Hunting for Primordial Non-Gaussianity

f_{NL}

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What is f_{NL}?

- For a pedagogical introduction to f_{NL}, see **Komatsu, astro-ph/0206039**
- In one sentence: "f_{NL} is a **quantitative measure of the magnitude of primordial non-Gaussianity** in curvature perturbations.*"

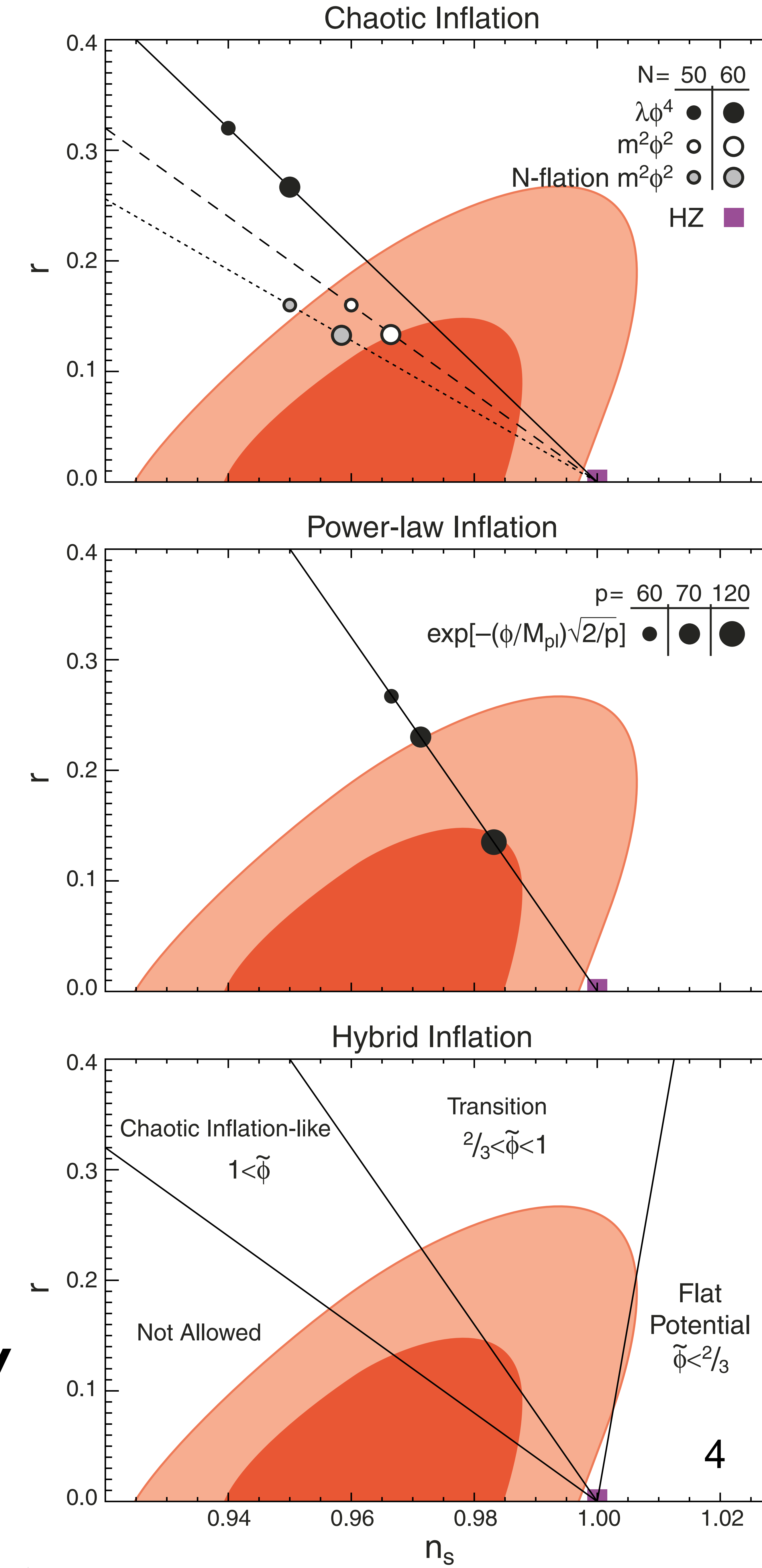
* where a positive curvature perturbation gives a negative CMB anisotropy in the Sachs-Wolfe limit

Why is Non-Gaussianity Important?

- Because a detection of f_{NL} has a best chance of ruling out the **largest** class of early universe models.
- Namely, it will rule out inflation models based upon
  - a single scalar field <u>with</u>
  - the canonical kinetic term <u>that</u>
  - rolled down a smooth scalar potential slowly, <u>and</u>
  - was initially in the Banch-Davies vacuum.
- ***Detection of non-Gaussianity would be a major breakthrough in cosmology.***

We have *r* and *n_s*. Why Bother?

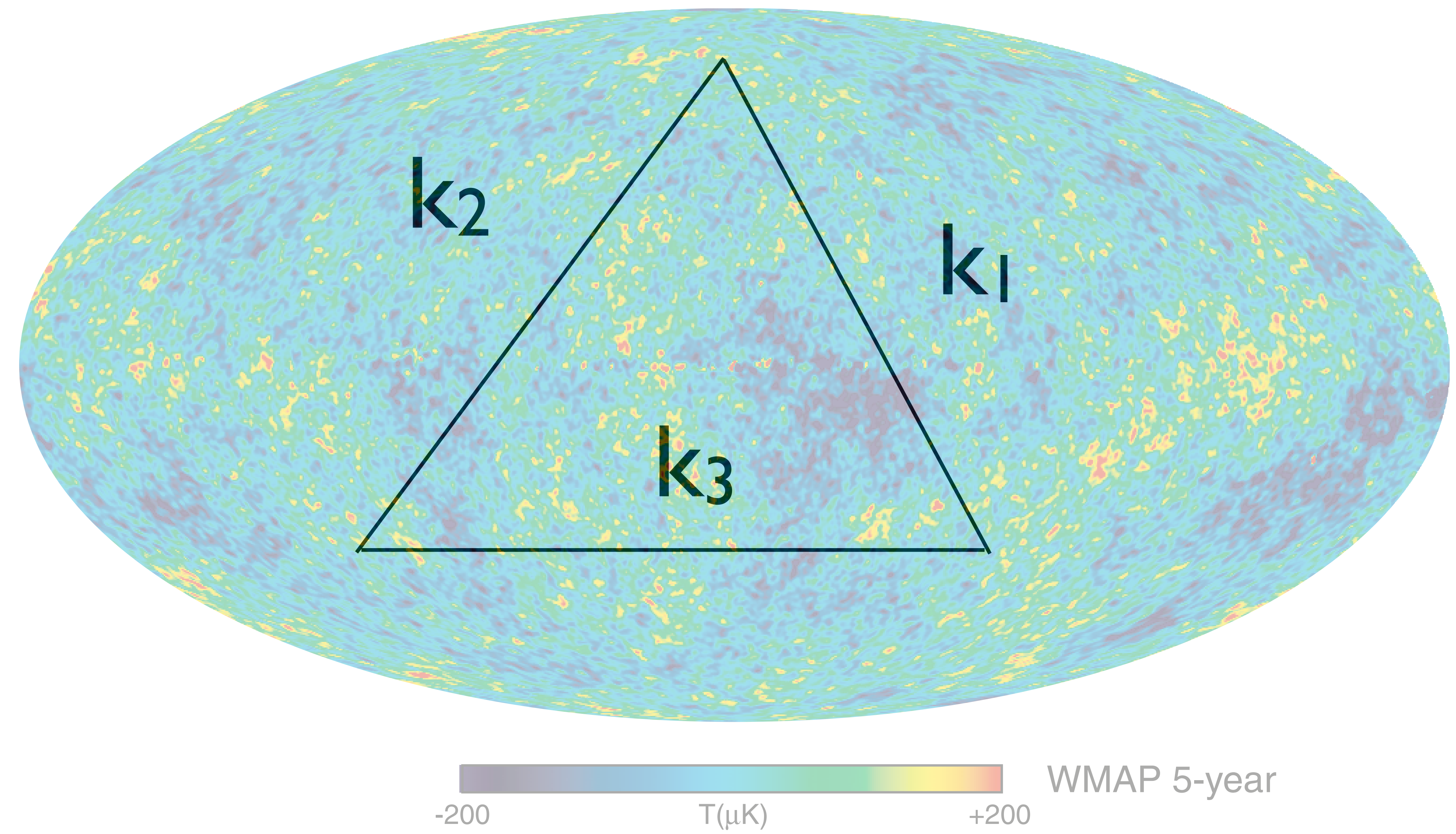
- While the current limit on the power-law index of the primordial power spectrum, **n_s**, and the amplitude of gravitational waves, **r**, have ruled out many inflation models already, many still survive (which is a good thing!)
- A convincing detection of f_{NL} would rule out most of them regardless of n_s or r.
- f_{NL} offers more ways to test various early universe models!



What if f_{NL} /= 0?

- A single field, canonical kinetic term, slow-roll, and/or Banch-Davies vacuum, must be modified.
  - Multi-field (curvaton)
  - Non-canonical kinetic term (k-inflation, DBI)
  - Temporary fast roll (features in potential; Ekpyrotic fast roll)
  - Departures from the Banch-Davies vacuum
- It will give us a lot of clues as to what the correct early universe models should look like.

So, what is f_{NL}?



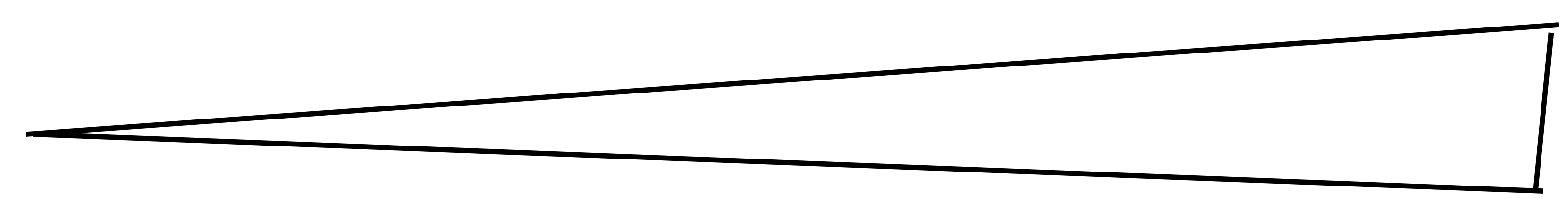
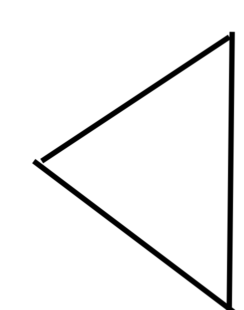
- **f_{NL} = the amplitude of three-point function**, or also known as the "bispectrum," $B(k_1,k_2,k_3)$, which is
  - $=\langle\Phi(k_1)\Phi(k_2)\Phi(k_3)\rangle = f_{NL}^{(i)}(2\pi)^3\delta^3(k_1+k_2+k_3)b^{(i)}(k_1,k_2,k_3)$
  - where $\Phi(k)$ is the Fourier transform of the curvature perturbation, and $b(k_1,k_2,k_3)$ is a model-dependent function that defines the shape of triangles predicted by various models.

Why Bispectrum?

- **The bispectrum <u>vanishes</u>** for Gaussian random fluctuations.
- Any non-zero detection of the bispectrum indicates the presence of (some kind of) non-Gaussianity.
- A very sensitive tool for finding non-Gaussianity.

