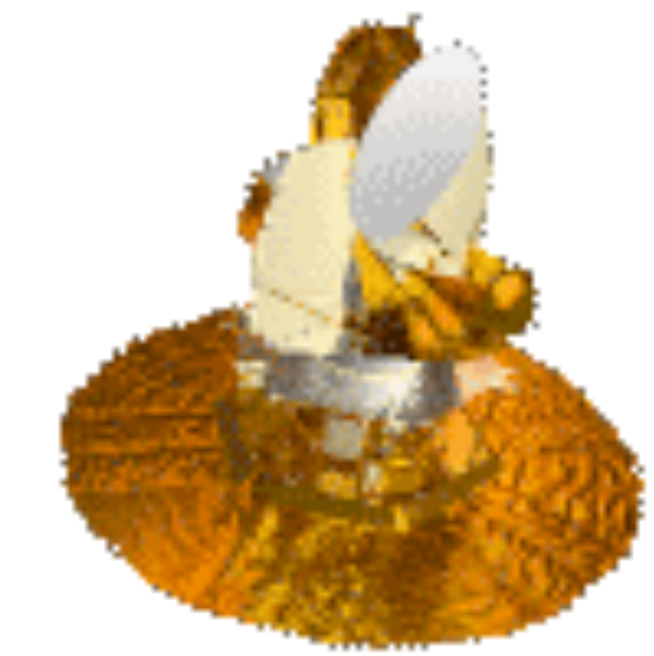


What WMAP taught us about inflation, and what to expect from Planck

Eiichiro Komatsu (Texas Cosmology Center, UT Austin)

Aspects of Inflation, UT-TAMU Workshop, April 8, 2011

How Do We Test Inflation?

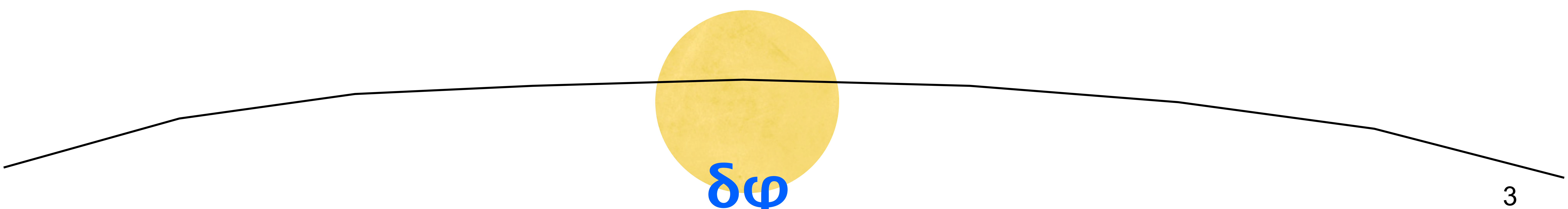
- How can we answer a simple question like this:
  - "*How were primordial fluctuations generated?*"

Stretching Micro to Macro

H^{-1} = Hubble Size

Quantum fluctuations on microscopic scales

INFLATION!

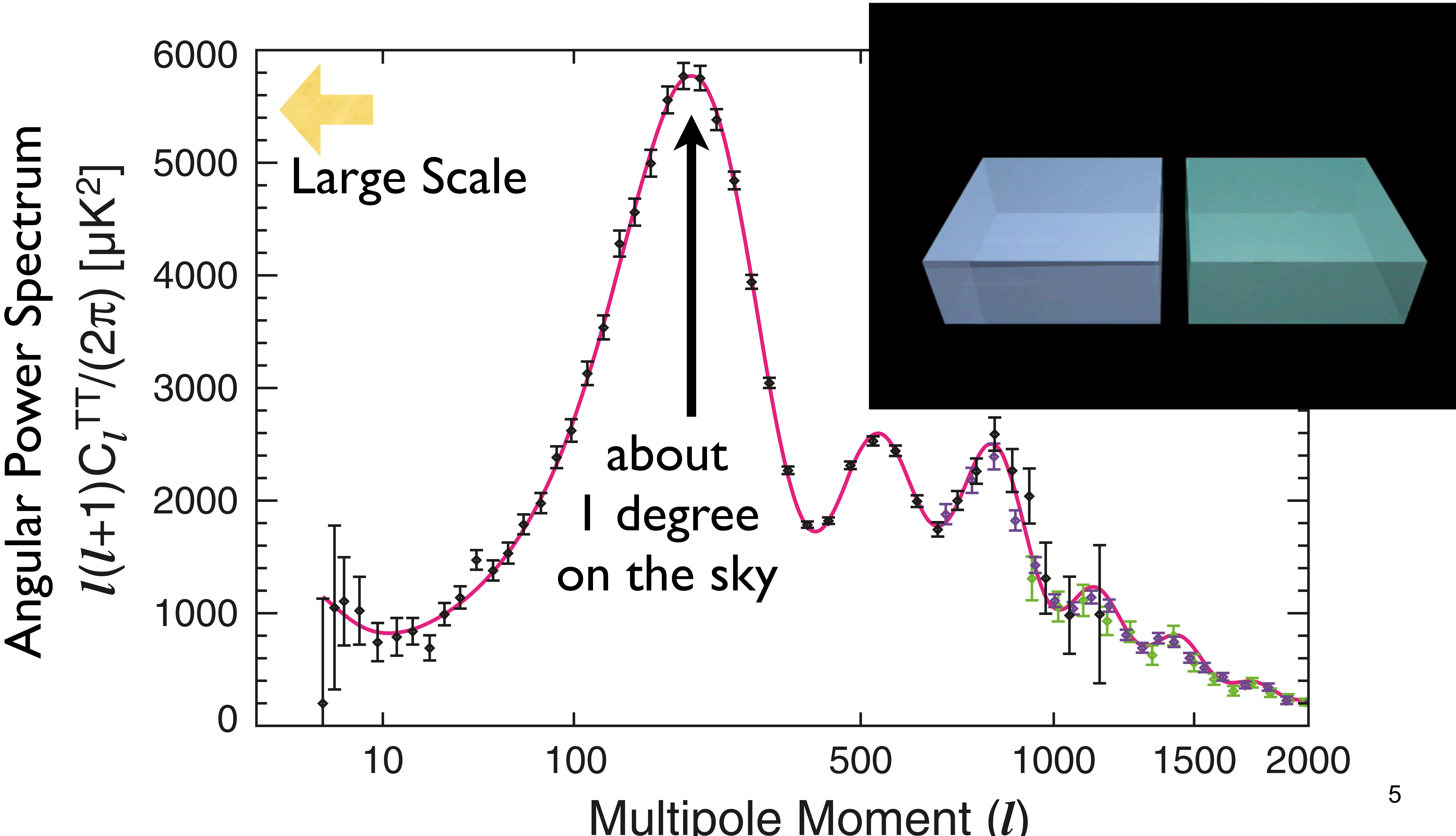


Quantum fluctuations cease to be quantum, and become observable

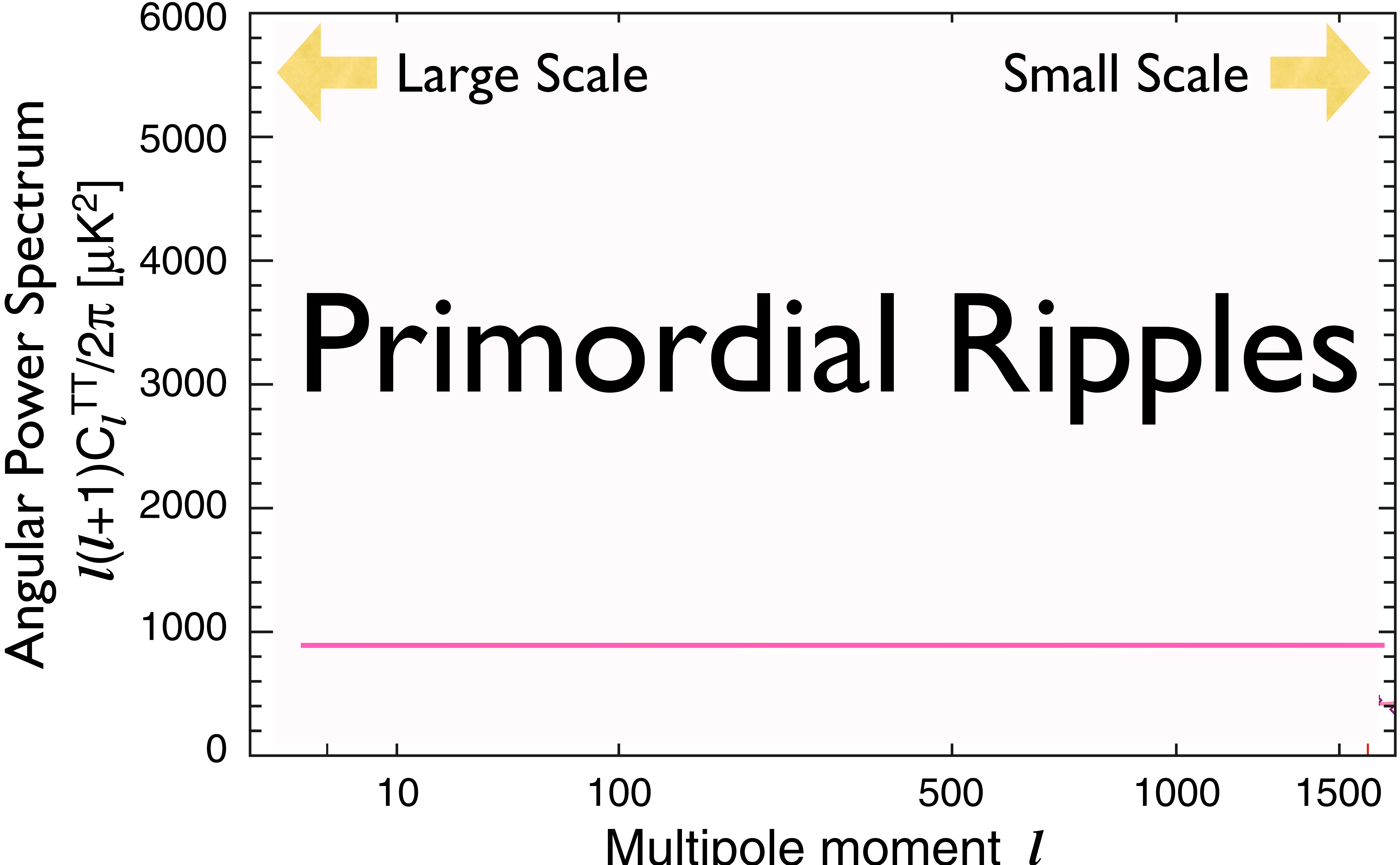
Power Spectrum

- A very successful explanation (Mukhanov & Chibisov; Guth & Pi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner) is:
  - *Primordial fluctuations were generated by quantum fluctuations of the scalar field that drove inflation.*
  - The prediction: a nearly scale-invariant power spectrum in the curvature perturbation, $\zeta = -(H dt/d\varphi)\delta\varphi$
    - **$P_\zeta(k) = \langle|\zeta_k|^2\rangle = A/k^{4-n_s} \sim A/k^3$**
    - where $n_s \sim 1$ and A is a normalization.

