Squeezed-limit bispectrum, Non-Bunch-Davies vacuum, Scale-dependent bias, and Multi-field consistency relation

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Cook's Branch, March 23, 2012

This talk is based on...

- Squeezed-limit bispectrum
  - *Ganc & Komatsu, JCAP, 12, 009 (2010)*
- Non-Bunch-Davies vacuum
  - *Ganc, PRD 84, 063514 (2011)*
- Scale-dependent bias [and μ-distortion]
  - *Ganc & Komatsu, in preparation*
- Multi-field consistency relation
  - *Sugiyama, Komatsu & Futamase, PRL, 106, 251301 (2011)*

Motivation

- Can we falsify inflation?

Falsifying "inflation"

- We still need inflation to explain the flatness problem!
  - (Homogeneity problem can be explained by a bubble nucleation.)
- However, the observed fluctuations may come from different sources.
- So, what I ask is, "can we rule out inflation as a mechanism for generating the observed fluctuations?"

First Question:

- Can we falsify **single-field** inflation?

An Easy One: Adiabaticity

- Single-field inflation = One degree of freedom.
  - Matter and radiation fluctuations originate from a single source.

$$\mathcal{S}_{c,\gamma} \equiv \underbrace{\frac{\delta\rho_c}{\rho_c}}_{\text{Cold Dark Matter}} - \underbrace{\frac{3\delta\rho_\gamma}{4\rho_\gamma}}_{\text{Photon}} = 0$$

* A factor of 3/4 comes from the fact that, in thermal equilibrium, $\rho_c \sim (1+z)^3$ and $\rho_\gamma \sim (1+z)^4$.

Komatsu et al. (2011)

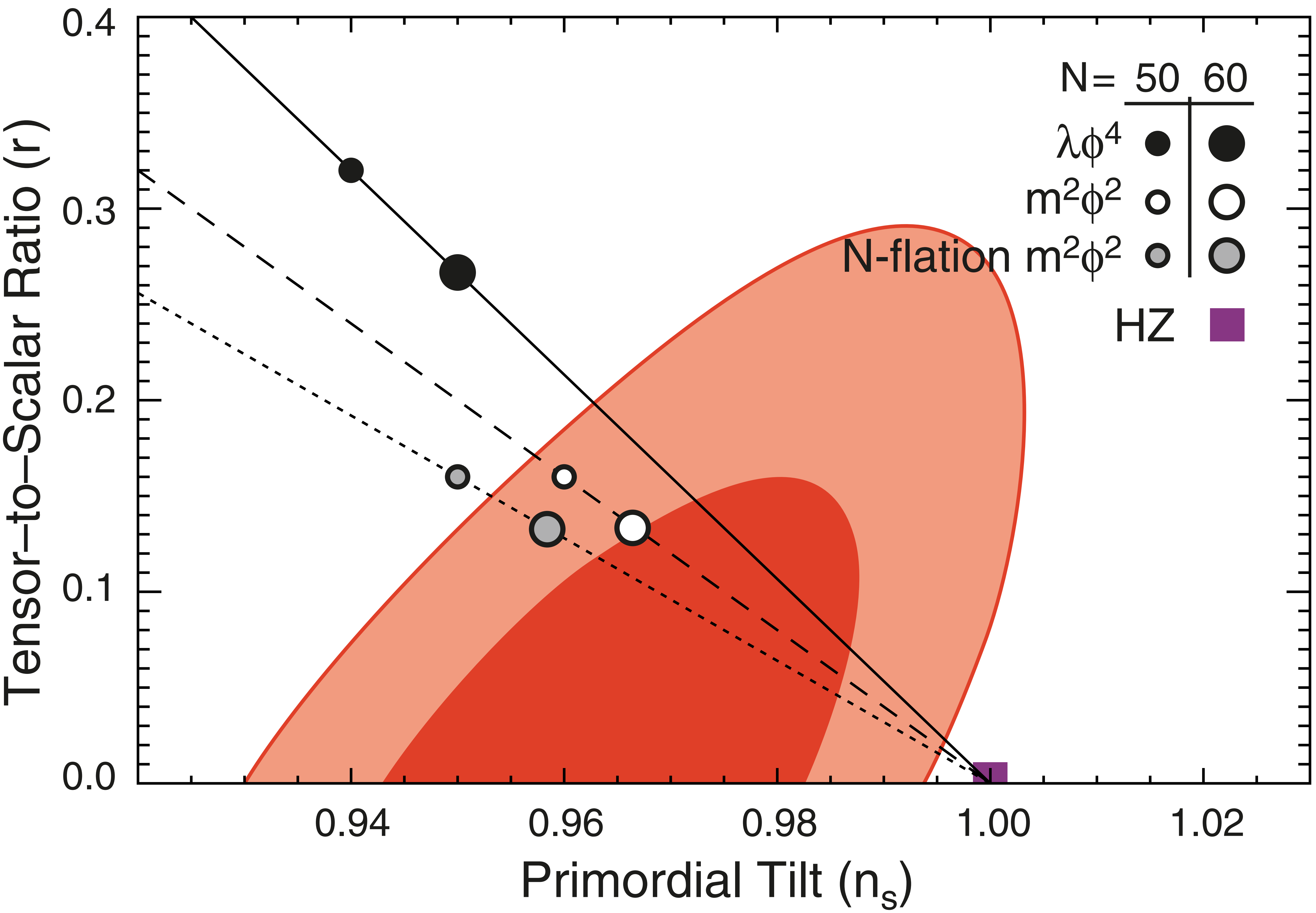
Non-adiabatic Fluctuations

- Detection of non-adiabatic fluctuations immediately rule out single-field inflation models.

The data are consistent with adiabatic fluctuations:

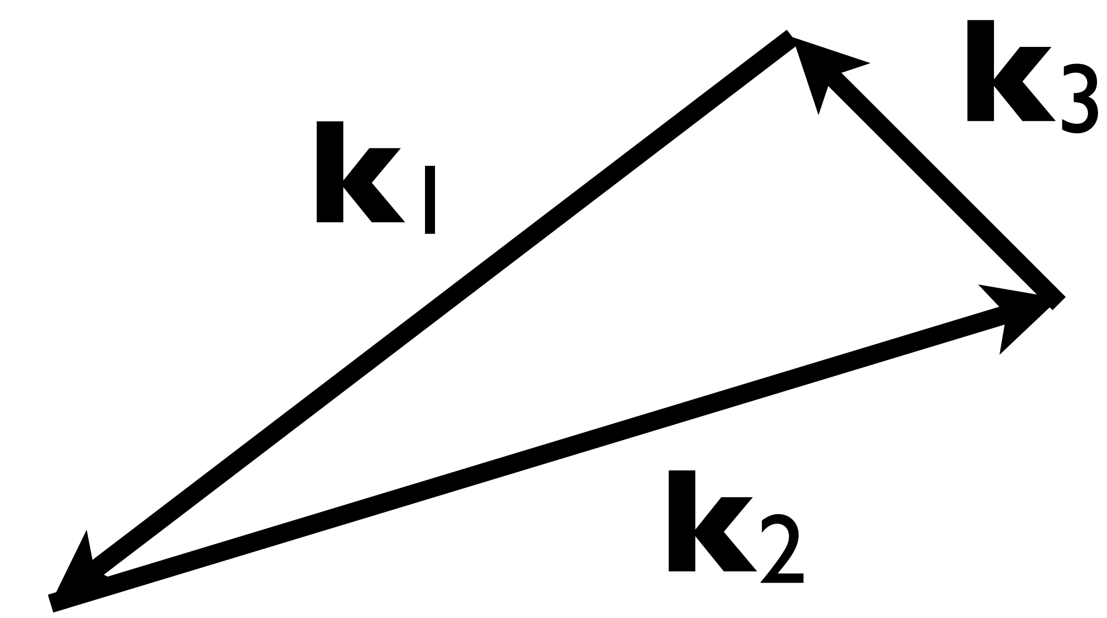
$$\frac{|\delta\rho_c/\rho_c - 3\delta\rho_\gamma/(4\rho_\gamma)|}{\frac{1}{2}[\delta\rho_c/\rho_c + 3\delta\rho_\gamma/(4\rho_\gamma)]} < 0.09 \quad \text{(95\% CL)}$$

Single-field inflation looks good (in 2-point function)

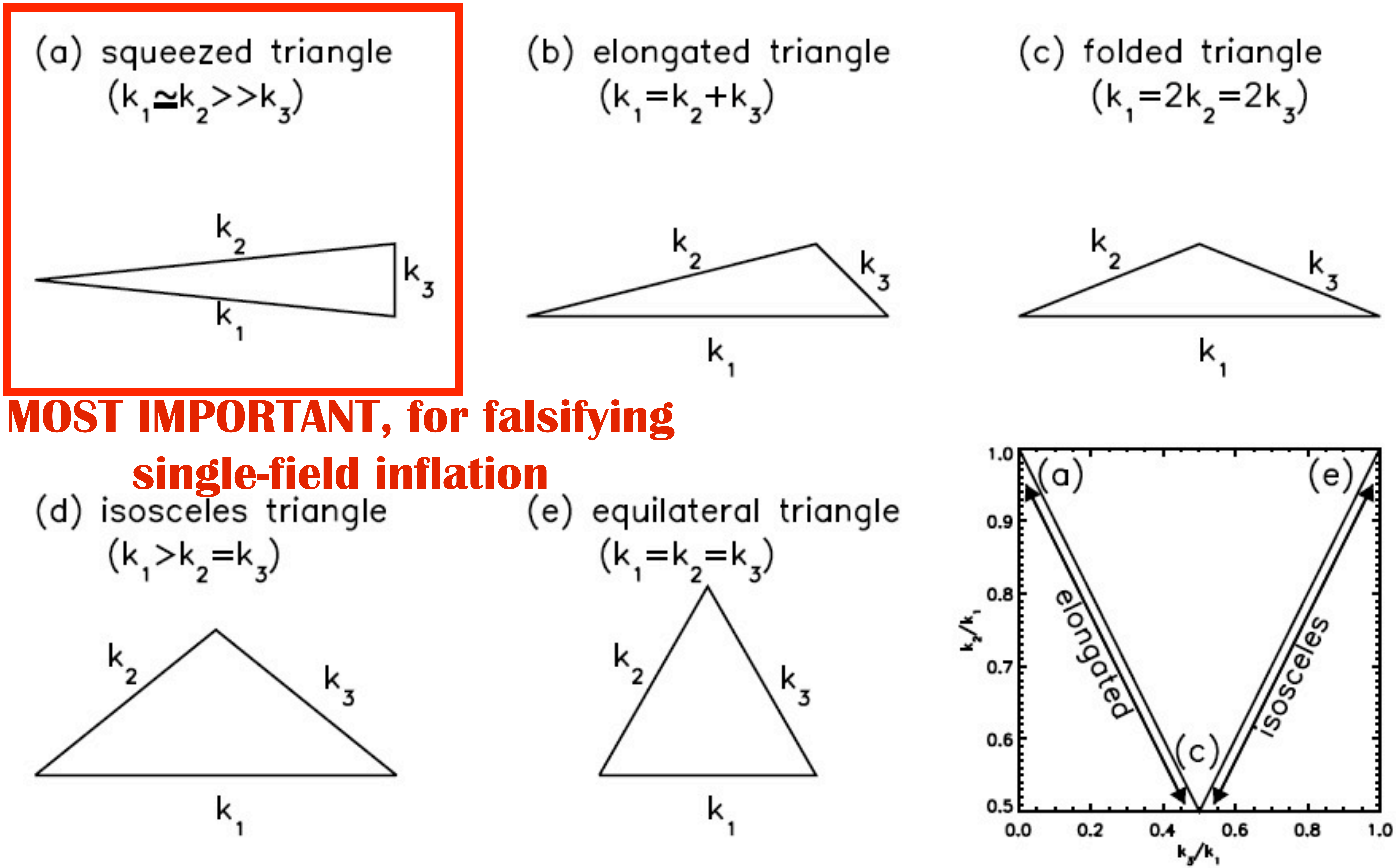


- $P_{scalar}(k) \sim k^{4-\mathbf{ns}}$
  - **n_s=0.968±0.012** (68%CL; WMAP7+BAO+H_0)
- **r**=$4P_{tensor}(k)/P_{scalar}(k)$
  - **r < 0.24** (95%CL; WMAP7+BAO+H_0)