Komatsu & Spergel (2001); Babich, Creminelli & Zaldarriaga (2004)

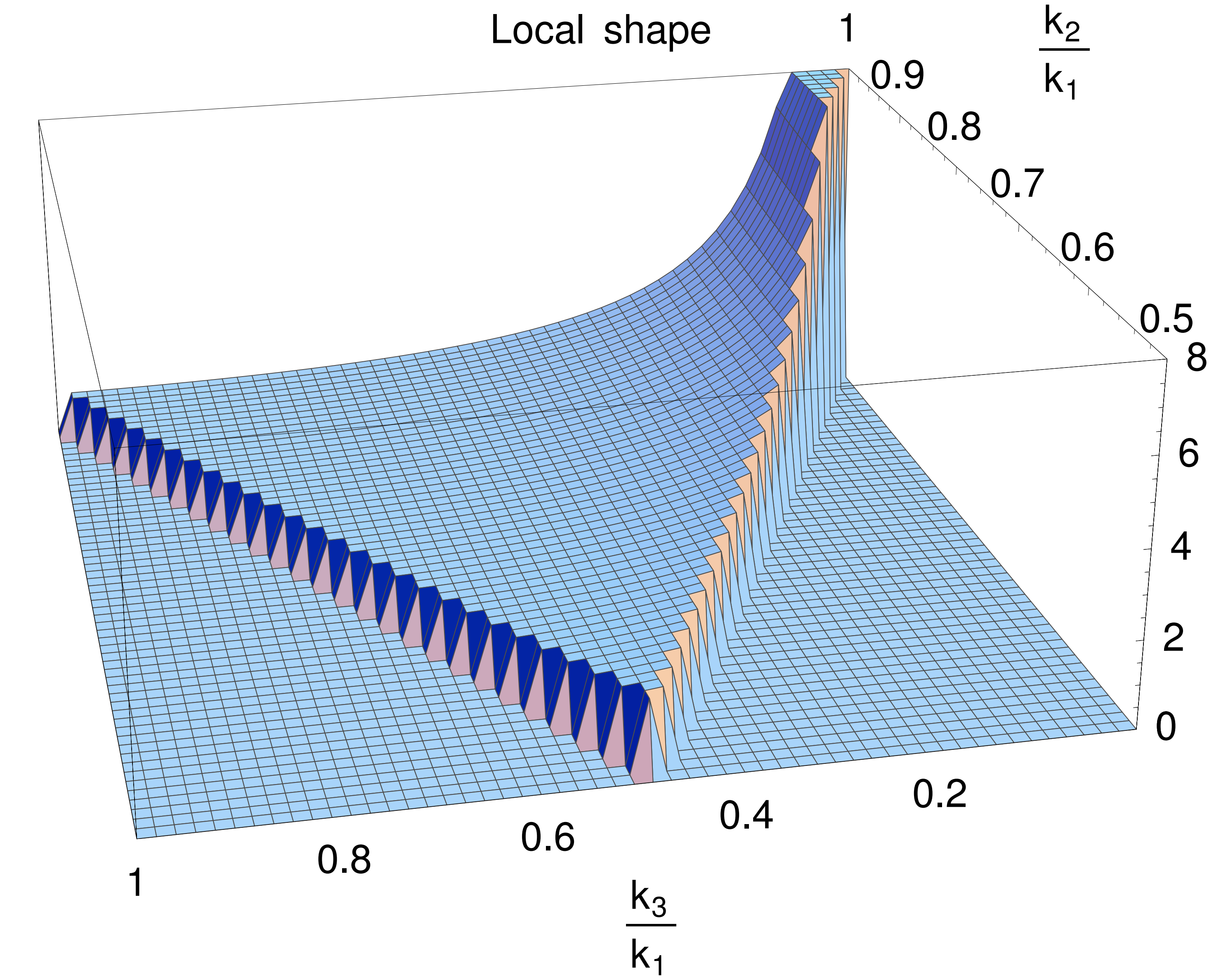
Two f_{NL}'s

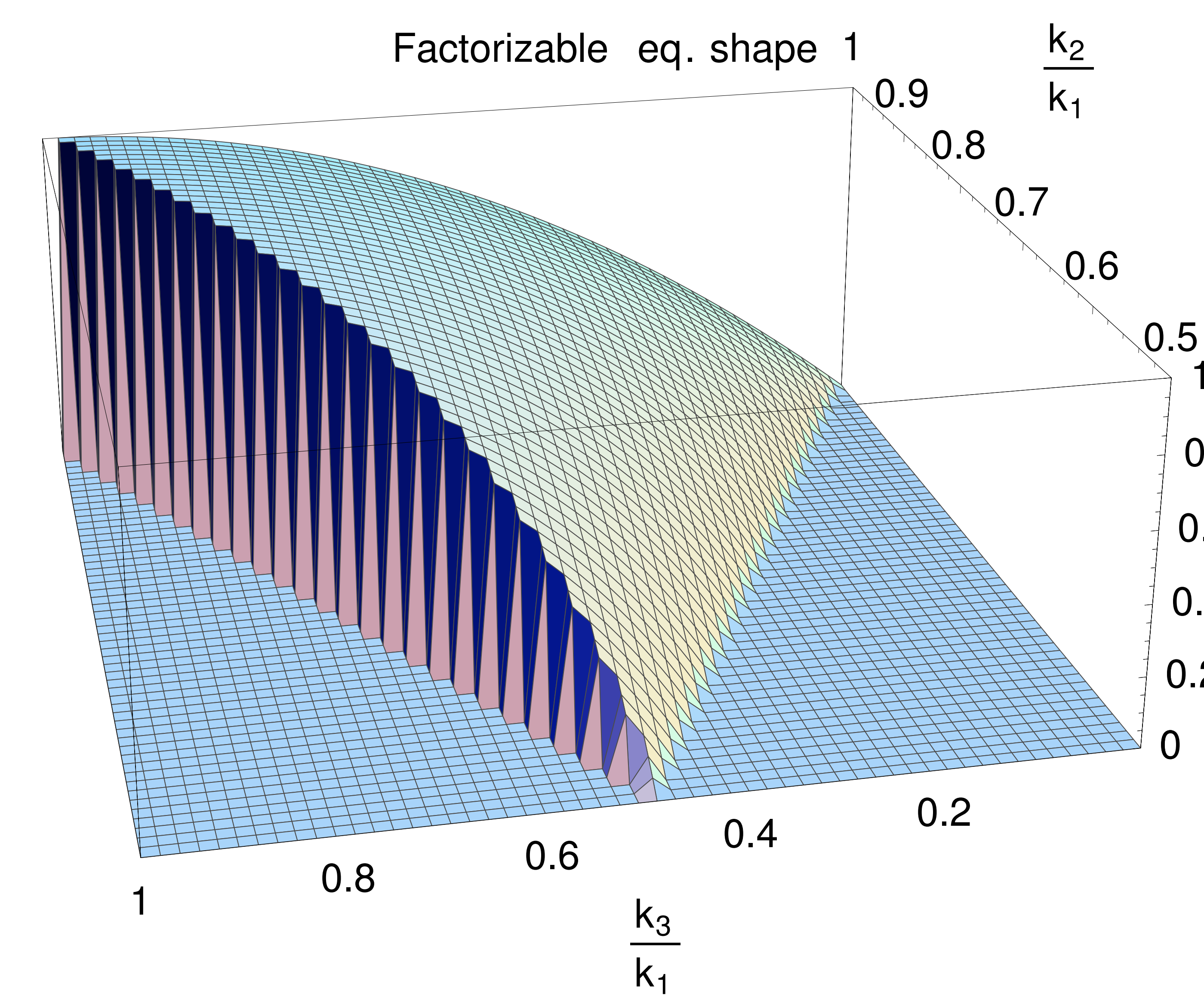
- Depending upon the shape of triangles, one can define various f_{NL}'s:
- "Local" form 
  - which generates non-Gaussianity locally (i.e., at the same location) via $\Phi(x)=\Phi_{gaus}(x)+f_{NL}^{local}[\Phi_{gaus}(x)]^2$
- "Equilateral" form 
  - which generates non-Gaussianity in a different way (e.g., k-inflation, DBI inflation)

Earlier work on the local form:
Salopek&Bond (1990); Gangui et al. (1994);
Verde et al. (2000); Wang&Kamionkowski (2000)

Forms of $b(k_1,k_2,k_3)$

- Local form (Komatsu & Spergel 2001)
  - $b^{local}(k_1,k_2,k_3) = 2[P(k_1)P(k_2)+cyc.]$
- Equilateral form (Babich, Creminelli & Zaldarriaga 2004)
  - $b^{equilateral}(k_1,k_2,k_3) = 6\{-[P(k_1)P(k_2)+cyc.] - 2[P(k_1)P(k_2)P(k_3)]^{2/3} + [P(k_1)^{1/3}P(k_2)^{2/3}P(k_3)+cyc.]\}$





Journal on f_{NL}

- Local
  - $-3500 < f_{NL}^{local} < 2000$ [COBE 4yr, l_{max}=20] Komatsu et al. (2002)
  - $-58 < f_{NL}^{local} < 134$ [WMAP 1yr, l_{max}=265] Komatsu et al. (2003)
  - $-54 < f_{NL}^{local} < 114$ [WMAP 3yr, l_{max}=350] Spergel et al. (2007)
  - **$-9 < f_{NL}^{local} < 111$ [WMAP 5yr, l_{max}=500]** Komatsu et al. (2008)
- Equilateral
  - $-366 < f_{NL}^{equil} < 238$ [WMAP 1yr, l_{max}=405] Creminelli et al. (2006)
  - $-256 < f_{NL}^{equil} < 332$ [WMAP 3yr, l_{max}=475] Creminelli et al. (2007)
  - **$-151 < f_{NL}^{equil} < 253$ [WMAP 5yr, l_{max}=700]** Komatsu et al. (2008)

Methodology

- I am not going to bother you too much with methodology...
  - Please read Appendix A of Komatsu et al., if you are interested in details.
- We use a well-established method developed over the years by: Komatsu, Spergel & Wandelt (2005); Creminelli et al. (2006); Yadav, Komatsu & Wandelt (2007)
  - There is still a room for improvement (Smith & Zaldarriaga 2006)

Data Combination

- We mainly use V band (61 GHz) and W band (94 GHz) data.
  - The results from Q band (41 GHz) are discrepant, probably due to a stronger foreground contamination
- These are *foreground-reduced maps*, delivered on the LAMBDA archive.
  - We also give the results from the raw maps.

