COIN

Cosmostatistics Initiative

http://cointoolbox.github.io/

Max-Planck-Institut
für Astrophysik

CAPES

Approximate Bayesian Computation in Astronomy

Emille E. O. Ishida
MPA – Germany

Bayes Forum, June 2015

arXiv:1504.06129v2

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Important remark

Parameter inference

Parameter inference

1. Physical phenomenon

Parameter inference

1. Physical phenomenon

2. Physical reasoning

Parameter inference

1. Physical phenomenon

2. Physical reasoning

3. Model

$\{z_i,\xi_i\}$ for i=1,2...,N $\leftarrow \mathscr{M}(\Omega_m$, w, …)

Parameter inference

1. Physical phenomenon

2. Physical reasoning

3. Model

$\{z_i,\xi_i\}$ for $i=1,2...,N \leftarrow \mathscr{M}(\Omega_m, w, \dots)$

4. Collect data

Parameter inference

1. Physical phenomenon

2. Physical reasoning

3. Model

$\{z_i,\xi_i\}$ for $i=1,2...,N \leftarrow \mathscr{M}(\Omega_m, w, \ldots)$

4. Collect data

5. Catalog

z	ξ
2.5	0.9
5.6	2.1
7.4	2.9
10.5	4.2
..	...

Parameter inference

1. Physical phenomenon

2. Physical reasoning

3. Model

$\{z_i,\xi_i\}$ for i=1,2...,N ← $\mathscr{M}(\Omega_m, w, \ldots)$

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5.6	2.1
7.4	2.9
10.5	4.2
..	...

6. Find parameters

$\Omega_m = \ldots$

$w = \ldots$

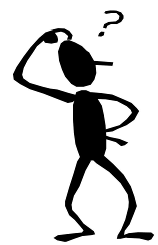
Parameter inference

1. Physical phenomenon

2. Physical reasoning

3. Model

$\{z_i, x_i\}$ for i=1,2...,N ← $\mathscr{M}(\Omega_m, w, \ldots)$



4. Collect

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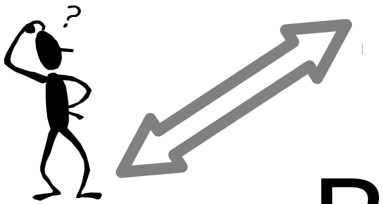
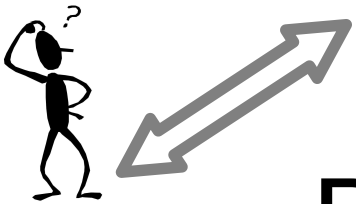
$w = \ldots$



Bayes theorem

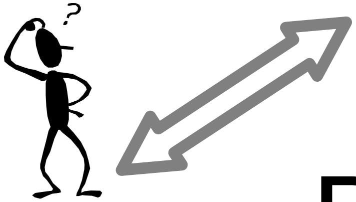
$$p(\boldsymbol{\theta}|\mathcal{D}) = \frac{p(\mathcal{D}|\boldsymbol{\theta})\,p(\boldsymbol{\theta})}{p(\mathcal{D})}$$



Bayes theorem

Can be too complicated!

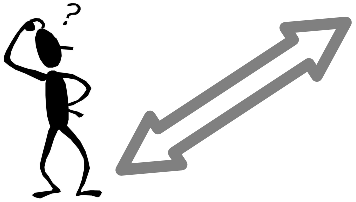
$$p(\boldsymbol{\theta}|\mathcal{D}) = \frac{p(\mathcal{D}|\boldsymbol{\theta})\,p(\boldsymbol{\theta})}{p(\mathcal{D})}$$



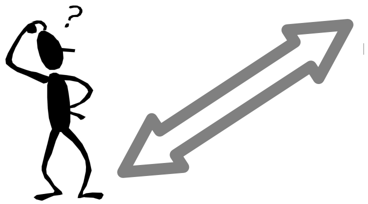
Bayes theorem

$$p(\boldsymbol{\theta}|\mathcal{D}) = \frac{p(\mathcal{D}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathcal{D})}$$

Can be problematic!



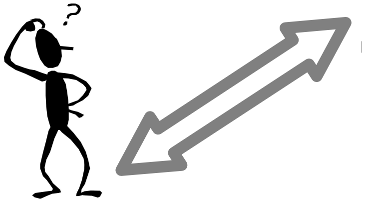
Can simulations help?



Comparing catalogs (or basic ABC)

Observed

z	ξ
2.5	0.9
5.6	2.1
7.4	2.9
10.5	4.2
..	...



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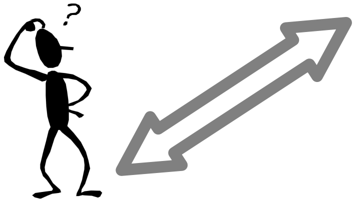

Comparing catalogs (or basic ABC)

Observed

z	ξ
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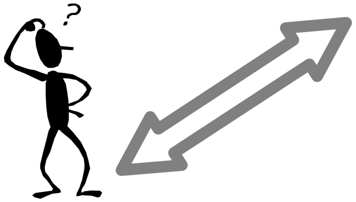
Draw $\{\Omega_{m1}, w_1\}$ from priors



Comparing catalogs (or basic ABC)

Model + $\{\Omega_{m1}, w_1\}$

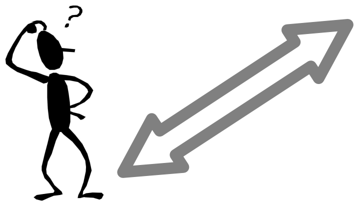
z	ξ
2.0	1.8
11.5	10.5
13.1	12.9
25.4	26.7
..	...

Observed

z	ξ
2.5	0.9
5.6	2.1
7.4	2.9
10.5	4.2
..	...

Simulate

Draw $\{\Omega_{m1}, w_1\}$ from priors



Comparing catalogs (or basic ABC)

Model + {Ω_{m1}, w_1}

z	ξ
2.0	1.8
11.5	10.5
13.1	12.9
25.4	26.7
..	...

Observed

z	ξ
2.5	0.9
5.6	2.1
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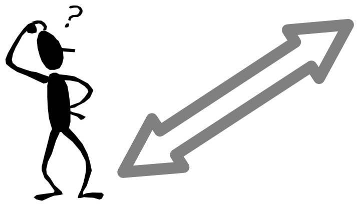
Simulate

Draw {Ω_{m1}, w_1} from priors

Compare



Comparing catalogs (or basic ABC)

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Simulate

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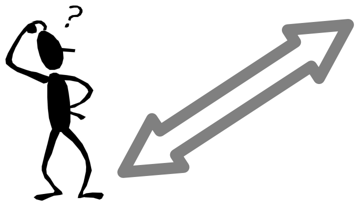

Comparing catalogs (or basic ABC)

Simulate

Draw $\{\Omega_{m1}, w_1\}$
from priors

Model + $\{\Omega_{m1}, w_1\}$

z	ξ
2.0	1.8
11.5	10.5
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..	...