So, let's use 3-point function



- Three-point function!
- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= \langle \zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3} \rangle$ = (amplitude) x $(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$
  model-dependent function


(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)
k_2
k_3
k_1
(b) elongated triangle
($k_1 = k_2 + k_3$)
k_2
k_3
k_1
(c) folded triangle
($k_1 = 2k_2 = 2k_3$)
k_2
k_3
k_1
MOST IMPORTANT, for falsifying
single-field inflation
(d) isosceles triangle
($k_1 > k_2 = k_3$)
k_2
k_3
k_1
(e) equilateral triangle
($k_1 = k_2 = k_3$)
k_2
k_3
k_1
1.0
0.9
0.8
0.7
0.6
0.5
k_2/k_1
0.0
0.2
0.4
0.6
0.8
1.0
k_3/k_1
(a)
(e)
(c)
elongated
isosceles

Curvature Perturbation

- In the gauge where the energy density is uniform, $\delta\rho=0$, the metric on super-horizon scales ($k<<aH$) is written as

$$ds^2 = -N^2(x,t)dt^2 + a^2(t)e^{2\zeta(x,t)}dx^2$$

  - We shall call ζ the "curvature perturbation."
  - This quantity is independent of time, $\zeta(x)$, on super-horizon scales for single-field models.
  - The lapse function, $N(x,t)$, can be found from the Hamiltonian constraint.

Action

- Einstein's gravity + a canonical scalar field:
- $S=(1/2)\int d^4x\sqrt{-g}\,[R-(\partial\Phi)^2-2V(\Phi)]$

Maldacena (2003)

Quantum-mechanical Computation of the Bispectrum

$$\langle \zeta^3(\bar{t}) \rangle = -i \int_{-(1-i\epsilon)\infty}^{\bar{t}} dt' \langle 0 | [\zeta^3(\bar{t}), H_I^{(3)}(t')] | 0 \rangle$$

$$\partial^2 \chi = \frac{\dot\phi^2}{2\dot\rho^2}\dot\zeta$$

$$H \equiv \dot\rho$$

$$S_{\rm int}^{(3)} = \int \frac{1}{4}\frac{\dot\phi^4}{\dot\rho^4}[e^{3\rho}\dot\zeta^2\zeta + e^{\rho}(\partial\zeta)^2\zeta] - \frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\dot\zeta\partial_i\chi\partial_i\zeta +$$
$$- \frac{1}{16}\frac{\dot\phi^6}{\dot\rho^6}e^{3\rho}\dot\zeta^2\zeta + \frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\dot\zeta\zeta^2\frac{d}{dt}\left[\frac{1}{2}\frac{\ddot\phi}{\dot\phi\dot\rho} + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}\right] + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\partial_i\partial_j\chi\partial_i\partial_j\chi\zeta$$
$$+ f(\zeta)\left.\frac{\delta L}{\delta\zeta}\right|_1$$

Initial Vacuum State

$$\zeta_{\mathbf{k}}(t) = u_k(t)a_{\mathbf{k}} + u_k^*(t)a^\dagger_{-\mathbf{k}}$$

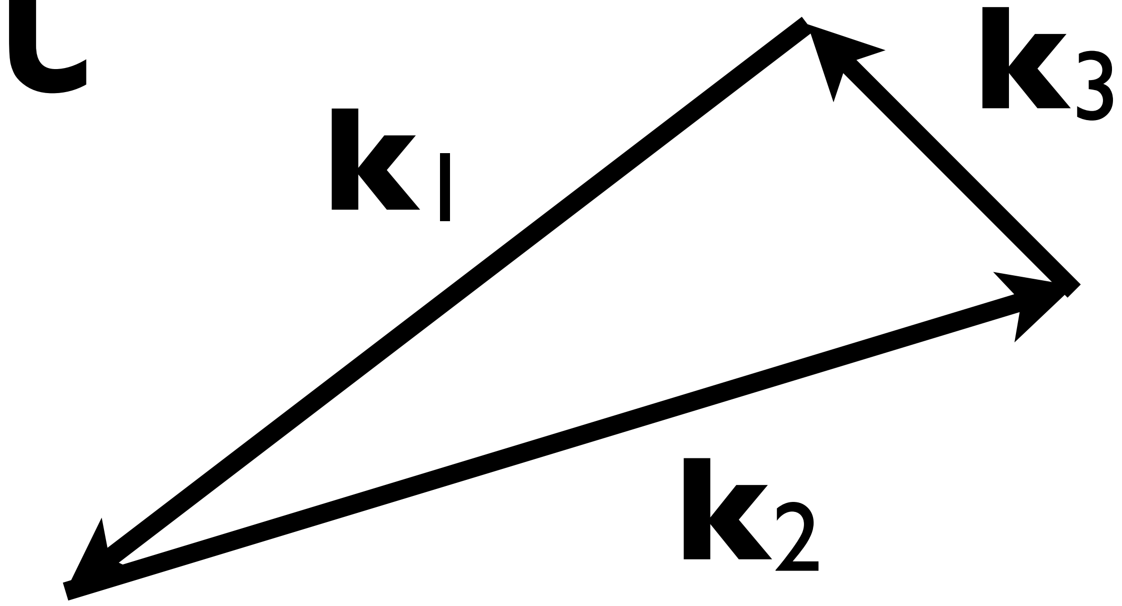
- Bunch-Davies vacuum, $a_{\mathbf{k}}|0\rangle=0$:

$$u_k(\eta) = \frac{H^2}{\dot{\phi}}\frac{1}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}$$

[η: conformal time]

Maldacena (2003)

Result



- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= \langle \zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3}\rangle$ = (amplitude) x $(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$

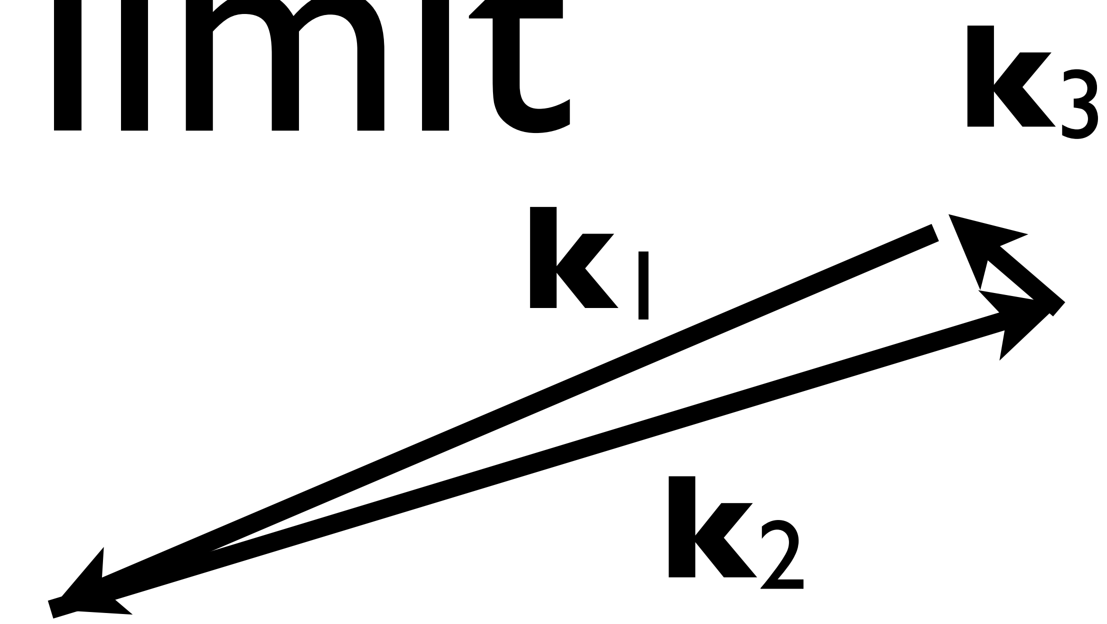
- $b(k_1,k_2,k_3)= \frac{\dot{\rho}_*^4}{\dot{\phi}_*^4}\frac{H_*^4}{M_{pl}^4}\frac{1}{\prod_i(2k_i^3)}$

$$\times\left\{2\frac{\ddot{\phi}_*}{\dot{\phi}_*\dot{\rho}_*}\sum_i k_i^3+\frac{\dot{\phi}_*^2}{\dot{\rho}_*^2}\left[\frac{1}{2}\sum_i k_i^3+\frac{1}{2}\sum_{i\neq j}k_ik_j^2+4\frac{\sum_{i>j}k_i^2k_j^2}{k_t}\right]\right\}$$

Complicated? But...

Maldacena (2003)

Taking the squeezed limit ($k_3 << k_1 \approx k_2$)



- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= <\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}> = (\text{amplitude}) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$

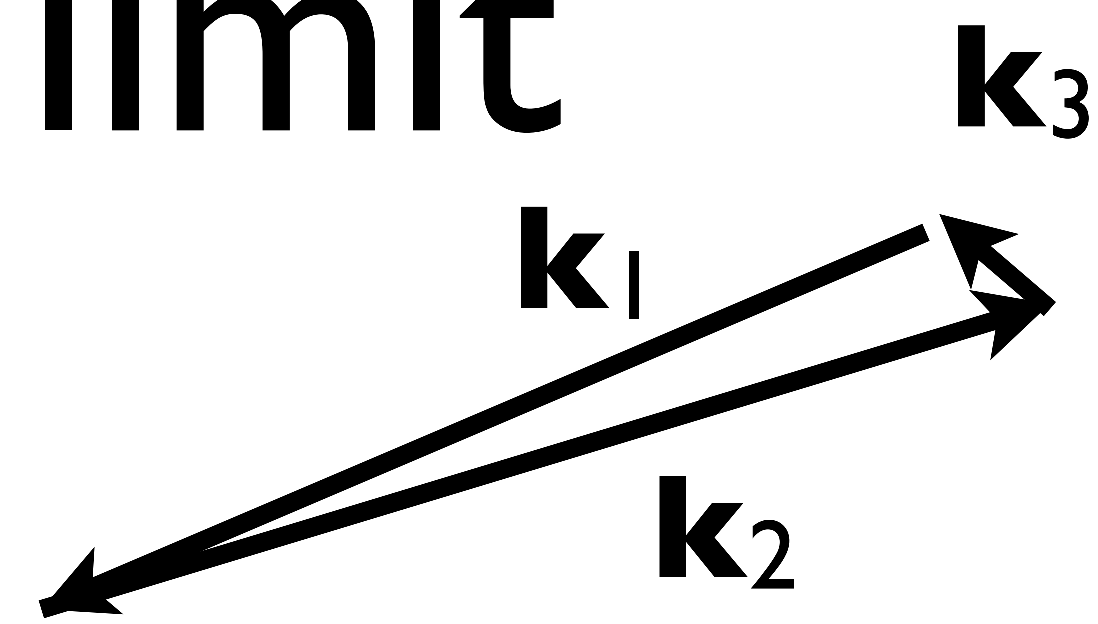
- $b(k_1,k_1,k_3\text{->}0) = \dfrac{\dot\rho_*^4}{\dot\phi_*^4}\dfrac{H_*^4}{M_{pl}^4}\dfrac{1}{\prod_i(2k_i^3)}$

$$\times\left\{2\frac{\ddot\phi_*}{\dot\phi_*\dot\rho_*}\sum_i k_i^3 + \frac{\dot\phi_*^2}{\dot\rho_*^2}\left[\frac{1}{2}\sum_i k_i^3 + \frac{1}{2}\sum_{i\neq j} k_i k_j^2 + 4\frac{\sum_{i>j} k_i^2 k_j^2}{k_t}\right]\right\}$$

