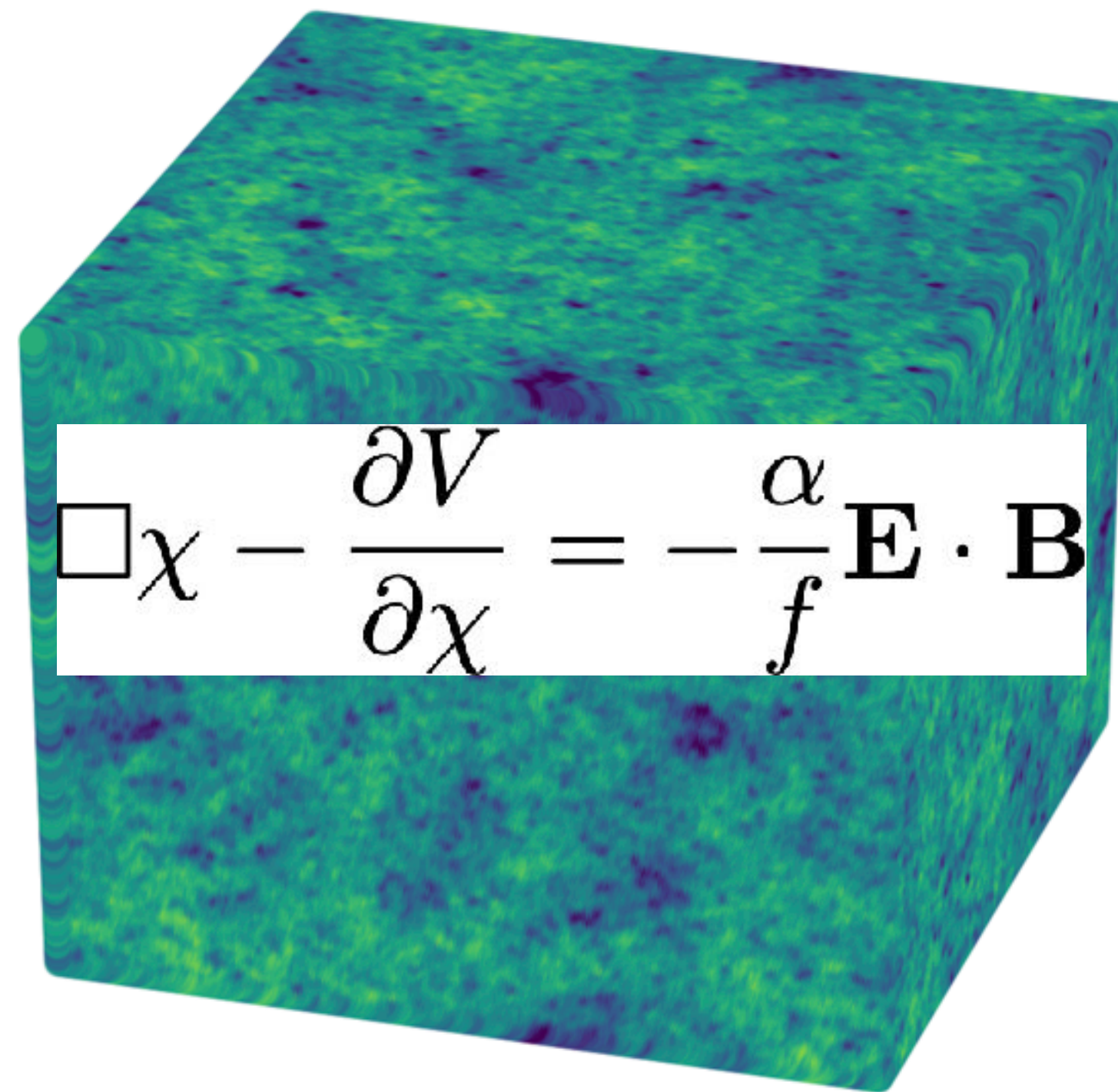
- Instability occurs when $\omega_{+}^2 < 0$ or $\omega_{-}^2 < 0$. In other words, $-k\tau < 2|\xi|$.
- The mode function for *one of the helicity states* is **amplified** on large scales (small $-k\tau$) **relative to the vacuum solution, $e^{-ik\tau}/\sqrt{2k}$** .
- The right-handed (+ helicity) state is amplified for $\xi > 0$, whereas the left-handed (- helicity) state remains close to the vacuum solution.
- **Parity violation!**

Truly *ab initio* simulation!

World's first lattice simulation of inflation



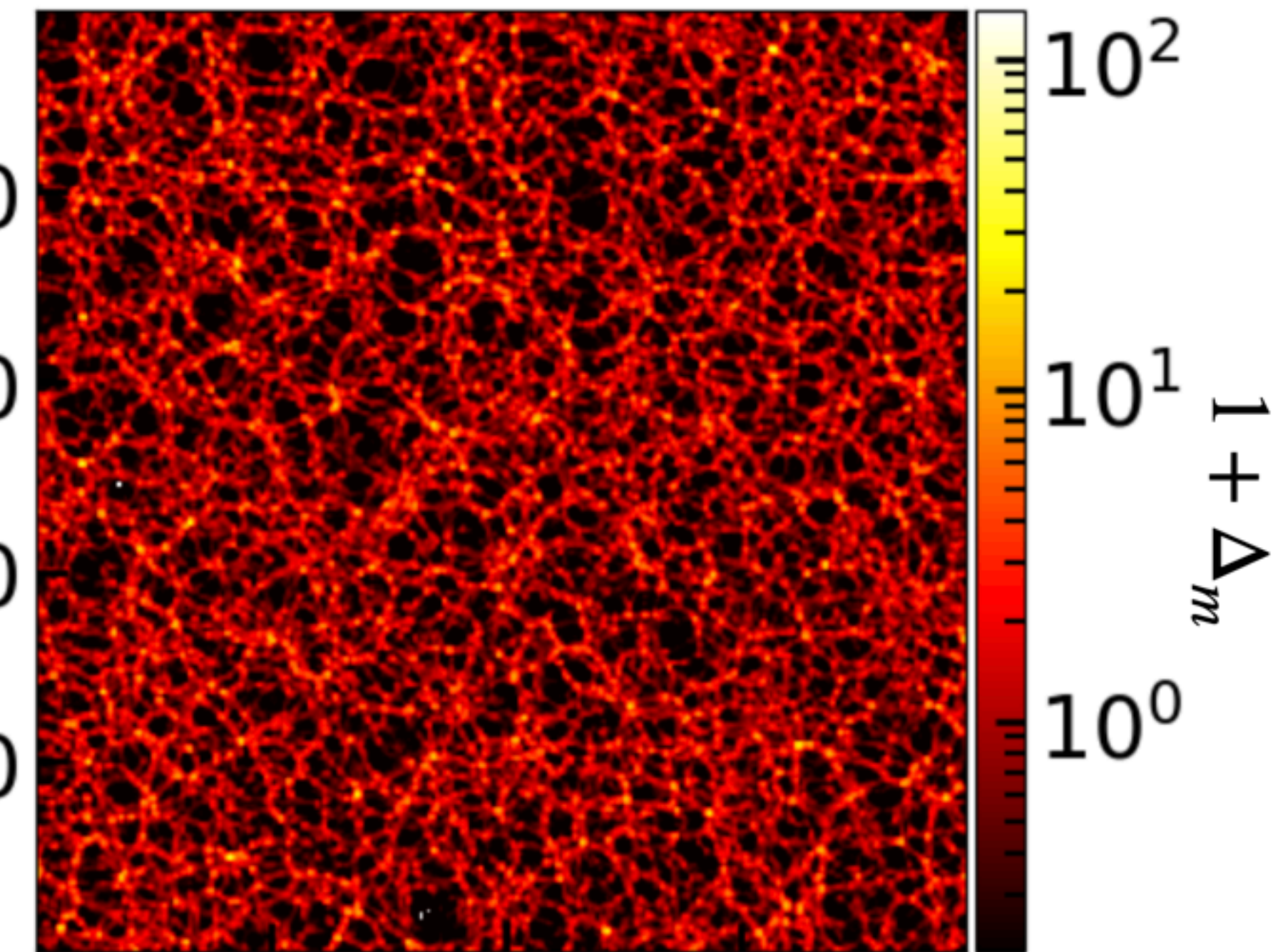
Angelo Caravano



y [Mpc h^{-1}]
800
600
400
200



Drew Jamieson



250 500 750
x [Mpc h^{-1}]

- (Left) Parity-violating and non-Gaussian density fluctuation during inflation.
- (Right) Outcome of N-body simulation at $z=0$, using the left panel as the initial condition.

