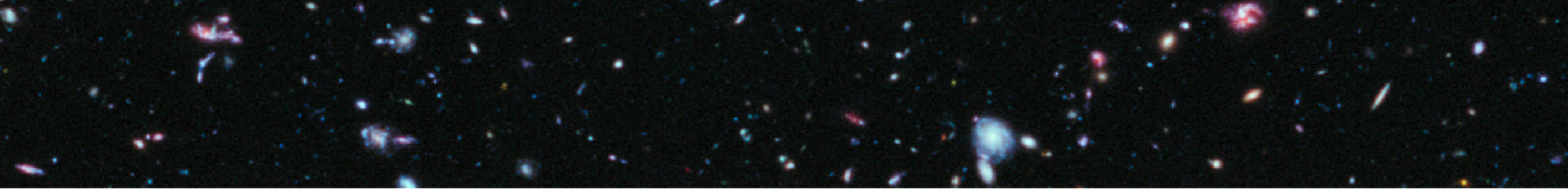
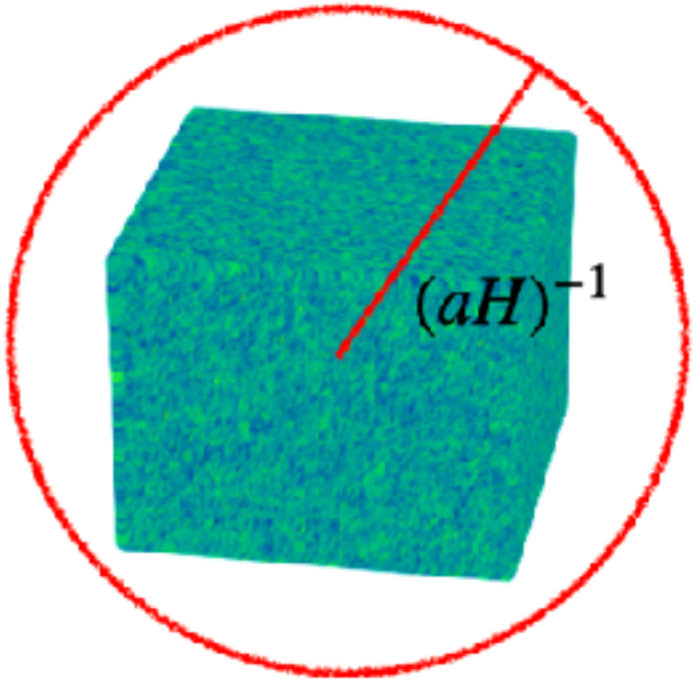
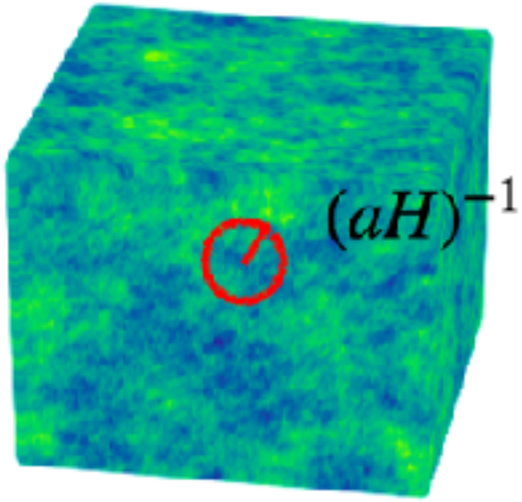
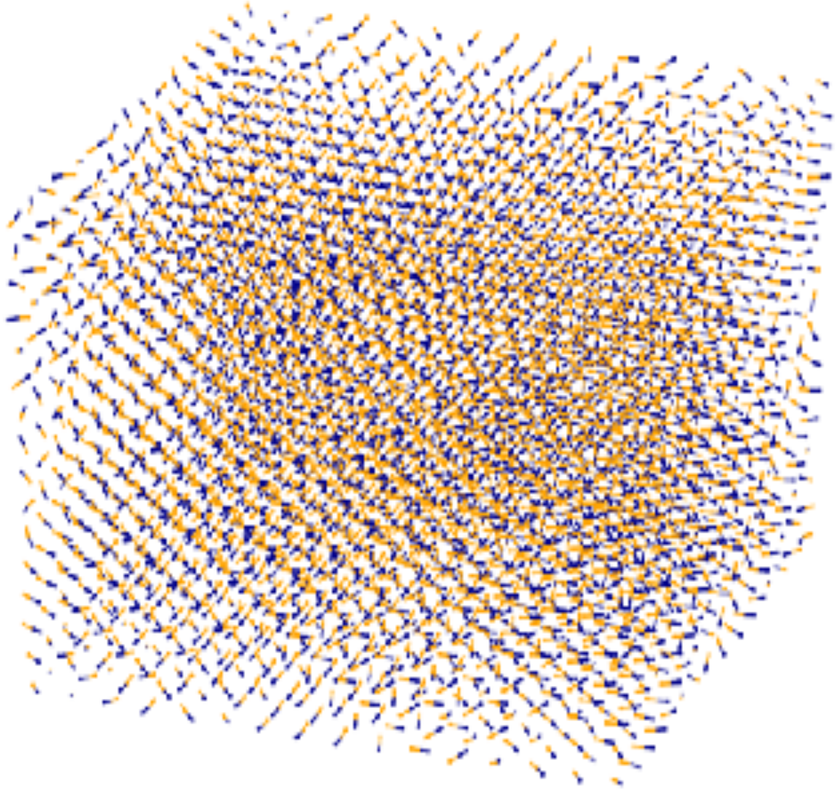


Lattice Simulations of Axion-U(1) Inflation

Angelo Caravano







Axion-U(1) inflation

Adding an electromagnetic U(1) field that interacts with the inflation:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_{\mathrm{Pl}}^2 \mathscr{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \boxed{- \frac{\alpha}{4f} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}} \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Ingredients:

- Pseudoscalar (axion) inflaton ϕ
- U(1) gauge field A_μ
- Interaction $\phi F \tilde{F}$

Axion-U(1) inflation

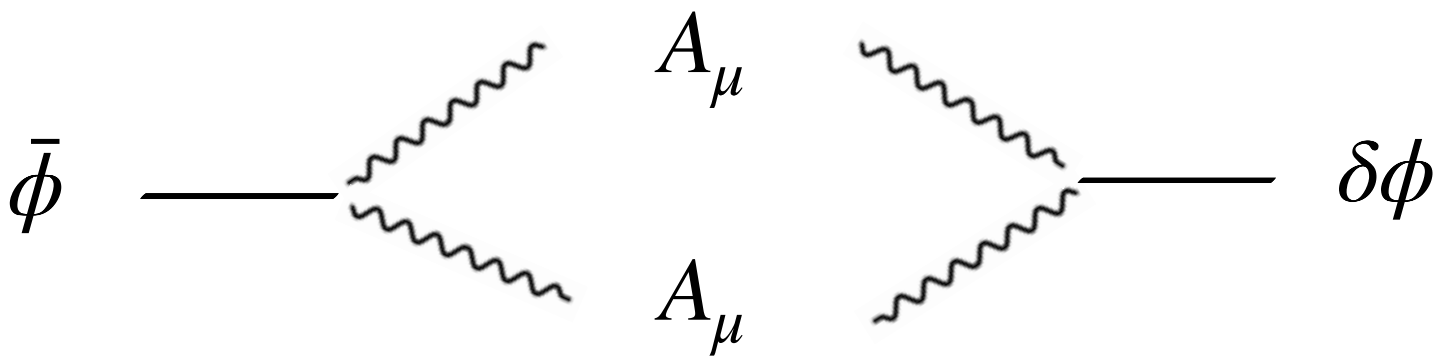
$$S = \int d^4x \sqrt{-g}\left(\frac{1}{2}M_{\mathrm{Pl}}^2\mathscr{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\boxed{- \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}}\right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

<u>Consequences</u> of the interaction:

1. Production of gauge field particles.

2. ⇒ these act as a source for inflation perturbation (and GWs)



This particle production is observable

Scalar perturbations:

For $k \ll aH$

$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

- $\langle \delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k}'}\rangle \sim \frac{H^2}{2k^3}\left(\underset{\substack{\uparrow \\ \text{vacuum}}}{1} + \underset{\substack{\uparrow \\ \text{sourced}}}{f_2(\xi)e^{4\pi\xi}}\right)\delta(\mathbf{k}+\mathbf{k}')$

- $\langle \delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_2}\delta\phi_{\mathbf{k}_3}\rangle \neq 0 \quad \sim \frac{9}{80}(2\pi)^{5/2}\frac{H^6}{k^6}f_3\left(\xi, \frac{k_2}{k_1}, \frac{k_3}{k_1}\right)e^{6\pi\xi}\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$

This particle production is observable

Scalar perturbations:

