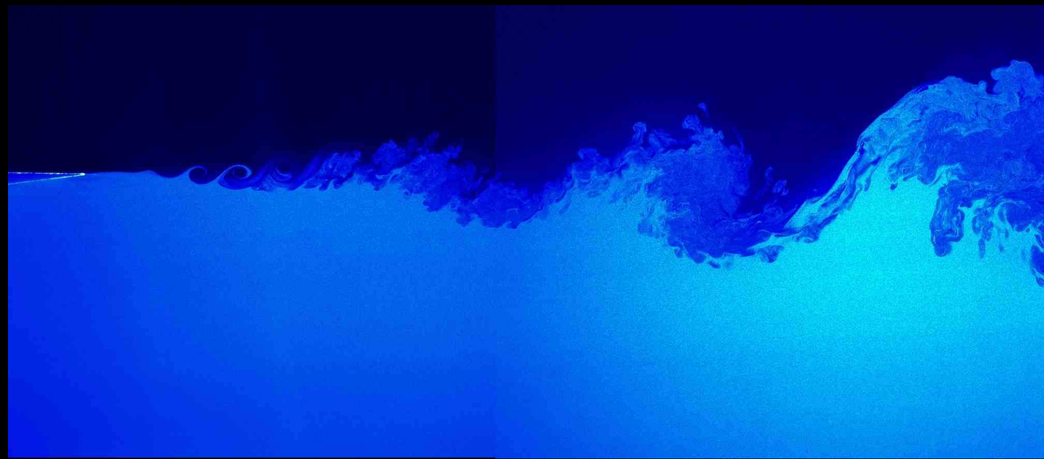
Turbulence modelling and control using maximum entropy methods

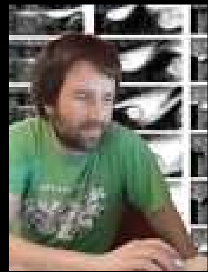
→



Bernd Noack   **& friends**
PPRIME/CNRS, Poitiers   *in Poitiers + elsewhere*

— supported by ANR, DFG, ERC, & ADFA@UNSW —

Friends / team

Machine learning	Closed-loop turbulence control — theory	Statistical physics
	   	
M. Abel **M. Segond** *Ambrosys*	**L. Cordier, T. Duriez, E. Kaiser, B. Noack, M. Schlegel, et al.** *PPRIME, Poitiers + TU Berlin*	**Robert Niven** *UNSW* *Australia*
Control theory	**Closed-loop turbulence control — experimental demonstrators**	**CFD + Stab.anal.**
	 	
S. Brunton *U Washington*	**V. Parezanovic** **J.-P. Bonnet** *PPRIME* **Rudi King** *TU Berlin*	**Marek Morzyński** *TU Poznań*

Overview

1. Introduction to turbulence control
. . . . A research area with impact of epic proportion

2. Reduced-order modelling
. . . . . Towards online control in experiment

3. MaxEnt & Finite-time thermodynamics closure
. . . . Understanding the nonlinear mode sociology

4. Examples of closed-loop turbulence control
. . . . . Drag reduction, lift increase, . . .

5. Conclusions and outlook
. . . . . Bayes/MaxEnt's potential role in turbulence control

Overview

Turbulence control $\mapsto$ transport vehicles

Control goals

- lift increase
- drag reduction
- noise reduction
- mixing/combustion control

Control strategies

- aerodynamic design
- passive (e.g. riblets)
- active, open-loop
  (e.g. periodic blowing)
- active, closed-loop
  (largest opportunities!)

Turbulence control ↦ other applications

Navier-Stokes equation

Incompressible flow: $\mathbf{x} = (x, y, z), t \mapsto \mathbf{u} = (u, v, w), p$

Normalization:
U: characteristic velocity
D: characteristic size
ρ: density

Reynolds number: $Re = UD/\nu$, ν= kinematic viscosity

Mass conservation: $\nabla \cdot \mathbf{u} = 0$

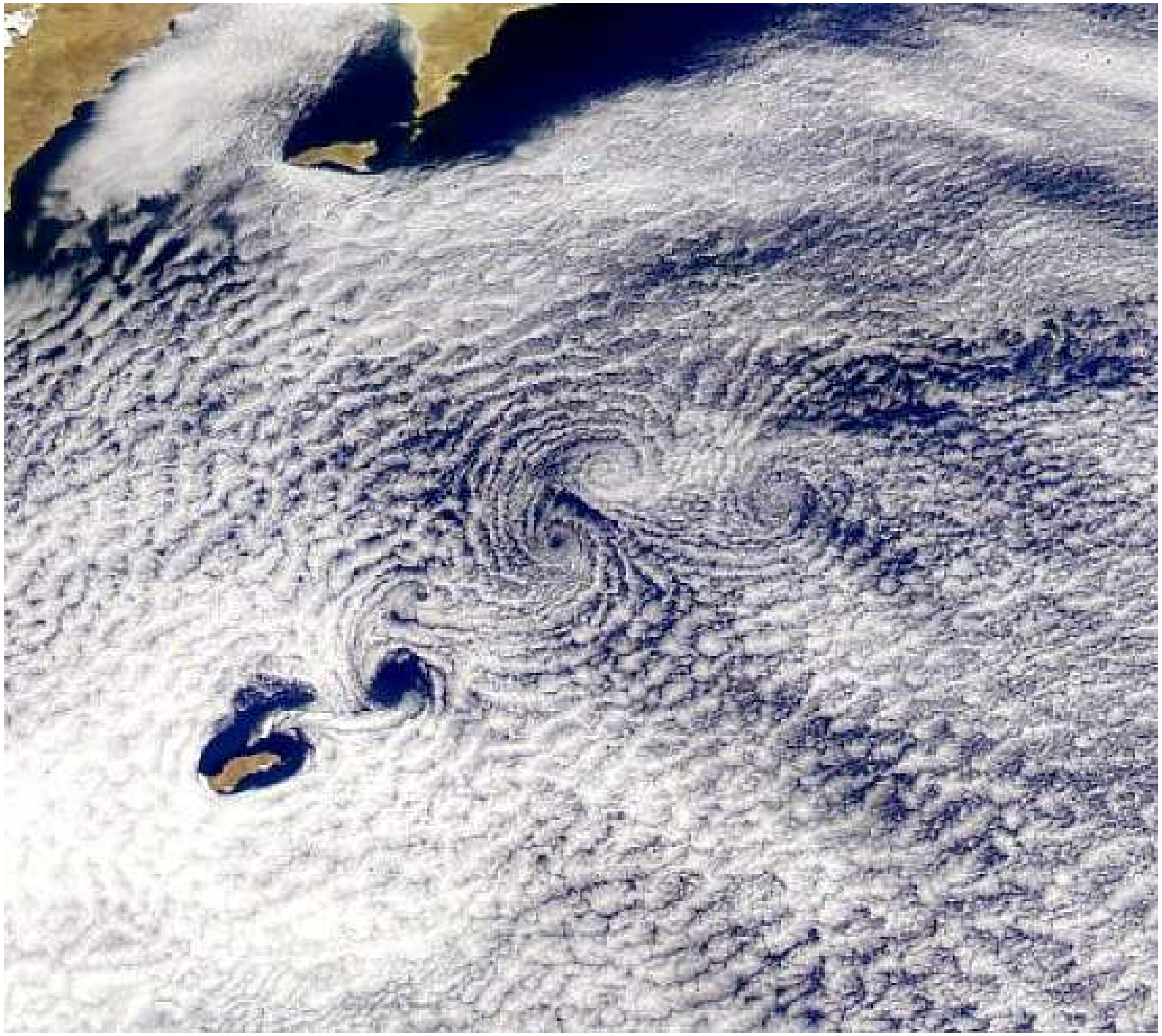
Momentum balance:

$$\underbrace{\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}}_{\text{acceleration}} = \underbrace{-\nabla p}_{\text{pressure force}} + \underbrace{\frac{1}{Re} \triangle \mathbf{u}}_{\text{viscous force}}$$

Solution:

- Flow has a single attractor
- Turbulence attractor $N \sim Re^{9/4}$, e.g., $Re = 10^6 \Rightarrow N \sim 3 \cdot 10^{13}$

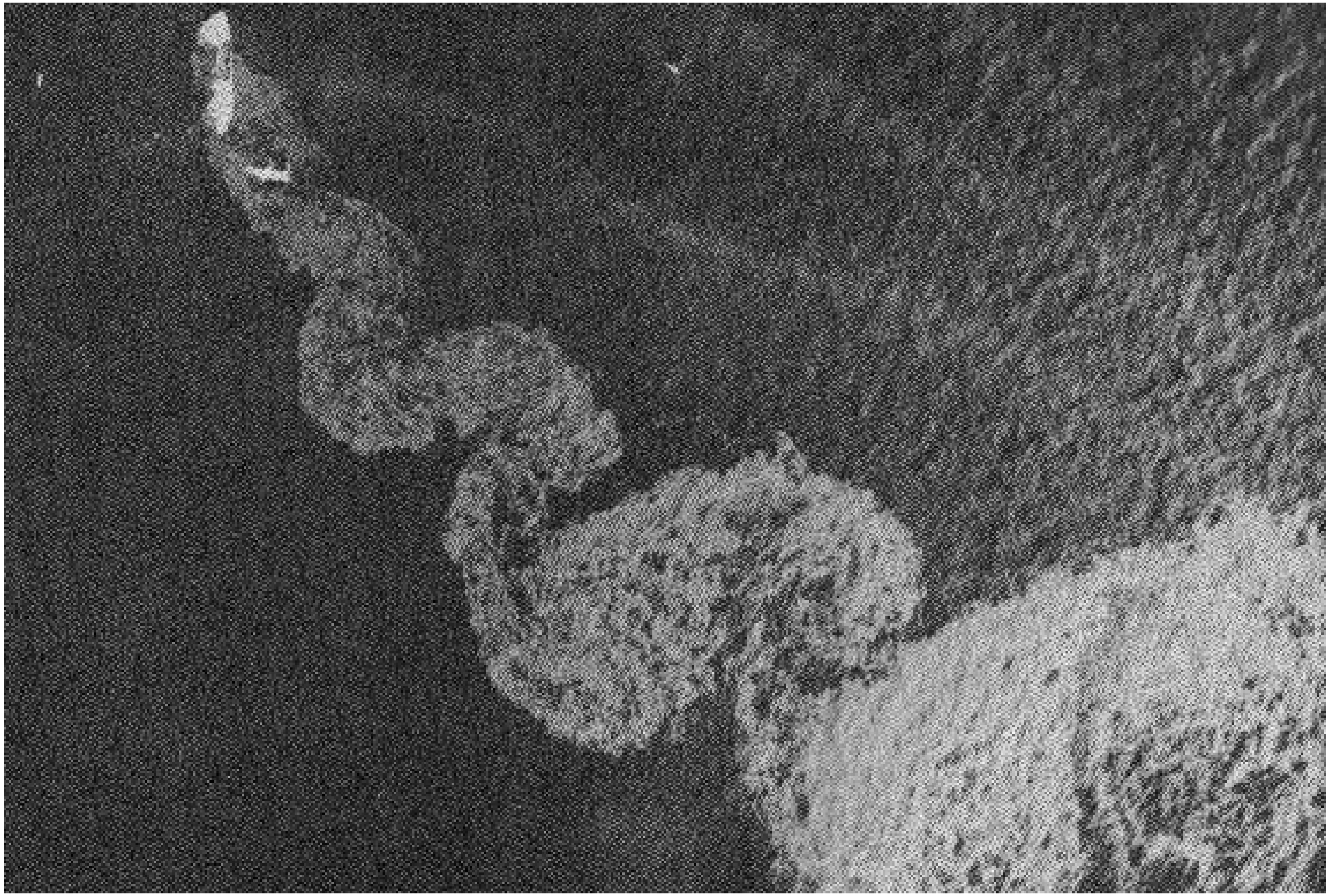
von Kármán vortex street in nature



Rear side of the island Guadalupe (20 Aug. 1999)

von Kármán vortex street in technology

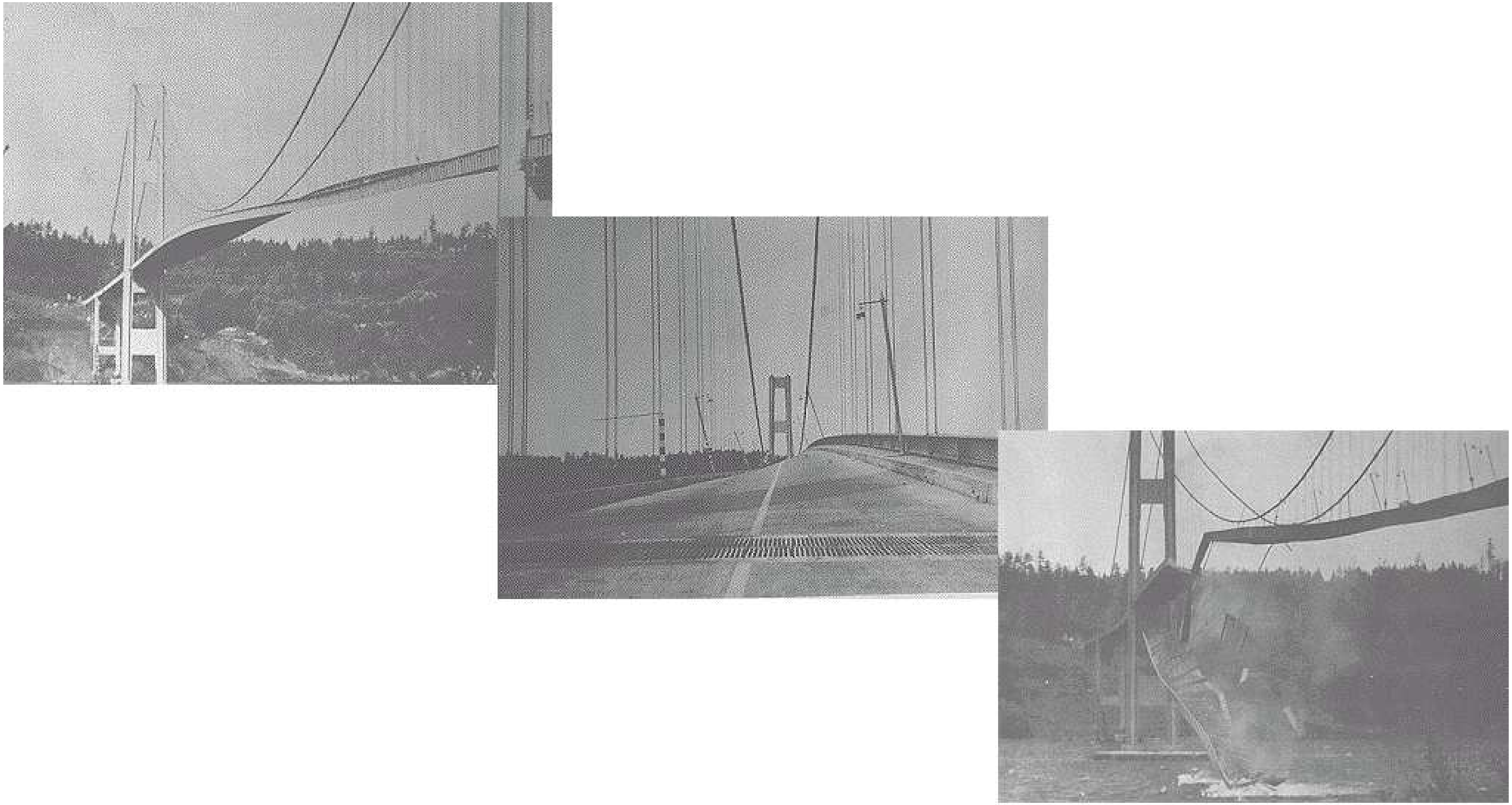
Damaged tanker — oil visualization



Van Dyke (1982), page 100

von Kármán vortex street in technology

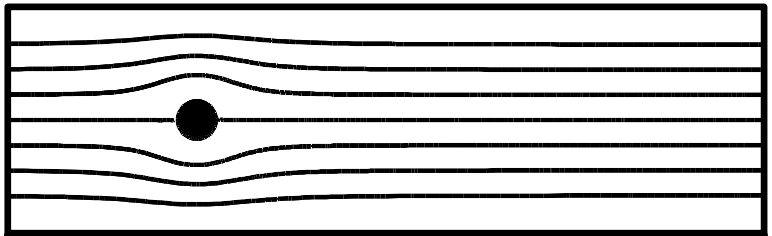
Tacoma Narrows Bridge (7 Nov. 1940)