GR + Maxwell (+ Chern-Simons)

$$\square = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu)$$

$$= \frac{1}{a^2} \left(-\frac{\partial^2}{\partial \tau^2} - 2 \frac{a'}{a} \frac{\partial}{\partial \tau} + \nabla^2 \right)$$

where $g^{\mu\nu} = a^{-2} \text{diag}(-1, \mathbf{1})$

$$I = \int d\tau d^3 \mathbf{x} \sqrt{-g} \left(\frac{R}{16\pi G} - \frac{1}{4} F^2 - \frac{\alpha}{4f} \chi F \tilde{F} \right) \sqrt{-g} = a^4$$

- The F^2 term contributes to the equation of motion for the GW via the stress-energy tensor (this is the second-order fluctuation).

$$\square h_{ij} = 16\pi G (E_i E_j + B_i B_j)^{\text{TT}} \text{ "Transverse and Traceless"}$$

- The $F\tilde{F}$ term does **not** contribute directly to the equation of motion for the GW.
 - But, it creates a parity violation in \mathbf{E} and \mathbf{B} , which also creates a parity violation in the GW.

Helicity basis to probe parity symmetry

Circular polarization states of GW. GW's helicity is $\lambda = \pm 2$.

- Just like for EM waves,

$$A_{\pm} = \frac{A_{\mathbf{k}}^1 \mp i A_{\mathbf{k}}^2}{\sqrt{2}}$$

A_+ : Right-handed state

A_- : Left-handed state

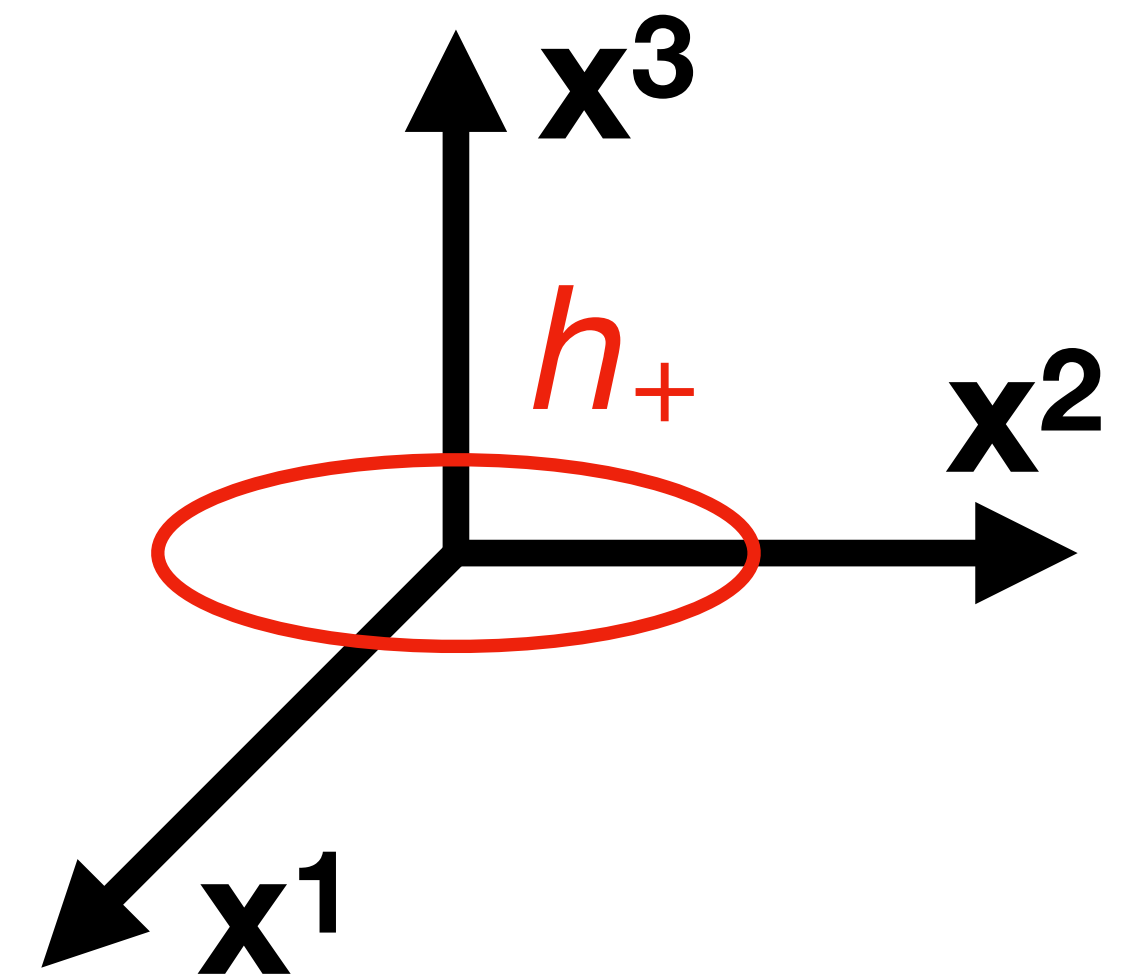
$$h_{ij} = \begin{pmatrix} h_+ & h_{\times} & 0 \\ h_{\times} & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

