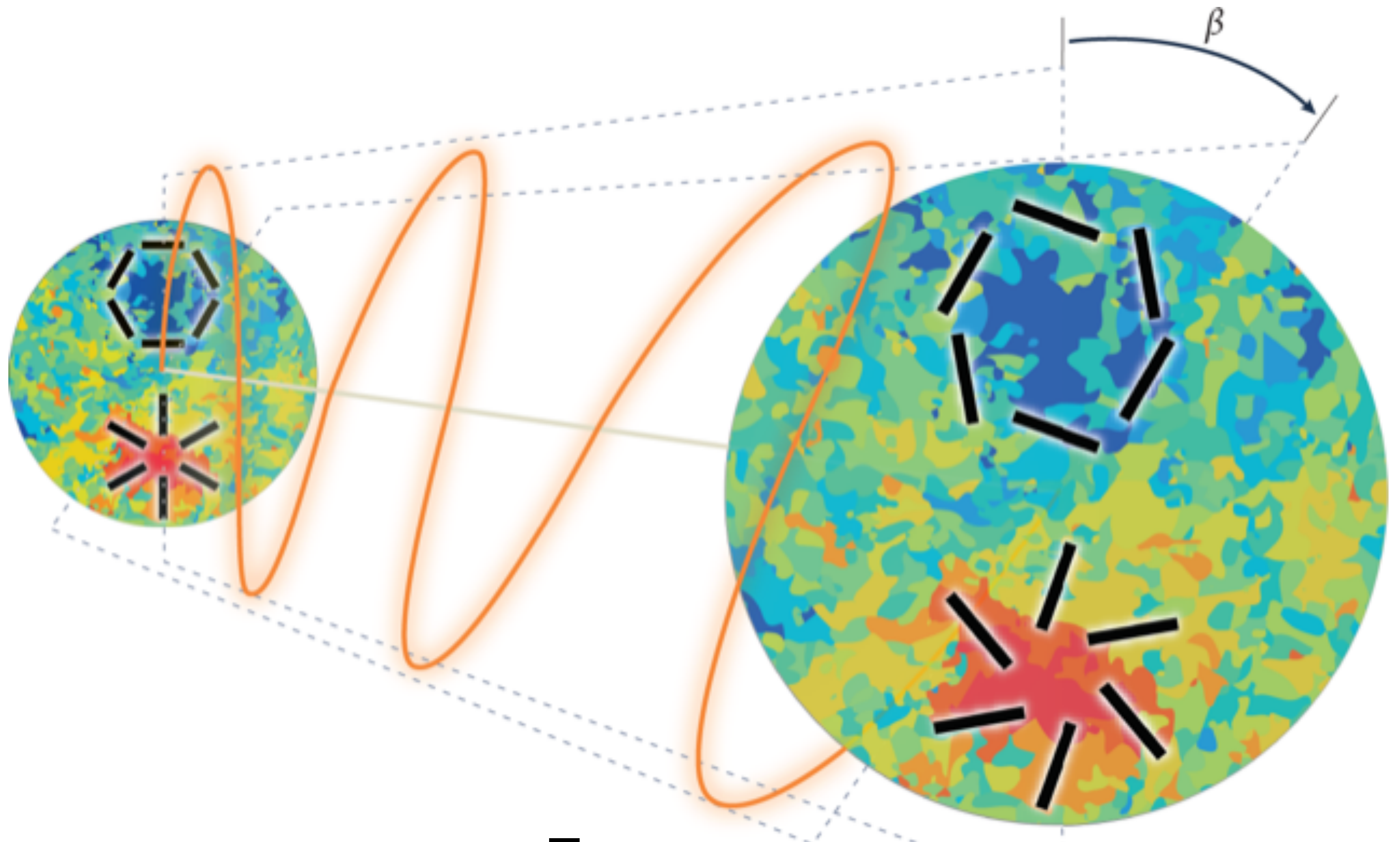
$$I_{\rm CS} = \int \mathrm{d}^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\tilde{F}\right)$$



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Parity Violation in Cosmology

In search of new physics for the Universe

The lecture slides are available at
https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Day 3