Phenomenogram of cylinder wake

Reynolds number $Re = \frac{UD}{\nu}$

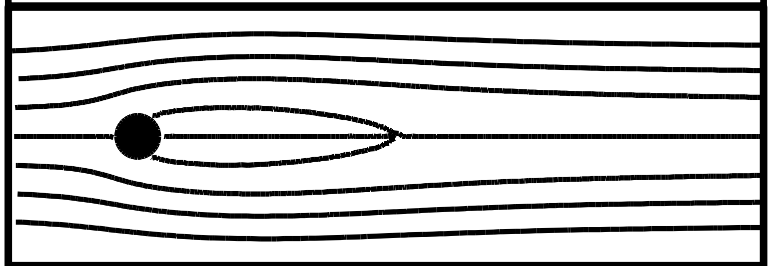
$Re < 4$



2D steady flow

without vortex pair

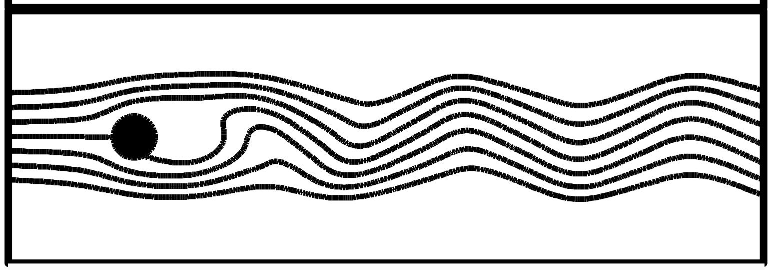
$Re < 47$



2D steady flow

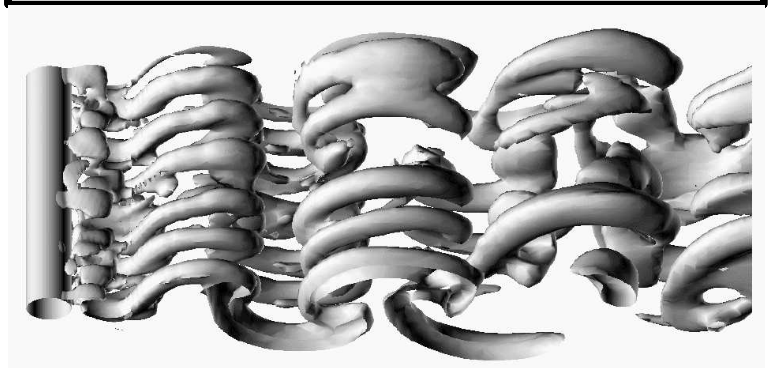
with vortex pair

$Re < 180$



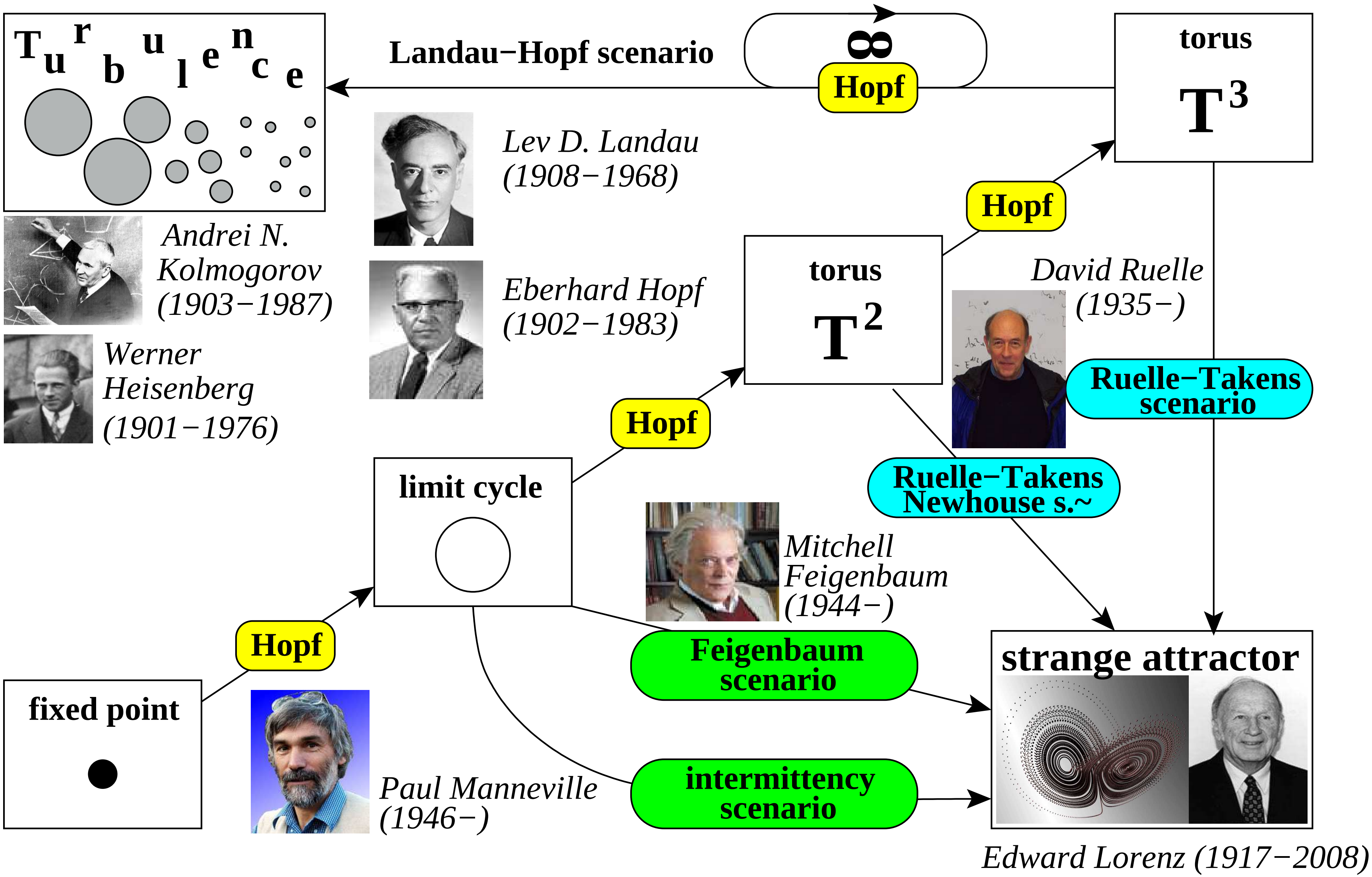
2D vortex shedding

$180 < Re$



2D vortex shedding superimposed by 3D modes / fluctuations

Dream [$20s - 80s$]: Instabilities $\mapsto$ turbulence



Werner Heisenberg 1948

As soon as one puts energy into a liquid **without friction**, this energy will be distributed among all degrees of freedom, and what finally results is a certain equilibrium distribution, corresponding to the **Maxwellian distribution** in gases ... **Turbulence is an essentially statistical problem of the same type as one meets in statistical mechanics,** since it is the problem of distribution of energy among a very large number of degrees of freedom.

Open questions: . . . .

- **Which state space?**
- **Which entropy?**
- **Which constraints?**

Dream [50*s*]: Statistical physics ↦ turbulence

Ludwig Boltzmann
(1840–1906)

Equivalent subsystems:

1877: Entropy

$S = k\, lnW$

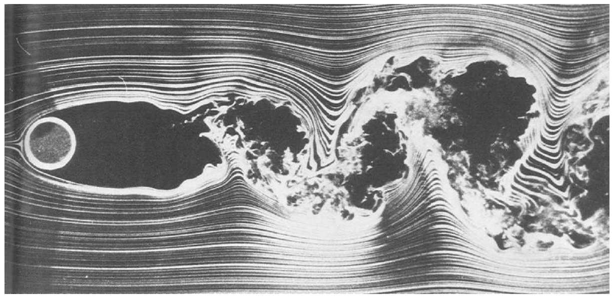
Lars Onsager
(1903–1976)

Particle/vortex picture:

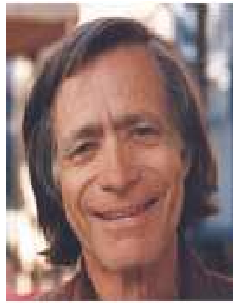
1949: point vortices in 2D flows
= thermodyn. degree of freedom

Hans W Liepmann (1914-2009)

WARNING:
Don't forget →



Robert H Kraichnan
(1928–2008)



Wave/Galerkin picture:

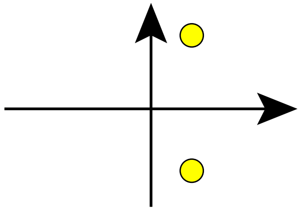
1955: Fourier modes
= thermodyn. degrees of freedom
(absolute equilibrium ensemble)

**How to partition the flow in equivalent subsystems (atoms)
(= thermodynamic degrees of freedom)???**

Dream $[ongoing]$: Control $\mapsto$ turbulence

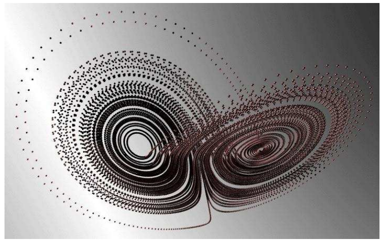
linear dynamics

da/dt = A a + B b



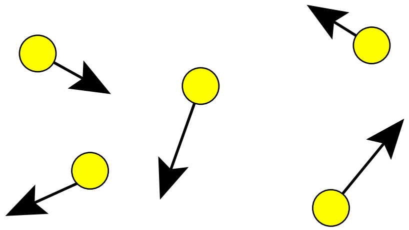
strange attractor

da/dt = f (a,b)



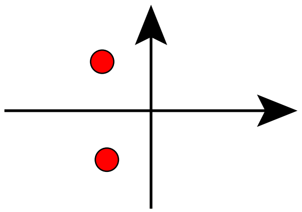
statistical physics

S = k ln W



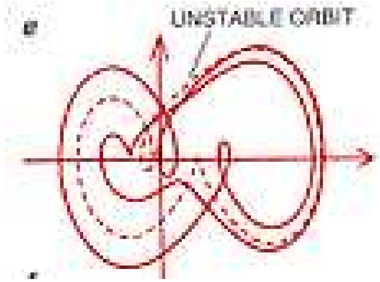
linear control

b =K a



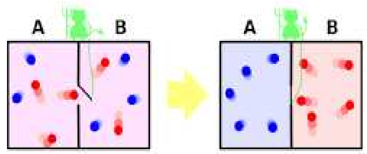
Anno
Dazumal

chaos control



Ott, Grebogi, Yorke
1990 PRL

Maxwell's deamon

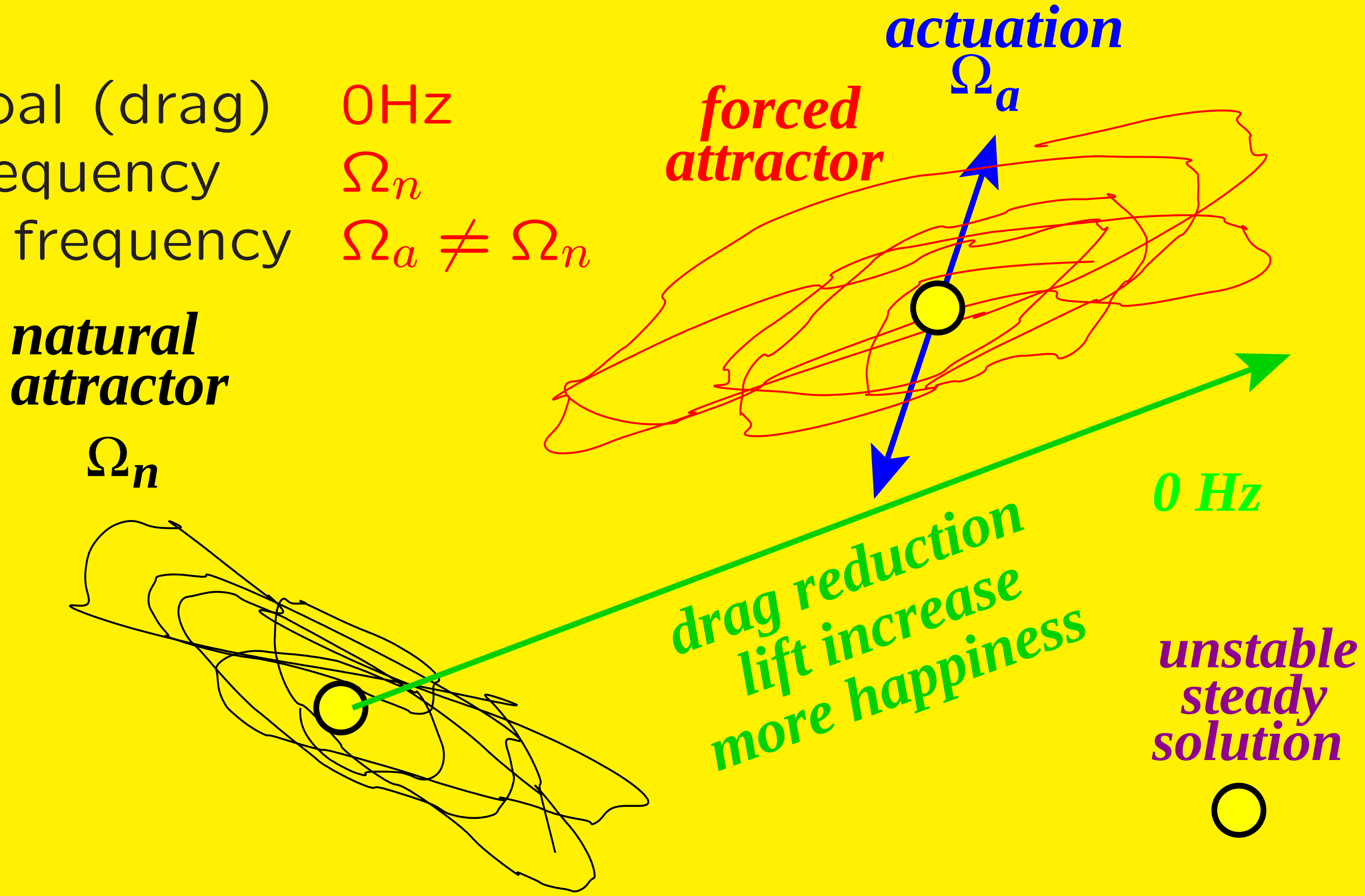


Maxwell 1867
Wiener 1948

Turbulence control = attractor control

Phase space

- Control goal (drag) 0Hz
- Natural frequency Ω_n
- Actuation frequency $\Omega_a \neq \Omega_n$



- Steady solution cannot be stabilized
- No meaningful linearization

Closure for first and second moments needed!

Turbulence control = attractor control

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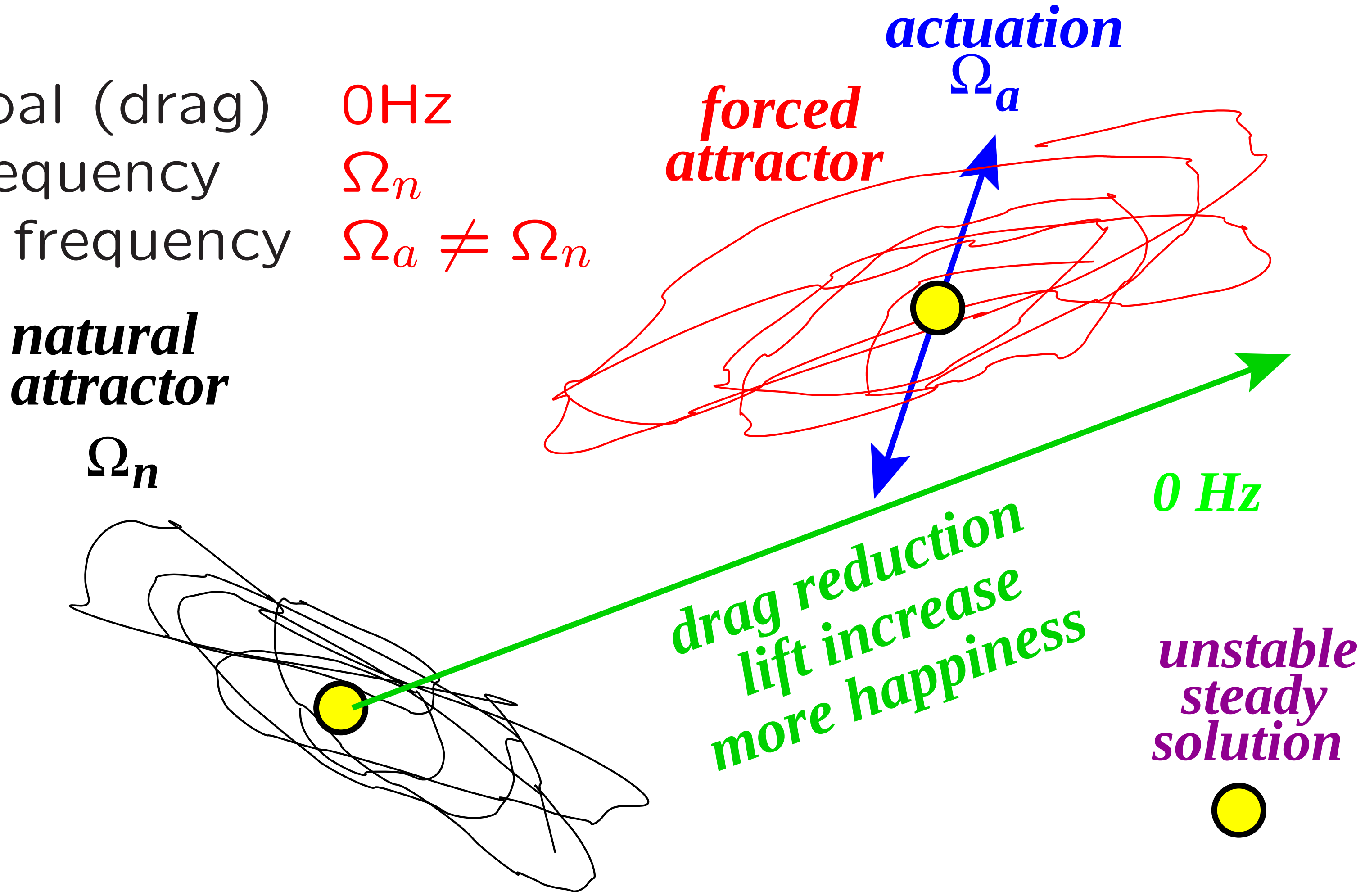

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   . . . . *Understanding the nonlinear mode sociology*

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   . . . . *Drag reduction, lift increase, . . .*

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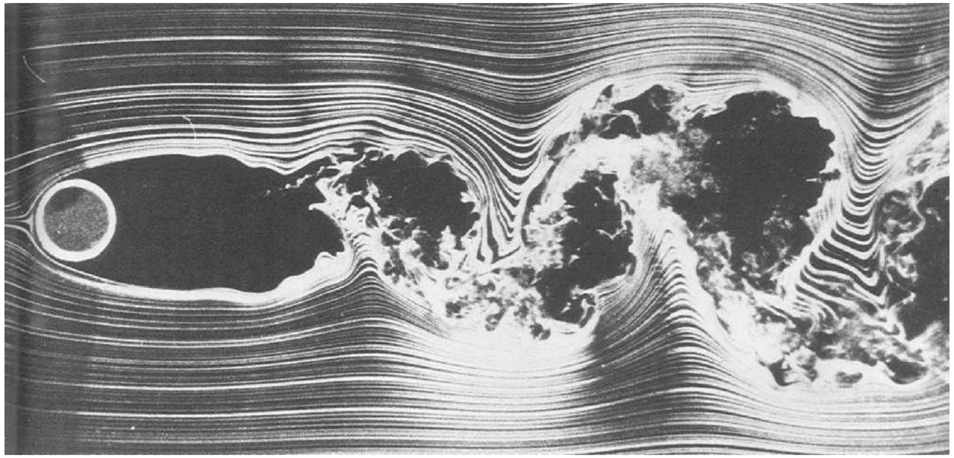
Path to low-dimensional models

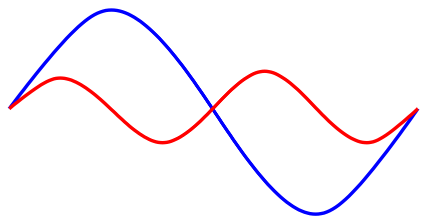
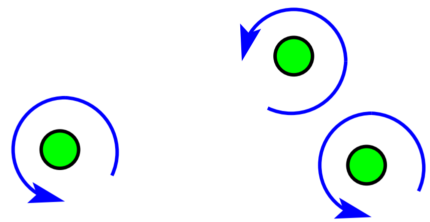
Low-dimensional coherent structures

