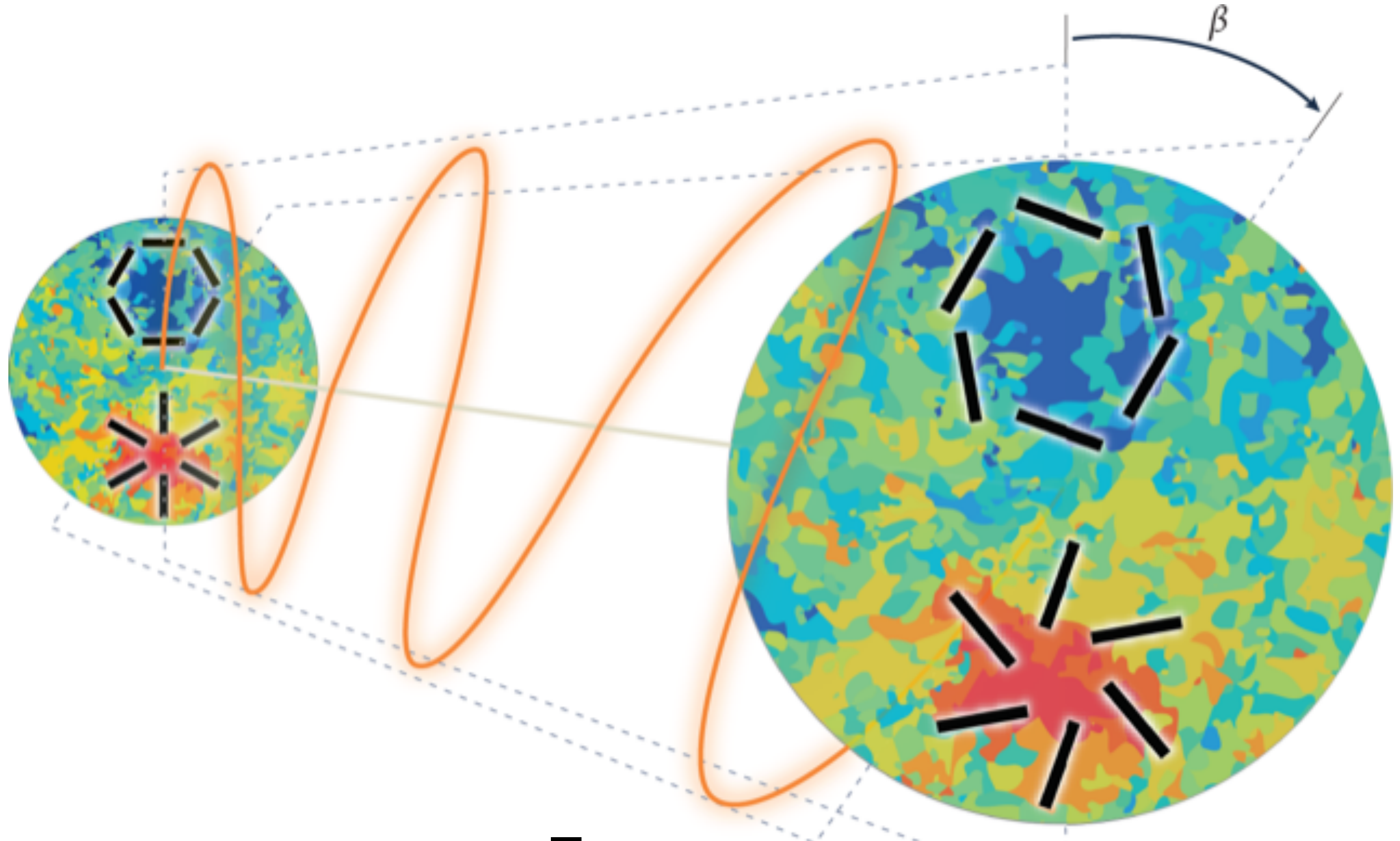
$$I_{\rm CS} = \int d^4x \sqrt{-g} \left(-\frac{\alpha}{4f} \chi F \tilde{F} \right)$$



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Parity Violation in Cosmology

In search of new physics for the Universe

The lecture slides are available at
https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Eiichiro Komatsu (Max Planck Institute for Astrophysics)
Nagoya University, June 6–30, 2023

Day 4

Topics

From the syllabus

1. What is parity symmetry?

2. Chern-Simons interaction

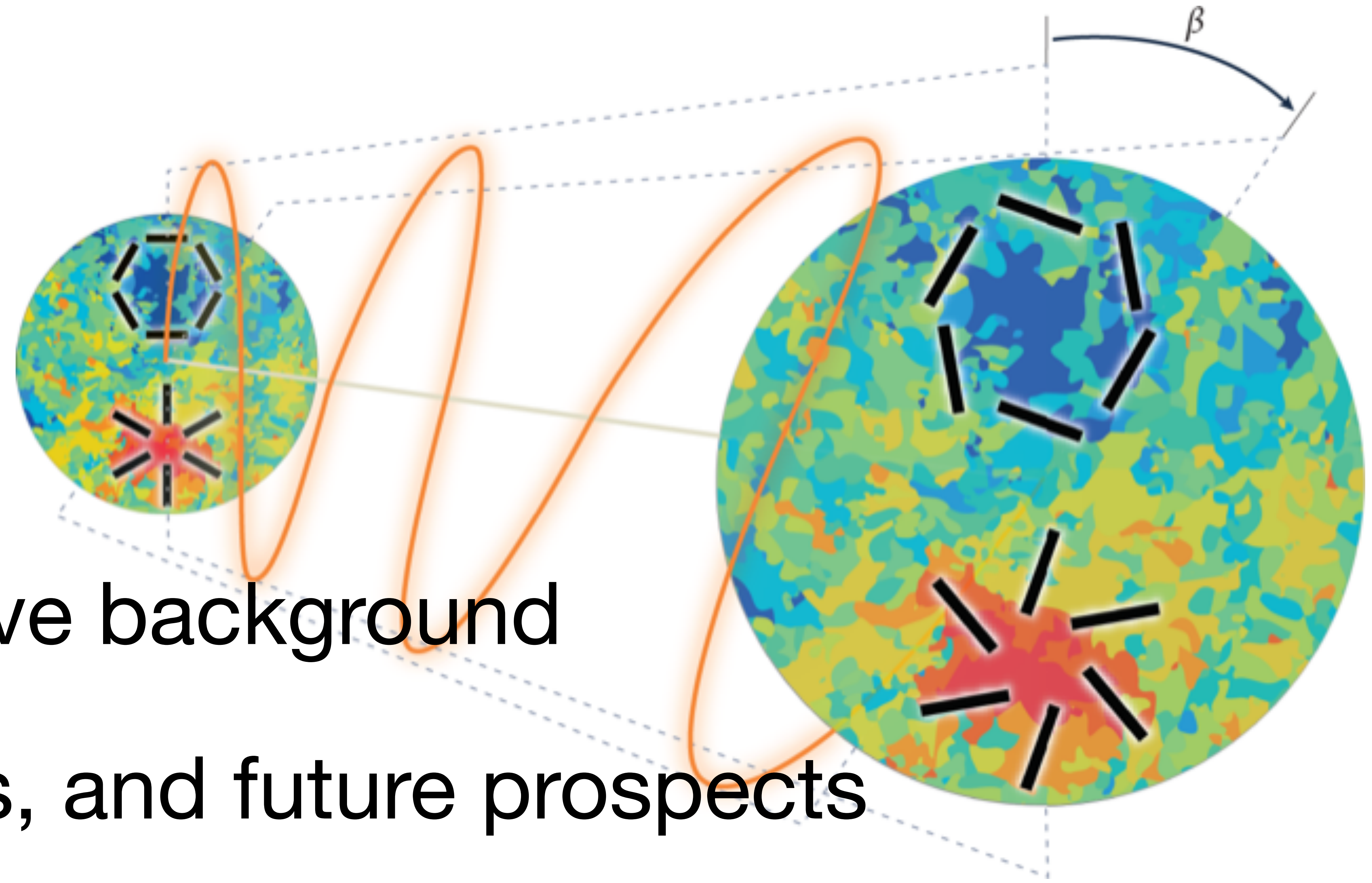
$$I_{\mathrm{CS}} = \int \mathrm{d}^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\tilde{F}\right)$$

3. Parity violation 1: Cosmic inflation (continued)

4. Parity violation 2: Dark matter

5. Parity violation 3: Dark energy

6. Light propagation: birefringence



7. Physics of polarization of the cosmic microwave background

8. Recent observational results, their implications, and future prospects

3.5 Quantization of χ during inflation

$$\Box\chi - \frac{\partial V}{\partial \chi} = -\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B}$$

The full action
Observational consequences

$$\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu) = \frac{1}{a^2}\left(-\frac{\partial^2}{\partial\tau^2} - 2\frac{a'}{a}\frac{\partial}{\partial\tau} + \nabla^2\right)$$

where $g^{\mu\nu} = a^{-2}\mathrm{diag}(-1, \mathbf{1})$

$\sqrt{-g} = a^4$

$$I = I_{\text{inflation}}$$ [no one understands this]

$$+\int d\tau d^3\mathbf{x}\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}(\partial\chi)^2 - V(\chi) - \frac{1}{4}F^2 - \frac{\alpha}{4f}\chi F\tilde{F}\right]$$

$(\partial\chi)^2 = g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$

$\theta = \frac{\chi}{f}$

Gravitational waves

→ $\Box h_{ij} = 16\pi G(E_iE_j + B_iB_j)^{\mathrm{TT}}$

Scalar fluctuations

→ $\Box\chi - \frac{\partial V}{\partial\chi} = -\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B}$

- ***f*: "decay constant"** of χ. It is 184 MeV for a pion, but it is much, much larger for the field that we will discuss in this lecture.

Second-order action for δχ

Without the Chern-Simons term

$$-\sqrt{-g}(\partial\chi)^2 = -\sqrt{-g}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$$

where $g^{\mu\nu} = a^{-2}\mathrm{diag}(-1, 1)$

$$\sqrt{-g} = a^4$$

$$I = \int d\tau d^3\mathbf{x}\ a^4 \left[\frac{1}{2a^2}\left(\chi'^2 - \nabla\chi\cdot\nabla\chi\right) - V(\chi)\right]$$

- We expand the action to the second-order in the fluctuation: $\chi = \bar{\chi} + \delta\chi$

$$I^{(2)} = \int d\tau d^3\mathbf{x} \left[\frac{a^2}{2}\left(\delta\chi'^2 - \nabla\delta\chi\cdot\nabla\delta\chi\right) - \frac{1}{2}a^4 m^2 \delta\chi^2\right]$$

where

$$m^2 = \frac{\partial^2 V}{\partial\chi^2}$$

- *δχ* is **not** the canonical variable for quantization due to $a^2(\tau)$!
- Change the variable to ***v = a(τ)δχ***.

Second-order action for δχ

The canonical variable for quantization: *v = a(τ)δχ*

- Change the variable to ***v = a(τ)δχ***. Hint: Use integration by parts for *-vv'a'/a* –> *v*²*(a'/a)'*

$$I^{(2)} = \int d\tau d^3\mathbf{x} \left[\frac{1}{2} \left(v'^2 - \nabla v \cdot \nabla v + \boxed{\frac{a''}{a} v^2} \right) - \frac{1}{2} a^2 m^2 v^2 \right]$$

This term indicates that (√-*g*)(∂χ)²
is **not** invariant under the
conformal transformation!

- The equation of motion is

$$v'' - \nabla^2 v + \left(m^2 a^2 - \frac{a''}{a} \right) v = 0$$

Non-expanding case

$$\left(\Longleftrightarrow \delta\chi'' - \nabla^2 \delta\chi + m^2 \delta\chi = 0 \right)$$

Second-order action for δχ

The canonical variable for quantization: $v = a(\tau)\delta\chi$

- Change the variable to $v = a(\tau)\delta\chi$.

- The solution admits a growing mode (instability) if $k^2 + m^2a^2 - a''/a < 0$!

- The equation of motion is

$$v'' - \nabla^2 v + \left(m^2 a^2 - \frac{a''}{a}\right) v = 0$$

In Fourier space

$$\Rightarrow v''_{\mathbf{k}} + \left(k^2 + m^2 a^2 - \frac{a''}{a}\right) v_{\mathbf{k}} = 0$$

Inflation can produce massive spin-0 particles!