we write the helicity states of GW in Fourier space as

$$h_{\pm 2} = \frac{h_{+, \mathbf{k}} \mp i h_{\times, \mathbf{k}}}{\sqrt{2}}$$

h_{+2} : Right-handed state

h_{-2} : Left-handed state



GW's helicity is $\lambda=\pm 2$

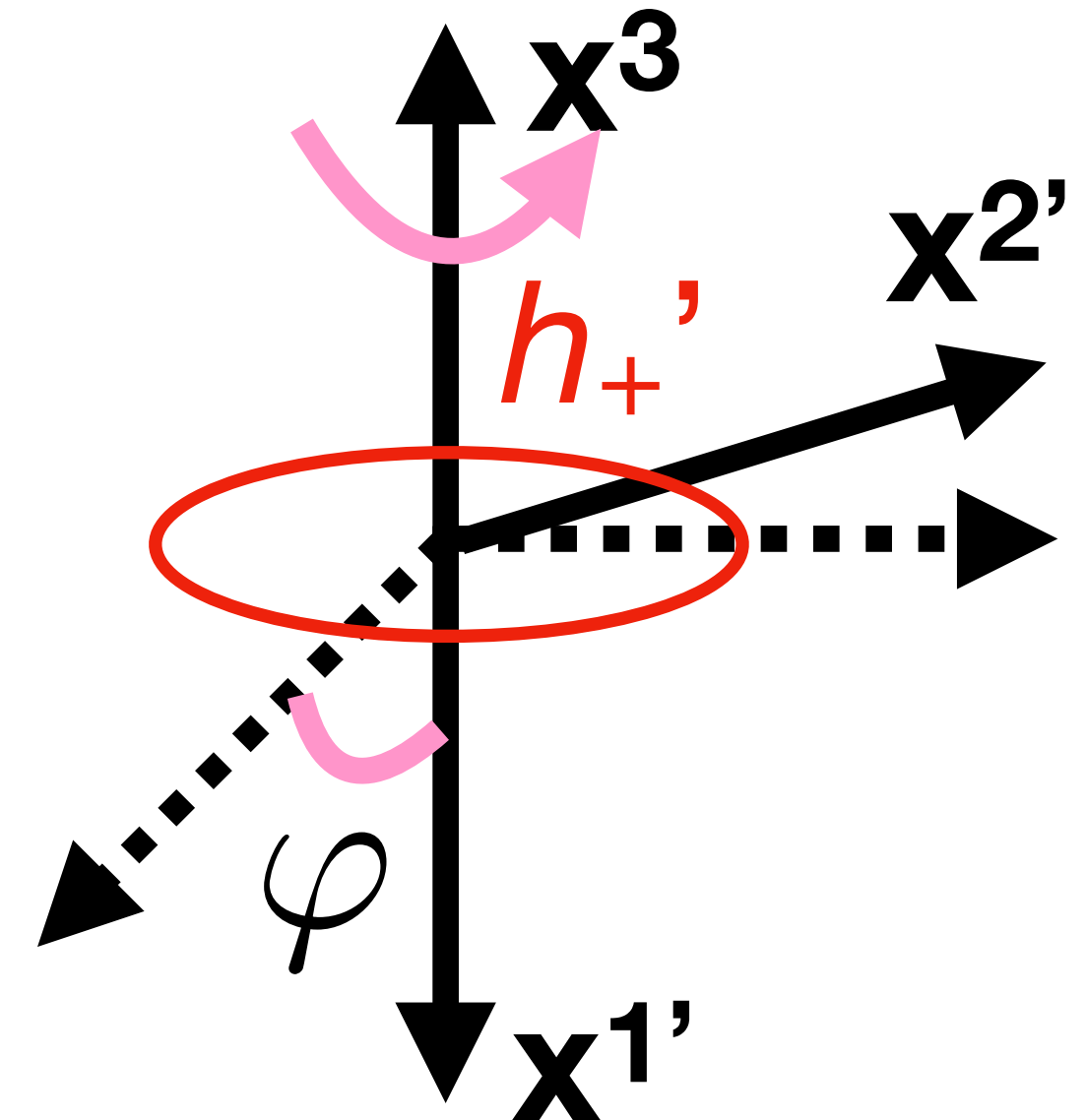
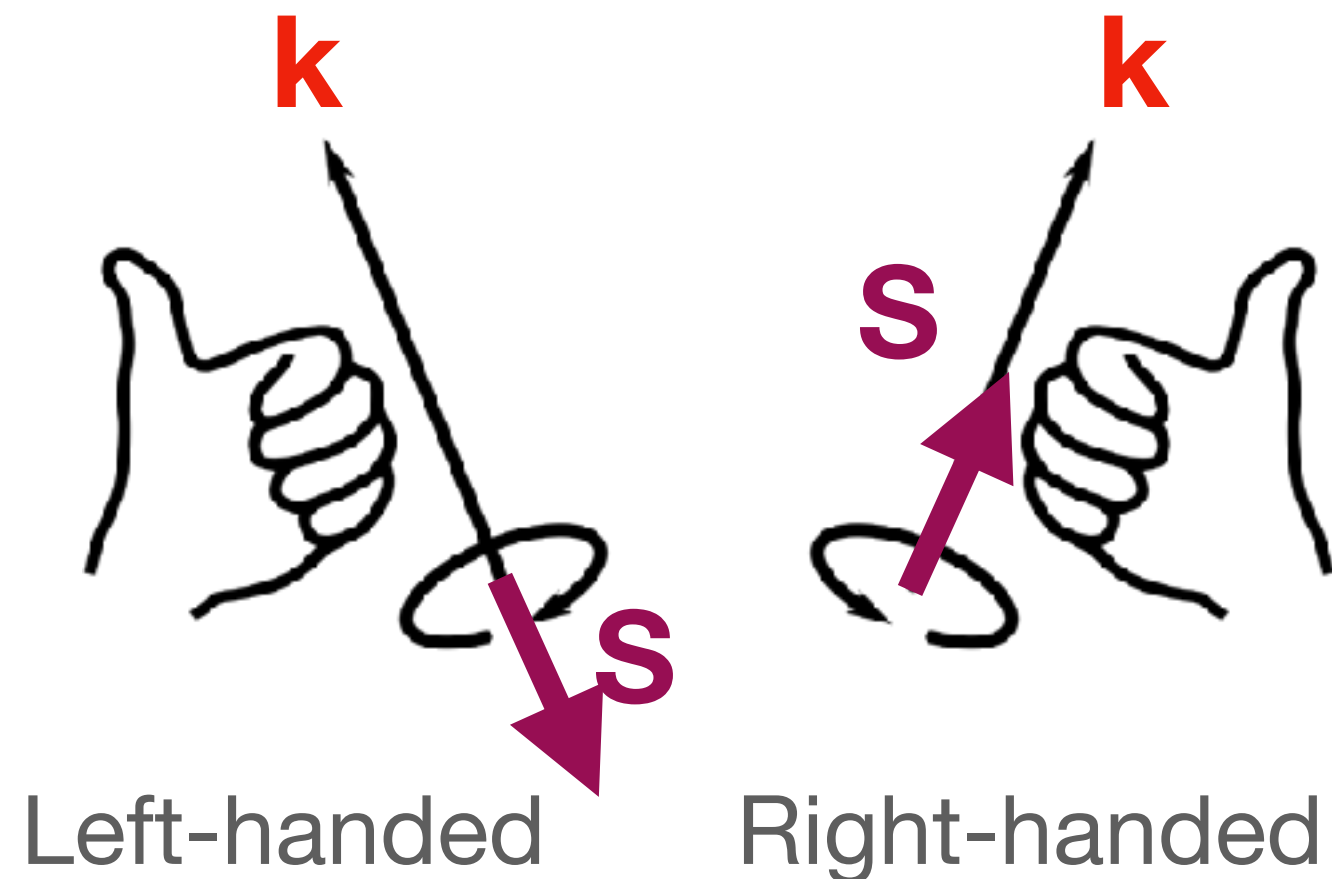
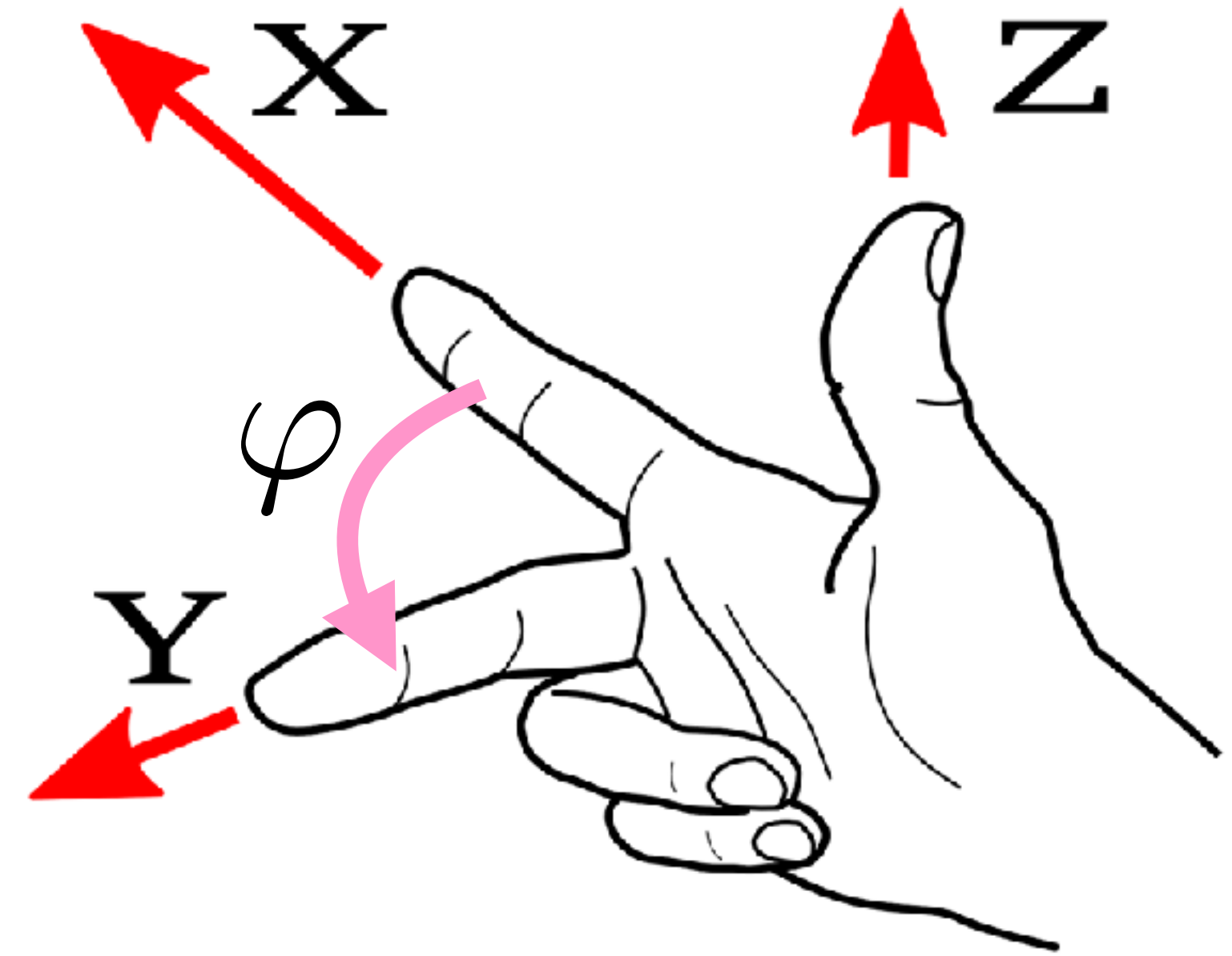
Gravitons are massless spin-2 particles!