$2k_1^3$ k_1^3 k_1^3 $2k_1^3$

Maldacena (2003)

Taking the squeezed limit ($k_3 << k_1 \approx k_2$)



- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
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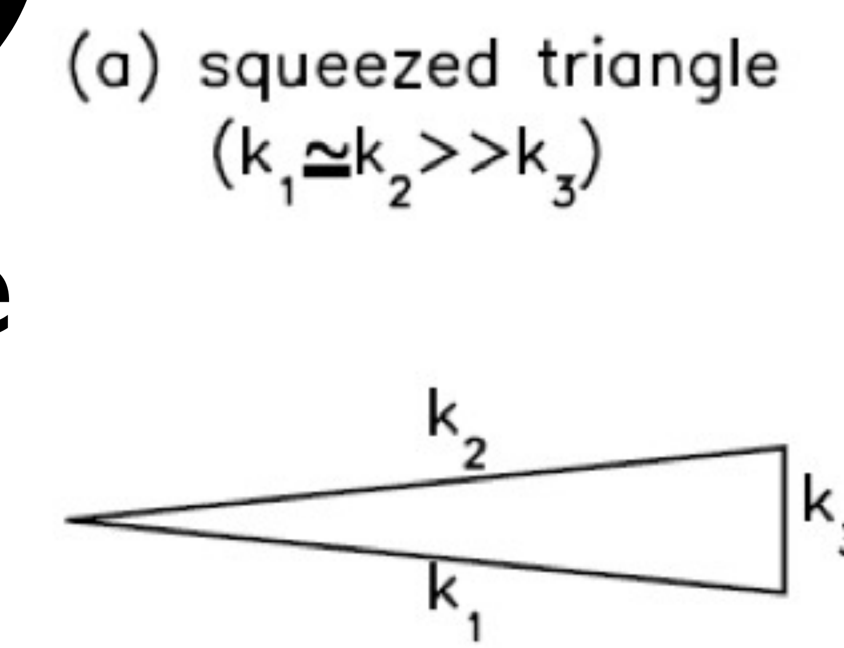
- $$b(k_1,k_1,k_3\text{->}0) = \frac{\dot\rho_*^4}{\dot\phi_*^4}\frac{H_*^4}{M_{pl}^4}\underbrace{2\left[\frac{\ddot\phi_*}{\dot\phi_*\dot\rho_*}+\frac{\dot\phi_*^2}{\dot\rho_*^2}\right]}_{=1-n_s}\frac{1}{k_1^3k_3^3}$$

$$= (1-n_s)P_\zeta(k_1)P_\zeta(k_3)$$

Maldacena (2003); Seery & Lidsey (2005); Creminelli & Zaldarriaga (2004)

Single-field Theorem (Consistency Relation)



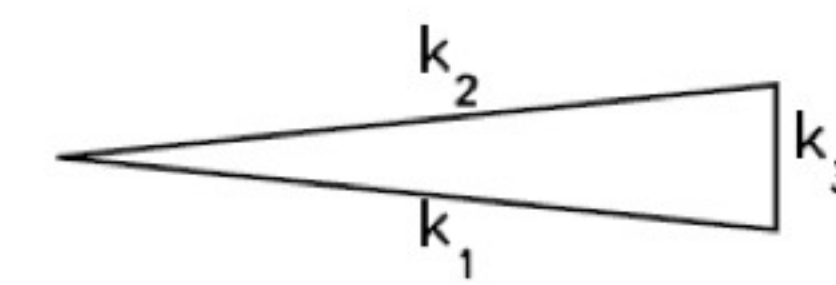
- For **<u>ANY</u>** single-field models*, the bispectrum in the squeezed limit ($k_3 << k_1 \approx k_2$) is given by
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_1,\mathbf{k}_3\text{->}0) = (1-n_s) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$

* for which the single field is solely responsible for driving inflation and generating observed fluctuations.

Maldacena (2003); Seery & Lidsey (2005); Creminelli & Zaldarriaga (2004)

Single-field Theorem (Consistency Relation)

(a) squeezed triangle ($k_1 \simeq k_2 >> k_3$)



- For **ANY** single-field models*, the bispectrum in the squeezed limit ($k_3 << k_1 \approx k_2$) is given by
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_1,\mathbf{k}_3\to 0) = (1-n_s) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$

$$\frac{6}{5} f_{NL} \equiv \frac{B_\zeta(k_1,k_2,k_3)}{P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_3)P_\zeta(k_1)}$$

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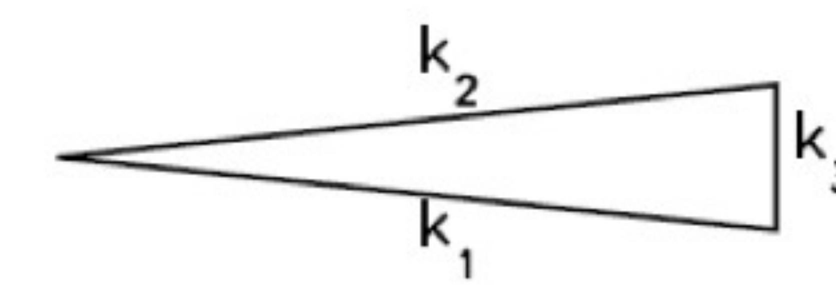
Single-field Theorem (Consistency Relation)

(a) squeezed triangle ($k_1 \simeq k_2 >> k_3$)



- For **ANY** single-field models*, the bispectrum in the squeezed limit ($k_3 << k_1 \approx k_2$) is given by
  - $B_\zeta(\mathbf{k}_1, \mathbf{k}_1, \mathbf{k}_3 \to 0) = (1 - n_s) \times (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times P_\zeta(k_1) P_\zeta(k_3)$
  - Therefore, all single-field models predict $f_{NL} \approx (5/12)(1 - n_s)$.
  - With the current limit $n_s = 0.96$, f_{NL} is predicted to be 0.017.

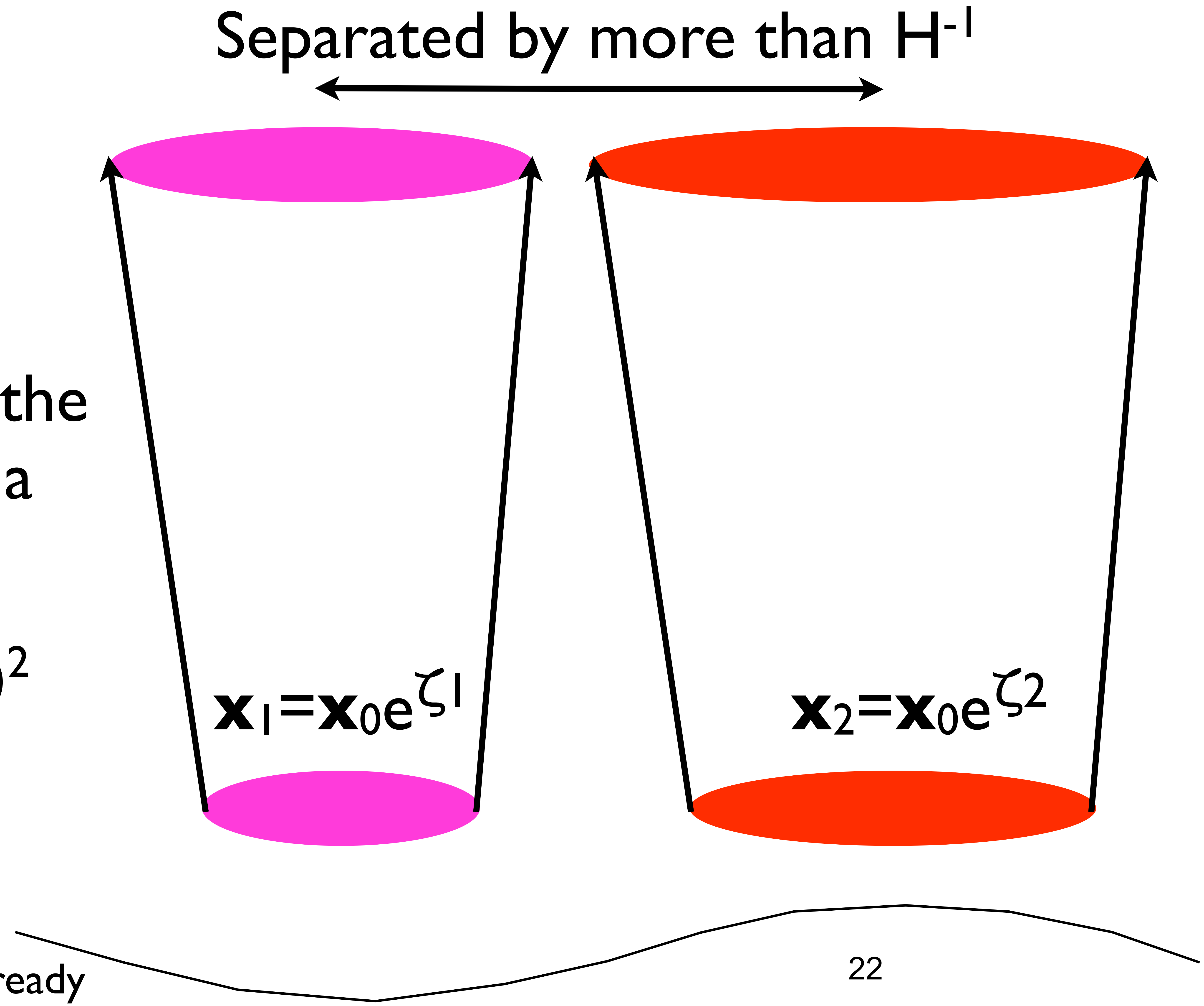
* for which the single field is solely responsible for driving inflation and generating observed fluctuations.

Understanding the Theorem

- First, the squeezed triangle correlates one very long-wavelength mode, k_L (=k_3), to two shorter wavelength modes, k_S (=$k_1 \approx k_2$):
  - $<\zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3}> \approx <(\zeta_{\mathbf{k}S})^2\zeta_{\mathbf{k}L}>$
- Then, the question is: "why should $(\zeta_{\mathbf{k}S})^2$ ever care about $\zeta_{\mathbf{k}L}$?"
  - The theorem says, "it doesn't care, if $\zeta_{\mathbf{k}}$ is exactly scale invariant."