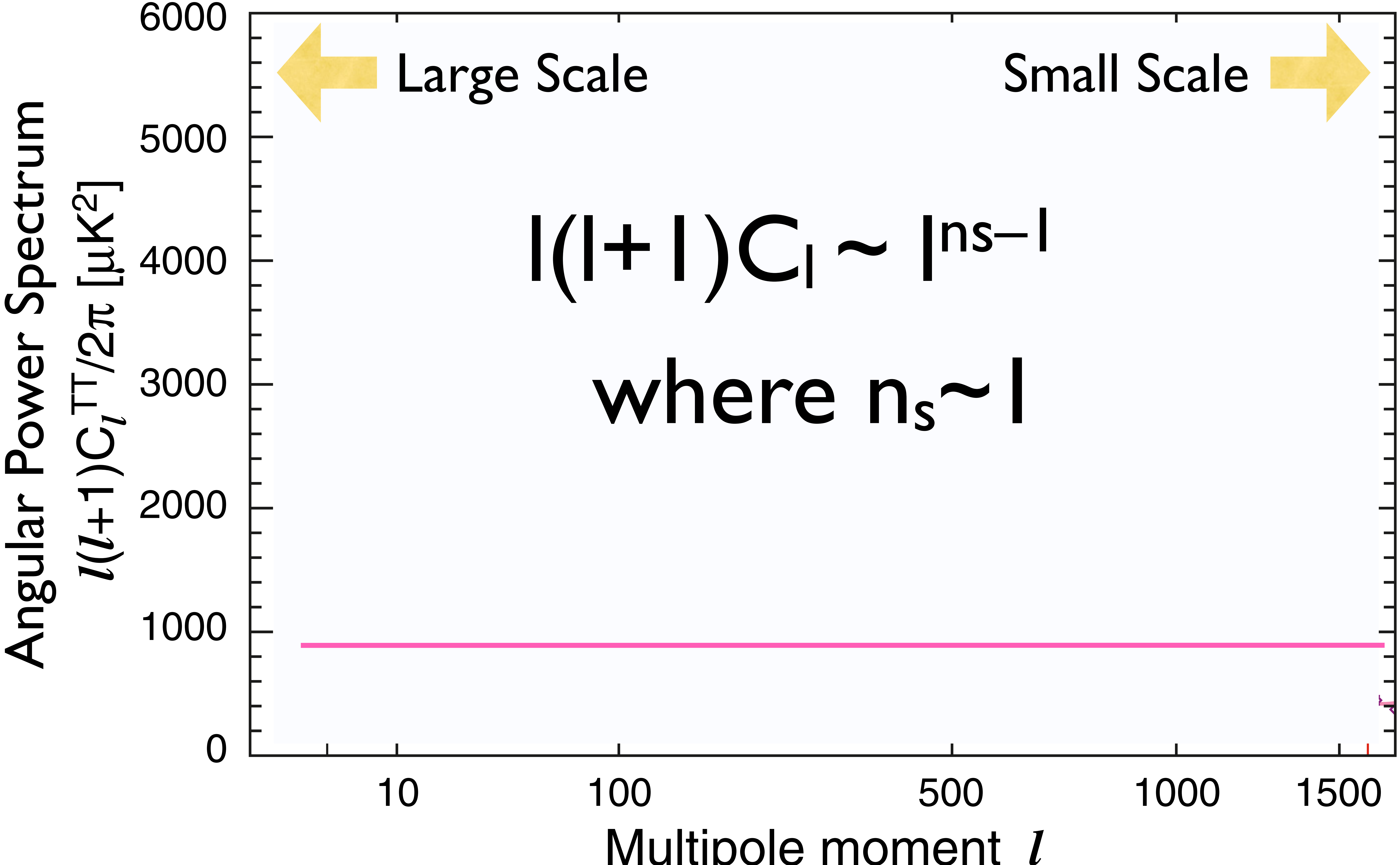
WMAP Power Spectrum



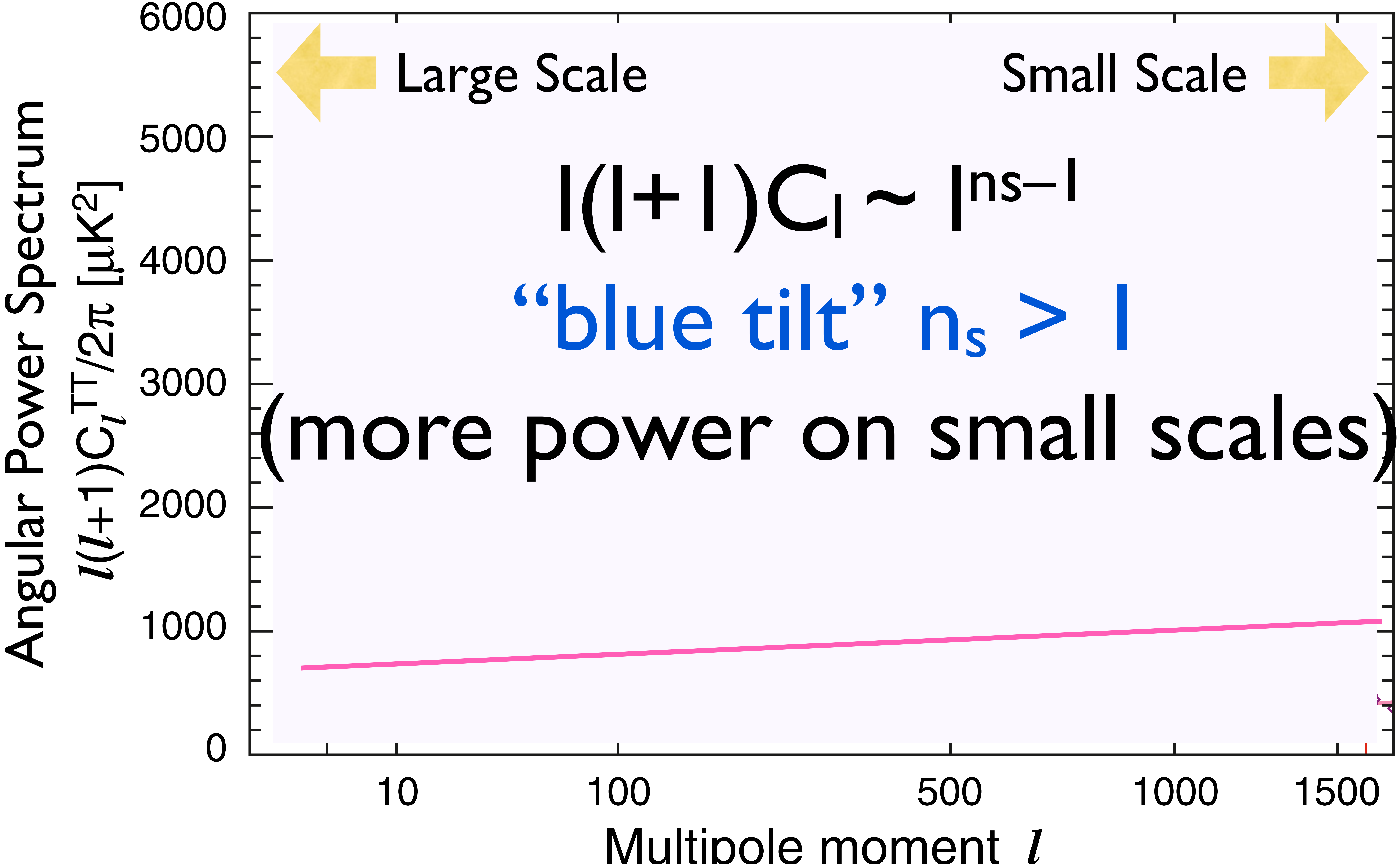
Getting rid of the Sound Waves



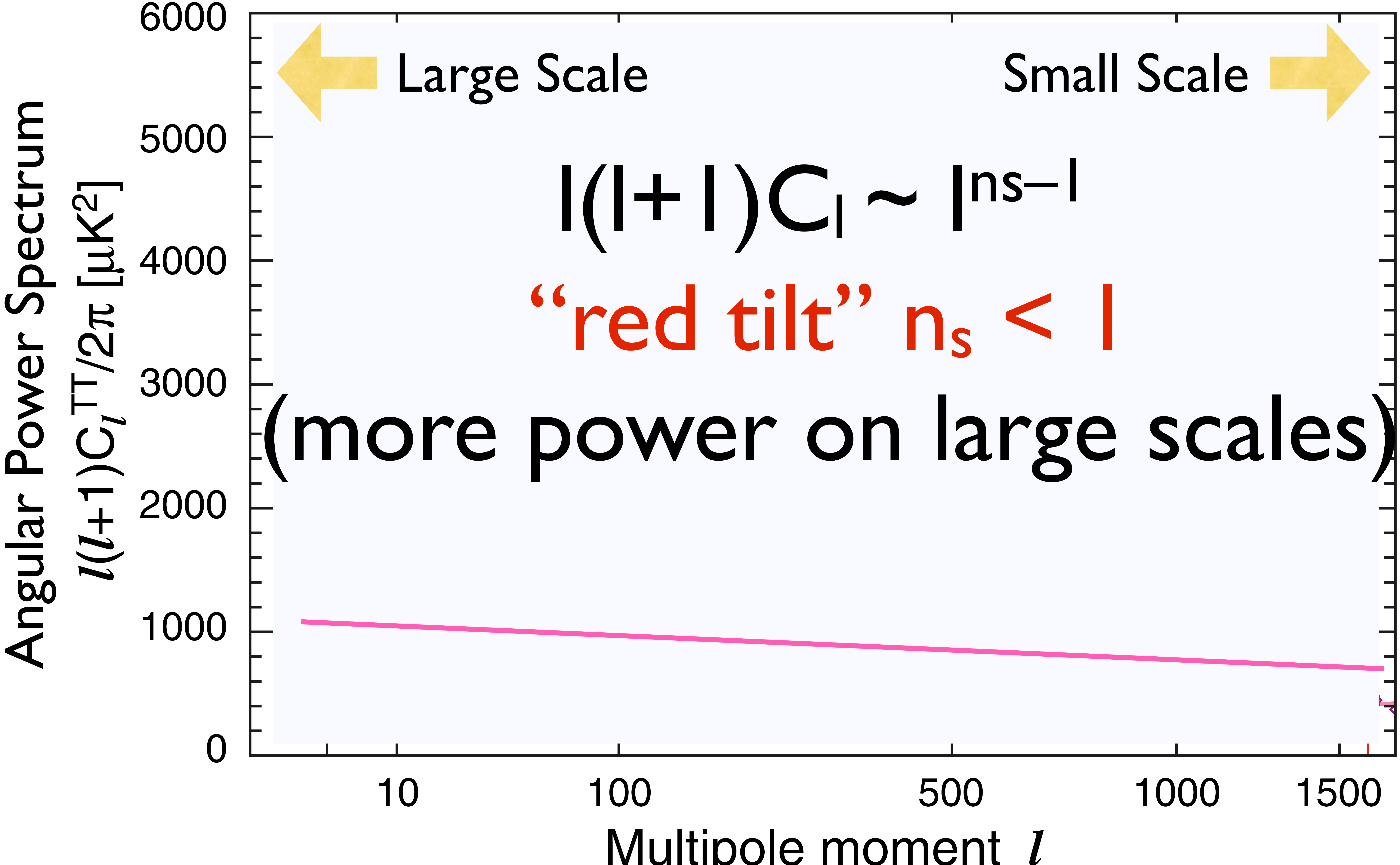
Inflation Predicts:



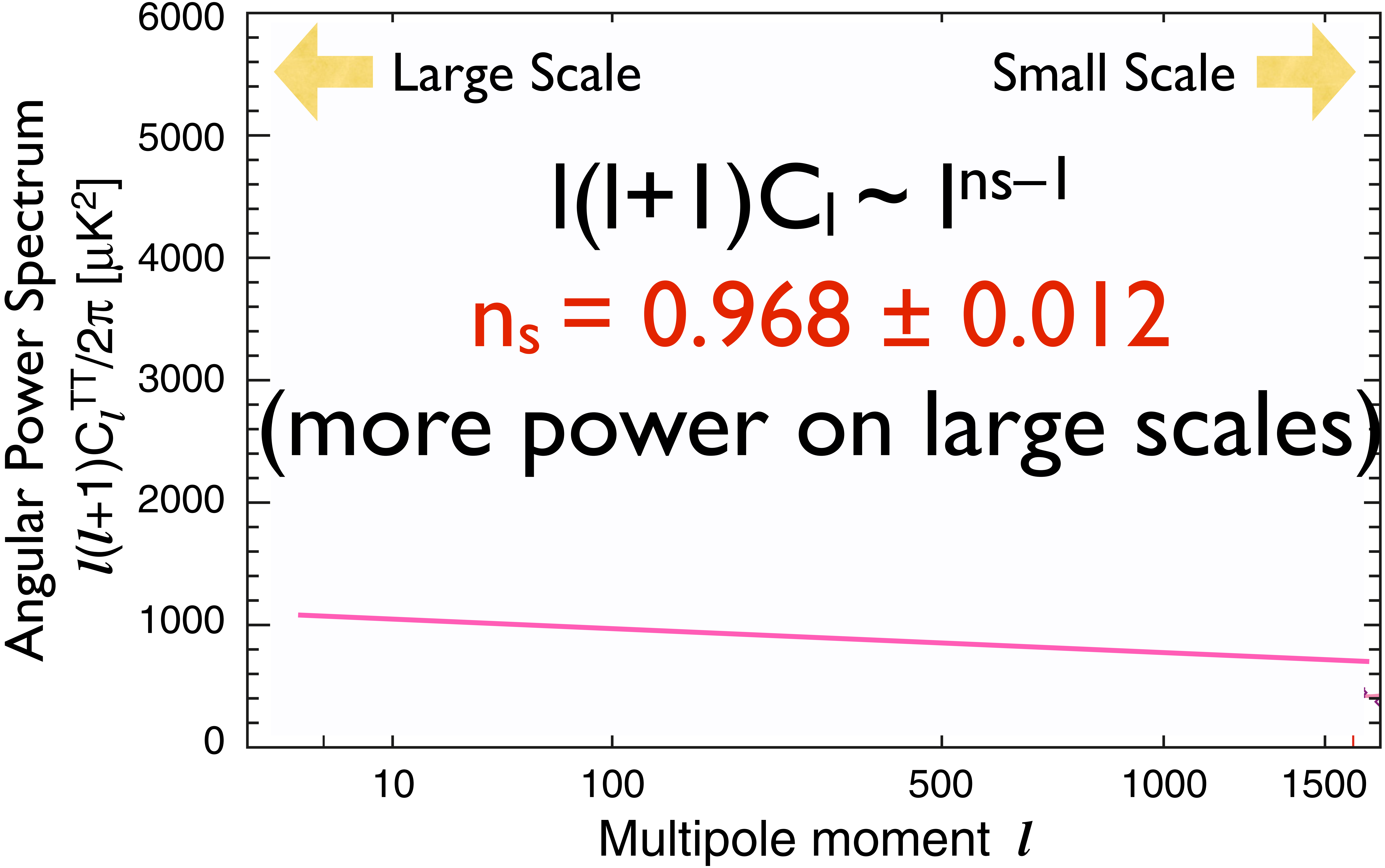
Inflation may do this



...or this


Large Scale
Small Scale
l(l+1)C_l ~ l^{ns−1}
“red tilt” n_s < 1
(more power on large scales)
Angular Power Spectrum
l(l+1)C_l^TT/2π [μK²]
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
1500
Multipole moment l

WMAP 7-year Measurement (Komatsu et al. 2011)



After 9 years of observations...