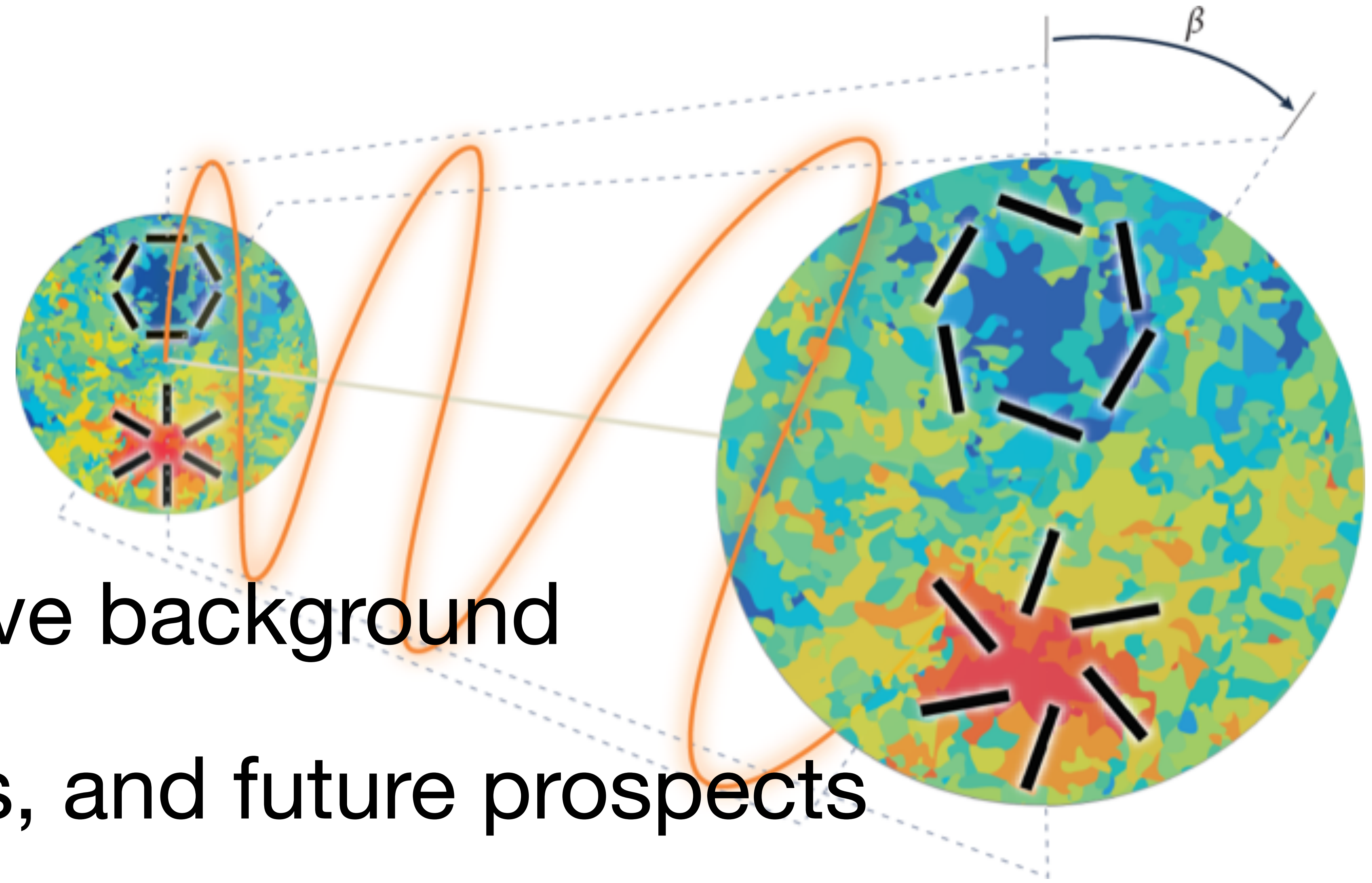
Eiichiro Komatsu (Max Planck Institute for Astrophysics)
Nagoya University, June 6–30, 2023

Topics

From the syllabus

1. What is parity symmetry?
2. Chern-Simons interaction
3. **Parity violation 1: Cosmic inflation**
4. Parity violation 2: Dark matter
5. Parity violation 3: Dark energy
6. Light propagation: birefringence
7. Physics of polarization of the cosmic microwave background
8. Recent observational results, their implications, and future prospects

$$I_{\mathrm{CS}} = \int \mathrm{d}^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\tilde{F}\right)$$



3.1 Cosmic Inflation


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years
NASA/WMAP Science Team

Cosmic Inflation: Key Features

More than 40 years of research in a single slide

- Inflation is the period of **accelerated** expansion in the very early Universe.
  - If the distance between two points increases as $a(t)$, $d^2a/dt^2 > 0$.