For $k \ll aH$

- $\langle \delta\phi_{\mathbf{k}} \delta\phi_{\mathbf{k}'} \rangle \sim \frac{H^2}{2k^3} \left(1 + f_2(\xi) e^{4\pi\xi} \right) \delta(\mathbf{k} + \mathbf{k}')$

$$\xi = \frac{\alpha \dot{\phi}}{2fH}$$

vacuum

sourced

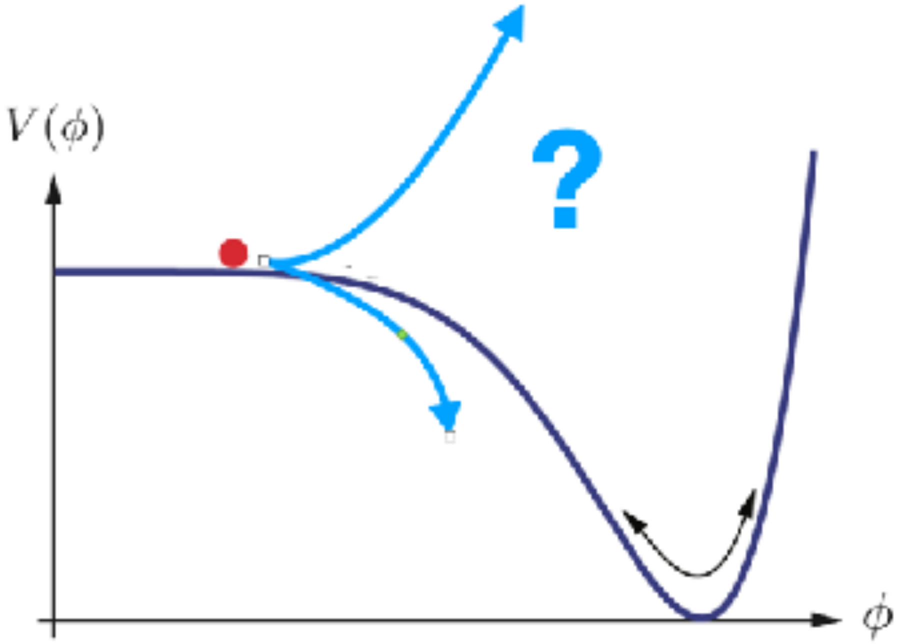
- $\langle \delta\phi_{\mathbf{k}_1} \delta\phi_{\mathbf{k}_2} \delta\phi_{\mathbf{k}_3} \rangle \neq 0 \quad \sim \frac{9}{80} (2\pi)^{5/2} \frac{H^6}{k^6} f_3 \left(\xi, \frac{k_2}{k_1}, \frac{k_3}{k_1} \right) e^{6\pi\xi} \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$

Backreaction

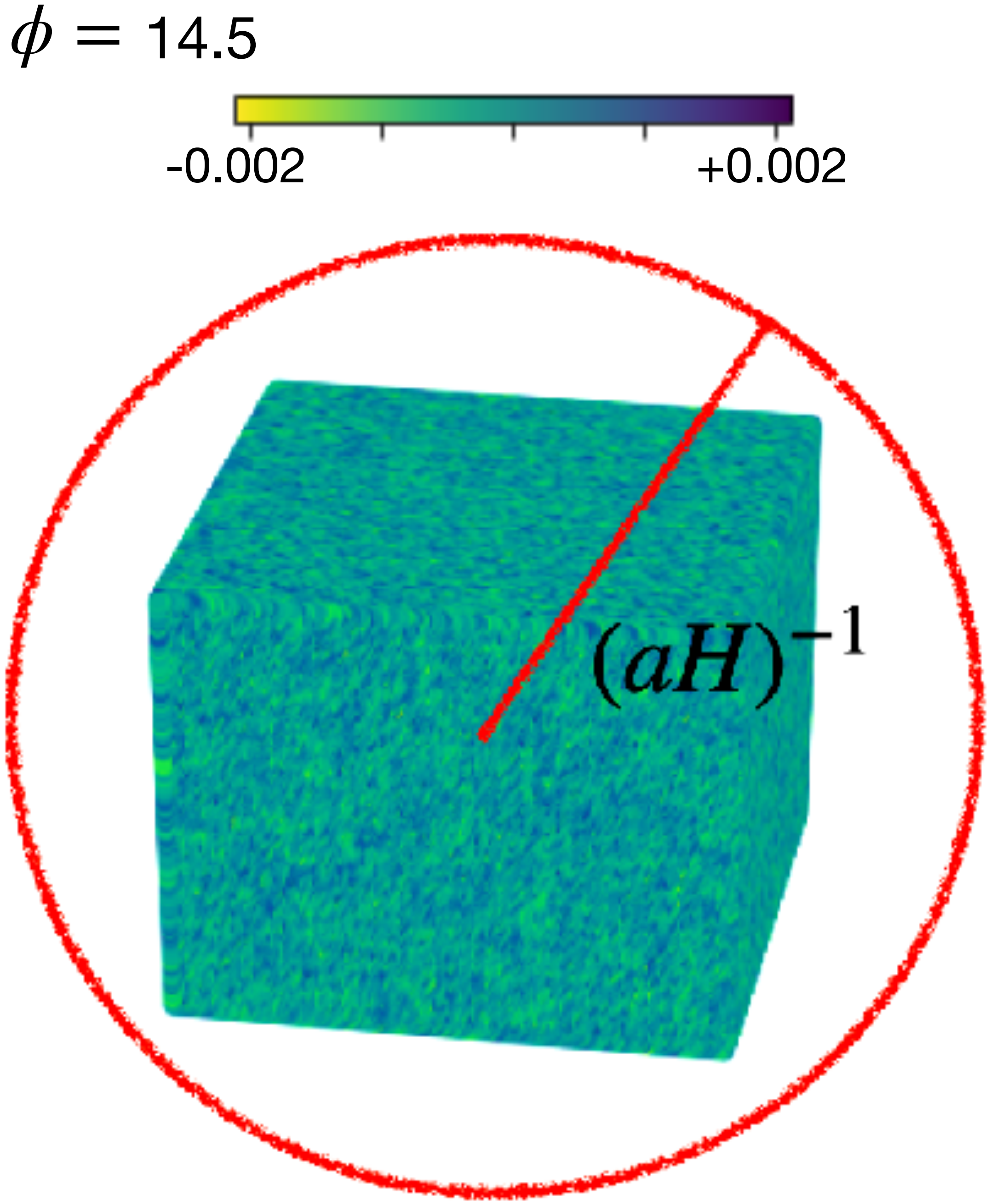
If ξ is large, perturbation theory breaks.

→ Need nonlinear tools

$$\frac{H^2}{26\pi|\dot{\phi}|}\xi^{-3/2}e^{\pi\xi} \cancel{\ll} 1.$$



1. Start with a **sub-horizon** patch



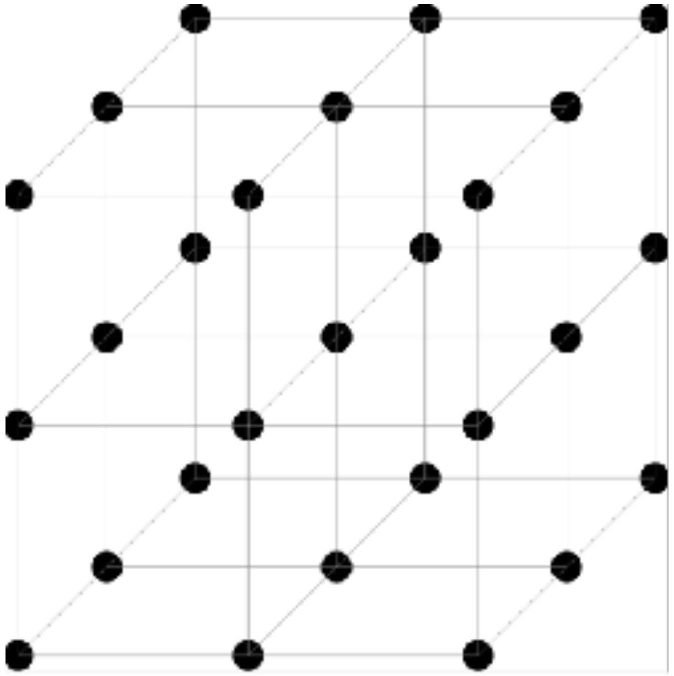
2. Evolve the system with the full equations of motion

Solve numerically for all lattice points:

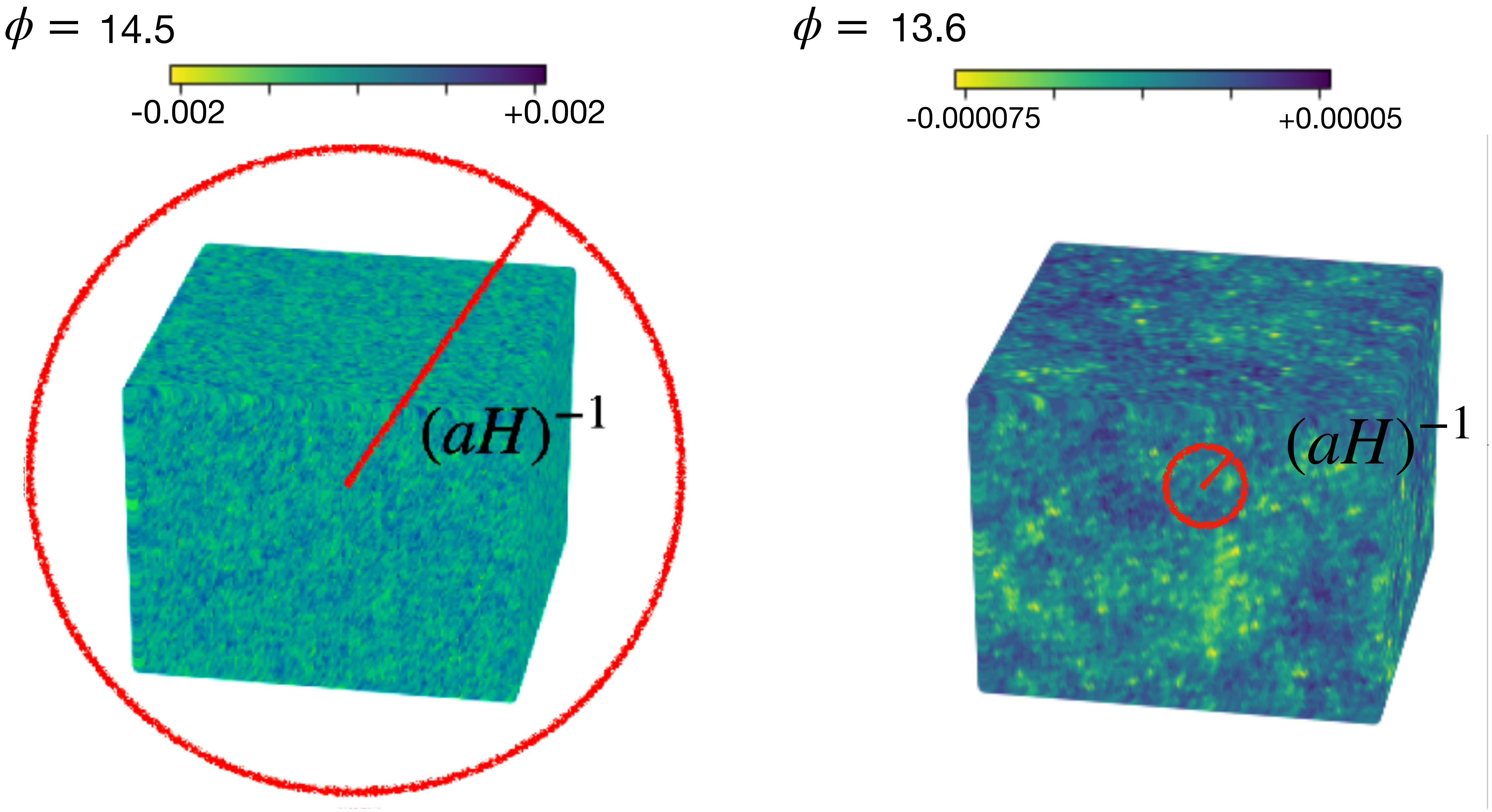
$$\phi'' + 2H\phi' - \partial_j\partial_j\phi + a^2\frac{\partial V}{\partial\phi} = -a^2\frac{\alpha}{4f}F_{\mu\nu}\tilde{F}^{\mu\nu},$$

$$A_0'' - \partial_j\partial_j A_0 = \frac{\alpha}{f}\epsilon_{ijk}\partial_k\phi\partial_i A_j,$$

$$A_i'' - \partial_j\partial_j A_i = \frac{\alpha}{f}\epsilon_{ijk}\phi'\partial_j A_k - \frac{\alpha}{f}\epsilon_{ijk}\partial_j\phi(A_k' - \partial_k A_0)$$

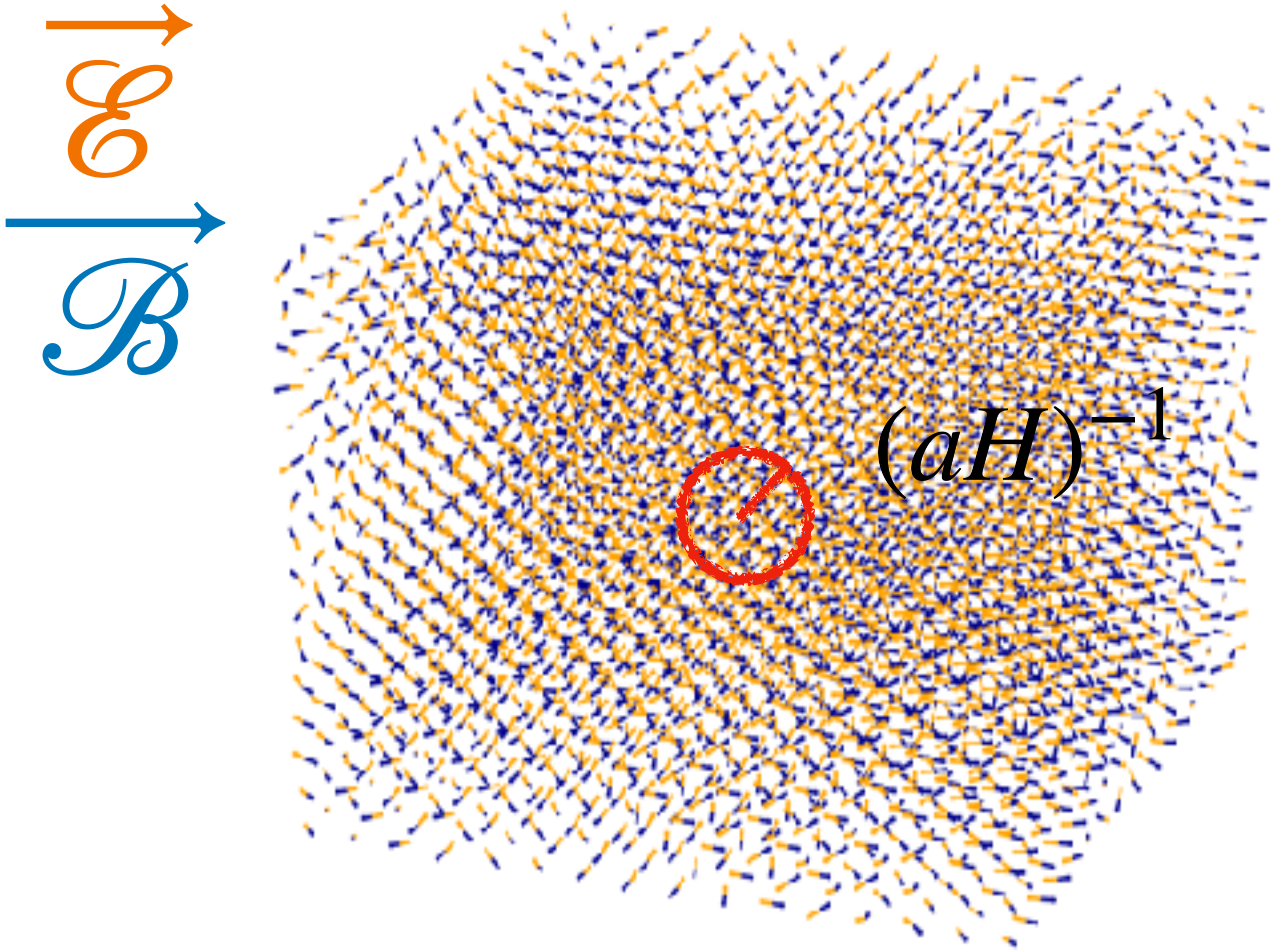


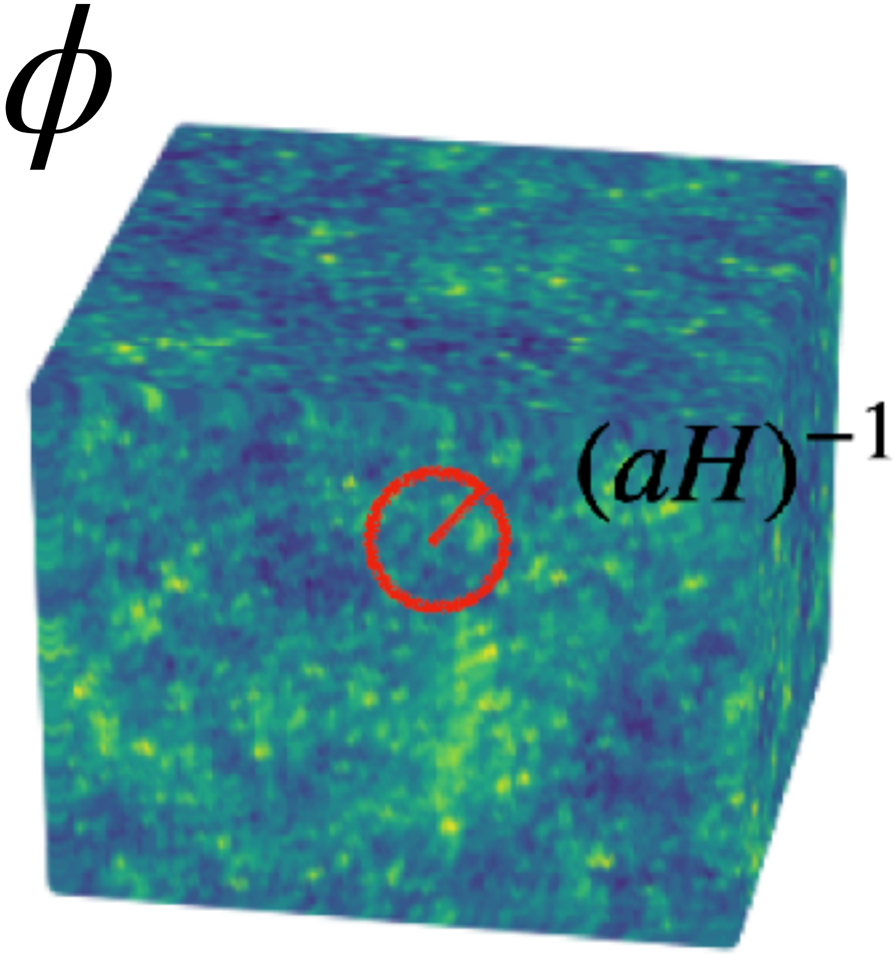
Assuming unperturbed FLRW universe $ds^2 = a^2(-d\tau^2 + d\vec{x}^2)$