WMAP taught us:

- **All of the basic predictions of single-field and slow-roll inflation models are consistent with the data**
    - But, not all models are consistent (i.e., $\lambda\varphi^4$ is out unless you introduce a non-minimal coupling)

Testing Single-field by Adiabaticity

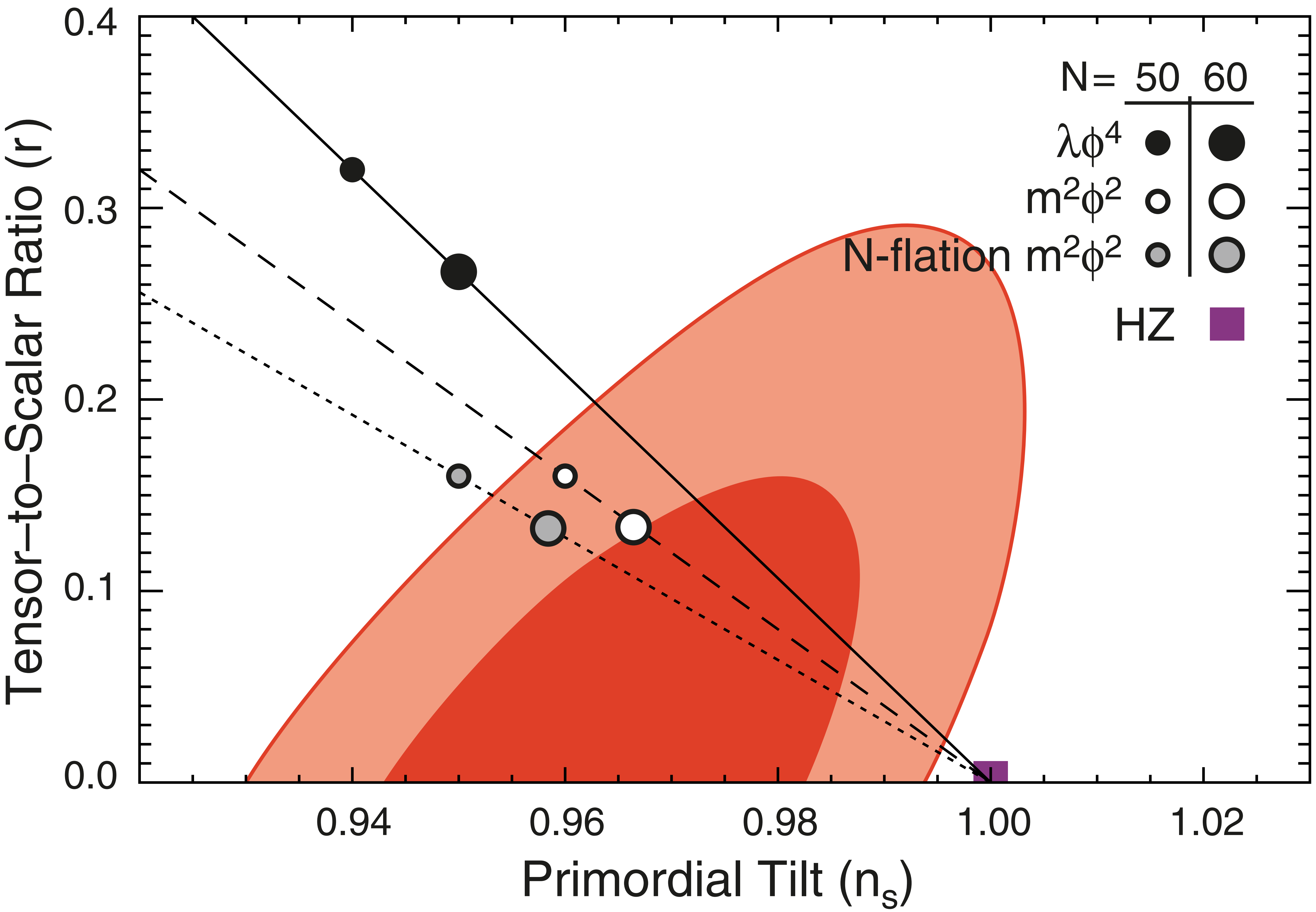
- Within the context of single-field inflation, all the matter and radiation originated from a single field, and thus there is a particular relation (adiabatic relation) between the perturbations in matter and photons:

$$\mathcal{S}_{c,\gamma} \equiv \frac{\delta\rho_c}{\rho_c} - \frac{3\delta\rho_\gamma}{4\rho_\gamma} = 0$$

The data are consistent with S=0:

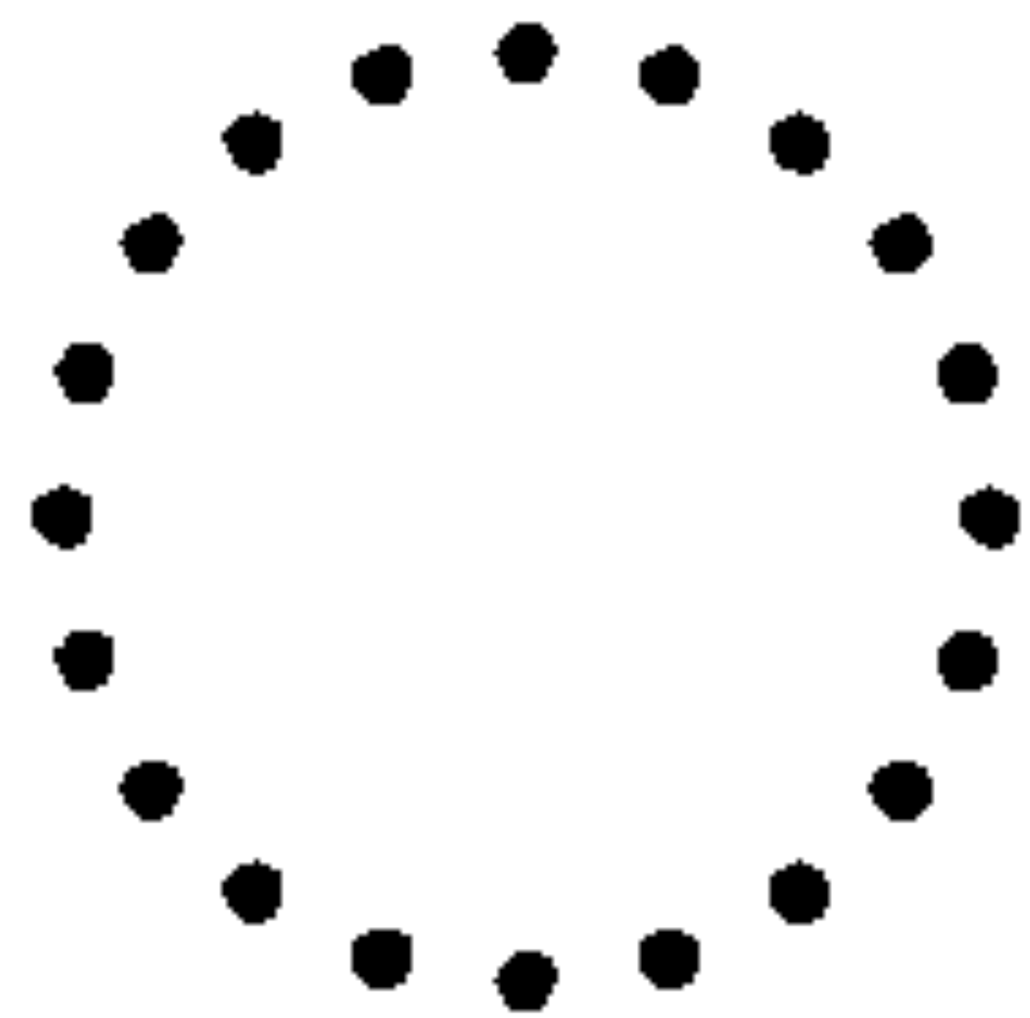
$$\frac{|\delta\rho_c/\rho_c - 3\delta\rho_\gamma/(4\rho_\gamma)|}{\frac{1}{2}[\delta\rho_c/\rho_c + 3\delta\rho_\gamma/(4\rho_\gamma)]} < 0.09 \quad (95\%\ \mathrm{CL})$$

Inflation looks good



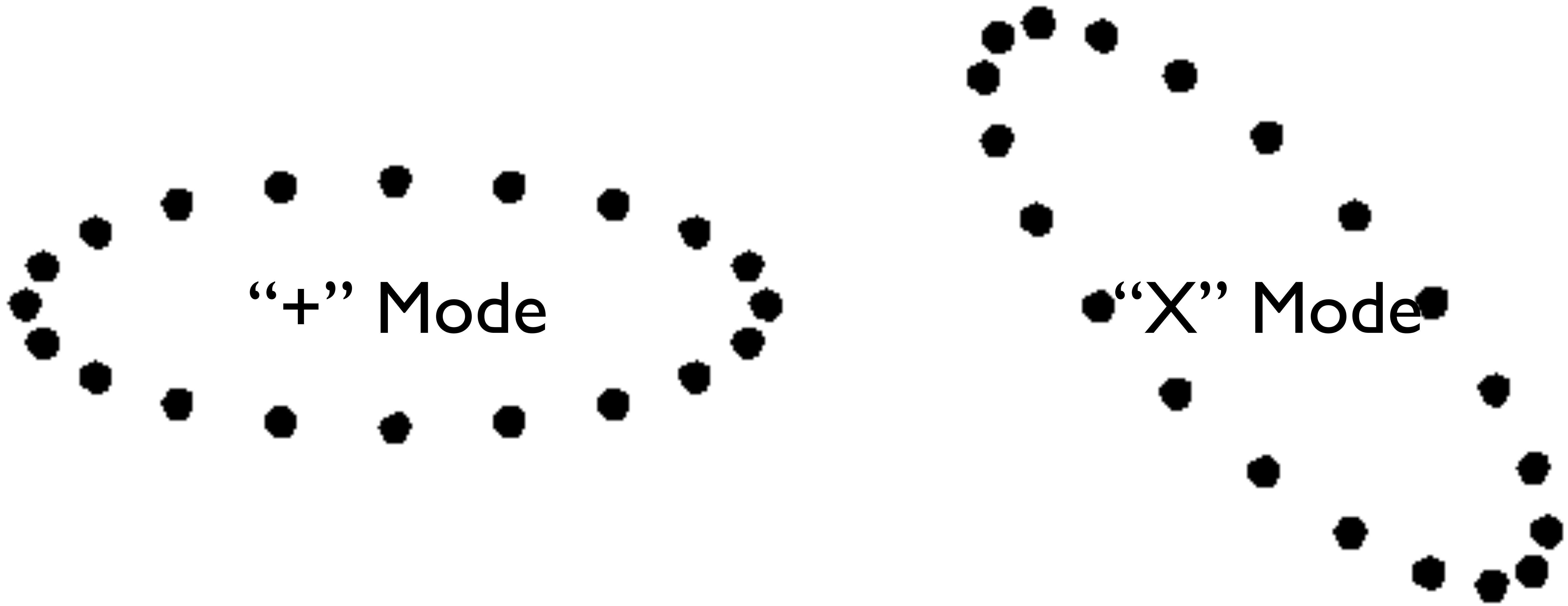
- Joint constraint on the primordial tilt, n_s, and the tensor-to-scalar ratio, r.
  - **r < 0.24** (95%CL; WMAP7+BAO+H_0)

Gravitational waves are coming toward you... What do you do?



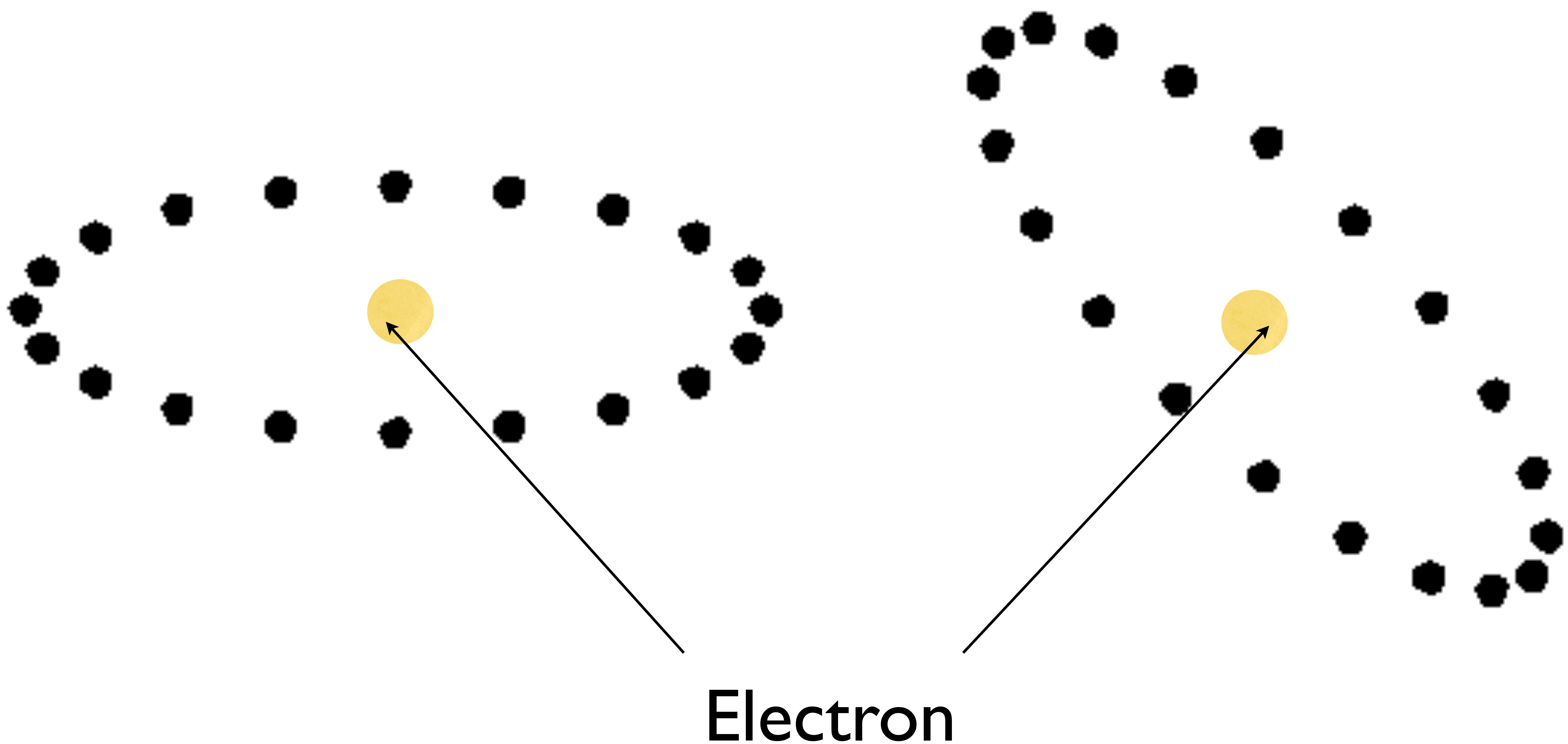
- Gravitational waves stretch space, causing particles to move.