This is the definition of inflation.

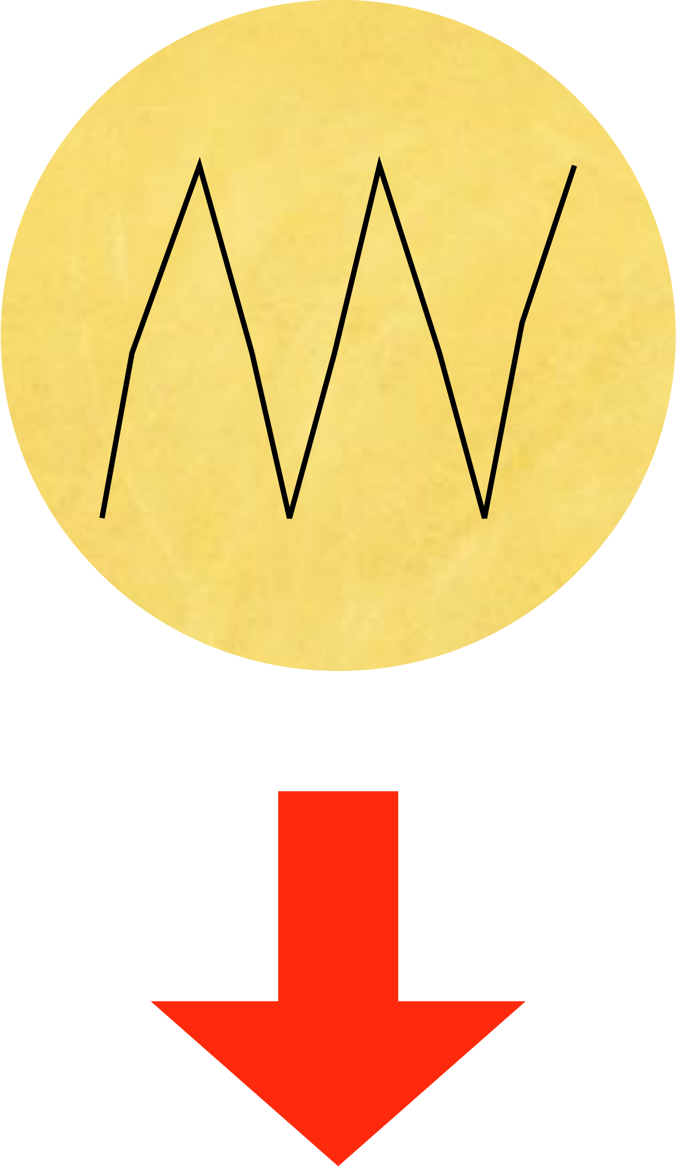
- *Primordial fluctuations* are generated **quantum mechanically**.
  - Scalar modes: Density fluctuations –> The origin of all cosmic structure.
  - Tensor modes: Gravitational waves –> Yet to be discovered.
  - Vector modes: ?
- **A New Paradigm**: Sourced contributions (this lecture)

Gravity + Quantum

= The origin of all cosmic structure in the Universe

Starobinsky (1980); Sato (1981); Guth (1981);
Linde (1982); Albrecht & Steinhardt (1982)

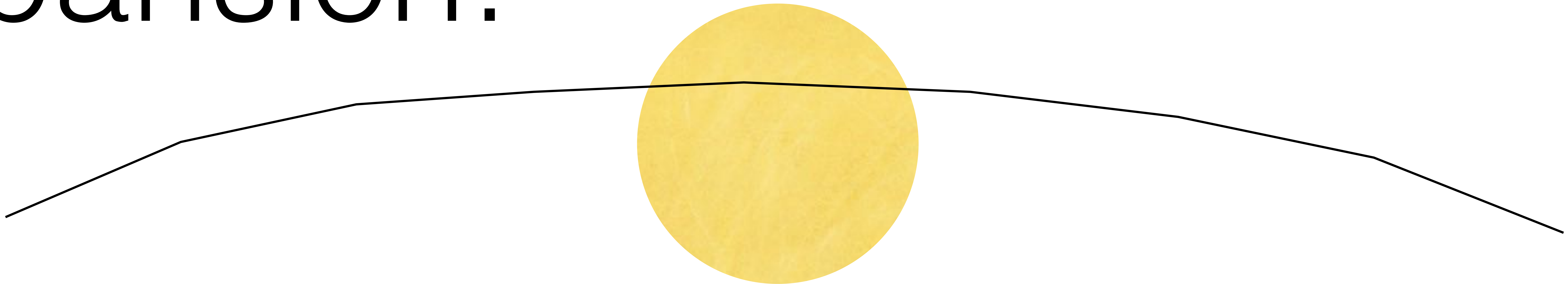
Cosmic Inflation



Quantum-mechanical fluctuation on microscopic scales

Mukhanov & Chibisov (1981);
Hawking (1982); Starobinsky (1982);
Guth & Pi (1982);
Bardeen, Turner & Steinhardt (1983)

Exponential Expansion!