Sub-horizon box $\xrightarrow[a_f/a_i = 10^3]{\text{Evolution}}$ **Super-horizon** box

We are interested in the statistical properties of the super-horizon box:





Results of the simulation:

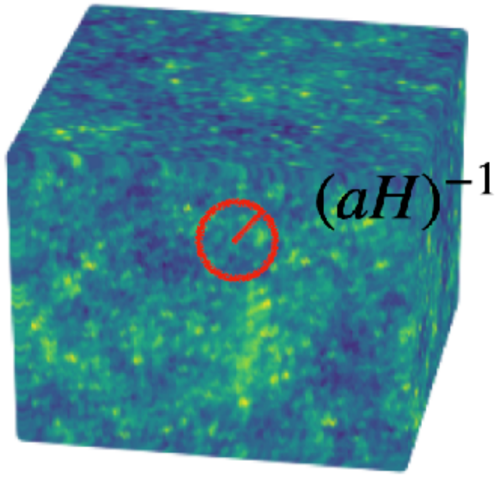
1. Linear regime

small $\xi = \dfrac{\alpha\dot{\phi}}{2fH}$

2. Nonlinear regime

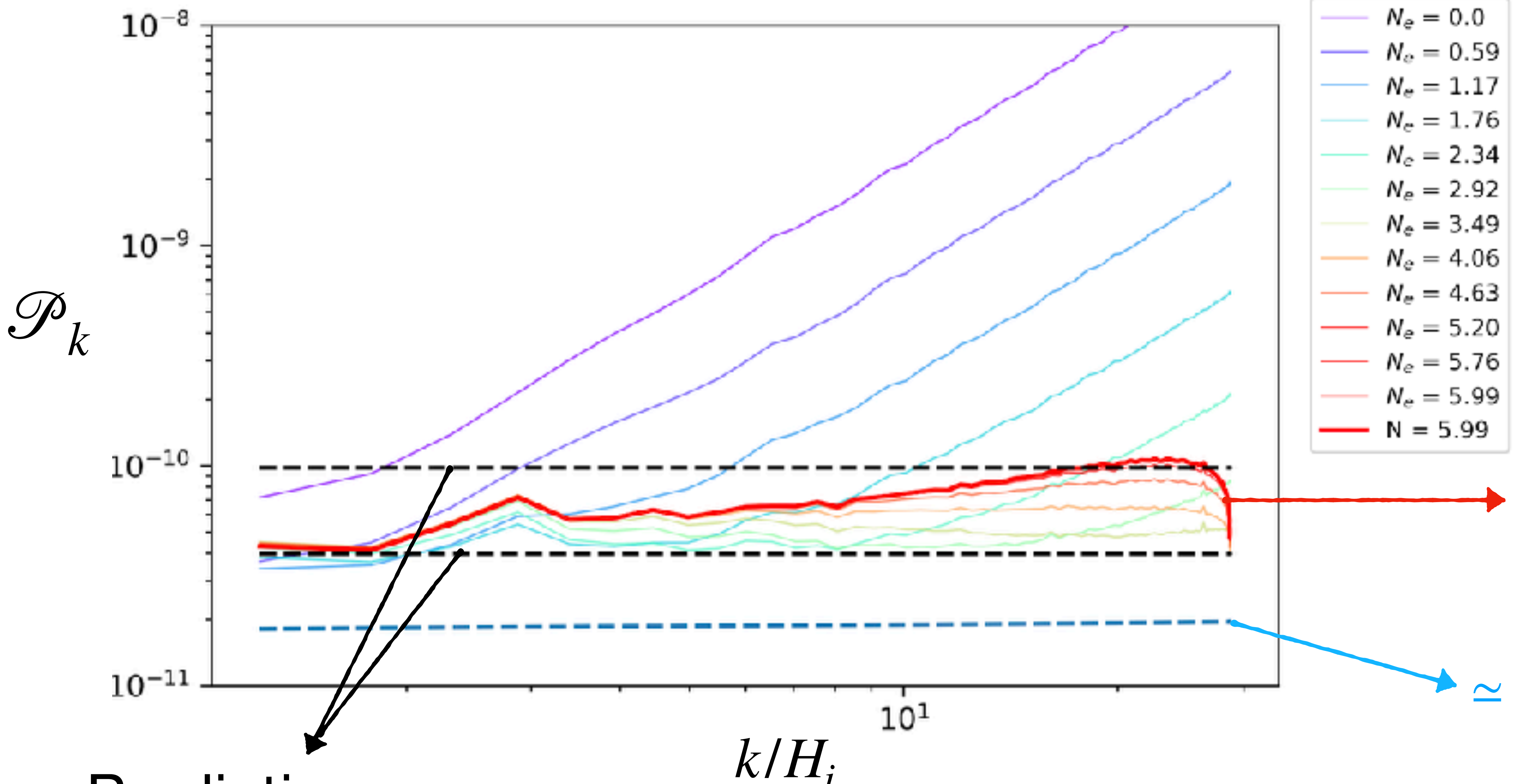
large $\xi = \dfrac{\alpha\dot{\phi}}{2fH}$

The linear case:



Power spectrum:

$$\mathscr{P}_k = \frac{k^3}{2\pi^2}\langle \delta\varphi_{\mathbf{k}}\delta\varphi_{\mathbf{k}'}\rangle \sim \frac{H^2}{4\pi^2}\left(1 + f_2(\xi)e^{4\pi\xi}\right)$$

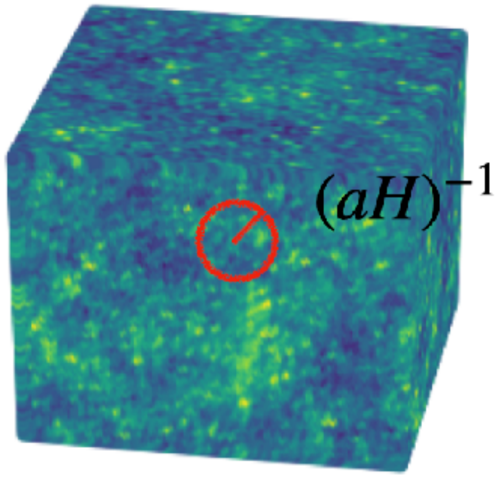


Lattice

$$\simeq \left(\frac{H}{2\pi}\right)^2$$

Prediction

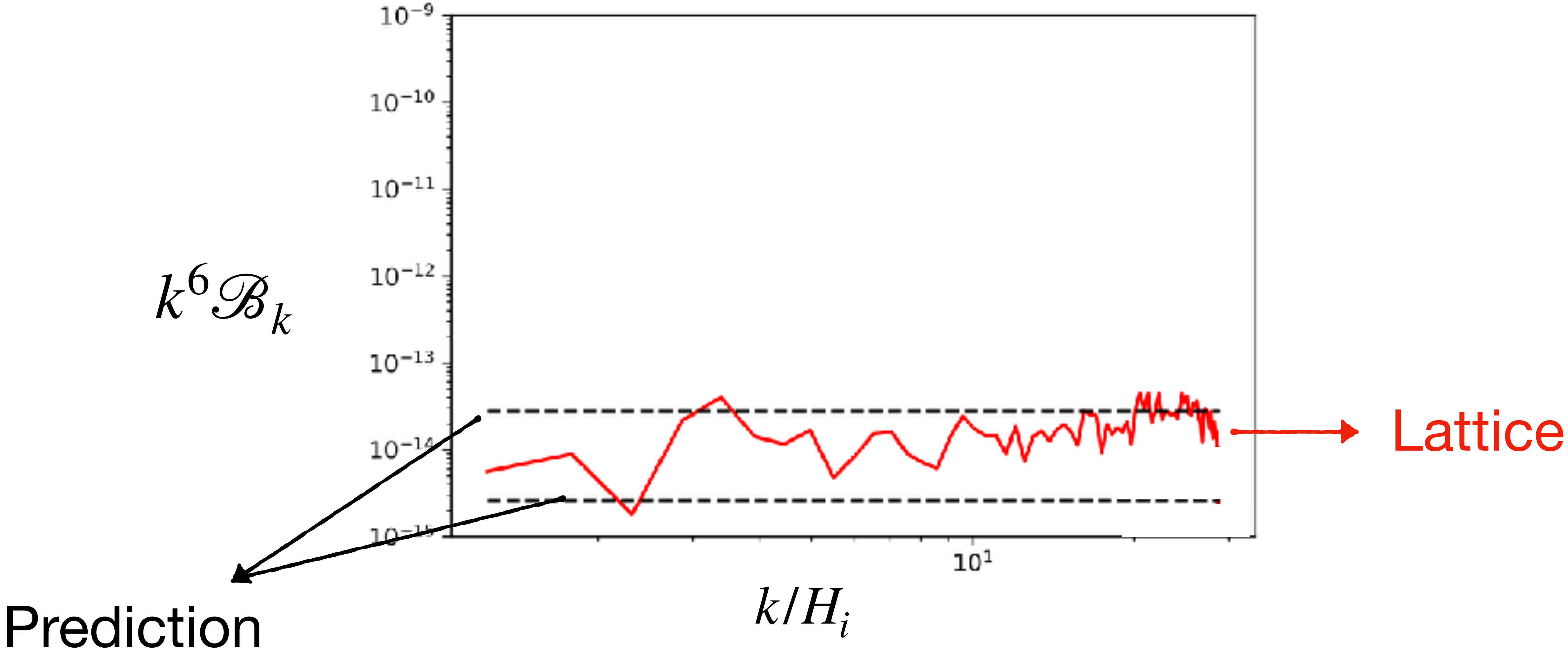
The linear case:



"Equilateral" bispectrum:

$$|\mathbf{k_1}| = |\mathbf{k_2}| = |\mathbf{k_3}| \equiv k$$

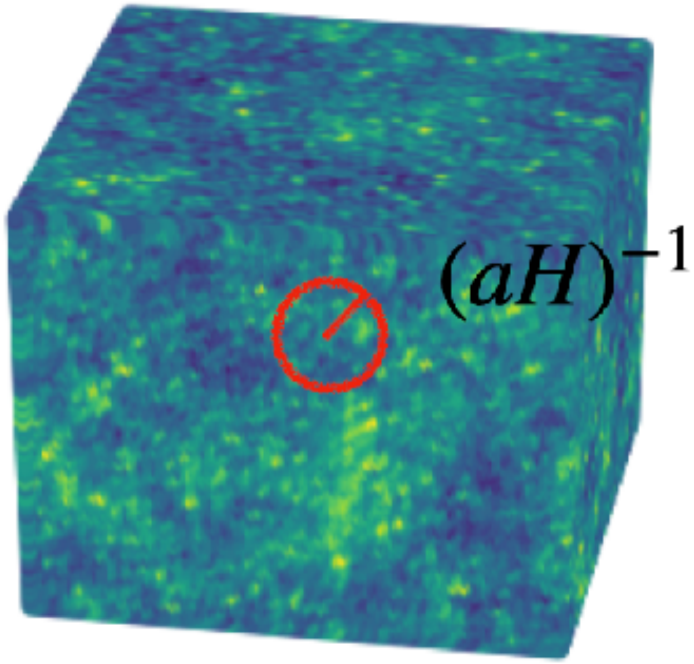
$$\mathscr{B}_k = \langle \delta\varphi_{\mathbf{k}_1} \delta\varphi_{\mathbf{k}_2} \delta\varphi_{\mathbf{k}_3} \rangle \sim \frac{9}{80} (2\pi)^{5/2} \frac{H^6}{k^6} f_3\left(\xi\right) e^{6\pi\xi}$$

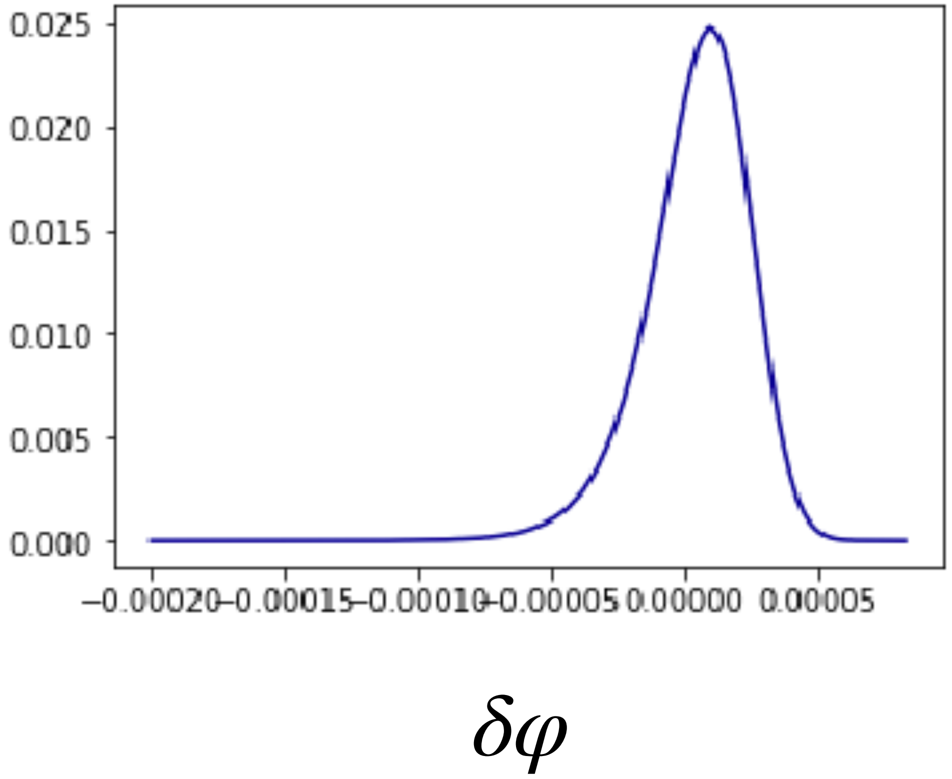


The linear case: what's new?

Thanks to the lattice,

we know the full distribution of $\delta\varphi(\mathbf{x})$ in real space!



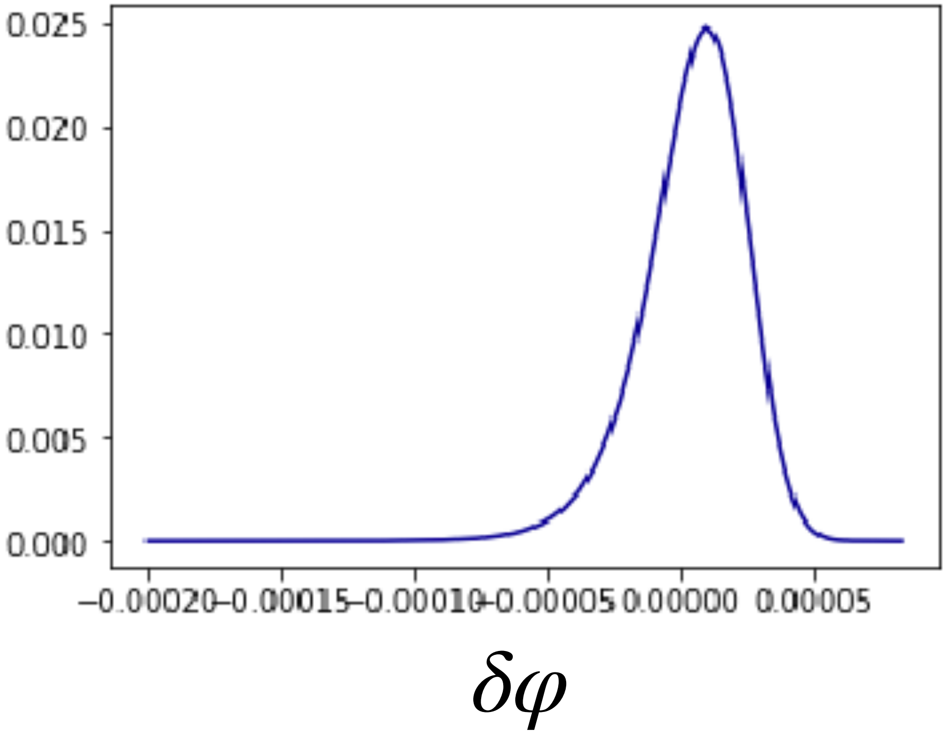


The linear case: what's new?

Define cumulants:

$$\kappa_n = \frac{\langle \delta\varphi^n \rangle_c}{\sigma^n}$$

κ_3 "skewness", κ_4 "kurtosis", etc.

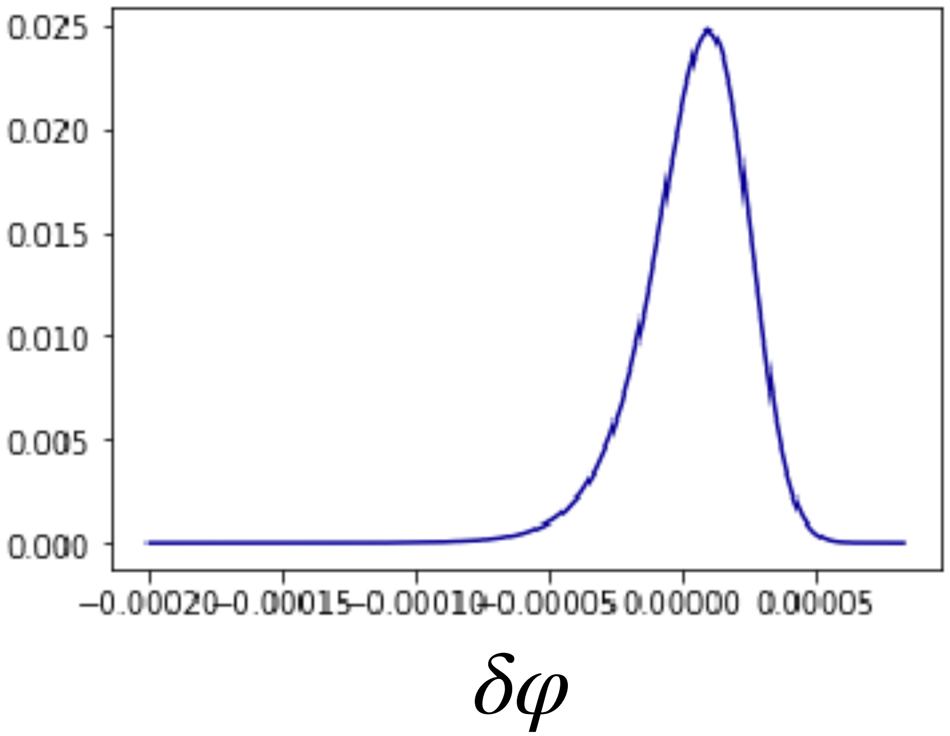


The linear case: what’s new?