Two Polarization States of GW

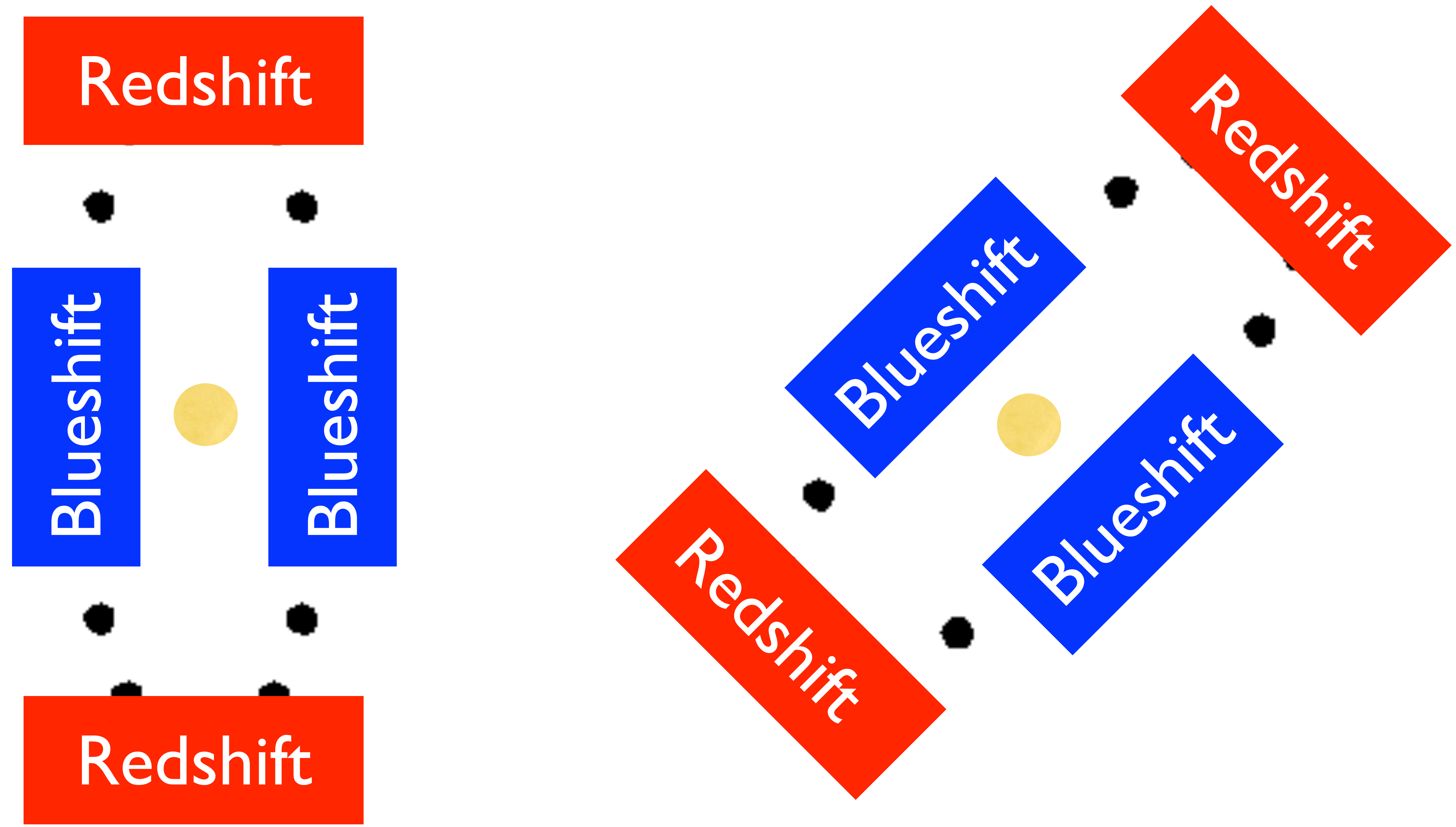


- This is great - this will automatically generate quadrupolar temperature anisotropy around electrons!

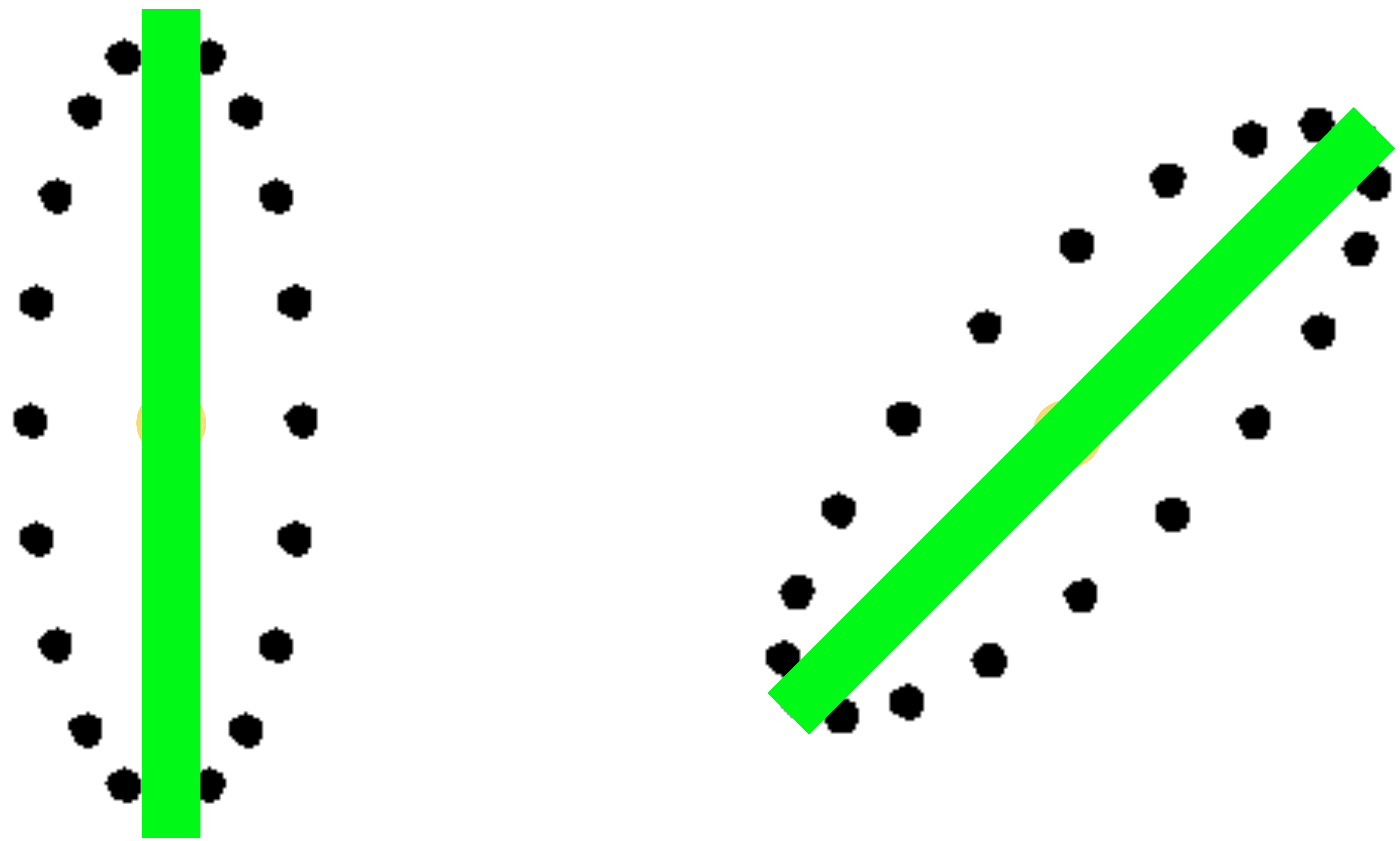
From GW to CMB Polarization



From GW to CMB Polarization



From GW to CMB Polarization



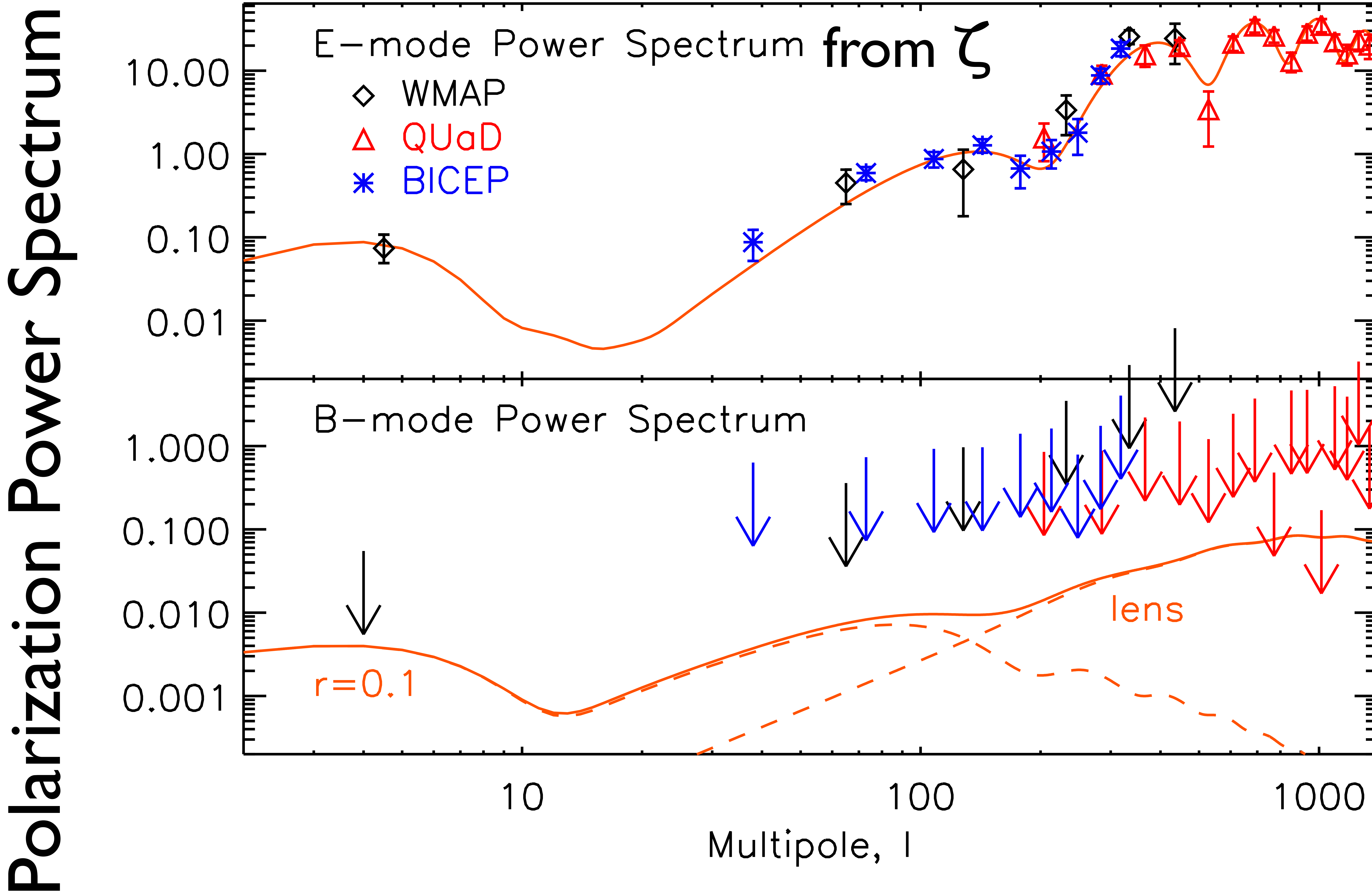
"Tensor-to-scalar Ratio," r

$$r \equiv \frac{2\langle |h_{\mathbf{k}}^{+}|^2 + |h_{\mathbf{k}}^{\times}|^2 \rangle}{\langle |\zeta_{\mathbf{k}}|^2 \rangle}$$

In terms of the slow-roll parameter:

$$r = 16\varepsilon$$

where $\varepsilon = -(dH/dt)/H^2 = 4\pi G(d\varphi/dt)^2/H^2 \approx (16\pi G)^{-1}(dV/d\varphi)^2/V^2$



- No detection of polarization from gravitational waves (B-mode polarization) yet.