Observed

z	ξ
2.5	0.9
5.6	2.1
7.4	2.9
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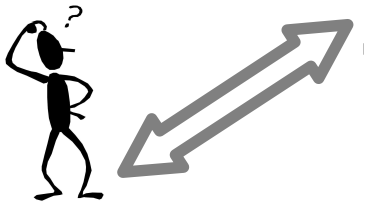
Compare

If too different

Throw away



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Model + $\{\Omega_{m1}, w_1\}$

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Observed

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2.5	0.9
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..	...

Simulate

Draw $\{\Omega_{m1}, w_1\}$ from priors

Compare

If too different

If close enough

Throw away

Store $\{\Omega_{m1}, w_1\}$

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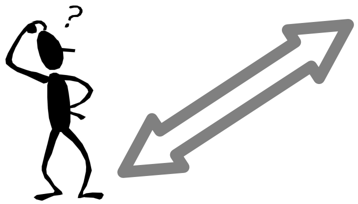

Comparing catalogs (or basic ABC)

Model + {Ω_{m1}, w_1}

z	ξ
2.0	1.8
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..	...

Observed

z	ξ
2.5	0.9
5.6	2.1
7.4	2.9
10.5	4.2
..	...

Simulate

Draw {Ω_{m1}, w_1} from priors

Compare

If too different

Throw away

If close enough

Store {Ω_{m1}, w_1}

Start again

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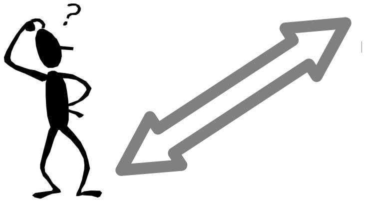
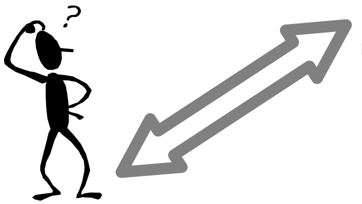
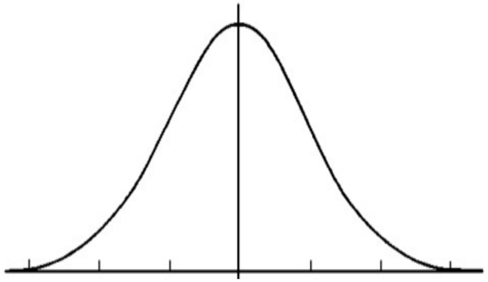


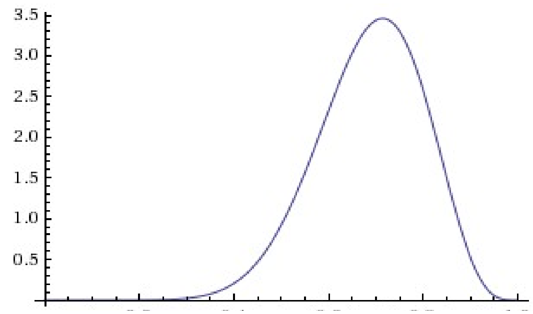
Posterior





0.27

Ω_m



-1.0

w

Posterior

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Approximate Bayesian Computation (ABC)

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Ingredients

1. Priors

2. Simulator (cheap)

3. Distance function
   (or summary statistics)

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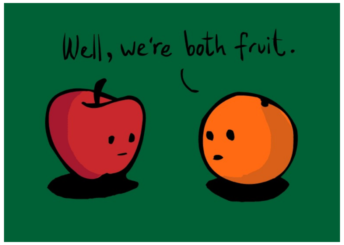

Approximate Bayesian Computation (ABC)

Ingredients

1. Priors

2. Simulator (cheap)

3. Distance function
   (or summary statistics)

Crucial and case dependent.
But before that...

Further developments in ABC

Further developments in ABC

Particle System

(parameter values
+ distances)

Further developments in ABC

Particle System

(parameter values + distances)

Simulate

Model + $\{\Omega_{m1}, w_1\}$

z	ξ
2.0	1.8
11.5	10.5
13.1	12.9
25.4	26.7
..	...

Draw $\{\Omega_{m1}, w_1\}$ from priors

Calculate distance

Store $\{\Omega_{m1}, w_1, d_1\}$

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Further developments in ABC

Particle System
(parameter values
+ distances)

Take the N
parameter vectors
with smallest
distances

Determine
distance
threshold, ε

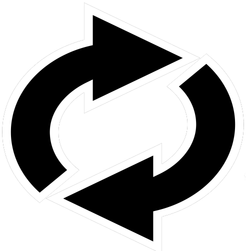
Further developments in ABC

Particle System
(parameter values
+ distances)

Take the N parameter vectors with smallest distances

Determine distance threshold, ε

Repeat basic ABC algorithm, accept if d < ε



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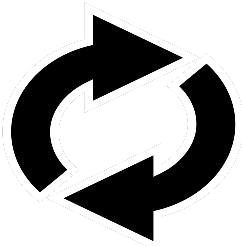

Use the new
particle system as
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subsequent
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Further developments in ABC

Particle System

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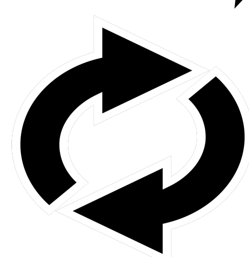

Use the new particle system as a guide to subsequent drawings

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Further developments in ABC

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(parameter values + distances)

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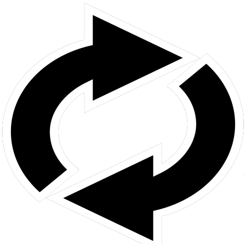

Use the new particle system as a guide to subsequent drawings

Further developments in ABC

Importance sampling: guiding draws

Use the new particle system as a guide to subsequent drawings

Further developments in ABC

Importance sampling: guiding draws

Determine ε

Further developments in ABC

Importance sampling: guiding draws

Associate a weight
with each particle

$$W_t^j = \frac{p(\boldsymbol{\theta}_t^j)}{\sum_{i=1}^N W_{t-1}^i \mathcal{N}(\boldsymbol{\theta}_t^j; \boldsymbol{\theta}_{t-1}^i, C_{t-1})},$$

Determine ε

Further developments in ABC

Importance sampling: guiding draws

Draw $\{\Omega_{m1}, w_1\}$ from previous weighted particle system

Associate a weight with each particle

$$W_t^j = \frac{p(\boldsymbol{\theta}_t^j)}{\sum_{i=1}^{N} W_{t-1}^i \mathcal{N}(\boldsymbol{\theta}_t^j; \boldsymbol{\theta}_{t-1}^i, C_{t-1})},$$