- **Exponential expansion stretches the wavelength of quantum fluctuations to cosmological scales.**

Kavli IPMU News No. 41 March 2018

KAVLI IPMU NEWS

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The University of Tokyo Institutes for Advanced Study
東京大学国際高等研究所

FEATURE

Kavli IPMU Principal Investigator **Eiichiro Komatsu**
Research Area: Theoretical Physics

https://www.ipmu.jp/sites/default/files/imce/news/41E_Feature.pdf

Quantum Fluctuation

The 20th century has seen the remarkable development of the Standard Model of elementary particles and fields. The last piece, the Higgs particle, was discovered in 2012. In the 21st century, we are witnessing the similarly remarkable development of the Standard Model of cosmology. In his 2008 book on "*Cosmology*" Steven Weinberg, who led the development of particle physics, wrote: "This new excitement in cosmology came as if on cue for elementary particle physicists. By the 1980s the

is not well known to the public. This ingredient is not contained in the name of ΛCDM, but is an indispensable part of the Standard Model of cosmology. It is the idea that our ultimate origin is the quantum mechanical fluctuation generated in the early Universe. However remarkable it may sound, this idea is consistent with all the observational data that have been collected so far for the Universe. Furthermore, the evidence supporting this idea keeps accumulating and is strengthened as we collect more

What caused cosmic inflation?

No one knows!

- Einstein's field equation tells us that d^2a/dt^2 is given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

- ρ: Total energy density.
- P: Total pressure.
- **$d^2a/dt^2 > 0$ requires $P < -\rho/3$.**

- Such an energy component, $P<-\rho/3$, is not included in the standard model of elementary particles and fields. That is, **cosmic inflation requires physics beyond the standard model.**

- **No one knows what caused inflation.** In this lecture, we will simply assume that inflation occurred and that space expanded nearly exponentially in the early Universe.

Accelerated expansion –> Slow-roll parameter

The most important quantity: *H*(*t*) = dln(*a*)/dt = *a*$^{-1}$ da/dt

- The most important quantity in cosmology is the expansion rate of space, called the "**Hubble expansion rate**", defined by

$$H(t) = \frac{\dot{a}}{a}$$

- The accelerated expansion ($d^2a/dt^2 > 0$) implies

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0 \quad \Rightarrow \quad \epsilon = -\frac{\dot{H}}{H^2} < 1$$

"Slow-roll parameter"

ε << 1 –> Hubble expansion rate changes slowly.

ε << 1 –> Nearly exponential expansion

The most important quantity: *H*(*t*) = dln(*a*)/dt = *a*$^{-1}$ da/dt

- The most important quantity in cosmology is the expansion rate of space, called the "**Hubble expansion rate**", defined by

$$\frac{\dot{a}}{a} = H(t) \rightarrow a(t) = \exp\left[\int^t dt' \; H(t')\right] \simeq e^{Ht} \begin{pmatrix} -\infty < t < \infty \\ 0 < a(t) < \infty \end{pmatrix}$$

During inflation, *a*(*t*) grows nearly exponentially

- The accelerated expansion ($d^2a/dt^2 > 0$) implies

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0 \quad \Rightarrow \quad \epsilon = -\frac{\dot{H}}{H^2} < 1$$

"Slow-roll parameter"

ε << 1 –> Hubble expansion rate changes slowly.

In this lecture, we will study quantum mechanics in expanding space with *H*(*t*) ~ constant.

We do not ask, "*What caused inflation?*". Many smart people have studied this problem. No one yet knows the answer. It is certainly too difficult for me to answer.
My approach: Only experiments will tell us the answer. Until then, let's do what we can do!