Mask

- We have upgraded the Galaxy masks.
  - 1yr and 3yr release
    - "Kp0" mask for Gaussianity tests (76.5%)
    - "Kp2" mask for the C_l analysis (84.6%)
  - 5yr release
    - "KQ75" mask for Gaussianity tests (71.8%)
    - "KQ85" mask for the C_l analysis (81.7%)

Gold et al. (2008)

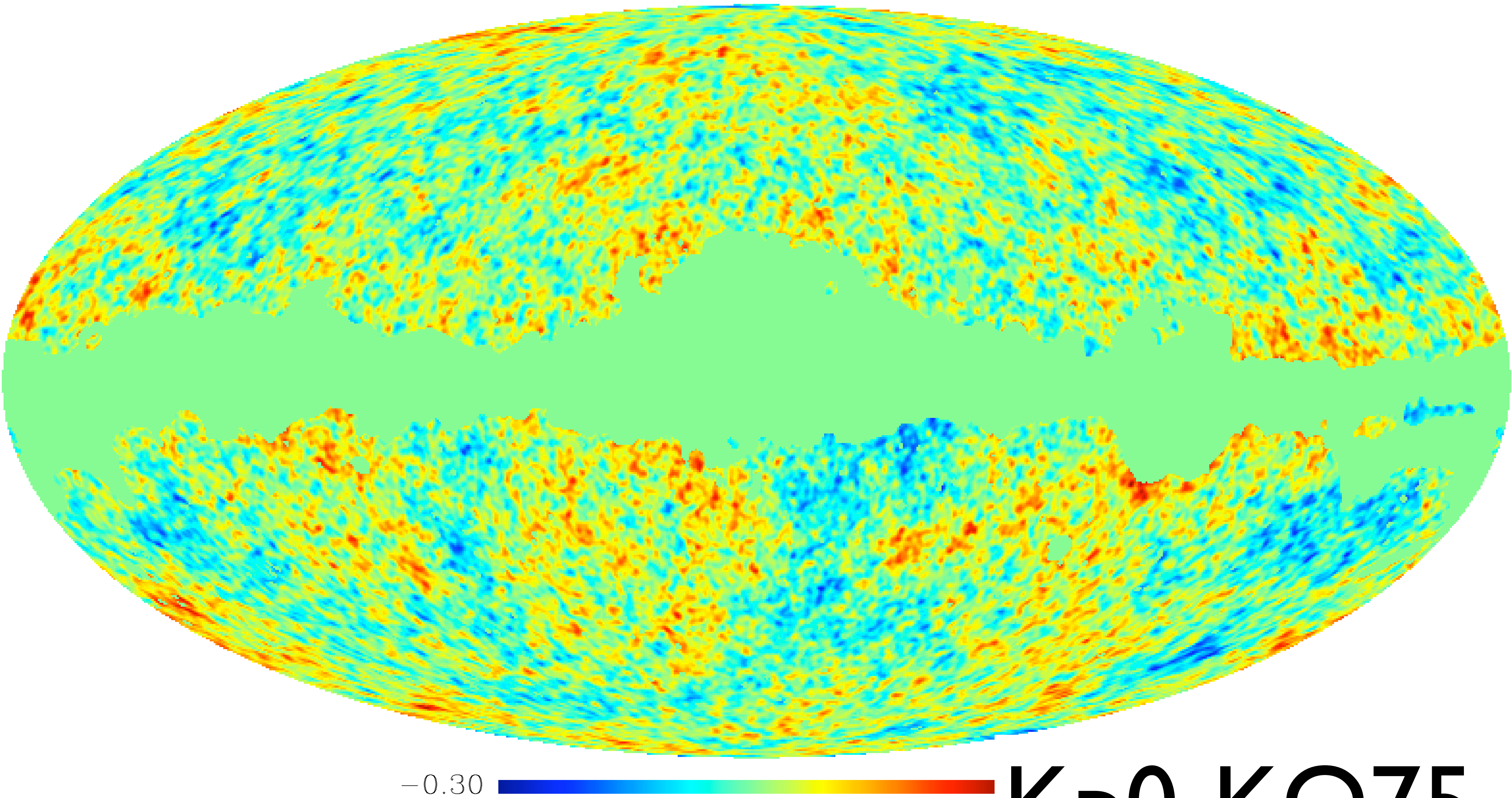
- What are the KQx masks?
  - The previous KpN masks identified the bright region in the K band data, which are contaminated mostly by the synchrotron emission, and masked them.
    - "p" stands for "plus," and N represents the brightness level above which the pixels are masked.
  - The new KQx masks identify the bright region in the K band minus the CMB map from Internal Linear Combination (the CMB picture that you always see), as well as the bright region in the Q band minus ILC.
  - Q band traces the free-free emission better than K.
  - x represents a fraction of the sky retained in K or Q.

Gold et al. (2008)

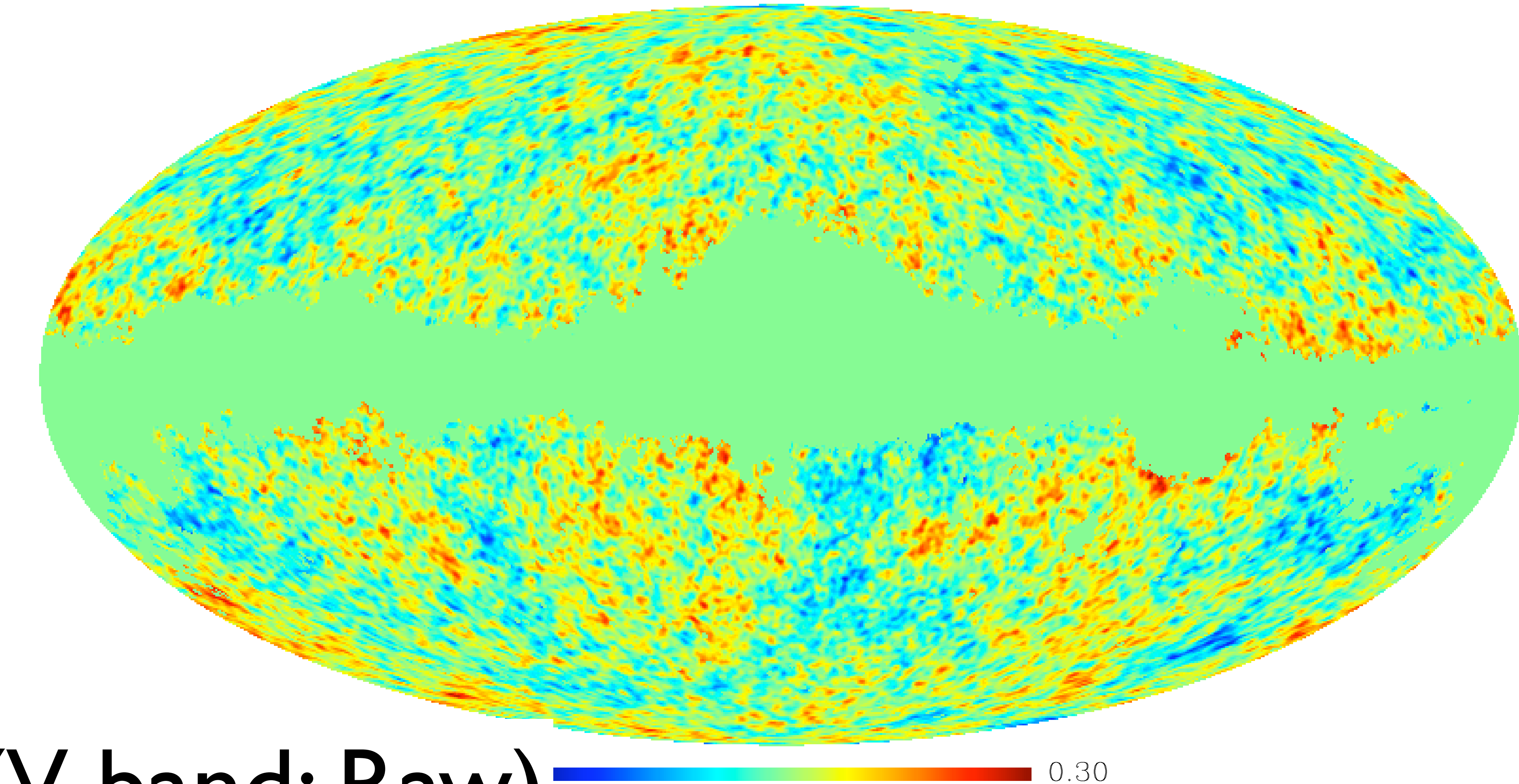
Why KQ75?

- The KQ75 mask removes the pixels that are contaminated by the free-free region better than the Kp0 mask.
- CMB was absent when the mask was defined, as the masked was defined by the K (or Q) band map minus the CMB map from ILC.
- The final mask is a combination of the K mask (which retains 75% of the sky) and the Q mask (which also retains 75%). Since Q masks the region that is not masked by K, the final KQ75 mask retains less than 75% of the sky. (It retains 71.8% of the sky for cosmology.)

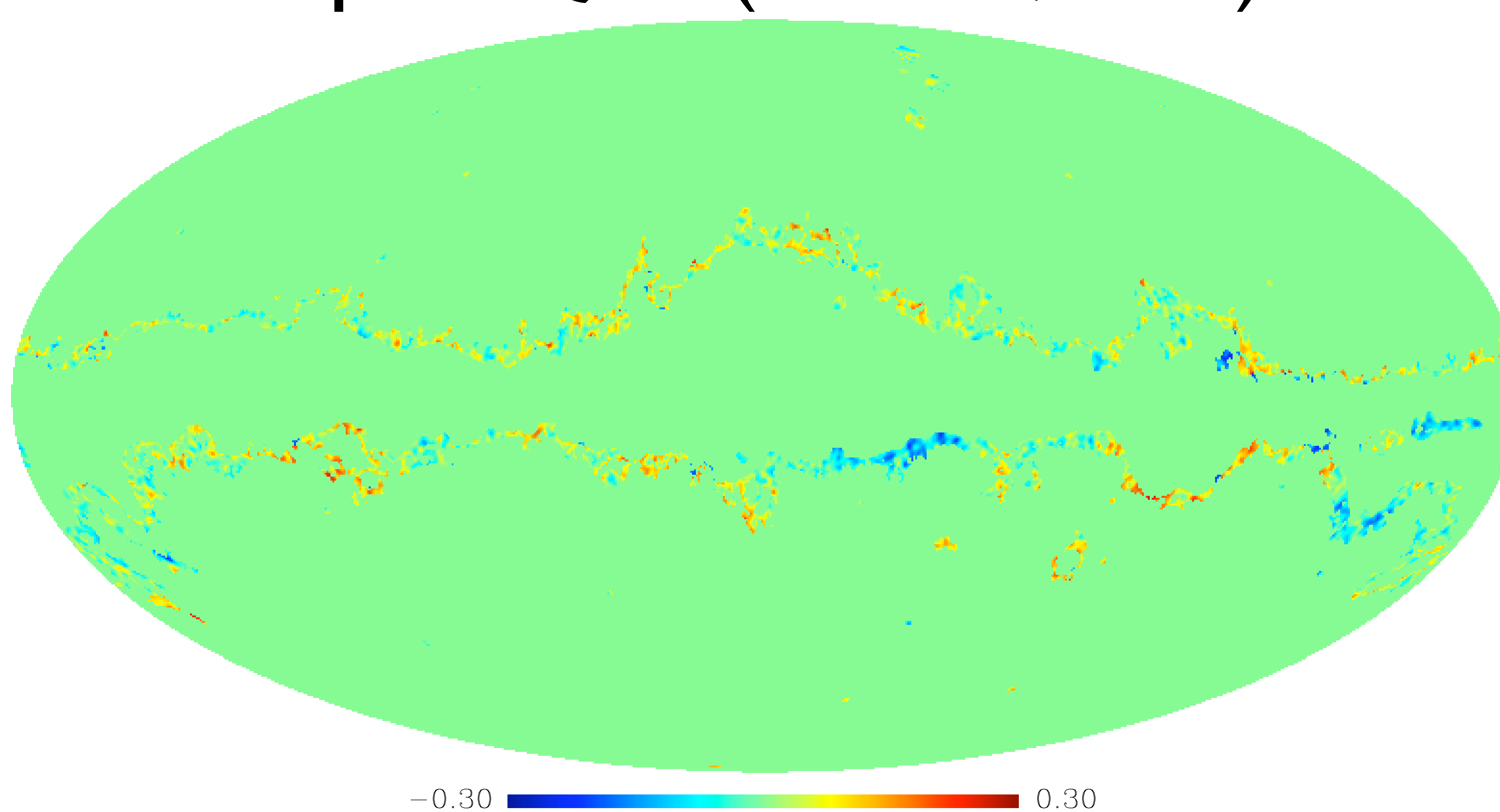
Kp0 (V band; Raw)



