Relations between *four* different dipole modulation measurements reported by the Planck collaboration

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Modeling the observed temperature anisotropy as $T(\hat{n}) \to T(\hat{n})[1 + A\hat{n}\cdot\hat{p}]$, where $\hat{p}$ is some preferred direction in the sky, the isotropy paper claims a detection of A with $A \approx 0.07$. In terms of the power spectrum of the modulating field, it is $C_1^f = (4\pi/9)A^2 \approx 7\times 10^{-3}$. This result is based on two methods: the direct pixel-based fitting and the bipolar spherical harmonics (BipoSH) analysis, which are applied to low-resolution maps at $N_{\rm side} = 32$. On the other hand, using the full-resolution Planck maps, the non-Gaussianity paper and the Doppler-boost paper find $C_1^f \approx 2\times 10^{-5}$, which is consistent with the expected signal due to the Solar System motion with respect of the rest frame of the cosmic microwave background: $A \approx 2.5(v/c) \approx 3\times 10^{-3}$. The apparent contradiction between these two numbers comes from the multipole ranges used in these analyses: the former is based on the low-resolution maps, whereas the latter is based on the full-resolution maps. In other words, a large signal reported by the isotropy paper is not scale-invariant, and does not appear to extend to high multipoles. If this (unknown) signal were to extend to higher multipoles, it should not be in conflict with the stringent constraints reported by the non-Gaussianity and Doppler-boost papers.

I. SETTING UP NOTATION

The Planck collaboration claims that they see hemispherical asymmetry of the power spectrum of temperature anisotropy of the cosmic microwave background.

One way to phenomenologically parametrize this is to model the observed temperature anisotropy as

$$T(\hat{n}) \to T(\hat{n})[1 + f(\hat{n})], \tag{1}$$

where $f(\hat{n})$ is a modulating field, which may be expanded in spherical harmonics as $f(\hat{n}) = \sum_{LM} f_{LM} Y_{LM}(\hat{n})$.

Let us define the power spectrum of f as

$$C_L^f \equiv \frac{1}{2L+1}\sum_M |f_{LM}|^2. \tag{2}$$

Assuming that only the dipole modulation, $L = 1$, is important, we write the above equation as

$$\begin{aligned} T(\hat{n}) &\to T(\hat{n})[1 + \sum_M f_{1M} Y_{1M}(\hat{n})] \\ &\equiv T(\hat{n})[1 + A\hat{n}\cdot\hat{p}], \end{aligned} \tag{3}$$

where $\hat{p}$ is a preferred direction in the sky. The coefficient A is equal to $\tilde{f}_{10}\sqrt{3/(4\pi)}$, where $\tilde{f}_{10}$ is defined in some rotated coordinates in which $Y_{10} = \sqrt{3/(4\pi)}\hat{n}\cdot\hat{p}$. This means that

$$C_L^f = \frac{1}{3}|\tilde{f}_{10}|^2 = \frac{4\pi}{9}A^2. \tag{4}$$

The Planck collaboration discusses this modulation in (at least) four different sections in three different papers. Our goal is to relate these four results and understand their consistency.

II. NON-GAUSSIANITY AND DOPPLER-BOOST PAPERS

The non-Gaussianity paper (Planck 2013 XXIV) and the Doppler-boost paper (Planck 2013 XXVII) discuss constraints on C_L^f and A.

The non-Gaussianity paper discusses this within the context of constraints on the local-form trispectrum, $\tau_{\rm NL}$, while the Doppler-boost paper discusses this within the context of the dipole modulation caused by the Solar System motion with respect to the rest frame of the cosmic microwave background.

In the non-Gaussianity paper, they use the estimator of f_{LM} given by Eq. (70) on page 15 of the non-Gaussianity paper:

$$\begin{aligned} f_{LM} \propto \int d^2\hat{n}\; Y_{LM}^*(\hat{n}) &\sum_{l_1 m_1} (C_{\rm tot}^{-1}T)_{l_1 m_1} Y_{l_1 m_1}(\hat{n}) \\ \times &\sum_{l_2 m_2} C_{l_2} (C_{\rm tot}^{-1}T)_{l_2 m_2} Y_{l_2 m_2}(\hat{n}), \end{aligned} \tag{5}$$

up to a constant normalization factor. Here, $C_{\rm tot}$ is the total signal-plus-noise covariance matrix, and C_l is the signal-only temperature power spectrum.

However, they do not use the proper $C_{\rm tot}^{-1}$ filtering, which is not diagonal in either harmonic or real space. Instead, (probably to save computational time), they use the diagonal weighting in harmonic space, i.e.,

$$(C_{\rm tot}^{-1}T)_{lm} \to \frac{T_{lm}}{C_{l,\rm tot}}, \tag{6}$$

where $C_{l,\rm tot} = C_l + N_l$ and N_l is the noise power spectrum. Both the non-Gaussianity and Doppler-boost papers use this diagonal weighting.

Using this estimator up to $l_{\rm max} = 2000$, they find

$$C_1^f \approx 2\times 10^{-5},$$

with high statistical significance. (See the data point at $L = 1$ of the purple dashed line of Fig. 16 in their paper). To avoid noise bias, they use the 143 GHz data for one of the maps in the above estimator, and the 217 GHz data for another.