Smoke visualization at $Re = 10000$

[van Dyke, Album of Fluid Motion]



Modelling approaches

	Eulerian view	**Lagrangian view**
coherent structures		
variable	velocity $\mathbf{u}$	vorticity ω
kinematics	$\mathbf{u} = \sum a_i(t)\mathbf{u}_i(\mathbf{x})$ Galerkin approximation	$\omega = \sum \Gamma_i \boldsymbol{\Omega}(\mathbf{x} - \mathbf{x}_i)$ vortex configuration
dynamics	$\frac{da_i}{dt} = f_i(a_1, \ldots)$ **Galerkin model**	$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}(\{\Gamma_i, \mathbf{x}_i\})$ **vortex model**

'Traditional' Galerkin method

— *Fletcher 1984 Computational Galerkin Methods, Springer* —

Galerkin method $\quad \nu = 1/Re$

$$\begin{array}{ccccccccc} \mathbf{u} & \rightarrow & \partial_t \mathbf{u} & = \nu \triangle \mathbf{u} & & -\nabla(\mathbf{u}\mathbf{u}) & & -\nabla p \\ \downarrow & & \downarrow & \downarrow & & \downarrow & & \downarrow \\ \mathbf{u}^{[N]} = \sum_{i=0}^{N} a_i \mathbf{u}_i & \rightarrow & \frac{da_i}{dt} & = \nu \sum_{j=0}^{N} l_{ij} a_j & + & \sum_{j,k=0}^{N} (q^c_{ijk} & & + q^\pi_{ijk})\, a_j\, a_k \end{array}$$

Inner product . . . . $(\mathbf{u}, \mathbf{v})_\Omega := \int_\Omega dV\, \mathbf{u} \cdot \mathbf{v}$

Galerkin approximation with orthon. modes $\left(\mathbf{u}_i, \mathbf{u}_j\right)_\Omega = \delta_{ij}$

Galerkin projection exemplified for $\partial_t \mathbf{u}$

$$\left(\mathbf{u}_i, \partial_t \mathbf{u}^{[N]}\right)_\Omega = \left(\mathbf{u}_i, \partial_t \left[\sum_{j=0}^{N} a_j \mathbf{u}_j\right]\right)_\Omega = \sum_{j=1}^{N} \frac{da_j}{dt} \underbrace{\left(\mathbf{u}_i, \mathbf{u}_j\ \right)_\Omega}_{\delta_{ij}} = \frac{d}{dt} a_i$$

'Traditional' Galerkin method

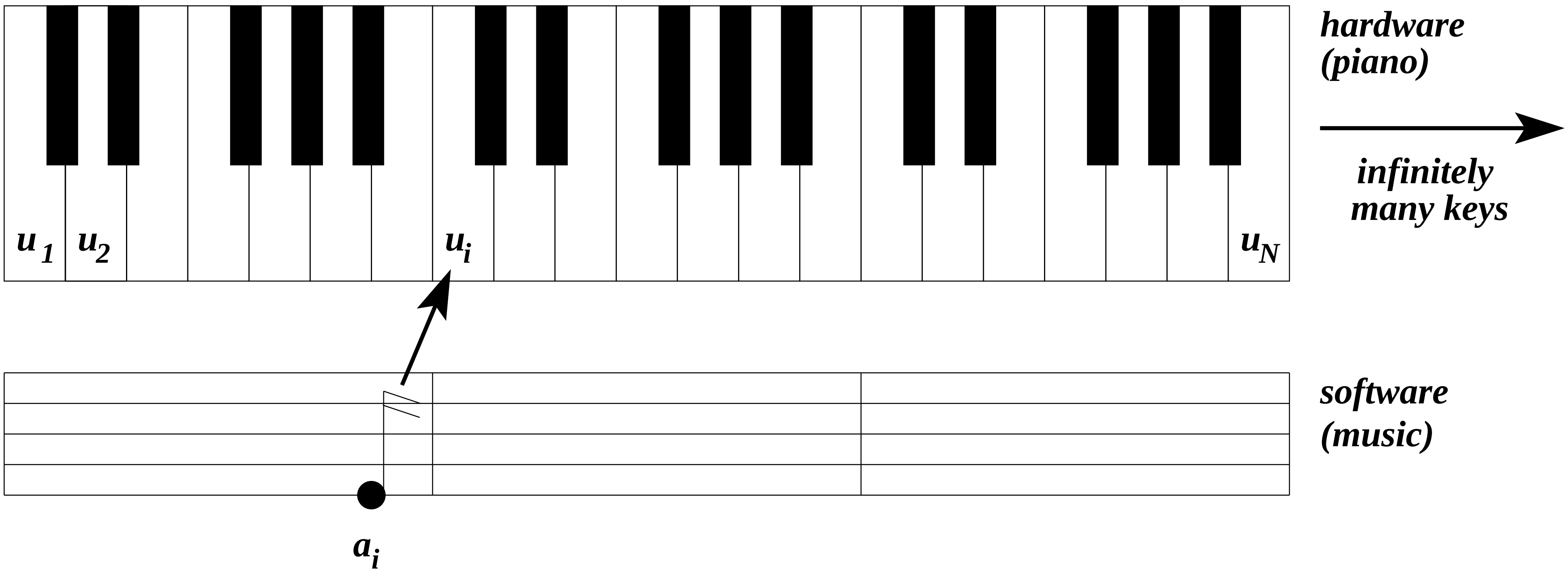
— *Fletcher 1984 Computational Galerkin Methods, Springer* —

Galerkin method

$$\mathbf{u}(\mathbf{x},\mathbf{t}) \quad \rightarrow \quad \partial_t \mathbf{u} = \frac{1}{Re}\triangle \mathbf{u} - \nabla(\mathbf{u}\mathbf{u}) - \nabla p$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\mathbf{u}^{[\mathrm{N}]} = \sum_{i=0}^{N} a_i(t)\mathbf{u}_i(\mathbf{x}) \quad \rightarrow \quad \frac{da_i}{dt} = c_i + \sum_{j=1}^{N} l_{ij}\, a_j + \sum_{j,k=1}^{N} q_{ijk}\, a_j\, a_k$$

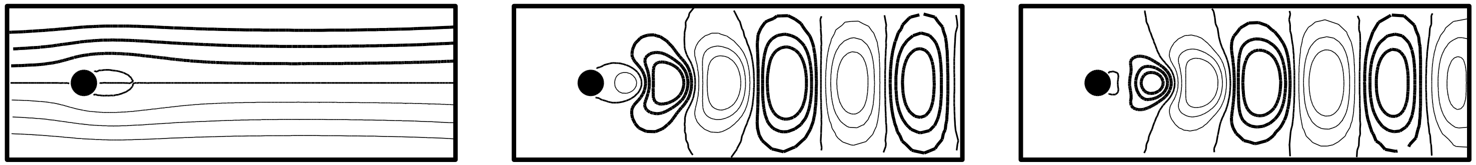


Transient dynamics of wake

— Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM —

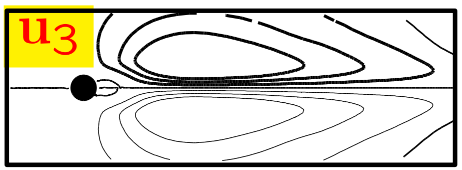
Operating condition II

on attractor



↑

transient on paraboloid



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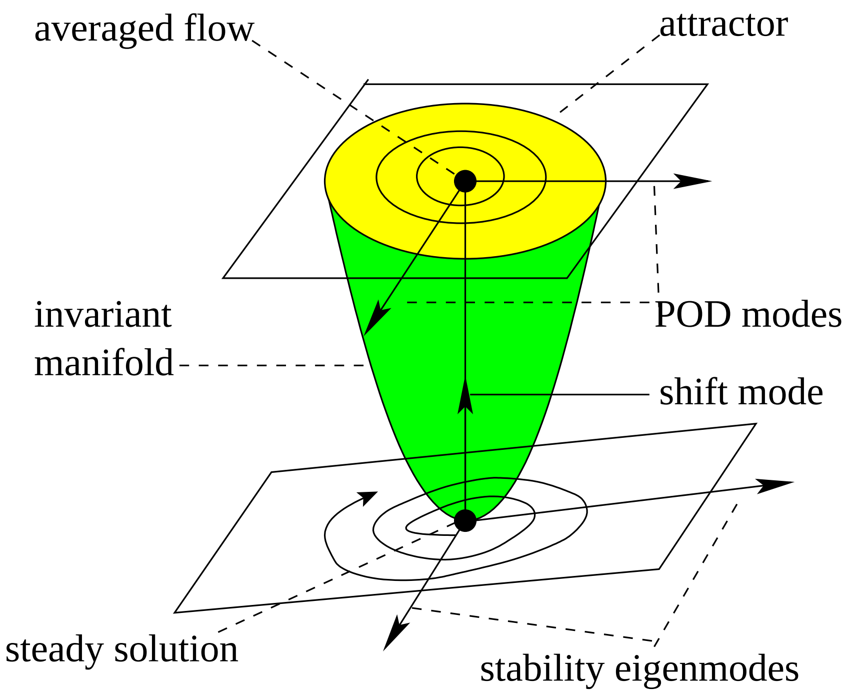


$$\frac{d}{dt}a_1 = \sigma a_1 - \omega a_2$$
$$\frac{d}{dt}a_2 = \sigma a_2 + \omega a_1$$
$$\frac{d}{dt}a_3 = -\sigma_3 a_3 + cA^2$$

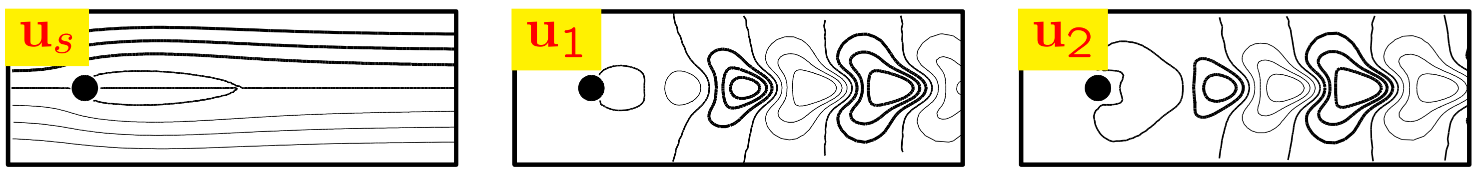
$$\sigma = \sigma_1 - \beta a_3$$
$$\omega = \omega_1 + \gamma a_3$$
$$A^2 = a_1^2 + a_2^2$$

Landau equation

$$\frac{d}{dt}A = \sigma_1 A - \beta^\star A^3$$

Operating condition I

near fixed point



Example: Wake stabilization in DNS

Gerhard, Pastoor, King, Noack, Dillmann, Morzyński & Tadmor (2003) AIAA

Kinematics *(3 modes)*

$$\mathbf{u}(\mathbf{x},t)=\mathbf{u}_s(\mathbf{x})+\sum_{i=1}^{3} a_i(t)\,\mathbf{u}_i(\mathbf{x})$$

Dynamics *(from Navier-Stokes eq.)*

$$\frac{d}{dt}a_i = f_i(a_1,a_2,a_3) + g_i b$$

Dynamic observer estimates state.

$$\frac{d}{dt}\hat{a}_i = f_i(\hat{a}_1,\hat{a}_2,\hat{a}_3) + g_i b + L_i(s-\hat{s})$$

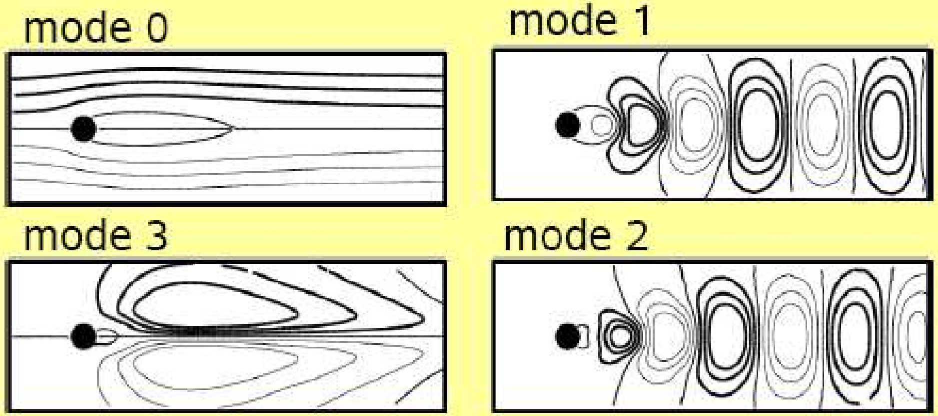
Energy-based controller enforces

$$\frac{d}{dt}K = -\sigma K, \qquad K=(a_1^2+a_2^2)/2.$$

Key enabler: choice of modes

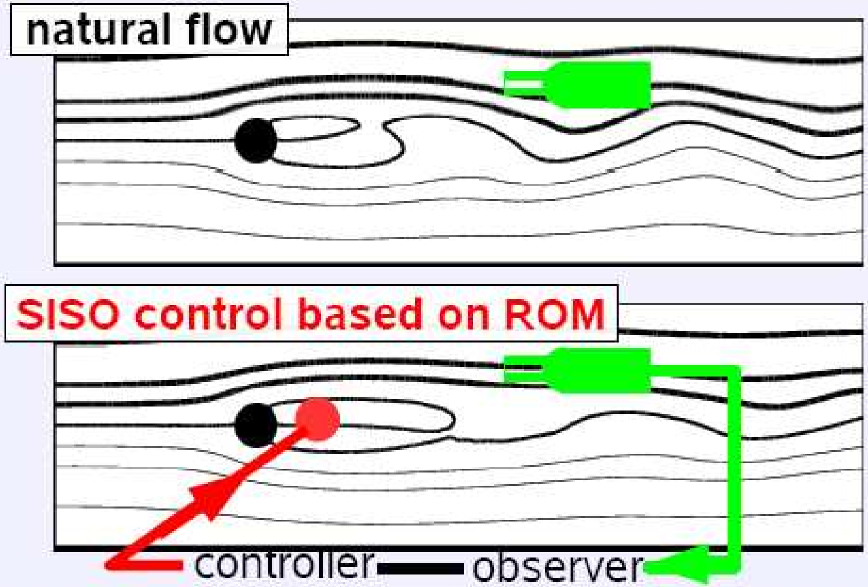
Resolved base flow is time-dependent.

95% of fluctuation energy is resolved by 2 modes depending on mean flow.



Strengths of our approach:

- Wake almost stabilized *in* ***direct numerical simulation (DNS).***
- „Least-order“ model has a ***tune-able phase controller for application in experiment.***
- Control is ***robust*** and ***online capable in experiment.***



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StatPhys imports in fluid dynamics

Eying & Sreenivasan 2006 RMP, ...

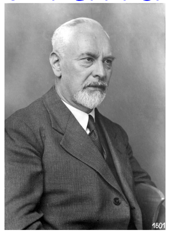
Gas kinetics

Maxwell 1860

$\nu = \frac{1}{3}\,\lambda\, u_{molecule}$

$\mapsto$ **fluid mechanics**

Prandtl 1927



$\nu_t = \frac{1}{3}\, l_m\, u_m$

Entropy principles

Boltzmann 1877

$S = k\,\ln W$

$\mapsto$ **fluid mechanics**

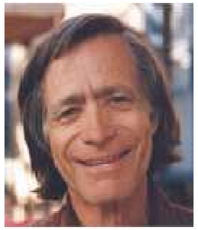
Onsager 1949

Gibbs formalism

$\mapsto$ vortices

Lee 1951

Kraichnan 1955



Equipart. of energy

$\mapsto$Fourier modes

Example for Jaynes' MaxEnt principle — I

— ≡ *E.T. Jaynes (1957,...,2003)* —

Edwin Thompson Jaynes (1922-1998)

State space .... $\boldsymbol{a} = (a_1, \ldots, a_N)$

Dynamics (not used) .... $\frac{d}{dt}\boldsymbol{a} = \boldsymbol{f}(\boldsymbol{a})$

Ergodic measure (PDF) .... $p(\boldsymbol{a})$

Ergodic average .... $\langle F(\boldsymbol{a})\rangle := \int d\boldsymbol{a}\, p(\boldsymbol{a})\, F(\boldsymbol{a})$

We know

(C0) normalization condition: .... $\langle 1 \rangle := \int d\boldsymbol{a}\, p(\boldsymbol{a}) = 1$

(C1) energy preservation: .... $\left\langle \sum_{i=1}^{N} \frac{a_i^2}{2} \right\rangle = E$

What is the least-biased/informative choice for $p(a)$?