Define cumulants:

$$\kappa_n = \frac{\langle \delta\varphi^n \rangle_c}{\sigma^n}$$

κ_3 “skewness”, κ_4 “kurtosis”, etc.



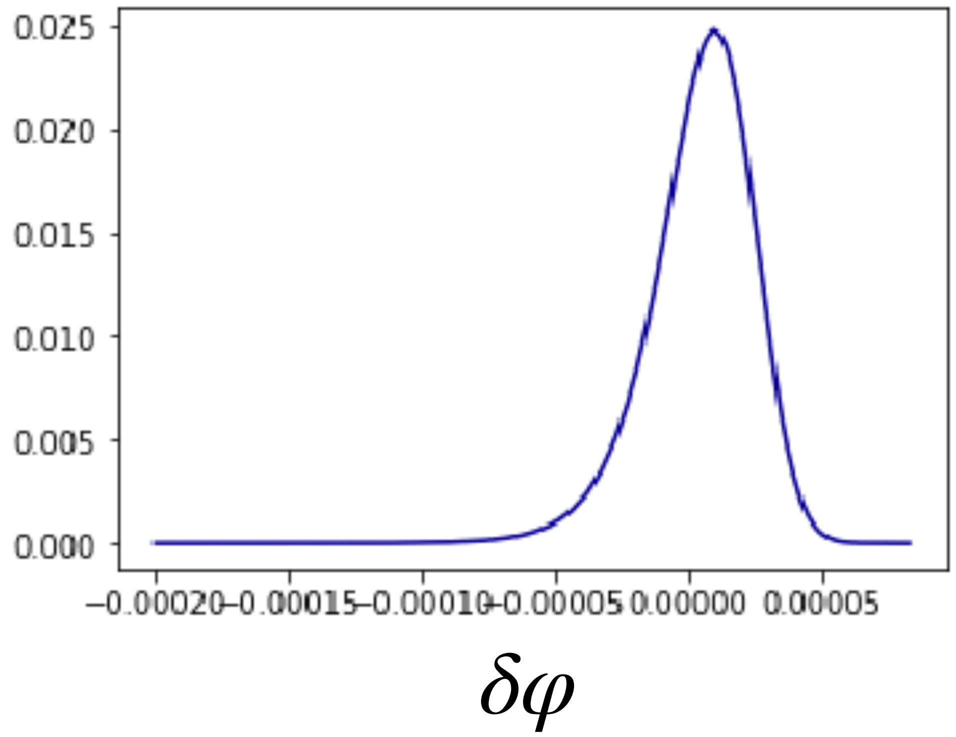
$$\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3 > 1$$

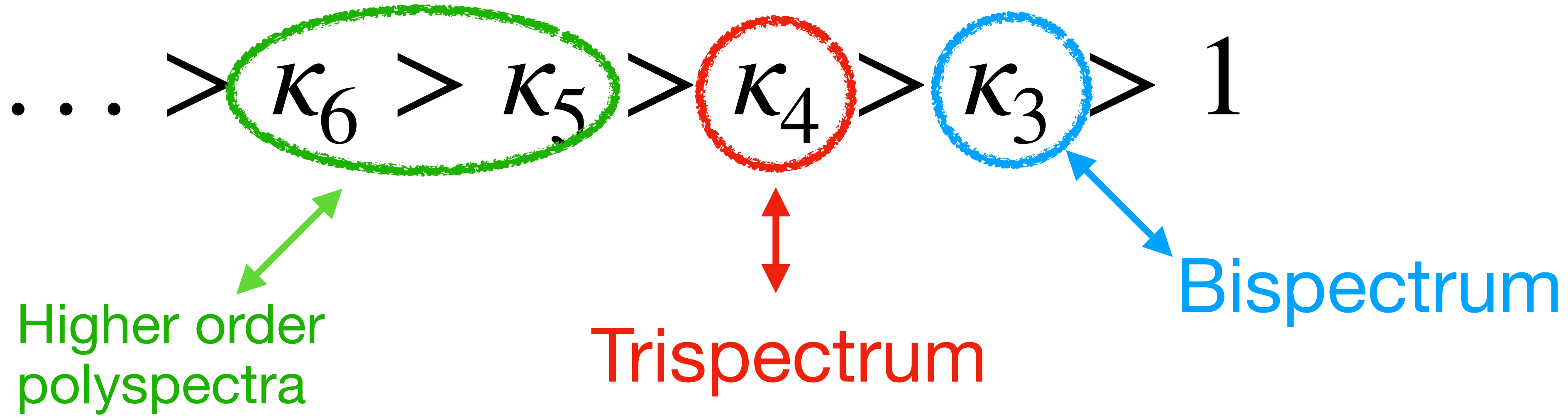
The linear case: what's new?

Define cumulants:

$$\kappa_n = \frac{\langle \delta\varphi^n \rangle_c}{\sigma^n}$$

κ_3 “skewness”, κ_4 “kurtosis”, etc.





$$\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3 > 1$$

Nonlinear case

Study transition linear ⟶ nonlinear

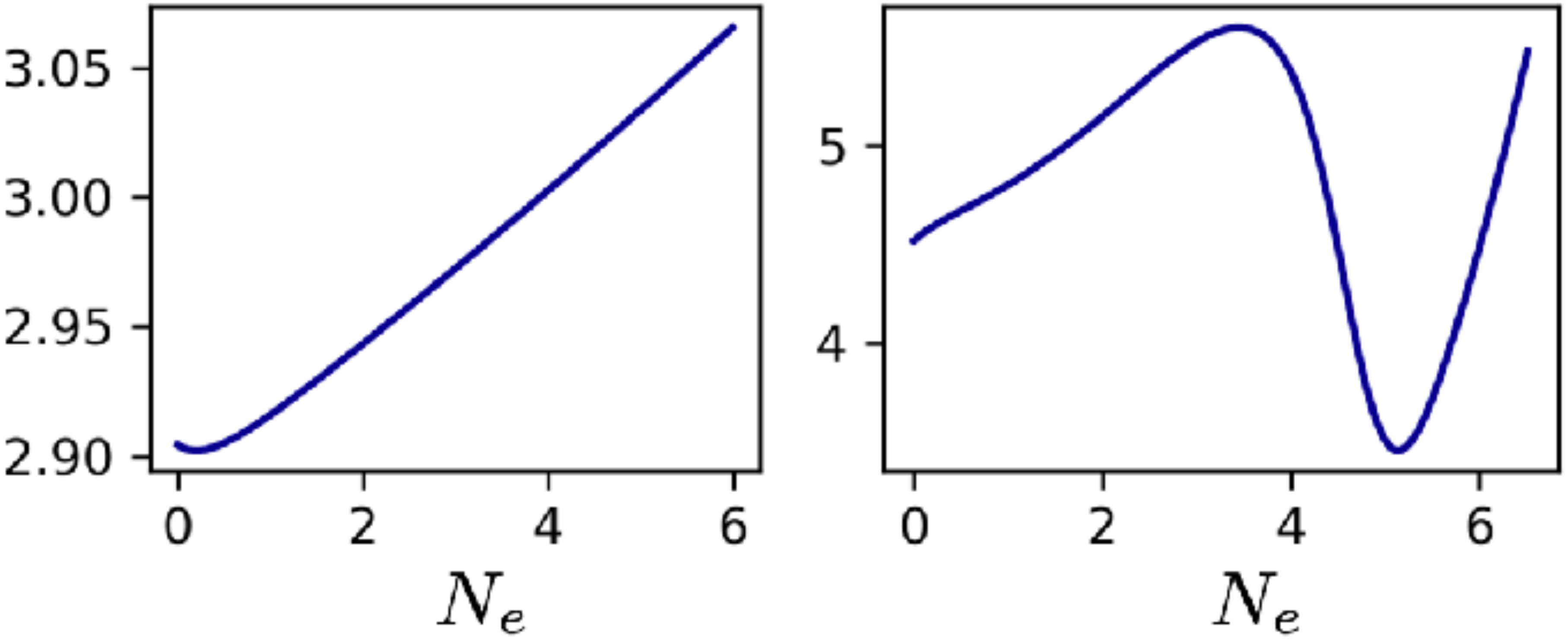
$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