Wavelength vs. the Hubble length

Massless case (m=0)

$$v''_{\mathbf{k}} + \left(k^2 - \frac{2}{\tau^2}\right) v_{\mathbf{k}} = 0$$

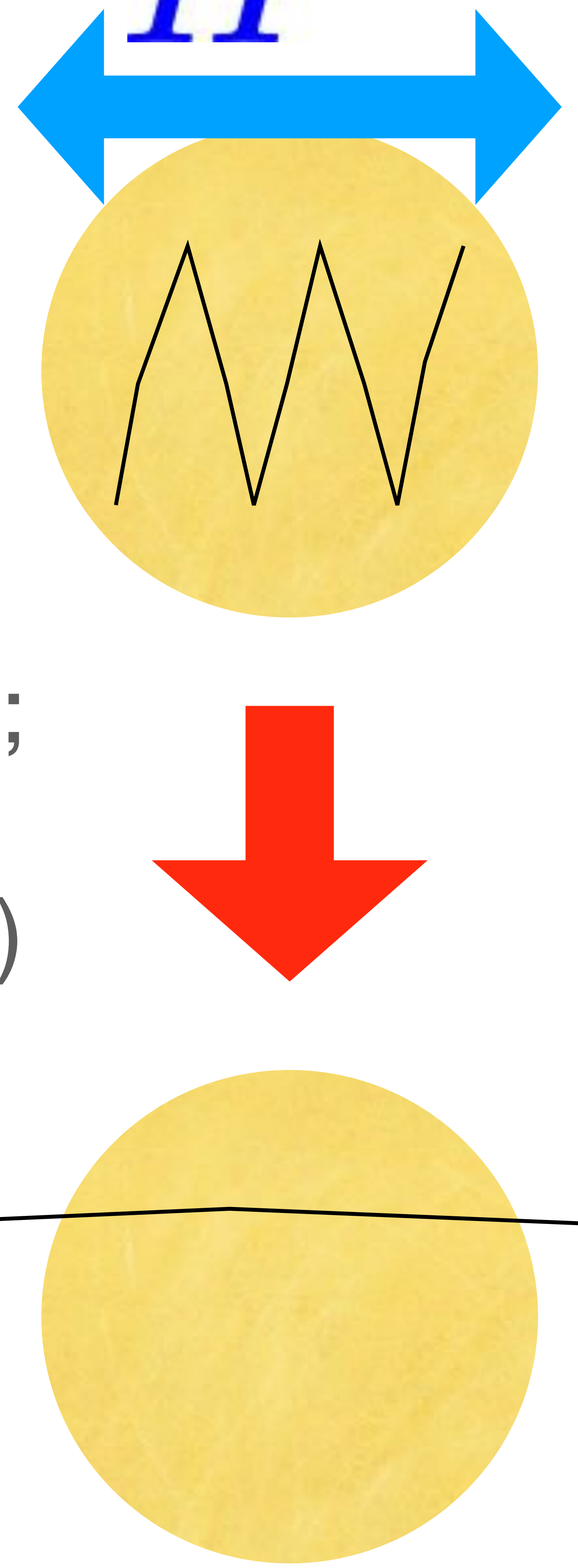
Hint: $\frac{a''}{a} = \frac{2}{\tau^2}$ for $a(\tau) = -(H\tau)^{-1}$

A growing mode for $-k\tau < \sqrt{2}$.

- **k** is the *comoving* wavenumber, derived from the comoving coordinates **x.** It is therefore independent of time.
  - The ***physical* wavelength**, stretched by the expansion in proportion to $a(\tau)$, is $\boldsymbol{L = 2\pi\, a(\tau)/k}$
- $-k\tau < \sqrt{2}$ gives $k < \sqrt{2}\, aH$ *and* $L > (\sqrt{2}\, \pi H)^{-1}$ ~ the Hubble length, H^{-1} (c=1).
  - The mode function behaves in a different way above and below H^{-1}.

Starobinsky (1980); Sato (1981); Guth (1981); Linde (1982); Albrecht & Steinhardt (1982)

Cosmic Inflation



$$H^{-1}$$

Quantum-mechanical fluctuation on microscopic scales

Mukhanov & Chibisov (1981);
Hawking (1982); Starobinsky (1982);
Guth & Pi (1982);
Bardeen, Turner & Steinhardt (1983)

$$L \ll H^{-1}$$

$$k \gg aH \;\; (-k\tau \gg 1)$$

Exponential Expansion!

$$v_{\mathbf{k}}'' + \left(k^2 - \frac{2}{\tau^2}\right) v_{\mathbf{k}} = 0$$

$$L \gg H^{-1}$$

$$k \ll aH \;\; (-k\tau \ll 1)$$

A growing mode for $-k\tau < \sqrt{2}$.

- **Exponential expansion stretches the wavelength of quantum fluctuations to cosmological scales.**

Quantization: Massless case (m=0)

Let's get some massless spin-0 particles out of the vacuum.

$$v''_{\mathbf{k}} + \left(k^2 - \frac{2}{\tau^2}\right) v_{\mathbf{k}} = 0$$

A growing mode for $-k\tau < \sqrt{2}$.

Hint: $\frac{a''}{a} = \frac{2}{\tau^2}$ for $a(\tau) = -(H\tau)^{-1}$

- Writing $v_{\mathbf{k}}$ in terms of creation and annihilation operators,

$$v_{\mathbf{k}}(\tau) = u_k(\tau)\hat{a}_{\mathbf{k}} + u_k^*(\tau)\hat{a}^\dagger_{-\mathbf{k}}$$

with $[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = \delta_D(\mathbf{k} - \mathbf{k}')$

- The mode function has a solution

$$u_k(\tau) = C_k \left[\cos(k\tau) - \frac{\sin(k\tau)}{k\tau}\right] + D_k \left[\frac{\cos(k\tau)}{k\tau} + \sin(k\tau)\right]$$

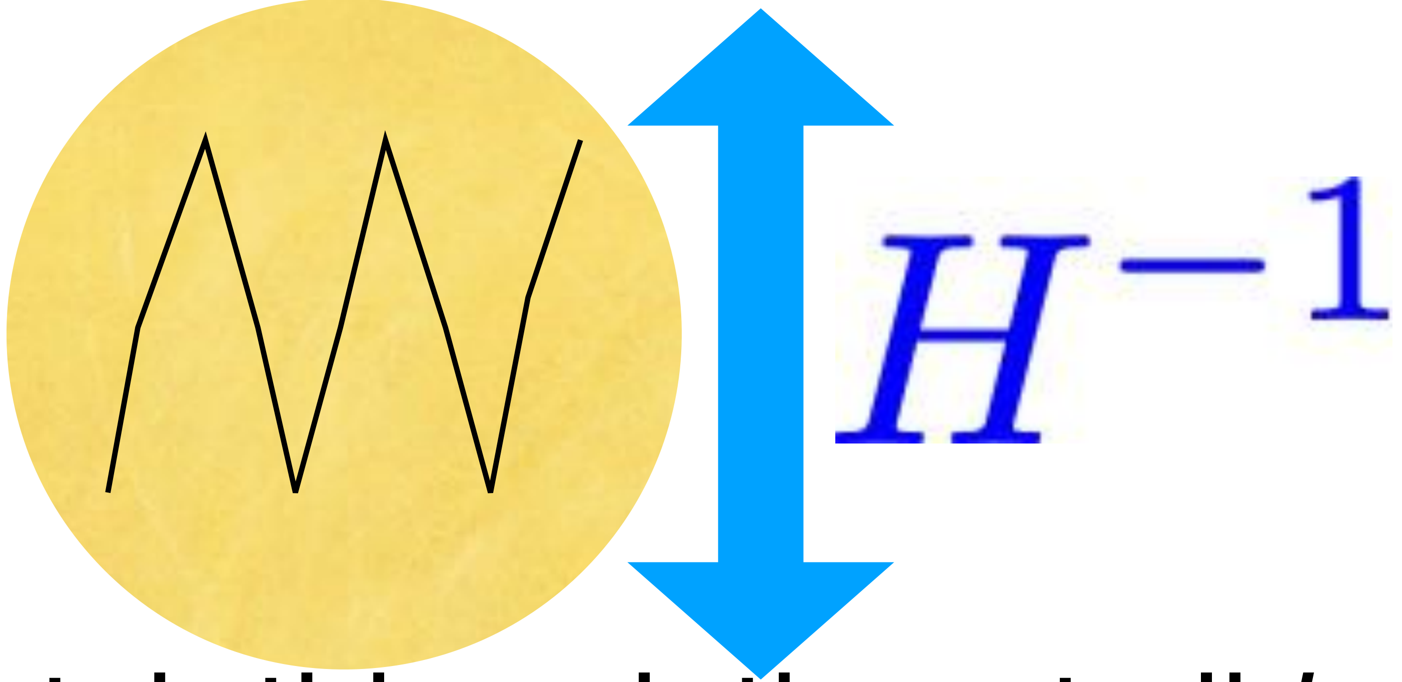
where C_k and D_k are integration constants. How do we determine them?

The vacuum state

The choice of a vacuum state determines the normalization of u_k

- Choose C_k and D_k to match the solution to the vacuum solution (Day 3)

$$u_k(\tau) \rightarrow \frac{e^{-ik\tau}}{\sqrt{2k}}$$



Quantum-mechanical fluctuation on micro scales

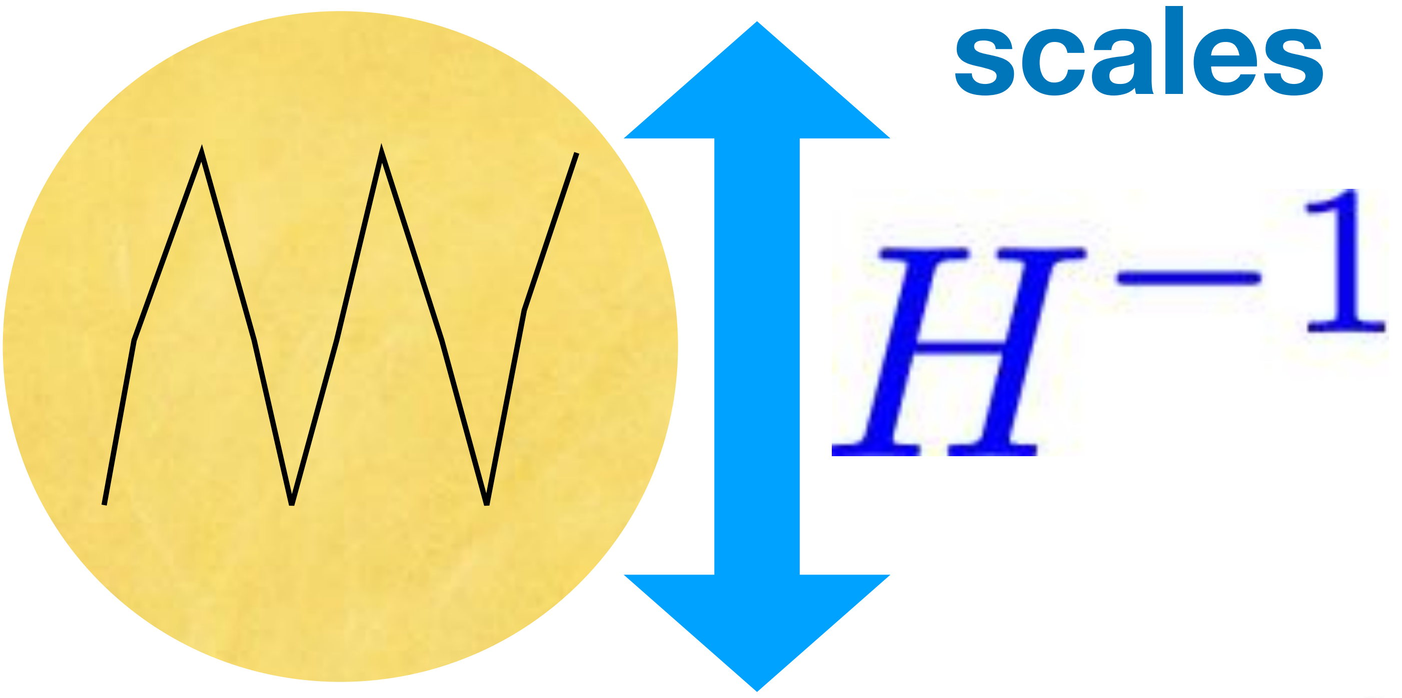
- However, it is not possible to match this solution at all k. **This is due to particle production by the inflationary expansion!**
    - **The idea:** At very short wavelengths (large values of k), the quantum state is still in the vacuum. In the limit $-k\tau \to \infty$, the solution approaches $u_k \to C_k \cos(k\tau) + D_k \sin(k\tau)$. Therefore, $\boldsymbol{C_k = 1/\sqrt{2k}}$ and $\boldsymbol{D_k = -i/\sqrt{2k}}$.