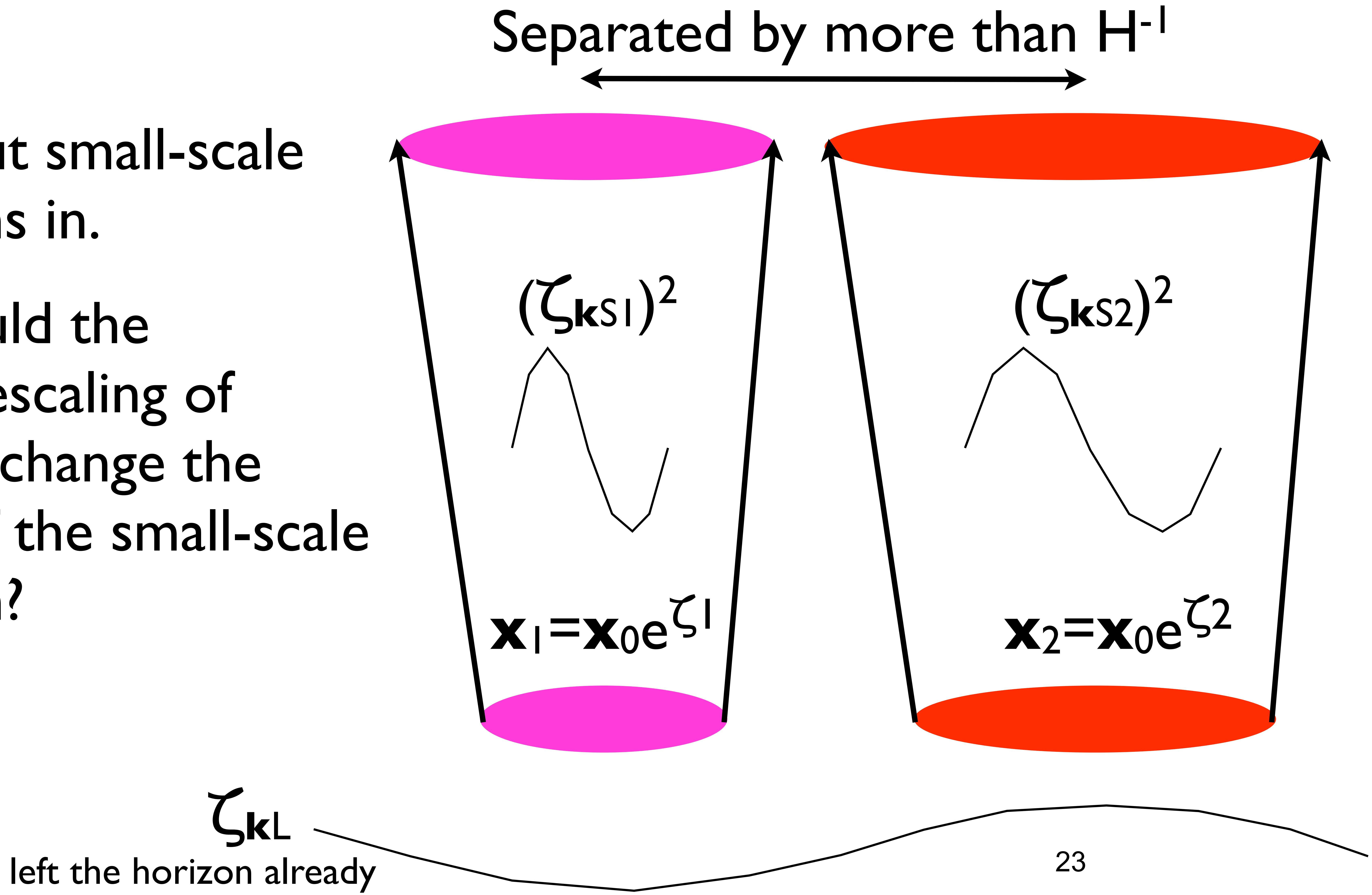
$\zeta_{\mathbf{k}L}$ rescales coordinates

- The long-wavelength curvature perturbation rescales the spatial coordinates (or changes the expansion factor) within a given Hubble patch:
  - $ds^2 = -dt^2 + [a(t)]^2 e^{2\zeta} (d\mathbf{x})^2$



$\zeta_{\mathbf{k}L}$ rescales coordinates

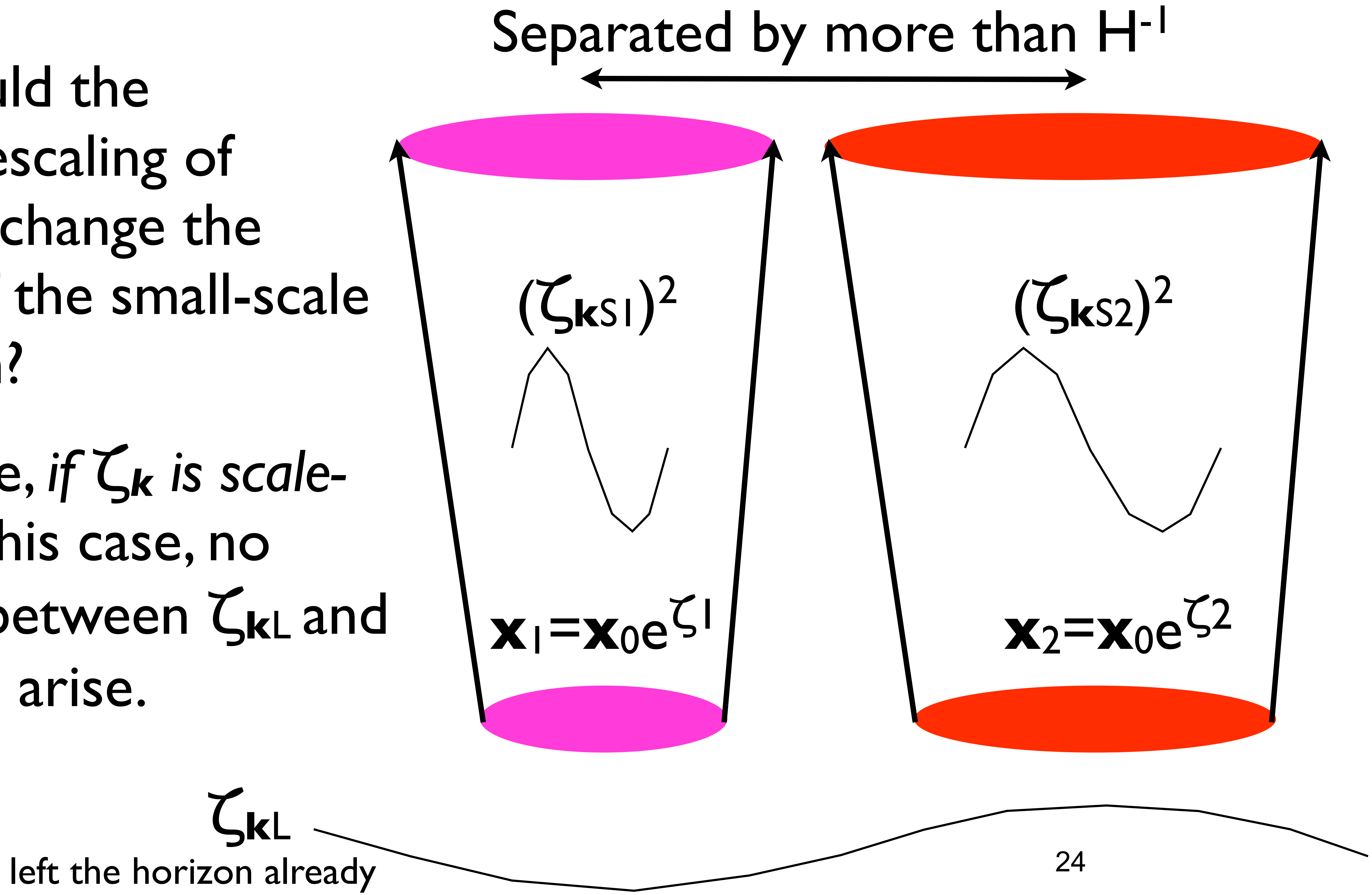
- Now, let's put small-scale perturbations in.
- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?



$\zeta_{\mathbf{k}L}$ rescales coordinates

- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?

- A. No change, *if $\zeta_{\mathbf{k}}$ is scale-invariant*. In this case, no correlation between $\zeta_{\mathbf{k}L}$ and $(\zeta_{\mathbf{k}S})^2$ would arise.

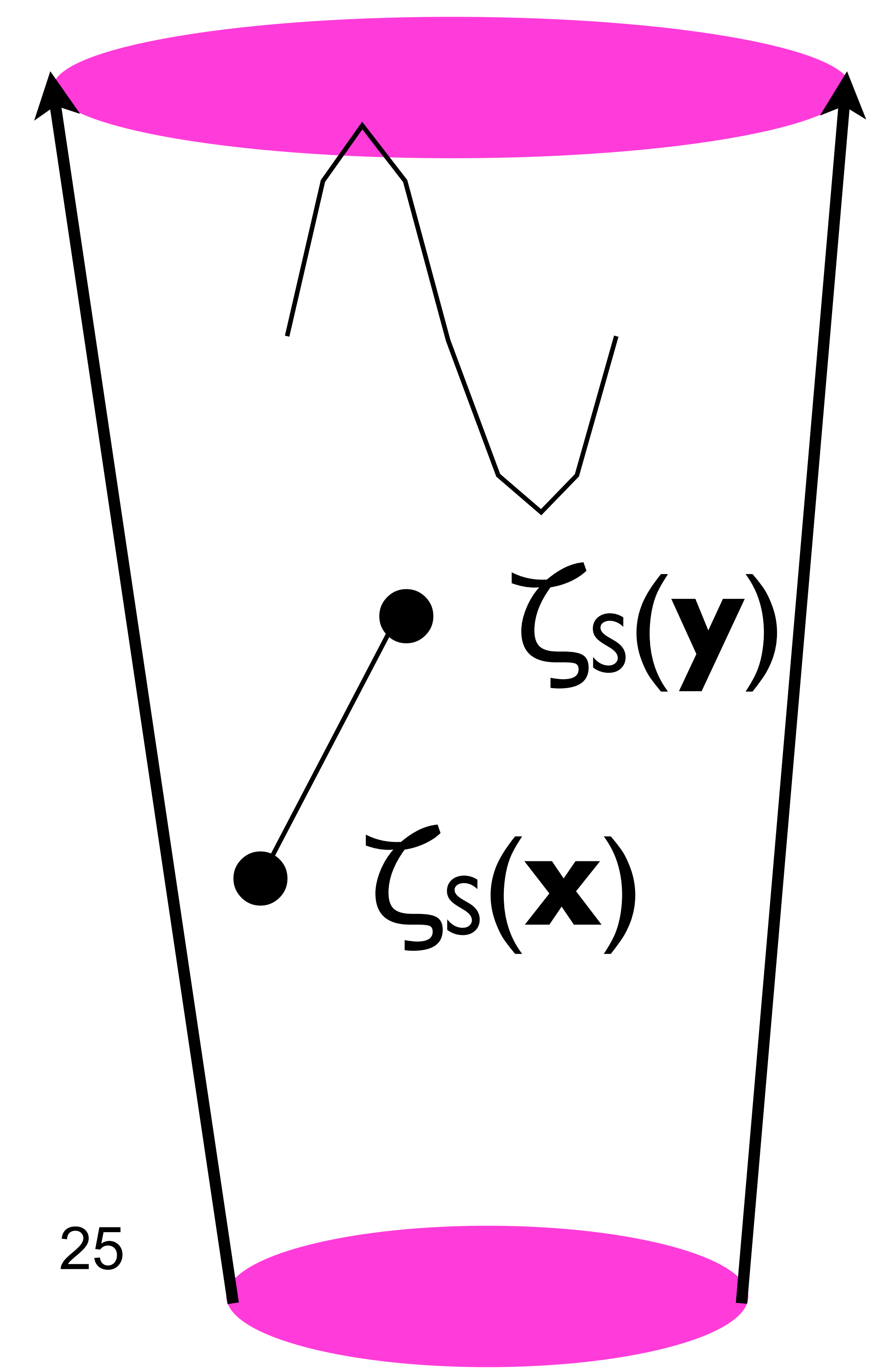


Creminelli & Zaldarriaga (2004); Cheung et al. (2008)

