PD Dr. Torsten Enßlin
Vishal Johnson, Ming Huang

Exercise sheet 5

Exercise 5 - 1

Assume that you are measuring a field ψ with symmetric statistics, i.e.

$$\mathcal{P}(\psi) = \mathcal{P}(-\psi)\ \forall \psi, \tag{1}$$

with a perfect instrument, i.e.

$$d = \psi. \tag{2}$$

You are interested in the power of the field, i.e.

$$s = \psi^2. \tag{3}$$

a) Calculate the signal response and the noise using the definition

$$R(s) = \langle d \rangle_{(d|s=\psi_0^2)} \tag{4}$$

of the signal response (1 point).

b) Do the same for a new data set d' that is the square of the old data set, $d' = d^2$ (1 point).

Exercise 5 - 2

A signal is observed by an instrument with Gaussian point spread function (PSF) in u-dimensional space $\mathbb{R}^u$, so that $d = R\,s + n$ with $R_{xy} = \mathcal{G}(x-y, B)$. Assume $B_{ij} = l_i^2\,\delta_{ij}$ and find the Fourier transformed PSF (2 points).
Hint: Express R in terms of $r := x - y$.

Exercise 5 - 3

Consider a random field s that is statistically homogeneous and isotropic,

$$\langle s(x)s(y)\rangle_{\mathcal{P}(s)} = C(|x-y|), \quad \text{with } x \in \mathbb{R}. \tag{5}$$

a) Show that the Fourier transformed autocorrelation function has the form

$$\langle s(k)s(q)^*\rangle_{\mathcal{P}(s)} = (2\pi)\delta(k-q)\,f(|k|) \tag{6}$$

(note that f depends only on the absolute of k). (2 points)

b) Show that if s follows Gaussian statistics all Fourier components are independent, i.e. show that

$$\mathcal{P}(s(k)|s(q)) = \mathcal{P}(s(k)) \quad \text{for} \quad k \neq q. \tag{7}$$

(3 points)

Hint: You are allowed to drop normalization factors and to be sloppy in case you discretize integrals.

This exercise sheet will be discussed during the exercises.
Group 01, Wednesday 18:00 - 20:00, Theresienstr. 37, A 449,
Group 02, Thursday, 10:00 - 12:00, Theresienstr. 37, A 249,

https://wwwmpa.mpa-garching.mpg.de/ ensslin/lectures/lectures.html