However

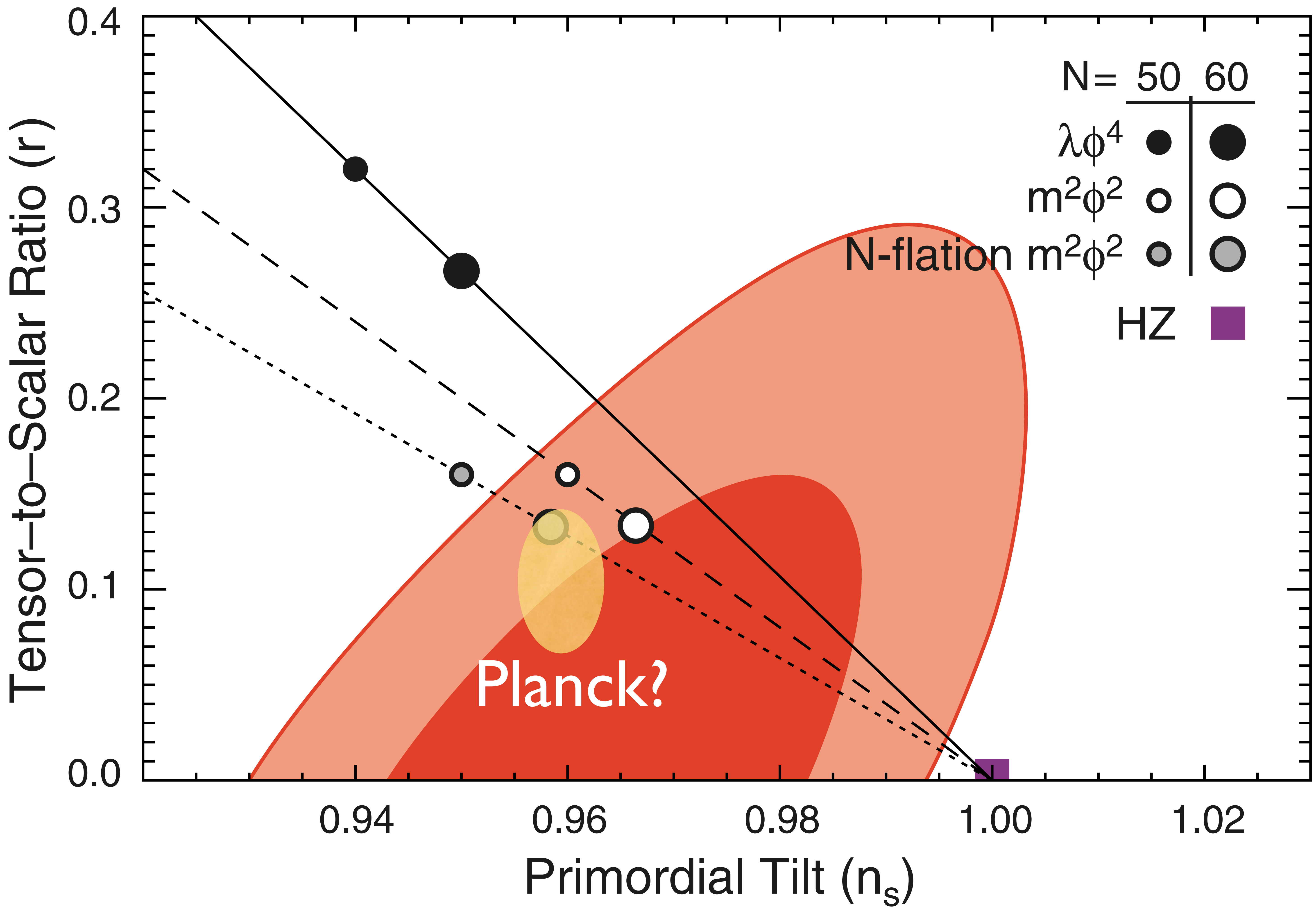
- We cannot say, just yet, that we have definite evidence for inflation.
- *Can we ever prove, or disprove, inflation?*

Planck may:

- Prove inflation by detecting the effect of primordial gravitational waves on polarization of the cosmic microwave background (i.e., detection of r)
- Rule out single-field inflation by detecting a particular form of the 3-point function called the "local form" (i.e., detection of f_{NL}^{local})

- Challenge the inflation paradigm by detecting a violation of inequality that should be satisfied between the local-form 3-point and 4-point functions

Planck might find gravitational waves (if r~0.1)



If found, this would give us a pretty convincing proof that inflation did indeed happen.

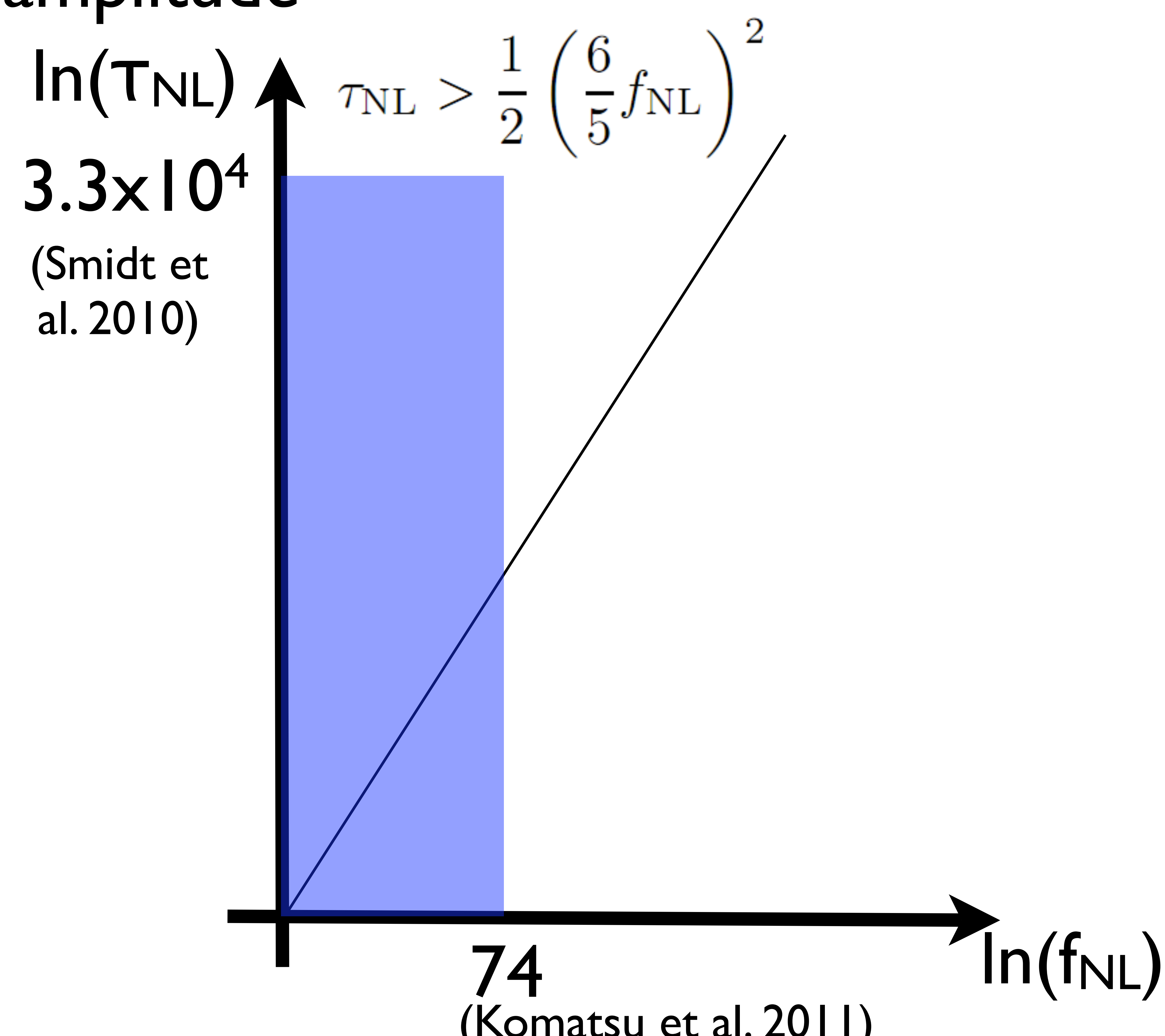
And...

- Typical "inflation data review" talks used to end here, but we now have exciting new tools: **non-Gaussianity**
- To characterize a departure of primordial fluctuations from a Gaussian distribution, we use the 3-point function (bispectrum) and 4-point function (trispectrum)

Main Conclusions First:

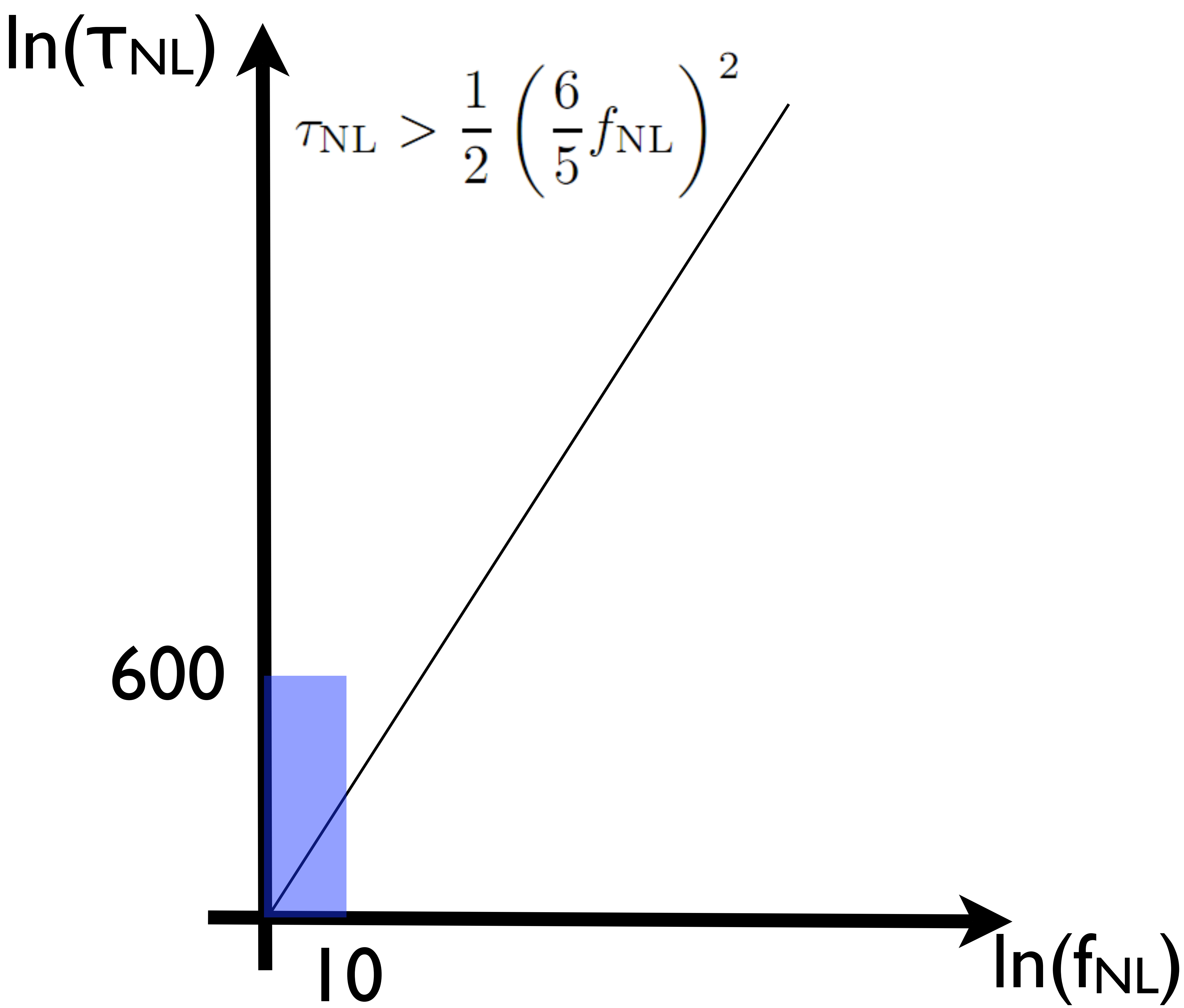
Eye-catchers

(Don't worry if you don't understand what I am talking about here: I will explain it later.)



- The current limits from WMAP 7-year are consistent with single-field or multi-field models.
- So, let's play around with the future.

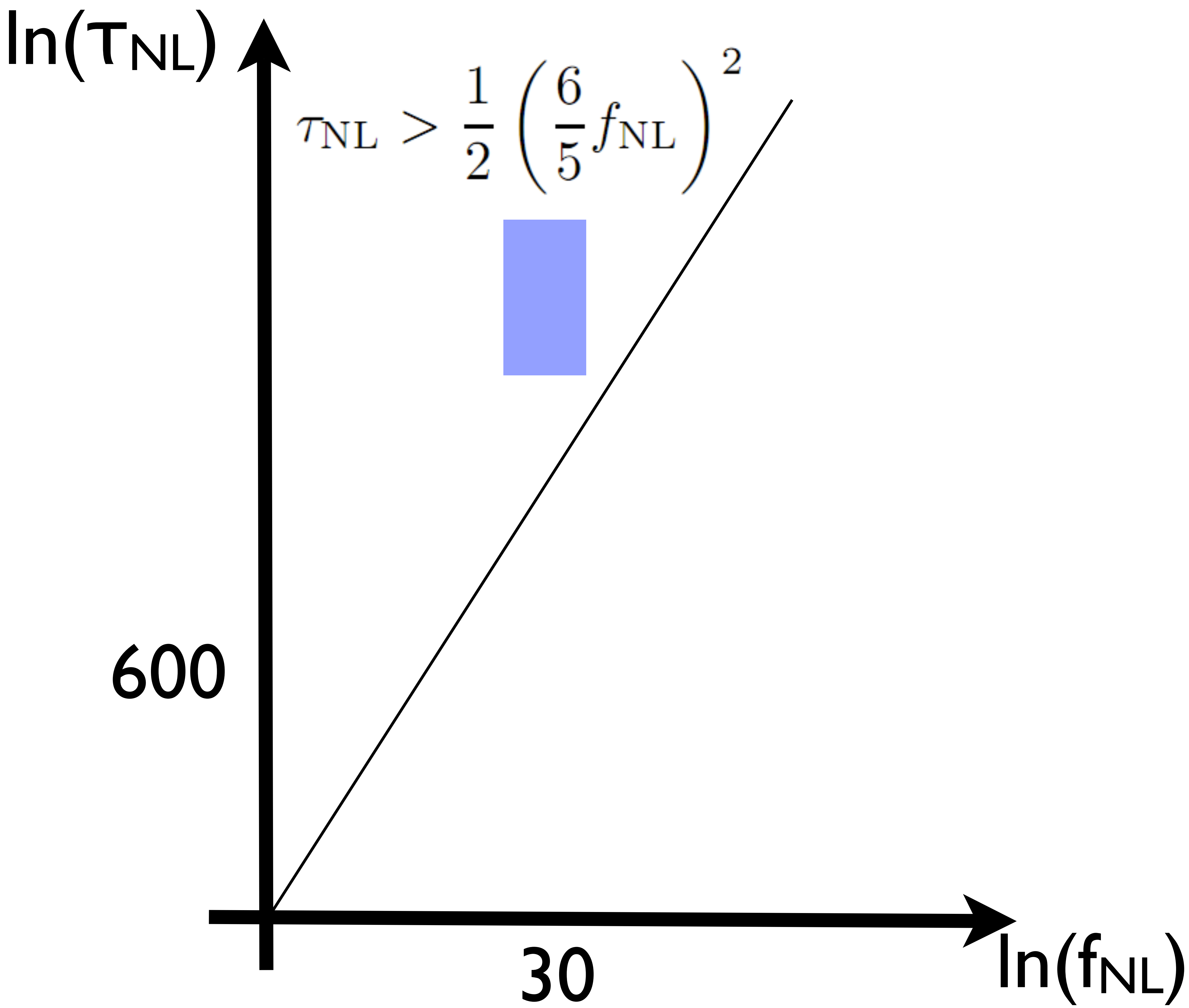
Case A: Single-field Happiness



- No detection of anything (f_{NL} or τ_{NL}) after Planck. Single-field survived the test (for the moment: the future galaxy surveys can improve the limits by a factor of ten).