3.2 EM in expanding space

Recap: Maxwell's equations in vacuum

Non-expanding space (see Day 2), in Heaviside units and c=1

"Lorenz gauge condition"

- Maxwell's equations in vacuum $\partial_\nu F^{\mu\nu} = 0$ and $\partial_\nu A^\nu = 0$ gives

  $$\Box A^\mu = 0$$ → The equation for a wave traveling at the speed of light!

  where

  $$\Box = \eta^{\alpha\beta}\partial_\alpha\partial_\beta = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

  $$\eta^{\alpha\beta} = \mathrm{diag}(-1, \mathbf{1})$$

  $$A^\mu = \eta^{\mu\alpha}A_\alpha = (\phi, \mathbf{A})$$ with $\dot{\phi} + \nabla \cdot \mathbf{A} = 0$

- With $\phi = 0$, (in vacuum) $\nabla \cdot \mathbf{A} = 0$ ("Coulomb gauge condition"), one obtains

  $$\ddot{\mathbf{A}} - \nabla^2 \mathbf{A} = 0$$

Maxwell's equations in vacuum

Expanding space, in Heaviside units and *c*=1

- In expanding space, one obtains

$$\mathbf{A}'' - \nabla^2 \mathbf{A} = 0 \quad \text{where} \quad ' = \frac{\partial}{\partial \tau} = a\frac{\partial}{\partial t}$$

- Remarkably, it takes the same form as in non-expanding space, except for **the change of variables** from the physical time, *t*, to the **conformal time, τ.**

- The distance between two points in 4d spacetime:

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu = -dt^2 + d\mathbf{x}^2 \quad \text{[Non-expanding space]}$$

$$\to -dt^2 + a^2(t)d\mathbf{x}^2 = a^2(\tau)[-d\tau^2 + d\mathbf{x}^2] \quad \text{[Expanding space]}$$

$$= g_{\mu\nu}dx^\mu dx^\nu \quad \text{with } x^\mu = (\tau, \mathbf{x}) \qquad g_{\mu\nu} = a^2\mathrm{diag}(-1, \mathbf{1}) = a^2\eta_{\mu\nu}$$

Conformal transformation

Rescaling the metric tensor, $g_{\mu\nu} \to g_{\mu\nu}' = \Omega^2 g_{\mu\nu}$

- The metric tensor describing a homogeneous, isotropic, spatially flat, and expanding background is **conformal** to the metric tensor describing the Minkowski space.

$$g_{\mu\nu} = a^2(\tau)\eta_{\mu\nu}\,, \qquad g^{\mu\nu} = a^{-2}(\tau)\eta^{\mu\nu}\,, \qquad g = \det(g_{\mu\nu}) = -a^8$$

$$g_{\mu\nu} = a^2 \mathrm{diag}(-1, \mathbf{1}) = a^2\eta_{\mu\nu}$$

- How does the action for EM transform?

$$I = -\frac{1}{4}\int d\tau d^3\mathbf{x}\,\sqrt{-g}F^2 = -\frac{1}{4}\int d\tau d^3\mathbf{x}\,\sqrt{-g}F_{\mu\nu}g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta}$$

Conformal transformation

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$$g_{\mu\nu} = a^2(\tau)\eta_{\mu\nu}\,, \quad g^{\mu\nu} = a^{-2}(\tau)\eta^{\mu\nu}\,, \quad g = \det(g_{\mu\nu}) = -a^8$$

- How does the action for EM transform?

$$I \rightarrow -\frac{1}{4}\int d\tau d^3\mathbf{x}\; F_{\mu\nu}\eta^{\mu\alpha}\eta^{\nu\beta}F_{\alpha\beta}$$

$\sqrt{-g}F^2$ remains invariant under conformal transformation.

The equation of motion remains the same as in the Minkowski spacetime, **except for the change of variables, $t \rightarrow \tau$.**

Problem Set 3

Action for the vector potential

Hint: Use integration by parts and

$$\phi = 0, \quad \nabla \cdot \mathbf{A} = 0$$

in vacuum "Coulomb gauge condition"

- Using $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $x^\mu = (\tau, \mathbf{x})$, show that

$$I = -\frac{1}{4}\int d\tau d^3\mathbf{x}\,\sqrt{-g}F^2 = \frac{1}{2}\int d\tau d^3\mathbf{x}\,\left[(\mathbf{A}')^2 - (\nabla\mathbf{A})^2\right]$$

where $\begin{cases} (\mathbf{A}')^2 = \sum_i (A^{i\prime})^2 \\ (\nabla\mathbf{A})^2 = \sum_{ij} (\partial_j A^i)^2 \end{cases}$

- When $a(t) = e^{Ht}$ (for $-\infty < t < \infty$)

  show that $a(\tau) = (-H\tau)^{-1}$ (for $-\infty < \tau < 0$)

Hint: $d\tau = \dfrac{dt}{a(t)}$

E and B fields in expanding space

How are they related to the vector potential?