- To show that $h_{\pm 2}$ represents the helicity states, rotate the spatial coordinates around the z axis in the right-handed system by an angle φ .
- The helicity states, $\lambda=\pm 2$, transform as

$$h_{\lambda} \rightarrow h'_{\lambda} = e^{i\lambda\varphi} h_{\lambda}$$

Helicity

h_{+2} : Right-handed state
 h_{-2} : Left-handed state



Parity Violation in GW

For a slowly varying $\xi > 0$

$$\xi = \frac{\alpha \dot{\theta}}{2H} = \frac{\alpha \dot{\chi}}{2H f}$$

$$\frac{k^3 P_{+2}(k)}{2\pi^2} \simeq \frac{2}{M_{\text{Pl}}^2} \left(\frac{H}{2\pi}\right)^2 \left[1 + 8.6 \times 10^{-7} \frac{H^2}{M_{\text{Pl}}^2} \frac{e^{4\pi\xi}}{\xi^6} \right]$$

$$\frac{k^3 P_{-2}(k)}{2\pi^2} \simeq \frac{2}{M_{\text{Pl}}^2} \left(\frac{H}{2\pi}\right)^2 \left[1 + 1.8 \times 10^{-9} \frac{H^2}{M_{\text{Pl}}^2} \frac{e^{4\pi\xi}}{\xi^6} \right]$$

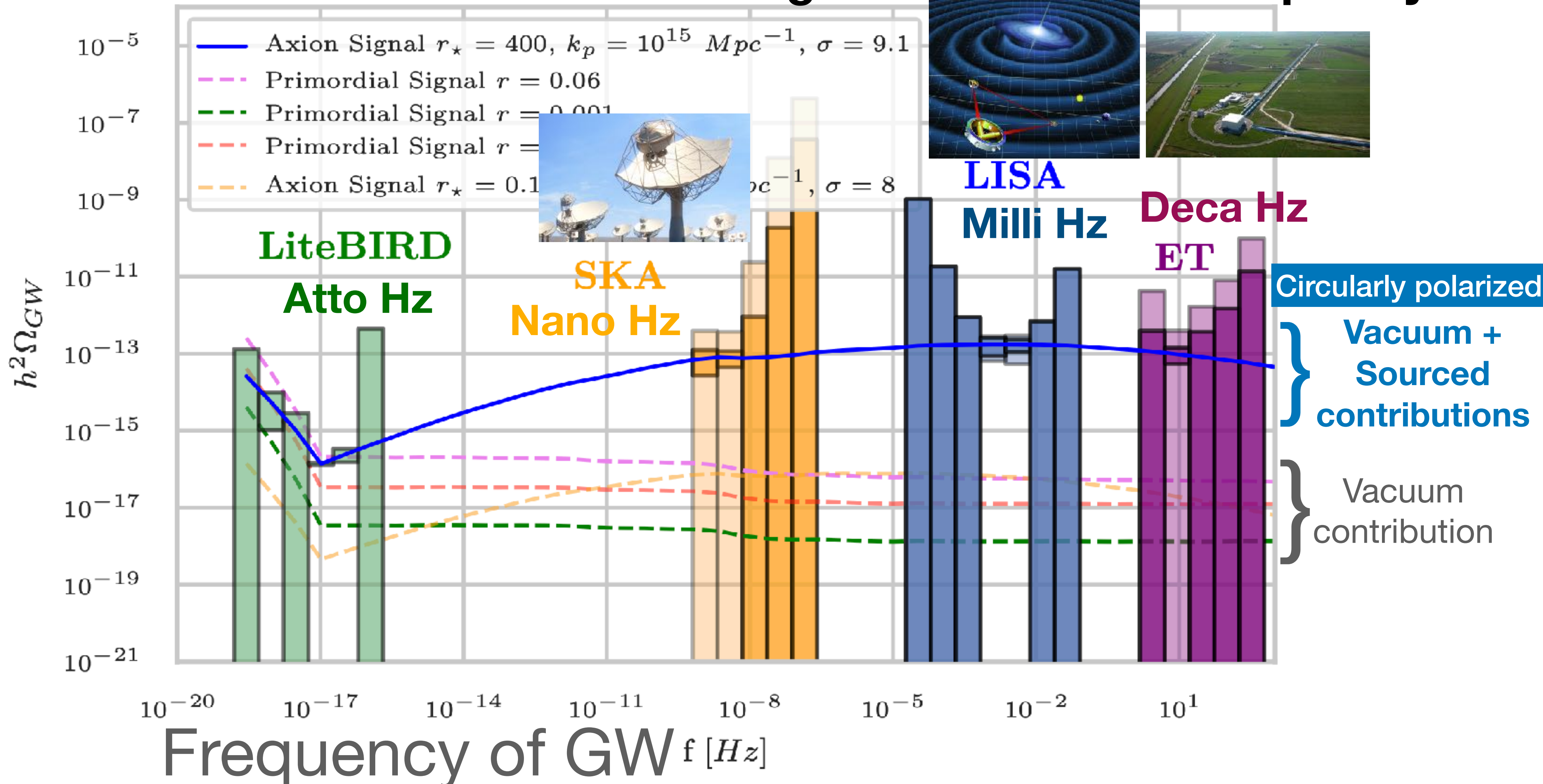
↓ Vacuum contribution ↓ Parity Violation!

- The sourced contributions are almost perfectly circularly polarized.
- The sum of the vacuum and sourced contributions is partially circularly polarized. **This can be observationally tested!** (Seto 2006; Seto, Taruya 2007)

GWs from the early Universe are everywhere!