The full solution for *δχ*

Different behavior above and below the Hubble length, *H^{-1}*

$$u_k(\tau) = \frac{i}{(-\tau)} \frac{e^{-ik\tau}}{\sqrt{2k^3}} (1 + ik\tau)$$

Quantum-mechanical fluctuation on micro scales



- Recalling $\delta\chi = v/a = v(-H\tau)$, one obtains

$$(-H\tau)u_k(\tau) = iH \frac{e^{-ik\tau}}{\sqrt{2k^3}} (1 + ik\tau) \rightarrow (-H\tau)\frac{e^{-ik\tau}}{\sqrt{2k}}$$

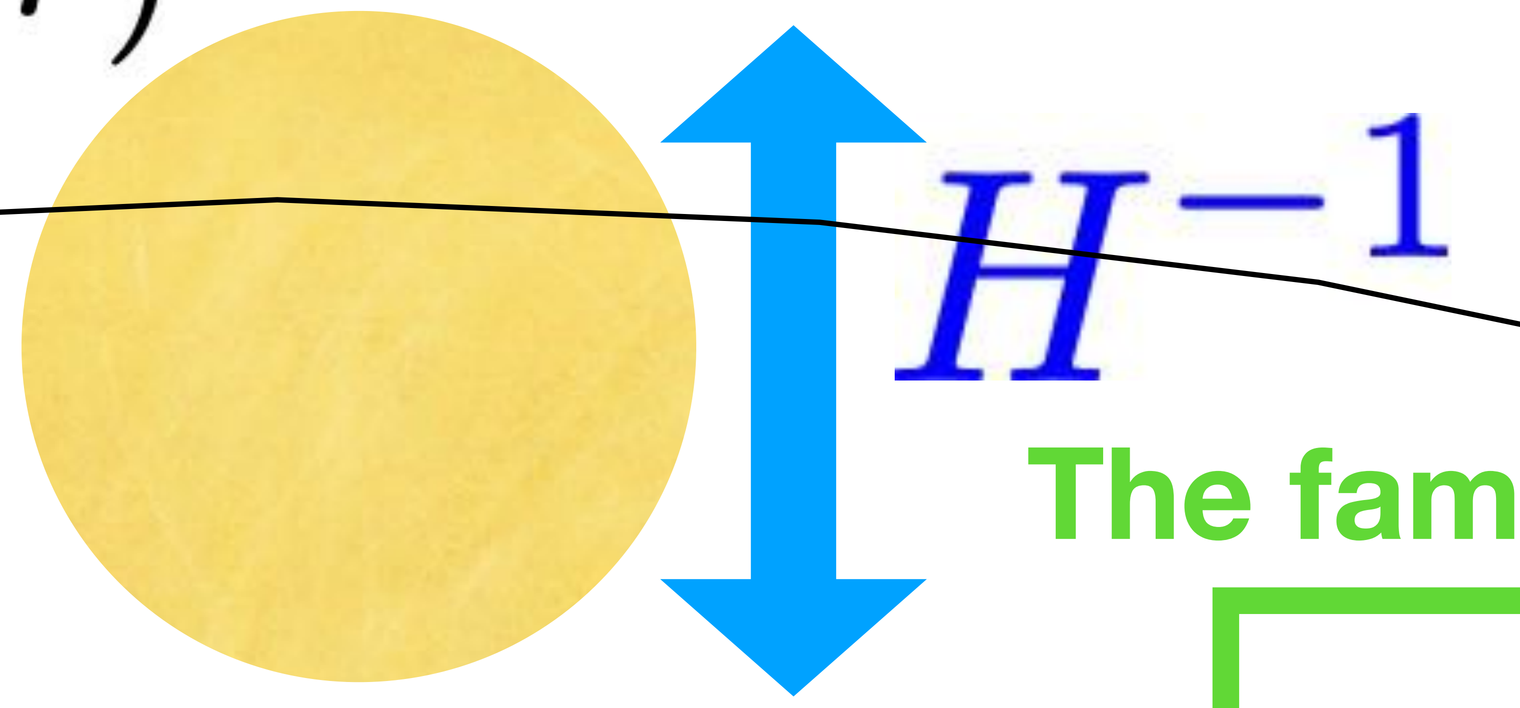
Short wavelength ($-k\tau \rightarrow \infty$)

The vacuum solution.

The full solution for *δχ*

$$u_k(\tau) = \frac{i}{(-\tau)} \frac{e^{-ik\tau}}{\sqrt{2k^3}} (1 + ik\tau)$$

Expansion stretches the wavelength of quantum fluctuations to cosmological scales.



- Recalling $\delta\chi = v/a = v(-H\tau)$, one obtains

$$(-H\tau)u_k(\tau) = iH \frac{e^{-ik\tau}}{\sqrt{2k^3}} (1 + ik\tau) \;\rightarrow\; i\frac{H}{\sqrt{2k^3}}$$

The famous result

Long wavelength ($-k\tau \rightarrow 0$)

This is nothing like
The vacuum solution.
Particle production
during inflation!

The 2-point correlation function

One step closer to the observable quantity.

$$\delta\chi(\tau,\mathbf{x}) = \frac{1}{a(\tau)}\int \frac{d^3\mathbf{k}}{(2\pi)^{2/3}}\left[u_k(\tau)\hat{a}_{\mathbf{k}} + u_k^*(\tau)\hat{a}^\dagger_{-\mathbf{k}}\right]e^{i\mathbf{k}\cdot\mathbf{x}}$$

- The 2-point correlation function, $\xi(r)$, is defined by

$$\xi(\tau,r) = \langle 0|\delta\chi(\tau,\mathbf{x})\delta\chi(\tau,\mathbf{x}+\mathbf{r})|0\rangle$$
$$= \frac{1}{a^2(\tau)}\int \frac{d^3\mathbf{k}}{(2\pi)^3}|u_k(\tau)|^2 e^{i\mathbf{k}\cdot\mathbf{r}}$$

Hint:

$$\langle 0|0\rangle = 1\,, \quad \hat{a}_{\mathbf{k}}|0\rangle = 0$$
$$\langle 0|\hat{a}_{\mathbf{k}}\hat{a}_{\mathbf{q}}|0\rangle = 0$$
$$\langle 0|\hat{a}^\dagger_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{q}}|0\rangle = 0$$
$$\langle 0|\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{q}}|0\rangle = 0$$
$$\langle 0|\hat{a}_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{q}}|0\rangle = \langle 0|[\hat{a}_{\mathbf{k}},\hat{a}^\dagger_{\mathbf{q}}]|0\rangle = \delta_D(\mathbf{k}-\mathbf{q})$$

$$
\begin{aligned}
\xi(\tau, r) &= \langle 0 | \delta\chi(\tau, \mathbf{x}) \delta\chi(\tau, \mathbf{x}+\mathbf{r}) | 0 \rangle \\
&= \frac{1}{a^2(\tau)} \int \frac{d^3\mathbf{k}}{(2\pi)^3} |u_k(\tau)|^2 e^{i\mathbf{k}\cdot\mathbf{r}} \\
&= \frac{1}{a^2(\tau)} \int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3} |u_k(\tau)|^2 \int_{-1}^{1} \frac{d\mu}{2} e^{ikr\mu} \\
&= \frac{1}{a^2(\tau)} \int_0^\infty \frac{dk}{k} \frac{k^3 |u_k(\tau)|^2}{2\pi^2} \frac{\sin(kr)}{kr} \\
&= \int_0^\infty \frac{dk}{k} \boxed{\left(\frac{H}{2\pi}\right)^2} \left[1 + (k\tau)^2\right] \frac{\sin(kr)}{kr}
\end{aligned}
$$

The famous result for a massless scalar field during inflation.

$$\delta\chi_{\rm rms} = \frac{H}{2\pi}$$

Problem Set 4

Scale-invariant power spectrum on large scales

- The Fourier transform of the 2-point correlation function is the power spectrum,

$$P(k) = \int d^3\mathbf{r}\ \xi(r) e^{-i\mathbf{k}\cdot\mathbf{r}}$$

Hint: $\int d^3\mathbf{r}\ e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} = (2\pi)^3\delta_D(\mathbf{k}-\mathbf{k}')$

  - Show that $\boldsymbol{k^3P(k)/(2\pi^2) = (H/2\pi)^2}$ on large scales, which is independent of the wavenumber. This is called the “scale invariant power spectrum”.

$$g^{\mu\nu} = a^{-2}\mathrm{diag}(-1, \mathbf{1})$$

$$\sqrt{-g} = a^4$$

- Adding the Chern-Simons term to the action for χ, show that

$$\Box\chi - \frac{\partial V}{\partial \chi} = -\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B} \quad \text{where} \quad F\tilde{F} = -4\mathbf{E}\cdot\mathbf{B}$$

$$\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu) = \frac{1}{a^2}\left(-\frac{\partial^2}{\partial\tau^2} - 2\frac{a'}{a}\frac{\partial}{\partial\tau} + \nabla^2\right)$$

Anber, Sorbo (2010); Barnaby, Peloso (2011)

Sourced contribution from the Chern-Simons term

Parity violation in $\delta\chi$

$$\Box\chi - \frac{\partial V}{\partial \chi} = -\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B}$$

- The right hand side is the second-order fluctuation.
  - **E** and **B** cannot have a uniform background, if we impose spatial isotropy (no preferred direction in space).
- The solution for the fluctuation, δχ, has two terms: $\delta\chi = \delta\chi_{\text{vacuum}} + \delta\chi_{\text{sourced}}$.
  - The first term is the homogeneous solution to the equation, $(\Box - m^2)\delta\chi_{\mathrm{vac}} = 0$
  - The second term is the particular solution, including $-(\alpha/f)\mathbf{E}\cdot\mathbf{B}$

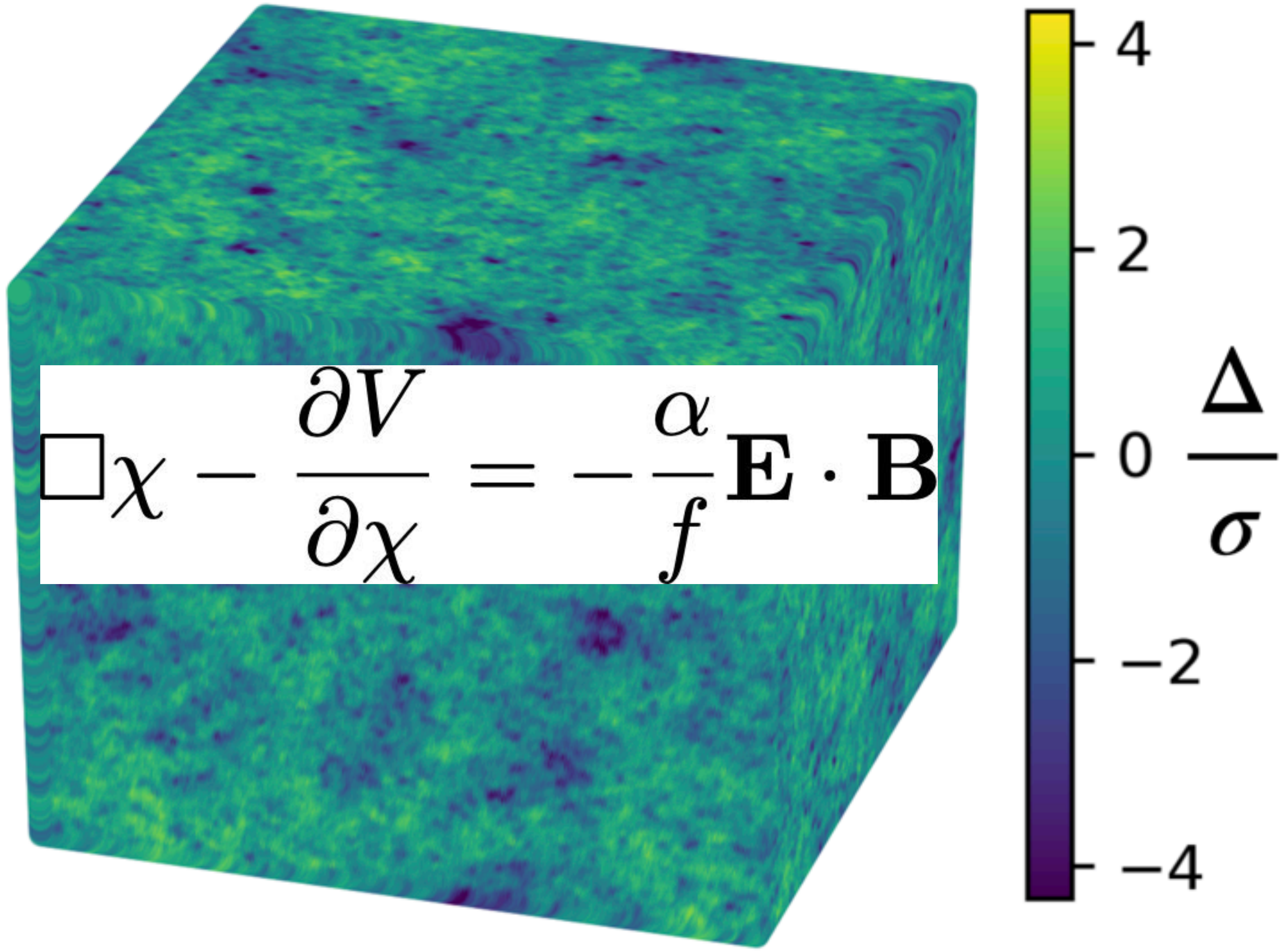
This is highly non-Gaussian and parity violating!