(Suyama & Yamaguchi 2008; Komatsu 2010; Sugiyama, Komatsu & Futamase 2011)

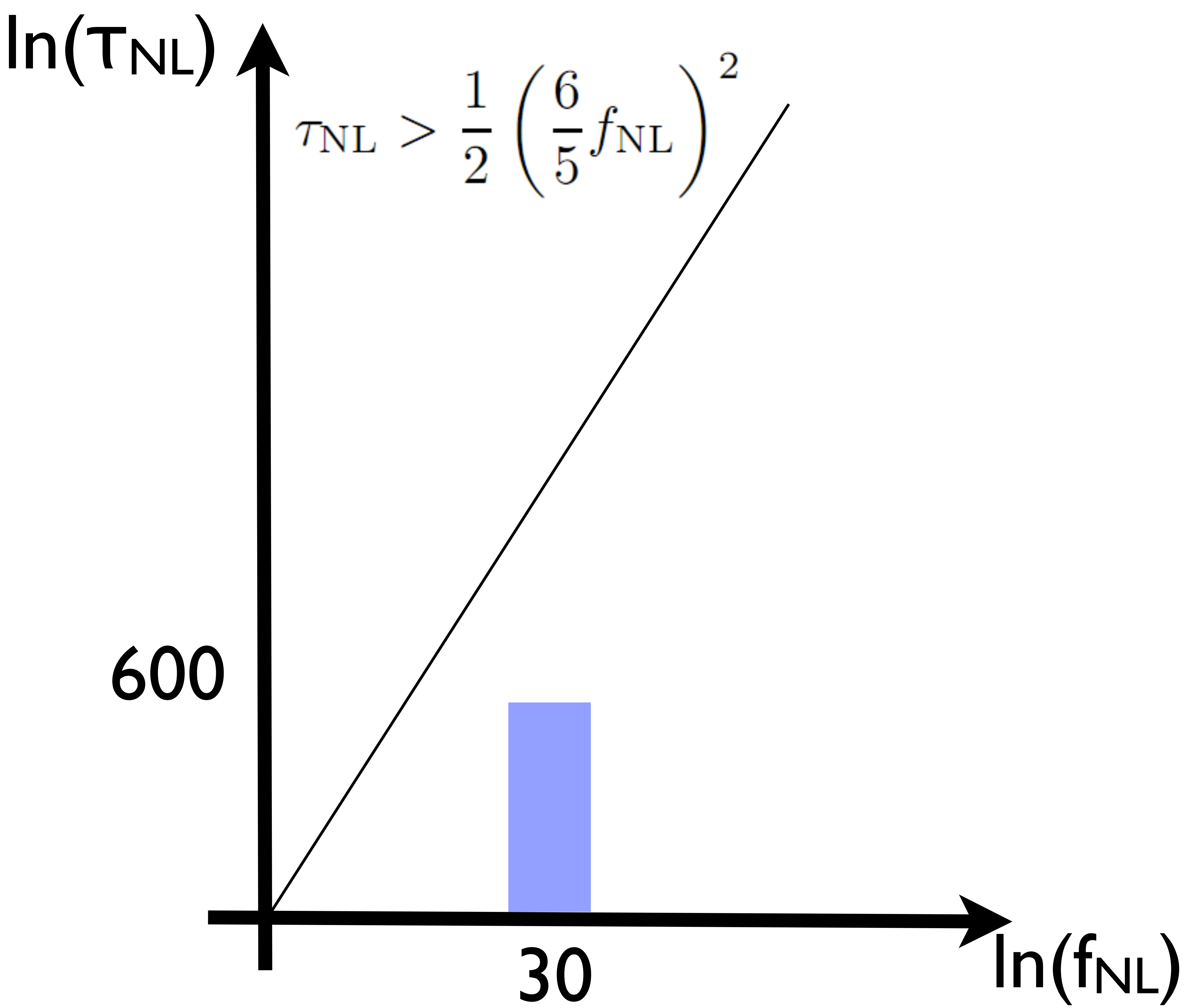
Case B: Multi-field Happiness(?)



- **f_{NL} is detected. Single-field is gone.**
- But, τ_{NL} is also detected, in accordance with **τ_{NL}>0.5(6f_{NL}/5)^2** expected from most multi-field models.

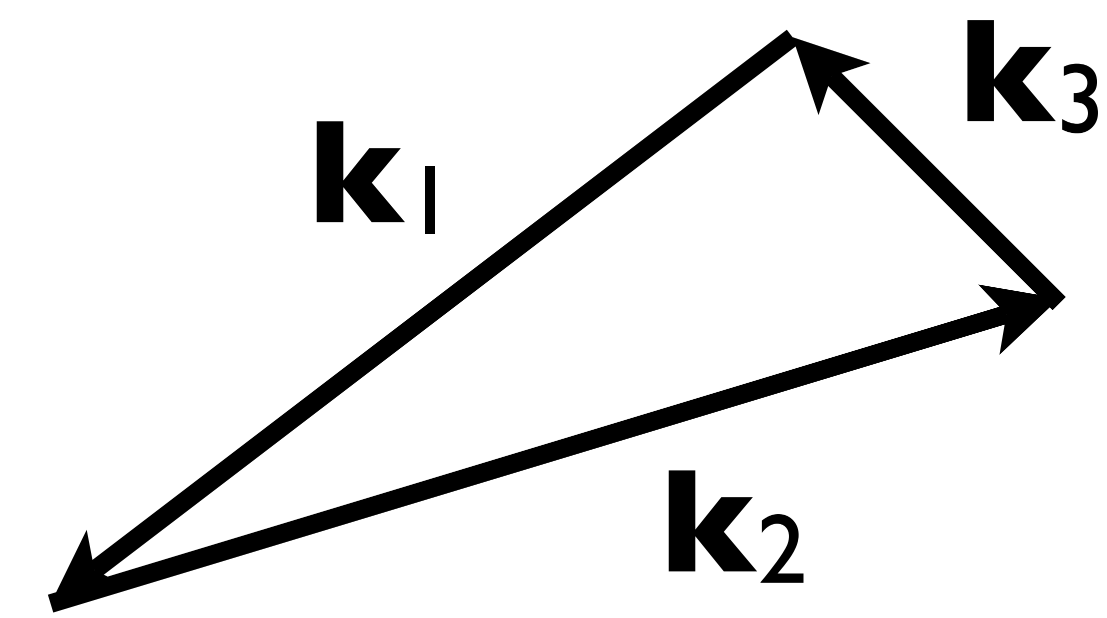
(Suyama & Yamaguchi 2008; Komatsu 2010; Sugiyama, Komatsu & Futamase 2011)

Case C: Madness



- f_{NL} is detected. Single-field is gone.
- But, τ_{NL} is not detected, or found to be negative, inconsistent with $\tau_{NL}>0.5(6f_{NL}/5)^2$.
- **Single-field <u>AND</u> most of multi-field models are gone.**

Bispectrum

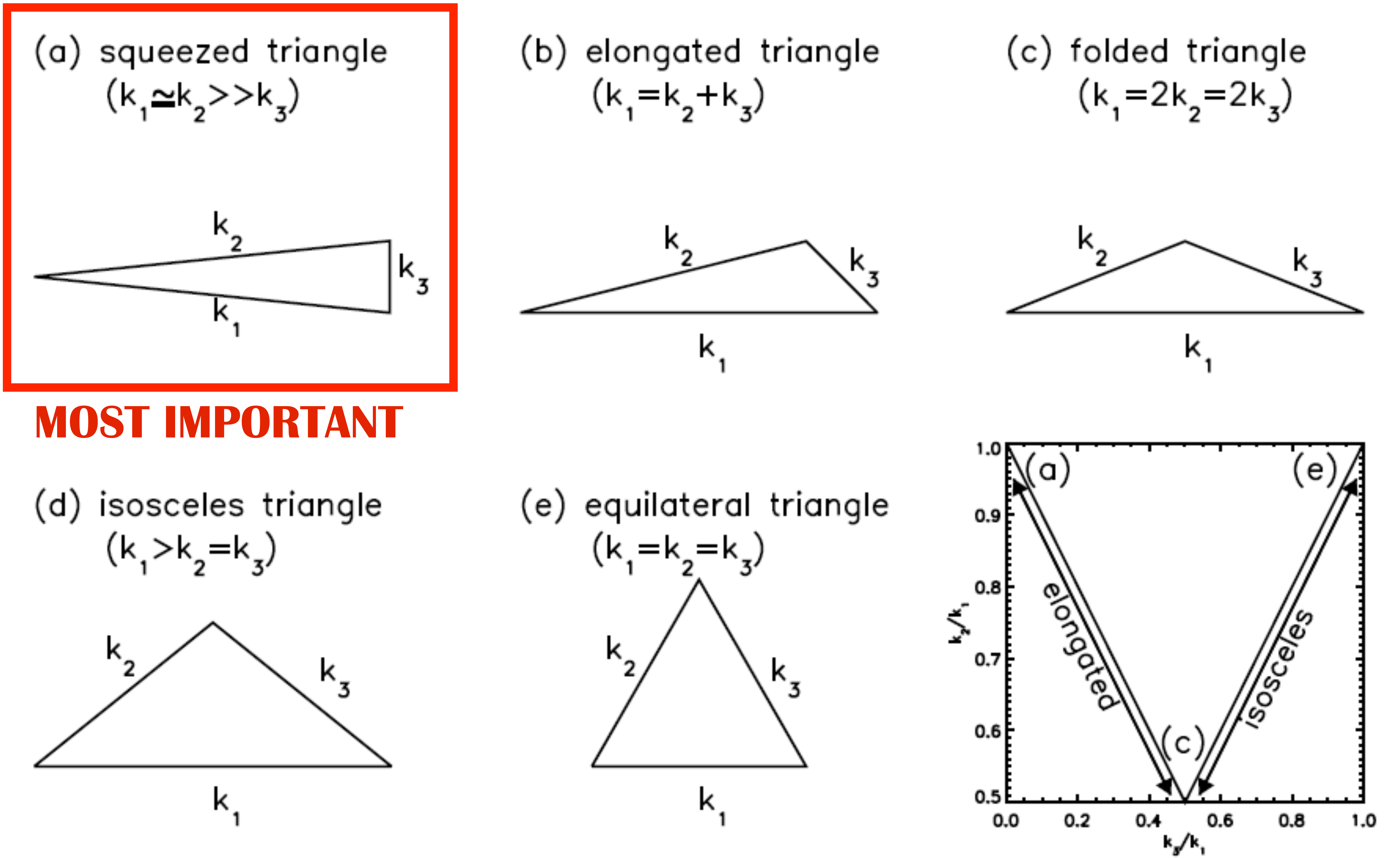


- Three-point function!
- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= <\zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3}> = (\text{amplitude}) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)F(k_1,k_2,k_3)$

model-dependent function

Primordial fluctuation

"f_{NL}"


(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)
k_2
k_3
k_1
MOST IMPORTANT
(b) elongated triangle
($k_1 = k_2 + k_3$)
k_2
k_3
k_1
(c) folded triangle
($k_1 = 2k_2 = 2k_3$)
k_2
k_3
k_1
(d) isosceles triangle
($k_1 > k_2 = k_3$)
k_2
k_3
k_1
(e) equilateral triangle
($k_1 = k_2 = k_3$)
k_2
k_3
k_1
1.0
0.9
0.8
0.7
0.6
0.5
k_2/k_1
(a)
(e)
(c)
elongated
isosceles
0.0
0.2
0.4
0.6
0.8
1.0
k_3/k_1

Probing Inflation (3-point Function)

- Inflation models predict that primordial fluctuations are very close to Gaussian.
  - In fact, ALL SINGLE-FIELD models predict the squeezed-limit 3-point function to have the amplitude of $f_{NL}=0.02$.
  - Detection of $f_{NL}>1$ would rule out ALL single-field models!
- No detection of this form of 3-point function of primordial curvature perturbations. The 95% CL limit is:
  - $-10 < f_{NL} < 74$
  - The WMAP data are consistent with the prediction of **simple single-field inflation** models: $1-n_s \approx r \approx f_{NL}$

A Non-linear Correction to Temperature Anisotropy

- The CMB temperature anisotropy, ΔT/T, is given by the curvature perturbation in the matter-dominated era, Φ.
  - One large scales (the Sachs-Wolfe limit), ΔT/T=–Φ/3.