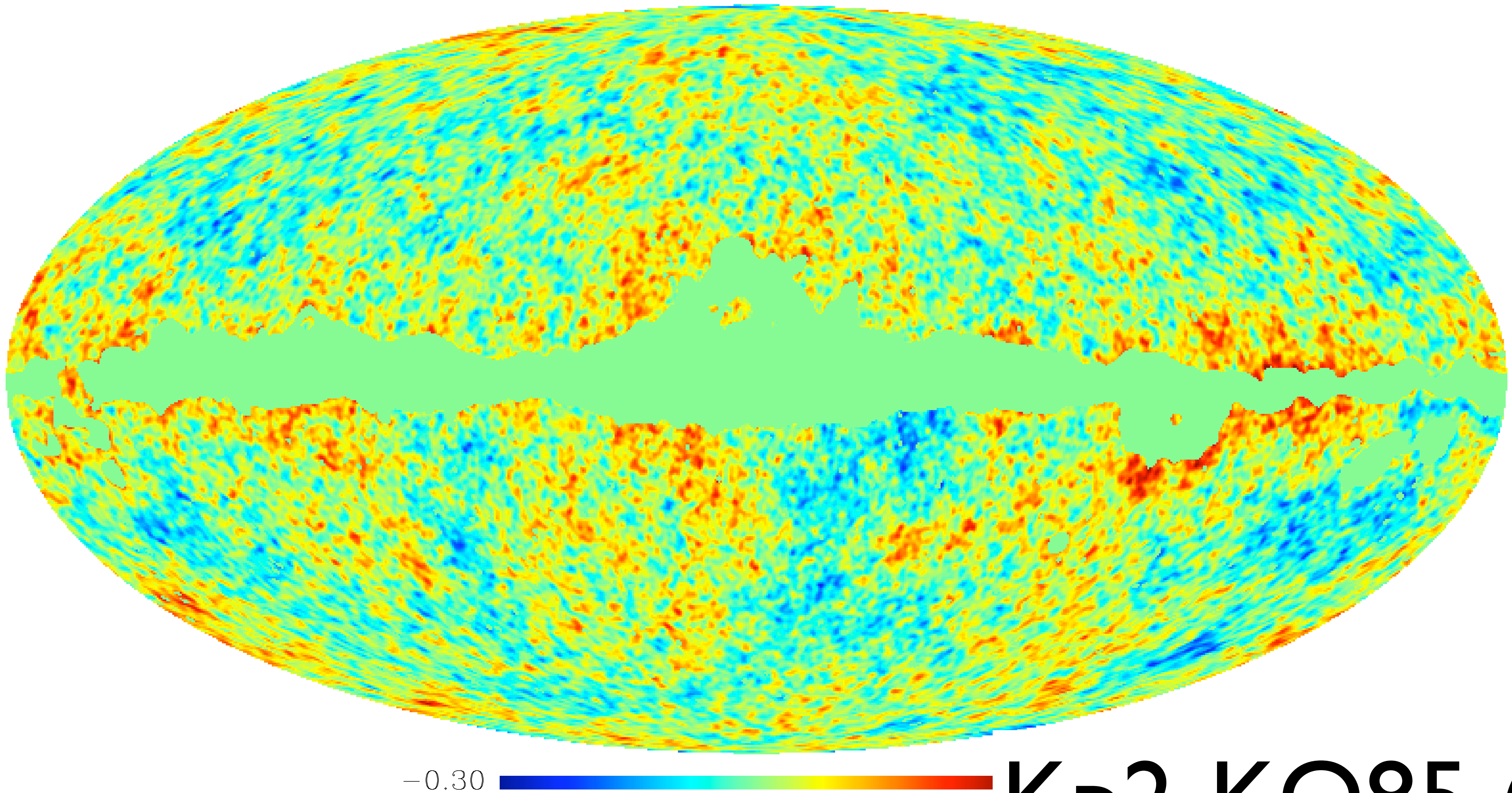
KQ75 (V band; Raw)



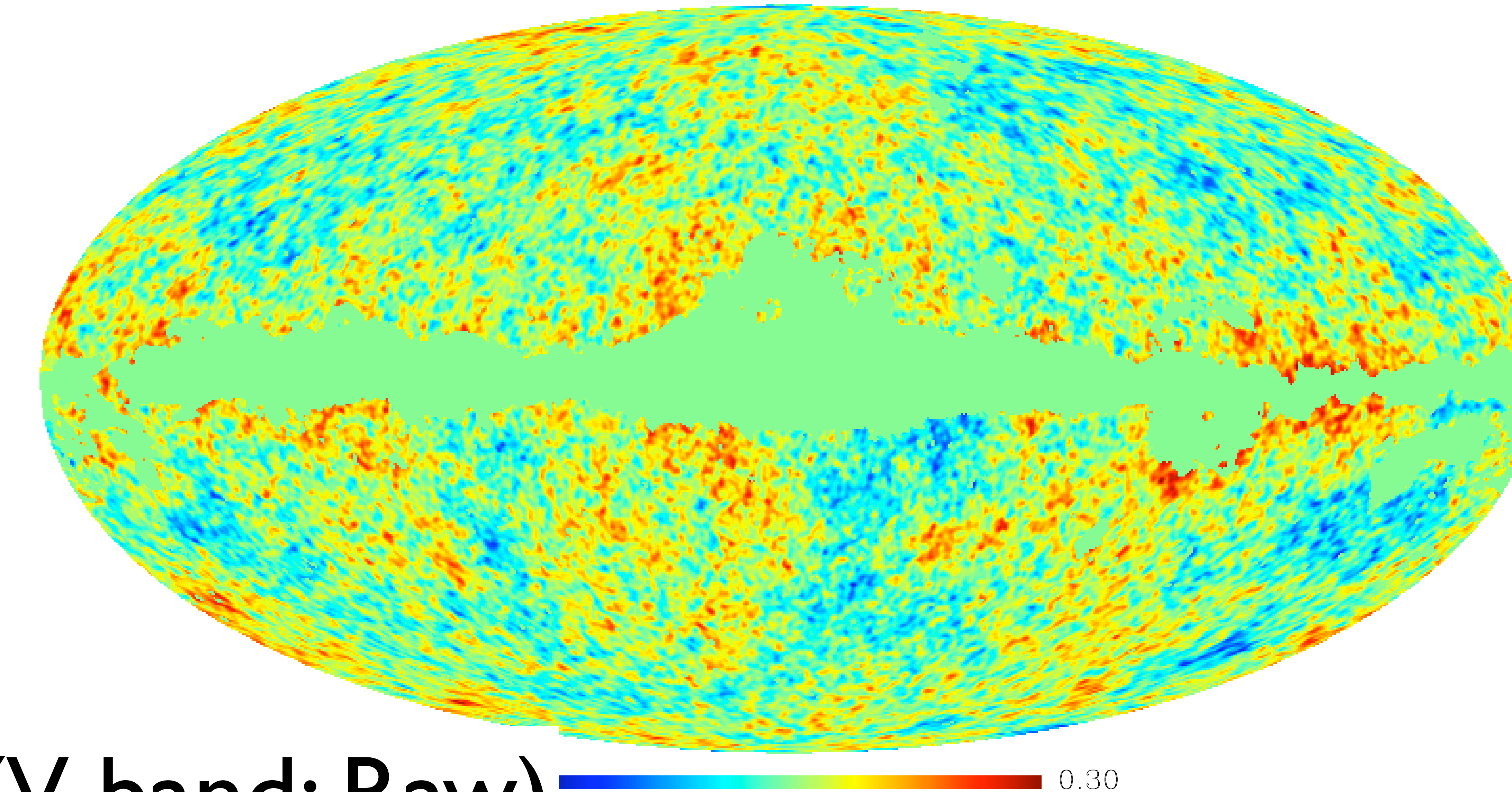
Kp0-KQ75 (V band; Raw)



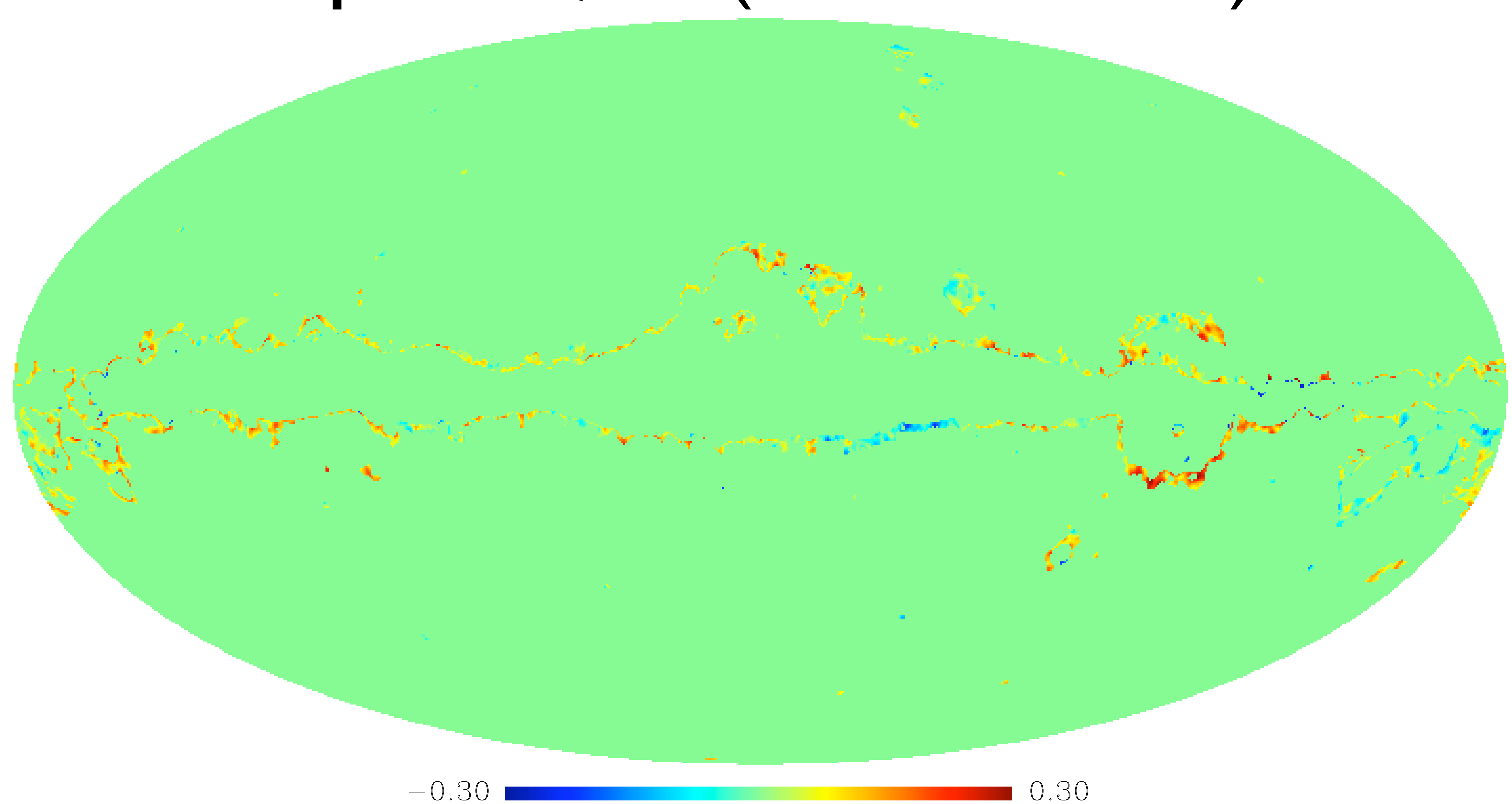
Kp2 (V band; Raw)



KQ85 (V band; Raw)



Kp2-KQ85 (V band; Raw)



Komatsu et al. (2008)

Main Result (Local)

Band	Mask	$l_{\max}$	$f_{NL}^{\rm local}$	$\Delta f_{NL}^{\rm local}$	b_{src}
V+W	*KQ85*	400	50 ± 29	1 ± 2	0.26 ± 1.5
V+W	*KQ85*	500	61 ± 26	2.5 ± 1.5	0.05 ± 0.50
V+W	*KQ85*	600	68 ± 31	3 ± 2	0.53 ± 0.28
V+W	*KQ85*	700	67 ± 31	3.5 ± 2	0.34 ± 0.20
V+W	*Kp0*	500	61 ± 26	2.5 ± 1.5	
V+W	*KQ75p1*[a]	500	53 ± 28	4 ± 2	
V+W	*KQ75*	400	47 ± 32	3 ± 2	-0.50 ± 1.7
V+W	*KQ75*	500	55 ± 30	4 ± 2	0.15 ± 0.51
V+W	*KQ75*	600	61 ± 36	4 ± 2	0.53 ± 0.30
V+W	*KQ75*	700	58 ± 36	5 ± 2	0.38 ± 0.21

- ~ 2 sigma "hint": **f_{NL}^{local} ~ 60 +/- 30 (68% CL)**
- 1.8 sigma for KQ75; 2.3 sigma for KQ85 & Kp0

Komatsu et al. (2008)

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V+W	*KQ75*	600	61 ± 36	4 ± 2	0.53 ± 0.30
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- The results are not sensitive to the maximum multipoles used in the analysis, l_{max}.