Caravano, EK, et al., arXiv:2204.12874

Truly *ab initio* simulation!

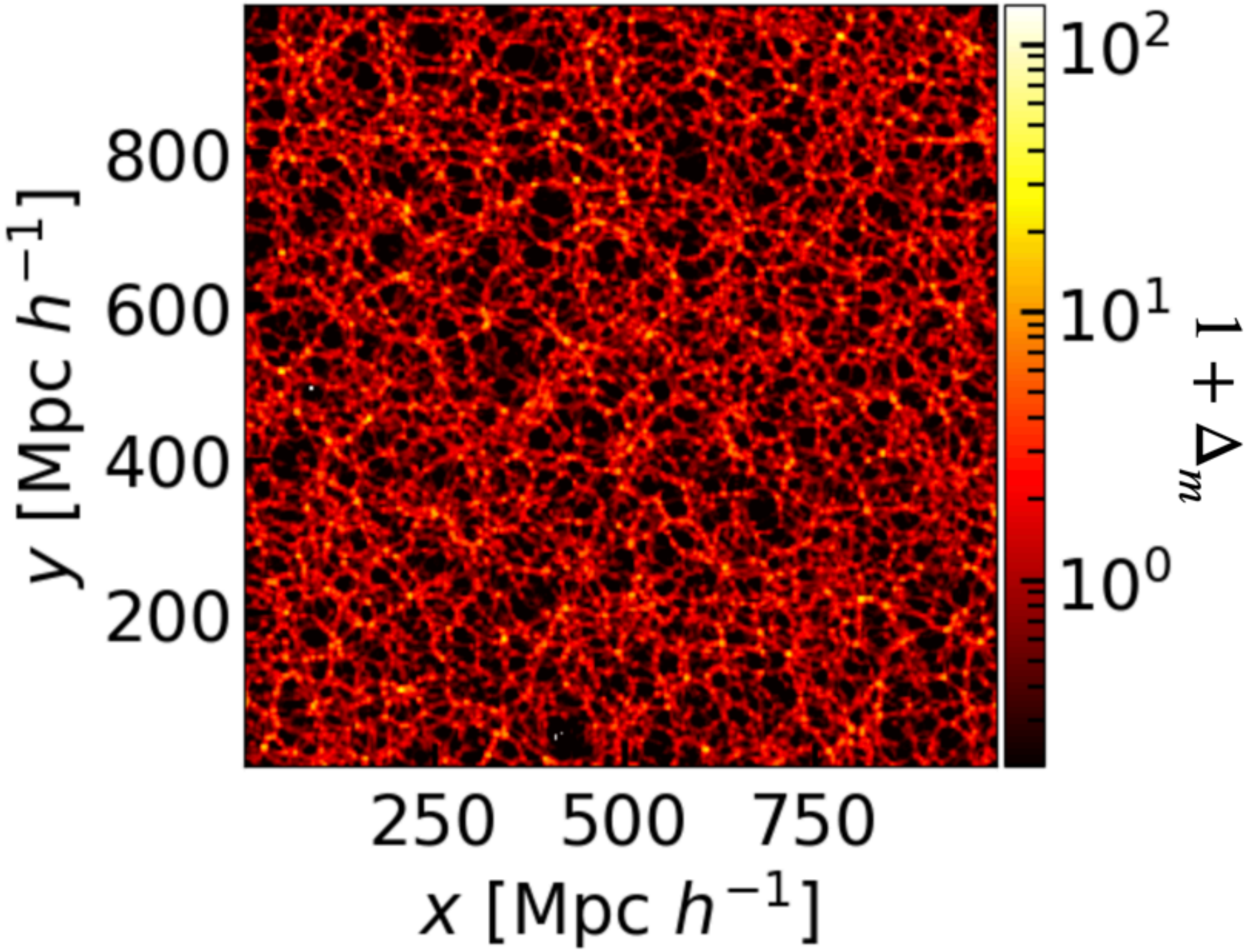
World's first lattice simulation of inflation

Angelo Caravano



$$\Box\chi - \frac{\partial V}{\partial \chi} = -\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B}$$

Drew Jamieson



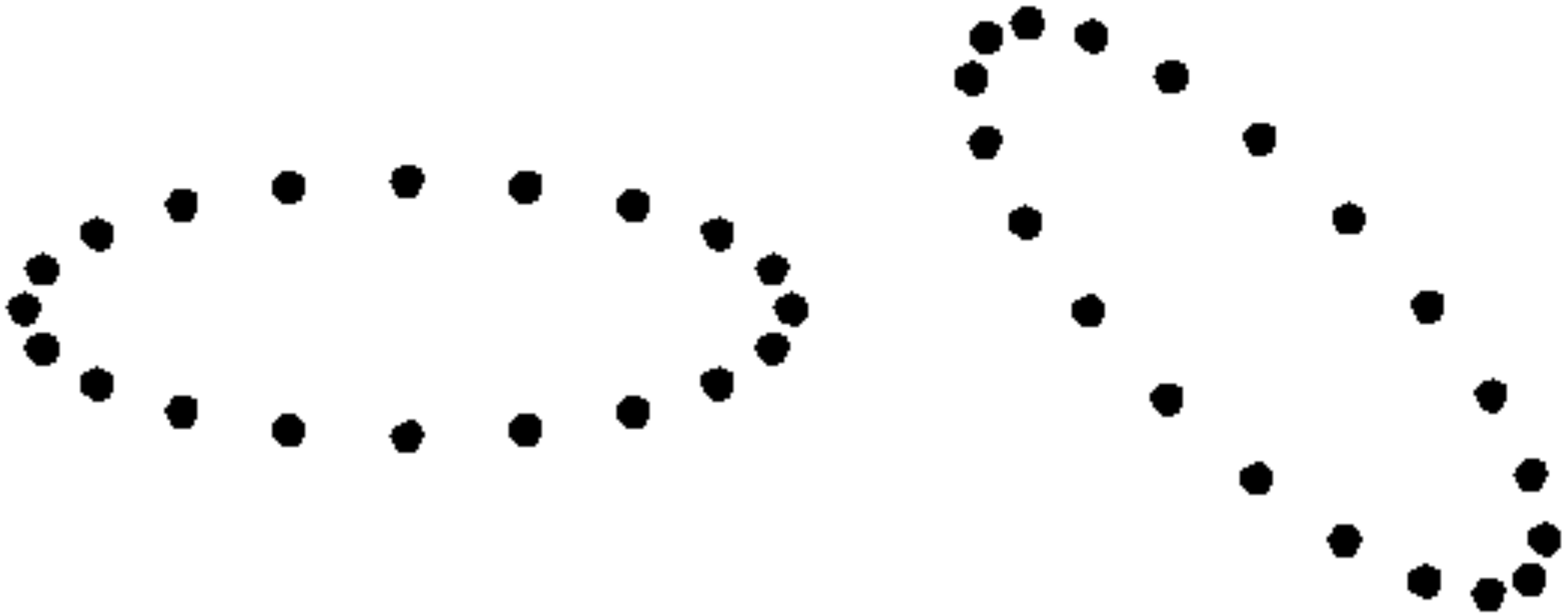
- (Left) Parity-violating and non-Gaussian density fluctuation during inflation.
- (Right) Outcome of N-body simulation at z=0, using the left panel as the initial condition.

3.6 Primordial Gravitational Waves

$$\Box h_{ij} = 16\pi G(E_i E_j + B_i B_j)^{\mathrm{TT}}$$

Gravitational waves are coming towards you!

To visualise the waves, watch motion of test particles.



Gravitational waves are coming towards you!

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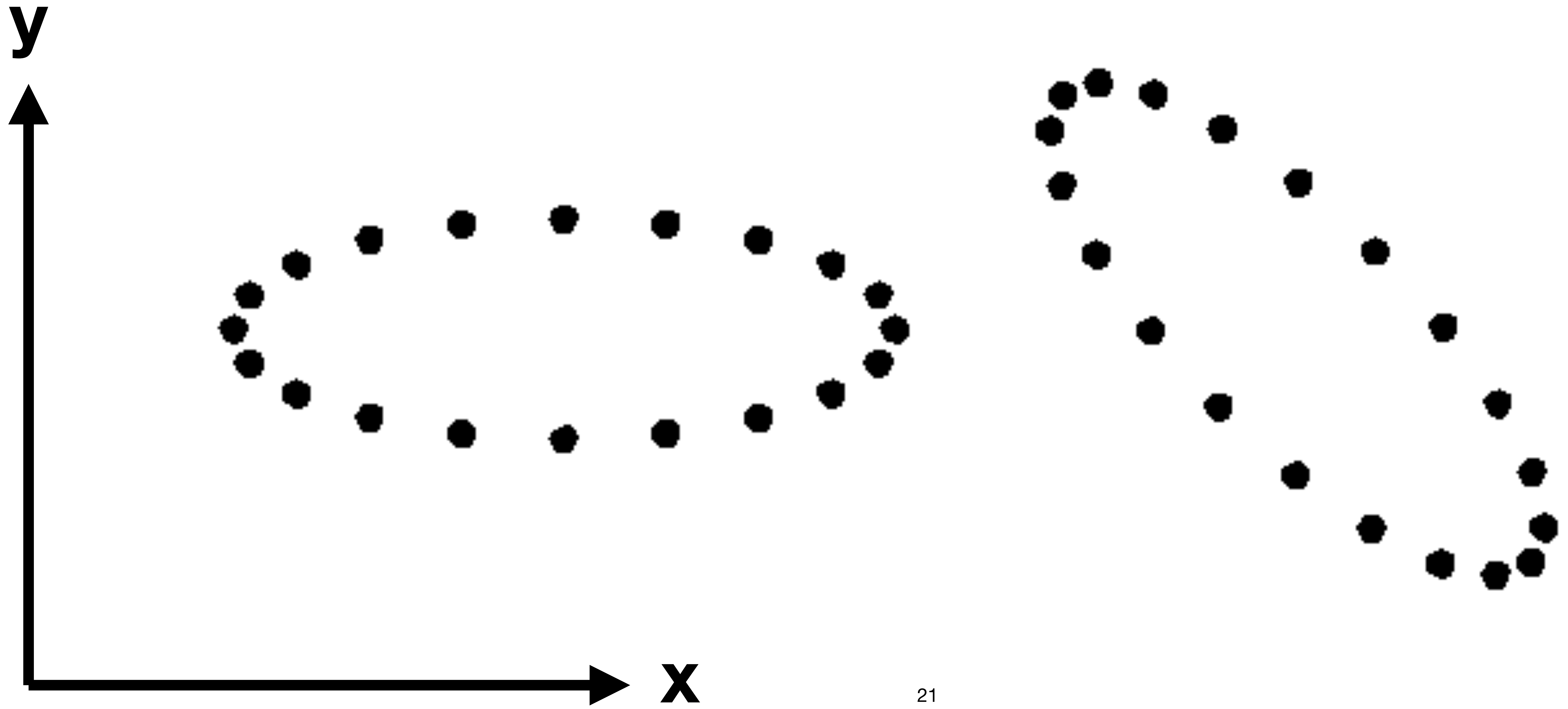