For the Schwarzschild metric, Φ=+GM/R.

- Add a non-linear correction to Φ:
  - $\Phi(\mathbf{x}) = \Phi_g(\mathbf{x}) + f_{NL}[\Phi_g(\mathbf{x})]^2$ (Komatsu & Spergel 2001)
  - f_{NL} was predicted to be small (~0.01) for slow-roll models (Salopek & Bond 1990; Gangui et al. 1994)

“Local Form” B_ζ

- Φ is related to the primordial curvature perturbation, ζ, as $\Phi=(3/5)\zeta$.

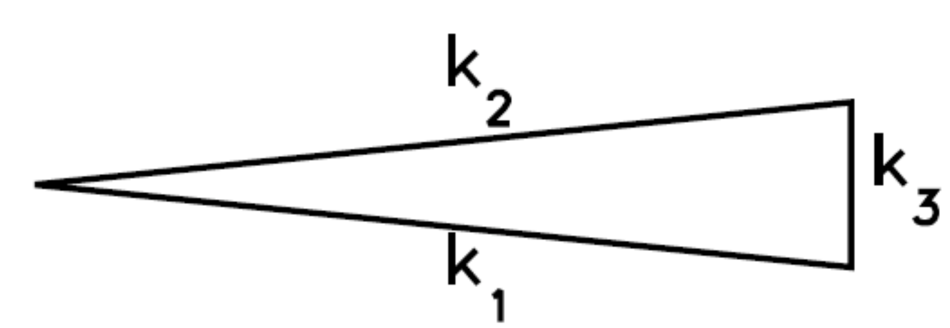
- $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2$

- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(6/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times [P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)]$

f_{NL}: Shape of Triangle

- For a scale-invariant spectrum, $P_\zeta(k)=A/k^3$,
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(6A^2/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times [1/(k_1k_2)^3 + 1/(k_2k_3)^3 + 1/(k_3k_1)^3]$
- Let's order k_i such that $k_3 \le k_2 \le k_1$. For a given k_1, one finds the largest bispectrum when the smallest k, i.e., k_3, is very small.
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ peaks when $k_3 << k_2 \sim k_1$
  - Therefore, the shape of f_{NL} bispectrum is the squeezed triangle!

(Babich et al. 2004)



B_ζ in the Squeezed Limit

- In the squeezed limit, the f_{NL} bispectrum becomes:
  $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \approx (12/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$

Maldacena (2003); Seery & Lidsey (2005); Creminelli & Zaldarriaga (2004)

Single-field Theorem (Consistency Relation)

- For **ANY** single-field models*, the bispectrum in the squeezed limit is given by
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \approx (1-n_s) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$
  - Therefore, all single-field models predict $f_{NL} \approx (5/12)(1-n_s)$.
  - With the current limit $n_s \sim 0.96$, f_{NL} is predicted to be ~ 0.02.

* for which the single field is solely responsible for driving inflation and generating observed fluctuations.

Suppose that single-field models are ruled out. Now what?

- We just don't want to be thrown into multi-field landscape without any clues...
- What else can we use?
  - Four-point function!

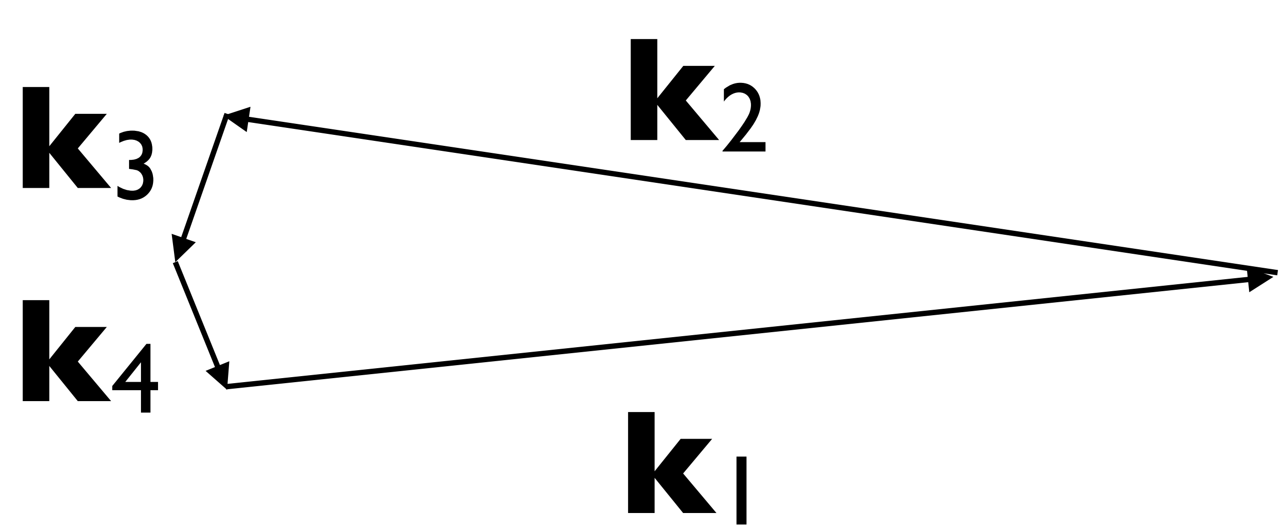
Trispectrum: Next Frontier

- The local form bispectrum, $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)f_{NL}$ $[(6/5)P_\zeta(k_1)P_\zeta(k_2)+\text{cyc.}]$
- is equivalent to having the curvature perturbation in position space, in the form of:
  - $\zeta(\mathbf{x})=\zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2$
- This can be extended to higher-order:
  - $\zeta(\mathbf{x})=\zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2 + (9/25)g_{NL}[\zeta_g(\mathbf{x})]^3$

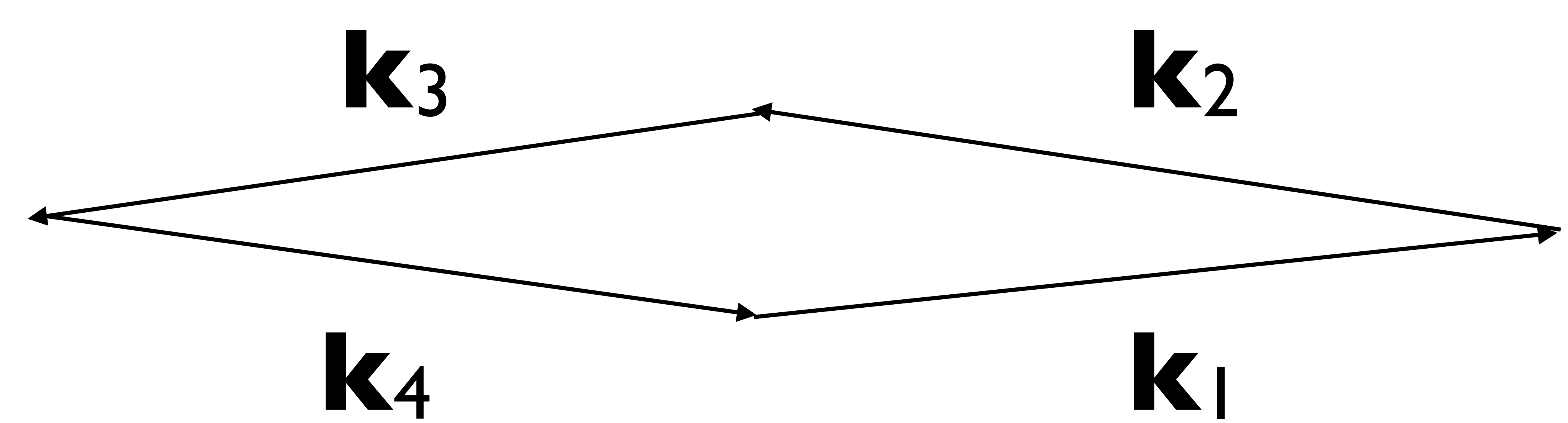
This term is probably too small to see, so I don’t talk much about it.

Local Form Trispectrum

- For $\zeta(\mathbf{x})=\zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2 + (9/25)g_{NL}[\zeta_g(\mathbf{x})]^3$, we obtain the trispectrum:
  - $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$ $\{\mathbf{g_{NL}}[(54/25)P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+\text{cyc.}] +\mathbf{(f_{NL})^2}[(18/25)P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\text{cyc.}]\}$



g_{NL}

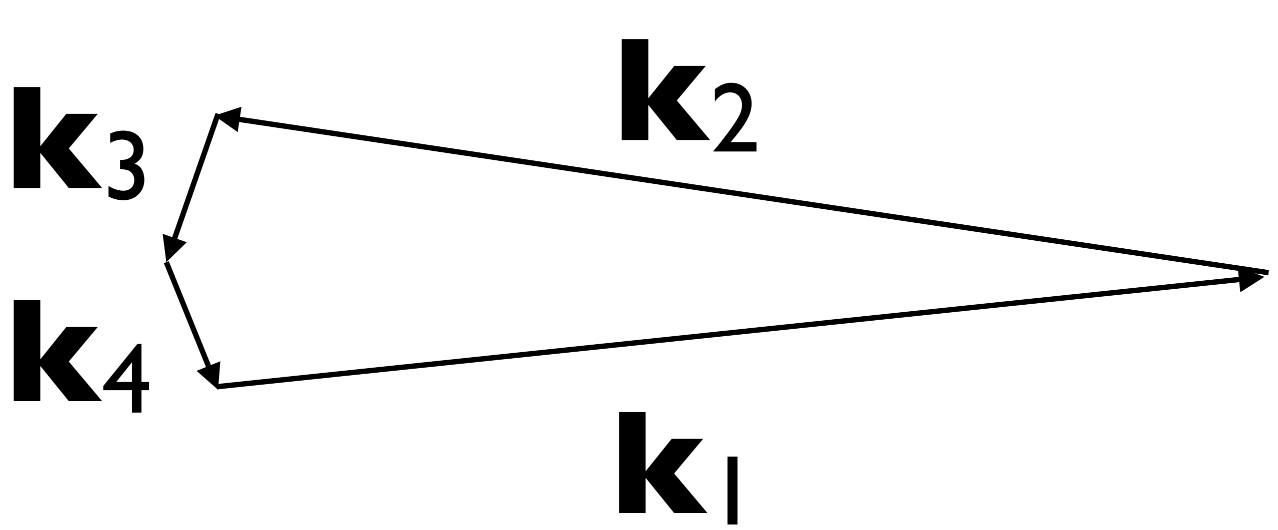


$f_{NL}{}^2$

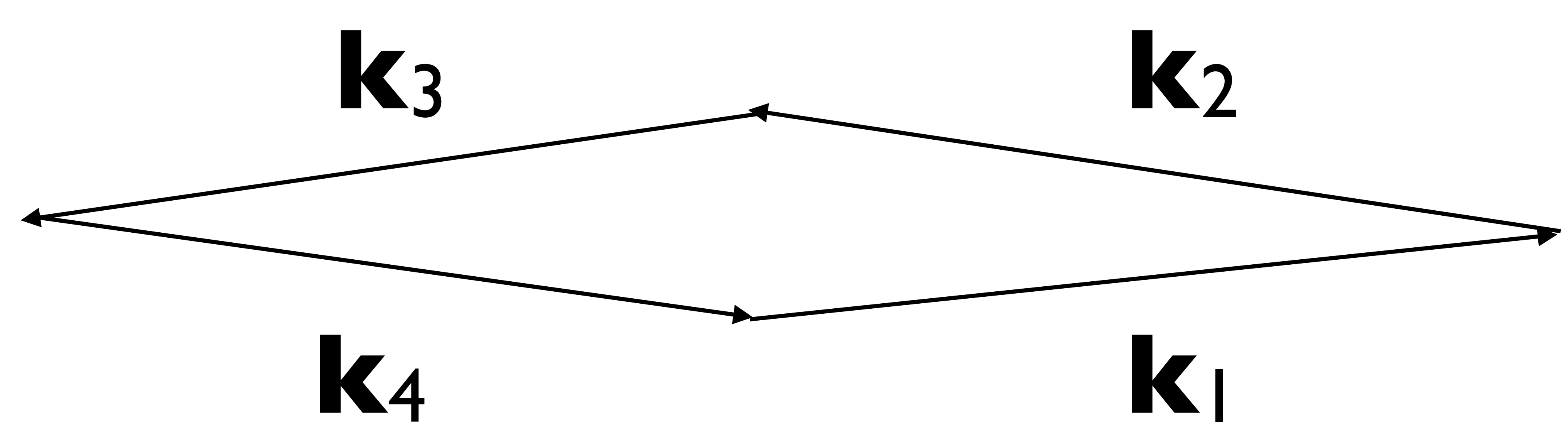
(Slightly) Generalized Trispectrum

- $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\ \{\mathbf{g_{NL}}[(54/25)P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+\mathrm{cyc.}]+\boldsymbol{\tau_{NL}}[P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\mathrm{cyc.}]\}$

The local form consistency relation, $\tau_{NL}=(6/5)(f_{NL})^2$, may not be respected – additional test of multi-field inflation!