Example for Jaynes' MaxEnt principle — II

— ≡ E.T. Jaynes (1957,...,2003) —

Improper prior (no a priori bias in state space) . $q(\boldsymbol{a}) \equiv 1$

Kullback-Leibler entropy .... $H := -\int d\boldsymbol{a}\, p(\boldsymbol{a}) \ln \left[\frac{p(\boldsymbol{a})}{q(\boldsymbol{a})}\right]$

Maximum entropy principle .... $H = \max$

Constraints: ....normalization (C0) & energy (C1)

Lagrange formalism:

... $L = H - \lambda_0 [\langle 1 \rangle - 1] - \lambda_1 \left[\left\langle \sum_{i=1}^{N} \frac{a_i^2}{2} \right\rangle - E\right] = \text{extremum}$

Solution: .... $p(\boldsymbol{a}) = \frac{1}{Z} \exp\left[-\lambda_1 \sum_{i=1}^{N} \frac{a_i^2}{2}\right]$

Partition function .... $Z = \int_{\mathcal{R}^N} d\boldsymbol{a} \exp\left[-\lambda_1 \sum_{i=1}^{N} \frac{a_i^2}{2}\right]$

(C0) leads to Z; (C1) determines λ_1

Jaynes' MaxEnt principle $\mapsto$ gas & flow

— ≡ *E.T. Jaynes (1957,...,2003),* ≡ *Noack & Niven 2012 JFM* —

	ideal gas	incompressible flow
state space $\boldsymbol{a} = (a_1, \ldots, a_N)$	M molecules $\boldsymbol{x}_l,\ \boldsymbol{u}_l,\ l = 1, \ldots, M$	**?**
evolution eq. $\dot{\boldsymbol{a}} = \boldsymbol{f}(\boldsymbol{a})$	Newton's law $m \frac{d}{dt} \boldsymbol{v}_l = \boldsymbol{F}_{l,coll}(\{\boldsymbol{x}_m\})$	**?**
constraint(s) $\boldsymbol{G}(\boldsymbol{a}) = 0$	Integral of motion $\sum \frac{1}{2} m \|\boldsymbol{v}_l\|^2 = E$	**?**
closure PDF	$p(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M, \boldsymbol{u}_1, \ldots \boldsymbol{u}_M)$	**?**
MaxEnt with contraint(s)	$\Rightarrow p(\boldsymbol{v}_l) \propto e^{-m v_l^2 / kT},$, $\Rightarrow$ Maxwell distribution $p(v_l) \propto v_l^2 e^{-m v_l^2 / kT}$, $v_l := \|\boldsymbol{u}_l\|$	**?**

Jaynes' MaxEnt principle $\mapsto$ gas & flow

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	ideal gas	incompressible flow
state space $\boldsymbol{a} = (a_1, \ldots, a_N)$	M molecules $\boldsymbol{x}_l$, $\boldsymbol{u}_l$, $l = 1, \ldots, M$	Galerkin expansion $\boldsymbol{u} = \boldsymbol{u}_0 + \sum_{i=1}^{N} a_i \boldsymbol{u}_i$
evolution eq. $\dot{\boldsymbol{a}} = \boldsymbol{f}(\boldsymbol{a})$	Newton's law $m \frac{d}{dt} \boldsymbol{v}_l = \boldsymbol{F}_{l,coll}(\{\boldsymbol{x}_m\})$	Galerkin system $\dot{a}_i = c_i + \sum l_{ij} a_j + \sum q_{ijk} a_j a_k$
constraint(s) $\boldsymbol{G}(\boldsymbol{a}) = 0$	integral of motion $\sum \frac{1}{2} m \|\boldsymbol{v}_l\|^2 = E$	power balance (pendant of TKE eq.)
closure PDF	$p(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M, \boldsymbol{u}_1, \ldots \boldsymbol{u}_M)$	$p(a_1, \ldots, a_N)$
MaxEnt with contraint(s)	$\Rightarrow p(\boldsymbol{v}_l) \propto e^{-m v_l^2/kT}$,, $\Rightarrow$ Maxwell distribution $p(v_l) \propto v_l^2 e^{-m v_l^2/kT}$, $v_l := \|\boldsymbol{u}_l\|$	$\Rightarrow m_i = \overline{a_i}$, $\Rightarrow E_i = \overline{a_i^2}/2$, etc.

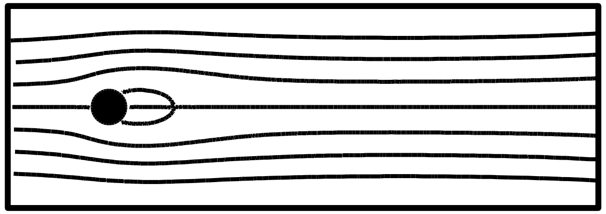
Galerkin expansion of cylinder wake

Noack, Afanasiev, Morzyński, Thiele & Tadmor 2003 JFM

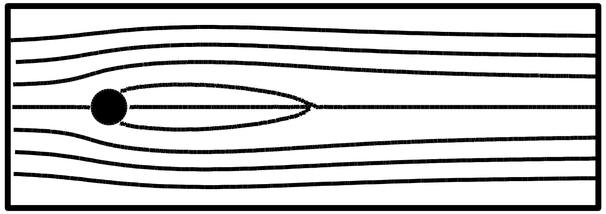
Base flow

$$\boldsymbol{u}^B = \boldsymbol{u}_s + a_7\, \boldsymbol{u}_7$$

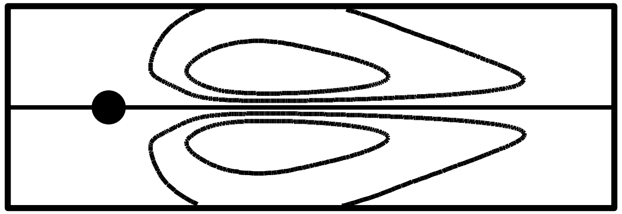
mode 0



steady solution



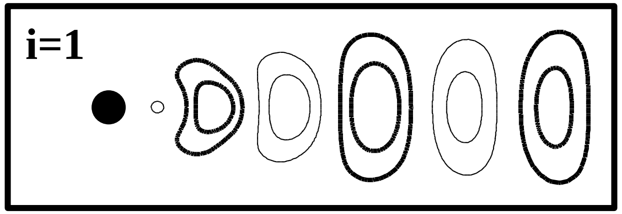
mode 7



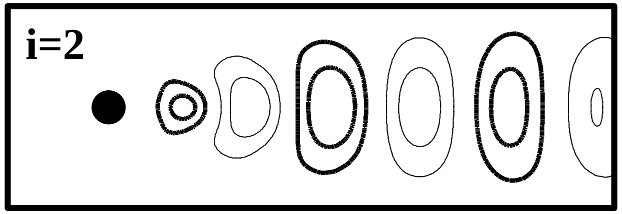
Fluctuation

$$\boldsymbol{u}' = \sum_{i=1}^{6} a_i\, \boldsymbol{u}_i$$ . . . . resolves 99.9% of TKE

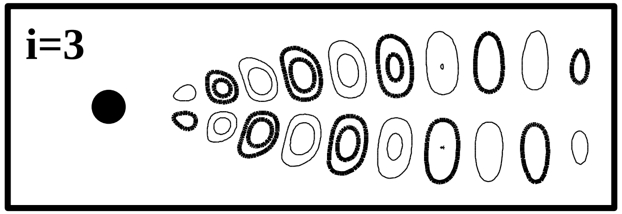
mode 1



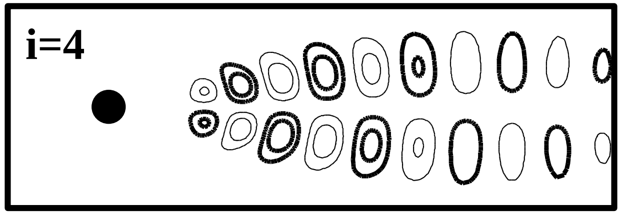
mode 2



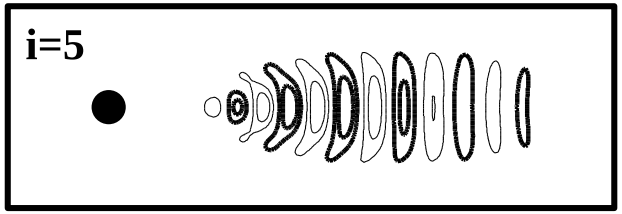
mode 3



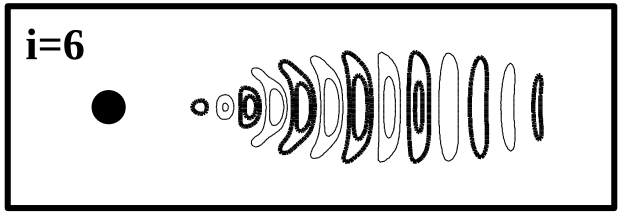
mode 4



mode 5



mode 6



POD Galerkin system of cylinder wake

≡ *Noack, Schlegel, Morzyński & Tadmor 2010 IJNMF; Noack & Niven 2012 JFM*

Galerkin expansion: $\boldsymbol{u} = \boldsymbol{u}_s + a_7 \boldsymbol{u}_\Delta + \sum_{i=1}^{6} a_i \boldsymbol{u}_i$

Galerkin system: ...simplified with Krylov-Bogoliubov etc.

$$
\begin{aligned}
da_1/dt &= \sigma a_1 - \omega a_2 + h_1 & \sigma &= \sigma_1 - \beta a_7 \\
da_2/dt &= \sigma a_2 + \omega a_1 + h_2 & \omega &= \omega_1 + \gamma a_7 \\
da_3/dt &= \sigma_2 a_3 - 2\omega a_4 + h_3 & & \\
da_4/dt &= \sigma_2 a_4 + 2\omega a_3 + h_4 & \sigma_1 &> 0 > \sigma_2 > \sigma_3 \\
da_5/dt &= \sigma_3 a_5 - 3\omega a_6 + h_5 & & \\
da_6/dt &= \sigma_3 a_6 + 3\omega a_5 + h_6 & & \\
0 = da_7/dt &= \sigma_0 a_7 + \alpha(a_1^2 + a_2^2) & \sigma_0 &< 0
\end{aligned}
$$

- The quadratic coupling $h_i = \sum_{j=1}^{6} \sum_{k=1}^{6} q_{ijk} a_j a_k$

  is energy preserving $\sum_{i=1}^{6} a_i h_i \equiv 0$ (≡ Kraichnan & Chen 1989).
- The shift mode a_7 is slaved to the oscillation amplitude.

MaxEnt $\mapsto$ generalized POD model

Noack & Niven 2012 JFM

Galerkin system: $\boldsymbol{a} = (a_1, a_2, \ldots, a_6)$, $\dot{\boldsymbol{a}} = \boldsymbol{f}(\boldsymbol{a})$

Kullback-Leibler entropy: . $H = -\int d\boldsymbol{a}\, p(\boldsymbol{a}) \ln \frac{p(\boldsymbol{a})}{q(\boldsymbol{a})} = \max$

Marginal stability prior . . . . $q = q(a_7) \approx 1$

Constraints: $\langle F(\boldsymbol{a}) \rangle := \int d\boldsymbol{a}\, p(\boldsymbol{a}) F(\boldsymbol{a})$

C0 (normalization) . . . . $\langle 1 \rangle = 1$

C2 (total TKE eq.) $P = \langle \frac{1}{2} d\|\boldsymbol{a}\|^2/dt \rangle = \langle \boldsymbol{a} \cdot \boldsymbol{f}(\boldsymbol{a}) \rangle = 0$

$$\Rightarrow P = \langle \sigma_1 r_1^2 - \beta_1^M\, r_1^4 + \sigma_2 r_2^2 + \sigma_3 r_3^2 \rangle = 0$$

where $r_1^2 = a_1^2 + a_2^2$, $r_2^2 = a_3^2 + a_4^2$, $r_3^2 = a_5^2 + a_6^2$

MaxEnt solution: $p(\boldsymbol{a}) = \frac{1}{Z_C} \exp\left[d_1 r_1^2 - e_1 r_1^4 + d_2 r_2^2 + d_3 r_3^2\right]$

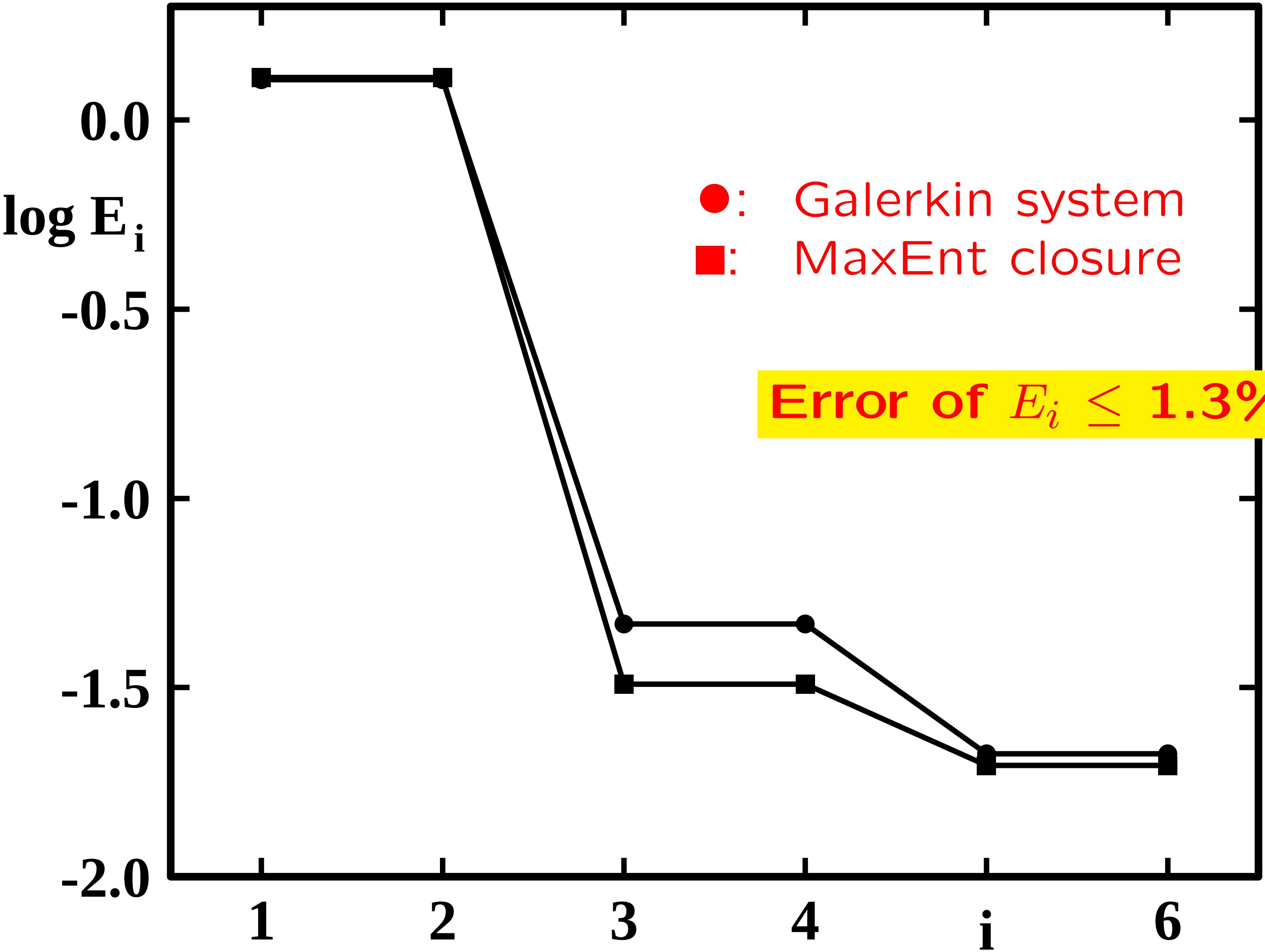
Partition fct. $Z_C = \int d\boldsymbol{a} \exp\left[d_1 r_1^2 - e_1 r_1^4 + d_2 r_2^2 + d_3 r_3^2\right]$

First moments: . . . . $\langle a_i \rangle = 0$

Second moments: . . . . $\langle a_i a_j \rangle = 2 E_i \delta_{ij}$

MaxEnt $\mapsto$ generalized POD model

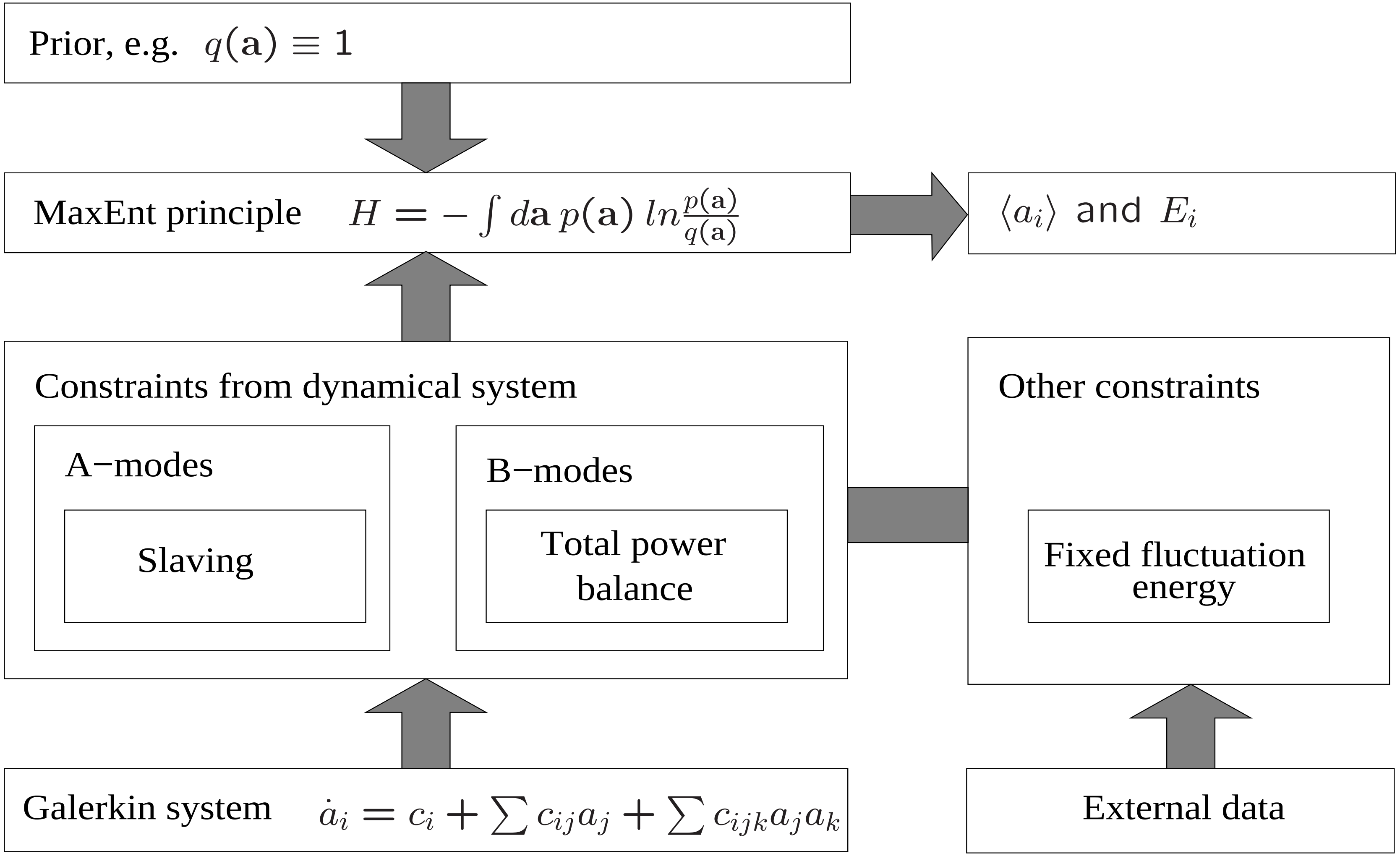
Noack & Niven 2012 JFM



Error of $E_i \leq$ 1.3% !!

MaxEnt closure strategy

Noack & Niven 2012 JFM



Compare with Jaynes' derivation of the Maxwell velocity distribution!