We can measure it across 21 orders of magnitude in the GW frequency

Energy Density of GW today

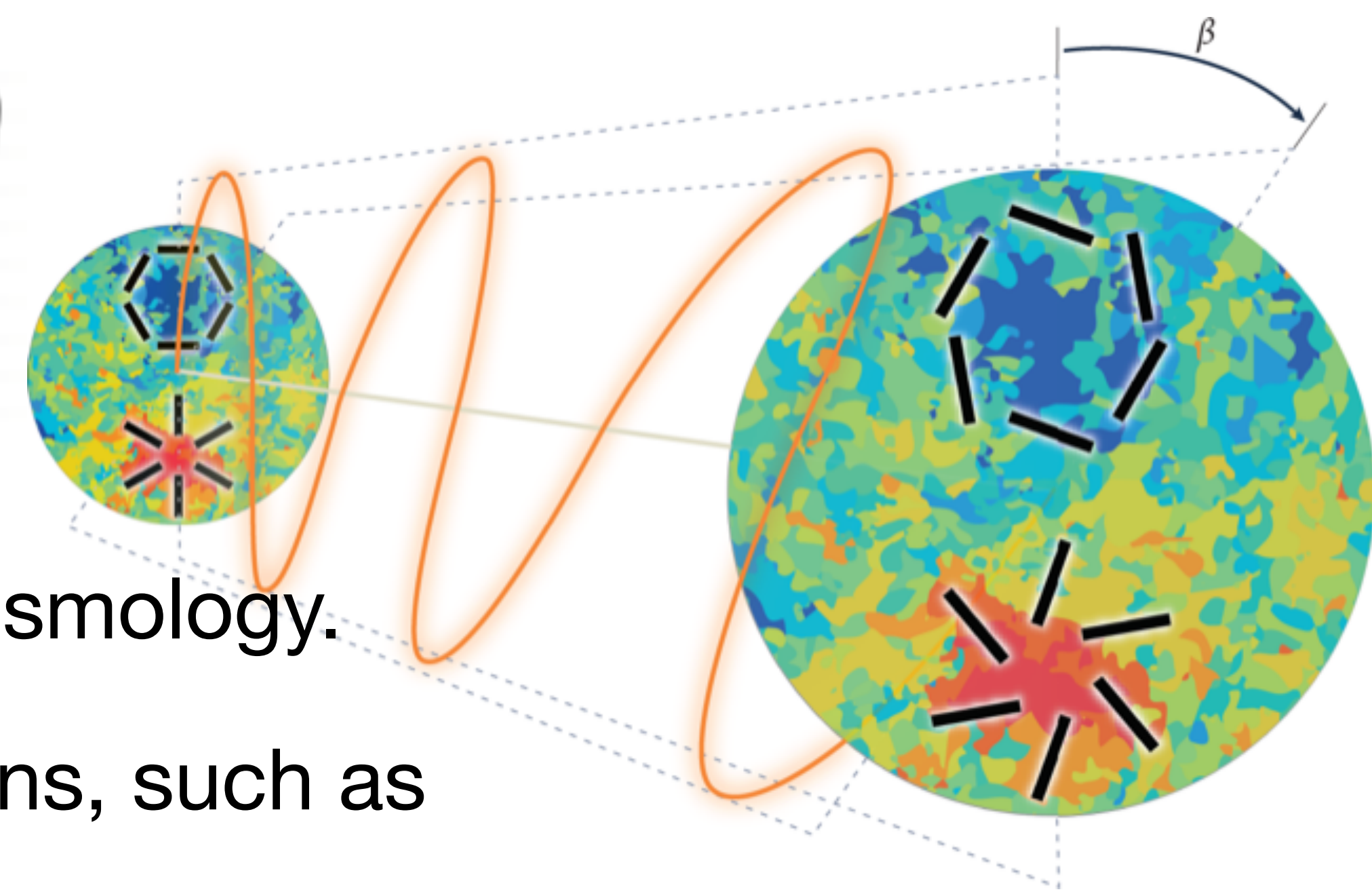


Summary

Jcap 20th

Let's find new physics!

2003-2023



- Violation of parity symmetry is a new topic in cosmology.
- It may hold the answers to fundamental questions, such as

- *What is Dark Matter and Dark Energy?*
- *What is the fundamental physics behind cosmic inflation?*

- Rich phenomenology of Chern-Simons term:
$$I_{CS} = \int d^4x \sqrt{-g} \left(-\frac{\alpha}{4f} \chi F \tilde{F} \right)$$

- Cosmic birefringence **3.6σ hint of the signal**

Abelian and non-Abelian gauge fields; Gravitational CS; ...

- Parity-violating and non-Gaussian gravitational waves and scalar fluctuations

- **What else should we look at? New and great topics of research.**

Back up slides

Parity violation in the density field?

What is right and left?

- The CMB polarization has directions from which one could construct parity eigenstates, such as E and B modes.
- **How can we construct a pseudoscalar** for the density field, which is a scalar field and has no directions?
- **Important:** We continue to assume that physics is invariant under spatial translation and rotation (homogeneity and isotropy).

Is the power spectrum sensitive to parity?

No.

- The power spectrum is related to the 2-point correlation function as

$$P(\mathbf{k}) = \int d^3\mathbf{r} \xi(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}}$$

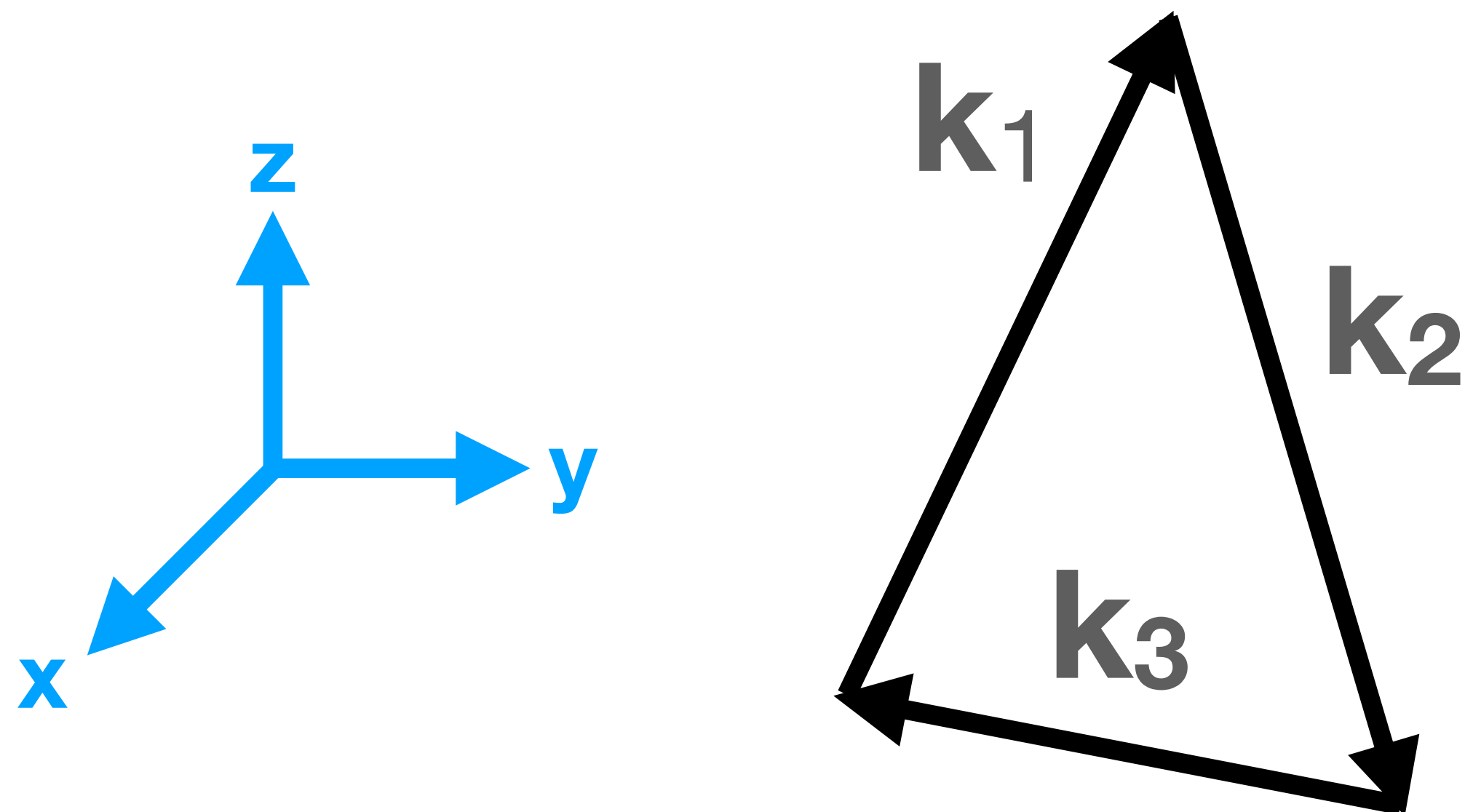
- Rotational invariance means that $\xi(\mathbf{r})$ does not depend on the direction of \mathbf{r} , but only on the magnitude, $r = |\mathbf{r}|$. Then the parity transformation, $\mathbf{k} \rightarrow -\mathbf{k}$, simply gives