Determine ε

Further developments in ABC

Importance sampling: guiding draws

Draw {a_1, b_1} from previous weighted particle system

Associate a weight with each particle

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Determine ε

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Further developments in ABC

Importance sampling: guiding draws

Draw $\{a_1, b_1\}$ from previous weighted particle system

if d < ε, store

$\{\Omega_{m1}, w_1\}$

Associate a weight with each particle

$$W_t^j = \frac{p(\boldsymbol{\theta}_t^j)}{\sum_{i=1}^N W_{t-1}^i \mathcal{N}(\boldsymbol{\theta}_t^j; \boldsymbol{\theta}_{t-1}^i, C_{t-1})},$$

Determine ε

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Further developments in ABC

Importance sampling: guiding draws

Draw $\{a_1, b_1\}$ from previous weighted particle system

Repeat until there are N particles stored

if $d < \varepsilon$, store

$\{\Omega_{m1}, w_1\}$

Associate a weight with each particle

$$W_t^j = \frac{p(\boldsymbol{\theta}_t^j)}{\sum_{i=1}^{N} W_{t-1}^i \mathcal{N}(\boldsymbol{\theta}_t^j; \boldsymbol{\theta}_{t-1}^i, C_{t-1})},$$

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Draw $\{a_1, b_1\}$ from previous weighted particle system

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if $d < \varepsilon$, store $\{\Omega_{m1}, w_1\}$

Associate a weight with each particle

$$W_t^j = \frac{p(\boldsymbol{\theta}_t^j)}{\sum_{i=1}^N W_{t-1}^i \mathcal{N}(\boldsymbol{\theta}_t^j; \boldsymbol{\theta}_{t-1}^i, C_{t-1})},$$

Determine ε

Example

Fiducial model: $Y \sim \mathit{Gaussian}(\mu, \sigma)$

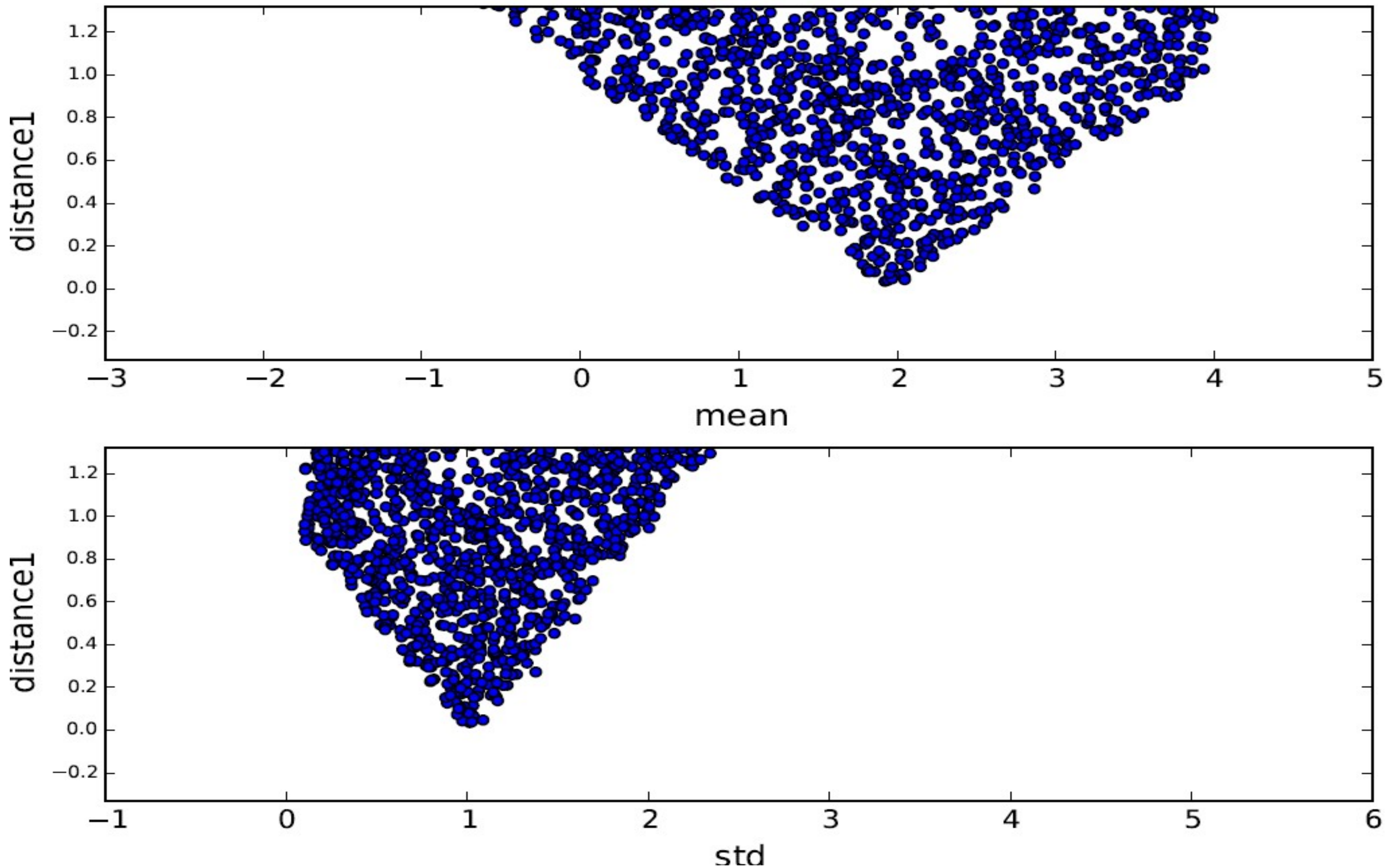
Priors: $-2.0 < \mu < 4.0$ and $0.1 < \sigma < 5.0$

Distance:

$$\rho = \mathrm{abs}\left(\frac{\bar{\mathcal{D}} - \bar{\mathcal{D}}_S}{\bar{\mathcal{D}}}\right) + \mathrm{abs}\left(\frac{\sigma_{\mathcal{D}} - \sigma_{\mathcal{D}_S}}{\sigma_{\mathcal{D}}}\right).$$

Y
11.2
15.3
200.0
0.5
14.3

Example: distance preview



Example: particle system evolution

http://cosmoabc.readthedocs.org/en/latest/

in Astronomy

2012

Cameron & Petit

Mon. Not. R. Astron. Soc. **425,** 44–65 (2012)

Approximate Bayesian Computation for astronomical model analysis: a case study in galaxy demographics and morphological transformation at high redshift

E. Cameron* and A. N. Pettitt

School of Mathematical Sciences (Statistical Science), Queensland University of Technology (QUT), GPO Box 2434, Brisbane 4001, QLD, Australia