POD Galerkin system of cylinder wake

Noack, Afanasiev, Morzyński, Thiele & Tadmor (2003) JFM

Dynamical system:

$$
\begin{aligned}
da_1/dt &= \sigma a_1 - \omega a_2 + h_1 \\
da_2/dt &= \sigma a_2 + \omega a_1 + h_2 \\
da_3/dt &= \sigma_2 a_3 - 2\omega a_4 + h_3 \\
da_4/dt &= \sigma_2 a_4 + 2\omega a_3 + h_4 \\
da_5/dt &= \sigma_3 a_5 - 3\omega a_6 + h_5 \\
da_6/dt &= \sigma_3 a_6 + 3\omega a_5 + h_6 \\
da_7/dt &= \sigma_4 a_7 - 4\omega a_8 + h_7 \\
da_8/dt &= \sigma_4 a_8 + 4\omega a_7 + h_8 \\
h_i &= \sum_{j=1}^{8} \sum_{k=1}^{8} q_{ijk} a_j a_k
\end{aligned}
$$

Modal energy: $E_i = \overline{a_i^2}/2$

$$
\begin{aligned}
0 &= 2\sigma E_1 + T_1 \\
0 &= 2\sigma E_2 + T_2 \\
0 &= 2\sigma_2 E_3 + T_3 \\
0 &= 2\sigma_2 E_4 + T_4 \\
0 &= 2\sigma_3 E_5 + T_5 \\
0 &= 2\sigma_3 E_6 + T_6 \\
0 &= 2\sigma_4 E_7 + T_7 \\
0 &= 2\sigma_4 E_8 + T_8 \\
T_i &= \sum_{j=1}^{8} \sum_{k=1}^{8} q_{ijk} \overline{a_i a_j a_k}
\end{aligned}
$$

- Dynamical system = harmonically related oscillators with growth rates $\sigma > 0 > \sigma_2 > \sigma_3 > \sigma_4$.
- Quadratic coupling is energy preserving: $\sum T_i = 0$

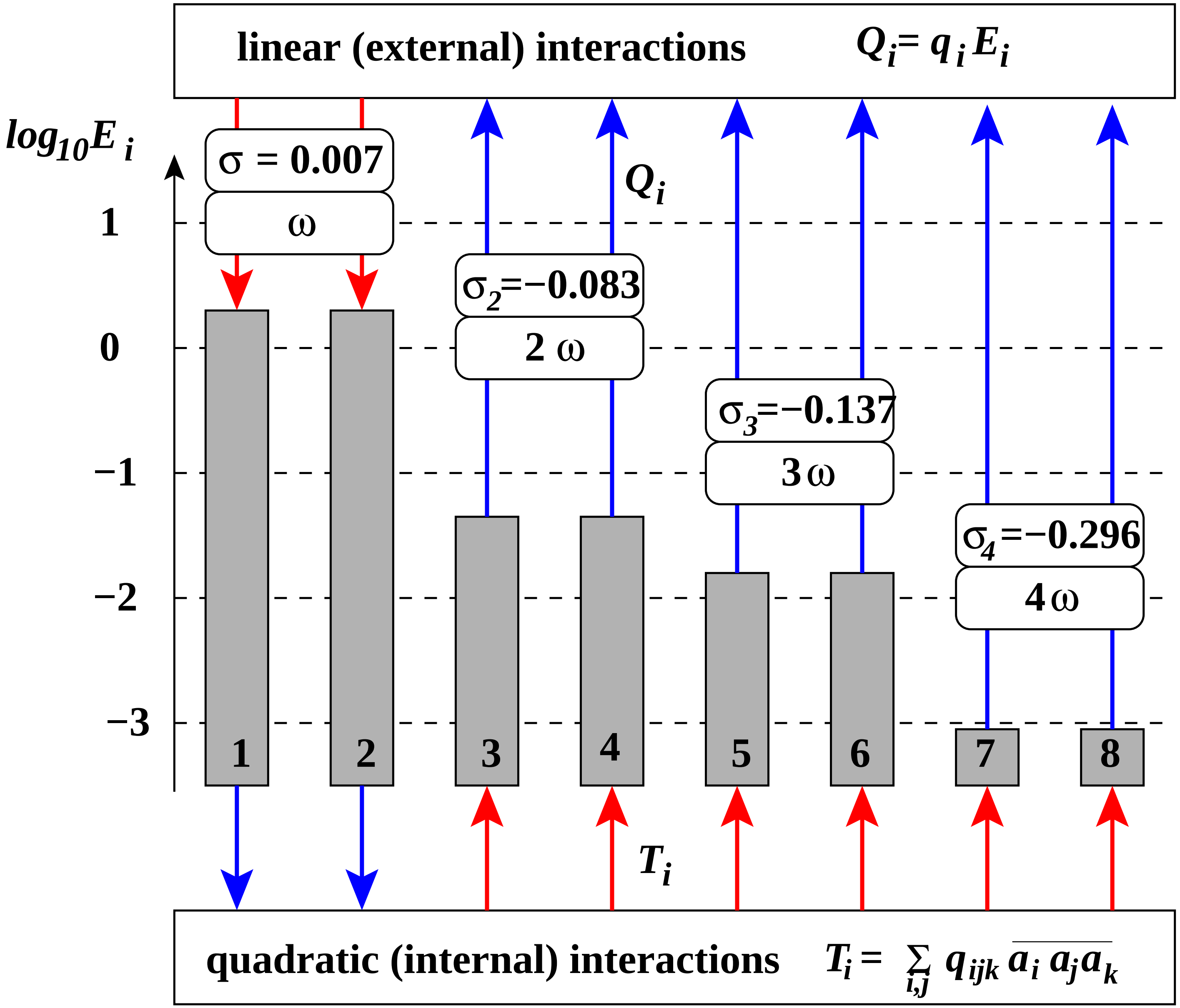
POD Galerkin system of cylinder wake

Noack, Afanasiev, Morzyński, Thiele & Tadmor (2003) JFM

Modal energetics:

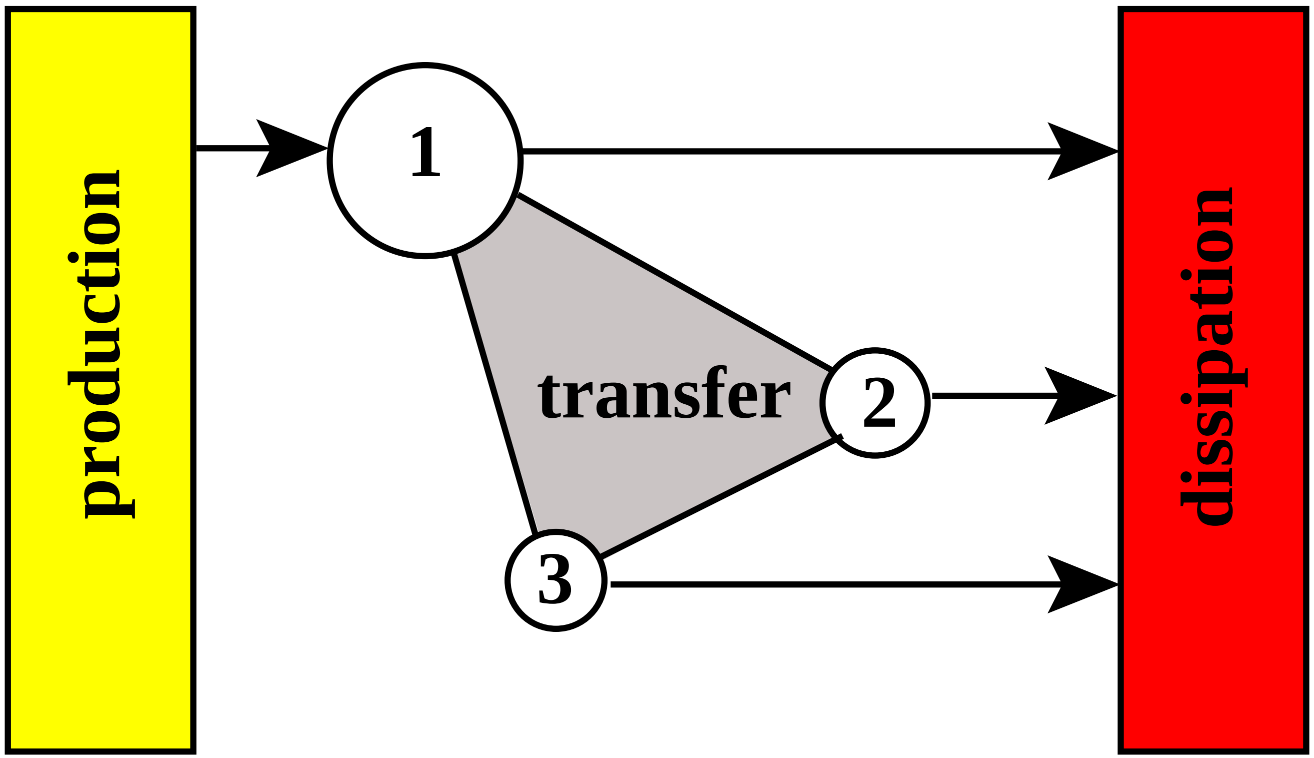
$0 = Q_i + T_i$

$Q_i = q_i E_i$



Modal enery flow analysis (simplified)

— ≡ *Noack, Papas & Monkewitz (2005) JFM* —



$$\begin{array}{ccc} P & = & \sum P_i \\ + & & + \\ D & = & \sum D_i \\ + & & + \\ T & = & \sum T_i \\ + & & + \\ 0 & = & 0 \end{array}$$

linear term . . . . $Q_i = P_i + D_i = q_i \, E_i$

quadratic term . . . . $T_i = \sum_{j=1}^{N} \sum_{k=1}^{N} q_{ijk} \, \overline{a_i a_j a_k}$

Finite-time thermodynamics formalism

Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET

dynamical system

constant
$\frac{da_i}{dt} = c_i + \ldots$

linear term
$+ \sum_j l_{ij} a_j + \ldots$

energy preserving

quadr. term
$+ \sum q_{ijk} a_j a_k$

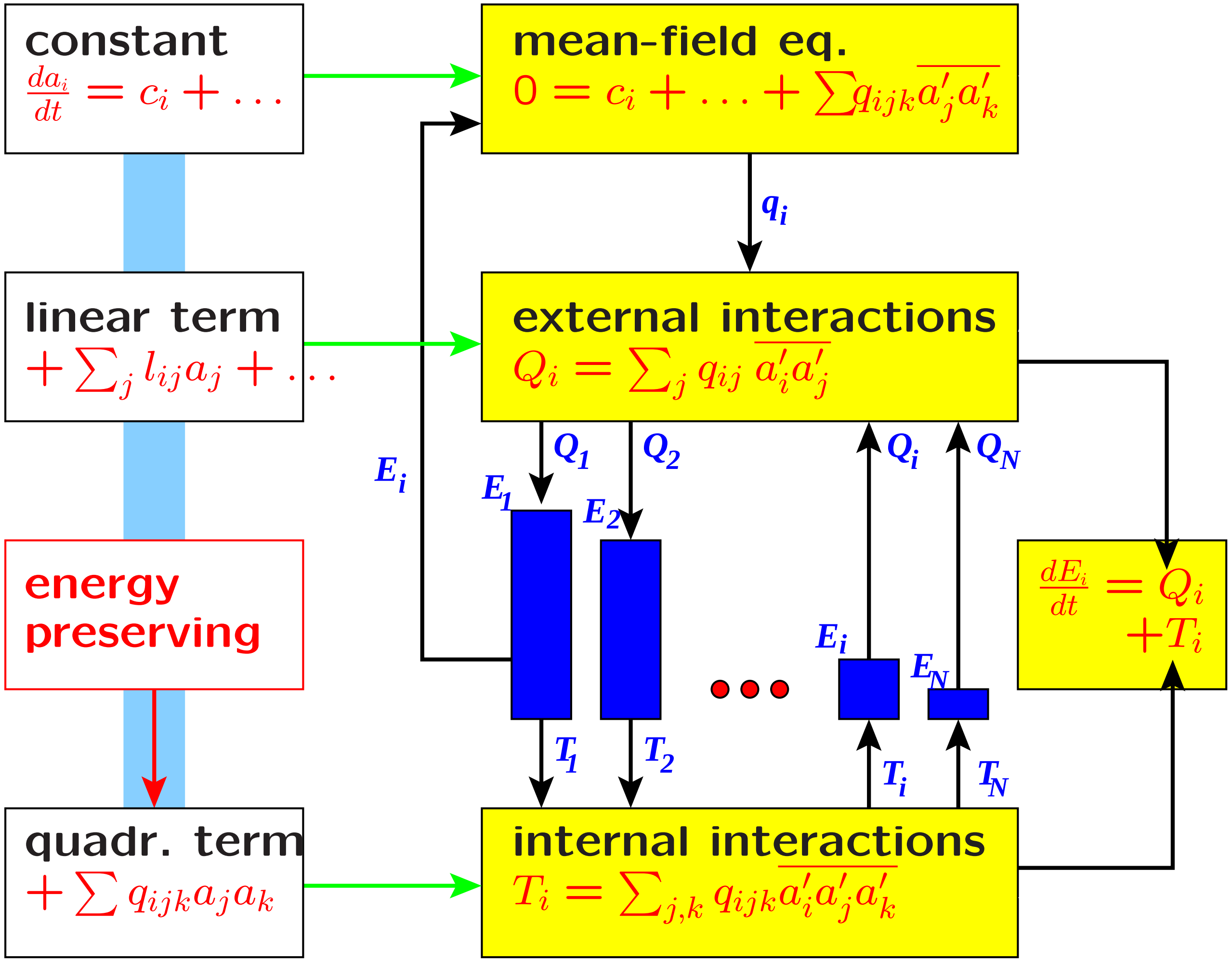
Finite-time thermodynamics formalism

Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET

dynamical system

averaged equations

$a_i = \overline{a_i} + a'_i,\ E_i = \overline{(a'_i)^2}/2$



Finite-time thermodynamics formalism

≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET

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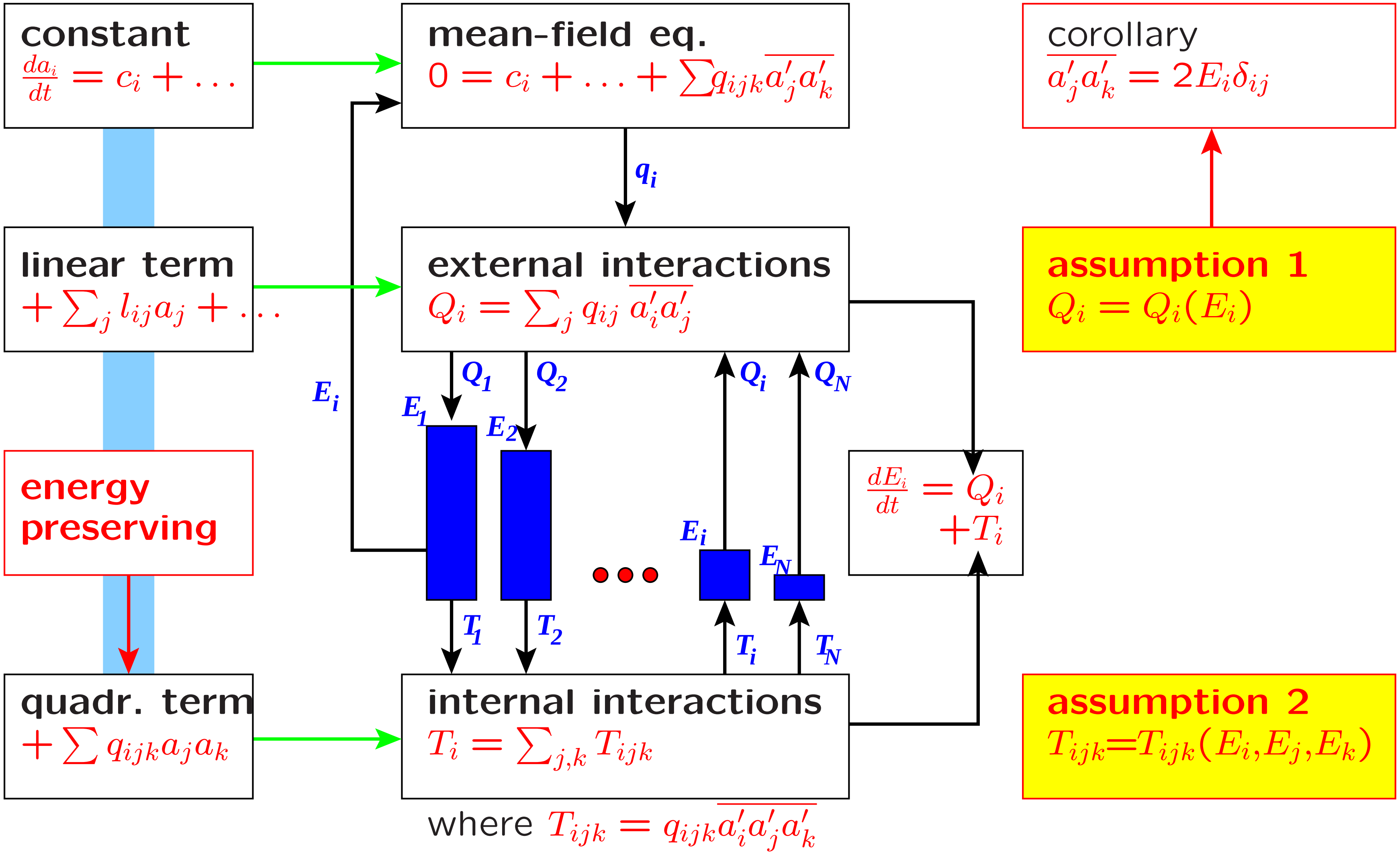
dynamical system | **averaged equations** | **closure assumptions**

$a_i = \overline{a_i} + a_i',\ E_i = \overline{(a_i')^2}/2$



Fick's law for triadic interactions

≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET*

Ansatz

$$T_{ijk} = T_{ijk}(E_i, E_j, E_k)$$

Properties from analysis of $T_{ijk} = q_{ijk}\, \overline{a_i a_j a_k}$

(1) Homogeneity . . . . $T_{ijk}(\lambda E_i, \lambda E_j, \lambda E_k) = \lambda^{3/2}\, T_{ijk}(E_i, E_j, E_k)$

(2) Zeros . . . . $T_{ijk}(E_i, E_j, 0) = T_{ijk}(E_i, 0, E_k) = T_{ijk}(0, E_j, E_k) = 0$

(3) Symmetry . . . . $T_{ijk} = T_{ikj}$

(4) Monotonicity . . . . $E_i < \min\{E_j, E_k\} \quad \Rightarrow T_{ijk}(E_i, E_j, E_k) > 0$

(5) Energy preservation . . . . $T_{ijk} + T_{ikj} + T_{jik} + T_{jki} + T_{kij} + T_{kji} = 0$