Real-space Proof

- The 2-point correlation function of short-wavelength modes, $\xi = \langle\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})\rangle$, within a given Hubble patch can be written in terms of its vacuum expectation value (in the absence of ζ_L), ξ_0, as:

- $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,[d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\zeta_L]$
- $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,[d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\ln|\mathbf{x}-\mathbf{y}|]$
- $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,(1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)$



$$3\text{-pt func.} = \langle(\zeta_S)^2\zeta_L\rangle = \langle\xi_{\zeta L}\zeta_L\rangle$$
$$= (1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)\langle\zeta_L^2\rangle$$

This is great, but...

- The proof relies on the following Taylor expansion:
  - $<\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})>_{\zeta_L} = <\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})>_0 + \zeta_L\,[d<\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})>_0/d\zeta_L]$
- Perhaps it is interesting to show this explicitly using the in-in formalism.
  - Such a calculation would shed light on the limitation of the above Taylor expansion.
  - Indeed it did - we found a non-trivial "counter-example" (more later)

An Idea

- How can we use the in-in formalism to compute the two-point function of short modes, given that there is a long mode, <$\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})$>$_{\zeta_L}$?
- Here it is!

$$\left\langle \zeta_S^2(\bar{t}) \right\rangle_{\zeta_L} = -i \int_{-(1-i\epsilon)\infty}^{\bar{t}} dt' \langle 0 | [\zeta_S^2(\bar{t}), H_I^{(3)}(t')] | 0 \rangle$$

Ganc & Komatsu, JCAP, 12, 009 (2010)

Long-short Split of H_I

$$\langle \zeta_S^2(\bar{t}) \rangle_{\zeta_L} = -i \int_{-(1-i\epsilon)\infty}^{\bar{t}} dt' \langle 0 | [\zeta_S^2(\bar{t}), H_I^{(3)}(t')] | 0 \rangle$$

- Inserting $\zeta=\zeta_L+\zeta_S$ into the cubic action of a scalar field, and retain terms that have one ζ_L and two ζ_S's.

$$S_{\rm int}^{(3)} = \int d^4x \left[\left(\frac{1}{4}\frac{\dot\phi_0^4}{H^4} - \frac{1}{16}\frac{\dot\phi_0^6}{H^6} \right) a^3 \zeta_L \dot\zeta_S^2 + \frac{1}{4}\frac{\dot\phi_0^4}{H^4} a \zeta_L (\partial\zeta_S)^2 - \frac{\dot\phi_0^4}{2H^4} a^3 \dot\zeta_S \partial_i \zeta_S \partial_i \partial^{-2} \dot\zeta_L + \right.$$
$$+ \frac{1}{16}\frac{\dot\phi_0^6}{H^6} a^3 \partial_i\partial_j\partial^{-2}\dot\zeta_S \, \partial_i\partial_j\partial^{-2}\dot\zeta_S \, \zeta_L + 2\frac{\dot\phi_0^2}{H^2} a^3 \zeta_L \frac{d}{dt}\left[\frac{1}{2}\frac{\ddot\phi_0}{\dot\phi_0 H} + \frac{1}{4}\frac{\dot\phi_0^2}{H^2} \right] \dot\zeta_S \zeta_S$$
$$\left. - f(\zeta)\frac{\delta L_0}{\delta\zeta_S} \right],$$

Result

$$\langle \zeta_{S,\mathbf{k}_1} \zeta_{S,\mathbf{k}_2} \rangle_{\zeta_{\mathbf{k}_3}} = \zeta_{L,\mathbf{k}_1+\mathbf{k}_2} \left[K + \left(\frac{\ddot{\phi}_0}{\dot{\phi}_0 H} + \frac{1}{2} \frac{\dot{\phi}_0^2}{H^2} \right) P(k_1) \right]$$

- where

$$K \equiv i u_{k_1}^2(\bar{\eta}) \int_{-\infty(1-i\epsilon)}^{\bar{\eta}} d\eta \left[\frac{1}{2} \frac{\dot{\phi}_0^4}{H^4} a^2 u_{k_1}^{\prime *2}(\eta) + \frac{1}{2} \frac{\dot{\phi}_0^4}{H^4} a^2 k_1^2 u_{k_1}^{*2}(\eta) + \right.$$
$$\left. + 2 \frac{\dot{\phi}_0^2}{H^2} a^3 \frac{d}{dt} \left(\frac{\ddot{\phi}_0}{\dot{\phi}_0 H} + \frac{1}{2} \frac{\dot{\phi}_0^2}{H^2} \right) u_{k_1}^{\prime *}(\eta) u_{k_1}^{*}(\eta) \right] + \text{c.c.}$$

Result

- Although this expression looks nothing like $(1-n_S)P(k_1)\zeta_{kL}$, we have verified that it leads to the known consistency relation for (i) slow-roll inflation, and (ii) power-law inflation.
- But, there was a curious case – Alexei Starobinsky's exact $n_S=1$ model.
  - If the theorem holds, we should get a vanishing bispectrum in the squeezed limit.

Starobinsky's Model

- The famous Mukhanov-Sasaki equation for the mode function is

$$\frac{d^2 u_k}{d\eta^2} + \left(k^2 - \frac{1}{z}\frac{d^2 z}{d\eta^2}\right) u_k = 0$$

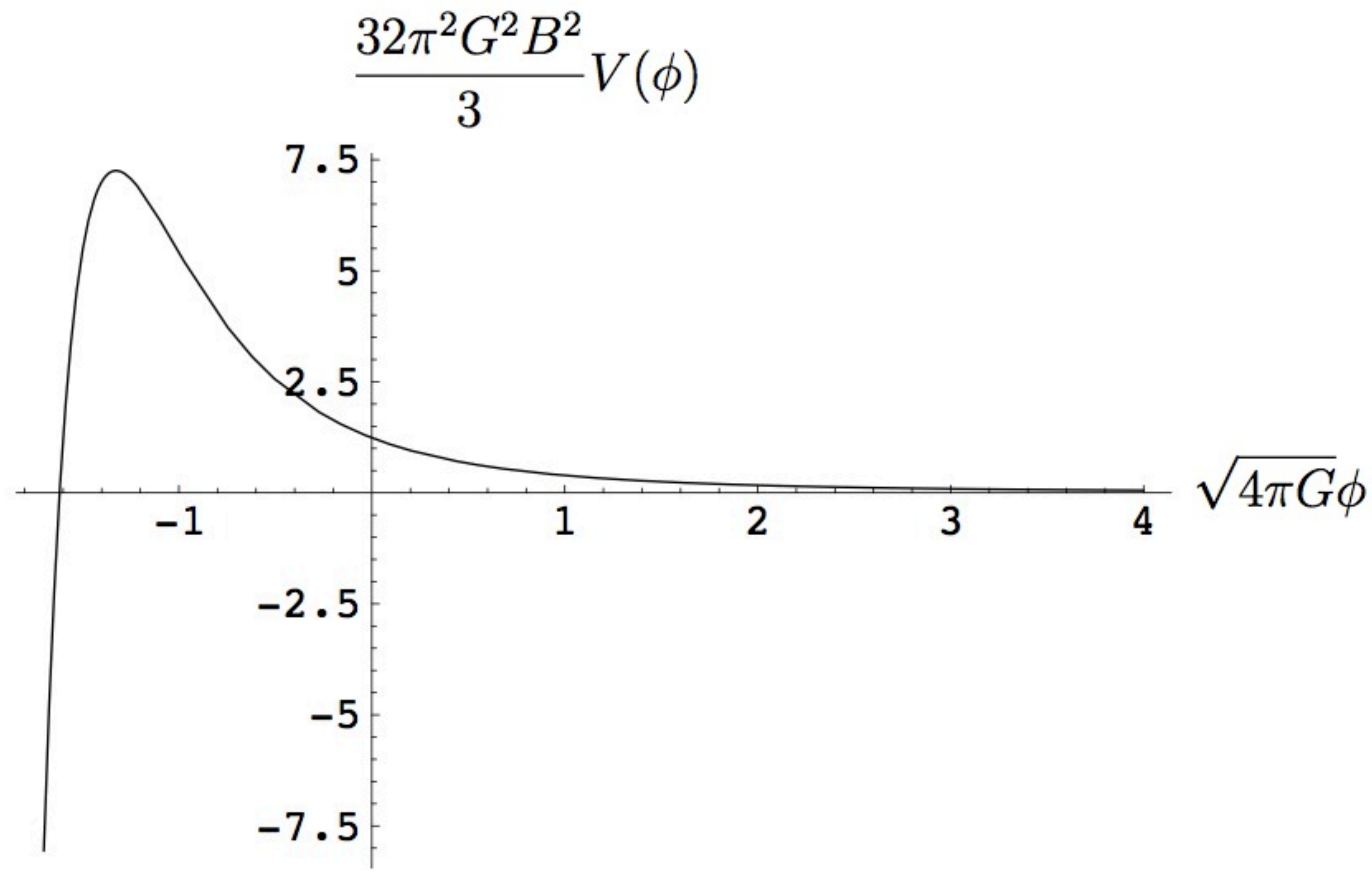
where

$$z = \frac{a\dot{\phi}}{H}$$

•The scale-invariance results when $\frac{1}{z}\frac{d^2 z}{d\eta^2} = \frac{2}{\eta^2}$

So, let's write **$z=B/\eta$**

Starobinsky's Potential



- This potential is a one-parameter family; this particular example shows the case where inflation lasts very long: φ_{end} -> ∞

Result

$$\langle \zeta_{S,\mathbf{k}_1} \zeta_{S,\mathbf{k}_2} \rangle_{\zeta_{\mathbf{k}_3}} = \zeta_{L,\mathbf{k}_1+\mathbf{k}_2} 4P(k_1)(k_1 \eta_{\rm start})^2 e^{-\frac{1}{2}\phi_{\rm end}^2}$$

- **It does not vanish!**
- But, it approaches zero when $\Phi_{\rm end}$ is large, meaning the duration of inflation is very long.
  - In other words, this is a condition that **the longest wavelength that we observe, k_3, is far outside the horizon.**
  - In this limit, the bispectrum approaches zero.

Initial Vacuum State?