- We continue to define the **E** and **B** fields from the field strength tensor (Day 2):

$$F^2 \equiv F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B}\cdot\mathbf{B} - \mathbf{E}\cdot\mathbf{E})$$

This is a *scalar* and is invariant under parity transformation.

- The action is

$$I = -\frac{1}{4}\int d\tau d^3\mathbf{x}\,\sqrt{-g}F^2 = \frac{1}{2}\int d\tau d^3\mathbf{x}\; a^4\,(\mathbf{E}\cdot\mathbf{E} - \mathbf{B}\cdot\mathbf{B})$$

E and B fields in expanding space

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- As shown in Problem Set 3,

$$= \frac{1}{2}\int d\tau d^3\mathbf{x}\left[(\mathbf{A}')^2 - (\nabla\mathbf{A})^2\right]$$

E and B fields in expanding space

How are they related to the vector potential?

- Therefore,

Non-expanding case (Day 2)

$$\mathbf{E} = -\frac{1}{a^2}\mathbf{A}', \quad \mathbf{B} = \frac{1}{a^2}\nabla \times \mathbf{A} \quad \Longleftrightarrow \quad \begin{aligned} \mathbf{E} &= -\nabla\phi - \dot{\mathbf{A}} \\ \mathbf{B} &= \nabla \times \mathbf{A} \end{aligned}$$

- The action is

$$I = -\frac{1}{4}\int d\tau d^3\mathbf{x}\ \sqrt{-g}F^2 = \frac{1}{2}\int d\tau d^3\mathbf{x}\ a^4\left(\mathbf{E}\cdot\mathbf{E} - \mathbf{B}\cdot\mathbf{B}\right)$$

- As shown in Problem Set 3,

$$= \frac{1}{2}\int d\tau d^3\mathbf{x}\left[(\mathbf{A}')^2 - (\nabla\mathbf{A})^2\right]$$

3.3 Quantization of A^μ during inflation

A note on terminology

“Photons” = Massless spin-1 particles

- We will study the evolution of A^μ during inflation with and without the Chern-Simons term in the action.
- Since inflation occurred long before the electroweak symmetry breaking, “photons” as we know them did not exist during inflation.
- Although we will continue to use the term “photons”, in this lecture we should think of them more generally as “**massless spin-1 particles**”.
- In this lecture, we will use the units so that $c = 1\,, \hbar = 1$

Vacuum fluctuations: Quantization

Let's get some photons out of the vacuum.

- The vacuum is not without particles. This is due to quantum mechanical zero-point fluctuations.

- The standard procedure for quantizing fields starts with the second-order action (as shown in Problem Set 3):

$$I = -\frac{1}{4}\int d\tau d^3\mathbf{x}\ \sqrt{-g}F^2 = \frac{1}{2}\int d\tau d^3\mathbf{x}\left[(\mathbf{A}')^2 - (\nabla\mathbf{A})^2\right]$$

  - This means that **A**(τ,**x**) is the correct variable for quantization (the "canonical variable").

Vacuum fluctuations: Quantization

Let's get some photons out of the vacuum.

- The vacuum is not without particles. This is due to quantum mechanical zero-point fluctuations.

- The standard procedure for quantizing fields starts with the second-order action. **With the helicity basis in Fourier space,** $\mathbf{A}(t,\mathbf{x}) = (2\pi)^{-3/2} \int d^3\mathbf{k}\ \mathbf{A}_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}$

$$I = \frac{1}{2} \sum_{\lambda=\pm1} \int d\tau d^3\mathbf{k} \left[|A'_{\lambda,\mathbf{k}}|^2 - k^2 |A_{\lambda,\mathbf{k}}|^2 \right]$$

  - This means that **A±(τ)** is also the correct variable for quantization (the "canonical variable").

Quantize!