(6) Realizability (strictly: $|T_{ijk}| \le |q_{ijk}|\, |a_i|\text{max}\, |a_j|\text{max}\, |a_k|\text{max}$)

$$|T_{ijk}| \lesssim |q_{ijk}|\, \sqrt{E_i\, E_j\, E_k}$$

Solution

$$T_{ijk} = \alpha \chi_{ijk}\, \sqrt{E_i E_j E_k}\, \left[1 - \frac{3E_i}{E_i + E_j + E_k}\right]$$

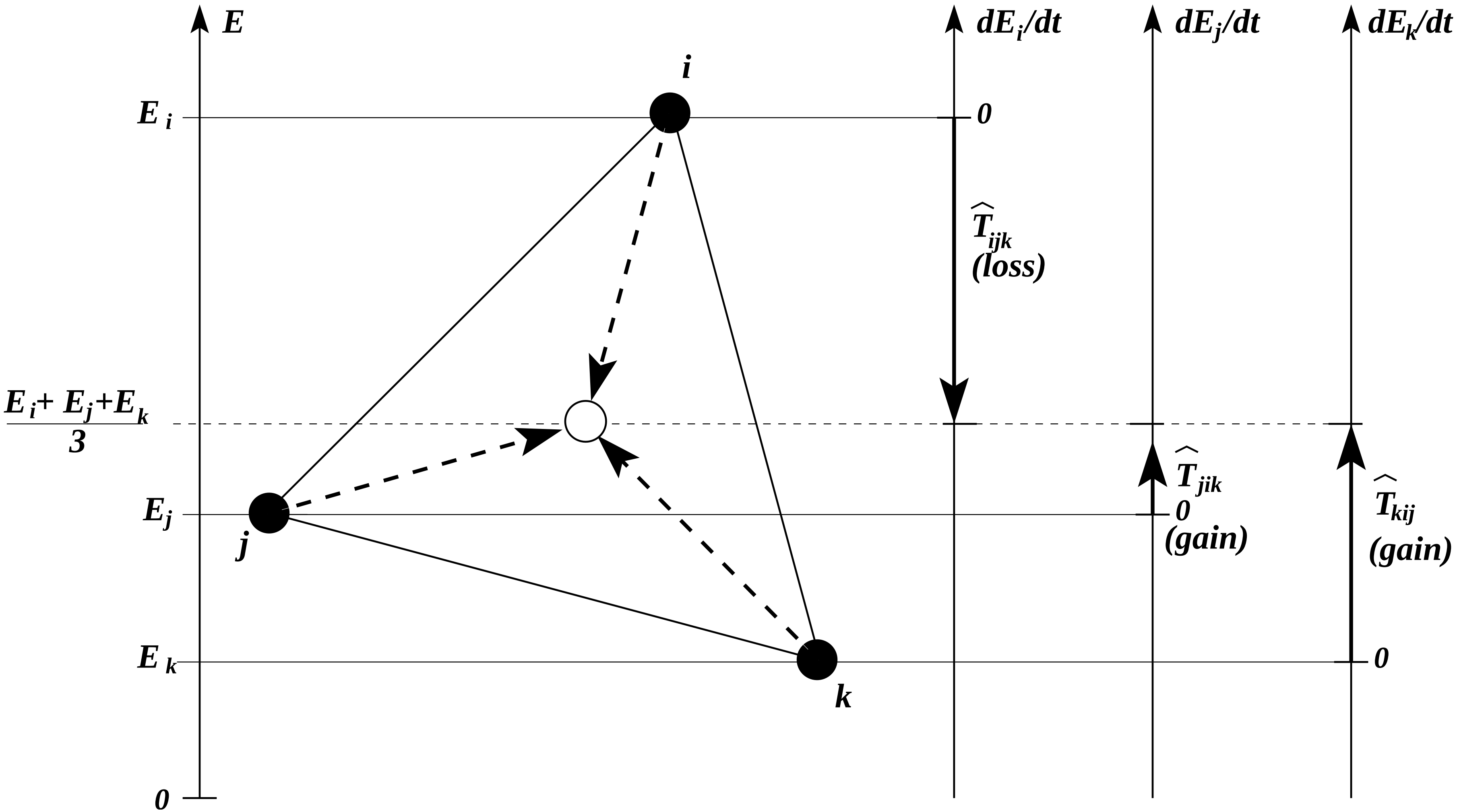
with the totally symmetric triadic structure function

$\chi_{ijk} := \frac{1}{6}\left(|q_{ijk}| + |q_{ikj}| + |q_{jik}| + |q_{jki}| + |q_{kij}| + |q_{kji}|\right)$ and α determined from energy flow consistency between donor and recipient modes.

Fick's law of triadic interactions

Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET

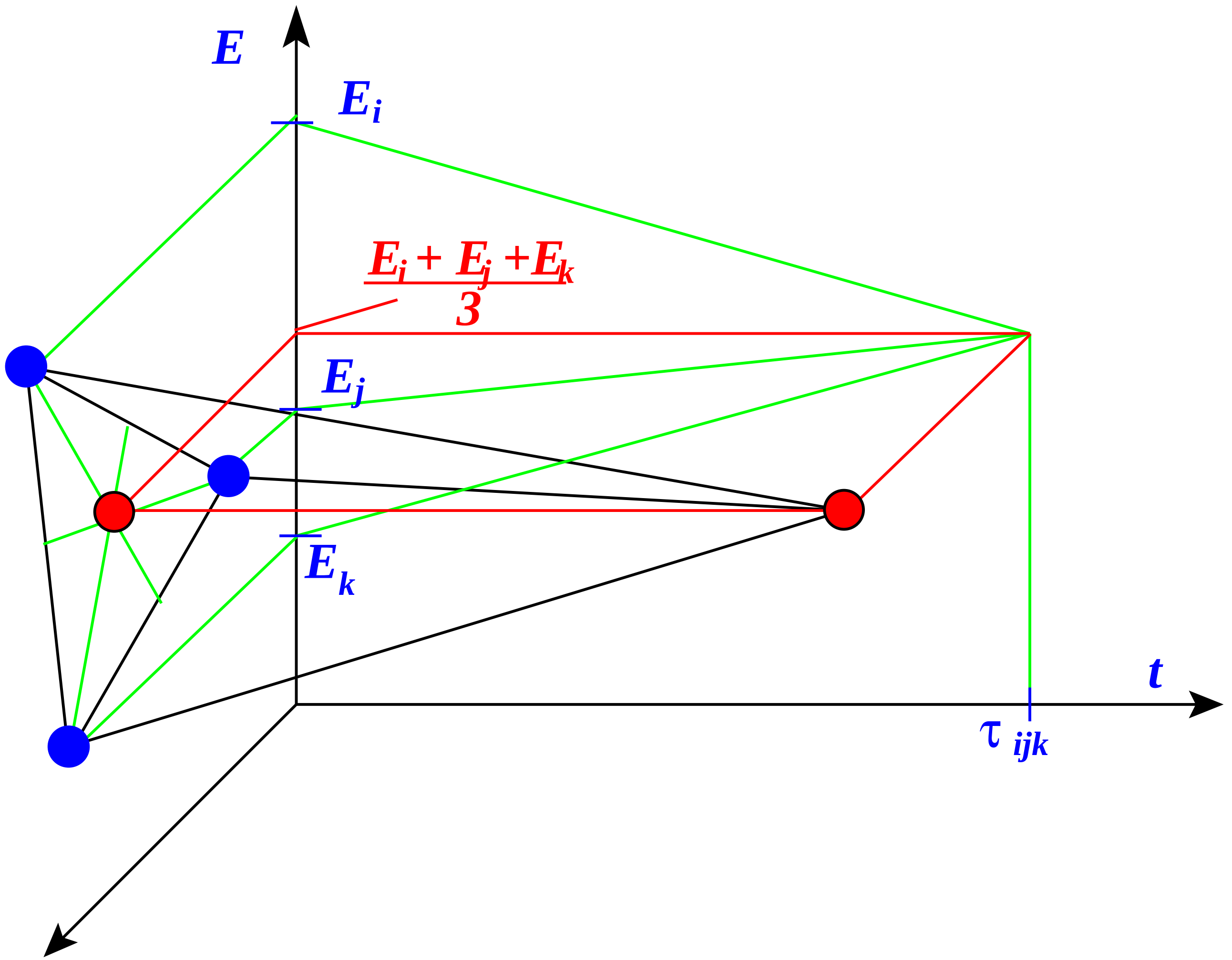
$$T_{ijk} = \sigma_{ijk} \left[1 - \frac{3E_i}{E_i + E_j + E_k} \right], \quad \text{where} \quad \sigma_{ijk} = \alpha\, \chi_{ijk} \sqrt{E_i E_j E_k}$$



Fick's law of triadic interactions

Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET

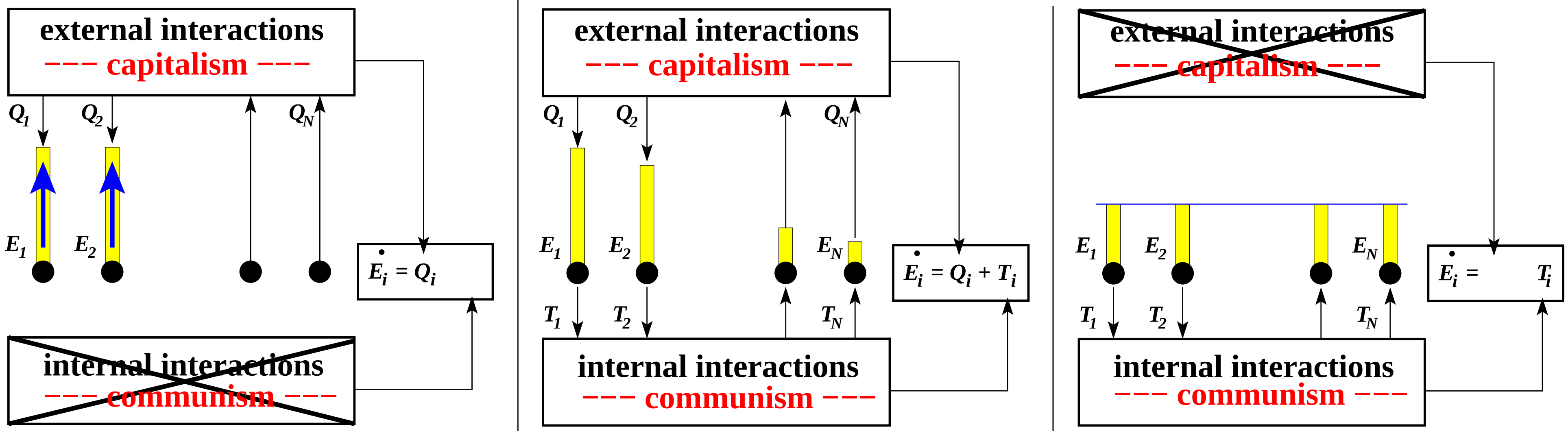
$$T_{ijk} = \sigma_{ijk} \left[1 - \frac{3E_i}{E_i + E_j + E_k} \right], \quad \text{where} \quad \sigma_{ijk} = \frac{1}{\tau_{ijk}} = \alpha \, \chi_{ijk} \sqrt{E_i E_j E_k}$$



FTT model — extremal limits

Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET

Linear dynamics	Partial LTE	LTE



theoretical fluid mechanics

Linear dynamics	Partial LTE	LTE
linear stability analysis	nonlinear dynamics	abs. equilibrium ensemble

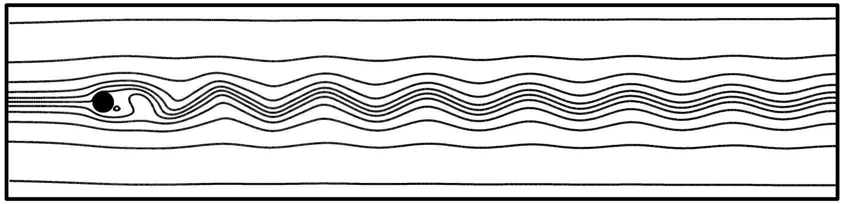
plasma physics analogy for charged particles [∼modes]

Linear dynamics	Partial LTE	LTE
no collisions $[T_{ijk} = 0]$	all interactions	no E-field $[\ Q_i = 0]$

Periodic cylinder wake ($Re = 100$)

≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET*

2D flow around circular cylinder (DNS)

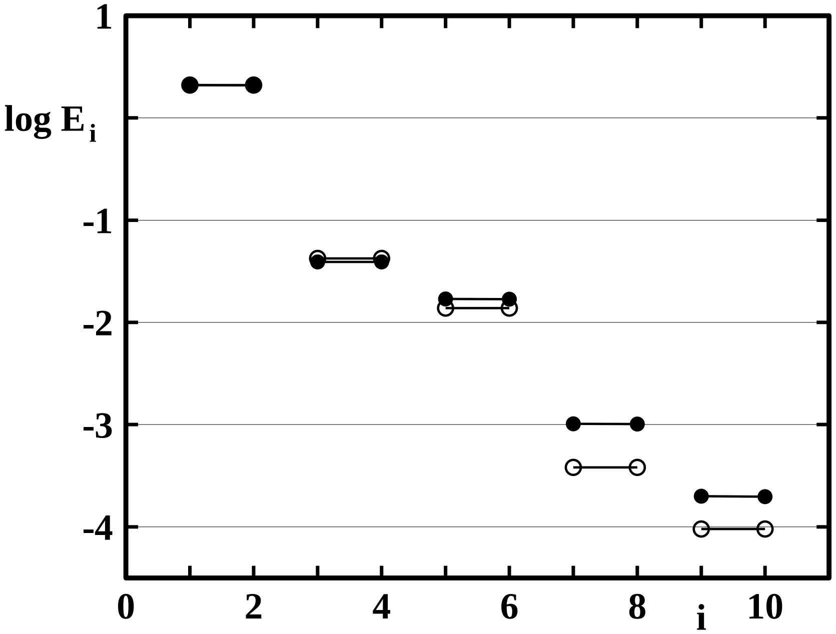


10-dim. Galerkin model

$$\mathbf{u} = \sum_{i=0}^{N} a_i \mathbf{u}_i \quad \text{(POD modes)}$$

$$\dot{a}_i = c_i + \sum_{j=1}^{N} l_{ij}\, a_j + \sum_{j,k=1}^{N} q_{ijk}\, a_j\, a_k$$

Energy distribution (computed and FTT predicted)

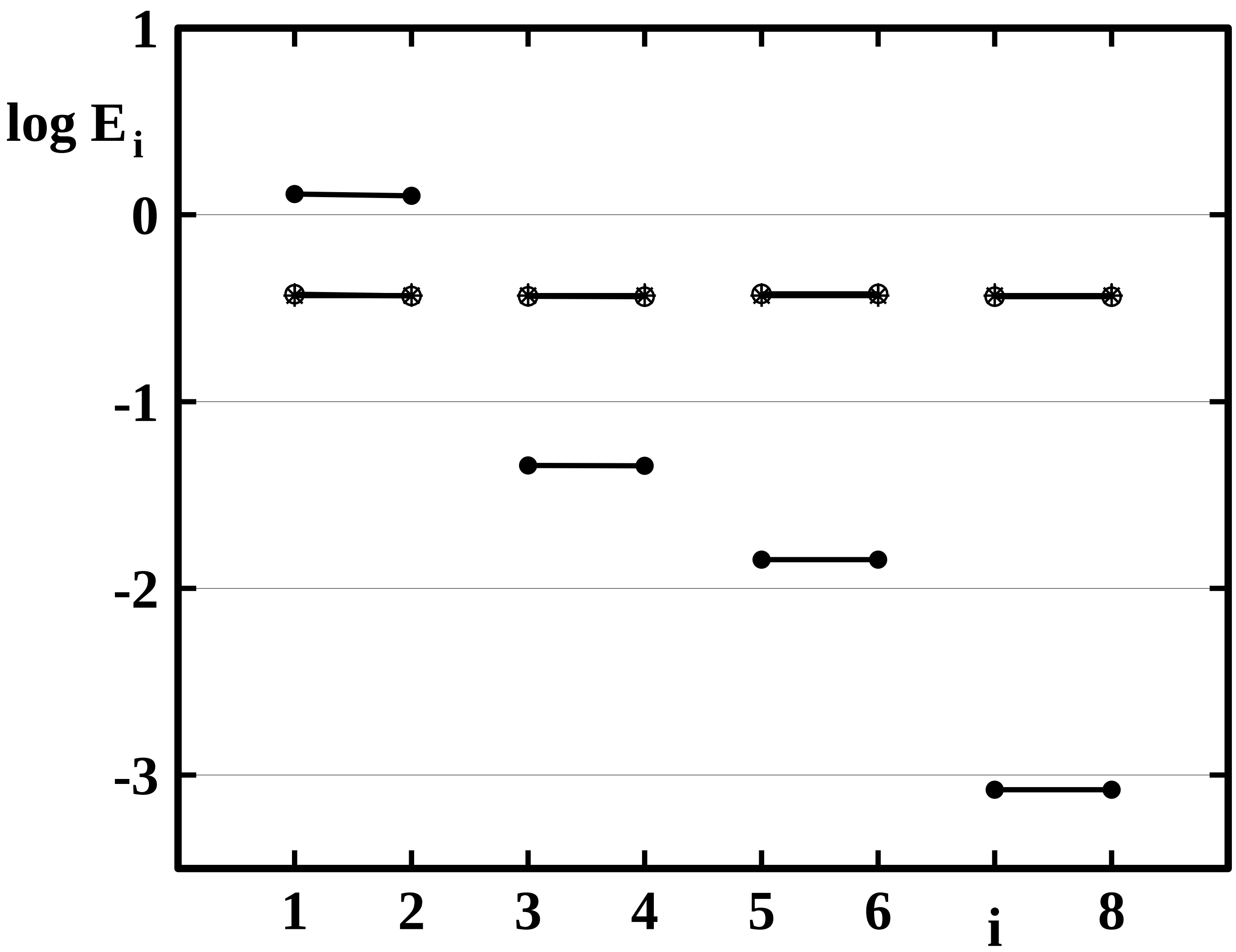


•: DNS; ∘: FTT

Good agreement between DNS and FTT prediction!