Distance between two points

Including *distortion* in space

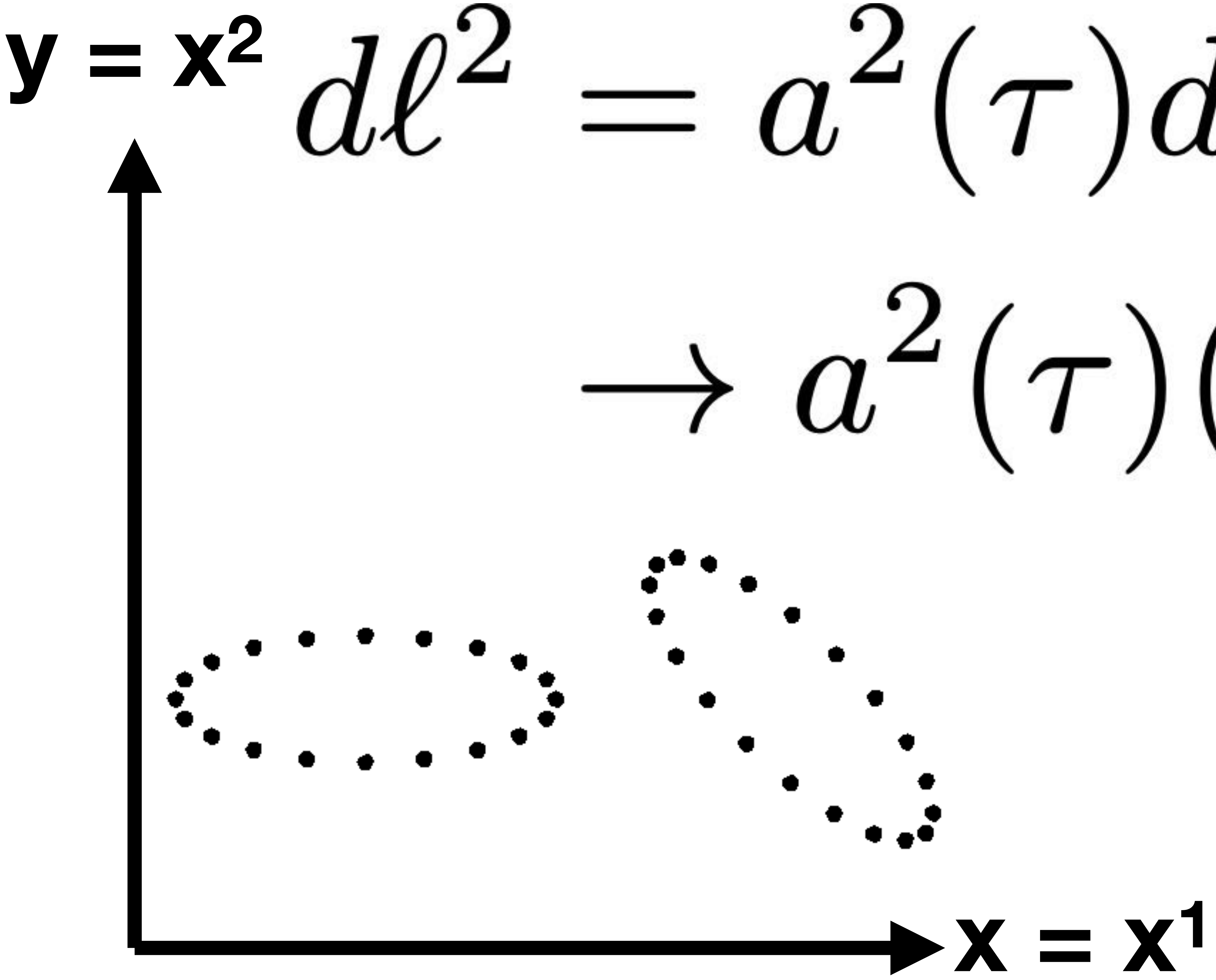
$\delta_{ij} = 1$ for $i{=}j$;
$\delta_{ij} = 0$ otherwise

$$d\ell^2 = a^2(\tau)d\mathbf{x}^2 = a^2(\tau)\delta_{ij}dx^i dx^j$$

$$\rightarrow a^2(\tau)(\delta_{ij} + h_{ij})dx^i dx^j$$

Distortion in space!

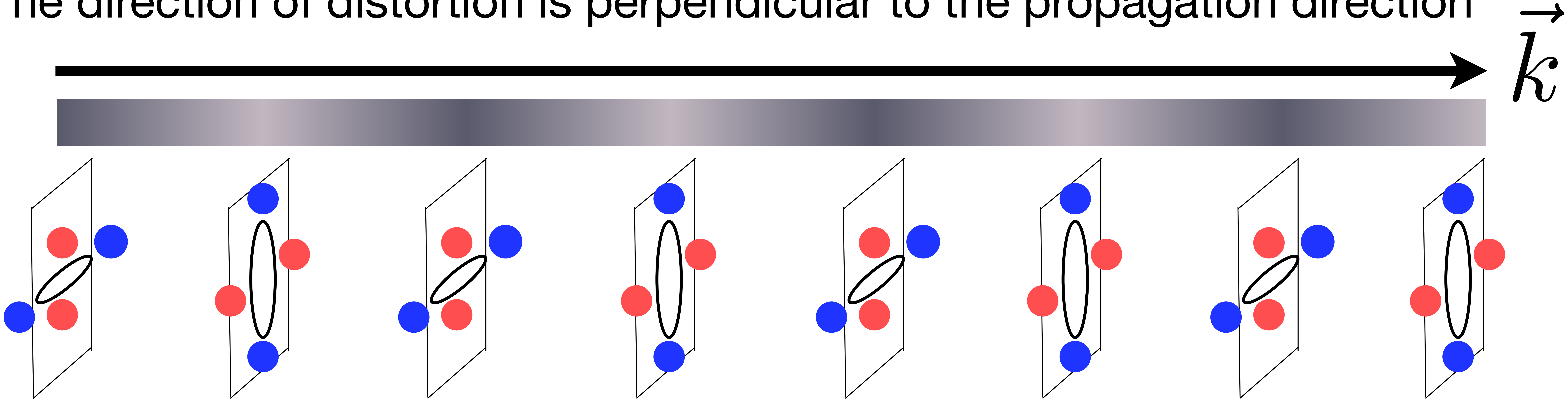
A real symmetric 3x3 matrix, h_{ij}, has **6 components**.



Four conditions for gravitational waves (GW)

Transverse (3 conditions)

- **The gravitational wave must be <u>transverse.</u>**
  - The direction of distortion is perpendicular to the propagation direction



Thus, $$\sum_{i=1}^{3} k^i h_{ij} = 0$$

3 conditions for h_{ij}

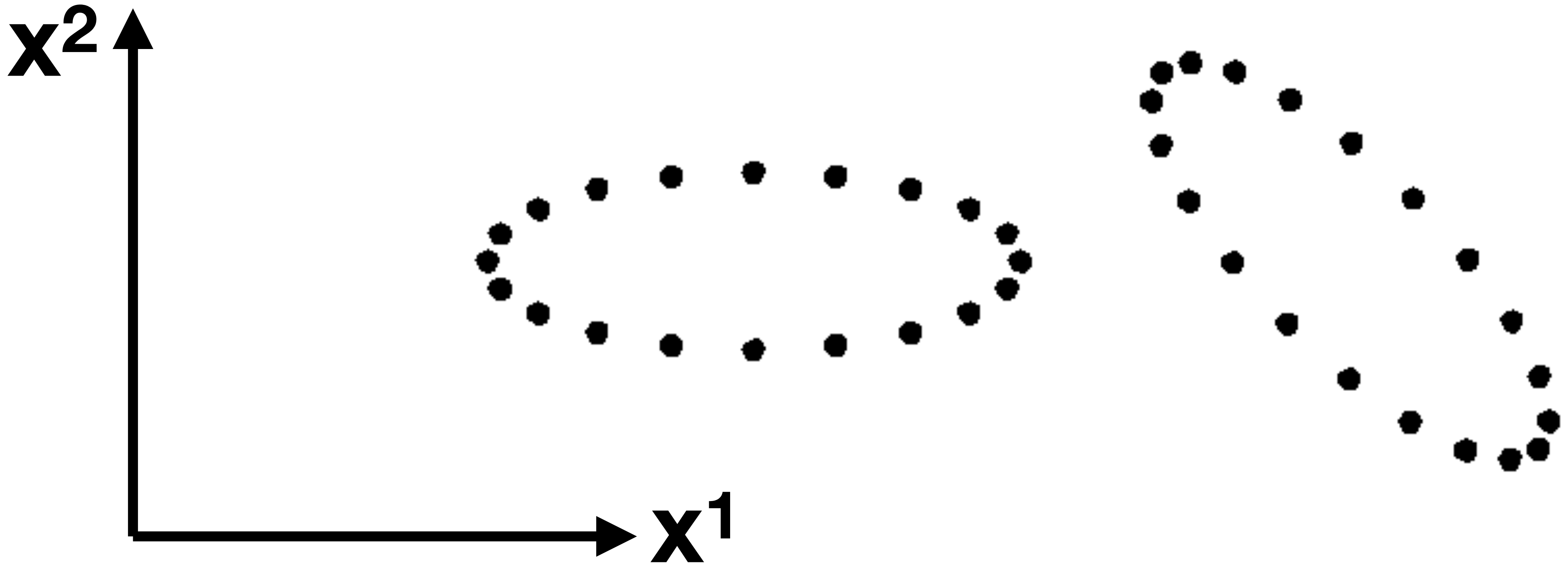
Four conditions for gravitational waves (GW)

Traceless (1 condition)

- **The gravitational wave must <u>not change the area</u>**
  - The determinant of $\delta_{ij}+h_{ij}$ is 1

$$d\ell^2 = a^2(\tau)(\delta_{ij} + h_{ij})dx^i dx^j$$

Det($\delta_{ij}+h_{ij}$) = 1



Thus, $$\sum_{i=1}^{3} h_{ii} = 0$$

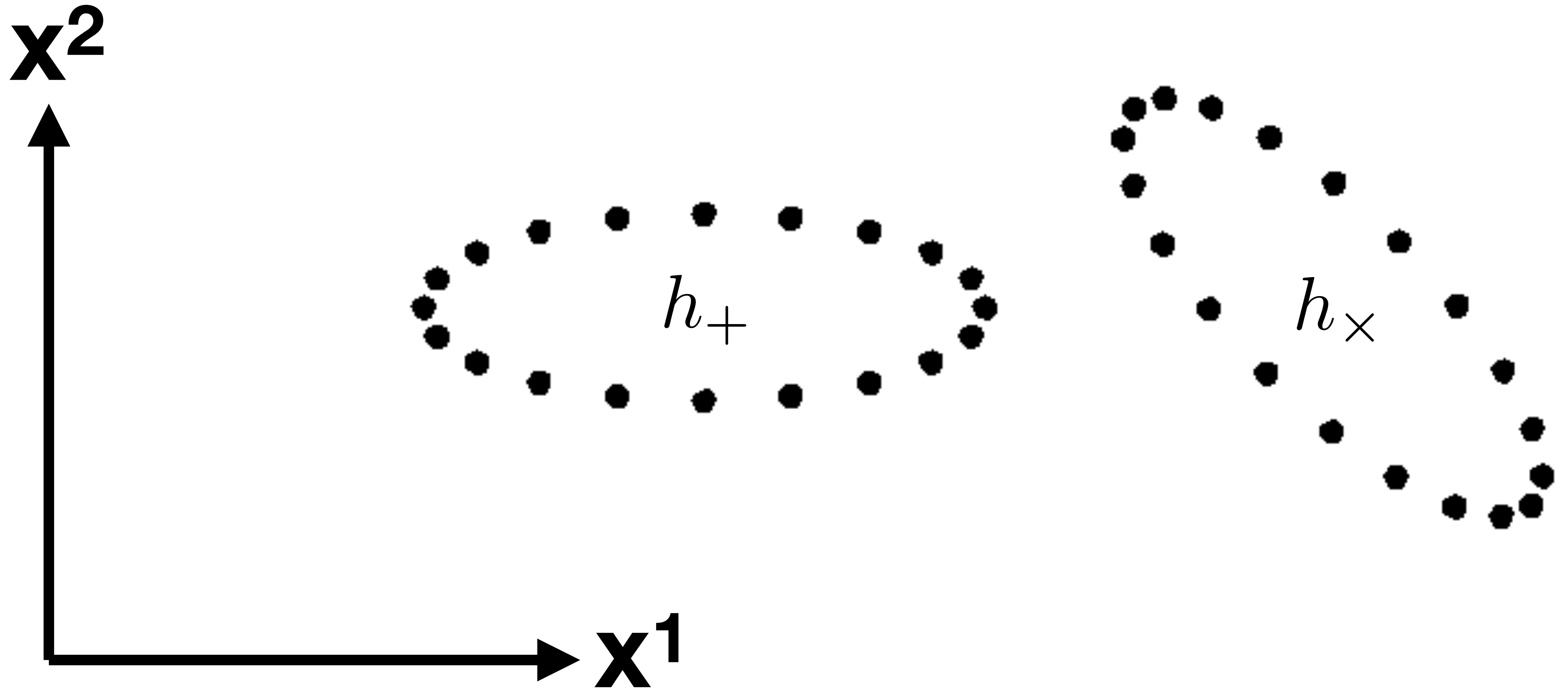
1 condition for h_{ij}

6 – 4 = 2 degrees of freedom for GW

We will call them “plus” and “cross” modes.