Komatsu et al. (2008)

Main Result (Local)

Band	Mask	$l_{\max}$	$f_{NL}^{\rm local}$	$\Delta f_{NL}^{\rm local}$	b_{src}
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V+W	*KQ75*	600	61 ± 36	4 ± 2	0.53 ± 0.30
V+W	*KQ75*	700	58 ± 36	5 ± 2	0.38 ± 0.21

- The estimated contamination from the point sources is small, if any. (Likely overestimated by a factor of ~2.)

Komatsu et al. (2008)

Null Tests

Band	Foreground	Mask	$f_{NL}^{\rm local}$
Q−W	Raw	$KQ75$	-0.53 ± 0.22
V−W	Raw	$KQ75$	-0.31 ± 0.23
Q−W	Clean	$KQ75$	0.10 ± 0.22
V−W	Clean	$KQ75$	0.06 ± 0.23

- **No signal in the difference of cleaned maps.**

Komatsu et al. (2008)

Frequency Dependence

Band	Foreground	Mask	$f_{NL}^{\rm local}$
Q	Raw	$KQ75$	-42 ± 48
V	Raw	$KQ75$	41 ± 35
W	Raw	$KQ75$	46 ± 35
Q	Clean	$KQ75$	10 ± 48
V	Clean	$KQ75$	50 ± 35
W	Clean	$KQ75$	62 ± 35

- **Q is very sensitive to the foreground cleaning.**

Komatsu et al. (2008)

V+W: Raw vs Clean (l_{max}=500)

Band	Foreground	Mask	f_{NL}^{local}
V+W	Raw	*KQ85*	9 ± 26
V+W	Raw	*Kp0*	48 ± 26
V+W	Raw	*KQ75p1*	41 ± 28
V+W	Raw	*KQ75*	43 ± 30

- Clean-map results:
  - KQ85; 61 +/- 26
  - Kp0; 61 +/- 26
  - KQ75p1; 53 +/- 28
  - KQ75; 55 +/- 30

Foreground contamination is not too severe.

The Kp0 and KQ85 results may be as clean as the KQ75 results.

Komatsu et al. (2008)

Our Best Estimate

- Why not using Kp0 or KQ85 results, which have a higher statistical significance?
- Given the profound implications and impact of non-zero f_{NL}^{local}, we have chosen a conservative limit from the KQ75 with the point source correction ($\Delta f_{NL}^{local}=4$, which is also conservative) as our best estimate.
  - The 68% limit: $f_{NL}^{local} = 51 \pm 30$ [1.7 sigma]
  - **The 95% limit: $-9 < f_{NL}^{local} < 111$**

Yadav & Wandelt (2008)

Comparison with Y&W

- Yadav and Wandelt used the raw V+W map from the 3-year data.
  - 3yr: f_{NL}^{local} = 68 +/- 30 for l_{max}=450 & Kp0 mask
  - 3yr: f_{NL}^{local} = 80 +/- 30 for l_{max}=550 & Kp0 mask
- Our corresponding 5-year raw map estimate is
  - 5yr: f_{NL}^{local} = 48 +/- 26 for l_{max}=500 & Kp0 mask
  - C.f. clean-map estimate: f_{NL}^{local} = 61 +/- 26
- With more years of observations, the values have come down to a lower significance.

Komatsu et al. (2008)

Main Result (Equilateral)

Band	Mask	$l_{\max}$	$f_{NL}^{\rm equil}$	$\Delta f_{NL}^{\rm equil}$
V+W	*KQ75*	400	77 ± 146	9 ± 7
V+W	*KQ75*	500	78 ± 125	14 ± 6
V+W	*KQ75*	600	71 ± 108	27 ± 5
V+W	*KQ75*	700	73 ± 101	22 ± 4

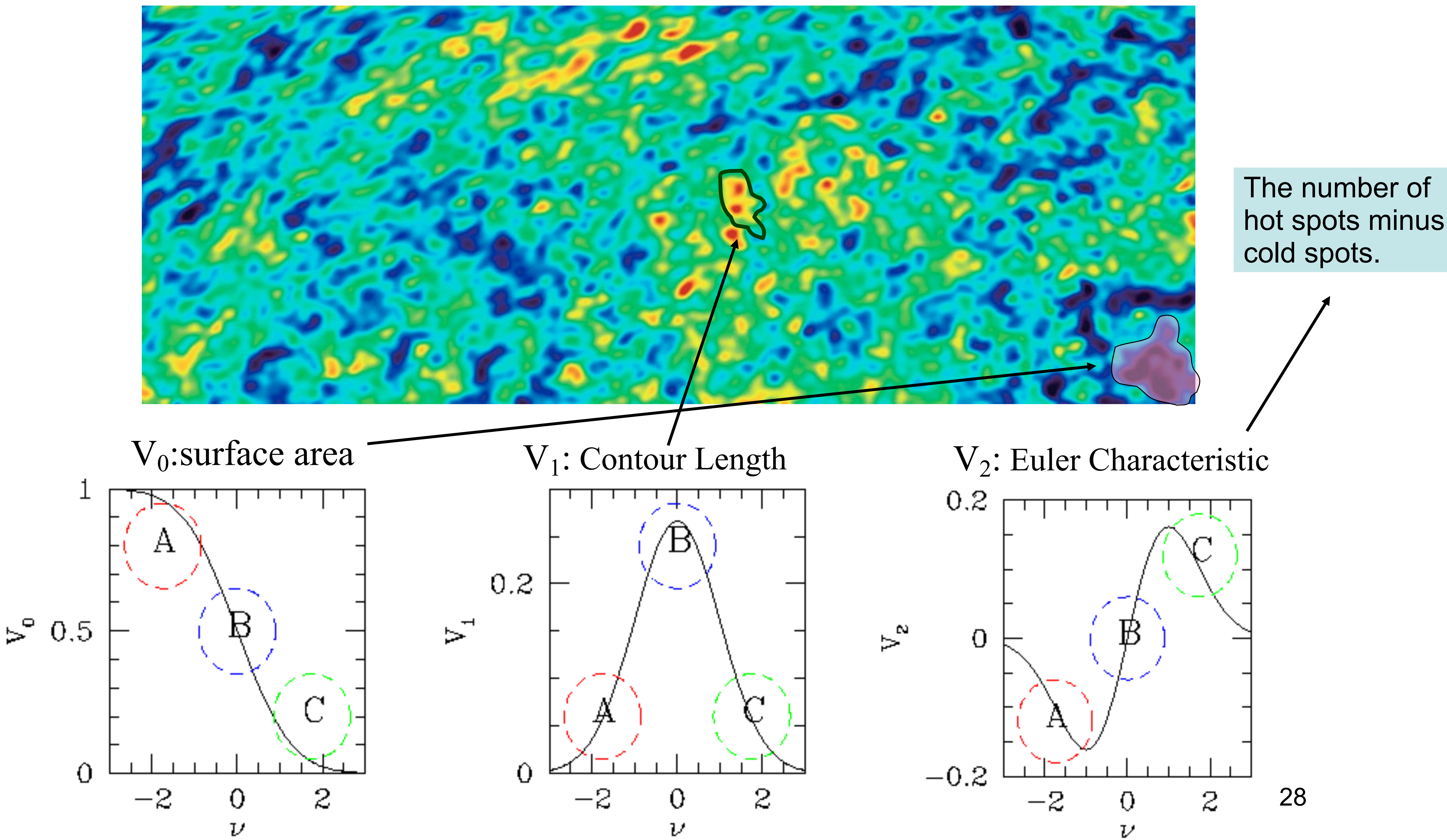
- The point-source correction is much larger for the equilateral configurations.
- Our best estimate from l_{max}=700:
  - The 68% limit: f_{NL}^{equil} = 51 +/- 101
  - **The 95% limit: -151 < f_{NL}^{equil} < 253**