Thermal equilibrium (TE) limit of wake model

Noack, Schlegel, Morzyński, & Tadmor (2010) IJNMF



Approximate thermal equilibrium ($E_1 = E_2 = \ldots = E_N$)
if energy sources are turned off ($Q_i = 0$, Hamiltonian dynamics)


Paris

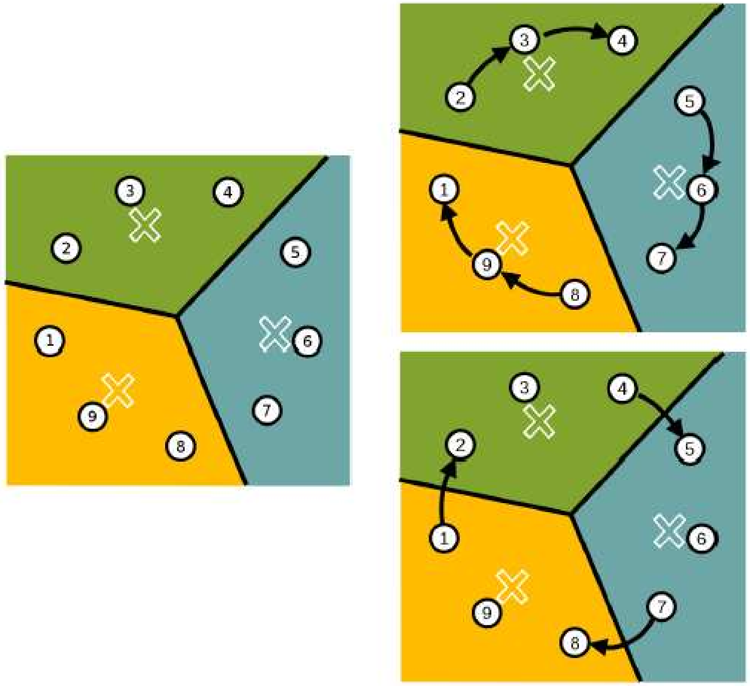
CROM as POD modelling alternative

Kaiser, Noack, Cordier, Spohn, Daviller, Segond, Abel & Niven (2013) JFM preprint

Dynamic system $\frac{d}{dt}\boldsymbol{a} = \boldsymbol{f}(\boldsymbol{a})$

$\Leftrightarrow$ Liouville equation $\partial_t p + \nabla \cdot (\boldsymbol{f} p) = 0$

Discretised state space (cluster analysis) → Count inner-cluster transitions and between clusters → Construct transition matrix → Markov model

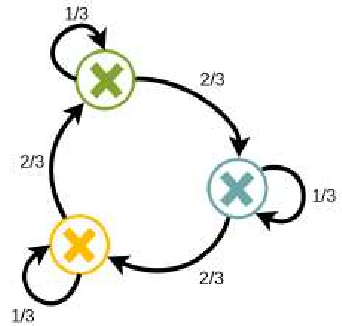


$$P_{jk} = \frac{n_{jk}}{n_k}$$

$$\boldsymbol{p}^{l+1} = \boldsymbol{P}\boldsymbol{p}^l$$

$$\boldsymbol{p}^l = \boldsymbol{P}^l \boldsymbol{p}^0$$

P_{jk}	k=1 1→j	k=2 2→j	k=3 3→j
j=1	2/3	0	1/3
j=2	1/3	2/3	0
j=3	0	1/3	2/3



↦ http://EurikaKaiser.com
↦ http://ClusterModelling.com

Overview

1. **Introduction to turbulence control**
. . . . A research area with impact of epic proportion

2. **Reduced-order modelling**
. . . . . Towards online control in experiment

3. **MaxEnt & Finite-time thermodynamics closure**
. . . . Understanding the nonlinear mode sociology

4. **Examples of closed-loop turbulence control**
. . . . . Drag reduction, lift increase, . . .

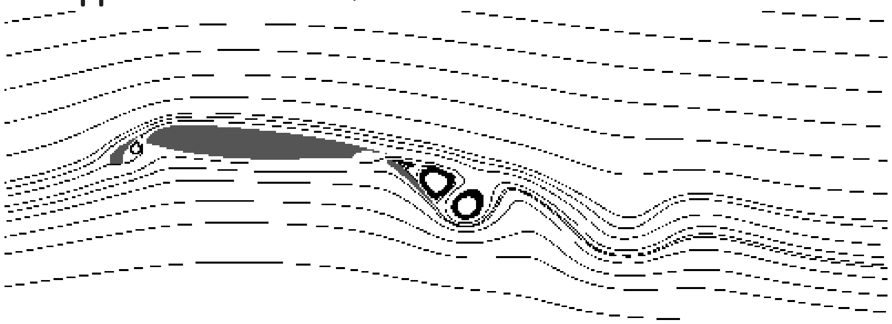
5. **Conclusions and outlook**
. . . . . Bayes/MaxEnt's potential role in turbulence control

Lift increase of an high-lift configuration

≡ *Luchtenburg, Günther, Noack, King & Tadmor (2009) JFM*

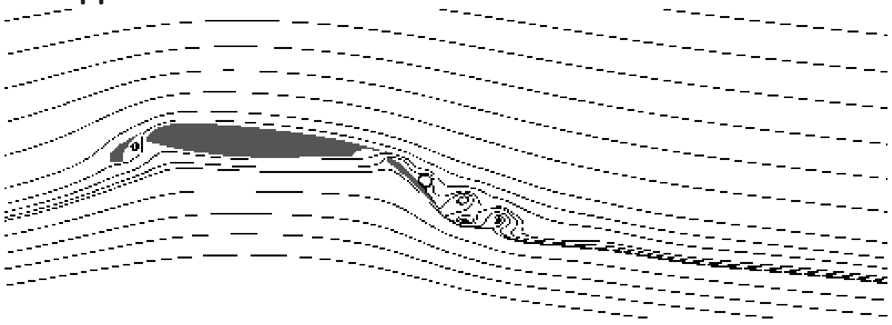
URANS $\mapsto$ natural flow

$St^n_{\mathsf{fl}} = f^n c_{\mathsf{fl}}/U_\infty = 0.32$



actuated flow

$St^a_{\mathsf{fl}} = f^a c_{\mathsf{fl}}/U_\infty = 0.6$

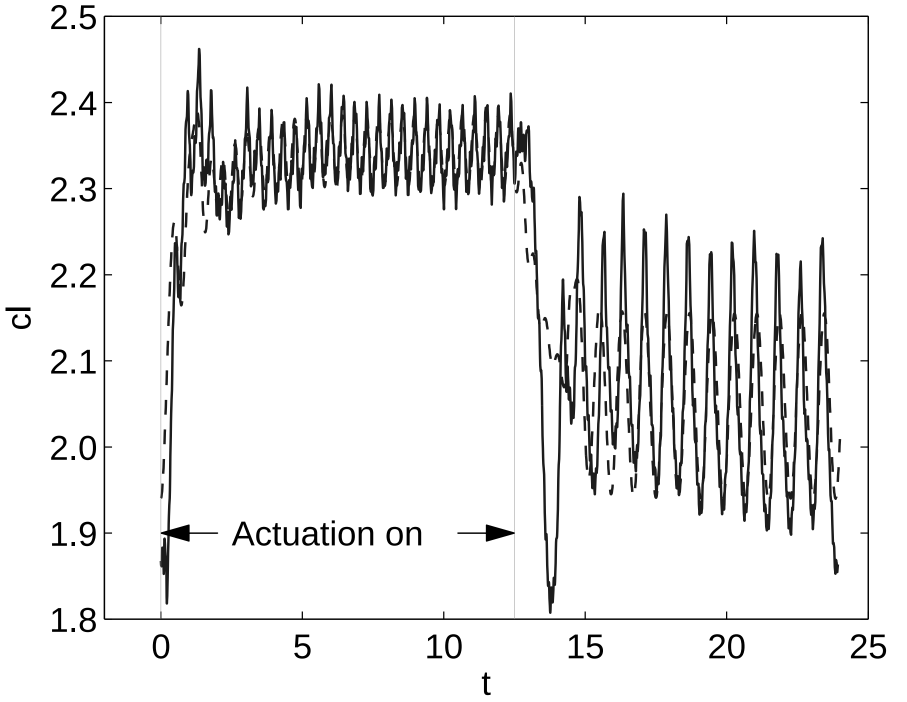


Galerkin model

$$\mathbf{u} = \mathbf{u}_0(\boldsymbol{a}) + \sum_{i=1}^{4} a_i\, \mathbf{u}_i$$

$$d\mathbf{a}/dt = f(\mathbf{a}, \mathbf{b}), \quad \mathbf{b} : \text{control}$$

$$c_L = c_L(\mathbf{a}) \text{ lift coefficient}$$

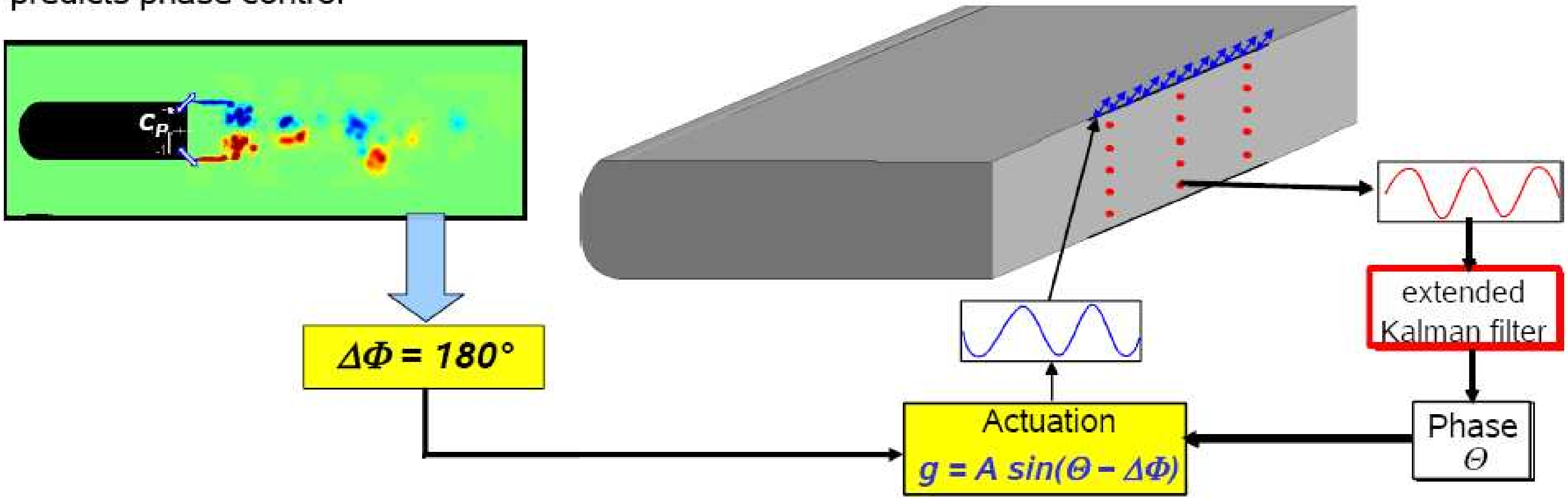


Drag reduction in experiment

Pastoor, Henning, Noack, King & Tadmor (2008) JFM

Reduced-order (vortex) model
predicts phase control

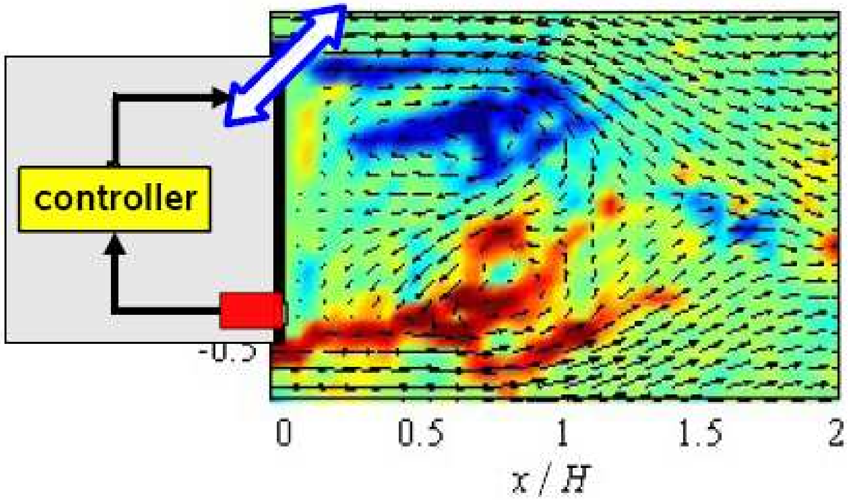
Phase controller in experiment Re_H = 20 000 ... 60 000



Strengths of our approach:

- **40% base pressure increase** (approx. 20% drag reduction)
- **40% actuation power saved** compared to open-loop actuation
- control with only **one actuator**
- **quick adaption** to other operating conditions / perturbations.