2012 2013

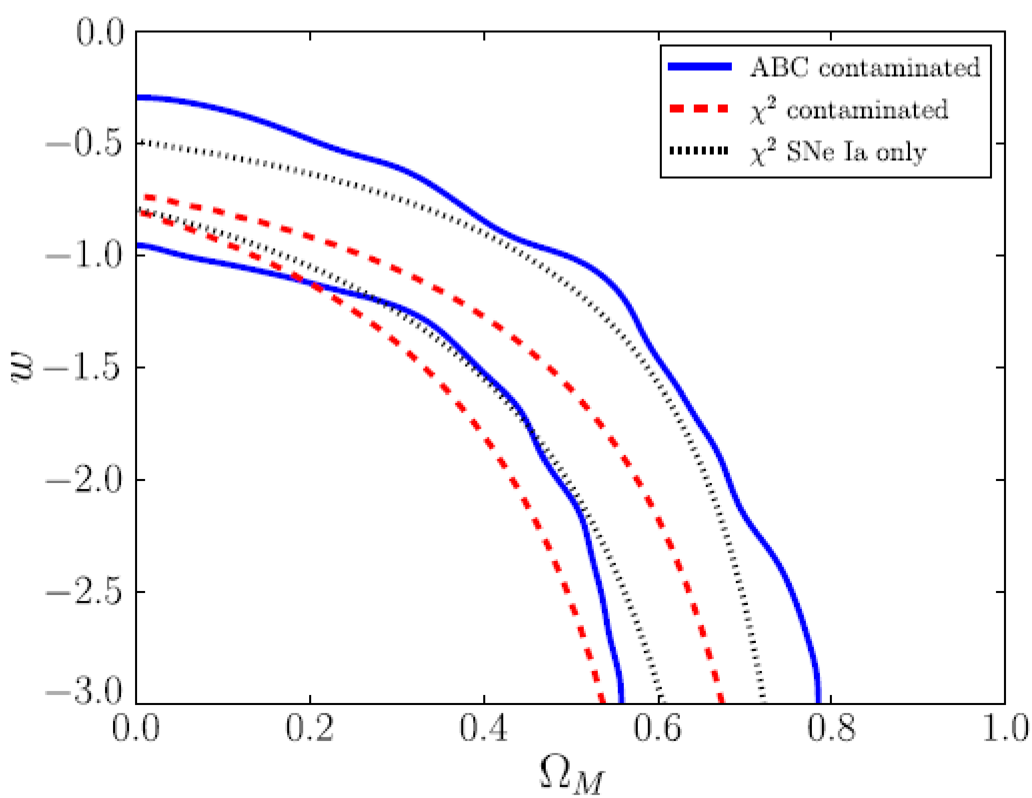
Cameron & Pettitt

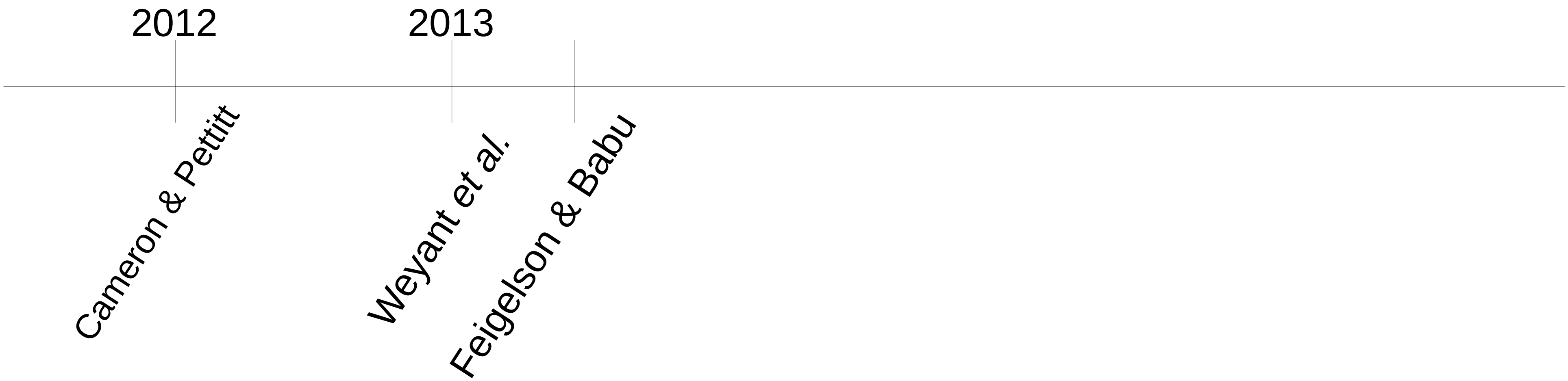
Weyant et al.

The Astrophysical Journal, 764:116 (15pp), 2013 February 20

LIKELIHOOD-FREE COSMOLOGICAL INFERENCE WITH TYPE Ia SUPERNOVAE: APPROXIMATE BAYESIAN COMPUTATION FOR A COMPLETE TREATMENT OF UNCERTAINTY

Anja Weyant[1], Chad Schafer[2], and W. Michael Wood-Vasey[1]





E.D. Feigelson and G.J. Babu (eds.), *Statistical Challenges in Modern Astronomy V*,
Lecture Notes in Statistics 209, DOI 10.1007/978-1-4614-3520-4_1,
© Springer Science+Business Media New York 2013

Chapter 1
Likelihood-Free Inference in Cosmology: Potential for the Estimation of Luminosity Functions

Chad M. Schafer and Peter E. Freeman

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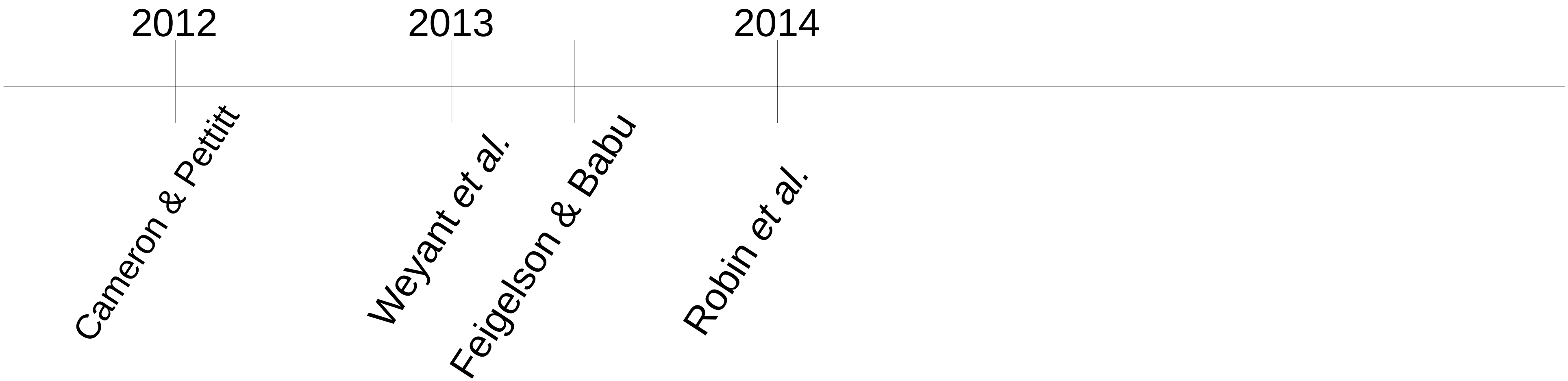