- Writing the helicity states in terms of creation and annihilation operators,

$$A_{\lambda,\mathbf{k}}(\tau) = u_{\lambda,k}(\tau)\hat{a}_{\lambda,\mathbf{k}} + u^*_{\lambda,k}(\tau)\hat{a}^\dagger_{\lambda,-\mathbf{k}}$$

This is quantization!

with the commutation relation given by $\left[\hat{a}_{\lambda,\mathbf{k}}, \hat{a}^\dagger_{\lambda',\mathbf{k}'}\right] = \delta_{\lambda\lambda'}\delta_D(\mathbf{k}-\mathbf{k}')$

where

  - $\hat{a}_{\lambda,\mathbf{k}}$: ***Annihilation operator***, to destroy a photon with λ and **k**.
  - $\hat{a}^\dagger_{\lambda,\mathbf{k}}$: ***Creation operator***, to create a photon with λ and **k**.
  - $u_{\lambda,\mathbf{k}}$: ***Mode function***, to describe the photon spectrum. It satisfies

$$u_{\lambda,k}u^{*\prime}_{\lambda,k} - u^*_{\lambda,k}u'_{\lambda,k} = i$$

Quantize!

- Writing the helicity states in terms of creation and annihilation operators,

$$A_{\lambda,\mathbf{k}}(\tau) = u_{\lambda,k}(\tau)\hat{a}_{\lambda,\mathbf{k}} + u^*_{\lambda,k}(\tau)\hat{a}^\dagger_{\lambda,-\mathbf{k}}$$

This is quantization!

with the commutation relation given by $\left[\hat{a}_{\lambda,\mathbf{k}}, \hat{a}^\dagger_{\lambda',\mathbf{k}'}\right] = \delta_{\lambda\lambda'}\delta_D(\mathbf{k}-\mathbf{k}')$

- The mode function, $u_{\lambda,k}(\tau)$, obeys the same equation of motion as $A_{\lambda,k}(\tau)$:

$$u''_{\lambda,k} + k^2 u_{\lambda,k} = 0 \quad \xrightarrow{\text{Solution}} \quad u_{\lambda,k} = C_{\lambda,k}e^{ik\tau} + D_{\lambda,k}e^{-ik\tau}$$

where $C_{\lambda,k}$ and $D_{\lambda,k}$ are integration constants. How do we determine them?

The vacuum state

The choice of a vacuum state determines the normalization of $u_{\lambda,k}$

- In quantum mechanics, we need to define a vacuum state, |0>. The annihilation operator acting on |0> leads to zero, i.e., $\hat{a}_{\lambda,\mathbf{k}}|0\rangle = 0$.

- The mode function satisfies the following equation in a vacuum state:

$$u'_{\lambda,k} + i\omega u_{\lambda,k} = 0 \quad \xrightarrow{\text{Solution}} \quad u_{\lambda,k} \propto e^{-i\omega\tau}$$

  where ω is the frequency.

- Thus, $\omega=k$, which is the dispersion relation for a massless particle, and $C_{\lambda,k}=0$. To determine $D_{\lambda,k}$, use the normalization condition for $u_{\lambda,k}{u^*_{\lambda,k}}' - u^*_{\lambda,k}u'_{\lambda,k} = i$

- The final result is

$$u_{\lambda,k}(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}$$

This determines the spectrum of photons created quantum-mechanically during inflation!

Turner, Widrow (1987)

Inflation cannot produce significant EM fields

…at least in Maxwell's theory.

- $|u_{\lambda,k}|^2 = 1/(2k)$
- As $\mathbf{E} = -\frac{1}{a^2}\mathbf{A}' , \quad \mathbf{B} = \frac{1}{a^2}\nabla \times \mathbf{A}$, both $\mathbf{E}^2$ and $\mathbf{B}^2$ redshift away as a^{-4}.
- The EM energy density also redshifts (dilutes) away as $\rho_{EM} = (\mathbf{E}^2+\mathbf{B}^2)/2 \sim a^{-4}$.

The EM fields are diluted exponentially!

- As a result, the EM fields described by Maxwell's action cannot be produced significantly during inflation. In other words, Maxwell's equations must be modified to produce astrophysically-relevant EM fields.
    - To produce interesting EM fields quantum mechanically during inflation, Maxwell's action must be modified.

3.4 Production of A^μ with $F\tilde{F}$

The mode function

- Change t –> τ in

$$\ddot{A}_\pm + \left(k^2 \mp k\alpha\dot{\bar{\theta}}\right) A_\pm = 0$$ (see Day 2)

Parity violation

The equation of motion depends on handedness!

- The mode function satisfies

$$u''_\pm + \left(k^2 \mp k\alpha\bar{\theta}'\right) u_\pm = 0$$

- The solution now admits a growing mode (instability) if $k^2 - k\alpha|\theta'| < 0$!