- The symmetric matrix h_{ij} has 6 components, but there are 4 conditions. This leaves **2 degrees of freedom**.
- If the GW propagates in the $x^3=z$ axis, non-vanishing components of h_{ij} are

$$h_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



propagation direction of GW $\vec{k}$
z
$h_+=\cos(kz)$
$h_\times=\cos(kz)$

Mukhanov, Feldman, Brandenberger (1992)

Second-order action for h_+ and $h_\times$

- Expand the Einstein-Hilbert action for General Relativity to second order in h_{ij}.

$$I_{\rm GR} = \int d\tau d^3\mathbf{x}\sqrt{-g}\,\frac{R}{16\pi G}$$

$$I_{\rm GR}^{(2)} = \int d\tau d^3\mathbf{x}\,\boxed{\frac{a^2(\tau)}{16\pi G}}\sum_{p=+,\times}\frac{1}{2}\left(h_p'^{\,2} - \nabla h_p \cdot \nabla h_p\right)$$

- h_p is **not** the canonical variable for quantization due to $a^2(\tau)/(16\pi G)$!
- Change the variable to $v_p = a(\tau)h_p/\sqrt{16\pi G}$.

Mukhanov, Feldman, Brandenberger (1992)

Second-order action for h_+ and $h_\times$

The canonical variable for quantization: $v_p = a(\tau)h_p/\sqrt{16\pi G}$

- Change the variable to $v_p = a(\tau)h_p/\sqrt{16\pi G}$. Hint: Use integration by parts for $-vv'a'/a \to v^2(a'/a)'$

$$I^{(2)}_{\rm GR} = \int d\tau d^3\mathbf{x} \sum_{p=+,\times} \left[\frac{1}{2}\left({v'_p}^2 - \nabla v_p \cdot \nabla v_p + \frac{a''}{a}v_p^2\right)\right]$$

This term indicates that GR is **not** invariant under the conformal transformation!

- The equation of motion is

$$v''_p - \nabla^2 v_p - \frac{a''}{a}v_p = 0$$

(Massive scalar field $\longleftrightarrow$ $v'' - \nabla^2 v + \left(m^2a^2 - \frac{a''}{a}\right)v = 0$)

A remarkable result:
v_p can be quantized like a massless scalar field.
We already did it!

Scale-invariant power spectrum for h_+ and $h_\times$

GWs from inflation are everywhere at all wavelengths!

$$M_{\rm Pl} = (8\pi G)^{-1/2} \simeq 2.4 \times 10^{18}\ {\rm GeV}$$

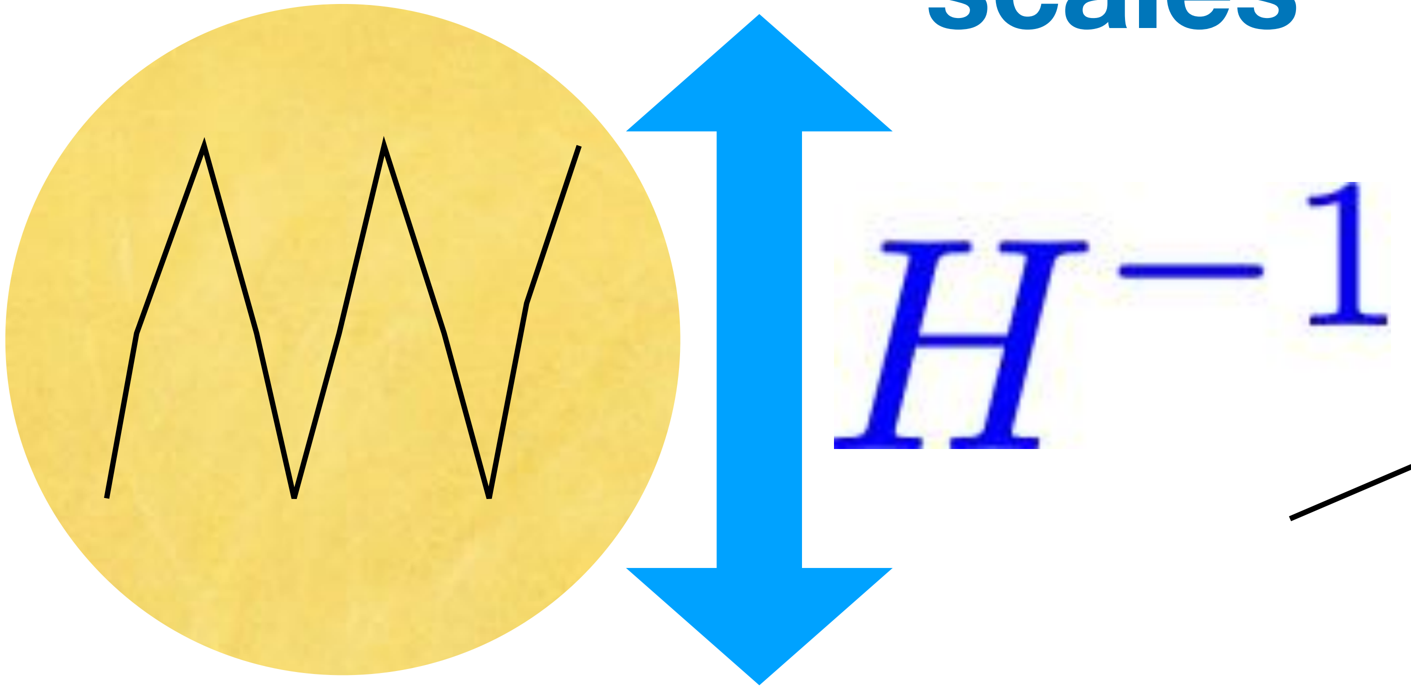
- The power spectrum is

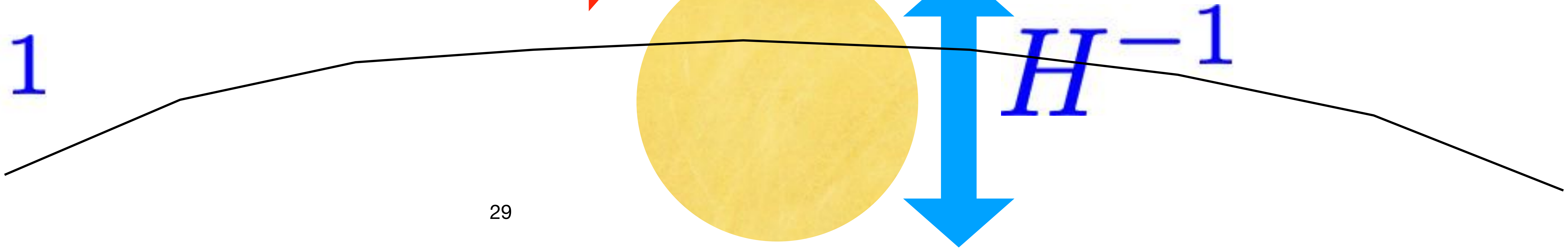
$$\frac{k^3 P_+(k)}{2\pi^2} = \frac{k^3 P_\times(k)}{2\pi^2} = 16\pi G \left(\frac{H}{2\pi}\right)^2 = \frac{2}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2$$

Quantum-mechanical fluctuation on micro scales

Exponential Expansion!

Expansion stretches the wavelength of quantum fluctuations to cosmological scales.

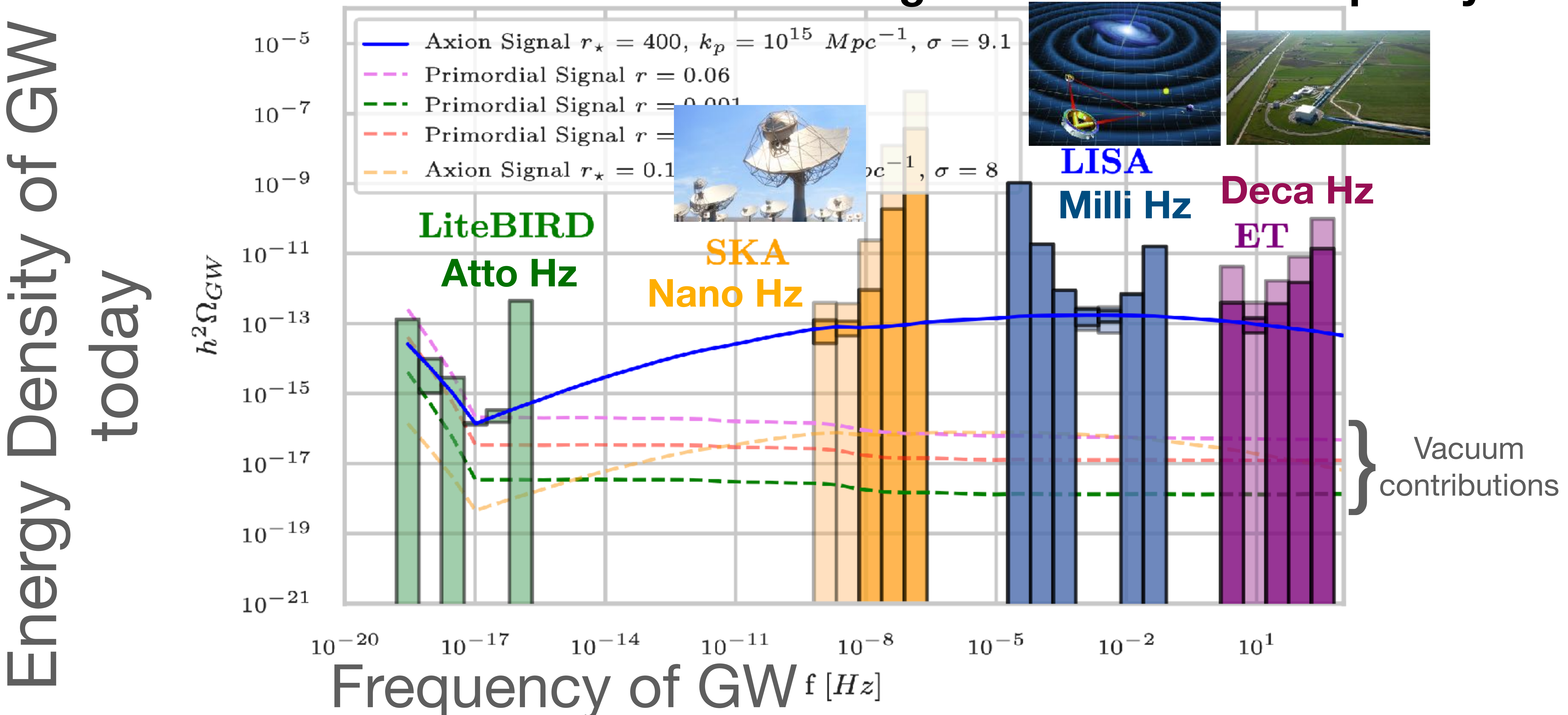




Campeti, EK, Poletti, Baccigalupi (2021)

GWs from the early Universe are everywhere!