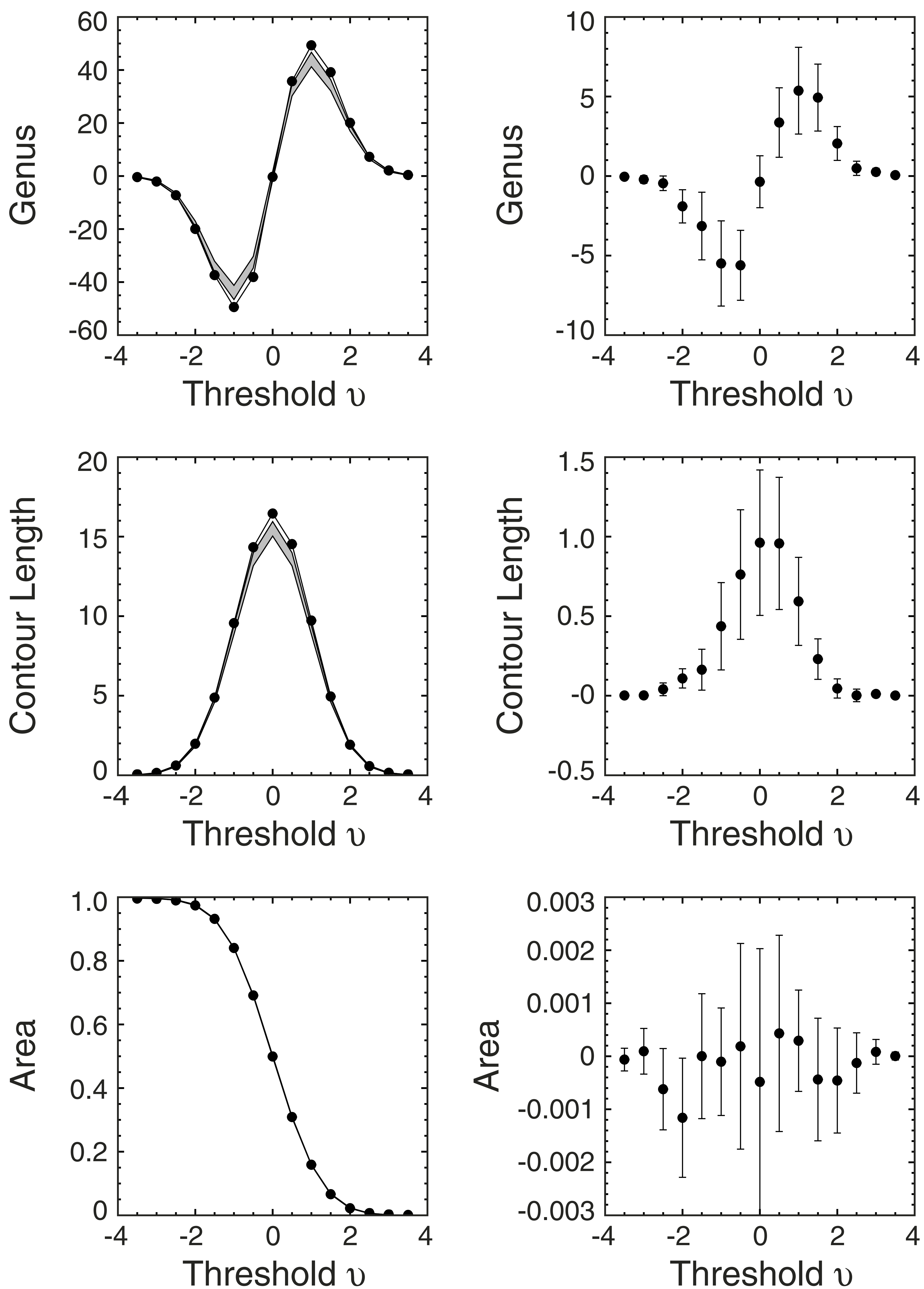
Forecasting 9-year Data

- The WMAP 5-year data do not show any evidence for the presence of f_{NL}^{equil}, but *do* show a (~2-sigma) hint for f_{NL}^{local}.
- Our best estimate is probably on the conservative side, but our analysis clearly indicates that more data are required to claim a firm evidence for $f_{NL}^{local}>0$.
- The 9-year error on f_{NL}^{local} should reach $\Delta f_{NL}^{local}=17$
  - If $f_{NL}^{local}\sim 50$, **we would see it at 3 sigma by 2011.**

(The WMAP 9-year survey, **recently funded**, will be complete in August 2010.)

Minkowski Functionals (MFs)





Komatsu et al. (2008)

MFs from *WMAP* *5-Year Data (V+W)*

Result from a single resolution
(N_{side}=128; 28 arcmin pixel)
[*analysis done by Al Kogut*]

f_{NL}^{local} = -57 +/- 60 (68% CL)

-178 < f_{NL}^{local} < 64 (95% CL)

Cf. Hikage et al. (2008) 3-year analysis using all the resolution:
f_{NL}^{local} = -22 +/- 43 (68% CL)
-108 < f_{NL}^{local} < 64 (95% CL)

"Tension?"

- **It is premature to worry about this**, but it is a little bit bothering to see that the bispectrum prefers a *positive* value, f_{NL}~60, whereas the Minkowski functionals prefer a *negative* value, f_{NL}~-60.
- These values are derived from the same data!
- What do the Minkowski functionals actually measure?

Hikage, Komatsu & Matsubara (2006)

Analytical formulae of MFs

Perturbative formulae of MFs (Matsubara 2003)

$$V_k(\nu) = \frac{1}{(2\pi)^{(k+1)/2}} \frac{\omega_2}{\omega_{2-k}\omega_k} \left(\frac{\sigma_1}{\sqrt{2}\sigma_0} \right)^k e^{-\nu^2/2} \{ H_{k-1}(\nu)$$

Gaussian term

$(k = 0,1,2)$

$$+ \left[\frac{1}{6} S^{(0)} H_{k+2}(\nu) + \frac{k}{3} S^{(1)} H_k(\nu) + \frac{k(k-1)}{6} S^{(2)} H_{k-2}(\nu) \right] \sigma_0 + O(\sigma_0^2) \}$$

leading order of Non-Gaussian term

$$\sigma_j^2 = \frac{1}{4} \sum_l (2l+1) [l(l+1)]^j C_l W_l^2 \qquad W_l\text{: smoothing kernel}$$

$$\omega_0 = 1, \omega_1 = 1, \omega_2 = \pi, \omega_3 = 4\pi/3 \qquad H_k : k\text{-th Hermite polynomial}$$

$S^{(a)}$: skewness parameters (a = 0,1,2)

In weakly non-Gaussian fields (σ_0<<1) , the non-Gaussianity in MFs is characterized by three skewness parameters $S^{(a)}$.

3 “Skewness Parameters”

- Ordinary skewness

$$S^{(0)} \equiv \frac{\langle f^3 \rangle}{\sigma_0^4},$$

- Second derivative

$$S^{(1)} \equiv -\frac{3}{4} \frac{\langle f^2 (\nabla^2 f) \rangle}{\sigma_0^2 \sigma_1^2},$$

- (First derivative)^2 x Second derivative

$$S^{(2)} \equiv -\frac{3d}{2(d-1)} \frac{\langle (\nabla f) \cdot (\nabla f) (\nabla^2 f) \rangle}{\sigma_1^4},$$

$$S^{(0)} = \frac{3}{2\pi\sigma_0^4} \sum_{2\le l_1 \le l_2 \le l_3} I^2_{l_1 l_2 l_3} b_{l_1 l_2 l_3} W_{l_1} W_{l_2} W_{l_3}, \quad (\!$$

$$S^{(1)} = \frac{3}{8\pi\sigma_0^2\sigma_1^2} \sum_{2\le l_1 \le l_2 \le l_3} [l_1(l_1+1) + l_2(l_2+1) + l_3(l_3+1)] \times I^2_{l_1 l_2 l_3} b_{l_1 l_2 l_3} W_{l_1} W_{l_2} W_{l_3}, \quad (\!$$

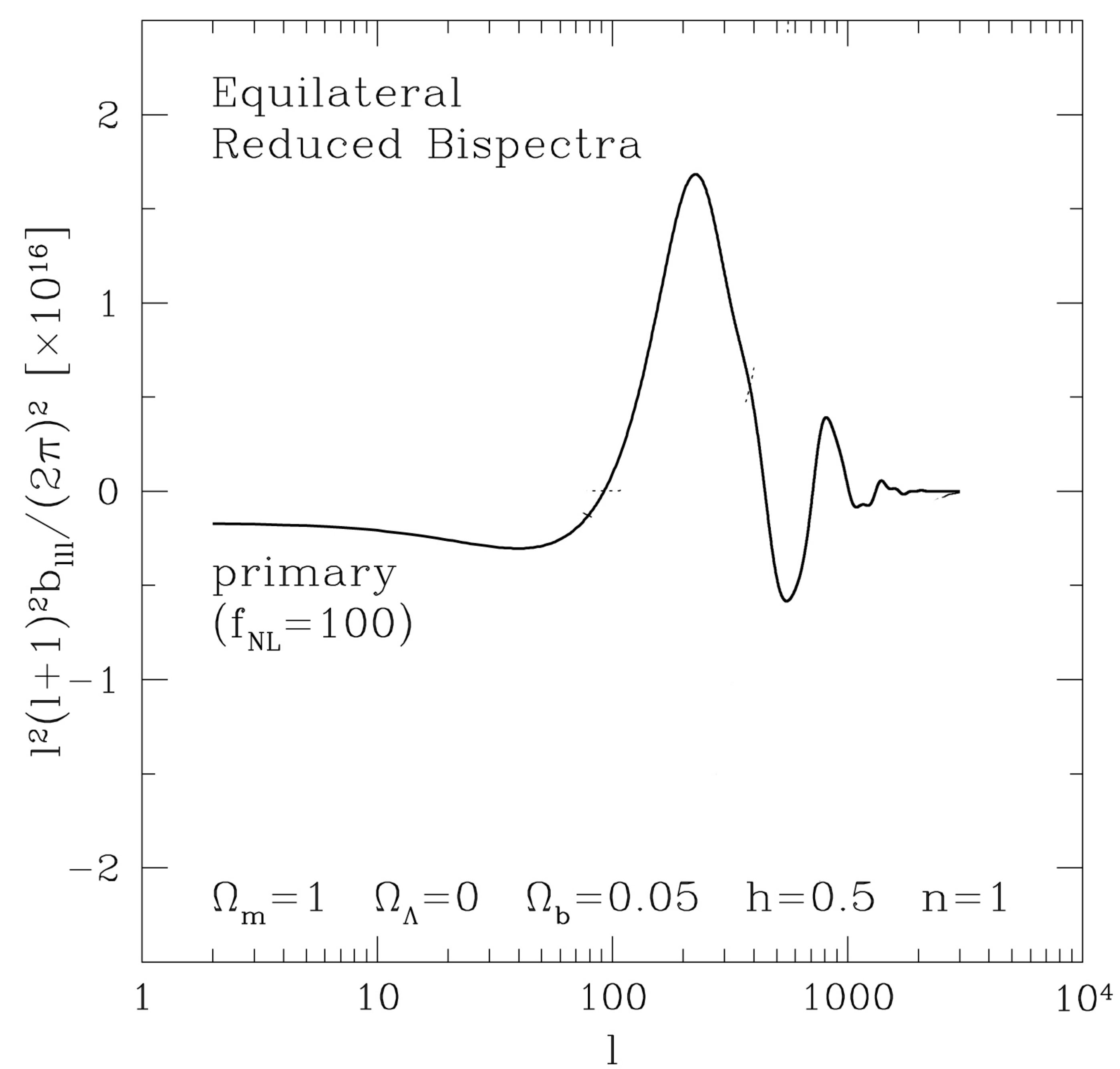
$$S^{(2)} = \frac{3}{4\pi\sigma_1^4} \sum_{2\le l_1 \le l_2 \le l_3} \{[l_1(l_1+1) + l_2(l_2+1) - l_3(l_3+1)] \times l_3(l_3+1) + (\text{cyc.})\} I^2_{l_1 l_2 l_3} b_{l_1 l_2 l_3} W_{l_1} W_{l_2} W_{l_3}, \quad (\!$$