→ $k < \alpha|\theta'|$

Anber, Sorbo (2010)

Particle production due to $\theta F\tilde{F}$ during inflation

Kinetic energy of θ is used to produce massless spin-1 particles

$$u''_\pm + \left(k^2 \mp \frac{2k\xi}{-\tau}\right) u_\pm = 0$$

$(-\infty < \tau < 0)$

where $\xi = \frac{(-\tau)\alpha\bar{\theta}'}{2} = \frac{\alpha\bar{\theta}'}{2aH} = \frac{\alpha\dot{\bar{\theta}}}{2H}$

$a(\tau) = -(H\tau)^{-1}$

(Problem Set 3)

- Instability occurs when $-k\tau < 2|\xi|$.
  - The mode function for *one of the helicity states* is **amplified** on large scales (small $-k\tau$) **relative to the vacuum solution, $e^{-ik\tau}/\sqrt{2k}$.**
  - The right-handed (+ helicity) state is amplified for $\xi>0$, whereas the left-handed (- helicity) state remains close to the vacuum solution.
  - **Parity violation!**

Anber, Sorbo (2010)

Particle production due to $\theta F\tilde{F}$ during inflation

Kinetic energy of θ is used to produce massless spin-1 particles

$$u''_{\pm} + \left(k^2 \mp \frac{2k\xi}{-\tau}\right)u_{\pm} = 0 \quad (-\infty < \tau < 0)$$

where $\xi = \frac{(-\tau)\alpha\bar{\theta}'}{2} = \frac{\alpha\bar{\theta}'}{2aH} = \frac{\alpha\dot{\bar{\theta}}}{2H}$

↑ $a(\tau) = -(H\tau)^{-1}$ (Problem Set 3)

- The + helicity state is amplified for ξ>0.
  - The mode function for one of the helicity states is **amplified** on large scales (small $-k\tau$) **relative to the vacuum solution, $e^{-ik\tau}/\sqrt{2k}$.**
  - The approximate solution for **ξ=constant** and **$-k\tau < 2\xi$** is given by
- **Parity violation!**

$$u_{+,k}(\tau) \simeq \frac{1}{\sqrt{2k}}\left(\frac{-k\tau}{2\xi}\right)^{1/4} e^{\pi\xi - \sqrt{2\xi(-k\tau)}}$$

Exponential amplification for πξ>1!

OK, that's enough.

What does all this mean for observations?

The full action

Observational consequences

$$\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$$
$$= \frac{1}{a^2}\left(-\frac{\partial^2}{\partial\tau^2} - 2\frac{a'}{a}\frac{\partial}{\partial\tau} + \nabla^2\right)$$

where $g^{\mu\nu} = a^{-2}\mathrm{diag}(-1, \mathbf{1})$

$\sqrt{-g} = a^4$

$I = I_{\mathrm{inflation}}$ [no one understands this]

$$+ \int d\tau d^3\mathbf{x}\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}(\partial\chi)^2 - V(\chi) - \frac{1}{4}F^2 - \frac{\alpha}{4f}\chi F\tilde{F}\right]$$

$(\partial\chi)^2 = g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$

$\theta = \frac{\chi}{f}$

Gravitational waves

$\Box h_{ij} = 16\pi G(E_iE_j + B_iB_j)^{\mathrm{TT}}$

Scalar fluctuations

$\Box\chi - \frac{\partial V}{\partial\chi} = -\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B}$

- ***f*: "decay constant"** of χ. It is 184 MeV for a pion, but it is much, much larger for the field that we will discuss in this lecture.

Recap: Day 3

- No one knows what caused cosmic inflation.
  - Therefore, we choose to study physics given the inflationary background.
  - The expansion rate, *H*(*t*), varies slowly during inflation: $\epsilon = -\dot{H}/H^2 \ll 1$
- The second-order action in the form of $I = \frac{1}{2}\int d\tau d^3\mathbf{x}\left[\dot{A}^2 - (\nabla A)^2\right]$

  is necessary to quantize a variable *A* (called the "canonical variable").
- The Chern-Simons term amplifies one of the helicity states of massless spin-1 particles relative to the vacuum –> **Parity violation**. $u''_\pm + \left(k^2 \mp k\alpha\bar{\theta}'\right) u_\pm = 0$
- **Observational consequences for the density fluctuation and gravitational waves!**