We can measure it across 21 orders of magnitude in the GW frequency

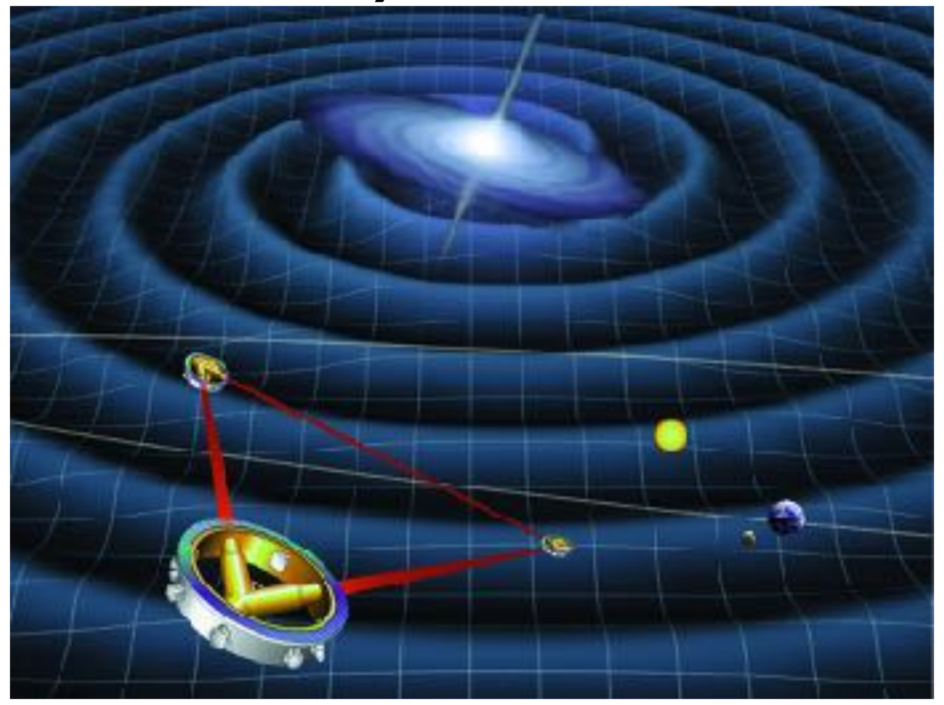


Let’s find Gravitational Waves (GW)!

But how? The detection method depends on the GW frequency.

- **Laser interferometers** on the ground: **deca- to kilo Hz** (*LIGO, VIRGO, …, ET*)
  - The wavelength ~ the size of Earth

- **Laser interferometers** in space: **milli Hz** (*LISA*), deci Hz (future mission?)
  - The wavelength ~ Astronomical Unit



- **Pulsar timing arrays**: **nano Hz** (*EPTA*, *SKA*)
  - The wavelength ~ light years

- **Cosmic microwave background**: **atto Hz** (*WMAP*, *Planck*, ***LiteBIRD***)
  - The wavelength ~ **billions of light years!**

3.7 Parity Violation in GW

$$\Box h_{ij} = 16\pi G(E_i E_j + B_i B_j)^{\mathrm{TT}}$$

GR + Maxwell (+ Chern-Simons)

$$\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$$
$$= \frac{1}{a^2}\left(-\frac{\partial^2}{\partial\tau^2} - 2\frac{a'}{a}\frac{\partial}{\partial\tau} + \nabla^2\right)$$

where $g^{\mu\nu} = a^{-2}\mathrm{diag}(-1, \mathbf{1})$

$\sqrt{-g} = a^4$

$$I = \int d\tau d^3\mathbf{x}\sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{4}F^2 - \frac{\alpha}{4f}\chi F\tilde{F}\right)$$

- The F^2 term contributes to the equation of motion for the GW via the stress-energy tensor (this is the second-order fluctuation).

$$\Box h_{ij} = 16\pi G(E_iE_j + B_iB_j)^{\mathrm{TT}}$$ "Transverse and Traceless"

- The $F\tilde{F}$ term does **not** contribute directly to the equation of motion for the GW.
  - But, it creates a parity violation in **E** and **B**, which also creates a parity violation in the GW.

Helicity basis to probe parity symmetry

Circular polarization states of GW. GW's helicity is $\lambda = \pm 2$.

- Just like for EM waves (Day 2),

$$A_\pm = \frac{A^1_{\mathbf{k}} \mp i A^2_{\mathbf{k}}}{\sqrt{2}}$$

A_+: Right-handed state

A_-: Left-handed state

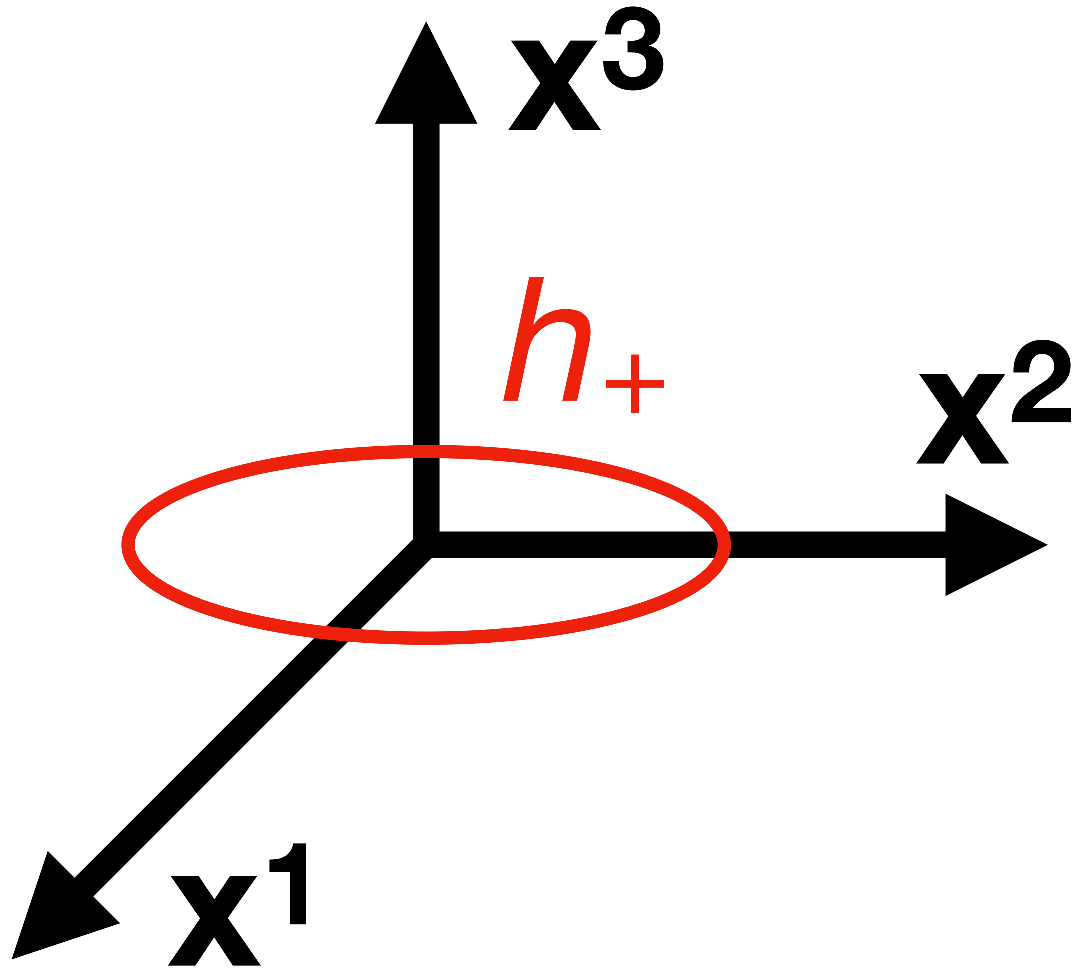
$$h_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

we write the helicity states of GW in Fourier space as

$$h_{\pm 2} = \frac{h_{+,\mathbf{k}} \mp i h_{\times,\mathbf{k}}}{\sqrt{2}}$$

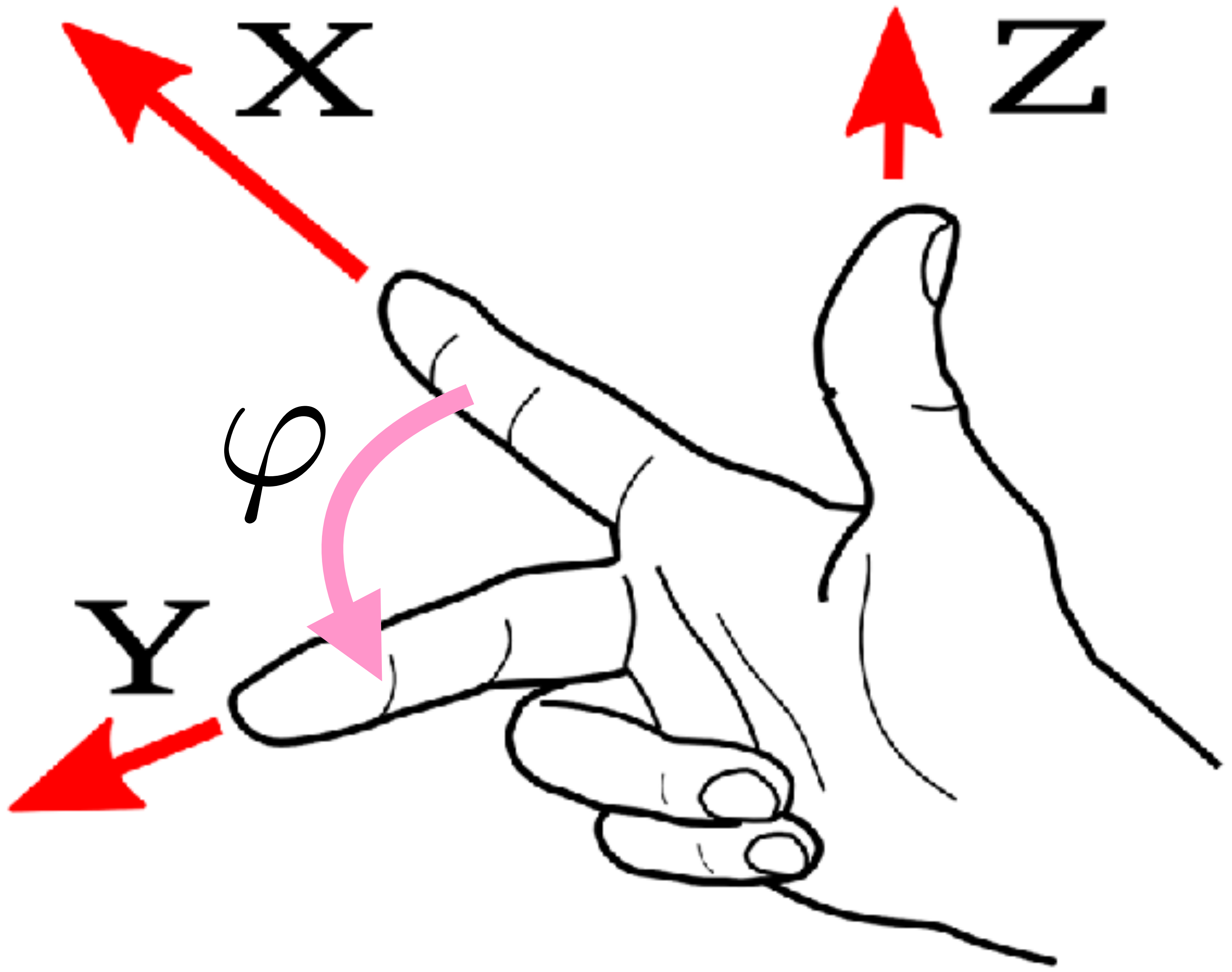
h_{+2}: Right-handed state

h_{-2}: Left-handed state



GW's helicity is λ=±2

Gravitons are massless spin-2 particles!