$S^{(0)}$: Simple average of b_{l1l2l3}

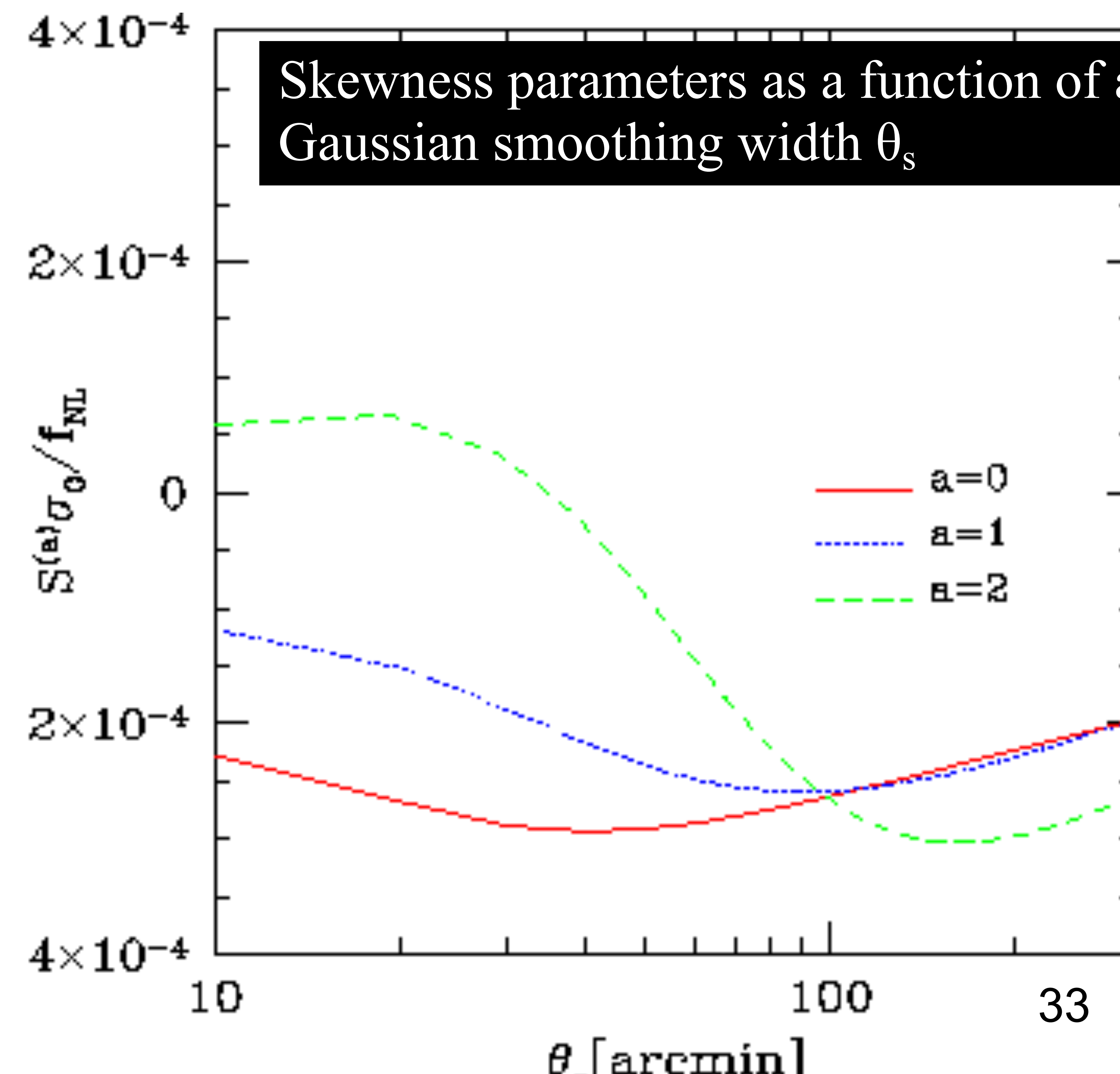
$S^{(1)}$: l^2 weighted average

$S^{(2)}$: l^4 weighted average

Analytical predictions of bispectrum at f_{NL}=100 (Komatsu & Spergel 2001)



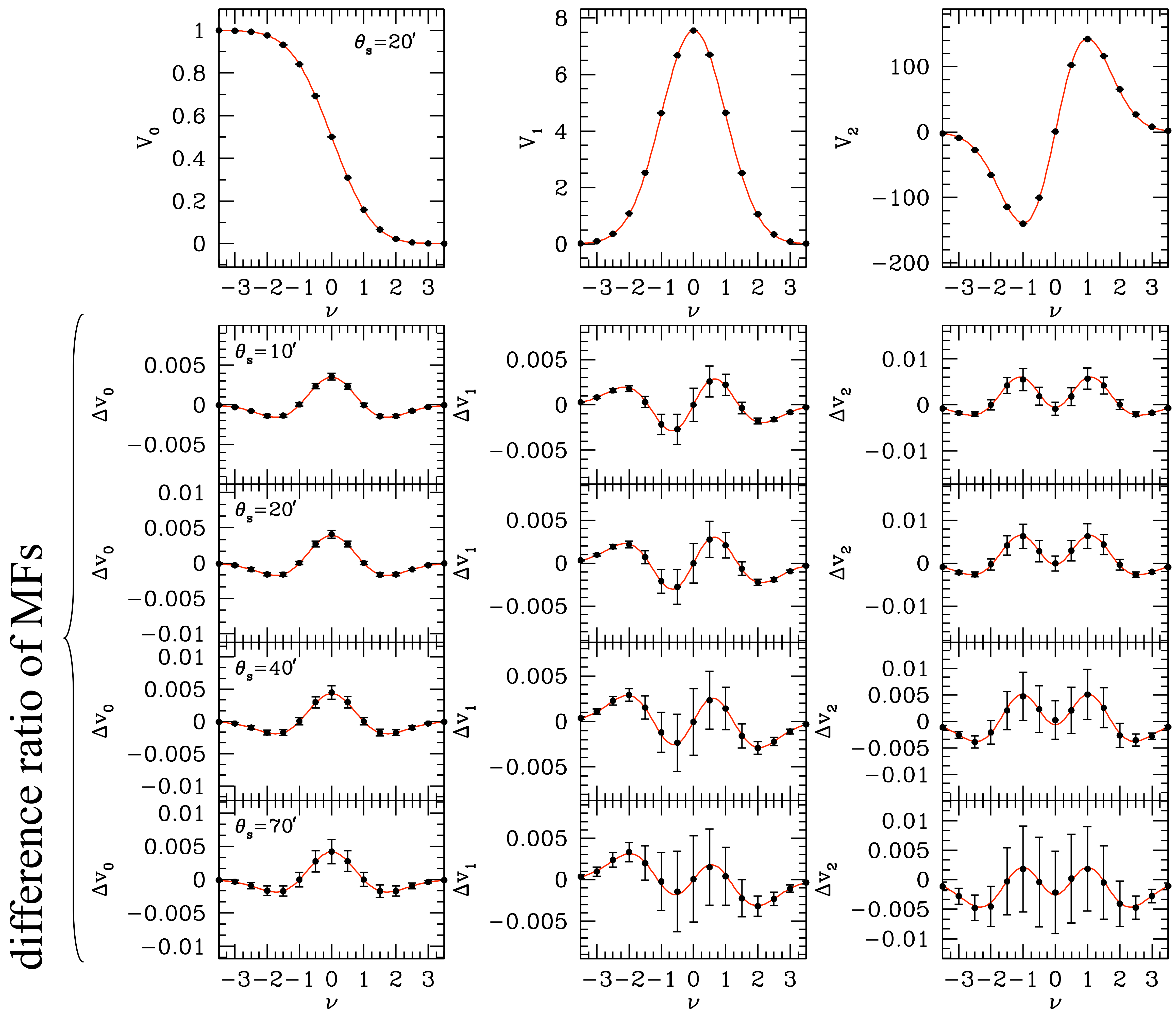
Skewness parameters as a function of a Gaussian smoothing width θ_s



Hikage et al. (2008)

Comparison of analytical formulae with Non-Gaussian simulations

Surface area Contour Length Euler Characteristic



Comparison of MFs between analytical predictions and non-Gaussian simulations with f_{NL}=100 at different Gaussian smoothing scales, $\boldsymbol{\theta_s}$

Simulations are done for WMAP.

Analytical formulae agree with non-Gaussian simulations very well.

Application of the Minkowski Functionals

- The skewness parameters are the direct observables from the Minkowski functionals.
- The skewness parameters can be calculated directly from the bispectrum.
- It can be applied to *any* form of the bispectrum!
  - Statistical power is weaker than the full bispectrum, but the application can be broader than the bispectrum estimator that is tailored for a very specific form of non-Gaussianity.

An Opportunity?

- This apparent "tension" should be taken as an opportunity to investigate the other statistical tools, such the Minkowski functionals, wavelets, etc., in the context of primordial non-Gaussianity.
- It is plausible that various statistical tools can be written in terms of the sum of the bispectrum with various weights, in the limit of weak non-Gaussianity.
- Different tools are sensitive to different forms of non-Gaussianity - this is an advantage.

Systematics!

- Why use different statistical tools, when we know that the bispectrum gives us the maximum sensitivity?
- Systematics! Systematics!! Systematics!!!
- **I don’t believe any detections, until different statistical tools give the same answer.**
    - That’s why it bothers me to see that the bispectrum and the Minkowski functionals give different answers at the moment.

Summary

- The best estimates of primordial non-Gaussian parameters from the bispectrum analysis of the WMAP 5-year data are
  - $-9 < f_{NL}^{local} < 111$ (95% CL)
  - $-151 < f_{NL}^{equil} < 253$ (95% CL)
- **9-year data are required to test $f_{NL}^{local} \sim 60$!**
- The other statistical tools should be explored more.
  - E.g., estimate the skewness parameters directly from the Minkowski functionals to find the source of "tension"

Future Prospects

- Future is always bright, right?

Gaussianity vs Flatness: Future

- **Flatness will never beat Gaussianity.**
  - In 5-10 years, we will know **flatness** to **0.1%** level.
  - In 5-10 years, we will know **Gaussianity** to **0.01%** level (f_{NL}~10), or even to **0.005%** level (f_{NL}~5), at 95% CL.
- However, a real potential of Gaussianity test is that **we might detect something at this level** (multi-field, curvaton, DBI, ghost cond., new ekpyrotic…)
  - Or, we might detect curvature first?
  - Is 0.1% curvature interesting/motivated?

Beyond Bispectrum: Trispectrum of Primordial Perturbations

- Trispectrum is the Fourier transform of four-point correlation function.
- Trispectrum(k_1,k_2,k_3,k_4)

  $=\langle\Phi(k_1)\Phi(k_2)\Phi(k_3)\Phi(k_4)\rangle$

  which can be sensitive to the higher-order terms:

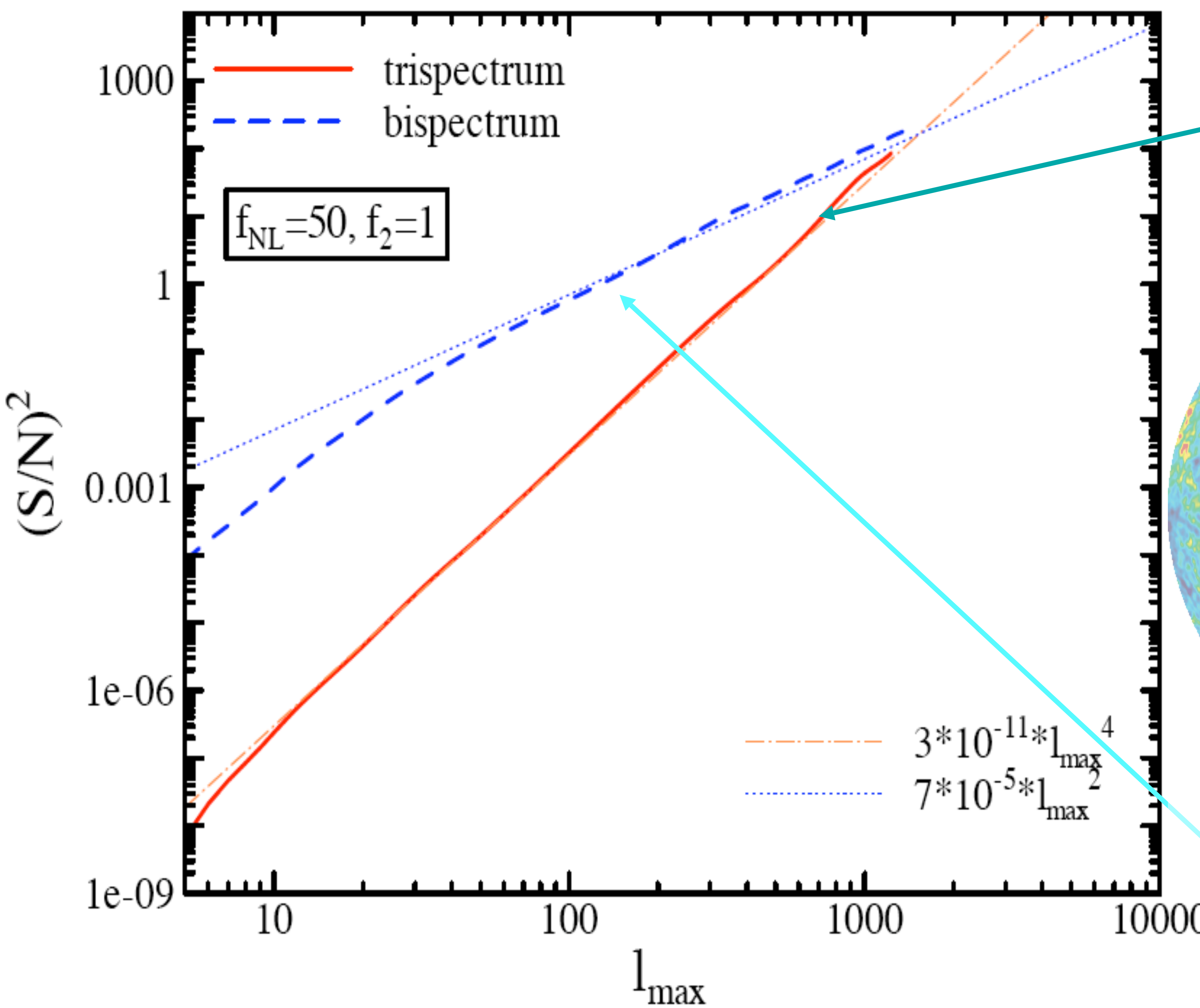
$$\Phi(\boldsymbol{x}) = \Phi_{\rm L}(\boldsymbol{x}) + f_{\rm NL}\left[\Phi_{\rm L}^2(\boldsymbol{x}) - \langle\Phi_{\rm L}^2(\boldsymbol{x})\rangle\right] + f_2\Phi_{\rm L}^3(\boldsymbol{x})$$