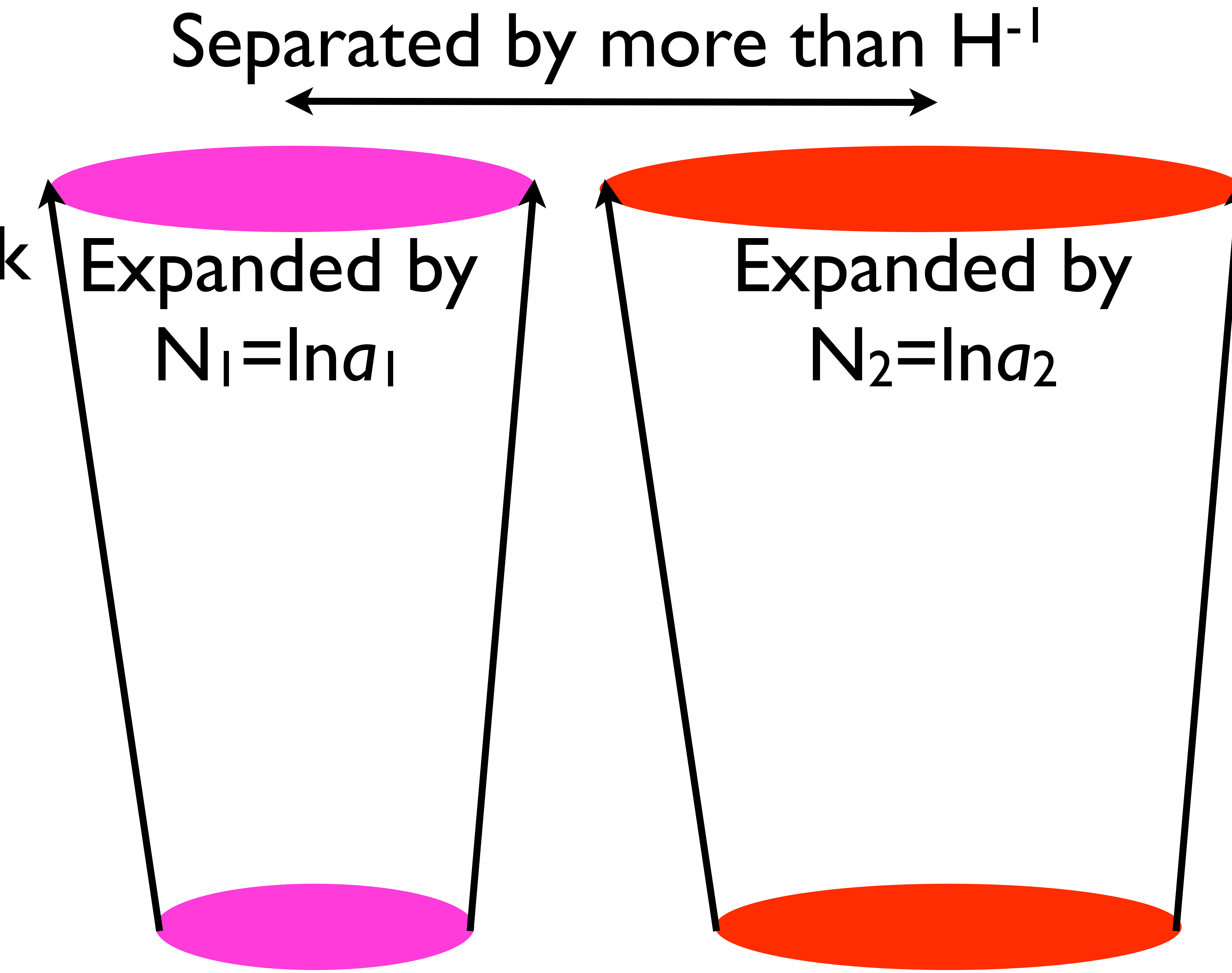
g_{NL}



τ_{NL}

The δN Formalism

- The δN formalism (Starobinsky 1982; Salopek & Bond 1990; Sasaki & Stewart 1996) states that the curvature perturbation is equal to the difference in $N=\ln a$.
  - $\zeta=\delta N=N_2-N_1$
  - where $N=\int H dt$



Getting the familiar result

- Single-field example at the linear order:
  - $\zeta = \delta\{\int H dt\} = \delta\{\int (H/\varphi')d\varphi\} \approx (H/\varphi')\delta\varphi$
  - Mukhanov & Chibisov; Guth & Pi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner

Extending to non-linear, multi-field cases

$$\zeta = \sum_I \frac{\partial N}{\partial \phi_I}\delta\phi_I + \frac{1}{2}\sum_{IJ}\frac{\partial^2 N}{\partial\phi_I\partial\phi_J}\delta\phi_I\delta\phi_J + \ldots$$

(Lyth & Rodriguez 2005)

- Calculating the bispectrum is then straightforward. Schematically:
  - $\langle\zeta^3\rangle = \langle(1\text{st})\times(1\text{st})\times(2\text{nd})\rangle \sim \langle\delta\varphi^4\rangle \neq 0$
  - $f_{NL} \sim \langle\zeta^3\rangle / \langle\zeta^2\rangle^2$

$$\frac{6}{5}f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,IJ}N_{,I}N_{,J}}{[\sum_I (N_{,I})^2]^2}$$

Extending to non-linear, multi-field cases

$$\zeta = \sum_I \frac{\partial N}{\partial \phi_I}\delta\phi_I + \frac{1}{2}\sum_{IJ}\frac{\partial^2 N}{\partial\phi_I \partial\phi_J}\delta\phi_I\delta\phi_J + \ldots$$

(Lyth & Rodriguez 2005)

- Calculating the trispectrum is also straightforward. Schematically:
  - $\langle\zeta^4\rangle = \langle(1\mathrm{st})^2(2\mathrm{nd})^2\rangle \sim \langle\delta\varphi^6\rangle \neq 0$
  - $f_{NL} \sim \langle\zeta^4\rangle / \langle\zeta^2\rangle^3$

$$\tau_{\mathrm{NL}} = \frac{\sum_{IJK} N_{,IJ}N_{,J}N_{,IK}N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ}N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

Now, stare at these.

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{[\sum_I (N_{,I})^2]^2},$$

$$\tau_{\rm NL} = \frac{\sum_{IJK} N_{,IJ} N_{,J} N_{,IK} N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ} N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

Change the variable...

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{[\sum_I (N_{,I})^2]^2},$$

$$\tau_{\rm NL} = \frac{\sum_{IJK} N_{,IJ} N_{,J} N_{,IK} N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ} N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

$$a_I = \frac{\sum_J N_{,IJ} N_{,J}}{[\sum_J (N_{,J})^2]^{3/2}}$$

$$b_I = \frac{N_{,I}}{[\sum_J (N_{,J})^2]^{1/2}}$$

$(6/5)f_{NL}=\sum_I a_I b_I$

$\tau_{NL}=(\sum_I a_I)^2(\sum_I b_I)^2$

Then apply the Cauchy-Schwarz Inequality

$$\left(\sum_I a_I^2\right)\left(\sum_J b_J^2\right) \geq \left(\sum_I a_I b_I\right)^2$$

- Implies (Suyama & Yamaguchi 2008)

$$\tau_{\rm NL} \geq \left(\frac{6 f_{\rm NL}^{\rm local}}{5}\right)^2$$

How generic is this inequality?

Be careful when 0=0

- The Suyama-Yamaguchi inequality does not always hold because the Cauchy-Schwarz inequality can be 0=0. For example:

$$\zeta = \frac{\partial N}{\partial \phi_1}\delta\phi_1 + \frac{1}{2}\frac{\partial^2 N}{\partial \phi_2^2}\delta\phi_2^2$$

In this harmless two-field case, the Cauchy-Schwarz inequality becomes 0=0 (both f_{NL} and τ_{NL} result from the second term).

We need more general results!

Assumptions

- Scalar fields are responsible for generating fluctuations.
- Fluctuations are Gaussian and scale-invariant at the horizon crossing.
    - All (local-form) non-Gaussianity was generated outside the horizon by δN
- We truncate δN expansion at $\delta\varphi^4$ (necessary for full calculations up to the "1-loop" order)

Starting point

$$\zeta(\mathbf{x},t) = N_a(t,t_*)\delta\varphi_*^a(\mathbf{x}) + \frac{1}{2}N_{ab}(t,t_*)\delta\varphi_*^a(\mathbf{x})\delta\varphi_*^b(\mathbf{x})$$
$$+ \frac{1}{3!}N_{abc}\delta\varphi_*^a\delta\varphi_*^b\delta\varphi_*^c + \frac{1}{4!}N_{abcd}\delta\varphi_*^a\delta\varphi_*^b\delta\varphi_*^c\delta\varphi_*^d$$

- Then, Fourier transform this and calculate the bispectrum and trispectrum...

Nao Sugiyama (a PhD student at Tohoku University in Sendai) did all the calculations!

Sugiyama, Komatsu & Futamase, arXiv:1101.3636

Here comes a simple result

(I have copies of our paper, so please feel free to take one if you are interested in how we derived this.)

$$\tau_{\rm NL} + (2\ \text{loop}) > \frac{1}{2}\left(\frac{6}{5}f_{\rm NL}\right)^2$$

- where (2 loop) denotes the following particular term:

$$\mathcal{P}_* \equiv (H_*/2\pi)^2$$

$$(2\ \text{loop}) = \frac{N_{ab}N_{abc}N_{cde}N_{de}\mathcal{P}_*^2 \ln^2(k_0 L)}{(N_a N_a)^3(1+\mathcal{P}_{\rm loop})^3}$$

$$\mathcal{P}_{\rm loop} \equiv \frac{\mathrm{Tr}(\tilde{N}^2)}{\tilde{N}_a\tilde{N}_a}\mathcal{P}_* \ln(kL)$$

Now, ignore this 2-loop term:

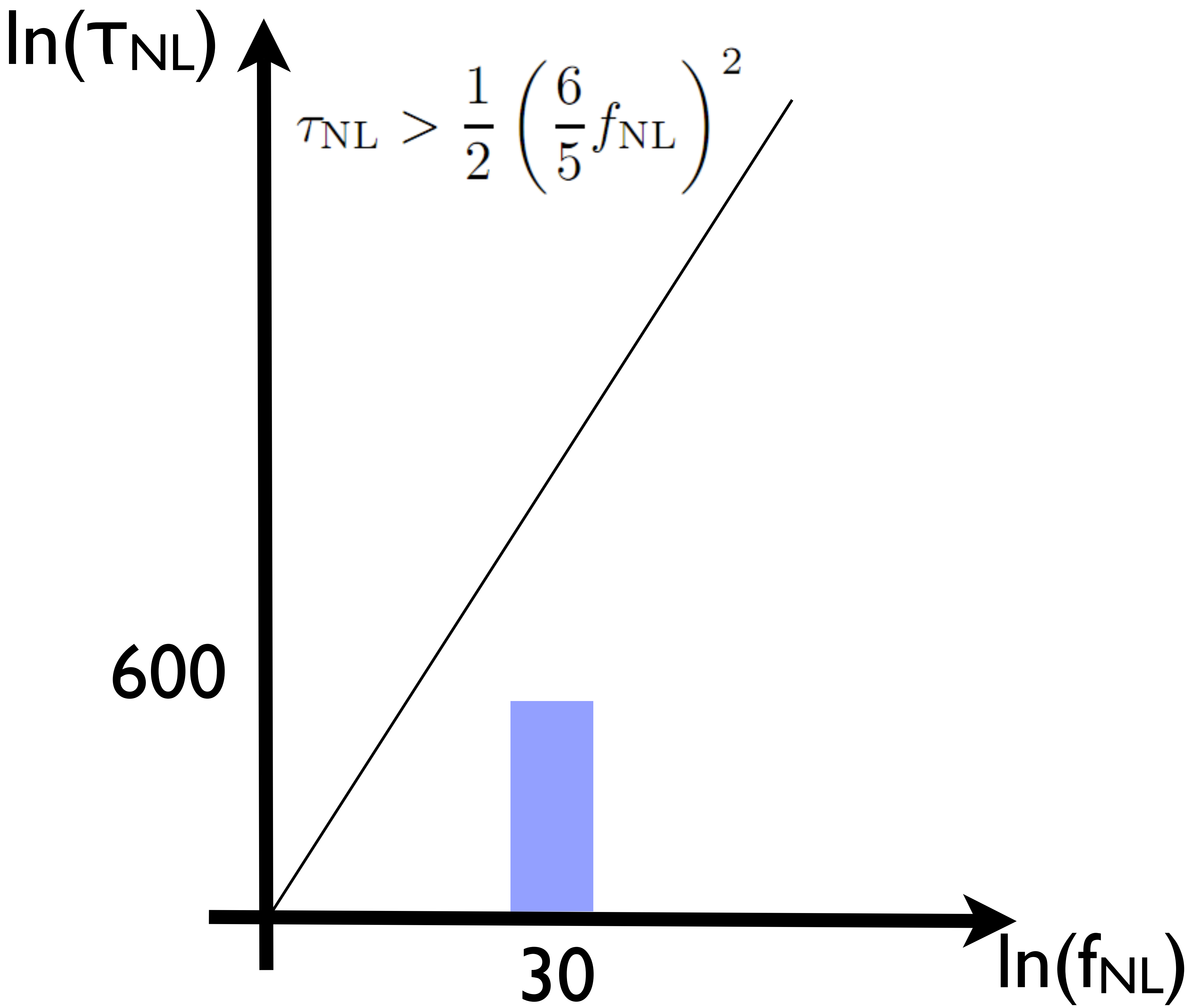
$$\tau_{\rm NL} > \frac{1}{2}\left(\frac{6}{5}f_{\rm NL}\right)^2$$

- The effect of including all 1-loop terms is to change the coefficient of Suyama-Yamaguchi inequality, $\tau_{NL} \geq (6f_{NL}/5)^2$

Recapping Assumptions

- Scalar fields are responsible for generating fluctuations.
- Fluctuations are Gaussian and scale-invariant at the horizon crossing.
  - All (local-form) non-Gaussianity was generated outside the horizon by δN
- We truncate δN expansion at $\delta\varphi^4$ (necessary for full calculations up to the "1-loop" order)
- We ignore 2-loop (and higher) terms

Looking Forward to "Interesting" Future...



- f_{NL} is detected. Single-field is gone.
- But, τ_{NL} is not detected, or found to be negative, inconsistent with $\tau_{NL}>0.5(6f_{NL}/5)^2$.
- **Single-field <u>AND</u> most of multi-field models are gone.**