A&A 569, A13 (2014)

Constraining the thick disc formation scenario of the Milky Way★

A. C. Robin[1], C. Reylé[1], J. Fliri[2,3], M. Czekaj[4], C. P. Robert[5], and A. M. M. Martins[1]

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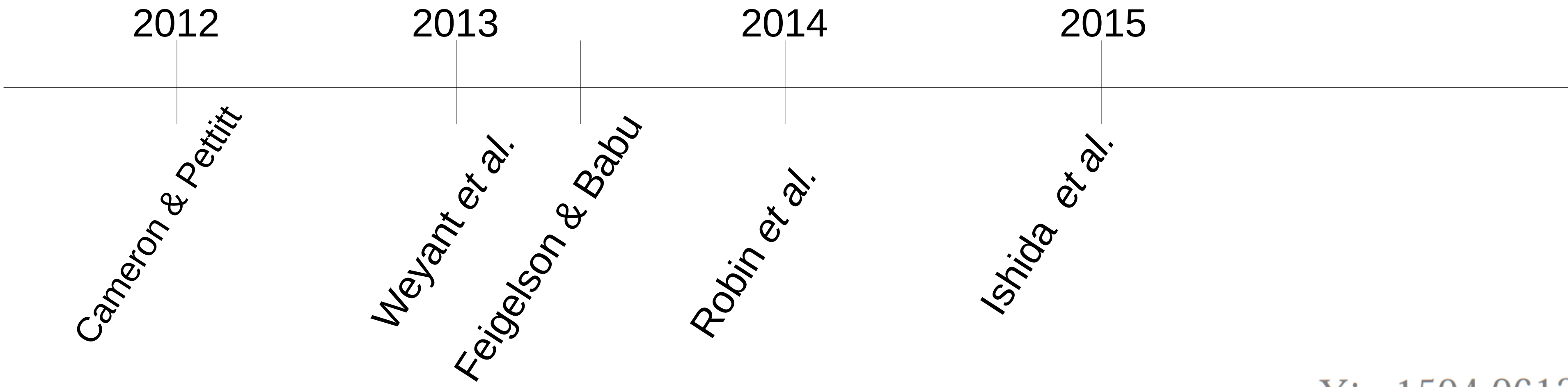


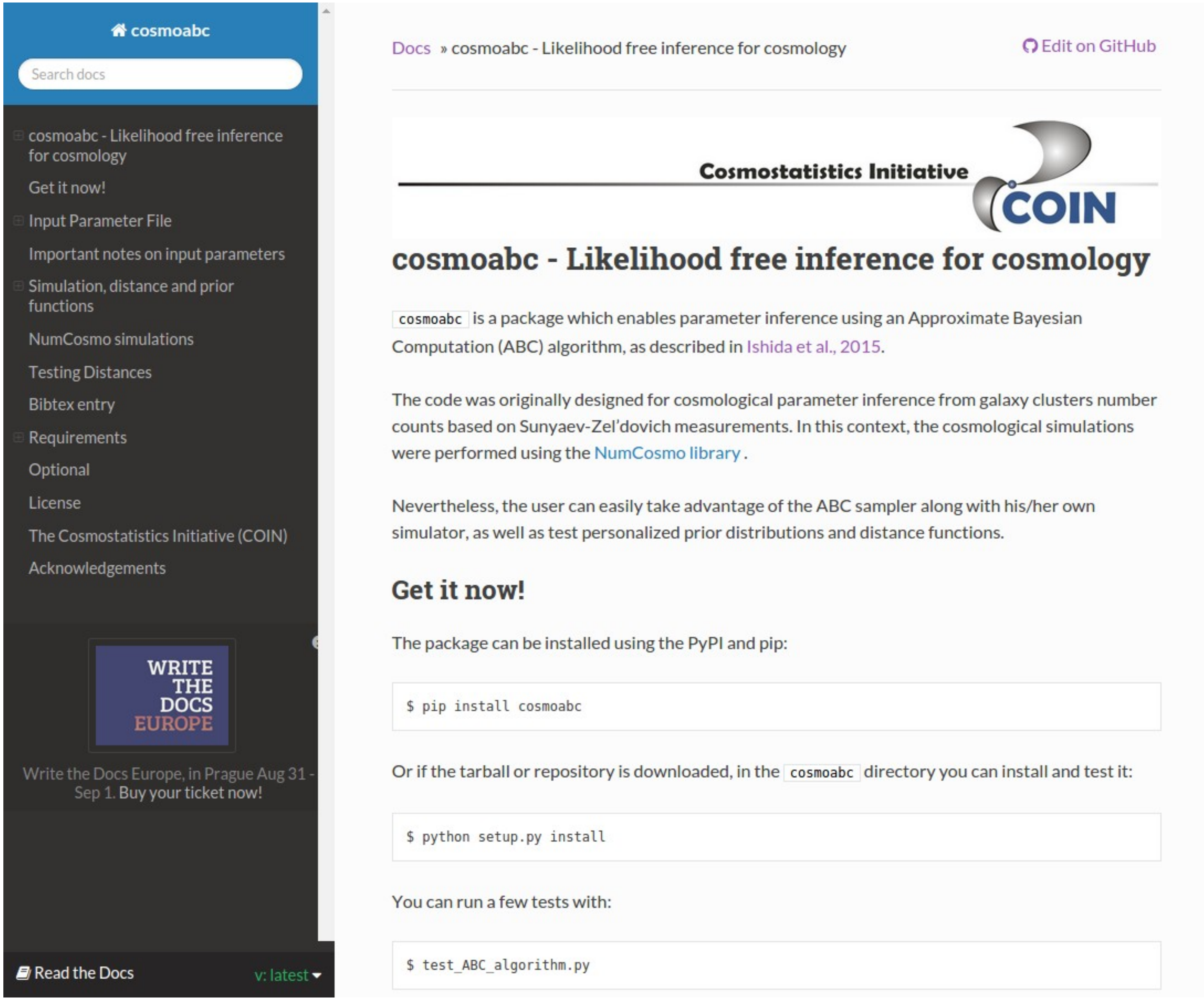
arXiv:1504.06129v2

cosmoabc: Likelihood-free inference via Population Monte Carlo Approximate Bayesian Computation

E. E. O. Ishida[1], S. D. P. Vitenti[2], M. Penna-Lima[3,4], J. Cisewski[5], R. S. de Souza[6], A. M. M. Trindade[7,8]
E. Cameron[9] and V. C. Busti[10], for the COIN collaboration

cosmoabc

arXiv:1504.06129v2



cosmoabc

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Docs » cosmoabc - Likelihood free inference for cosmology Edit on GitHub

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cosmoabc - Likelihood free inference for cosmology

`cosmoabc` is a package which enables parameter inference using an Approximate Bayesian Computation (ABC) algorithm, as described in Ishida et al., 2015.