$$P(-\mathbf{k}) = P(\mathbf{k}) = P(k) \quad \text{where} \quad k = |\mathbf{k}|$$

Higher-order Statistics (N -point functions)

Many wavenumber vectors \rightarrow Right- and left-handed?

- The 2-point function correlates 2 points in space. The 3-point function correlates 3 points in space.
- The Fourier transform of the 2-point function is the power spectrum. The Fourier transform of the 3-point function is the **bispectrum**.



$$\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2} \delta_{\mathbf{k}_3} \rangle \text{ with } \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$$

where

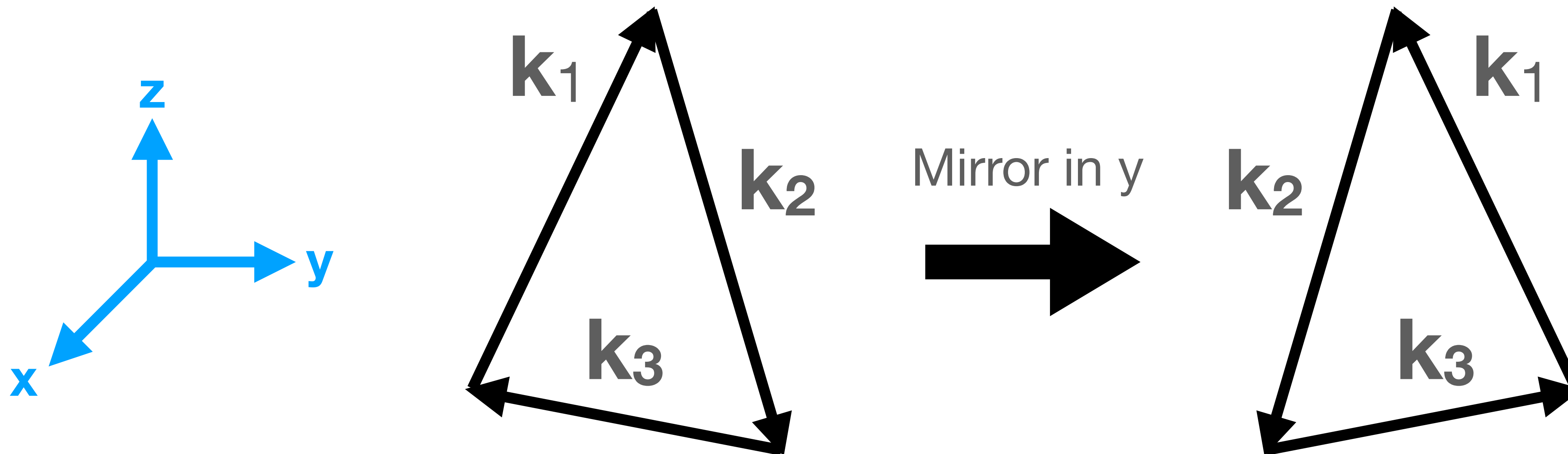
$$\delta(t, \mathbf{x}) = \frac{\rho(t, \mathbf{x}) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

Mass density fluctuations

Higher-order Statistics (N -point functions)