PIV of experiment



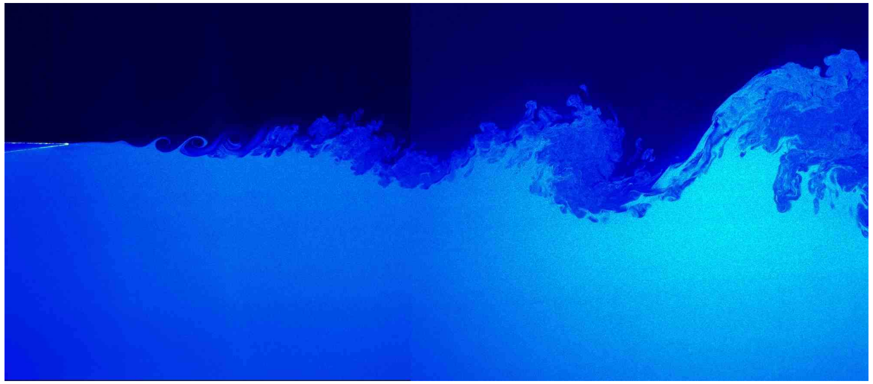
TUCOROM wind-tunnel

☰. *Parezanović, Laurentie, Fourment, Cordier, Noack, & Shaqarin (2013) TSFP8*

New
turbulence control
wind-tunnel at P' →

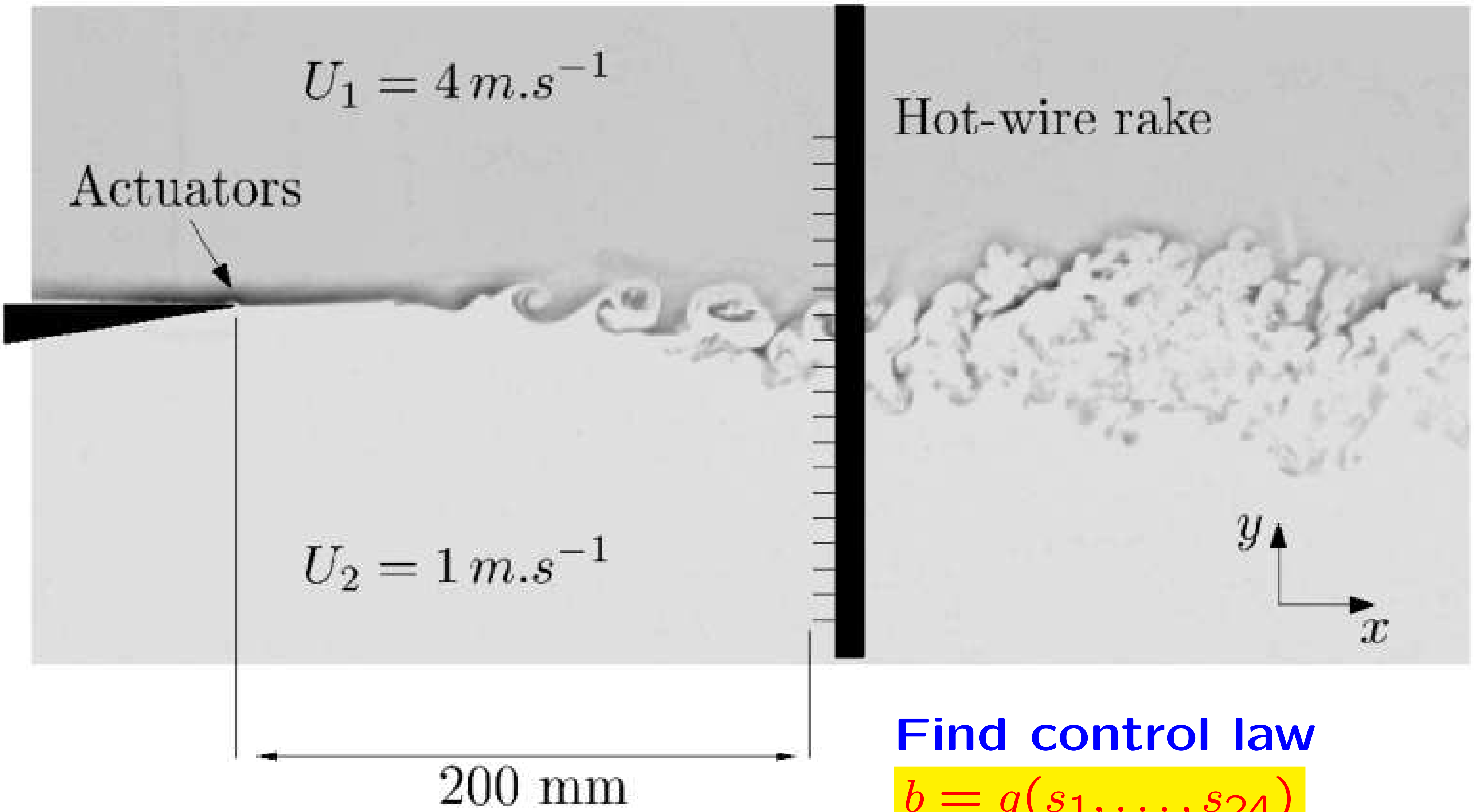
Control team
at control desk
↓

Flow visualization



TUCOROM mixing layer demonstrator

. Parezanović, Laurentie, Fourment, Cordier, Noack, & Shaqarin (2013) TSFP8



Find control law

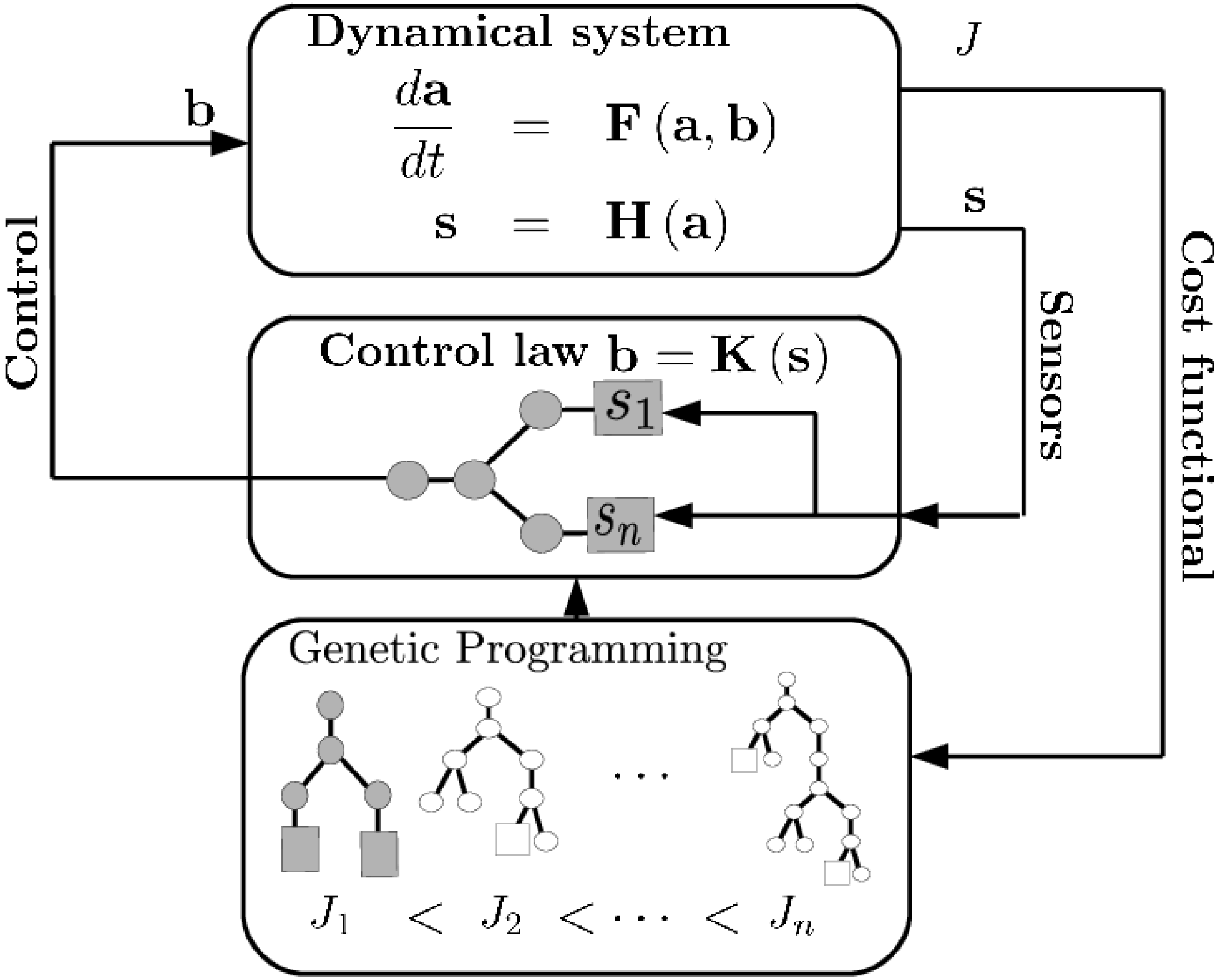
$b = g(s_1, \ldots, s_{24})$

so that

$J = \sum_{i=1}^{24} \overline{\left(s_i'\right)^2} = max$

Machine learning control I

≡. Duriez, Parezanović, Noack, Cordier, Segond & Abel (2013) PRL preprint, ≡ Wahde 2008



MLC = model-free optimization of control laws

Similar approaches exist for roboter missions, etc.

Machine learning control II

. *Duriez, Parezanović, Noack, Cordier, Segond & Abel (2013) PRL preprint*

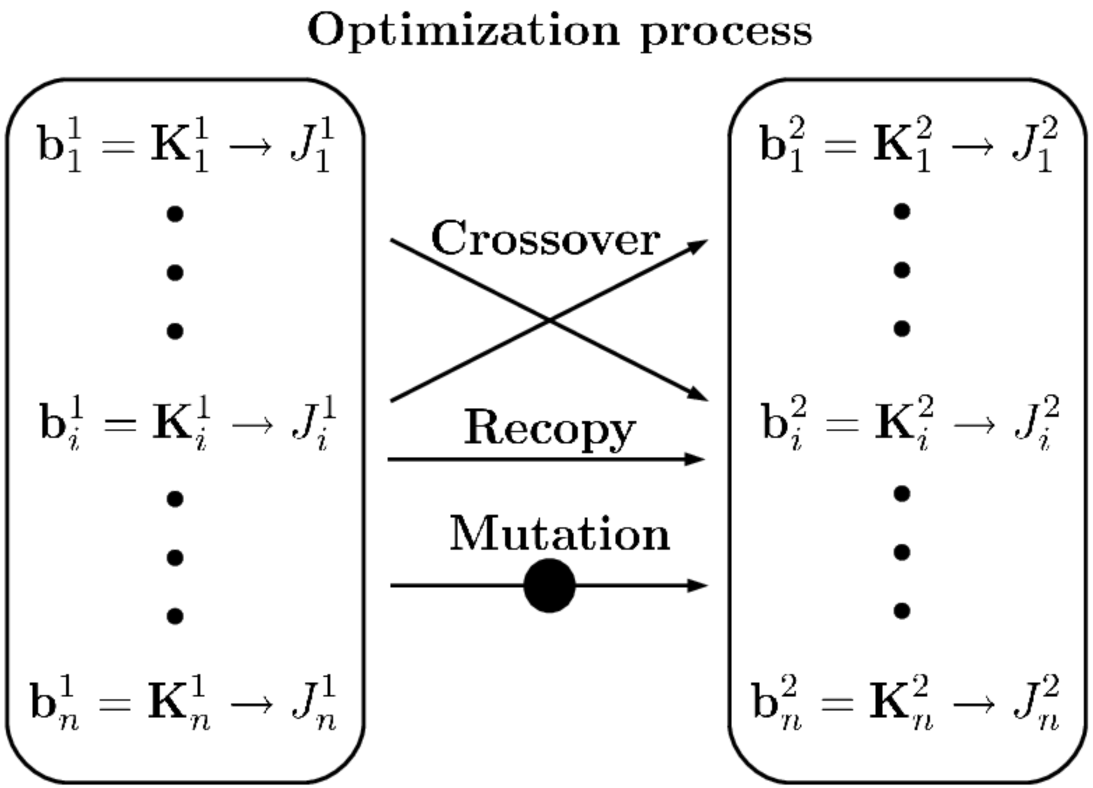
Step 1: 1st generation with random nonlinear control laws

$b_m^1 = K_m^1(s), m = 1, ..., 100$

Step 2–50:

Biologically inspired optimization of the control laws based on the 'fitness grades'

$J\left[b = K(s)\right]$



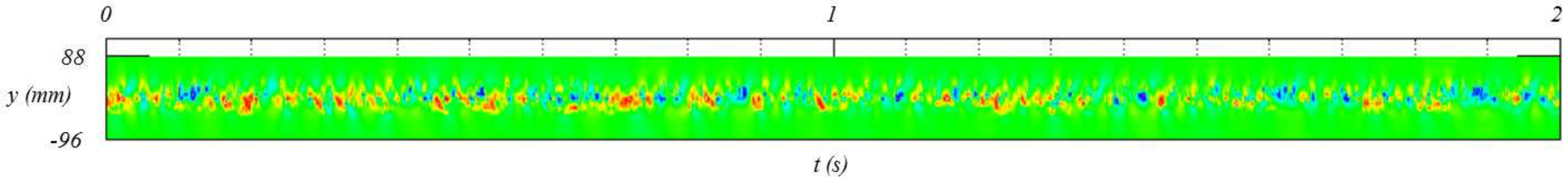
J.R. Koza 1992 Genetic Programming, The MIT Press

TUCOROM mixing layer control experiment

≡. *Duriez, Parezanović, Noack, Cordier, Segond & Abel (2013) PRL preprint*

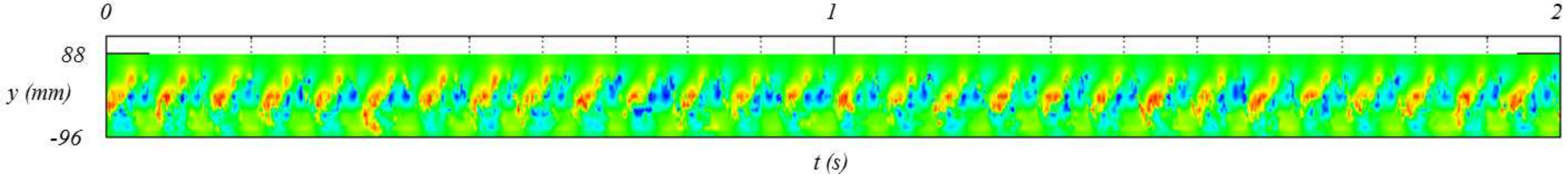
Unforced flow

$J = J_n$



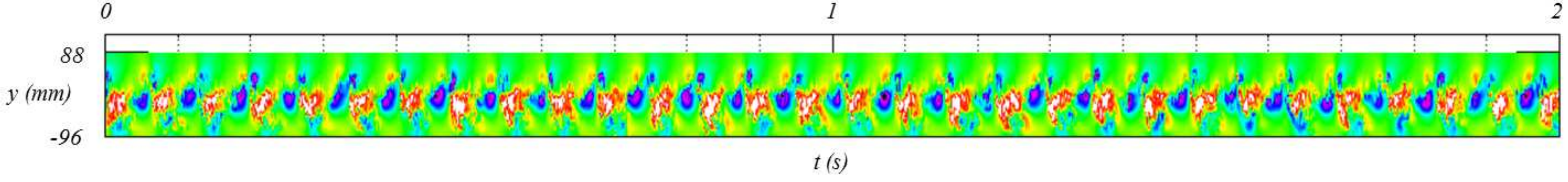
Best open-loop control

$J/J_n = 2.40$ (+140%)



Machine learning control

$J/J_n = 5.66$ (+466%)



MLC has found a very effective nonlinear low-frequency resonance mechanism!

Machine learning control
better with Bayesian inference/MaxEnt?

☰. Duriez, Parezanović, Noack, Cordier, Segond & Abel (2013) PRL preprint

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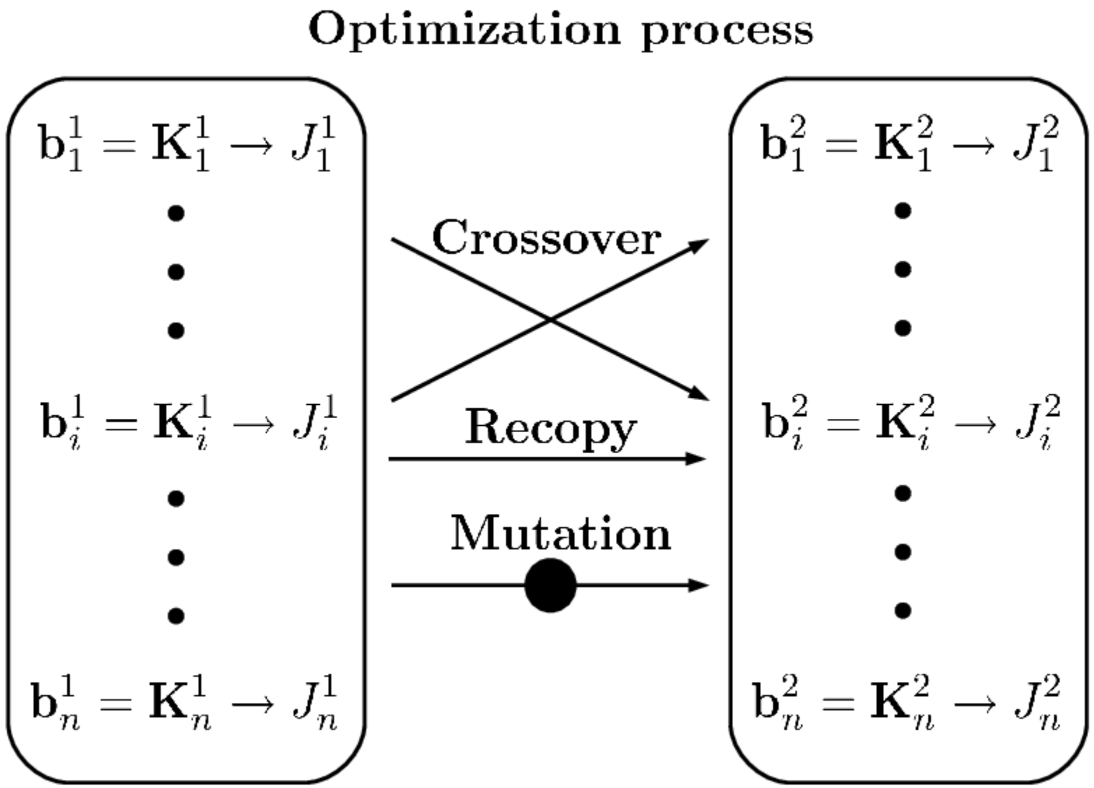
Step 1: 1st generation with random nonlinear control laws

$b^1_m = K^1_m(s), m = 1, ..., 100$

Step 2–50:

Biologically inspired optimization of the control laws based on the 'fitness grades'

$J\,[b = K(s)]$



- **MaxEnt framework for function tree**
- **Add control laws with Bayesian inference**

Overview

1. Introduction to turbulence control

. . . . A research area with impact of epic proportion

2. Reduced-order modelling

. . . . . Towards online control in experiment

3. MaxEnt & Finite-time thermodynamics closure

. . . . Understanding the nonlinear mode sociology

4. Examples of closed-loop turbulence control

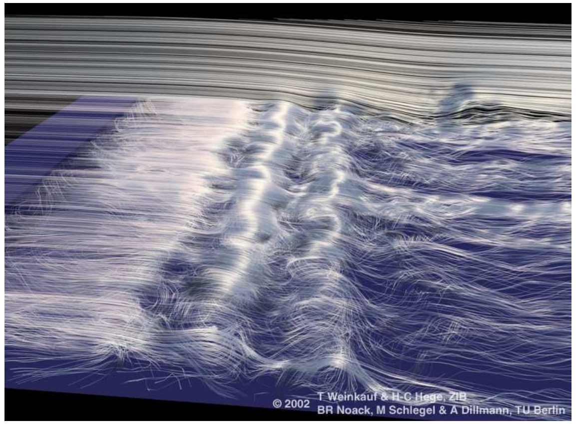
. . . . . Drag reduction, lift increase, . . .

5. Conclusions and outlook

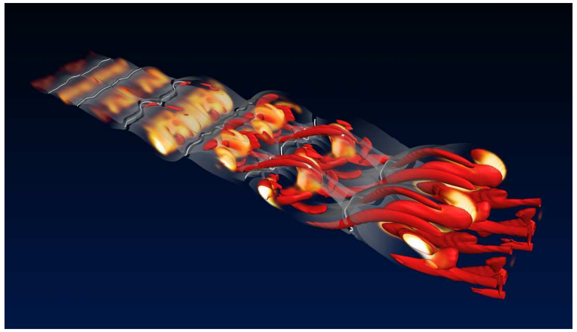
. . . . . Bayes/MaxEnt's potential role in turbulence control

Configurations

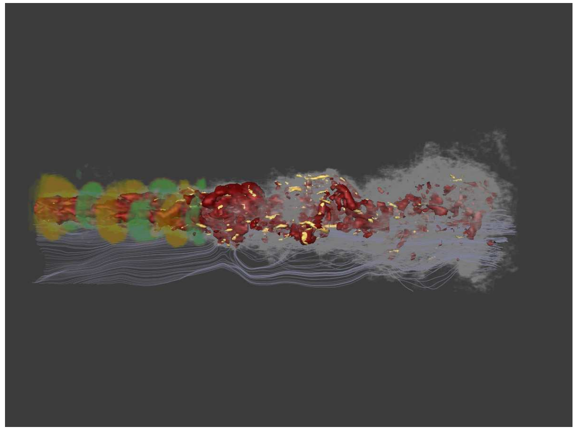
3D flow over a step



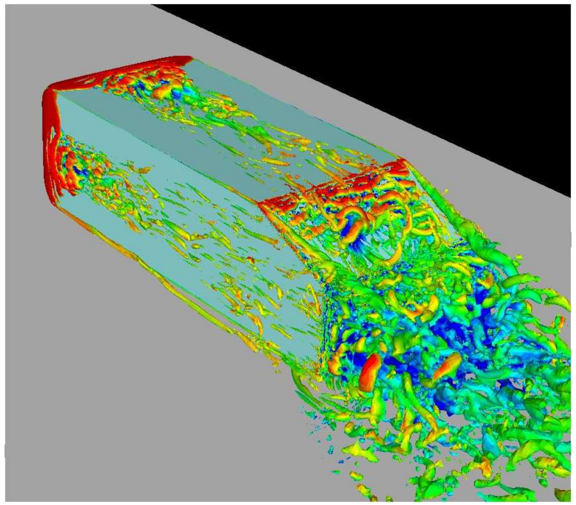
3D mixing layer



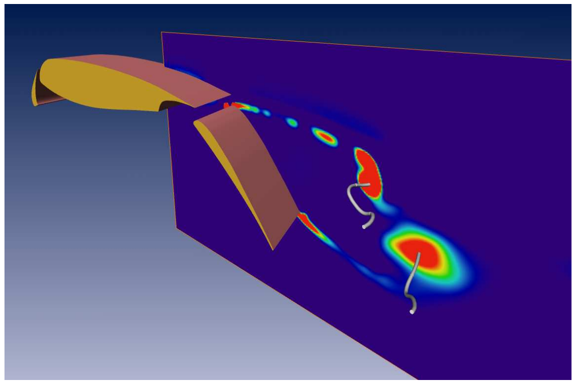
jet noise



Ahmed body



airfoil



wake
channel flow
combustor
cavity flow
. . .

Conclusions

Noack & Niven 2012 JFM, Duriez, Parezanovic, Cordier, Noack, Segond & Abel 2013 PRL

- **Turbulence control = attractor control**

  Physics mechanisms are strongly nonlinear.

  • D-shaped body, high-lift configuration, mixing layer, ...

- **Model-based control design → 1 or few frequencies**

  MaxEnt principle: • statistical closure of Galerkin model,

  • model identification, • POD derivation

- **Model-free machine learning control design**

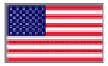
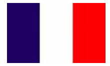
  Novel game changer → broadband turbulence

  • Outperforms any other method for mixing layer experiment and tested dynamical systems.

  • Task: further improvement of control law optimization by Bayesian inference / MaxEnt

More information or any ideas

Call 24h/7d

 	 	 	 	 
+61-2-62688330	+1-206-543-7124	+49-17682001688	+48-61-6652778	+33-549-366015

... or read

Noack & Niven 2012 JFM .... *MaxEnt closure*	Duriez+ 2013 arXiv ... *machine learning control*
Pastoor+ 2008 JFM .... *bluff-body control*	Luchtenburg+ 2009 JFM .... *airfoil control*
Noack+ 2011 'ROM for flow control', Springer	

... or ask now!!!

In any case, stay tuned in for news + publications:

- http://TurbulenceControl.com ....http://BerndNoack.com