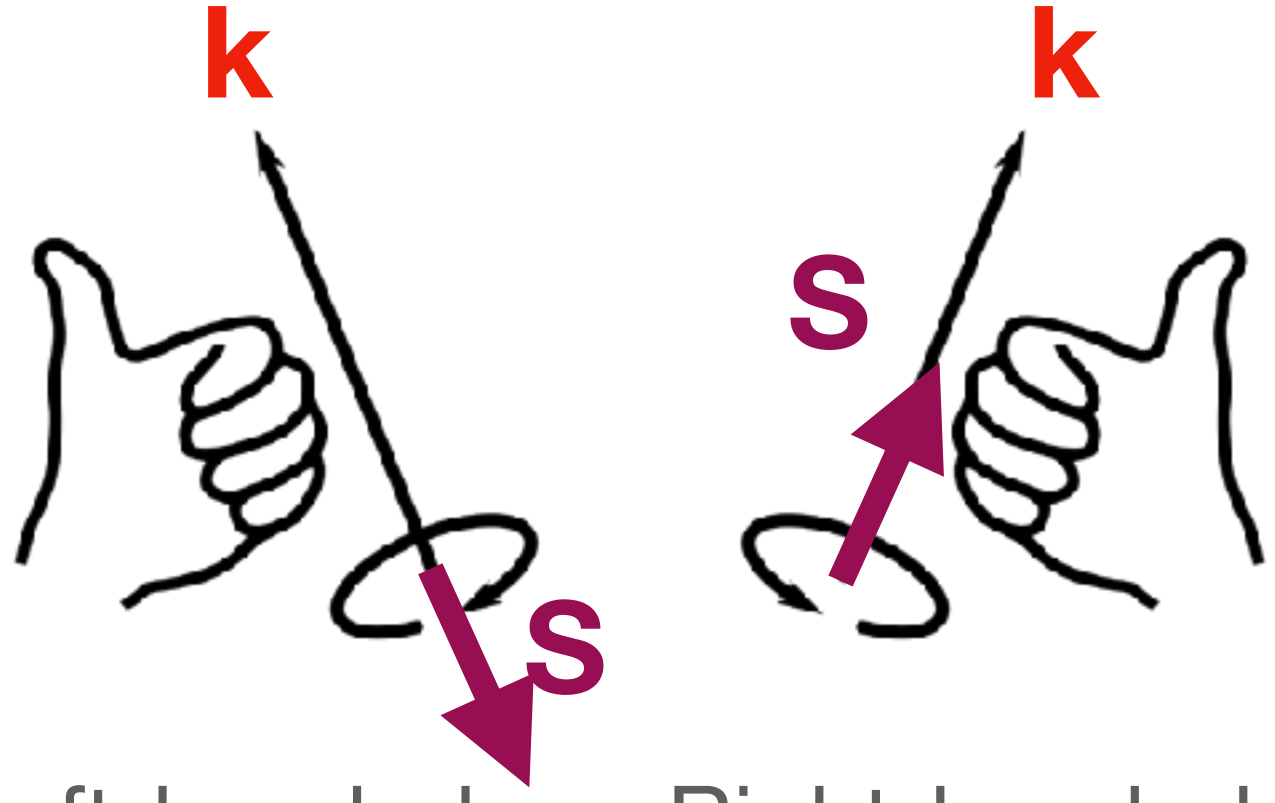
- To show that $h_{\pm 2}$ represents the helicity states, rotate the spatial coordinates around the z axis in the right-handed system by an angle φ .

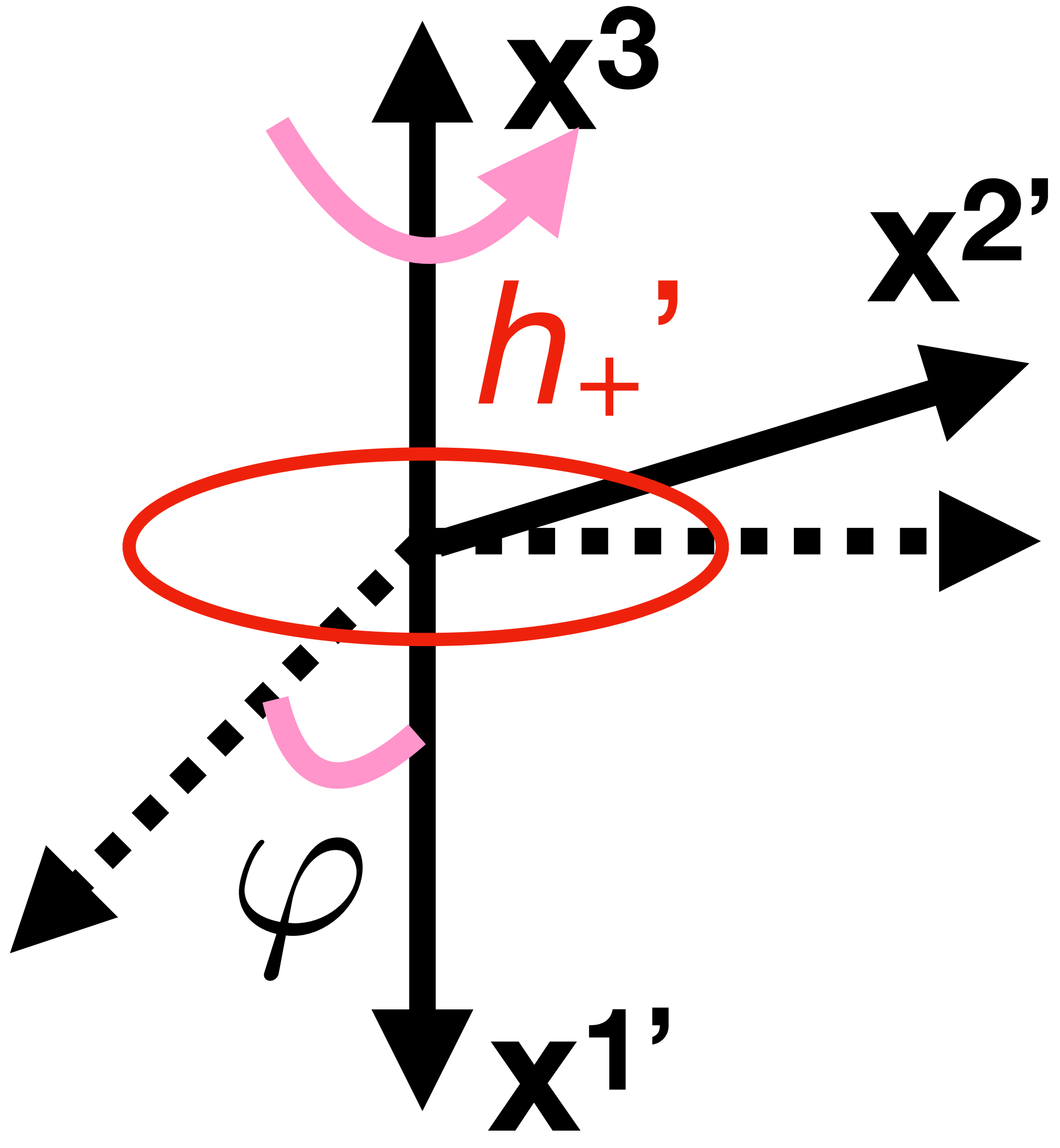
- The helicity states, λ=±2, transform as

$$h_\lambda \rightarrow h'_\lambda = e^{i\lambda\varphi} h_\lambda$$

Helicity

h_{+2}: Right-handed state
h_{-2}: Left-handed state





Sorbo (2011); Barnaby, Namba, Peloso (2011)

Parity Violation in GW

For a slowly varying ξ>0

$$\xi = \frac{\alpha\dot{\bar{\theta}}}{2H} = \frac{\alpha\dot{\chi}}{2Hf}$$

$$\frac{k^3 P_{+2}(k)}{2\pi^2} \simeq \frac{2}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2\left[1 + 8.6\times 10^{-7}\frac{H^2}{M_{\rm Pl}^2}\frac{e^{4\pi\xi}}{\xi^6}\right]$$

$$\frac{k^3 P_{-2}(k)}{2\pi^2} \simeq \frac{2}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2\left[1 + 1.8\times 10^{-9}\frac{H^2}{M_{\rm Pl}^2}\frac{e^{4\pi\xi}}{\xi^6}\right]$$

Vacuum contribution (the terms "1")

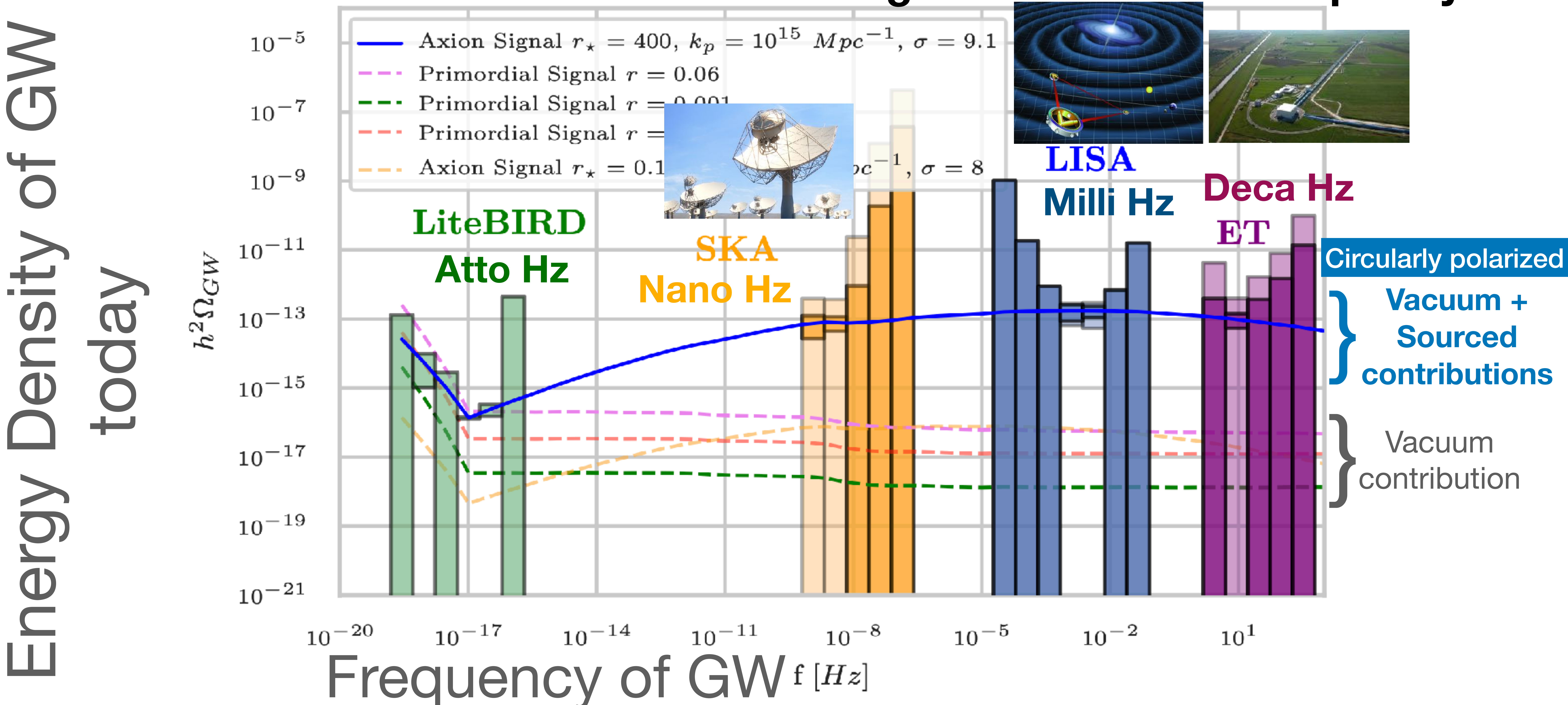
Parity Violation! (the terms 8.6×10^{-7} and 1.8×10^{-9})

- The sourced contributions are almost perfectly circularly polarized.
- The sum of the vacuum and sourced contributions is partially circularly polarized. **This can be observationally tested!** (Seto 2006; Seto, Taruya 2007)

Campeti, EK, Poletti, Baccigalupi (2021)

GWs from the early Universe are everywhere!

We can measure it across 21 orders of magnitude in the GW frequency



Recap: Day 4

- Quantum fluctuations in massless scalar fields and gravitational waves are amplified in the same way during inflation.
- Vacuum contributions are Gaussian, scale invariant, and parity symmetric, $k^3P(k)/(2\pi^2) = C(H/2\pi)^2$, where
  - C=1 for χ and C=16πG for h_+ and $h_\times$. No parity violation.
- Sourced contributions are **non-Gaussian**, **not scale invariant**, and **parity violating**! **This is a new paradigm in cosmology**.

$$\Box\chi - \frac{\partial V}{\partial \chi} = -\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B} \qquad \Box h_{ij} = 16\pi G(E_iE_j + B_iB_j)^{\mathrm{TT}}$$

  - These properties can be observationally tested using the density fluctuations (*scalar modes*) and gravitational waves (*tensor modes*).

Appendix: Conventions for the Power Spectrum of the GW

There is a factor of 2

- In the literature, the correlation function of the GW is often defined by summing over all indices of h_{ij}.

$$\xi_{\mathrm{t}}(\tau, r) = \sum_{ij} \langle 0 | h_{ij}(\tau, \mathbf{x}) h_{ij}(\tau, \mathbf{x} + \mathbf{r}) | 0 \rangle$$

"t" stands for "tensor modes"

$$= \boxed{2} \sum_{p=+,\times} \langle 0 | h_p(\tau, \mathbf{x}) h_p(\tau, \mathbf{x} + \mathbf{r}) | 0 \rangle$$

There is a factor of 2

- The power spectrum is given by

$$\mathcal{P}_{\rm t}(k) = \frac{k^3 P_{\rm t}(k)}{2\pi^2} = \boxed{2} \sum_{p=+,\times} \frac{k^3 P_p(k)}{2\pi^2}$$

$$= \frac{8}{M_{\rm Pl}^2} \left(\frac{H}{2\pi}\right)^2$$