The code was originally designed for cosmological parameter inference from galaxy clusters number counts based on Sunyaev-Zel'dovich measurements. In this context, the cosmological simulations were performed using the NumCosmo library .

Nevertheless, the user can easily take advantage of the ABC sampler along with his/her own simulator, as well as test personalized prior distributions and distance functions.

Get it now!

The package can be installed using the PyPI and pip:

```
$ pip install cosmoabc
```

Or if the tarball or repository is downloaded, in the `cosmoabc` directory you can install and test it:

```
$ python setup.py install
```

You can run a few tests with:

```
$ test_ABC_algorithm.py
```

http://cosmoabc.readthedocs.org/en/latest/

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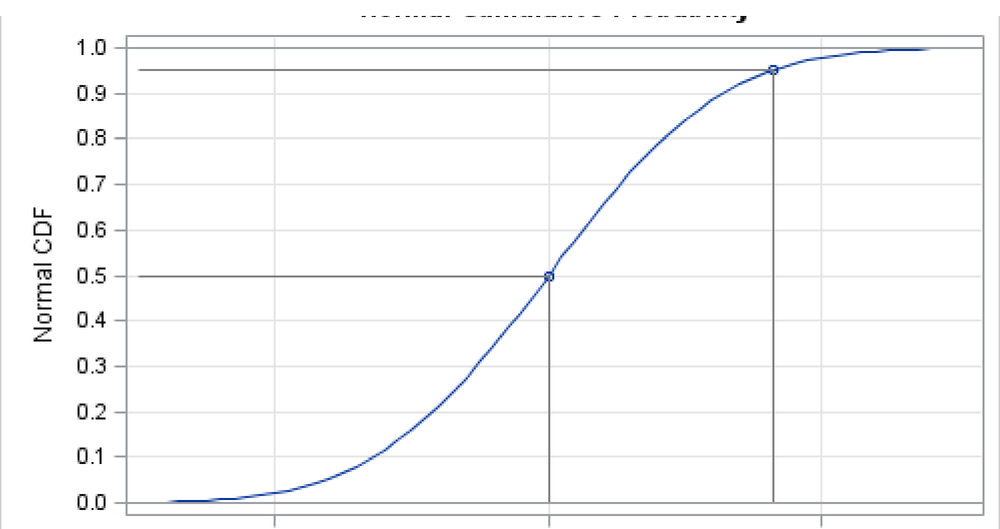
Distance