- What we learned so far:
  - The squeezed-limit bispectrum is proportional to $(1-n_S)P(k_1)P(k_3)$, provided that ζ_{k3} is far outside the horizon when k_1 crosses the horizon.
- What if the state that $\zeta_{\mathbf{k}3}$ sees is not a Bunch-Davies vacuum, but something else?
  - The exact squeezed limit (k_3->0) should still obey the consistency relation, but perhaps something happens when **k_3/k_1 is small but finite**.

Back to in-in

$$\langle \zeta^3(t^*) \rangle = -i \int_{t_0}^{t^*} dt' \langle 0 | [\zeta^3(t^*), H_I(t')] | 0 \rangle$$

$$B_\zeta(k_1, k_2, k_3) = 2i \frac{\dot{\phi}^4}{H^6} \sum_i \left(\frac{1}{k_i^2} \right) \tilde{u}_{k_1}(\bar{\eta}) \tilde{u}_{k_2}(\bar{\eta}) \tilde{u}_{k_3}(\bar{\eta}) \int_{\eta_0}^{\bar{\eta}} d\eta \frac{1}{\eta^3} u'^*_{k_1} u'^*_{k_2} u'^*_{k_3} + \text{c.c.}$$

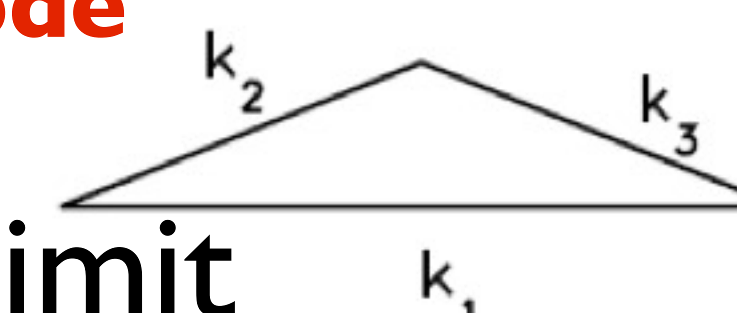
- The Bunch-Davies vacuum: $u_k' \sim \eta e^{-ik\eta}$ (positive frequency mode)
  - The integral yields $1/(k_1+k_2+k_3)$ -> $1/(2k_1)$ in the squeezed limit

Back to in-in

$$\langle \zeta^3(t^*) \rangle = -i \int_{t_0}^{t^*} dt' \langle 0 | [\zeta^3(t^*), H_I(t')] | 0 \rangle$$

$$B_\zeta(k_1, k_2, k_3) = 2i \frac{\dot\phi^4}{H^6} \sum_i \left(\frac{1}{k_i^2} \right) \tilde u_{k_1}(\bar\eta) \tilde u_{k_2}(\bar\eta) \tilde u_{k_3}(\bar\eta) \int_{\eta_0}^{\bar\eta} d\eta \frac{1}{\eta^3} u'^*_{k_1} u'^*_{k_2} u'^*_{k_3} + \text{c.c.}$$

- Non-Bunch-Davies vacuum: $u_k' \sim \eta(A_k e^{-ik\eta} + B_k e^{+ik\eta})$ **negative frequency mode**
- The integral yields $1/(k_1-k_2+k_3)$, peaking in the folded limit



Chen et al. (2007); Holman & Tolley (2008)

- The integral yields $1/(k_1-k_2+k_3) \rightarrow 1/(2k_3)$ in the squeezed limit

Enhanced by k_1/k_3: this can be a big factor!

Agullo & Parker (2011)

Agullo & Parker (2011)

How about the consistency relation?

$$B_\zeta(k_1,k_2,k_3) \underset{k_3/k_1<<1}{\longrightarrow} P_\zeta(k_1)P_\zeta(k_3)\left\{(1-n_s) + 4\frac{\dot{\phi}^2}{H^2}\frac{k_1}{k_3}\left[1-\cos(k_3\eta_0)\right]\right\}$$

- When k_3 is far outside the horizon at the onset of inflation, η_0 (whatever that means), $k_3\eta_0 \to 0$, and thus the above additional term vanishes.
  - The consistency relation is restored. *Sounds familiar!*

An interesting possibility:

- What if $k_3\eta_0 = O(1)$?
- The squeezed bispectrum receives an enhancement of order $\varepsilon k_1/k_3$, which can be sizable.
- Most importantly, **the bispectrum grows faster than the local-form toward k_1/k_3 -> 0!**
  - $B_\zeta(k_1,k_2,k_3) \sim 1/k_3^3$ [Local Form]
  - $B_\zeta(k_1,k_2,k_3) \sim 1/k_3^{\mathbf{4}}$ [non-Bunch-Davies]
- This has an observational consequence – particularly a scale-dependent bias.

Power Spectrum of Galaxies

- Galaxies do not trace the underlying matter density fluctuations perfectly. They are **biased tracers**.
- "Bias" is operationally defined as
  - $b_{galaxy}^2(k) = \langle|\delta_{galaxy,k}|^2\rangle / \langle|\delta_{matter,k}|^2\rangle$

Dalal et al. (2008); Matarrese & Verde (2008); Desjacques et al. (2011)

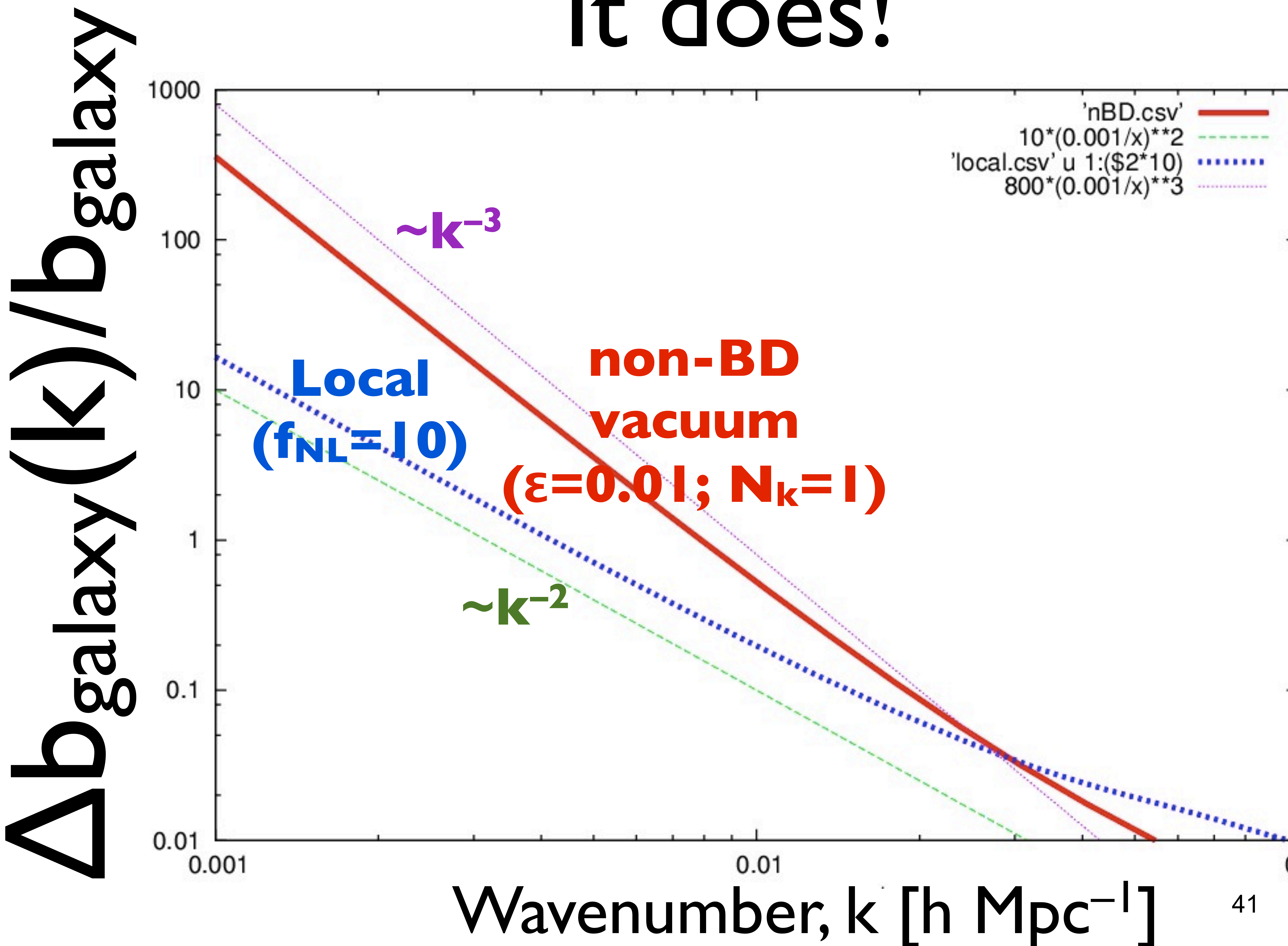
Scale-dependent Bias

$$\frac{\Delta b_h(k,R)}{b_h} = \frac{\delta_c}{D(z)\mathcal{M}_R(k)} \frac{1}{8\pi^2\sigma_R^2} \int_0^\infty dk_1\, k_1^2 \mathcal{M}_R(k_1) \times \int_{-1}^{1} d\mu\, \mathcal{M}_R\left(\sqrt{k^2+k_1^2+2kk_1\mu}\right) \frac{B\left(k_1, \sqrt{k^2+k_1^2+2kk_1\mu}, k\right)}{P_\zeta(k)}$$

- A rule-of-thumb:
  - For $B(k_1,k_2,k_3) \sim 1/k_3^{\mathbf{p}}$, the scale-dependence of the halo bias is given by $b(k) \sim 1/k^{\mathbf{p-1}}$
  - For a local-form (p=3), it goes like $b(k) \sim 1/k^2$
  - For a non-Bunch-Davies vacuum (p=4), would it go like $b(k) \sim 1/k^3$?

It does!

Ganc & Komatsu (in prep)



Ganc, PRD 84, 063514 (2011); Ganc and Komatsu, in prep

CMB?

- The expected contribution to f_{NL}^{local} as measured by CMB is typically $f_{NL}^{local} \approx 8(\varepsilon/0.01)$.
  - A lot bigger than $(5/12)(1-n_S)$, and could be detectable with Planck.

How about...

- Falsifying multi-field inflation?