The Doppler effect yields, among other things, a dipolar modulation with a y-type spectral distortion as

$$T(\hat{n}) \to T(\hat{n})[1 + b_\nu \hat{n}\cdot\vec{\beta}], \quad (7)$$

where b_ν has the frequency dependence that is similar to the y-distortion, and it gives $b_\nu \simeq 2$ and 3 for 143 and 217 GHz, respectively. The velocity term is given by $|\vec{\beta}| = v/c \simeq 1.2\times 10^{-3}$ for the Solar System motion with respect to the rest frame of the cosmic microwave background.

Their estimator for $b_\nu \hat{n}\cdot\vec{\beta}$ is given in Eq. (15) on page 4 of the Doppler-boost paper, which is identical to the above estimator used by the non-Gaussianity paper. Applying this estimator to the 143 GHz and 217 GHz data, they show that they detect the expected effect: namely, $A = b_\nu\beta \simeq 2.5\times 1.2\times 10^{-3} = 3\times 10^{-3}$. This measurement translates into C_1^f as

$$C_1^f = \frac{4\pi}{9}(b_\nu\beta)^2 \approx 1.3\times 10^{-5}.$$

Note that they find a different preferred direction when they restrict their analysis to lower multipoles, $l < 100$ (see Fig. 3 of their paper). This seems consistent with the finding of the isotropy paper that we discuss in the next section. In order to avoid potential contamination from a modulation in lower multipoles, they restrict their analysis to $l_{\rm min} \le l \le 2000$, where $l_{\rm min} = 500$. Such a $l_{\rm min}$ cut is not used in the non-Gaussianity paper.

The measurement reported in the non-Gaussianity paper, $C_1^f \approx 2\times 10^{-5}$, is consistent with the reported detection of the dipolar modulation by the Doppler boost. Subtracting the expected Doppler boost effect from the map, the non-Gaussianity paper reports $C_1^f \approx 0.2\times 10^{-5}$, which is consistent with zero to within 68% CL.

Therefore, the results reported in the non-Gaussianity paper and the Doppler boost paper agree well with each other.

III. ISOTROPY PAPER

The isotropy paper (Planck 2013 XXIII) discusses constraints on A using two different methods: a direct pixel-based likelihood (Section 5.5.2 from page 26), and the bipolar spherical harmonics (BipoSH) method (Section 5.6 from page 30).

Both methods find A which is much bigger than those found in the non-Gaussianity paper and the Doppler boost paper. They find $A \simeq 0.07$, which translates into

$$C_1^f \approx 7\times 10^{-3},$$

which is 340 times bigger than that reported by the non-Gaussianity paper. What is going on?

The answer is that, in the isotropy paper, they use the data only up to smaller multipoles. The pixel-based method uses maps smoothed to a 5 degree Gaussian beam at $N_{\rm side} = 32$, and the BipoSH method also uses maps at $N_{\rm side} = 32$, which limits the usable multipole range to $2 \le l \le 64$. On the other hand, both the non-Gaussianity paper and the Doppler boost paper use the maps at the full resolution, $N_{\rm side} = 2048$, with $l_{\rm max} = 2000$. However, the Doppler-boost paper removes lower multipoles and use $500 \le l \le 2000$, whereas the non-Gaussianity papers uses all the multipoles up to $l_{\rm max} = 2000$.

The BipoSH estimator for f_{LM} is also similar to (if not identical to) those used by the non-Gaussianity and Doppler-boost papers. It is given by Eq. (43) on page 30 of the isotropy paper:

$$f_{LM} \propto \int d^2\hat{n}\; Y_{LM}^*(\hat{n}) \sum_{l_1 l_2} \frac{C_{l_2}}{\sigma^2_{l_1 l_2, LM}} \times \sum_{m_1} T_{l_1 m_1} Y_{l_1 m_1}(\hat{n}) \sum_{m_2} T_{l_2 m_2} Y_{l_2 m_2}(\hat{n}), \quad (8)$$

where $\sigma^2_{l_1 l_2, LM}$ is the variance of the BipoSH coefficients estimated from simulations. In other words, the BipoSH estimator also uses the diagonal weighting rather than the proper $C_{\rm tot}^{-1}$ weighting of the data.

They also apply the BipoSH analysis to higher-resolution maps. Fig. 33 of the isotropy paper shows C_1^f/π as a function of the multipole bins used in the estimation. They find $C_1^f/\pi \approx 2.5\times 10^{-3}$, or $C_1^f \approx 8\times 10^{-3}$, for $2 \le l \le 64$, but find much smaller values for $l > 64$, up to $l_{\rm max} = 384$.

IV. CONCLUSION

The above observations give us the following plausible explanation: when the data at $l \le 64$ are used, the dipole modulation at the level of $A \approx 0.07$ or $C_1^f \approx 7\times 10^{-3}$ is found. The origin of this large signal is unknown.

However, when the analysis is extended to much higher multipoles, $l_{\rm max} = 2000$, this signal is diluted, as it does not extend to $l > 64$. Instead, another signal due to the Doppler boost appears and is detected at the expected level of $C_1^f \approx 2\times 10^{-5}$.

Now, we hear that the mysterious part of the dipole modulation actually extends to much higher multipoles of order 1000. However, such a modulation cannot be much greater than what is already constrained by the non-Gaussianity and Doppler-boost papers; namely, $A \approx 3\times 10^{-3}$ and $C_1^f \approx 2\times 10^{-5}$.