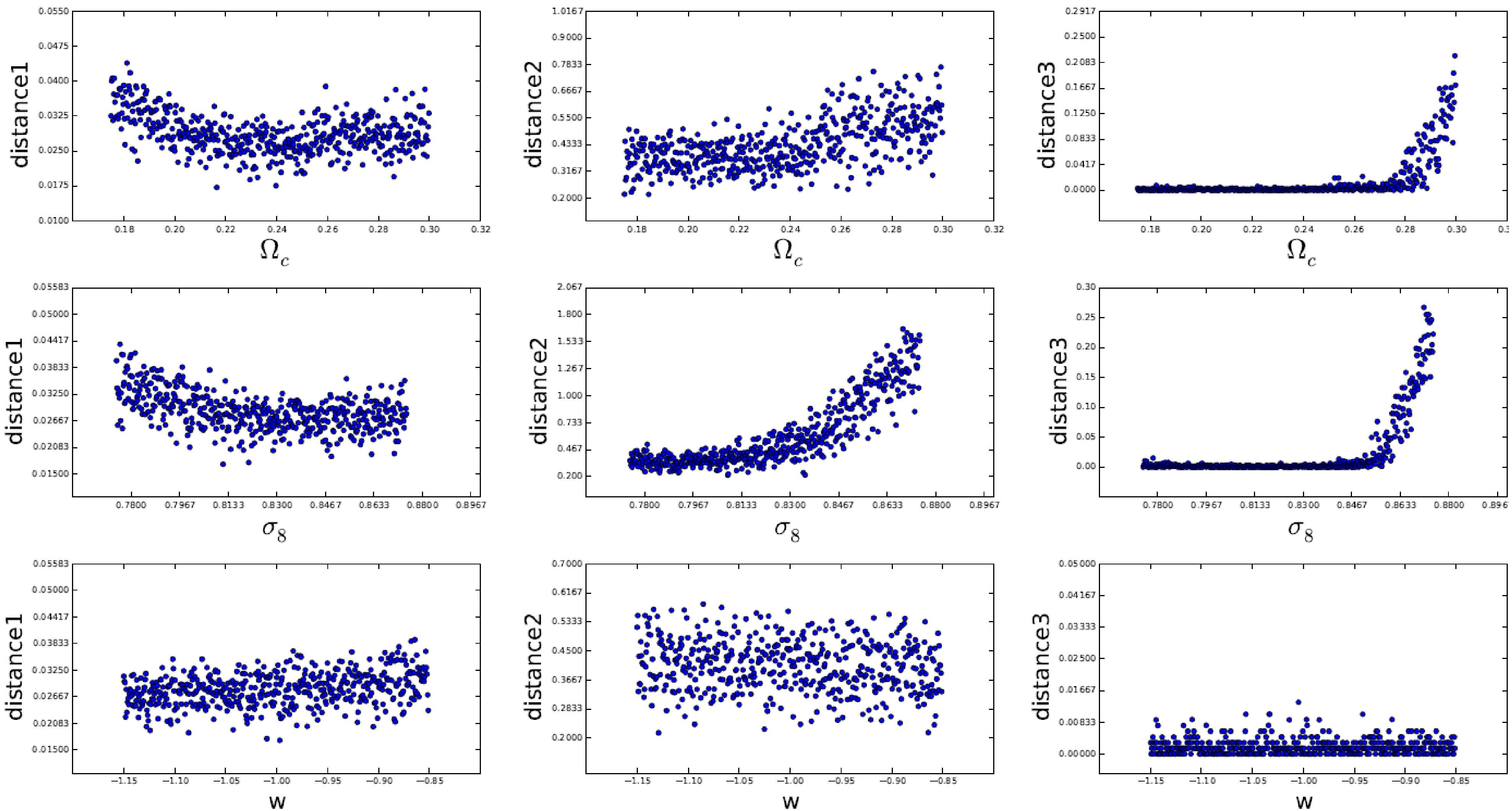
For z and ξ

+ total number of objects

cosmoabc

arXiv:1504.06129v2

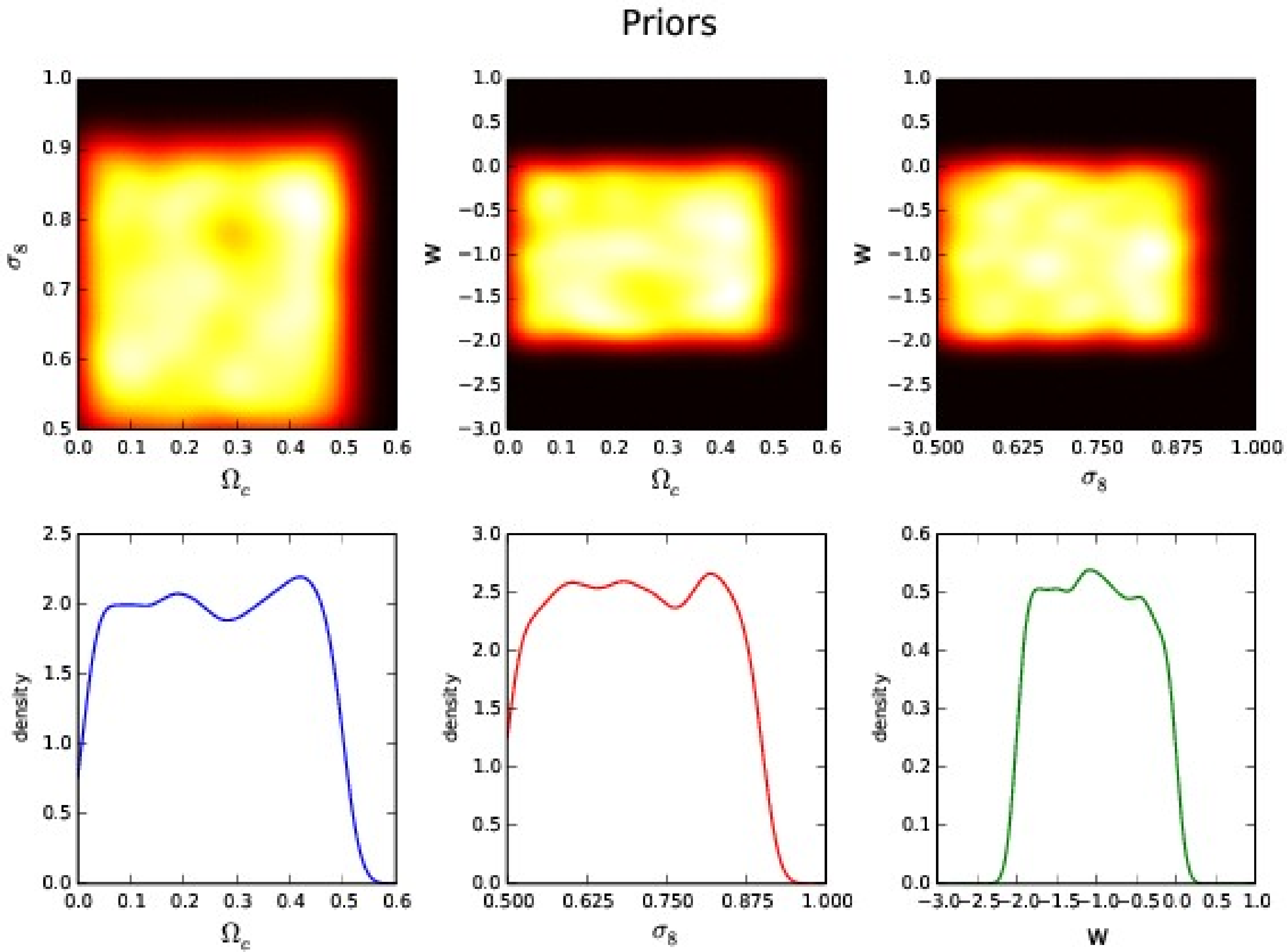
Distance



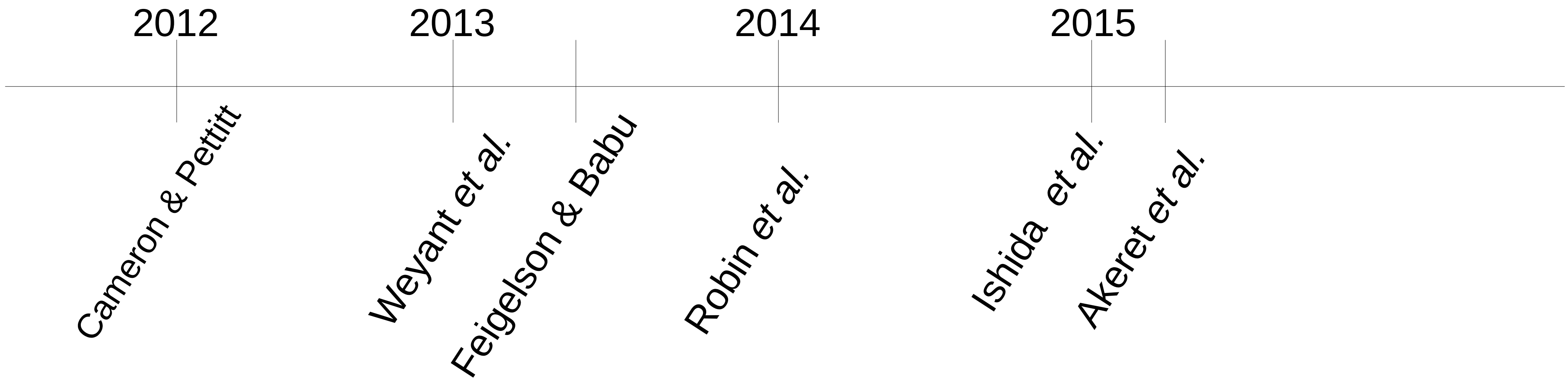
http://cosmoabc.readthedocs.org/en/latest/

cosmoabc

arXiv:1504.06129v2



See frames in the pdf arxiv (open with Adobe Reader)