Strategy

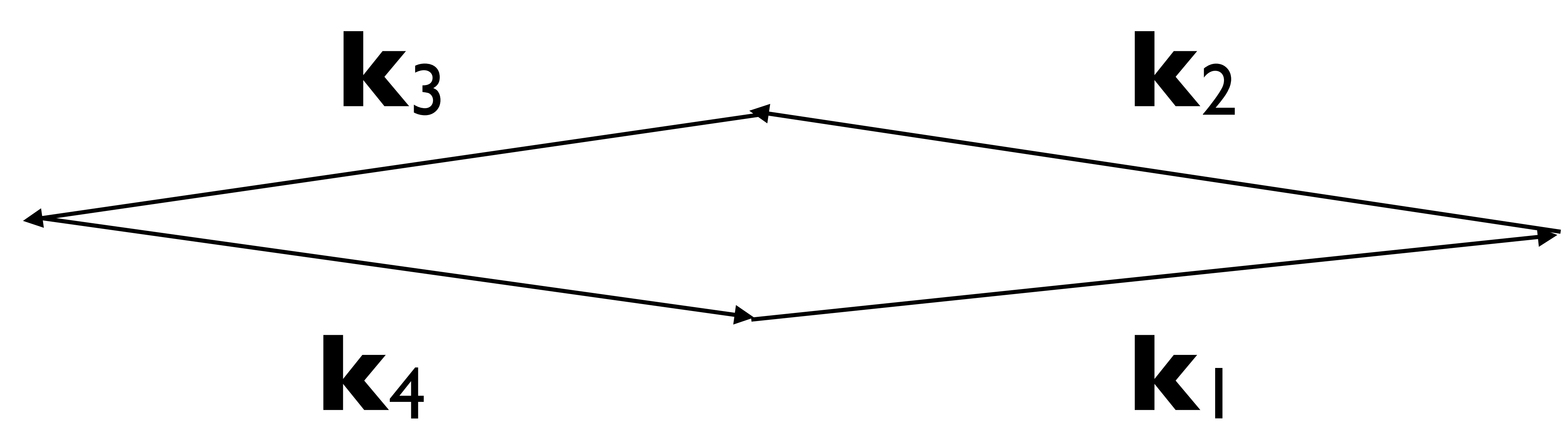
- We look at the local-form four-point function (trispectrum).
- Specifically, we look for a consistency relation between the local-form bispectrum and trispectrum that is respected by (almost) all models of multi-field inflation.
- We found one: $\tau_{\rm NL} > \frac{1}{2}\left(\frac{6}{5}f_{\rm NL}\right)^2$

provided that 2-loop and higher-order terms are ignored.

Trispectrum

- $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$

  $\times\tau_{NL}[P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\text{cyc.}]$



τ_{NL}

Tree-level Result (Suyama & Yamaguchi)

- Usual δN expansion to the second order

$$\zeta = \sum_I \frac{\partial N}{\partial \phi_I}\delta\phi_I + \frac{1}{2}\sum_{IJ}\frac{\partial^2 N}{\partial\phi_I \partial\phi_J}\delta\phi_I\delta\phi_J + \ldots$$

gives:

$$\frac{6}{5}f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,IJ}N_{,I}N_{,J}}{[\sum_I (N_{,I})^2]^2},$$

$$\tau_{\rm NL} = \frac{\sum_{IJK} N_{,IJ}N_{,J}N_{,IK}N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ}N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

Now, stare at these.

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{[\sum_I (N_{,I})^2]^2},$$

$$\tau_{\rm NL} = \frac{\sum_{IJK} N_{,IJ} N_{,J} N_{,IK} N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ} N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

Change the variable...

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{[\sum_I (N_{,I})^2]^2},$$

$$\tau_{\rm NL} = \frac{\sum_{IJK} N_{,IJ} N_{,J} N_{,IK} N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ} N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

$$a_I = \frac{\sum_J N_{,IJ} N_{,J}}{[\sum_J (N_{,J})^2]^{3/2}}$$

$$b_I = \frac{N_{,I}}{[\sum_J (N_{,J})^2]^{1/2}}$$

$(6/5) f_{NL} = \sum_I a_I b_I$

$\tau_{NL} = (\sum_I a_I^2)(\sum_I b_I^2)$

Then apply the Cauchy-Schwarz Inequality

$$\left(\sum_I a_I^2\right)\left(\sum_J b_J^2\right) \geq \left(\sum_I a_I b_I\right)^2$$

- Implies (Suyama & Yamaguchi 2008)

$$\tau_{\rm NL} \geq \left(\frac{6\,f_{\rm NL}^{\rm local}}{5}\right)^2$$

But, this is valid only at the tree level!

Harmless models can violate the tree-level result

- The Suyama-Yamaguchi inequality does not always hold because the Cauchy-Schwarz inequality can be 0=0. For example:

$$\zeta = \frac{\partial N}{\partial \phi_1}\delta\phi_1 + \frac{1}{2}\frac{\partial^2 N}{\partial \phi_2^2}\delta\phi_2^2$$

In this harmless two-field case, the Cauchy-Schwarz inequality becomes 0=0 (both f_{NL} and τ_{NL} result from the second term). In this case,

$$\tau_{\rm NL} \sim 10^3 (f_{\rm NL}^{\rm local})^{4/3}$$

(Suyama & Takahashi 2008)

"1 Loop"

$$\zeta = \frac{\partial N}{\partial \phi_1}\delta\phi_1 + \frac{1}{2}\frac{\partial^2 N}{\partial \phi_2^2}\delta\phi_2^2$$

Fourier transform this, and multiply 3 times

$$\int \frac{d^3p}{(2\pi)^3}\int \frac{d^3q}{(2\pi)^3}\int \frac{d^3s}{(2\pi)^3}\langle \delta\tilde{\phi}_2(\mathbf{k}_1-\mathbf{p})\delta\tilde{\phi}_2(\mathbf{p})\delta\tilde{\phi}_2(\mathbf{k}_2-\mathbf{q})\delta\tilde{\phi}_2(\mathbf{q})\delta\tilde{\phi}_2(\mathbf{k}_3-\mathbf{s})\delta\tilde{\phi}_2(\mathbf{s})\rangle$$

$$= \left(\frac{H^2}{2}\right)^3 (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\int \frac{d^3p}{(2\pi)^3}\frac{1}{p^3|\mathbf{k}_1-\mathbf{p}|^3|\mathbf{k}_3+\mathbf{p}|^3} + (\text{permutations})$$

$p_{min}=1/L$

$$\approx \left(\frac{H^2}{2}\right)^3 (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{8\ln(k_bL)}{2\pi^2}\left[\frac{1}{k_1^3k_3^2}+\frac{1}{k_2^3k_3^2}+\frac{1}{k_1^3k_2^2}\right]$$

- $k_b=\min(k_1,k_2,k_3)$

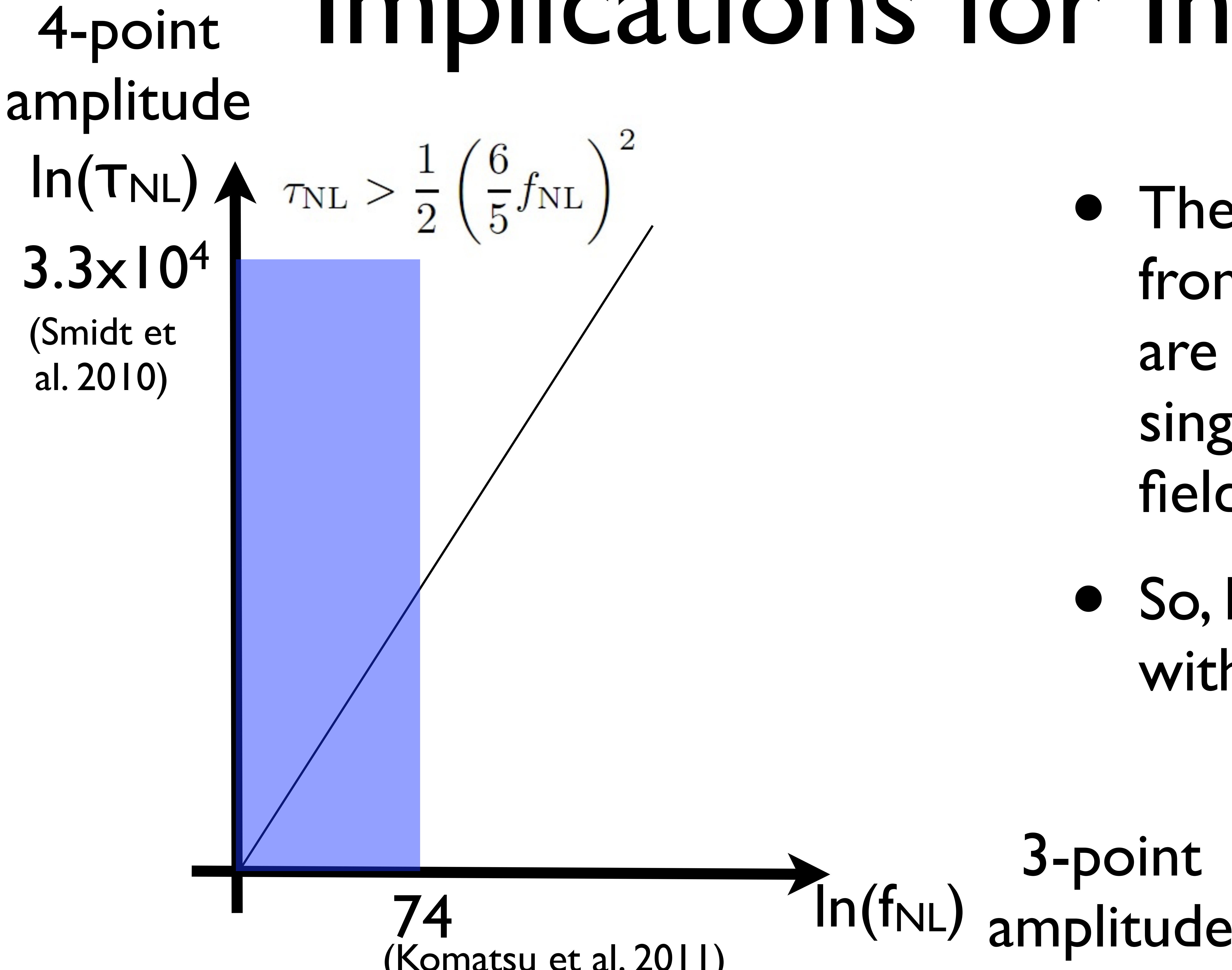
Ignoring details...

- I don't have time to show you the derivation (you can look it up in the paper), but the result is somewhat weaker than the Suyama-Yamaguchi inequality:

$$\tau_{\rm NL} > \frac{1}{2}\left(\frac{6}{5} f_{\rm NL}\right)^2$$

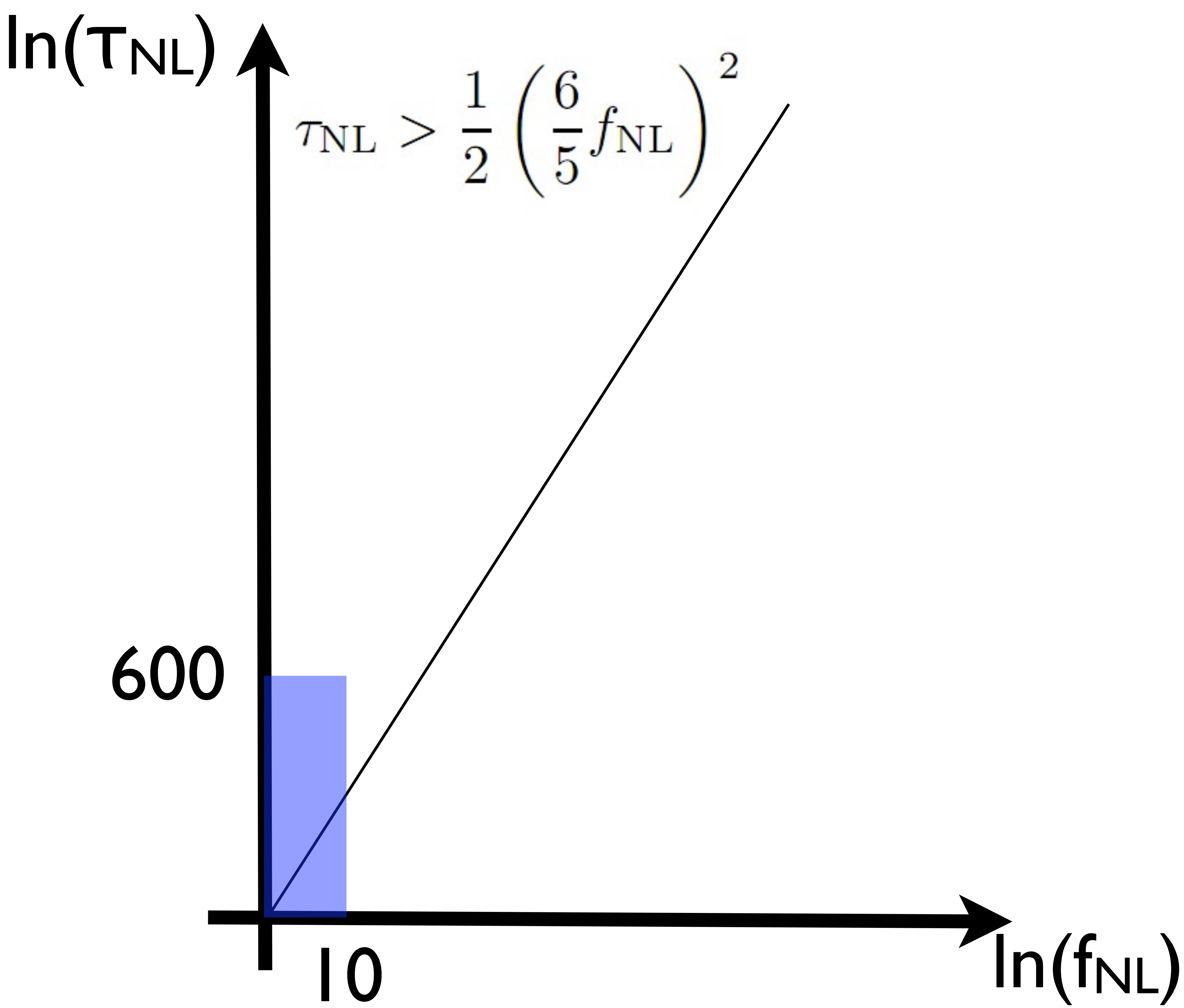
Detection of a violation of this relation can potentially falsify inflation as a mechanism for generating cosmological fluctuations.

Implications for Inflation



- The current limits from WMAP 7-year are consistent with single-field or multi-field models.
- So, let's play around with the future.

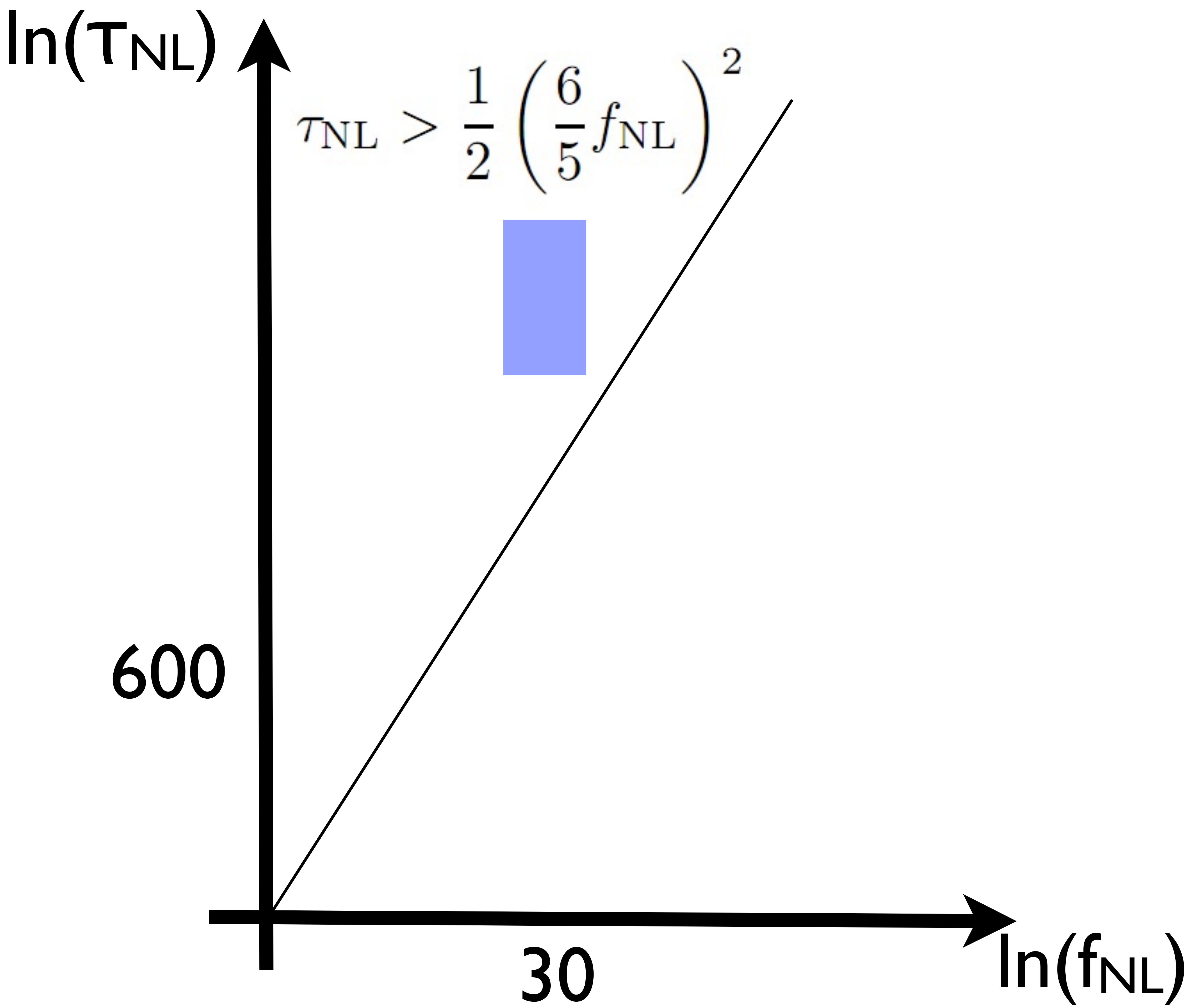
Case A: Single-field Happiness



- No detection of anything (f_{NL} or τ_{NL}) after Planck. Single-field survived the test (for the moment: the future galaxy surveys can improve the limits by a factor of ten).

(Suyama & Yamaguchi 2008; Komatsu 2010; Sugiyama, Komatsu & Futamase 2011)

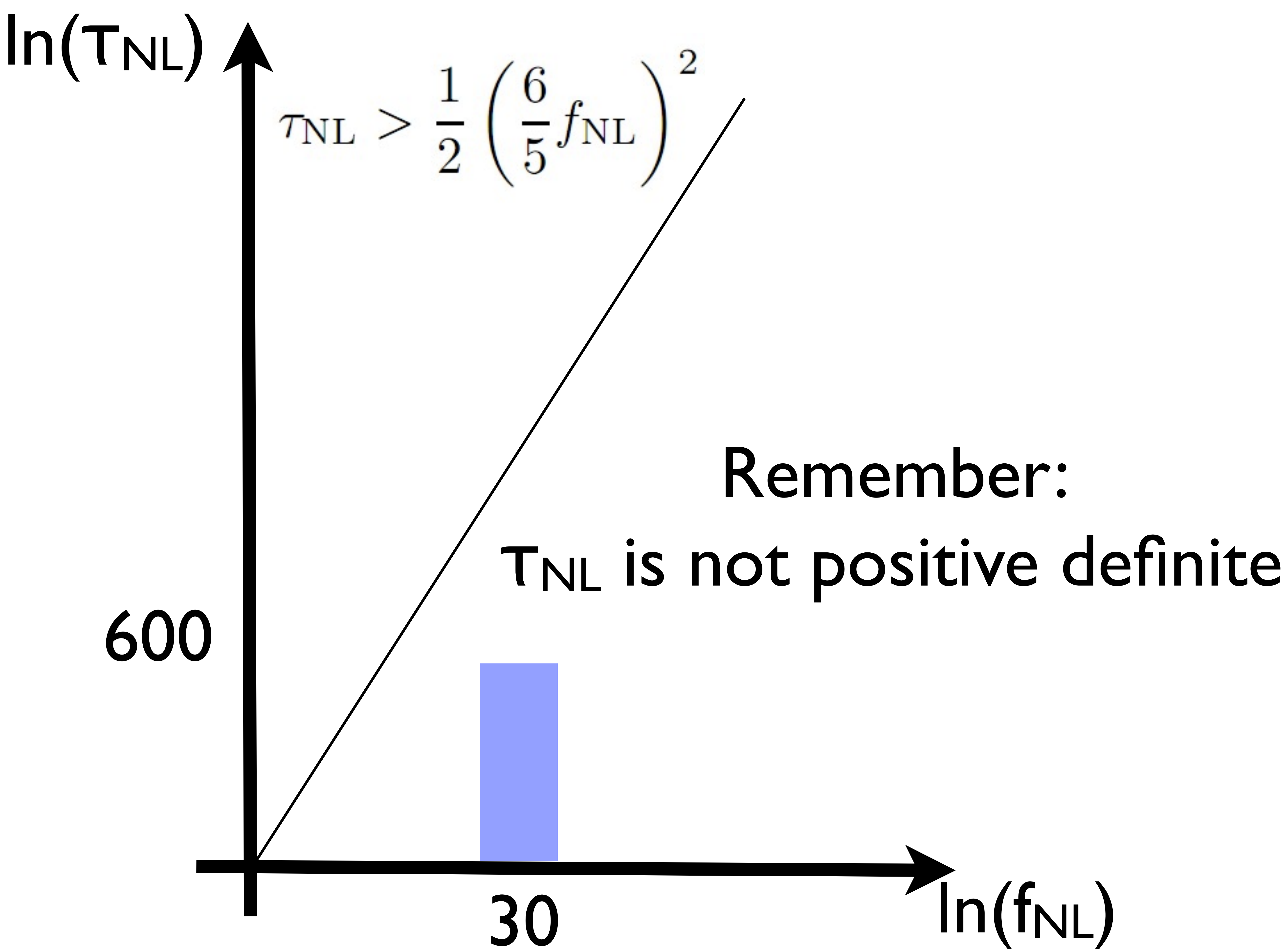
Case B: Multi-field Happiness(?)



- **f_{NL} is detected. Single-field is gone.**
- But, τ_{NL} is also detected, in accordance with **$\tau_{NL}>0.5(6f_{NL}/5)^2$** expected from most multi-field models.

(Suyama & Yamaguchi 2008; Komatsu 2010; Sugiyama, Komatsu & Futamase 2011)

Case C: Madness



- f_{NL} is detected. Single-field is gone.
- But, τ_{NL} is not detected, or **found to be negative**, inconsistent with $\tau_{NL}>0.5(6f_{NL}/5)^2$.
- **Single-field AND most of multi-field models are gone.**

Summary

- A more insight into the single-field consistency relation for the squeezed-limit bispectrum using in-in formalism.
- Non-Bunch-Davies vacuum can give an enhanced bispectrum in the $k_3/k_1 \ll 1$ limit, yielding a distinct form of the scale-dependent bias.
- Multi-field consistency relation between the 3-point and 4-point function can be used to rule out multi-field inflation, as well as single-field.