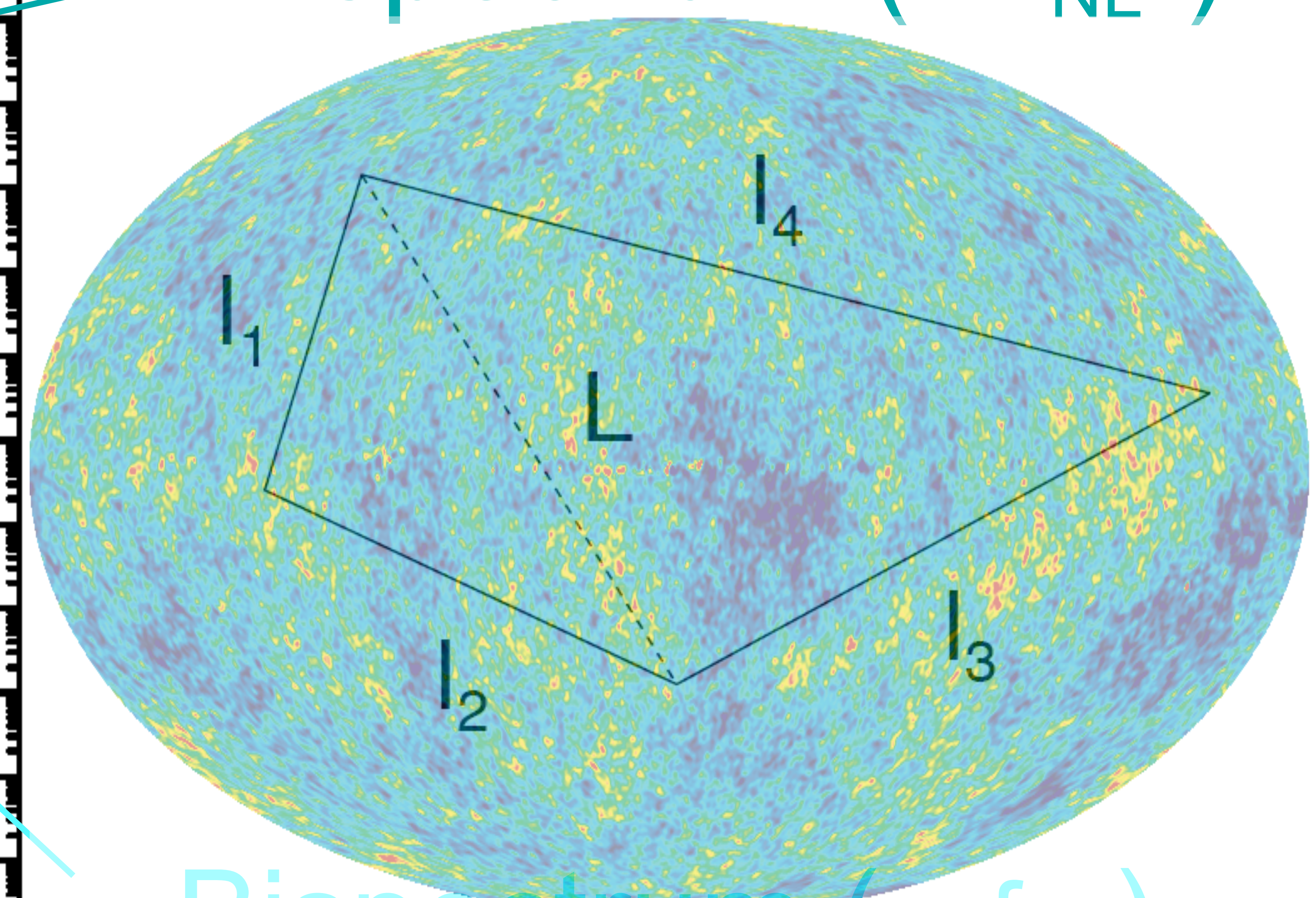
Measuring Trispectrum

- It's pretty painful to measure all the quadrilateral configurations.
  - Measurements from the COBE 4-year data (Komatsu 2001; Kunz et al. 2001)
- Only limited configurations measured from the WMAP 3-year data
  - Spergel et al. (2007)
- No evidence for non-Gaussianity, but f_{NL} has not been constrained by the trispectrum yet. (Work to do.)

Kogo & Komatsu (2006)

Trispectrum: Not useful for WMAP, but maybe useful for Planck, if f_{NL} is greater than ~50: Excellent Cross-check!





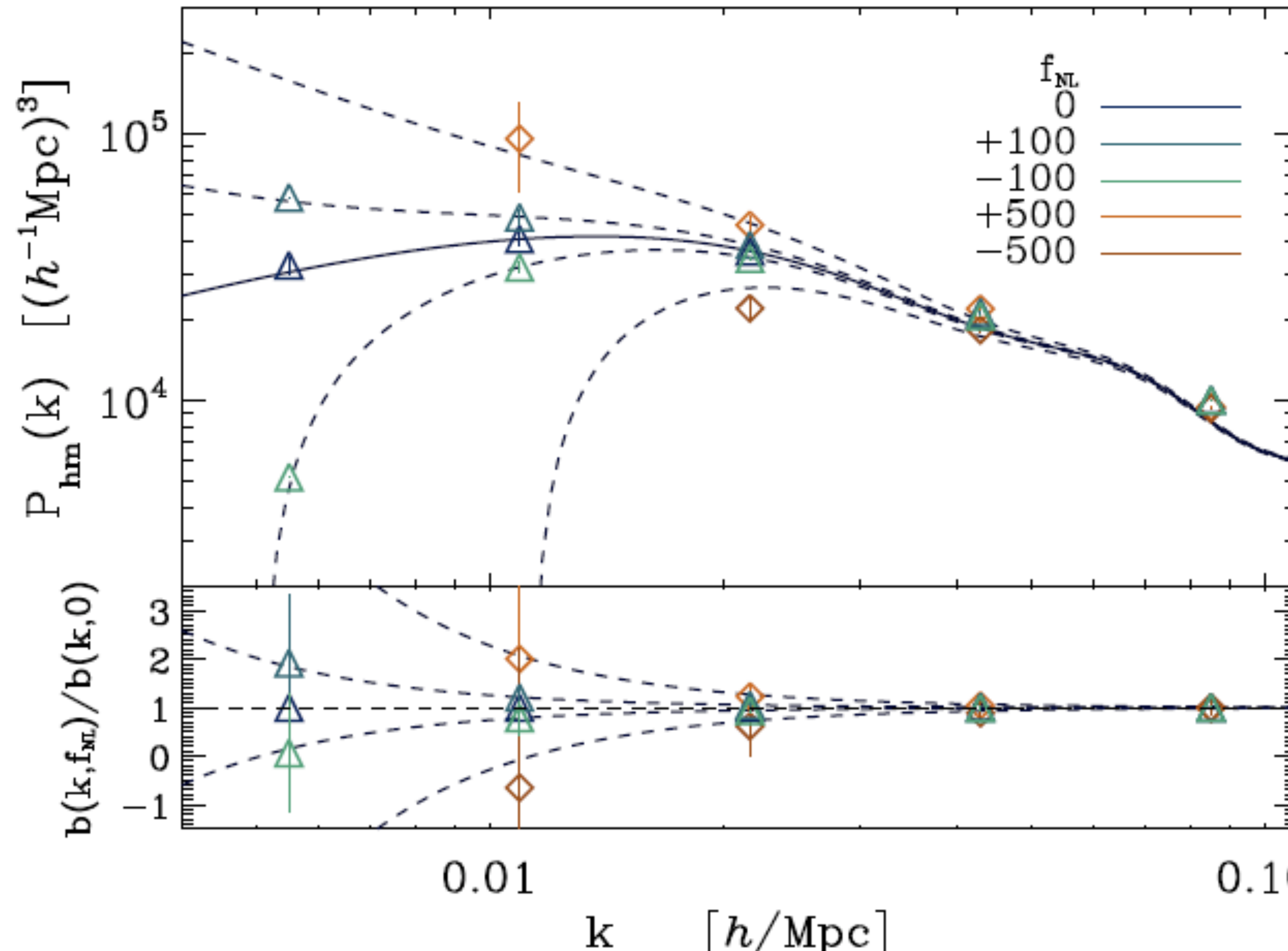
- Trispectrum ($\sim f_{NL}^2$)
- Bispectrum ($\sim f_{NL}$)

More On Future Prospects

- CMB: Planck (temperature + polarization): **Δf_{NL}(local)=6 (95%)**
  - –Yadav, Komatsu & Wandelt (2007)
- Large-scale Structure: e.g., ADEPT, CIP: **Δf_{NL}(local)=7 (95%); Δf_{NL}(equilateral)=90 (95%)**
  - –Sefusatti & Komatsu (2007)
- CMB and LSS are independent. By combining these two constraints, we get **Δf_{NL}(local)=4.5**.

New, Powerful Probe of f_{NL}!

- f_{NL} modifies the galaxy bias with a unique scale dependence
  - –Dalal et al.; Matarrese & Verde
  - –Mcdonald; Afshordi & Tolley
- The statistical power of this method is promising!
  - –SDSS: $-29 < f_{NL} < 70$ (95%CL); Slosar et al.
  - –Comparable to the WMAP limit already ($-9 < f_{NL} < 111$)
  - –Combined limit (SDSS+WMAP):
    - **$-1 < f_{NL} < 70$** (95%CL)



Where Should We Be Going?

- Explore different statistics (both CMB and LSS)
  - –Minkowski functionals, trispectrum, and others

- Go for the large-scale structure
  - –The large-scale structure of the Universe at high redshifts offers a definitive cross-check for the presence of primordial non-Gaussianity.
  - –If CMB sees primoridial non-Gaussianity, the **same** non-Gaussianity must also be seen by the large-scale structure!