arXiv:1504.07245v2

Approximate Bayesian Computation for Forward Modeling in Cosmology

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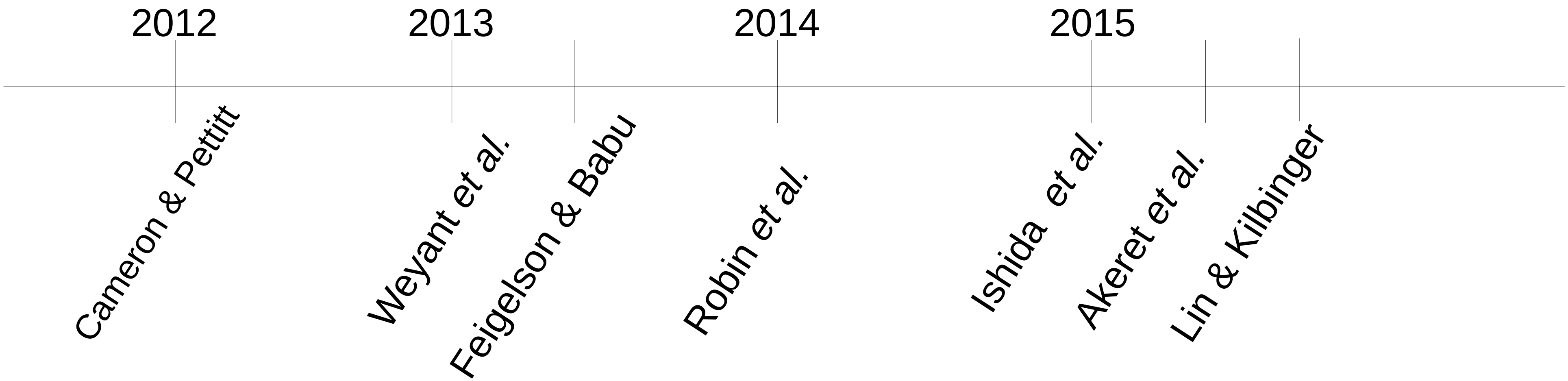

arXiv:1506.01076v1

A new model to predict weak-lensing peak counts

II. Parameter constraint strategies

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Cosmostatistics Initiative

COIN

Summary

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- ✓ Becomes more attractive with faster simulations.

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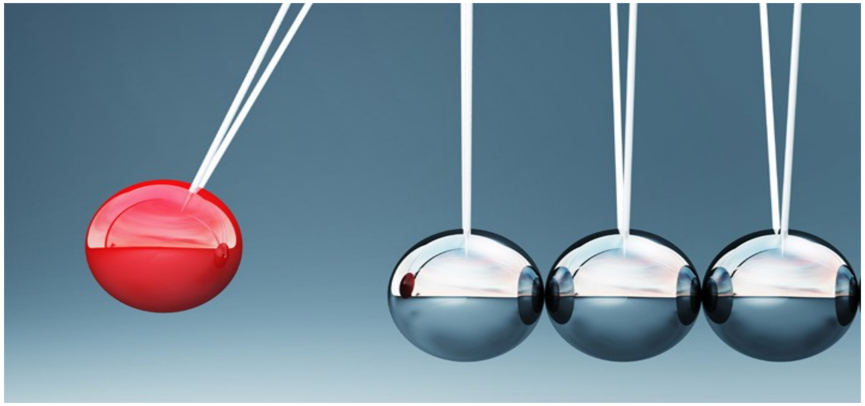

Promising perspective as it gains momentum inside the astronomical community!

Thank you!

Examples of ABC application

...The obvious class of problems is population studies that use population synthesis (i.e., simulating an astrophysical population). The earliest work I know of using ABC-like ideas is a thread of papers from the late 1990s/early 00s by Zaven Arzoumanian, David Chernoff, and Jim Cordes, using population synthesis (including Galactic orbital dynamics) to simulate the pulsar population in order to constrain the initial kick distribution. If you do an ADS search on Arzoumanian and Chernoff you'll find the string of papers. It culminated in this paper, which has had a big impact:

The Velocity Distribution of Isolated Radio Pulsars
http://adsabs.harvard.edu/abs/2002ApJ...568..289A

Tom Loredo, ASAIP Bayesian forum

Examples of ABC application

The hidden Potts model (used, e.g., in image segmentation Algorithms).

The model is easy to write down statistically in terms of a simple relationship between adjacent pixels, and it's easy to Gibbs sample from pixel-by-pixel at a fixed temperature (for which the normalization constant is unnecessary), BUT to perform inference of the temperature parameter too we need the proper likelihood which requires computation of the normalization constant (a sum over a huge combinatorial space) at each given temperature. When the image size is above ~1000x1000 pixels we simply don't have the computational power to make this calculation multiple times (as for an MCMC algorithm) so we consider the likelihood Intractable.

See section 4 of http://arxiv.org/pdf/1403.4359.pdf

Ewan Cameron, private communication

Algorithm

cosmoabc

Data: $\mathcal{D} \longrightarrow$ observed catalogue.
Result: ABC-posteriors distributions over the model parameters.

$t \longleftarrow 0$
$K \longleftarrow M$
for $J = 1, \ldots, M$ **do**
  Draw θ, from the prior, $p(\theta)$.
  Use θ to generate $\mathcal{D}_S$.
  Calculate distance, $\rho = \rho(\mathcal{D}_S, \mathcal{D})$.
  Store parameter and distance values,
  $\mathcal{S}_{ini} \leftarrow \{\theta, \rho\}$
end
Sort elements in $\mathcal{S}_{ini}$ by $|\rho|$.
Keep only the N parameter values with lower distance in $\mathcal{S}_{t=0}$.
$C_{t=0} \longleftarrow$ covariance matrix from $\mathcal{S}_{t=0}$
for $L = 1, \ldots, N$ **do**
  $W_1^L \longleftarrow 1/N$
end
while $N/K > \Delta$ **do**
  $K \longleftarrow 0$.
  $t \longleftarrow t + 1$.
  $\mathcal{S}_t \longleftarrow []$
  $\epsilon_t \longleftarrow 75^{th}$-quantile of distances in $\mathcal{S}_{t-1}$.
  while len($\mathcal{S}_t$) $<$ N **do**
    $K \longleftarrow K + 1$
    Draw θ_0 from $\mathcal{S}_{t-1}$ with weights $\hat{W}_{t-1}$.
    Draw θ, from $\mathcal{N}(\theta_0, C_{t-1})$.
    Use θ to generate $\mathcal{D}_S$.
    Calculate distance, $\rho = \rho(\mathcal{D}_S, \mathcal{D})$
    if $\rho \leq \epsilon_t$ **then**
      $\mathcal{S}_t \longleftarrow \{\theta, \rho, K\}$
      $K \longleftarrow 0$
    end
  end
  for $J = 1, \ldots, N$ **do**
    $\hat{W}_t^J \longleftarrow$ equation (3).
  end
  $W_t \longleftarrow$ normalized weights.
  $C_t \longleftarrow$ weighted covariance matrix from $\{\mathcal{S}_t, W_t\}$.
end

$$W_t^j = \frac{p(\boldsymbol{\theta}_t^j)}{\sum_{i=1}^{N} W_{t-1}^i \mathcal{N}(\boldsymbol{\theta}_t^j; \boldsymbol{\theta}_{t-1}^i, C_{t-1})}, \qquad (3)$$