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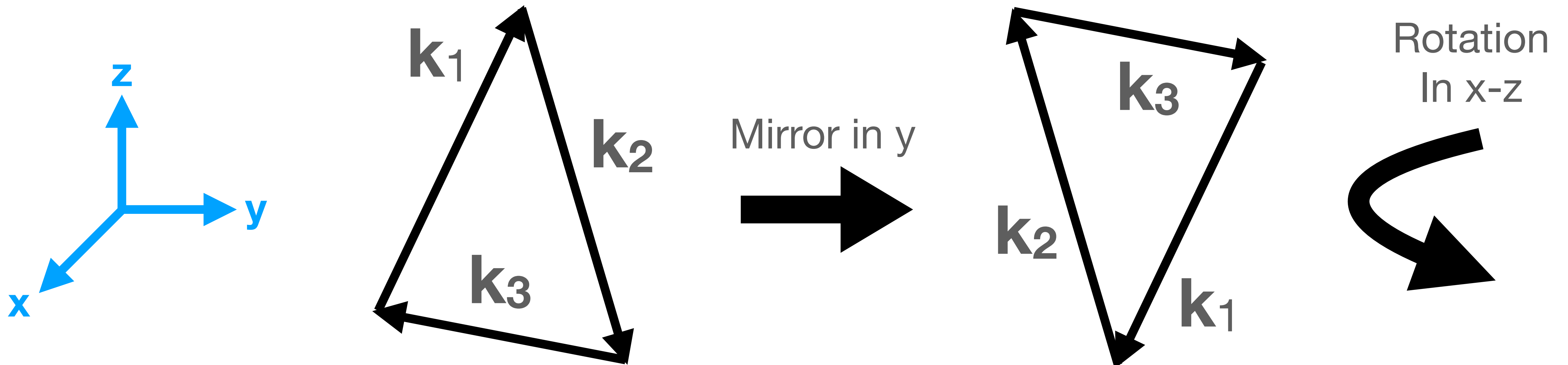
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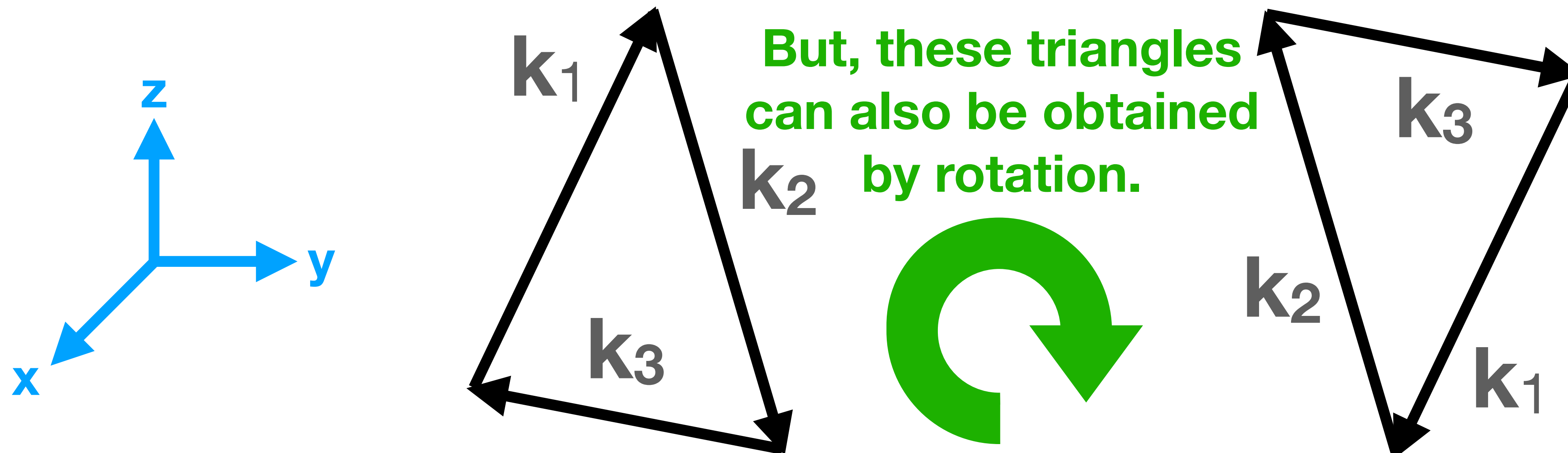
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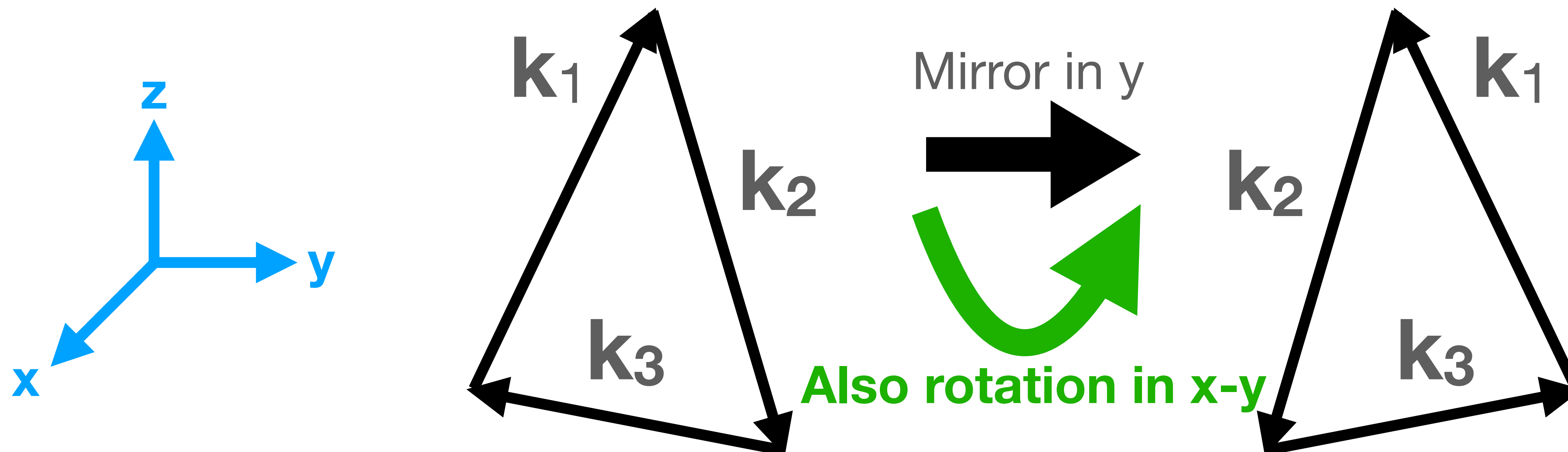


Rotational invariance in 3d = The bispectrum is not sensitivity to parity.

Higher-order Statistics (N -point functions)

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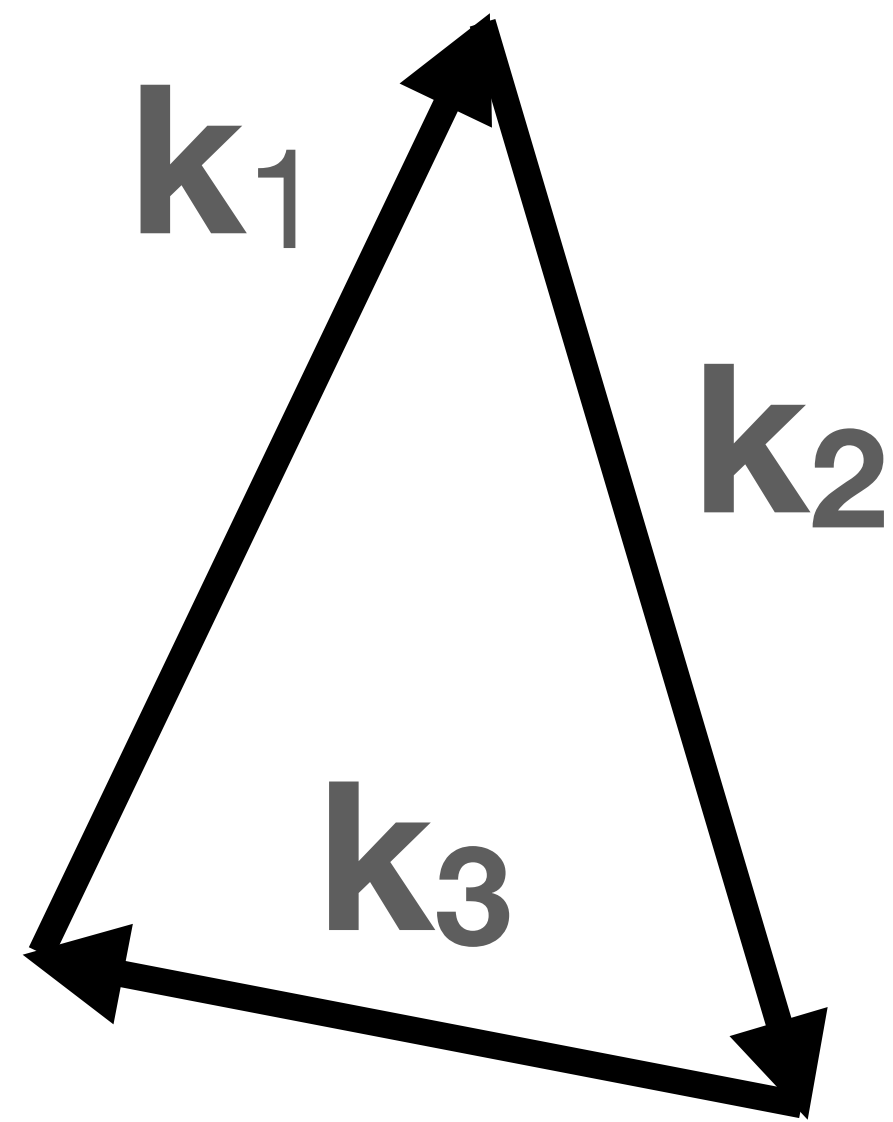
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Rotational invariance in 3d = The bispectrum is not sensitivity to parity.

Why is the 3d bispectrum insensitive to parity?