Linear

(no backreaction)

Linear-nonlinear transition

(strong backreaction)

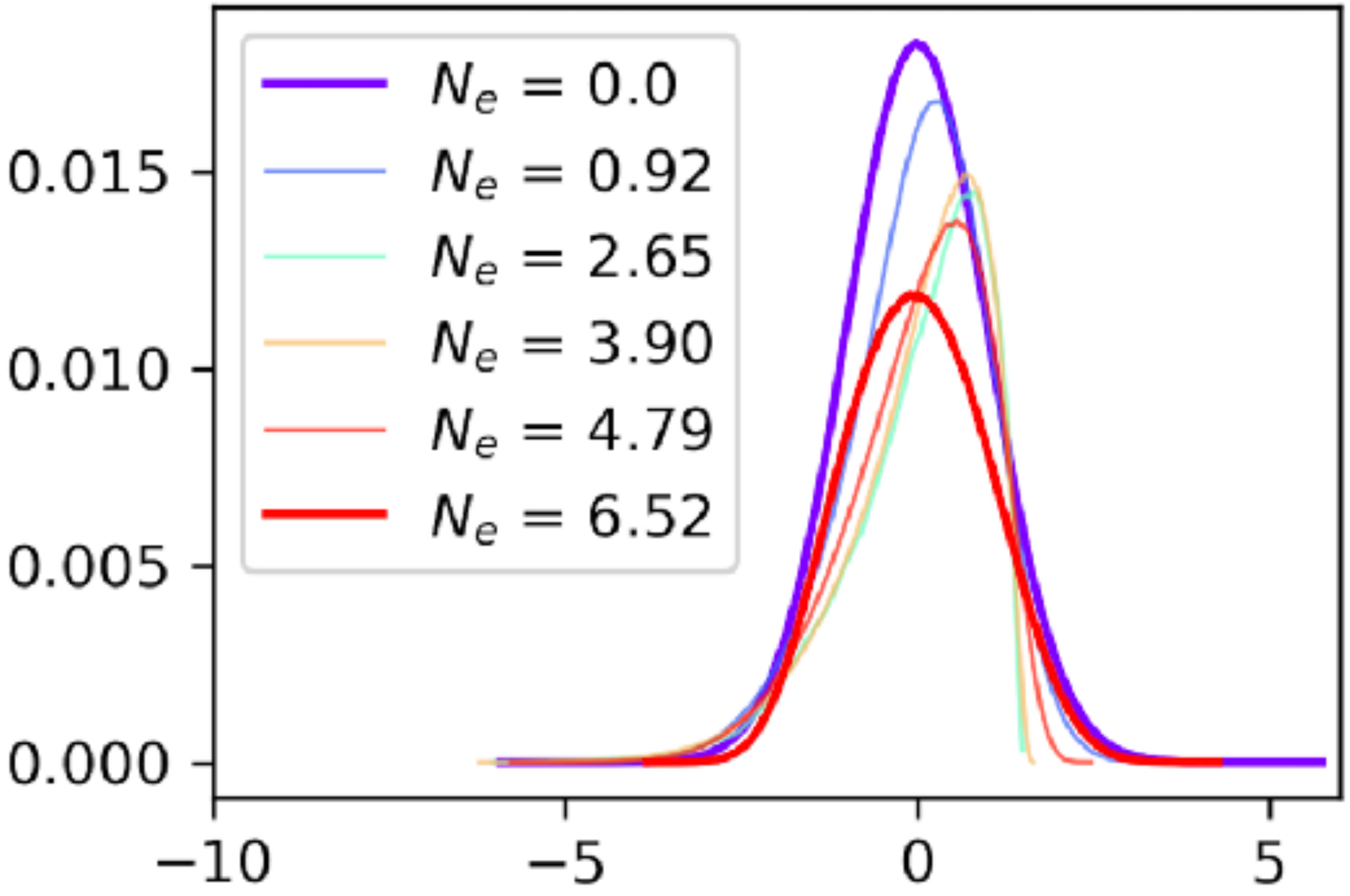


e-folds number (time)

Nonlinear case

Study transition linear ⟶ nonlinear

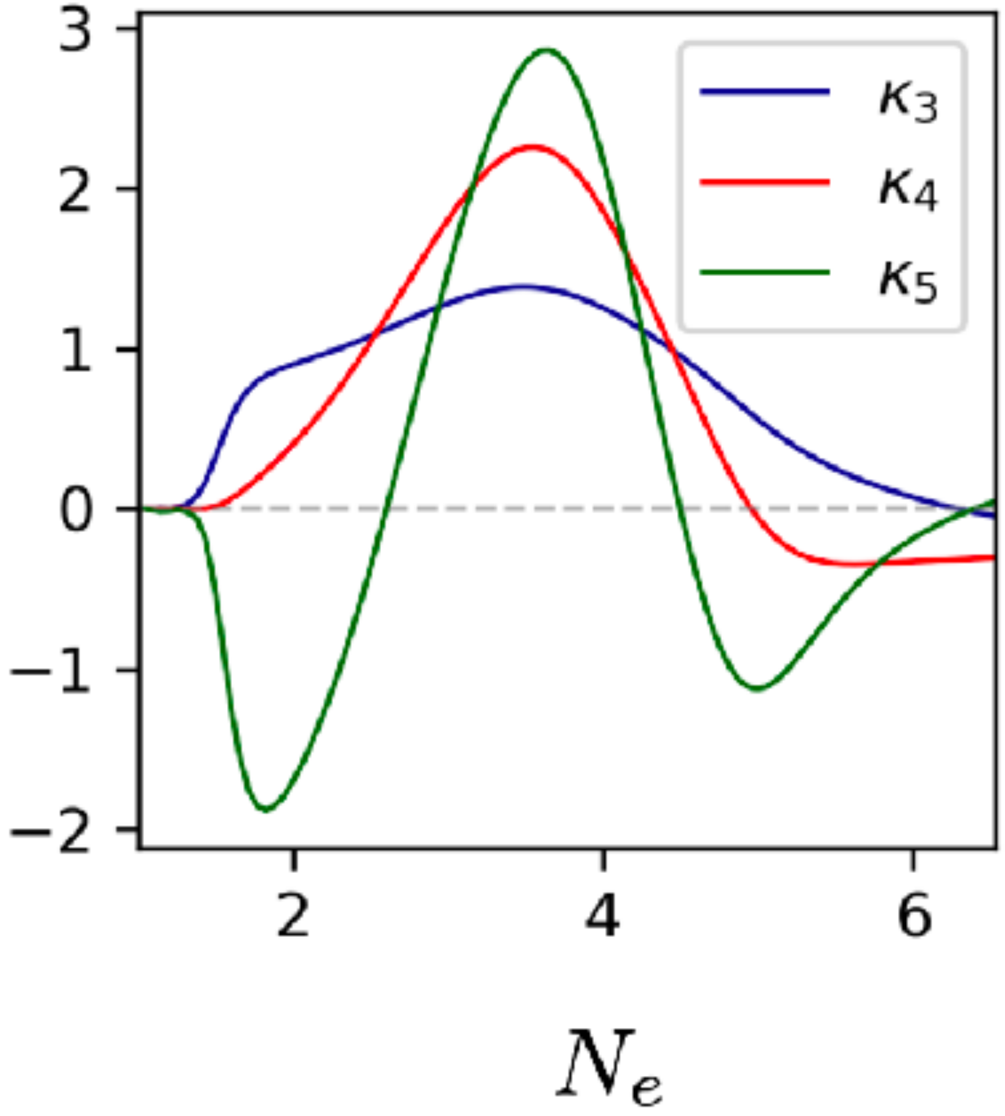
Non-Gaussianity is **suppressed** in the nonlinear regime.



Nonlinear case

Study transition linear ⟶ nonlinear

Non-Gaussianity is **suppressed** in the nonlinear regime.



Nonlinear case

A. Linde, S. Mooij, E. Pajer, arXiv:1212.1693

J. Garcia-Bellido, M. Peloso, C. Unal, arXiv:1212.1693

Before our study, it was believed that:

Large ξ $\longrightarrow$ large non-Gaussianity

Nonlinear case

A. Linde, S. Mooij, E. Pajer, arXiv:1212.1693

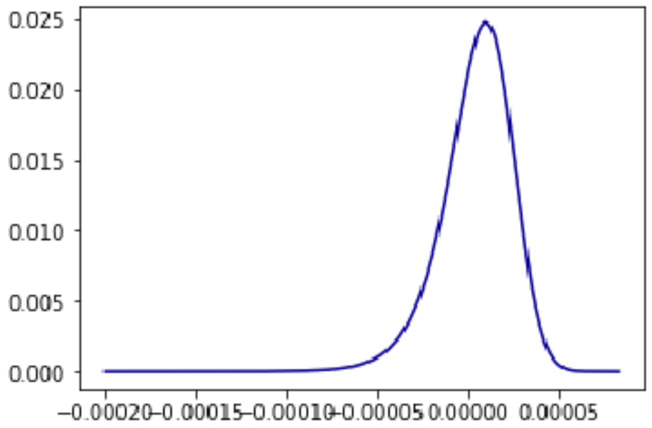
J. Garcia-Bellido, M. Peloso, C. Unal, arXiv:1212.1693

Before our study, it was believed that:

Large $\xi \longrightarrow$ large non-Gaussianity

$\downarrow$

Very efficient production of Primordial BH

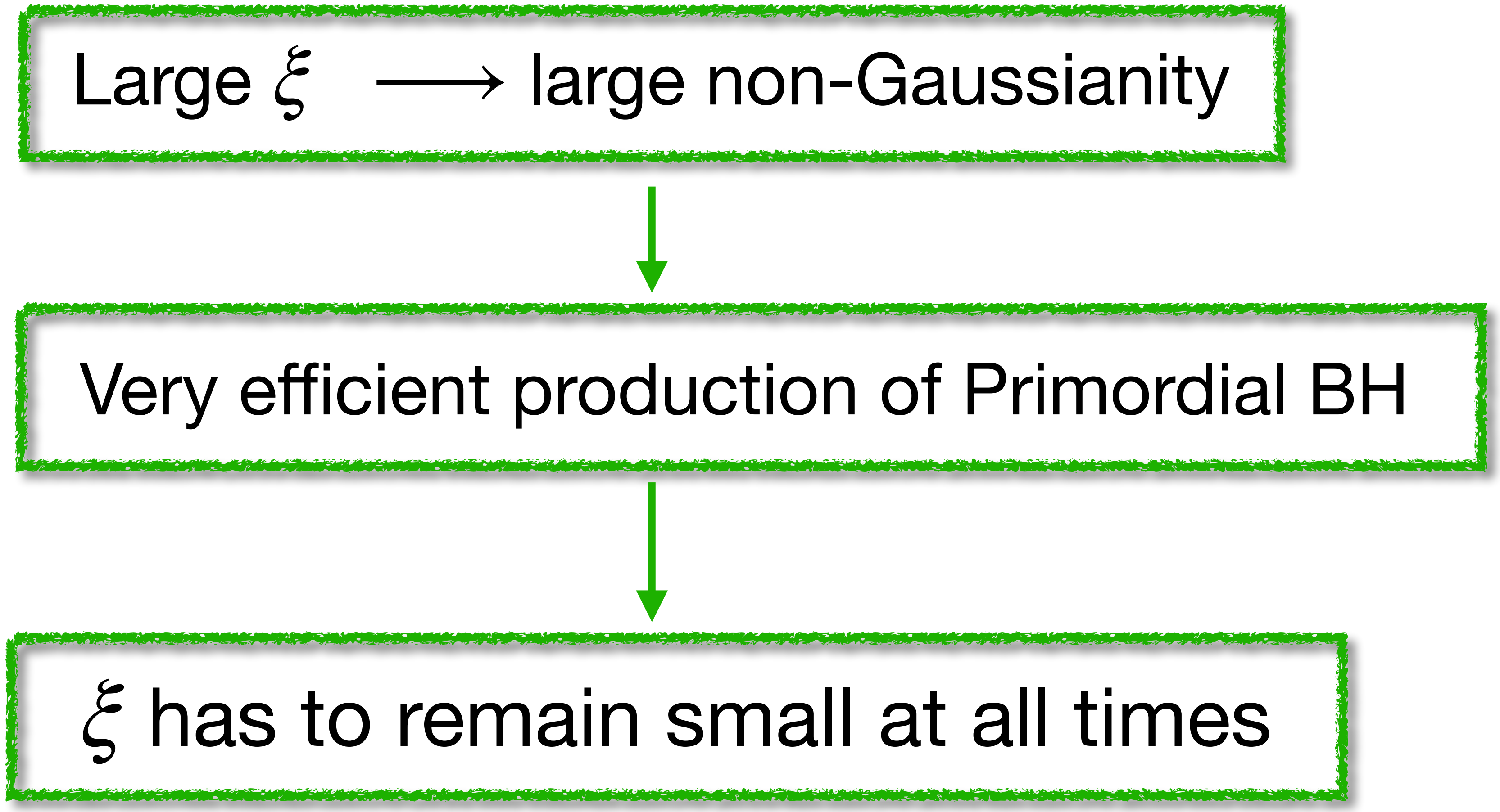


Nonlinear case

A. Linde, S. Mooij, E. Pajer, arXiv:1212.1693

J. Garcia-Bellido, M. Peloso, C. Unal, arXiv:1212.1693

Before our study, it was believed that:



Large $\xi \longrightarrow$ large non-Gaussianity

Very efficient production of Primordial BH

ξ has to remain small at all times

Nonlinear case

A. Linde, S. Mooij, E. Pajer,
arXiv:1212.1693
J. Garcia-Bellido, M. Peloso,
C. Unal, arXiv:1212.1693

Before our study, it was believed that:

Large ξ $\longrightarrow$ large non-Gaussianity

↓

Very efficient production of Primordial BH

↓

ξ has to remain small at all times

↓

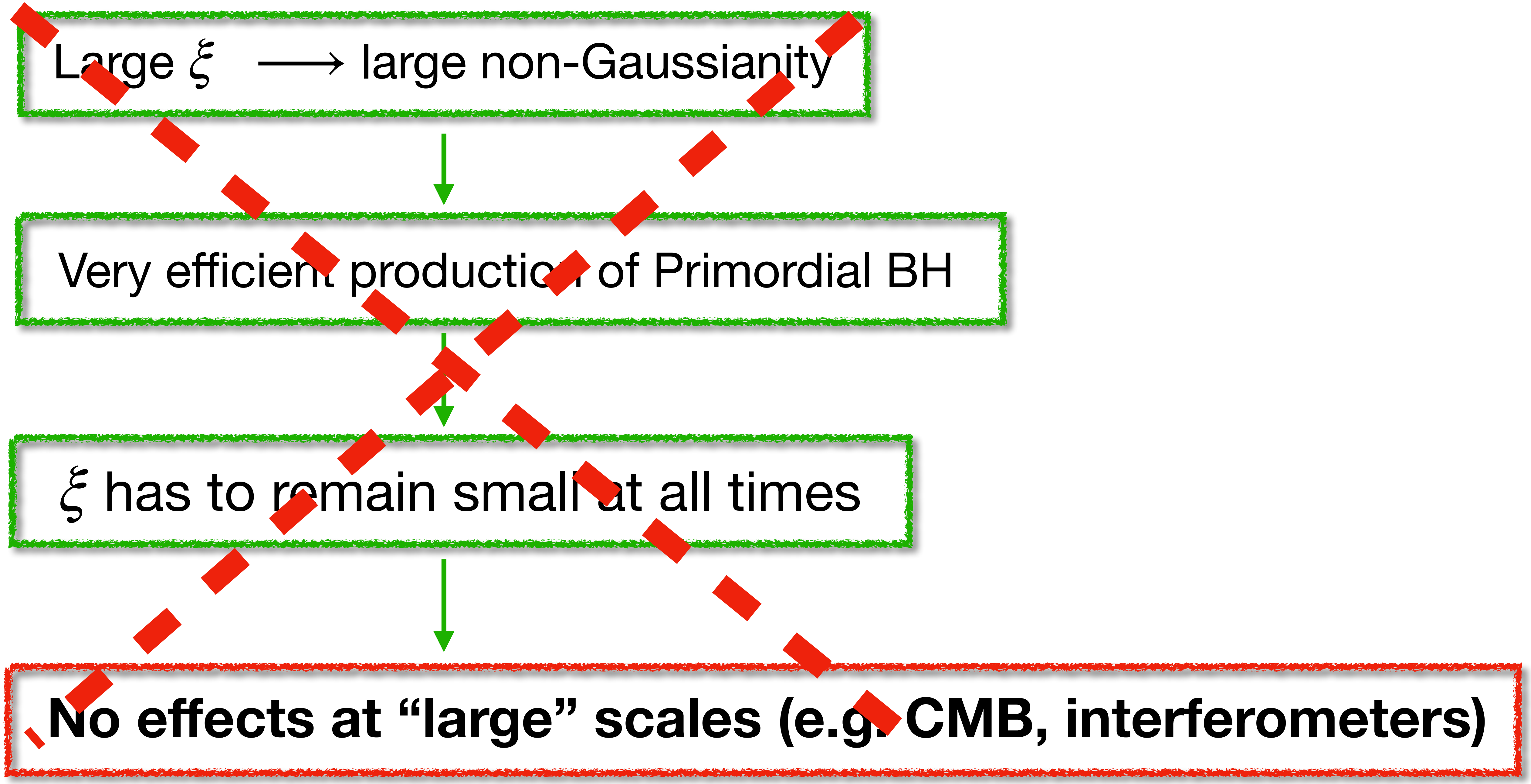
No effects at “large” scales (e.g. CMB, interferometers)

Nonlinear case

A. Linde, S. Mooij, E. Pajer, arXiv:1212.1693

J. Garcia-Bellido, M. Peloso, C. Unal, arXiv:1212.1693

Before our study, it was believed that:



Large ξ $\longrightarrow$ large non-Gaussianity

Very efficient production of Primordial BH

ξ has to remain small at all times

No effects at “large” scales (e.g. CMB, interferometers)

Conclusions:

- First simulation of axion-gauge model during inflation

Results:

- Linear regime:

  Providing a full characterisation of non-Gaussianity.

$$\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3 > 1$$

- Nonlinear regime:

  Perturbations become Gaussian.

  → Invalidate PBH bounds, allowing for interesting phenomenology at large scales.