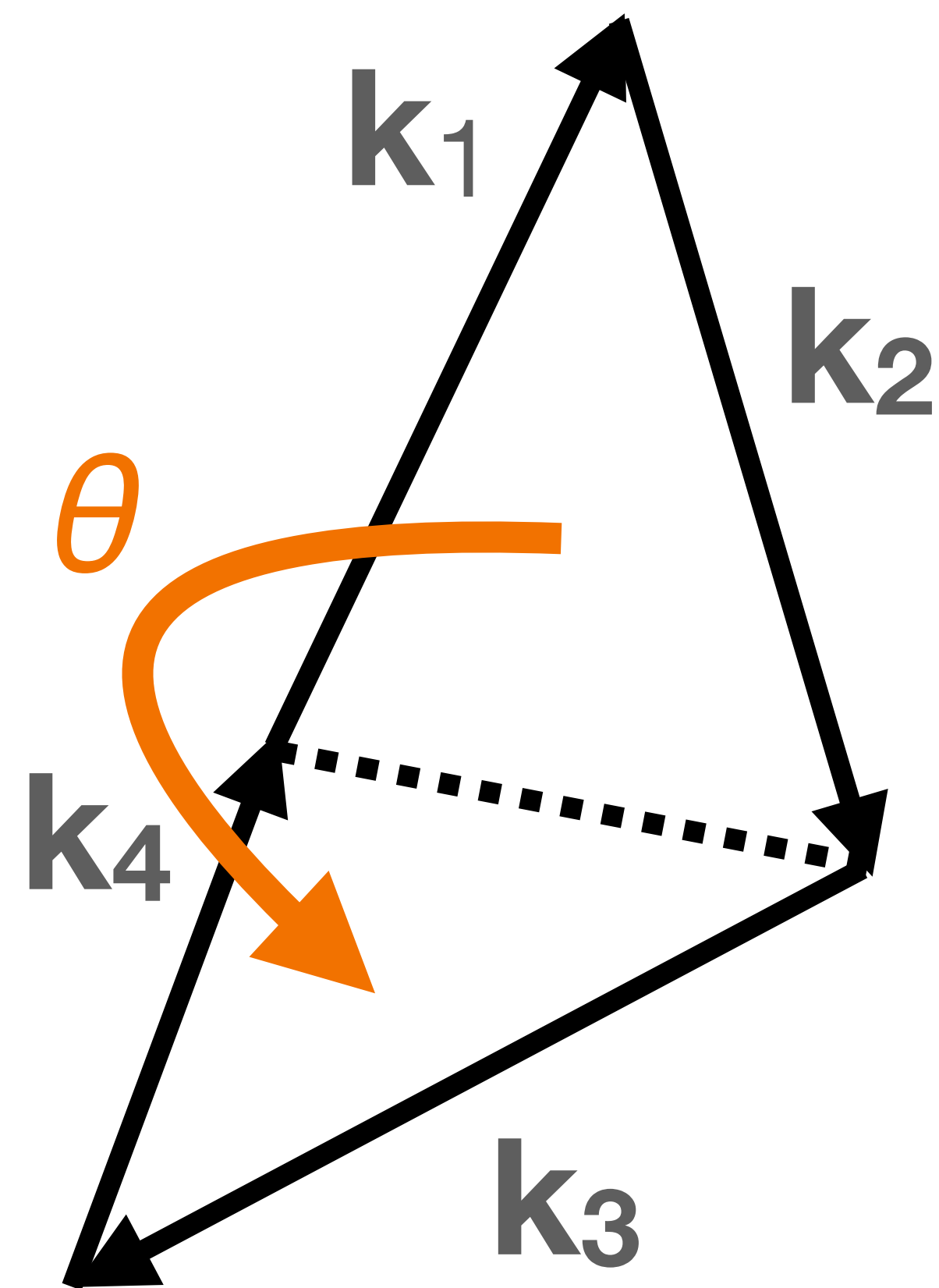
Because the triangle forms a plane.



- 3 vectors form a plane ($\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$).
- To define handedness, a pseudoscalar (like helicity) is required.
- The only possible pseudoscalar is $(\mathbf{k}_a \times \mathbf{k}_b) \cdot \mathbf{k}_c$.
- However, this vanishes because $\mathbf{k}_c = -\mathbf{k}_a - \mathbf{k}_b$!
- There is no unique handedness for triangles in 3d.
- **How about the 4-point function?**

4-point function in 3d is sensitive to parity

...unless it forms a plane.



- The Fourier transform of the 4-point function is the **trispectrum**.
- There are 4 vectors and one can form a pseudoscalar, $(\mathbf{k}_a \times \mathbf{k}_b) \cdot \mathbf{k}_c$, that does not vanish!
- ...unless it forms a plane, $\theta = 0$ or π .
- The 4-point function is the lowest-order statistics that is parity-sensitive in 3 dimensions.
- The Chern-Simons term can generate this via

Parity violation in the density fluctuation

$$\square_{\chi} - \frac{\partial V}{\partial \chi} = \boxed{-\frac{\alpha}{f} \mathbf{E} \cdot \mathbf{B}}$$

Parity-odd Trispectrum: Density Fluctuation

Imaginary part

- Fourier coefficients satisfy $\delta_{\mathbf{k}}^* = \delta_{-\mathbf{k}}$ for a real function $\delta(\mathbf{x})$.
- Under the parity transformation, $\mathbf{k} \rightarrow -\mathbf{k}$, and the trispectrum is transformed as

$$\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2} \delta_{\mathbf{k}_3} \delta_{\mathbf{k}_4} \rangle$$

$$\rightarrow \langle \delta_{-\mathbf{k}_1} \delta_{-\mathbf{k}_2} \delta_{-\mathbf{k}_3} \delta_{-\mathbf{k}_4} \rangle = \langle \delta_{\mathbf{k}_1}^* \delta_{\mathbf{k}_2}^* \delta_{\mathbf{k}_3}^* \delta_{\mathbf{k}_4}^* \rangle$$
- **The imaginary part, $\text{Im}(\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2} \delta_{\mathbf{k}_3} \delta_{\mathbf{k}_4} \rangle)$, is sensitive to parity violation.**

Observational hints?

New and exciting research area

PHYSICAL REVIEW D

covering particles, fields, gravitation, and cosmology

Probing parity violation with the four-point correlation function of BOSS galaxies


Oliver H. E. Philcox

Phys. Rev. D **106**, 063501 – Published 6 September 2022

In LOWZ, we find 3.1σ evidence for a non-zero parity-odd 4PCF, and in CMASS we detect a parity-odd 4PCF at 7.1σ .

These find similar results, with the rank test giving a detection probability of 99.6% (2.9σ).

JOURNAL ARTICLE

Measurement of parity-odd modes in the large-scale 4-point correlation function of Sloan Digital Sky Survey Baryon Oscillation Spectroscopic Survey twelfth data release CMASS and LOWZ galaxies 

Jiamin Hou , Zachary Slepian, Robert N Cahn

Monthly Notices of the Royal Astronomical Society, Volume 522, Issue 4, July 2023,

Pages 5701–5739, <https://doi.org/10.1093/mnras/stad1062>

Published: 22 May 2023 [Article history](#) 

Parity-odd Trispectrum: CMB Temperature

$$\ell_1 + \ell_2 + \ell_3 + \ell_4 = \text{odd}$$

- Under the parity transformation, $\hat{n} \rightarrow -\hat{n}$, the spherical harmonics coefficients of CMB temperature anisotropy, $\Delta T(\hat{n}) = \sum a_{\ell m} Y_{\ell}^m(\hat{n})$, are transformed as $a_{\ell m} \rightarrow (-1)^{\ell} a_{\ell m}$.
- Therefore, the temperature trispectrum is transformed as

$$\begin{aligned} & \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} a_{\ell_4 m_4} \rangle \\ & \rightarrow (-1)^{\ell_1 + \ell_2 + \ell_3 + \ell_4} \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} a_{\ell_4 m_4} \rangle \end{aligned}$$

- **The configuration with $\sum \ell_i = \text{odd}$ is sensitive to parity violation.**

Observational constraints

New and exciting research area

Do the CMB Temperature Fluctuations Conserve Parity?

Oliver H. E. Philcox^{1, 2, *}

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The measured trispectra can be used to constrain physical models of inflationary parity violation, including Ghost Inflation, Cosmological Collider scenarios, and Chern-Simons gauge fields. Considering eight such models, we find no evidence for new physics, with a maximal detection significance of 2.0σ .